

DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning

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Abstract

We introduce our first-generation reasoning models, DeepSeek-R1-Zero and DeepSeek-R1. DeepSeek-R1-Zero, a model trained via large-scale reinforcement learning (RL) without supervised fine-tuning (SFT) as a preliminary step, demonstrates remarkable reasoning capabilities. Through RL, DeepSeek-R1-Zero naturally emerges with numerous powerful and intriguing reasoning behaviors. However, it encounters challenges such as poor readability, and language mixing. To address these issues and further enhance reasoning performance, we introduce DeepSeek-R1, which incorporates multi-stage training and cold-start data before RL. DeepSeek-R1 achieves performance comparable to OpenAI-o1-1217 on reasoning tasks. To support the research community, we open-source DeepSeek-R1-Zero, DeepSeek-R1, and six dense models (1.5B, 7B, 8B, 14B, 32B, 70B) distilled from DeepSeek-R1 based on Qwen and Llama.

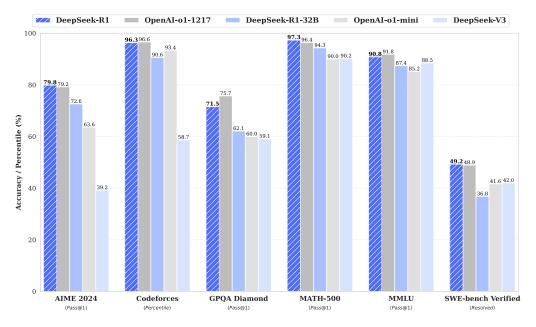


Figure 1 | Benchmark performance of DeepSeek-R1.

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1. Introduction

In recent years, Large Language Models (LLMs) have been undergoing rapid iteration and evolution (Anthropic 2024; Google 2024; OpenAI 2024a), progressively diminishing the gap towards Artificial General Intelligence (AGI).

Recently, post-training has emerged as an important component of the full training pipeline. It has been shown to enhance accuracy on reasoning tasks, align with social values, and adapt to user preferences, all while requiring relatively minimal computational resources against pre-training. In the context of reasoning capabilities, OpenAl's o1 (OpenAl 2024b) series models were the first to introduce inference-time scaling by increasing the length of the Chain-of-Thought reasoning process. This approach has achieved significant improvements in various reasoning tasks, such as mathematics, coding, and scientific reasoning. However, the challenge of effective test-time scaling remains an open question for the research community. Several prior works have explored various approaches, including process-based reward models (Lightman et al., 2023) Uesato et al., 2022 Wang et al., 2023), reinforcement learning (Kumar et al., 2024), and search algorithms such as Monte Carlo Tree Search and Beam Search (Feng et al., 2024). However, none of these methods has achieved general reasoning performance comparable to OpenAl's o1 series models.

In this paper, we take the first step toward improving language model reasoning capabilities using pure reinforcement learning (RL). Our goal is to explore the potential of LLMs to develop reasoning capabilities without any supervised data, focusing on their self-evolution through a pure RL process. Specifically, we use DeepSeek-V3-Base as the base model and employ GRPO (Shao et al.), [2024) as the RL framework to improve model performance in reasoning. During training, DeepSeek-R1-Zero naturally emerged with numerous powerful and interesting reasoning behaviors. After thousands of RL steps, DeepSeek-R1-Zero exhibits super performance on reasoning benchmarks. For instance, the pass@1 score on AIME 2024 increases from 15.6% to 71.0%, and with majority voting, the score further improves to 86.7%, matching the performance of OpenAI-o1-0912.

However, DeepSeek-R1-Zero encounters challenges such as poor readability, and language mixing. To address these issues and further enhance reasoning performance, we introduce DeepSeek-R1, which incorporates a small amount of cold-start data and a multi-stage training pipeline. Specifically, we begin by collecting thousands of cold-start data to fine-tune the DeepSeek-V3-Base model. Following this, we perform reasoning-oriented RL like DeepSeek-R1-Zero. Upon nearing convergence in the RL process, we create new SFT data through rejection sampling on the RL checkpoint, combined with supervised data from DeepSeek-V3 in domains such as writing, factual QA, and self-cognition, and then retrain the DeepSeek-V3-Base model. After fine-tuning with the new data, the checkpoint undergoes an additional RL process, taking into account prompts from all scenarios. After these steps, we obtained a checkpoint referred to as DeepSeek-R1, which achieves performance on par with OpenAI-o1-1217.

