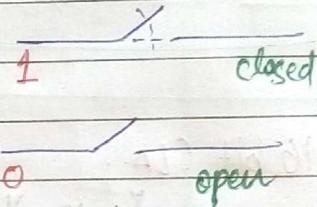
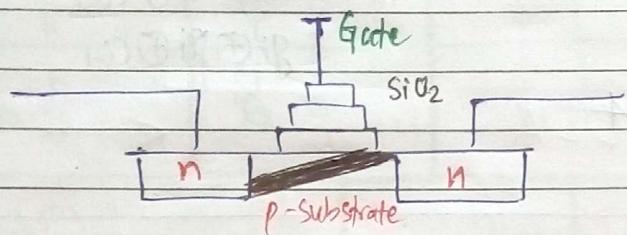
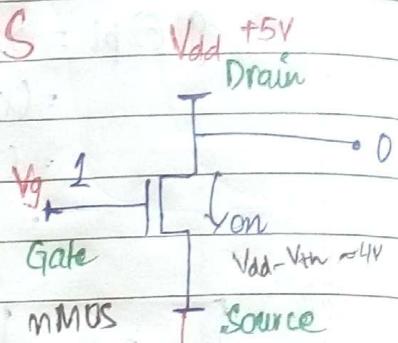


REALIZING LOGICS WITH CMOS

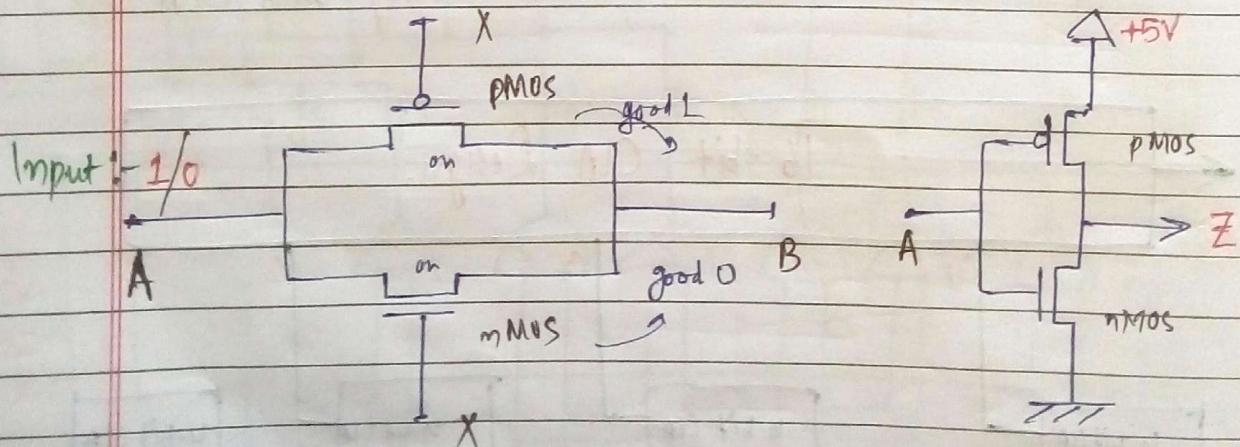
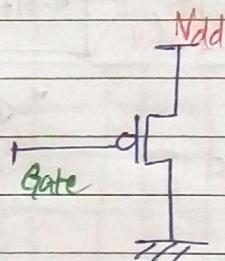
P MOS
n MOS

Can act as a switch



nMOS passes a good 0
nMOS passes a poor 1.

pMOS passes a good 1
pMOS passes a poor 0.

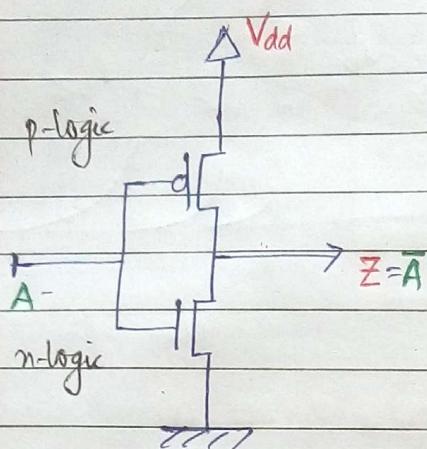


CMOS inverter	
mMOS	pMOS
off	on → passes good

Complemented Switch

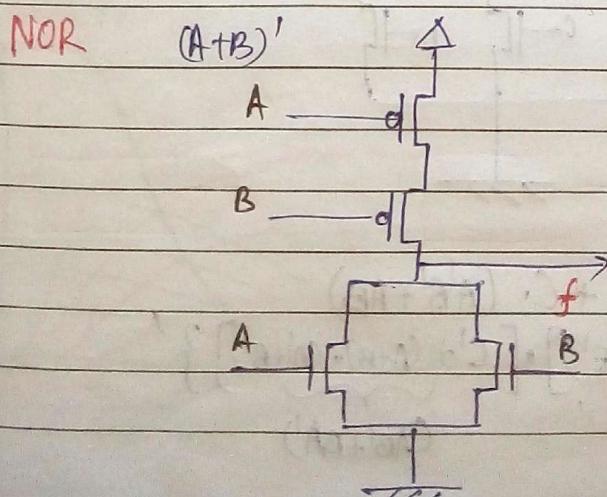
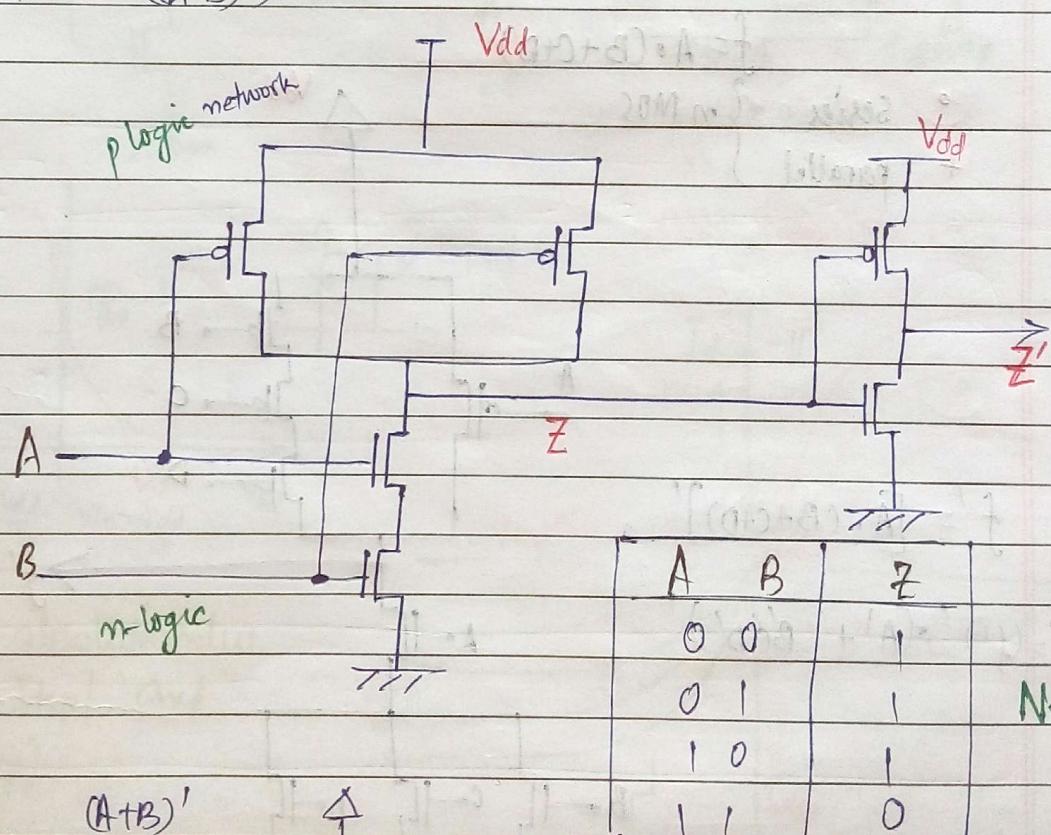
A = 0	Z = 1
A = 1	Z = 0

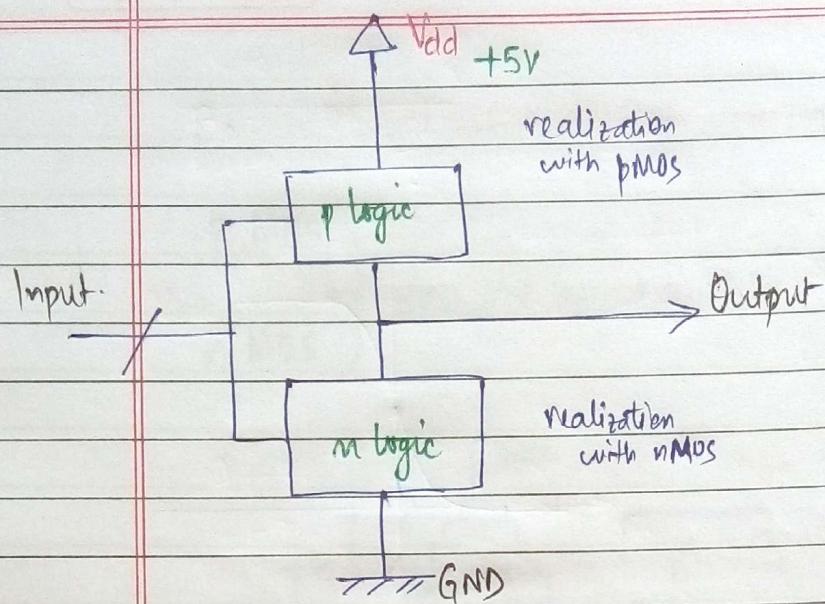
Passes good 0



2 - input NAND Gate

$$\text{And} - ((A \cdot B)')'$$



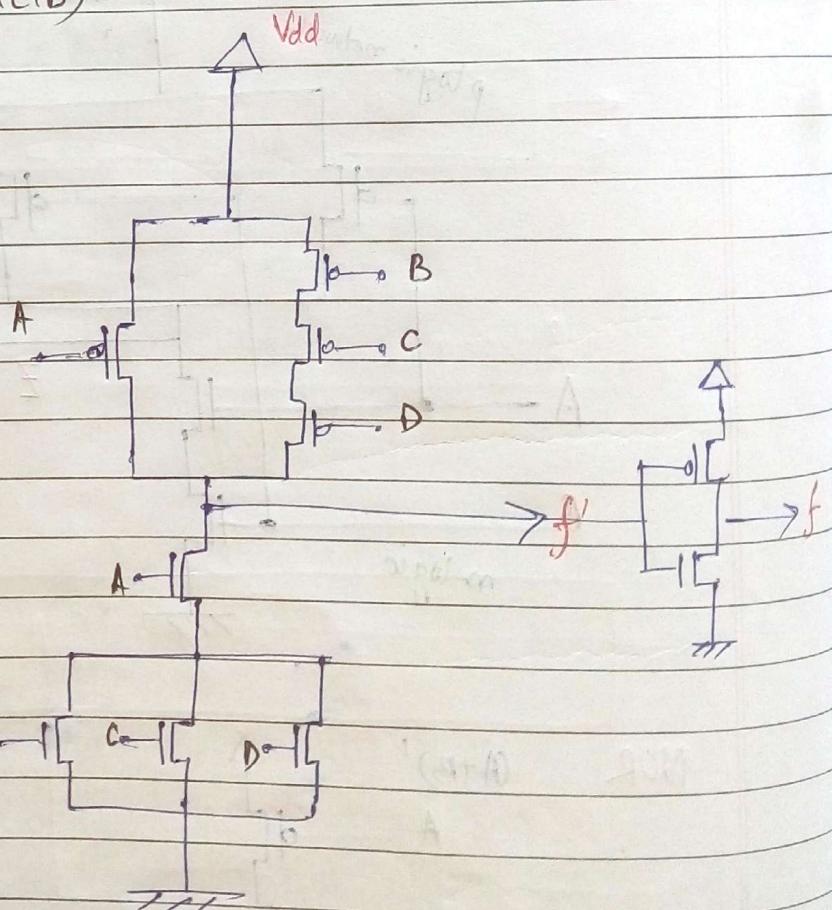


$$f = A \cdot (B + C + D)$$

• Series } n MOS
+ parallel

$$f' = [A \cdot (B + C + D)]'$$

$$f = (f')' = (A' + B'C'D')'$$

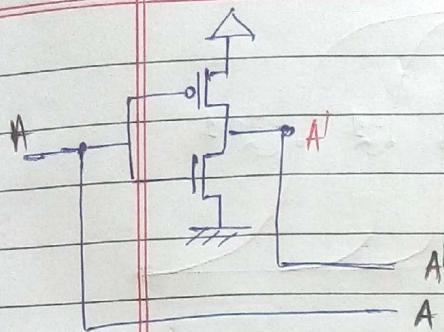


$$f = (A+B) \cdot (C+D+E) + C \cdot (A'B' + AB)$$

$$f = (f')' = \{ [(A'B') + (C'D'E')] \cdot [C' + (A+B) \cdot (A+B)'] \} '$$

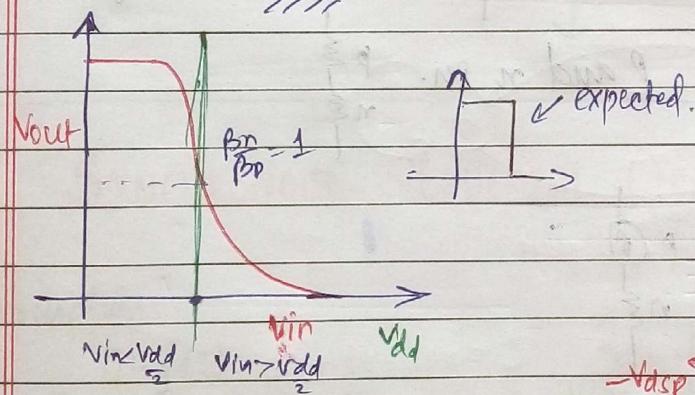
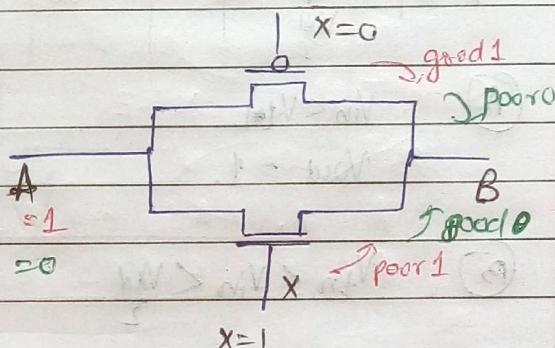
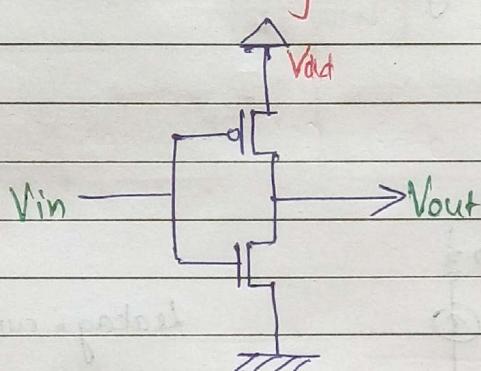
$\downarrow (A+B)$ $\downarrow (C+D+E)'$