We further explore distillation from DeepSeek-R1 to smaller dense models. Using Qwen2.5-32B Qwen, 2024b) as the base model, direct distillation from DeepSeek-R1 outperforms applying RL on it. This demonstrates that the reasoning patterns discovered by larger base models are crucial for improving reasoning capabilities. We open-source the distilled Qwen and Llama (Dubey et al., 2024) series. Notably, our distilled 14B model outperforms state-of-the-art open-source QwQ-32B-Preview Qwen 2024a) by a large margin, and the distilled 32B and 70B models set a new record on the reasoning benchmarks among dense models.

1.1. Contributions

Post-Training: Large-Scale Reinforcement Learning on the Base Model

- We directly apply RL to the base model without relying on supervised fine-tuning (SFT) as a preliminary step. This approach allows the model to explore chain-of-thought (CoT) for solving complex problems, resulting in the development of DeepSeek-R1-Zero. DeepSeek-R1-Zero demonstrates capabilities such as self-verification, reflection, and generating long CoTs, marking a significant milestone for the research community. Notably, it is the first open research to validate that reasoning capabilities of LLMs can be incentivized purely through RL, without the need for SFT. This breakthrough paves the way for future advancements in this area.
- We introduce our pipeline to develop DeepSeek-R1. The pipeline incorporates two RL stages aimed at discovering improved reasoning patterns and aligning with human preferences, as well as two SFT stages that serve as the seed for the model's reasoning and non-reasoning capabilities. We believe the pipeline will benefit the industry by creating better models.

Distillation: Smaller Models Can Be Powerful Too

- We demonstrate that the reasoning patterns of larger models can be distilled into smaller models, resulting in better performance compared to the reasoning patterns discovered through RL on small models. The open source DeepSeek-R1, as well as its API, will benefit the research community to distill better smaller models in the future.
- Using the reasoning data generated by DeepSeek-R1, we fine-tuned several dense models that are widely used in the research community. The evaluation results demonstrate that the distilled smaller dense models perform exceptionally well on benchmarks. DeepSeek-R1-Distill-Qwen-7B achieves 55.5% on AIME 2024, surpassing QwQ-32B-Preview. Additionally, DeepSeek-R1-Distill-Qwen-32B scores 72.6% on AIME 2024, 94.3% on MATH-500, and 57.2% on LiveCodeBench. These results significantly outperform previous open-source models and are comparable to o1-mini. We open-source distilled 1.5B, 7B, 8B, 14B, 32B, and 70B checkpoints based on Qwen2.5 and Llama3 series to the community.

1.2. Summary of Evaluation Results

- Reasoning tasks: (1) DeepSeek-R1 achieves a score of 79.8% Pass@1 on AIME 2024, slightly surpassing OpenAI-o1-1217. On MATH-500, it attains an impressive score of 97.3%, performing on par with OpenAI-o1-1217 and significantly outperforming other models. (2) On coding-related tasks, DeepSeek-R1 demonstrates expert level in code competition tasks, as it achieves 2,029 Elo rating on Codeforces outperforming 96.3% human participants in the competition. For engineering-related tasks, DeepSeek-R1 performs slightly better than DeepSeek-V3, which could help developers in real world tasks.
- Knowledge: On benchmarks such as MMLU, MMLU-Pro, and GPQA Diamond, DeepSeek-R1 achieves outstanding results, significantly outperforming DeepSeek-V3 with scores of 90.8% on MMLU, 84.0% on MMLU-Pro, and 71.5% on GPQA Diamond. While its performance is slightly below that of OpenAI-o1-1217 on these benchmarks, DeepSeek-R1 surpasses other closed-source models, demonstrating its competitive edge in educational tasks. On the factual benchmark SimpleQA, DeepSeek-R1 outperforms DeepSeek-V3, demonstrating its capability in handling fact-based queries. A similar trend is observed where OpenAI-o1 surpasses 40 on this benchmark.

Others: DeepSeek-R1 also excels in a wide range of tasks, including creative writing, general question answering, editing, summarization, and more. It achieves an impressive length-controlled win-rate of 87.6% on AlpacaEval 2.0 and a win-rate of 92.3% on ArenaHard, showcasing its strong ability to intelligently handle non-exam-oriented queries. Additionally, DeepSeek-R1 demonstrates outstanding performance on tasks requiring long-context understanding, substantially outperforming DeepSeek-V3 on long-context benchmarks.

2. Approach

2.1. Overview

Previous work has heavily relied on large amounts of supervised data to enhance model performance. In this study, we demonstrate that reasoning capabilities can be significantly improved through large-scale reinforcement learning (RL), even without using supervised fine-tuning (SFT) as a cold start. Furthermore, performance can be further enhanced with the inclusion of a small amount of cold-start data. In the following sections, we present: (1) DeepSeek-R1-Zero, which applies RL directly to the base model without any SFT data, and (2) DeepSeek-R1, which applies RL starting from a checkpoint fine-tuned with thousands of long Chain-of-Thought (CoT) examples. 3) Distill the reasoning capability from DeepSeek-R1 to small dense models.

2.2. DeepSeek-R1-Zero: Reinforcement Learning on the Base Model

Reinforcement learning has demonstrated significant effectiveness in reasoning tasks, as evidenced by our previous works Shao et al. 2024 Wang et al. 2023. However, these works heavily depended on supervised data, which are time-intensive to gather. In this section, we explore the potential of LLMs to develop reasoning capabilities without any supervised data, focusing on their self-evolution through a pure reinforcement learning process. We start with a brief overview of our RL algorithm, followed by the presentation of some exciting results, and hope this provides the community with valuable insights.

2.2.1. Reinforcement Learning Algorithm

Group Relative Policy Optimization In order to save the training costs of RL, we adopt Group Relative Policy Optimization (GRPO) (Shao et al.) 2024), which foregoes the critic model that is typically the same size as the policy model, and estimates the baseline from group scores instead. Specifically, for each question q, GRPO samples a group of outputs $\{o_1, o_2, \cdots, o_G\}$ from the old policy $\pi_{\theta_{old}}$ and then optimizes the policy model π_{θ} by maximizing the following objective:

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}\left[q \sim P(Q), \left\{o_{i}\right\}_{i=1}^{G} \sim \pi_{\theta_{old}}(O|q)\right]$$

$$\frac{1}{G} \sum_{i=1}^{G} \left(\min\left(\frac{\pi_{\theta}(o_{i}|q)}{\pi_{\theta_{old}}(o_{i}|q)}A_{i}, \operatorname{clip}\left(\frac{\pi_{\theta}(o_{i}|q)}{\pi_{\theta_{old}}(o_{i}|q)}, 1 - \varepsilon, 1 + \varepsilon\right)A_{i}\right) - \beta \mathbb{D}_{KL}\left(\pi_{\theta}||\pi_{ref}\right)\right), \tag{1}$$

$$\mathbb{D}_{KL}\left(\pi_{\theta}||\pi_{ref}\right) = \frac{\pi_{ref}(o_i|q)}{\pi_{\theta}(o_i|q)} - \log\frac{\pi_{ref}(o_i|q)}{\pi_{\theta}(o_i|q)} - 1,\tag{2}$$

where ε and β are hyper-parameters, and A_i is the advantage, computed using a group of rewards $\{r_1, r_2, \dots, r_G\}$ corresponding to the outputs within each group:

$$A_{i} = \frac{r_{i} - \text{mean}(\{r_{1}, r_{2}, \cdots, r_{G}\})}{\text{std}(\{r_{1}, r_{2}, \cdots, r_{G}\})}.$$
(3)

A conversation between User and Assistant. The user asks a question, and the Assistant solves it. The assistant first thinks about the reasoning process in the mind and then provides the user with the answer. The reasoning process and answer are enclosed within <think> </think> and <answer> </answer> tags, respectively, i.e., <think> reasoning process here </think> <answer> answer here </answer>. User: prompt. Assistant:

Table 1 | Template for DeepSeek-R1-Zero. prompt will be replaced with the specific reasoning question during training.