$\downarrow (AB' + BA')$

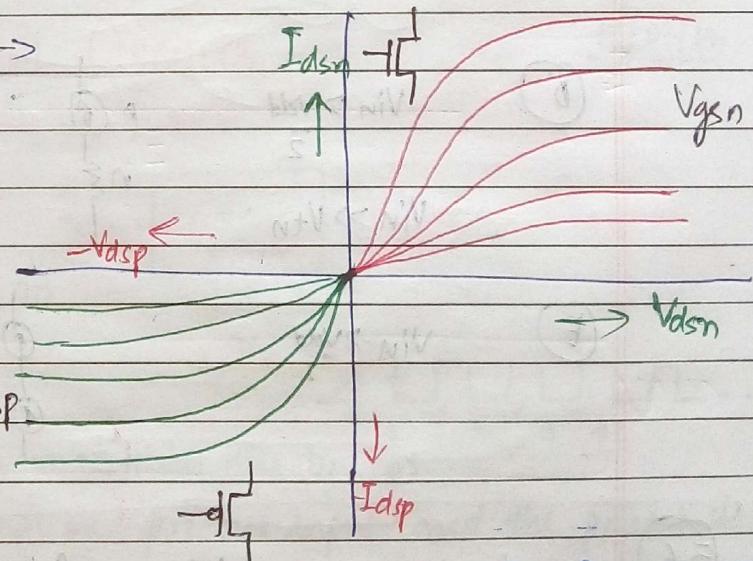


For each complemented input, 2 transistors are required.

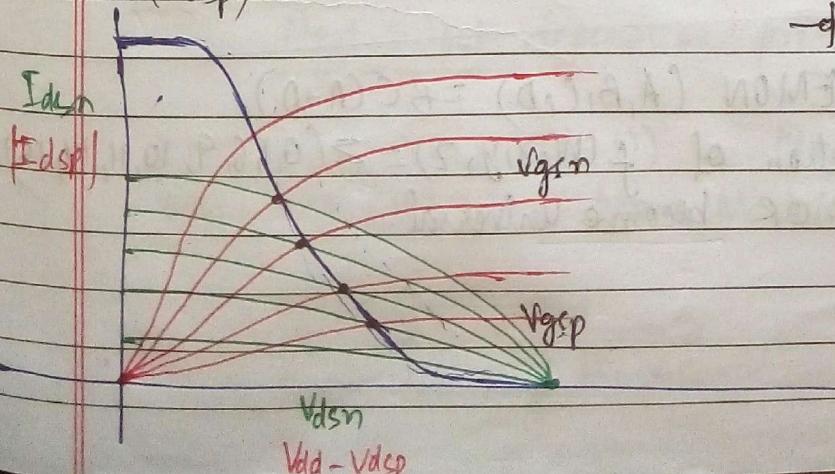
Transfer Characteristics of CMOS Inverter



Pass transistor



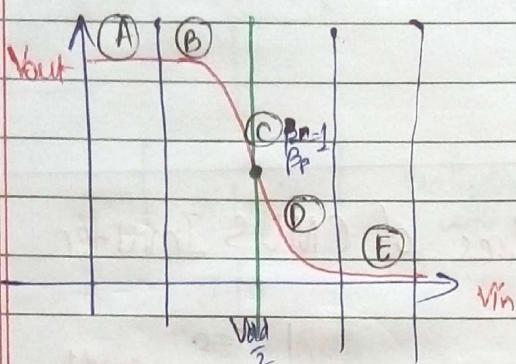
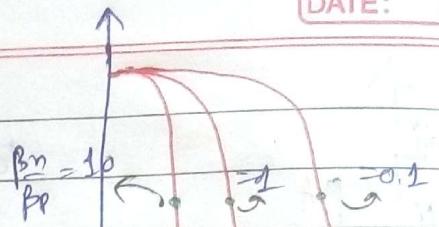
Take absolute value of $|Id_{sp}|$ and $(V_{ds_{sp}})$



JAIN BHAV
JAIN NOTES

Some same value for V_{gsp} and V_{gsn} which gives you V_{in} of a inverter.

$$\beta_n = \left(\frac{NE}{t} \right) \frac{W}{L}$$

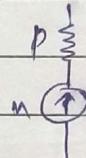


five regions in the curve space.

(A) $V_{in} < V_{tn}$

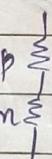
$V_{out} = 1$.

(B) $V_{tn} \leq V_{in} < \frac{V_{dd}}{2}$

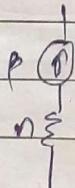


Leakage current.

(C) $V_{in} = \frac{V_{dd}}{2}$

p and n on. 

(D) $V_{in} > \frac{V_{dd}}{2}$



$V_{in} > V_{thn}$

(E) $V_{in} > \frac{V_{dd}}{2}$



5.6

i 4-in gate LEMON (A, B, C, D) $= B'C(A+D)$

(a) show a realization of $f(W, X, Y, Z) = \sum(0, 1, 6, 9, 10, 11, 14, 15)$

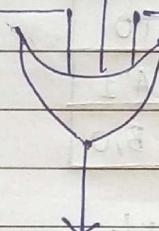
(b) can LEMON & OR become universal.

CD	AB	00	01	11	10
00	00	00	00	00	00
01	01	01	01	01	01
11	11	11	11	11	11
10	10	10	10	10	10

wx	00	01	11	10
00	1	0	0	0
01	1	0	0	1
11	0	0	1	1
10	0	1	1	1

- (1) LEMON ($wx'yz$)
 (2) LEMON ($w'xyz'$)
 (3) LEMON ($w'x'y'z$)

$wx'yz$ $wx'yz'$ $w'x'y'z$
 LEMON LEMON LEMON

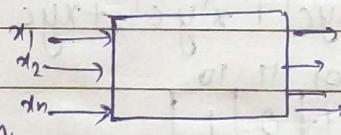


SYNCHRONOUS SEQUENTIAL CIRCUITS

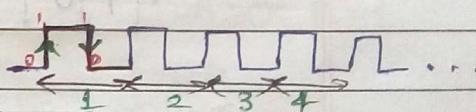
01/03/17

Combination

$$O = f(I)$$



Present output depends only on present input.



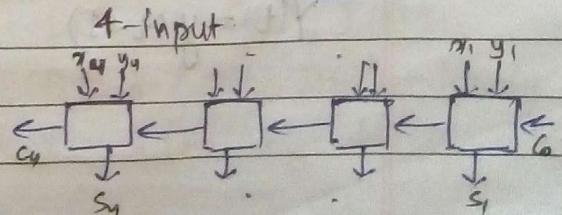
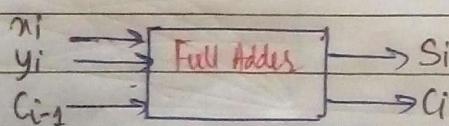
Sequential - Clock maintains the timings

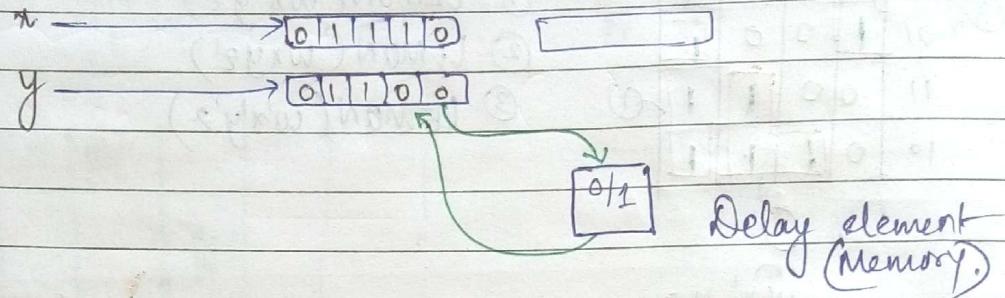
Next output depends on present input and the present state.

$$Z(t) = \lambda(S(t-1), I(t-1)) \quad - \text{Mealy Machine}$$

$$Z(t) = \lambda(S(t-1)) \quad - \text{Moore Machine}$$

ANUBHAV
JAIN
NOTES



Serial Adder

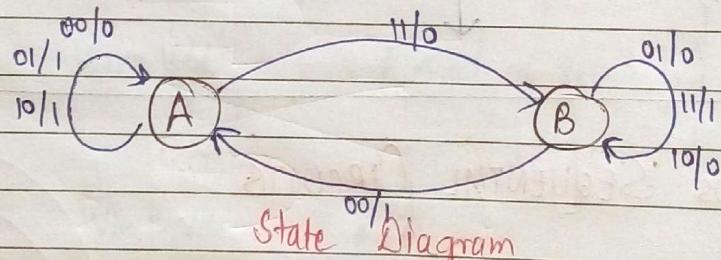
A → State where carryout is 0 State Assignment

B → State where carryout is 1.

PS: Present state NS: Next state Z: Output

State Table NS, Z

PS	00	01	11	10
A(0)	A, 0	A, 1	B, 0	A, 1
B(1)	A, 1	B, 0	B, 1	B, 0

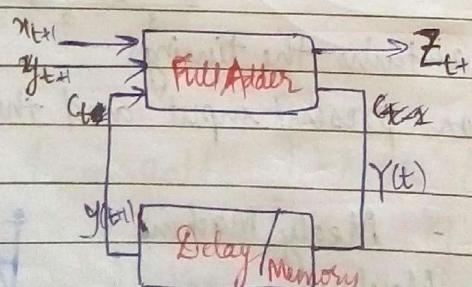


State Diagram

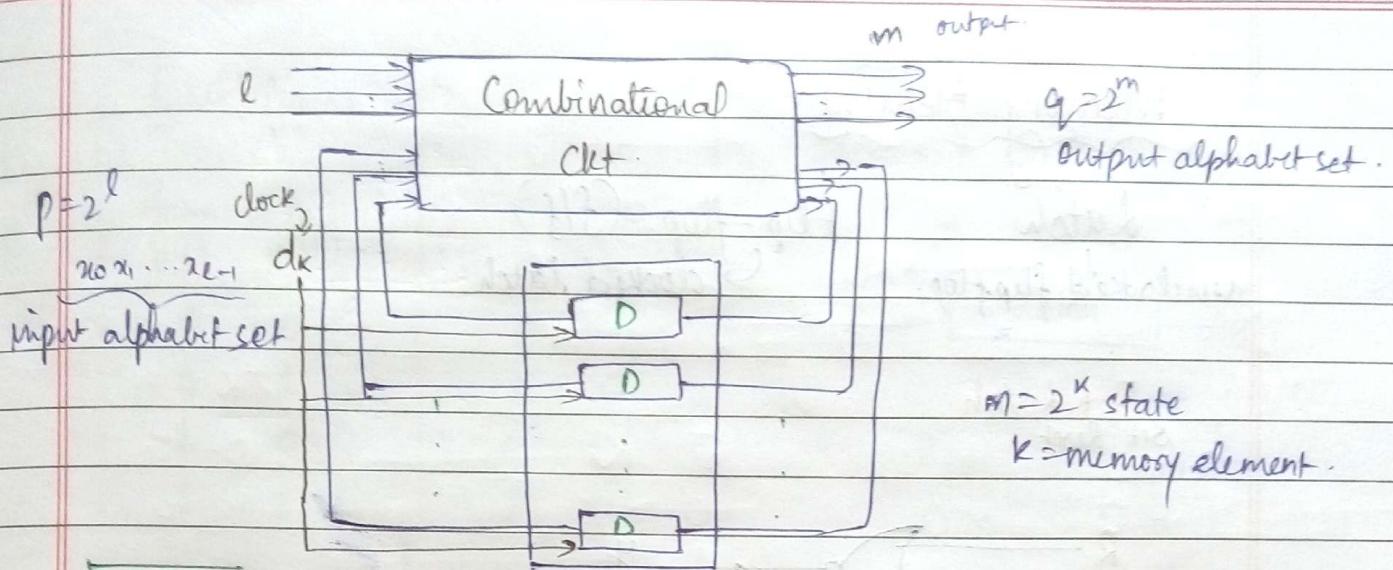
$$Z = x'y'c + xy'c' + x'y'c + xyc$$

c	x	y	00	01	11	10
0	0	1	0	1		
1	1	0	1	0		

$$C = xy + xc + yc$$



$$y(t) = y(t+1)$$



FSM

Finite State Model

Serial Adder

02/03/17

	x_1x_2	z
$ps=y$	00 01 11 10	
A	0 0 1 0 1	
B	1 1 0 1 0	

$ps=x_1x_2$	00	01	11	10
$A(0)$	A,0	A,1	B,0	A,1
$B(1)$	A,1	B,0	B,1	B,0

$$z = x_1'x_2y' + x_1x_2'y' + x_1'x_2'y + x_1x_2y$$

A: state with carry 0

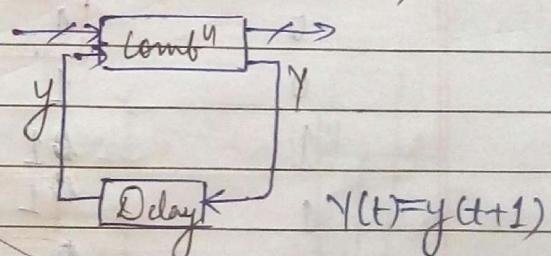
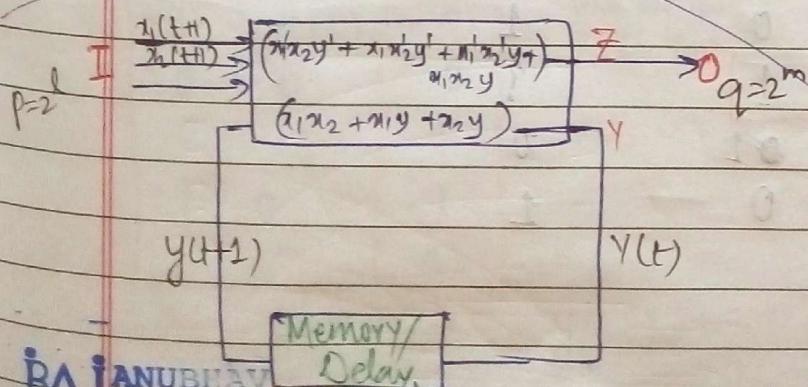
B: state with carry 1