2.2.2. Reward Modeling

The reward is the source of the training signal, which decides the optimization direction of RL. To train DeepSeek-R1-Zero, we adopt a rule-based reward system that mainly consists of two types of rewards:

- Accuracy rewards: The accuracy reward model evaluates whether the response is correct.
 For example, in the case of math problems with deterministic results, the model is required to provide the final answer in a specified format (e.g., within a box), enabling reliable rule-based verification of correctness. Similarly, for LeetCode problems, a compiler can be used to generate feedback based on predefined test cases.
- Format rewards: In addition to the accuracy reward model, we employ a format reward
 model that enforces the model to put its thinking process between '<think>' and '</think>'
 tags.

We do not apply the outcome or process neural reward model in developing DeepSeek-R1-Zero, because we find that the neural reward model may suffer from reward hacking in the large-scale reinforcement learning process, and retraining the reward model needs additional training resources and it complicates the whole training pipeline.

2.2.3. Training Template

To train DeepSeek-R1-Zero, we begin by designing a straightforward template that guides the base model to adhere to our specified instructions. As depicted in Table 1 this template requires DeepSeek-R1-Zero to first produce a reasoning process, followed by the final answer. We intentionally limit our constraints to this structural format, avoiding any content-specific biases—such as mandating reflective reasoning or promoting particular problem-solving strategies—to ensure that we can accurately observe the model's natural progression during the RL process.

2.2.4. Performance, Self-evolution Process and Aha Moment of DeepSeek-R1-Zero

Performance of DeepSeek-R1-Zero Figure 2 depicts the performance trajectory of DeepSeek-R1-Zero on the AIME 2024 benchmark throughout the RL training process. As illustrated, DeepSeek-R1-Zero demonstrates a steady and consistent enhancement in performance as the RL training advances. Notably, the average pass@1 score on AIME 2024 shows a significant increase, jumping from an initial 15.6% to an impressive 71.0%, reaching performance levels comparable to OpenAI-o1-0912. This significant improvement highlights the efficacy of our RL algorithm in optimizing the model's performance over time.

Table provides a comparative analysis between DeepSeek-R1-Zero and OpenAI's o1-0912 models across a variety of reasoning-related benchmarks. The findings reveal that RL empowers

Model	AIME 2024		MATH-500	GPQA Diamond	LiveCode Bench	CodeForces
	pass@1	cons@64	pass@1	pass@1	pass@1	rating
OpenAI-o1-mini	63.6	80.0	90.0	60.0	53.8	1820
OpenAI-o1-0912	74.4	83.3	94.8	77.3	63.4	1843
DeepSeek-R1-Zero	71.0	86.7	95.9	73.3	50.0	1444

Table 2 | Comparison of DeepSeek-R1-Zero and OpenAI o1 models on reasoning-related benchmarks.

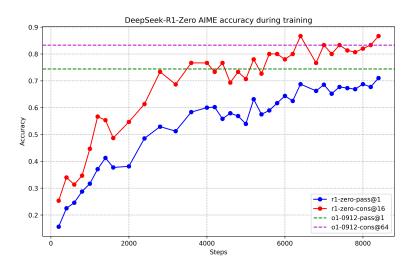


Figure 2 | AIME accuracy of DeepSeek-R1-Zero during training. For each question, we sample 16 responses and calculate the overall average accuracy to ensure a stable evaluation.

DeepSeek-R1-Zero to attain robust reasoning capabilities without the need for any supervised fine-tuning data. This is a noteworthy achievement, as it underscores the model's ability to learn and generalize effectively through RL alone. Additionally, the performance of DeepSeek-R1-Zero can be further augmented through the application of majority voting. For example, when majority voting is employed on the AIME benchmark, DeepSeek-R1-Zero's performance escalates from 71.0% to 86.7%, thereby exceeding the performance of OpenAI-o1-0912. The ability of DeepSeek-R1-Zero to achieve such competitive performance, both with and without majority voting, highlights its strong foundational capabilities and its potential for further advancements in reasoning tasks.