$ps=y$	x_1x_2	NS
	00 01 11 10	
A	0 0 1 0 0	
B	0 1 0 1 1	

$$NS = y = x_1x_2 + x_1y + x_2y$$

State Assignment

$$A=0, B=1$$



$$S(t+1) = S(S(t), I(t))$$

$$Z(t) = \lambda(S(t), I(t)) - \text{Mealy}$$

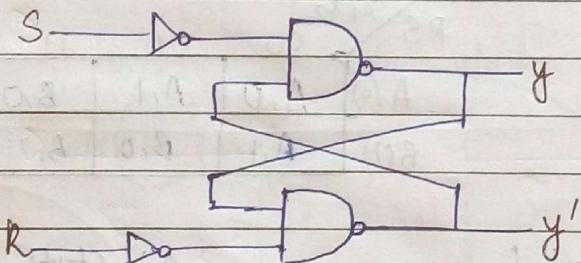
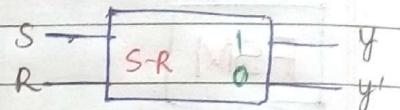
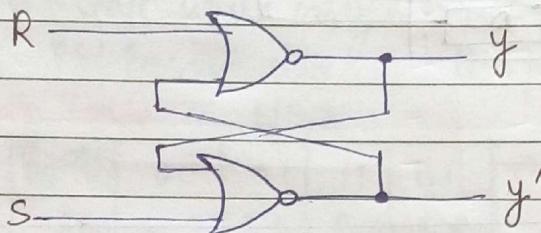
$$Z(t) = \lambda(S(t)) - \text{Moore}$$

$(I, O, S, \delta, \lambda, S)$

Memory Element

Latch, Flip-flop. (f/f)
 undelayed flip-flop. ↳ clocked latch.

S-R Latch
 Set Reset

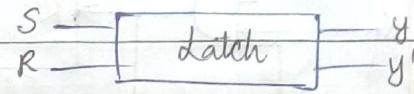


$y(t)$	$S(t)$	$R(t)$	$y(t+1)$
0	0	0	0
0	0	1	0
0	1	1	?
0	1	0	1
1	1	0	1
1	1	1	?
1	0	1	0
1	0	0	1

(Not allowed state) $\rightarrow y = y' = 0$.

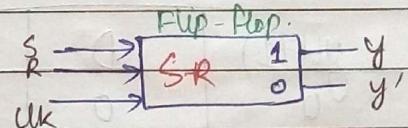
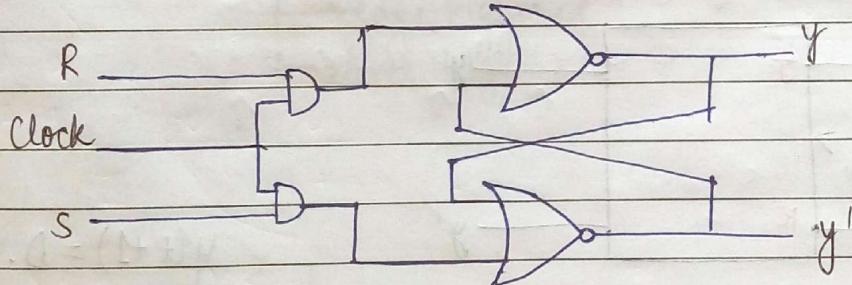
Excitation Table

Transition		Input	
from $y(t)$	to $y(t+1)$	S	R
0 0	0 -	0	-
0 1	1 0	1	0
1 0	0 1	0	1
1 1	- 0	-	0



$y(t)$	SR		$y(t+1)$	
	00	01	11	10
0	0 0	0 0	X	1
1	1 1	0 0	X	1

$$y(t+1) = S + R'y(t)$$

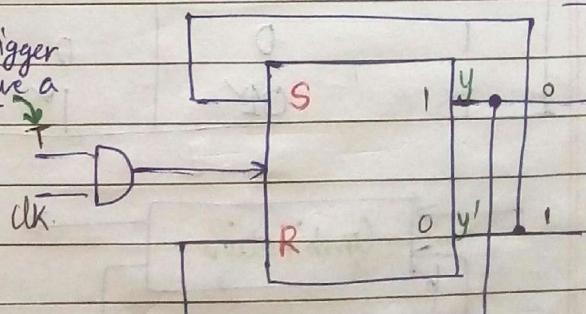


Toggle f.f.

03/03/17

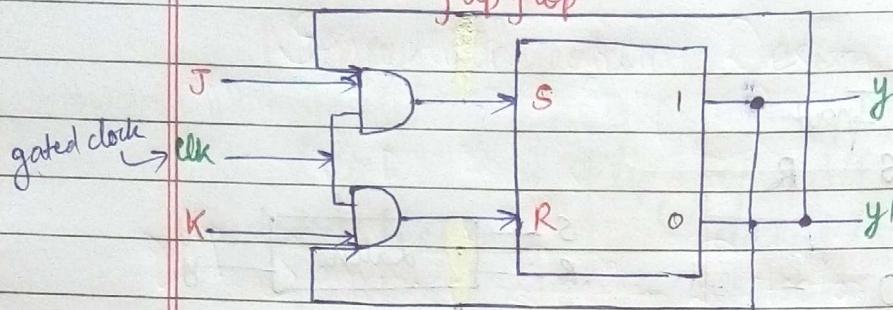
$y(t)$	$y(t+1)$	T
0 0	0	0
0 1	1	1
1 0	1	1
1 1	0	0

it will trigger when we give a input to T



$$y(t+1) = T y'(t) + T' y(t)$$

J-K flip-flop

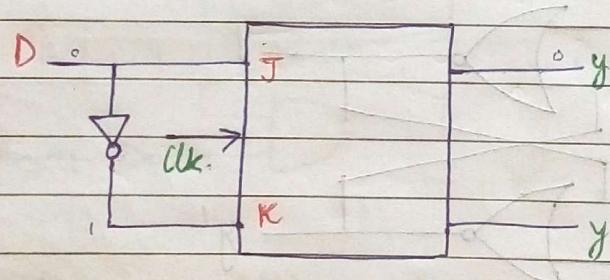


$$JK = 11$$

$y(t)$	$y(t+1)$	J	K	y
0	0	0	-0	0
0	1	1	-	1
1	0	-	1	0
1	1	-	0	1

$$\begin{matrix} 0 \rightarrow 1 \\ 1 \rightarrow 0 \end{matrix}$$

D flip-flop.

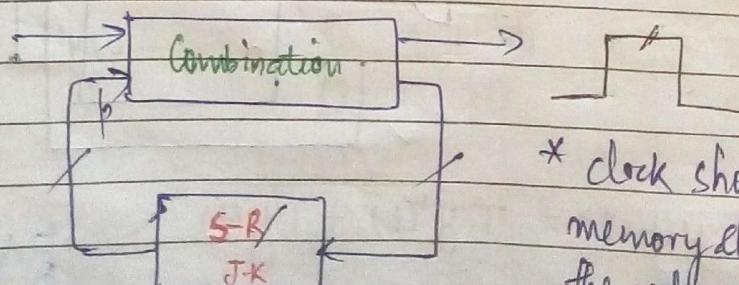


$$y(t+1) = D$$

$y(t)$	$y(t+1)$	D	J^D	K^D
0	0	0	0	1
0	1	1	1	0
1	0	0	0	1
1	01	1	1	0

D latch is
actual memory
element.

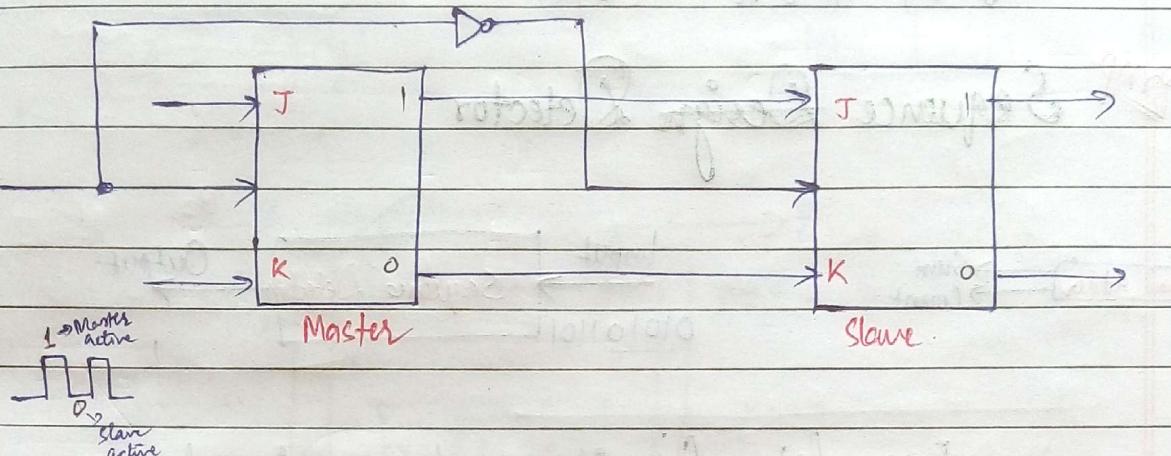
T = Toggle the prev.
state (0/1).



* clock should be long enough so that
memory element should store
the value (latch value).

Propagation through the combinational logic.

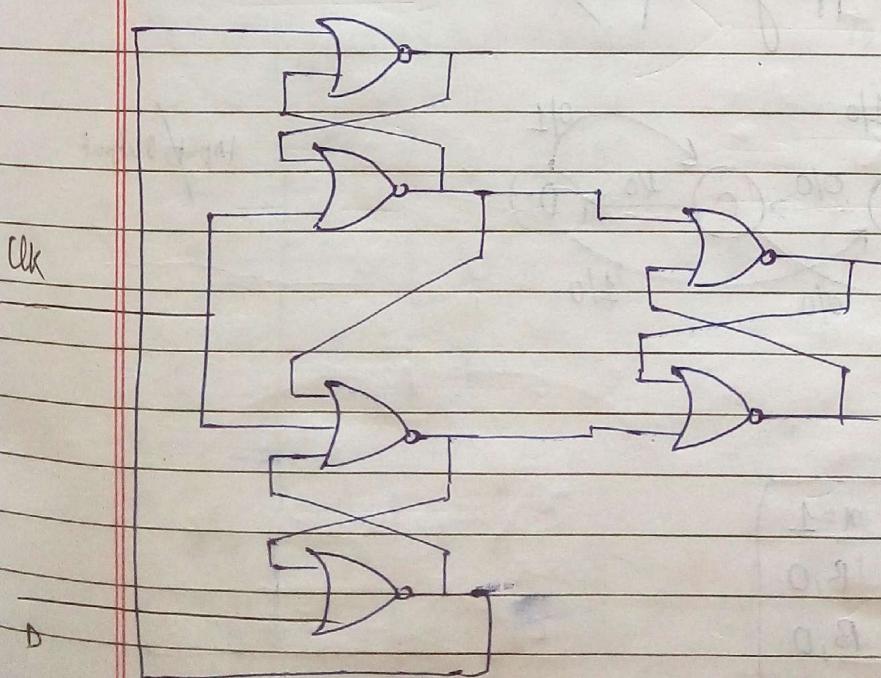
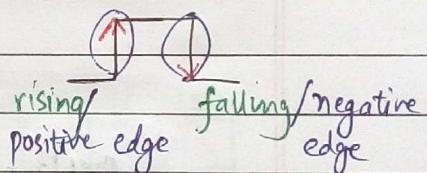
* clock should be short enough so that it is only stable.
Propagation through the combinational logic.



Synchronous Sequential Circuits

synchronized with clock

- edge triggered
- level triggered

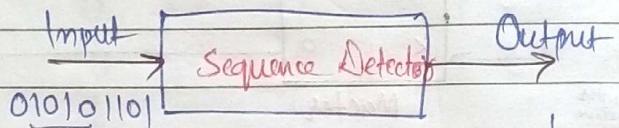
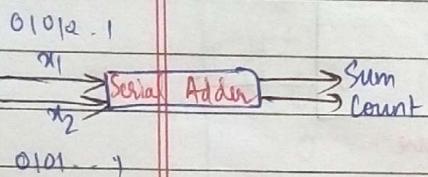


Negative edge triggered D f/f

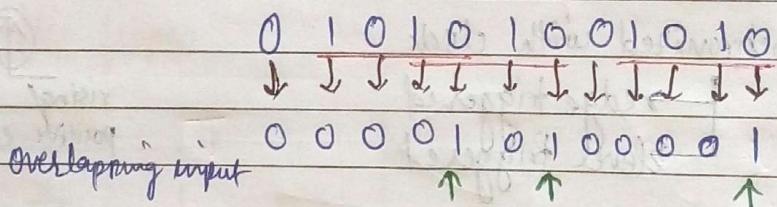
Sequential Circuit Design

Memory elements (Flip-flop)
D, T, JK (SR)

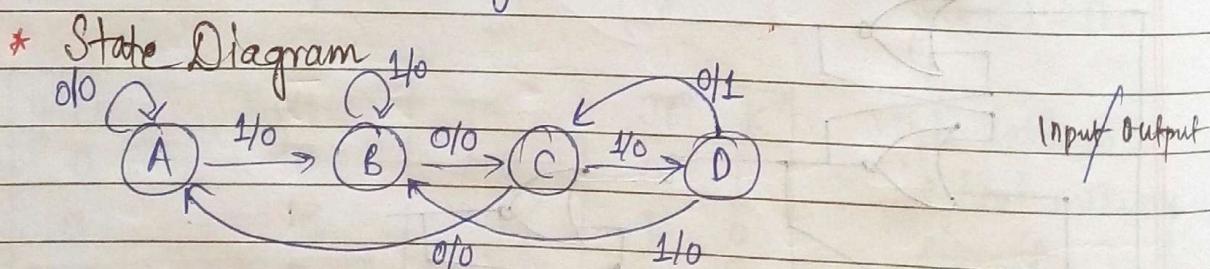
Sequence Design Detector



Example: Circuit produces a 1 whenever a sequence 1010 is detected.



(Consider the overlapping input.)



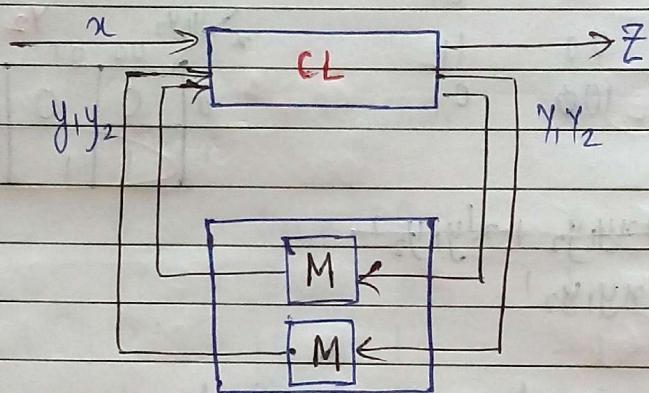
* State Table

State Assignment	Input	NS, Z	
		n=0	n=1
00	A	A, 0	B, 0
01	B	C, 0	B, 0
11	C	A, 0	D, 0
10	D	C, 1	B, 0

x	$y_1 y_2$	Z
0	00 01 11 10	
1	00 00 00 00	

$$Z = x'y_1 y_2'$$

$y_1 y_2$	$x=0$	$x=1$
00	00	01
01	11	01
11	00	10
10	11	01

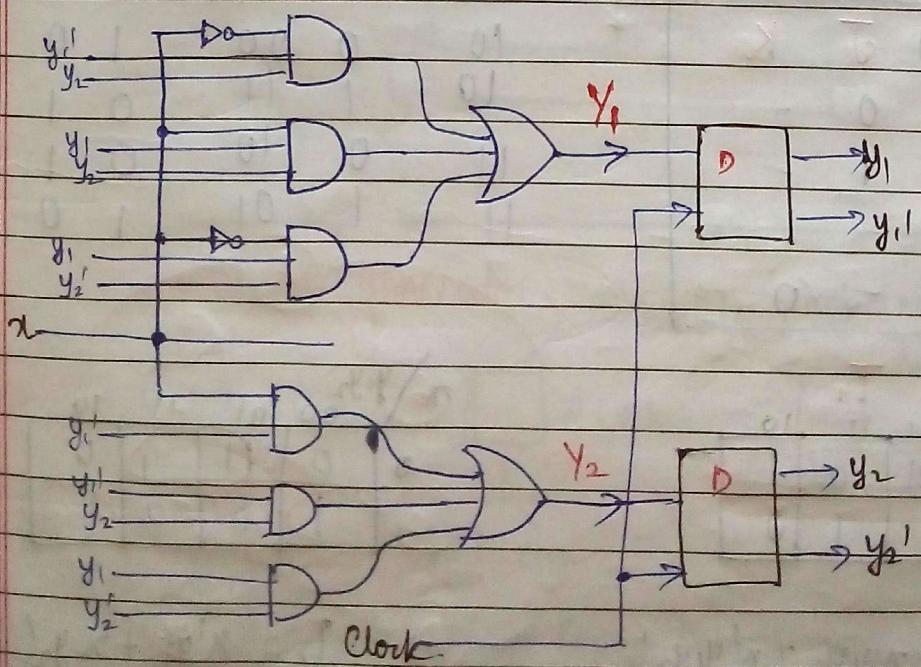


x	$y_1 y_2$	y_1
0	00 01 11 10	0 1 0 1
1	00 00 00 00	0 0 0 0

x	$y_1 y_2$	y_2
0	00 01 11 10	0 0 1 0
1	00 00 00 00	1 1 0 0

$$y_1 = x'y_1'y_2 + xy_1y_2 + x'y_1y_2'$$

$$y_2 = xy_1' + y_1'y_2 + y_1y_2'$$



Change State Assignments:-

A-00 B-01 C-0

D-11

\sim

\sim

		NS	
		$y_1 y_2$	$y_1 y_2'$
		$x=0$	$x=1$
00		00	01
01		10	01
10		00	11
11		100	01

		Y ₁			
		00	01	11	10
		0	0	1	000
		1	0	0	000
		0	0	0	001

		Y ₂			
		00	01	11	10
		0	0	0	0
		1	1	1	1
		1	1	1	1

$$Y_1 = x'y_1y_2' + xy_1y_2 + x'y_1y_2$$

$$Y_1 = x'y_2 + xy_1y_2'$$

$$Y_2 = x$$

Less no. of gates required

↳ More efficient design.

y(t) y(t+1)		T
00	0	
01	1	
10	1	
10	0	

PS	x	NS	T ₁ T ₂
00	0	00	0 0
00	1	01	0 1
01	0	10	1 1
01	1	01	0 0

y(t) y(t+1)		J K
00	0	-
01	1	-
10	-	1
11	-	0

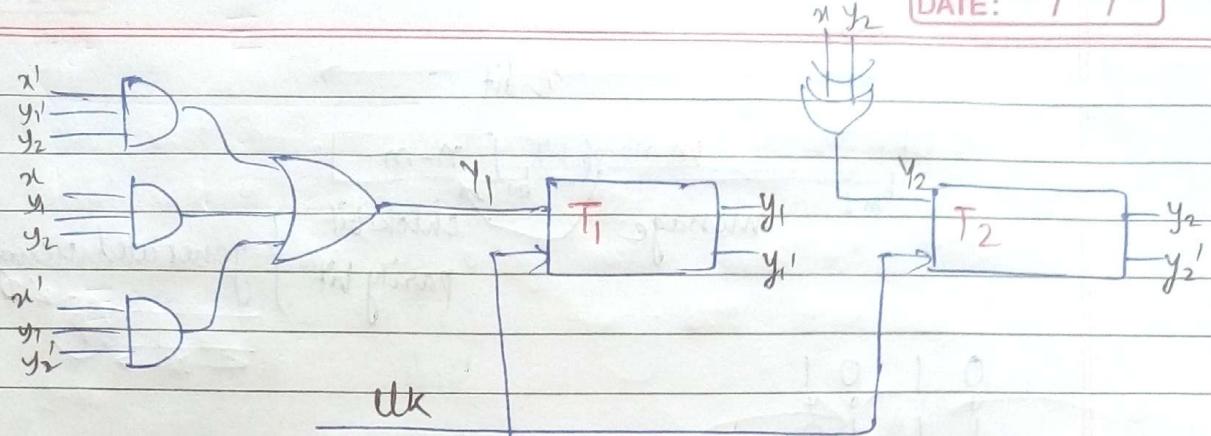
10	0	00	1 0
10	1	11	0 1
11	0	10	0 1
11	1	01	1 0

		T ₁
00	01	01
10	01	10

		T ₂
00	01	10
10	01	01

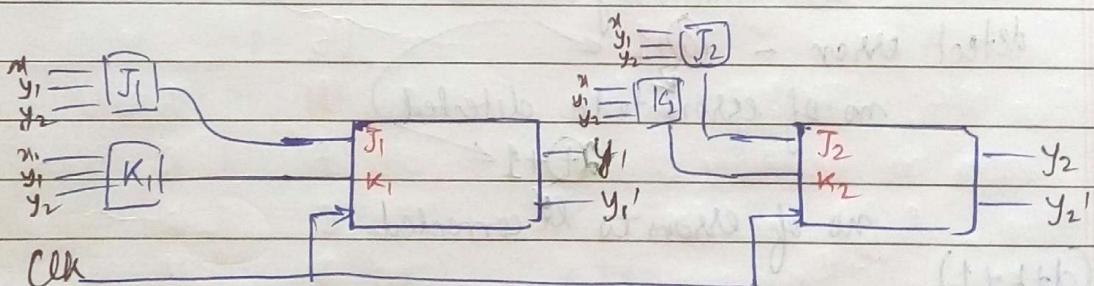
$$T_1 = x'y_1'y_2 + xy_1y_2 + x'y_1y_2'$$

$$T_2 = x'y_2 + xy_2'$$

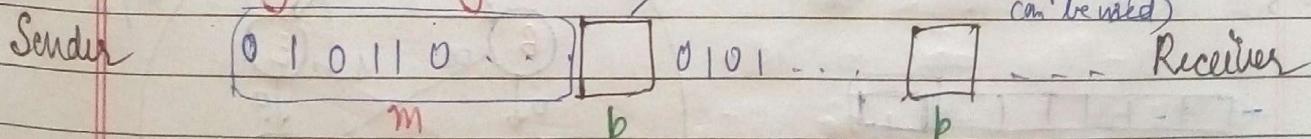


PS	Input x	NS	Excitation $J_1 K_1$	Excitation $J_2 K_2$
$y_1 y_2$				

00	0	00	0-	0-
00	1	01	0-	1-
01	0	10	1-	-1
01	1	01	0-	-0
10	0	00	-	0-
10	1	11	-0	1-
11	0	10	-0	-1
11	1	01	-1	-0



Parity Bit generator.



even parity
- The no. of

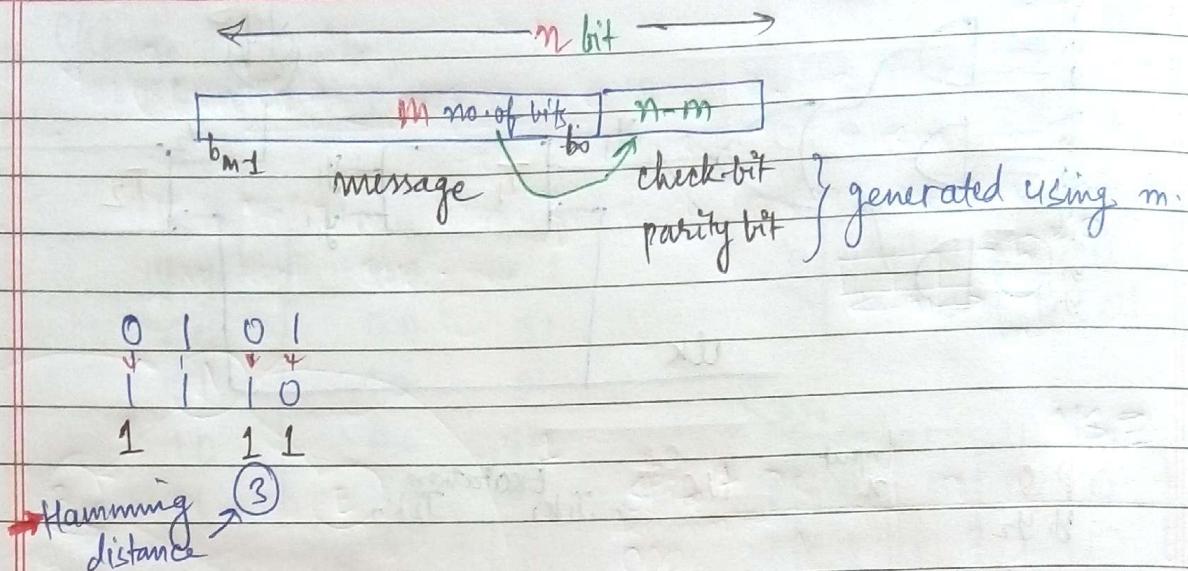
odd even

1's in m bits are odd. \Rightarrow Parity is 1 : so that together there are even no. of ones , else put 0.

instead of 1 only,
more no. of parity bits
can be used

09/03/17

ANUBHAV
JAIN
NOTES



Hamming weight = no. of 1's.

$$2^4 \left\{ \begin{array}{l} 0000 \\ 0001 \\ \vdots \\ 1111 \end{array} \right. \quad 2^5 = 32$$

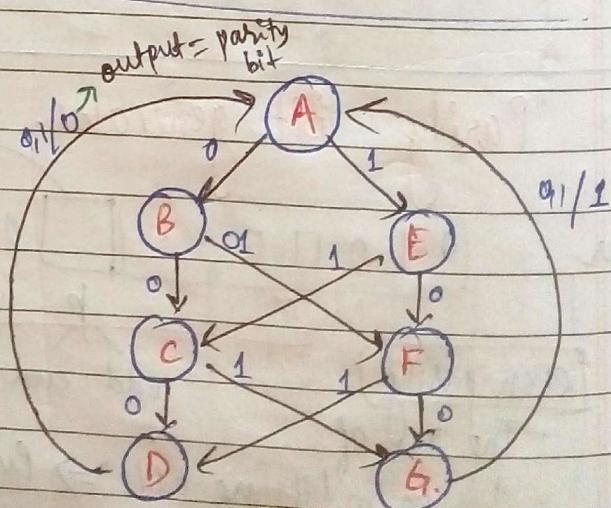
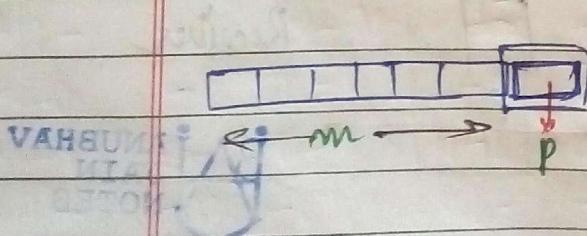
distance between any pair ≥ 2

$d+1$
 $t+1$
(let)

Hamming distance
detect error - $d+1$
no. of errors to be detected.
 $2t+1$
no. of errors to be corrected.
 $(d+t+1)$

Even parity bit generator

$$M=3$$



NS, Z

State Action	PS	$n=0$	$n=1$
0 000	A	B, X	E, X
2 010	B	C, X	F, X
3 011	C	D, X	G, X
6 110	D	A, 0	A, 0
7 111	E	F, X	C, X
4 100	F	G, X	D, X
5 101	G	A, 1	A, 1

Z

y_1	y_2	y_3	$n=0$	$n=1$
0 0 0			X 001	X
0 1 0			X 110	X
0 1 1			X 111	X
1 1 0			0 101	0
1 1 1			X	X
2 0 0			X	X
1 0 1			1	1

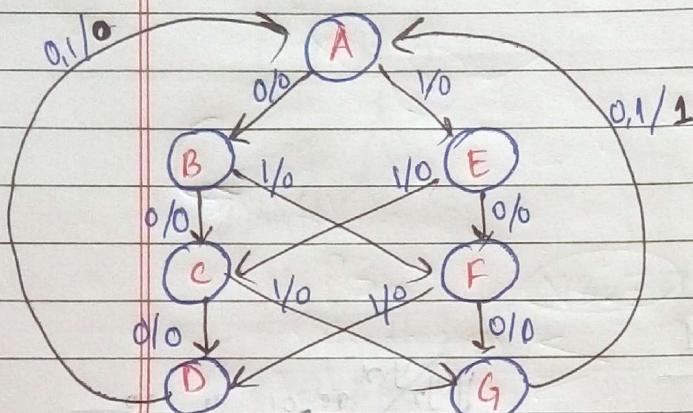
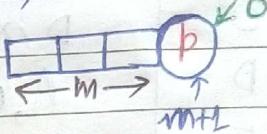
y_1, y_2, y_3, n	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	0	0	X	X
10	X	X	1	1

$$Z = y_3.$$

NS

y_1	y_2	y_3	$n=0$	$n=1$
A 0 0 0			B 0 1 0	E 1 1 1
B 0 1 0			C 0 1 1	F 1 0 0
C 0 1 1			D 1 0 0	G 1 0 1
D 1 1 0			A 0 0 0	A 0 0 0
E 1 1 1			F 1 0 0	C 0 1 1
F 1 0 0			G 1 0 1	D 1 1 0
G 1 0 1			A 0 0 0	A 0 0 0

Yesterday, for intermediate output, we took don't care. But conventionally, for even parity, intermediate bits are set to be zero and for odd parity, intermediate bits are kept one.