Self-evolution Process of DeepSeek-R1-Zero The self-evolution process of DeepSeek-R1-Zero is a fascinating demonstration of how RL can drive a model to improve its reasoning capabilities autonomously. By initiating RL directly from the base model, we can closely monitor the model's progression without the influence of the supervised fine-tuning stage. This approach provides a clear view of how the model evolves over time, particularly in terms of its ability to handle complex reasoning tasks.

As depicted in Figure 3 the thinking time of DeepSeek-R1-Zero shows consistent improve-

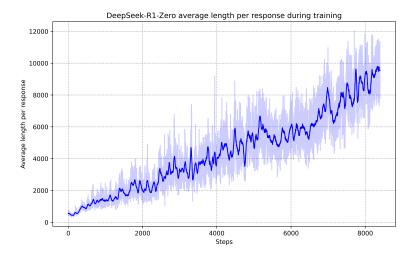


Figure 3 | The average response length of DeepSeek-R1-Zero on the training set during the RL process. DeepSeek-R1-Zero naturally learns to solve reasoning tasks with more thinking time.

ment throughout the training process. This improvement is not the result of external adjustments but rather an intrinsic development within the model. DeepSeek-R1-Zero naturally acquires the ability to solve increasingly complex reasoning tasks by leveraging extended test-time computation. This computation ranges from generating hundreds to thousands of reasoning tokens, allowing the model to explore and refine its thought processes in greater depth.

One of the most remarkable aspects of this self-evolution is the emergence of sophisticated behaviors as the test-time computation increases. Behaviors such as reflection—where the model revisits and reevaluates its previous steps—and the exploration of alternative approaches to problem-solving arise spontaneously. These behaviors are not explicitly programmed but instead emerge as a result of the model's interaction with the reinforcement learning environment. This spontaneous development significantly enhances DeepSeek-R1-Zero's reasoning capabilities, enabling it to tackle more challenging tasks with greater efficiency and accuracy.

Aha Moment of DeepSeek-R1-Zero A particularly intriguing phenomenon observed during the training of DeepSeek-R1-Zero is the occurrence of an "aha moment". This moment, as illustrated in Table [3] occurs in an intermediate version of the model. During this phase, DeepSeek-R1-Zero learns to allocate more thinking time to a problem by reevaluating its initial approach. This behavior is not only a testament to the model's growing reasoning abilities but also a captivating example of how reinforcement learning can lead to unexpected and sophisticated outcomes.