$y_3 y_2 y_1$	State Assignment	PS	$n=0$	$n=1$
000	A		B, 0	E, 0
010	B		C, 0	F, 0
110	C		D, 0	G, 0
100	D		A, 0	A, 0
011	E		F, 0	C, 0
111	F		G, 0	D, 0
101	G		A, 1	A, 1

Clearly, $\bar{z} = y_1 y_2 y_3$

For NS, JK flip-flop can be chosen as memory element.

Here, 3 JF will be required, one each for y_1, y_2 & y_3 .

$J_3 K_3$	$J_2 K_2$	$J_1 K_1$	$J_3 K_3$	$J_2 K_2$	$J_1 K_1$	$n=1$
A 0 -	1 -	0 -	0 -	1 -	1 -	
B 1 -	-0	0 -	1 -	-0	1 -	
C -0	-1	0 -	-0	1 -	1 -	
D -1	0 -	0 -	-1	0 -	0 -	
E 1 -	-0	-0	1 -	-0	-1	
F -0	-1	-0	-0	-1	-1	
G -1	0 -	-1	-1	0 -	-1	

PS	$n=0$	$n=1$
000 A	B, 0	E, 0
010 B	C, 0	F, 0
110 C	D, 0	G, 0
100 D	A, 0	A, 0
011 E	F, 0	C, 0
111 F	G, 0	D, 0
101 G	A, 1	A, 1

(We have an extra state $\rightarrow 001$ taking it as don't care.)

		y_3y_2	y_1x	00	01	11	10	J_1
		00	0	1	X	X		
		01	0	1	X	X		
		11	0	1	X	X		
		10	0	0	X	X		

		y_3y_2	y_1x	00	01	11	10	K_1
		00	X	X	X	X		
		01	X	X	1	0		
		11	X	X	1	0		
		10	X	X	1	1		

$$J_1 = y_3'x + y_2x \text{ or left term}$$

$$K_1 = x + y_2'$$

		y_3y_2	y_1x	00	01	11	10	J_2
		00	1	X	X			
		01	X	X	X	X		
		11	X	X	X	X		
		10	0	0	0	0		

		y_3y_2	y_1x	00	01	11	10	K_2
		00	X	X	X	X		
		01	0	0	0	0		
		11	1	1	1	1		
		10	X	X	X	X		

$$J_2 = y_3'$$

$$K_2 = y_3$$

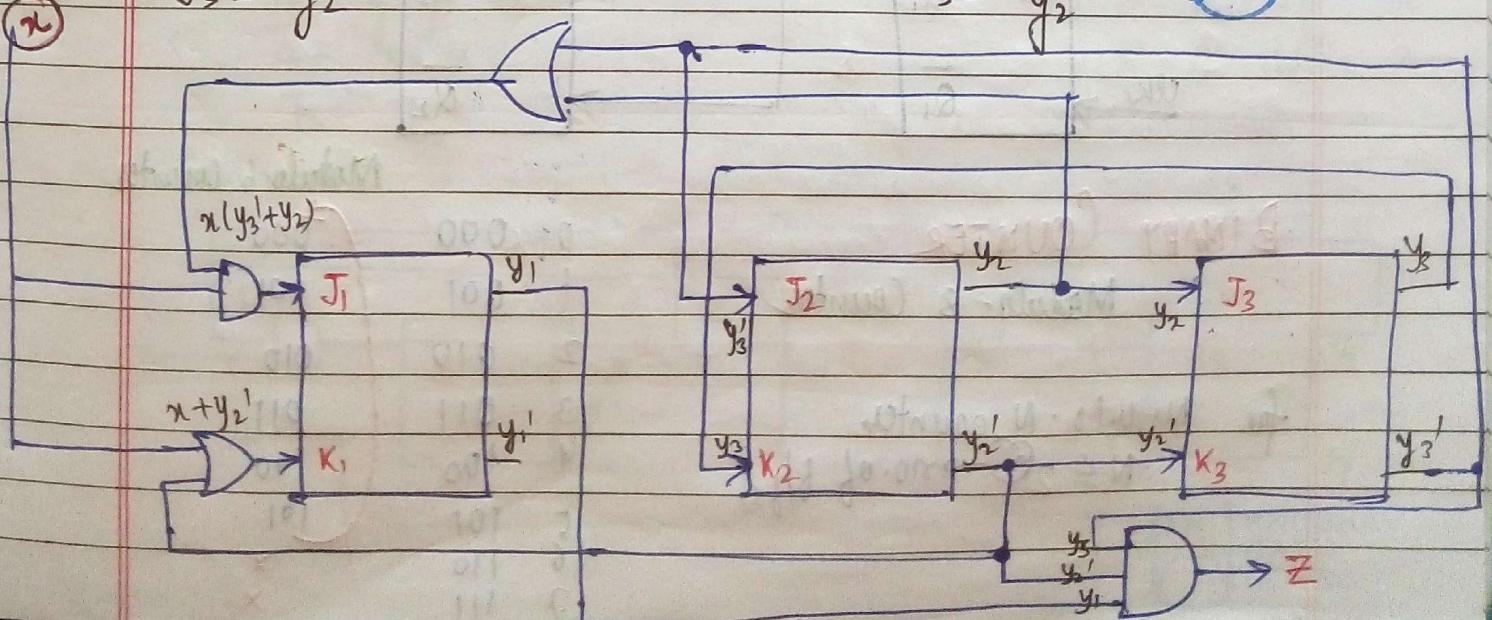
		y_3y_2	y_1x	00	01	11	10	J_3
		00	0	0	X	X		
		01	1	1	1			
		11	X	X	X	X		
		10	X	X	X	X		

		y_3y_2	y_1x	00	01	11	10	K_3
		00	*	X	X	X		
		01	X	X	X	X		
		11	0	0	0	0		
		10	1	1	1	1		

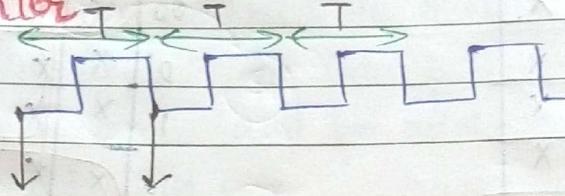
$$J_3 = y_2$$

$$K_3 = y_2'$$

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JAIN
NOTES



Counter



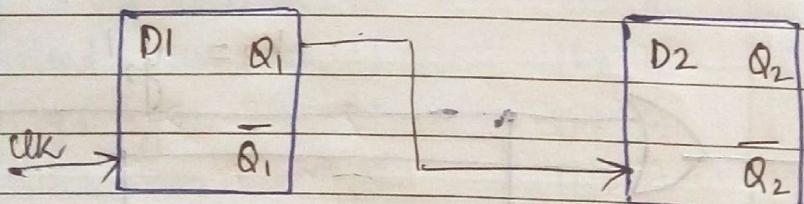
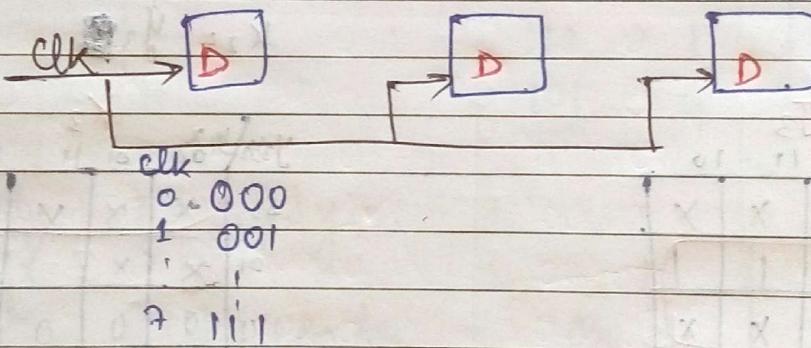
Combination of ffs and the no. of state (of ff) changes gives the count of clock pulse.

State of the ffs will directly give you the count value

Synchronous
Asynchronous

Counters

All ffs are triggered with same clock time



BINARY COUNTER

Modulo-8 Counter

for Modulo-N counter

$N \leq 2^k \rightarrow$ no. of ffs

0 000

1 001

2 010

3 011

4 100

5 101

6 110

7 111

Modulo-6 Counter

000

001

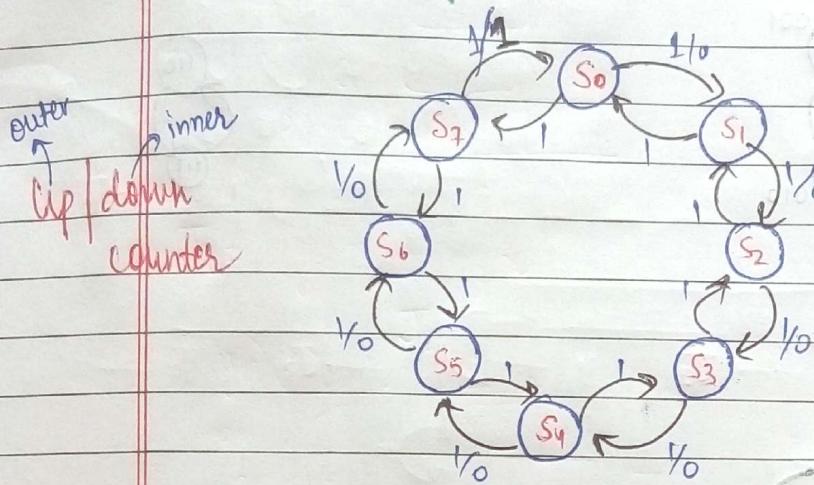
010

011

100

101

State Diagram



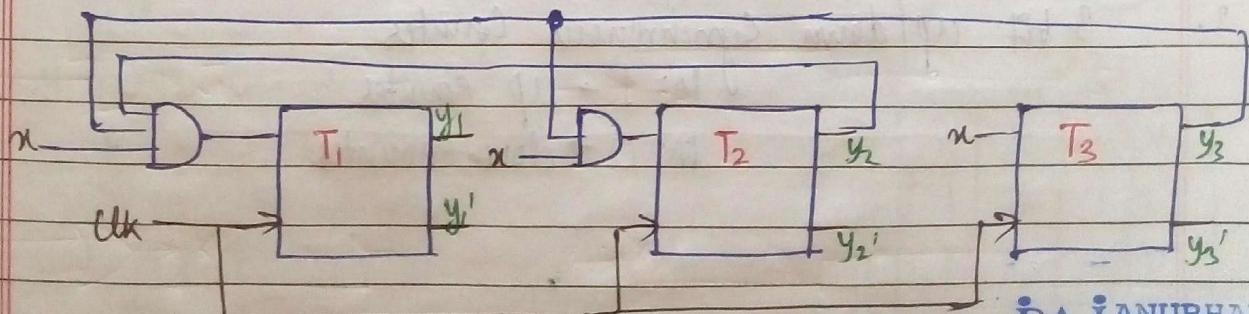
PS $y_1 \ y_2 \ y_3$	NS, Z	T
000	001, 0	0 001
001	010, 0	0 101
010	011, 0	0 001
011	100, 0	1 011
100	101, 0	0 100
101	110, 0	0 011
110	111, 0	0 01
111	000, 1	1 11

$$T_1 = y_2 y_3 \cdot x$$

$$T_2 = y_3 \cdot x$$

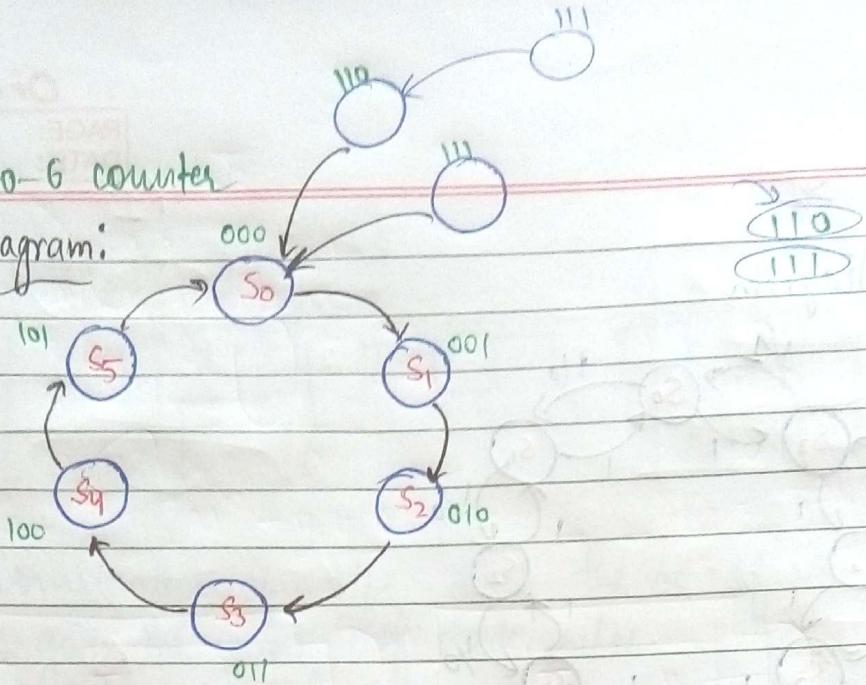
$$T_3 = 1 \cdot x$$

$M=1$ up x
 $M=0$ down \bar{x}



Modulo-6 counter

State Diagram:



* Lock-out state.

PS	NS, Z $M=1$ (up)	$M=0$ (down)
0 0 0	001, 0	101, 0
0 0 1	010, 0	000, 1 ✓ 100
0 1 0	011, 0	001, 0 . 010
0 1 1	100, 0	010, 0 . 110
1 0 0	101, 0	011, 0 001
1 0 1	000, 1 ✓	100, 0 . 101
1 1 0	000, 0	000, 0 . 011
1 1 1	000, 0	000, 0 . 111

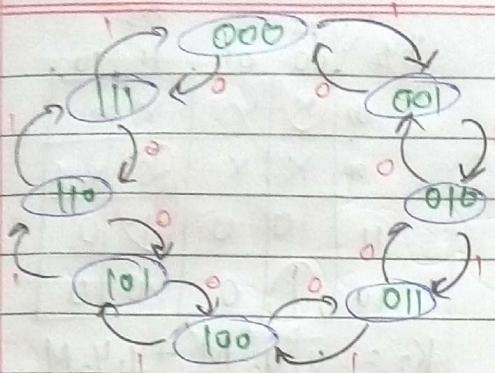
J-K.

15/03/17.

Synchronous Counter

1. Modulo-8 counter (up)
2. Modulo - 6 counter (up/down)
3. 3-bit up/down Synchronous Counter

 $M=1 \rightarrow$ up counter $M=0 \rightarrow$ down counter



PS	Mode	NS.	NS-implementation			
y_3	y_2	y_1	M	$J_3 K_3$	$J_2 K_2$	$J_1 K_1$
0	0	0	0	1 -	1 -	1 -
0	0	0	1	0 -	0 -	1 -
0	0	1	0	0 -	0 -	-1
0	0	1	1	0 -	1 -	-1
0	1	0	0	0 -	-1	1 -
0	1	0	1	0 -	-0	1 -
0	1	1	0	0 -	-0	-1
0	1	1	1	1 -	-1	-1
1	0	0	0	0 11	-1	1 -
1	0	0	1	1 01	-0	0 -
1	0	1	0	0 100	-0	0 -
1	0	1	1	1 10	-0	1 -
1	1	0	0	0 101	-0	-1
1	1	0	1	1 11	-0	-0
1	1	1	0	0 110	-0	-0
1	1	1	1	1 000	-1	-1

Clearly, $J_1 = 1$, $K_1 = 1$ (No need to draw K-map).

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JAIN NOTES

y_3	y_2	y_1	J_2
00	00 01	11 10	
01	X X	X X	
11	X X	X X	
10	1 0	1 0	

$$J_2 = y_1' M' + y_1 M$$

y_3	y_2	y_1	K_2
00	00 01	11 10	
01	X X	X X	
11	X X	X X	
10	1 0	1 0	

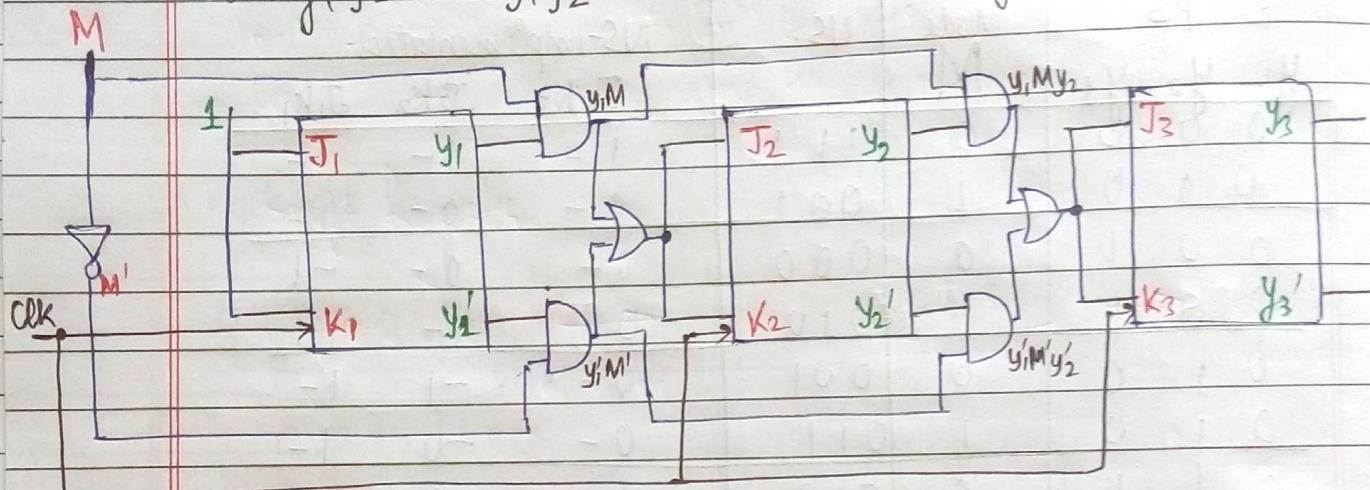
$$K_2 = y_1' M' + y_1 M = J_2$$

	y_3y_2	y_1M	00	01	11	10
00	U	0	0	0		
01	0	0	1	0		
11	X	X	X	X		
10	X	X	X	X		

	y_3y_2	y_1M	00	01	11	10
00	X	X	X	X		
01	X	X	X	X		
11	0	0	1	0		
10	0	0	0	0		

$$J_3 = y_1'y_2'M' + y_1y_2M = J_3$$

$$K_2 = y_1'y_2'M' + y_1y_2M = J_3$$



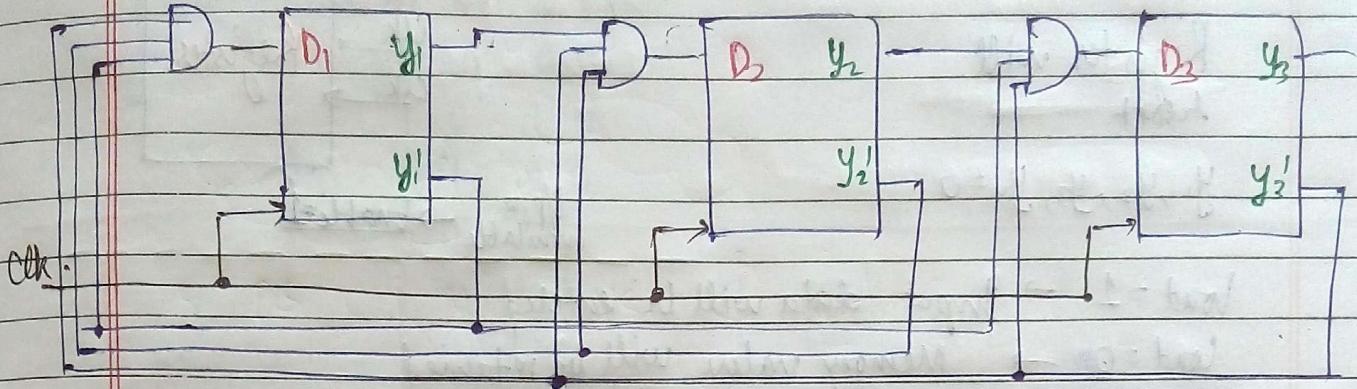
1. Design a synchronous ckt with D f/f's that goes through 0, 1, 2, 4, 0. All undesired states must go to 0-state with 1 clock pulse.

PS	NS	D-excitation		
		D ₃	D ₂	D ₁
000	000	001	0	0 1
000	010	010	1	0 0
000	100	100	0	0 0
000	000	000	0	0 0
000	000	000	0	0 0
000	000	000	0	0 0
000	000	000	0	0 0
000	000	000	0	0 0

$$D_1 = y_1'y_2'y_3'$$

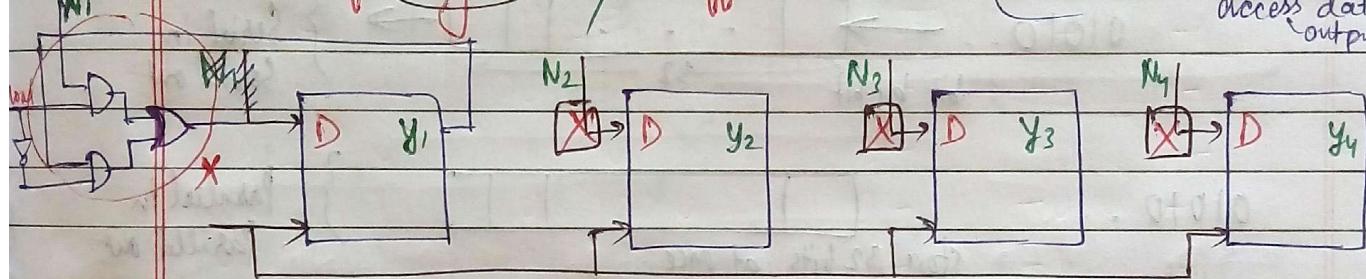
$$D_2 = y_1y_2'y_3'$$

$$D_3 = y_1'y_2y_3'$$

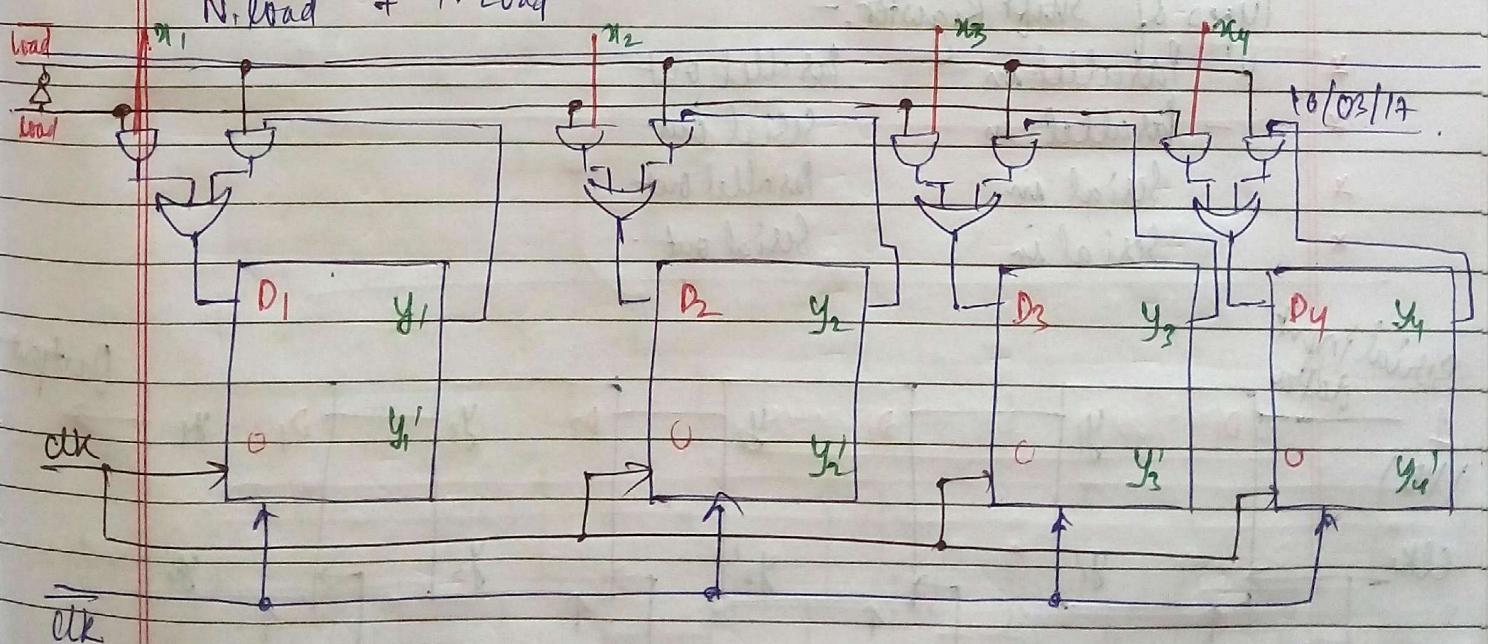


Shift Register / Buffer

(Either I store data or access data output.)



N. Load + Y. Load



$$D_i = N_i \cdot \text{Load} + Y_i \cdot \overline{\text{Load}}$$

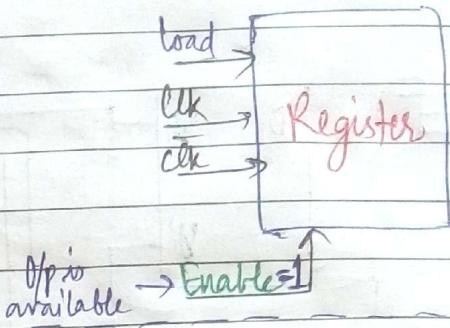
4 bit Register (buffer)