This moment is not only an "aha moment" for the model but also for the researchers observing its behavior. It underscores the power and beauty of reinforcement learning: rather than explicitly teaching the model on how to solve a problem, we simply provide it with the right incentives, and it autonomously develops advanced problem-solving strategies. The "aha moment" serves as a powerful reminder of the potential of RL to unlock new levels of intelligence in artificial systems, paving the way for more autonomous and adaptive models in the future.

```
Question: If a > 1, then the sum of the real solutions of \sqrt{a - \sqrt{a + x}} = x is equal to Response: <think>
To solve the equation \sqrt{a - \sqrt{a + x}} = x, let's start by squaring both \cdots \left(\sqrt{a - \sqrt{a + x}}\right)^2 = x^2 \implies a - \sqrt{a + x} = x^2. Rearrange to isolate the inner square root term: (a - x^2)^2 = a + x \implies a^2 - 2ax^2 + (x^2)^2 = a + x \implies x^4 - 2ax^2 - x + (a^2 - a) = 0 ...

Wait, wait. Wait. That's an aha moment I can flag here. Let's reevaluate this step-by-step to identify if the correct sum can be \cdots We started with the equation: \sqrt{a - \sqrt{a + x}} = x First, let's square both sides: a - \sqrt{a + x} = x^2 \implies \sqrt{a + x} = a - x^2 Next, I could square both sides again, treating the equation: \cdots ...
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Table 3 | An interesting "aha moment" of an intermediate version of DeepSeek-R1-Zero. The model learns to rethink using an anthropomorphic tone. This is also an aha moment for us, allowing us to witness the power and beauty of reinforcement learning.

Drawback of DeepSeek-R1-Zero Although DeepSeek-R1-Zero exhibits strong reasoning capabilities and autonomously develops unexpected and powerful reasoning behaviors, it faces several issues. For instance, DeepSeek-R1-Zero struggles with challenges like poor readability, and language mixing. To make reasoning processes more readable and share them with the open community, we explore DeepSeek-R1, a method that utilizes RL with human-friendly cold-start data.

2.3. DeepSeek-R1: Reinforcement Learning with Cold Start

Inspired by the promising results of DeepSeek-R1-Zero, two natural questions arise: 1) Can reasoning performance be further improved or convergence accelerated by incorporating a small amount of high-quality data as a cold start? 2) How can we train a user-friendly model that not only produces clear and coherent Chains of Thought (CoT) but also demonstrates strong general capabilities? To address these questions, we design a pipeline to train DeepSeek-R1. The pipeline consists of four stages, outlined as follows.

2.3.1. Cold Start

Unlike DeepSeek-R1-Zero, to prevent the early unstable cold start phase of RL training from the base model, for DeepSeek-R1 we construct and collect a small amount of long CoT data to fine-tune the model as the initial RL actor. To collect such data, we have explored several approaches: using few-shot prompting with a long CoT as an example, directly prompting models to generate detailed answers with reflection and verification, gathering DeepSeek-R1-Zero outputs in a readable format, and refining the results through post-processing by human annotators.

In this work, we collect thousands of cold-start data to fine-tune the DeepSeek-V3-Base as the starting point for RL. Compared to DeepSeek-R1-Zero, the advantages of cold start data

include:

- Readability: A key limitation of DeepSeek-R1-Zero is that its content is often not suitable for reading. Responses may mix multiple languages or lack markdown formatting to highlight answers for users. In contrast, when creating cold-start data for DeepSeek-R1, we design a readable pattern that includes a summary at the end of each response and filters out responses that are not reader-friendly. Here, we define the output format as | special_token | <reasoning_process> | special_token | <summary>, where the reasoning process is the CoT for the query, and the summary is used to summarize the reasoning results.
- Potential: By carefully designing the pattern for cold-start data with human priors, we observe better performance against DeepSeek-R1-Zero. We believe the iterative training is a better way for reasoning models.

2.3.2. Reasoning-oriented Reinforcement Learning

After fine-tuning DeepSeek-V3-Base on the cold start data, we apply the same large-scale reinforcement learning training process as employed in DeepSeek-R1-Zero. This phase focuses on enhancing the model's reasoning capabilities, particularly in reasoning-intensive tasks such as coding, mathematics, science, and logic reasoning, which involve well-defined problems with clear solutions. During the training process, we observe that CoT often exhibits language mixing, particularly when RL prompts involve multiple languages. To mitigate the issue of language mixing, we introduce a language consistency reward during RL training, which is calculated as the proportion of target language words in the CoT. Although ablation experiments show that such alignment results in a slight degradation in the model's performance, this reward aligns with human preferences, making it more readable. Finally, we combine the accuracy of reasoning tasks and the reward for language consistency by directly summing them to form the final reward. We then apply RL training on the fine-tuned model until it achieves convergence on reasoning tasks.

2.3.3. Rejection Sampling and Supervised Fine-Tuning

When reasoning-oriented RL converges, we utilize the resulting checkpoint to collect SFT (Supervised Fine-Tuning) data for the subsequent round. Unlike the initial cold-start data, which primarily focuses on reasoning, this stage incorporates data from other domains to enhance the model's capabilities in writing, role-playing, and other general-purpose tasks. Specifically, we generate the data and fine-tune the model as described below.

Reasoning data We curate reasoning prompts and generate reasoning trajectories by performing rejection sampling from the checkpoint from the above RL training. In the previous stage, we only included data that could be evaluated using rule-based rewards. However, in this stage, we expand the dataset by incorporating additional data, some of which use a generative reward model by feeding the ground-truth and model predictions into DeepSeek-V3 for judgment. Additionally, because the model output is sometimes chaotic and difficult to read, we have filtered out chain-of-thought with mixed languages, long parapraphs, and code blocks. For each prompt, we sample multiple responses and retain only the correct ones. In total, we collect about 600k reasoning related training samples.