BAJ ANUBHAU JAIN NOTES

if CLR = 0

Register will
reset

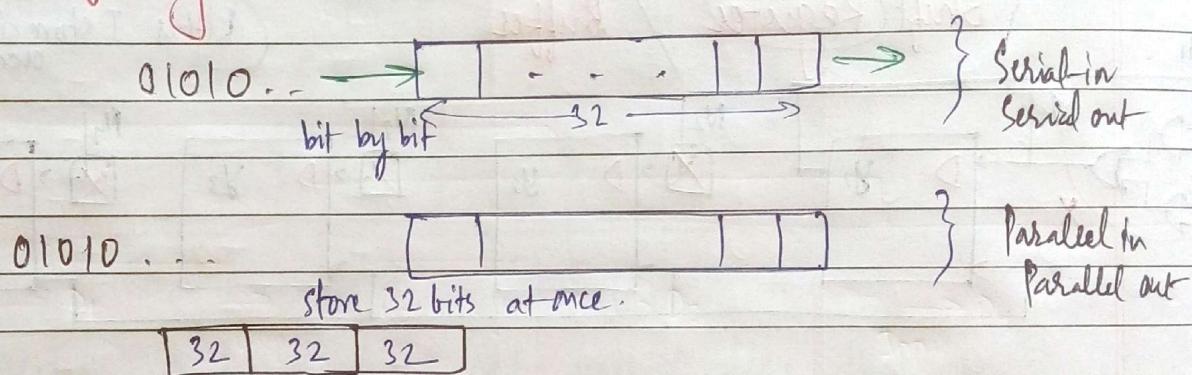
$$Y_1, Y_2, Y_3, Y_4 = 0$$



Load = 1 → Input data will be entered

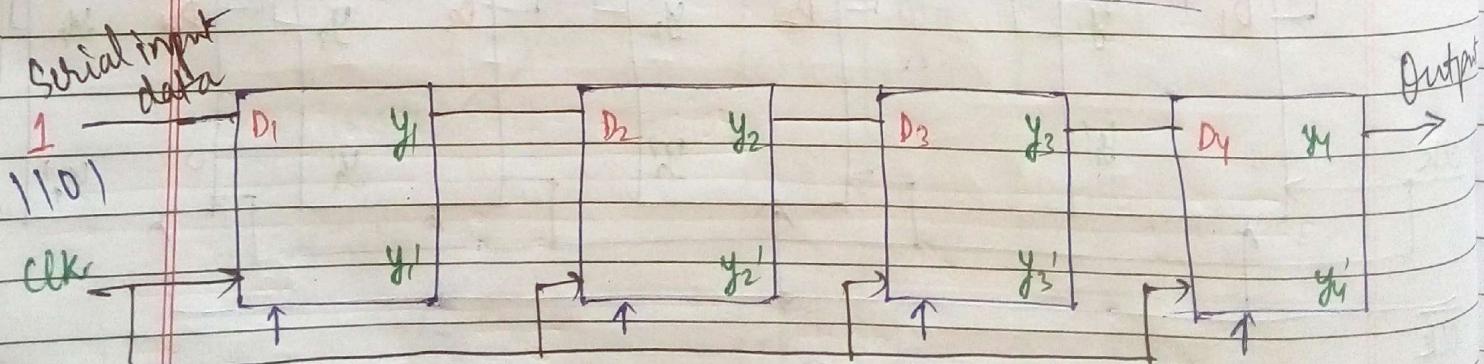
Load = 0 → Memory value will be retained

Shift Register



Types of Shift Register:-

- * Parallel-in Parallel-out
- * - Parallel-in Serial out
- * Serial in Parallel out
- * Serial in Serial out



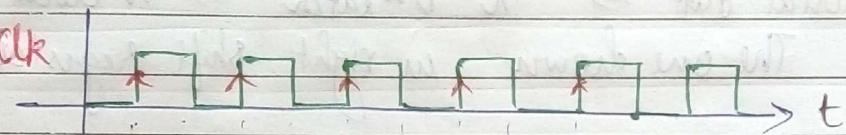
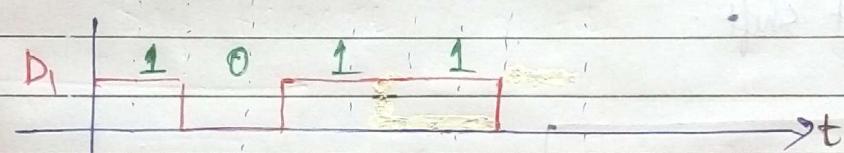
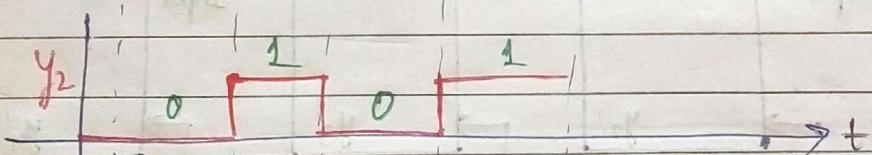
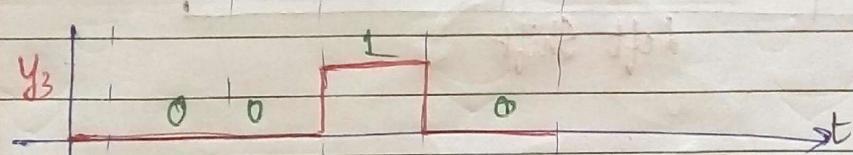
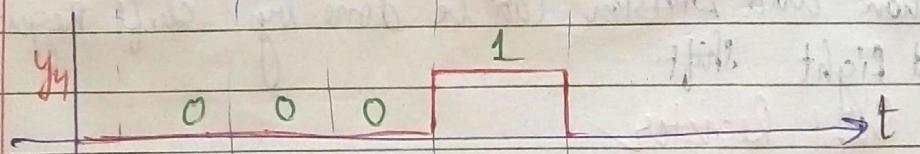
Serial in-Serial out

data-1101

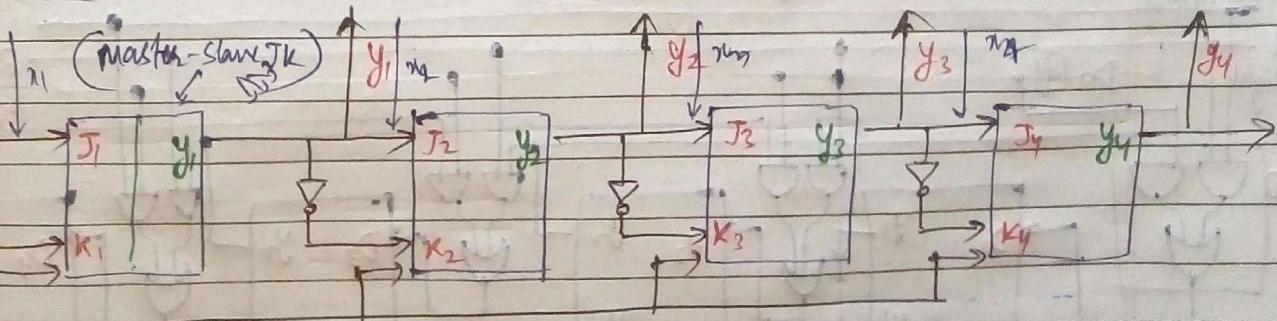
data. (register value)

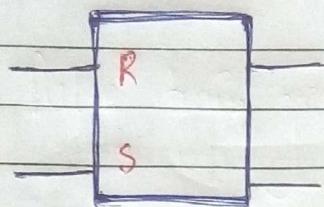
Clock.	D ₁	D ₂	D ₃	D ₄
0	0	0	0	0
1	1	0	0	0
2	0	1	0	0
3	1	0	1	0
4	1	1	0	1

Clk

D₁y₁y₂y₃y₄

data input





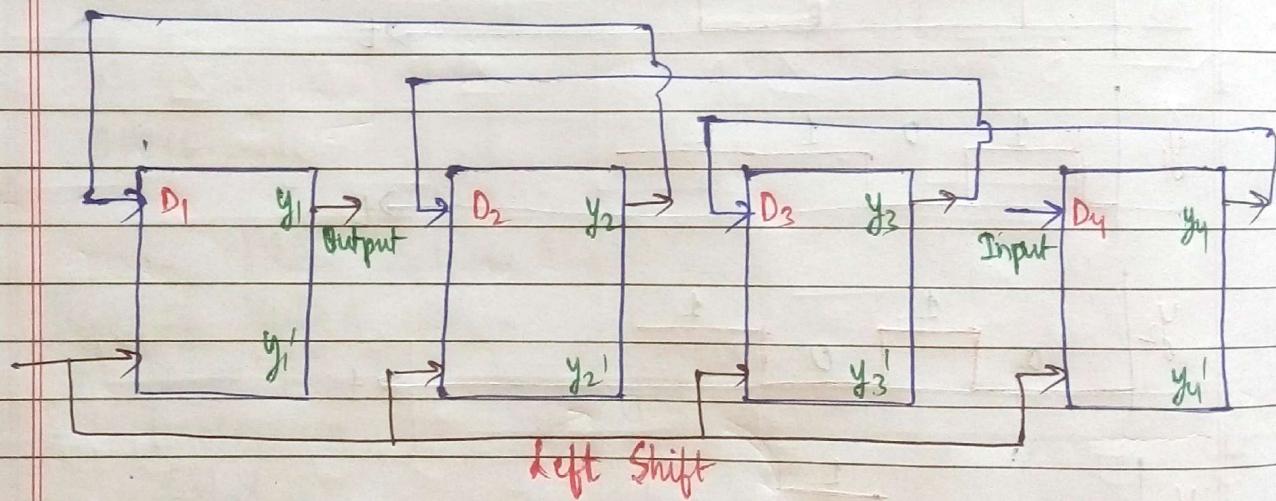
Serial in parallel out
parallel in parallel out

|| Input will never appear

17/3/17

Serial in Serial out \rightarrow n D-latch \rightarrow n clock pulses required. The one shown is right shift register.

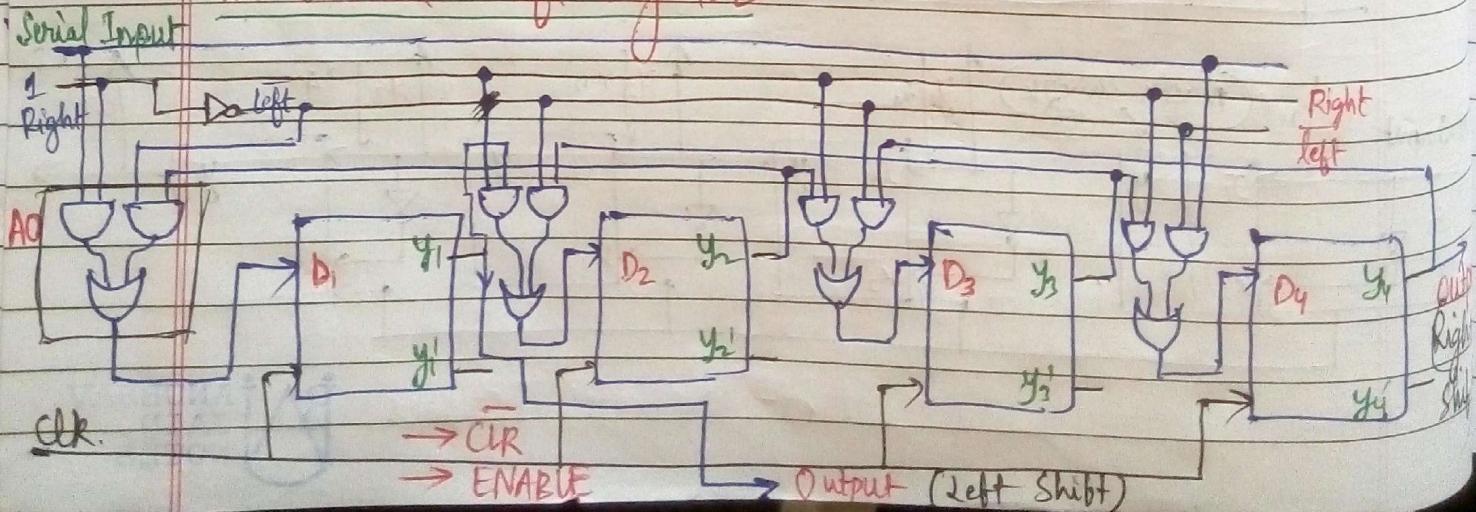
Make it left shift.

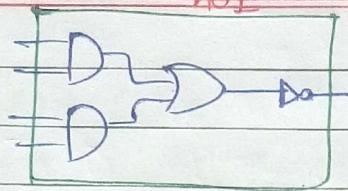


1010

Multiplication and Division can be done by shift Register.
Left and Right shift.

Bi-directional Shift Register



AOI

AOI And OR inverter

OR And inverter.

can directly be used.

Counters

1. Ring Counter

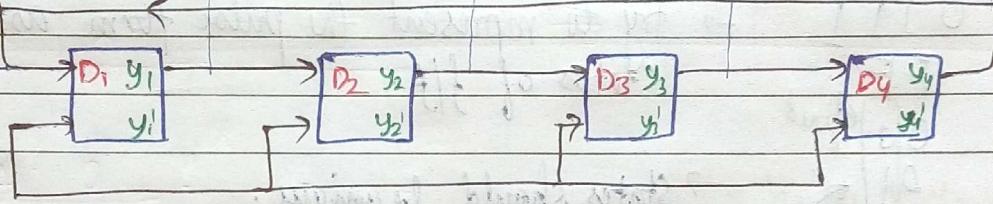
2. Johnson Counter

AND, OR,
NAND, NOR

logic

(switch)

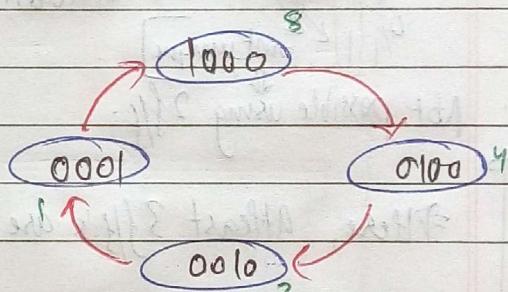
①



Clk

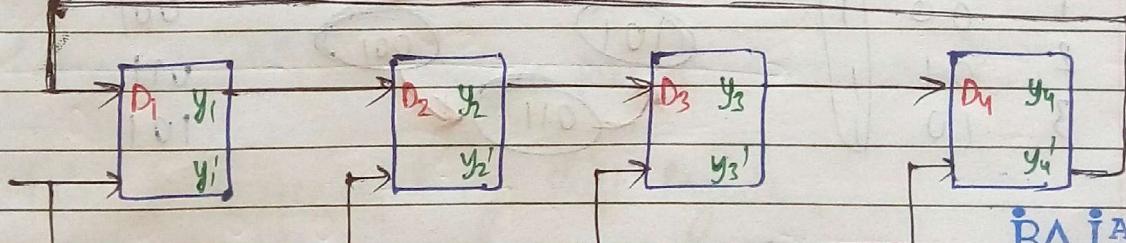
	D ₁	D ₂	D ₃	D ₄
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1

desired states → 1, 2, 4, 8



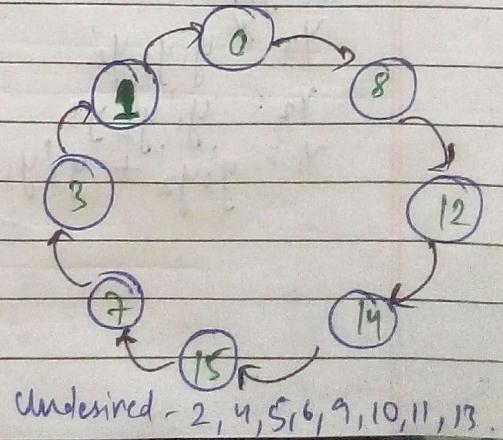
③, 5, 6, 7 → undesired states.

②

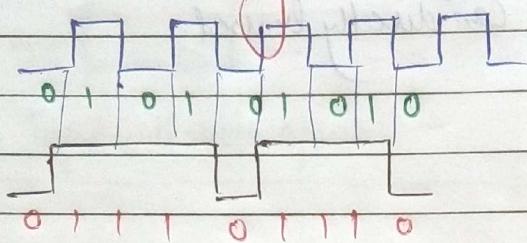


BAJANUBHAV JAIN NOTES

Clk.	D ₁	D ₂	D ₃	D ₄
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1
5	0	1	1	1
6	0	0	1	1
7	0	0	0	0



Pulse-train generator



0111 → try to represent the pulse form as the outputs or states of f/f.

FF
Qd fixed

01

→ States should be unique.

11

→ determine the no. of f/f's required.

01

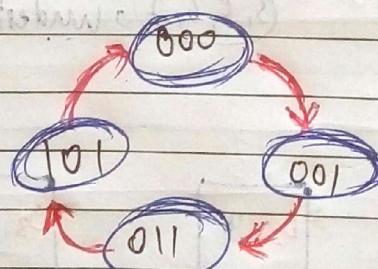
not unique

Not possible using 2 f/f.

Direct logic

∴ At least 3 f/f's are required.

clk	F	F	F
0	0	0	0
1	0	0	1
2	0	1	1
3	1	0	1



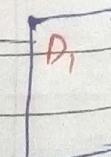
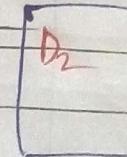
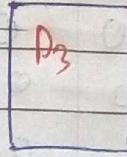
PS	NS
y ₃ y ₂ y ₁ 0 0 0	D ₃ D ₂ D ₁ 0 0 1
0 0 1	0 1 1
0 1 1	1 0 1
1 0 1	0 0 0

D f/f

$$y_3 = y_1 y_2 y_3$$

$$y_2 = y_1' y_2 y_3$$

$$y_1 = y_3 y_2' + y_2' y_1$$



Indirect logic

Use counter (Modulo)

pulse train - 01101 01101

$$N=5 \Rightarrow n=3$$

$$N < 2^n$$

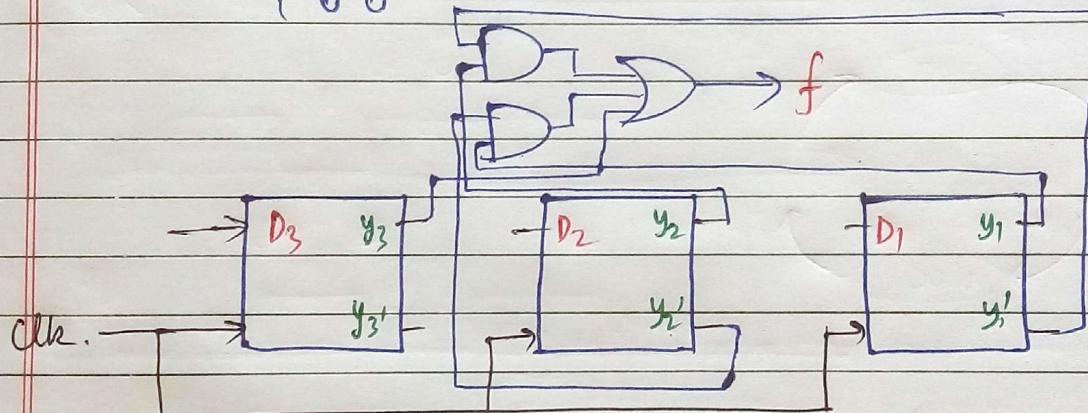
Modulo (N) - counter.

Modulo - 5 Counter

y_3	y_2	y_1	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1

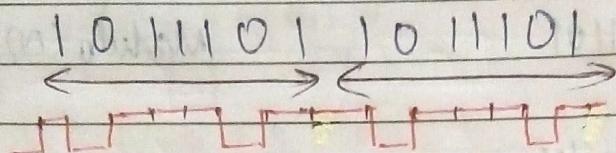
y_3	y_2	y_1	00	01	11	10
0	0	0	0	1	0	0
1	1	x	1	x	x	x

$$f = y_3 + y_2'y_1 + y_2y_1'$$

(Assuming $\text{Modulo } 5$ counter is present somewhere).

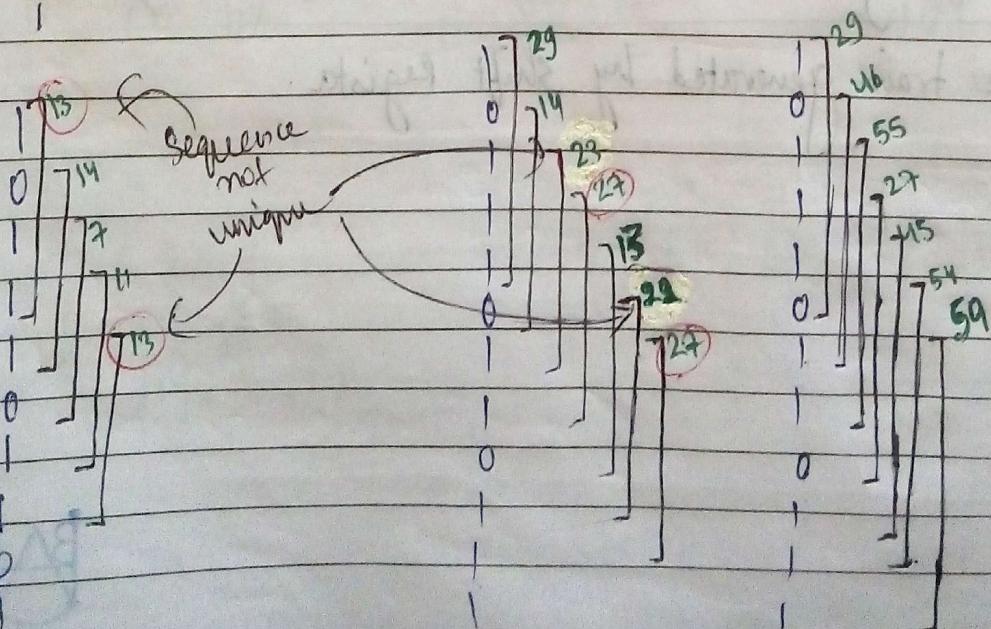
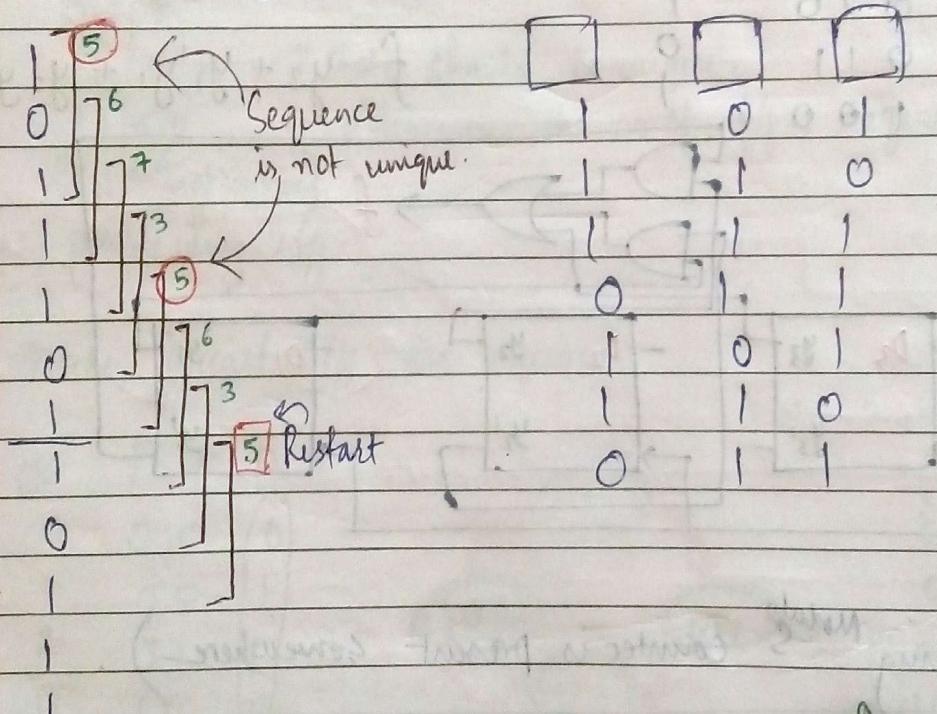
Pulse train generated by shift Register.

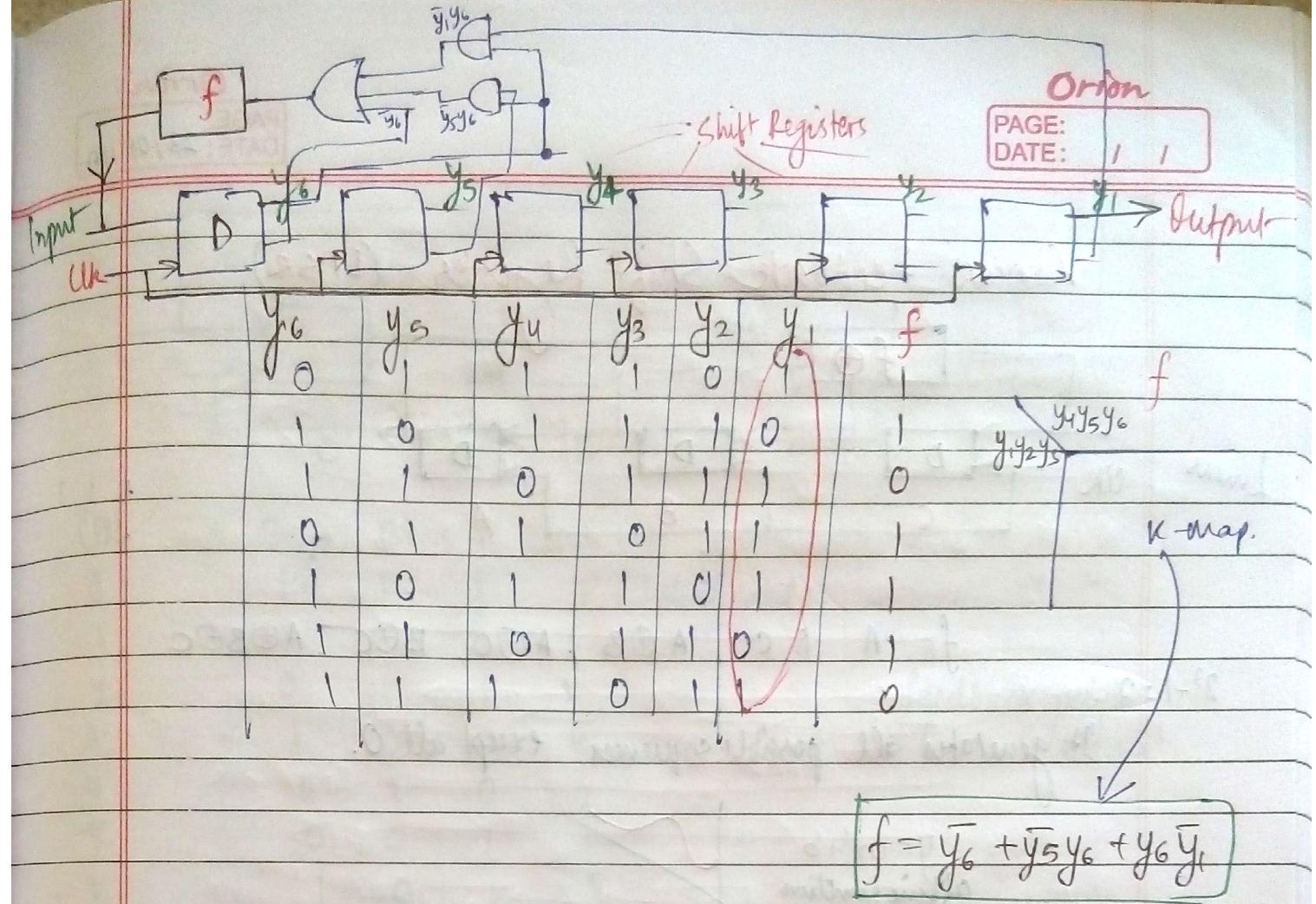
Pulse train generated using Shift Register



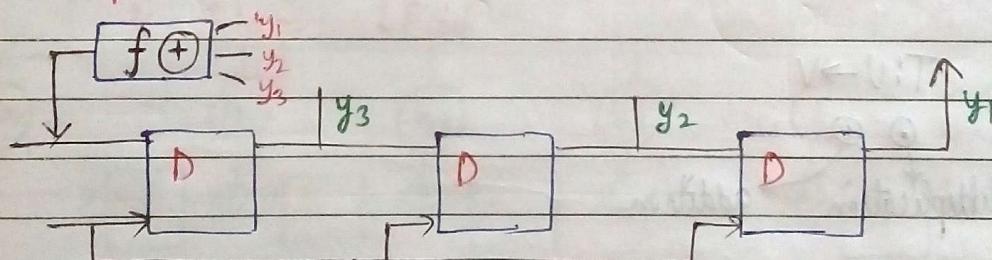
(V) How many shift Registers?

1. direct logic - synthesis of sequential cks.
 2. indirect logic - counter + addition logic.





Linear Sequence Generator



C	B	A	$A \oplus B$
1	1	1	0
0	1	1	0
0	0	1	1
1	0	0	0

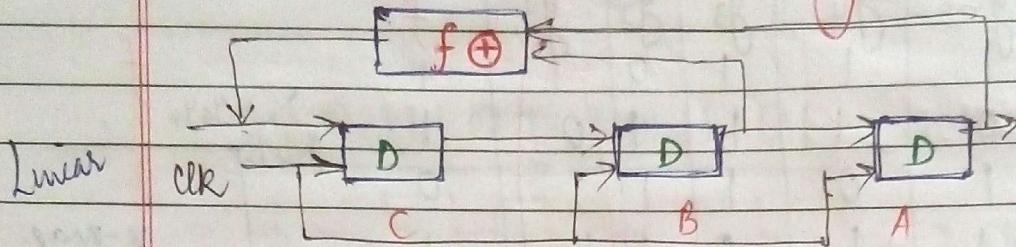
C	B	A	$A \oplus C$
1	1	1	0
0	1	1	1
1	0	1	0
0	0	1	1

C	B	A
0	0	0
0	0	1
0	1	0
1	0	1
1	0	0
1	1	0

Repeat.

Repeat

Linear Feedback Shift Register (LFSR)



$$f = A \oplus B \oplus C, A \oplus B, A \oplus C, B \oplus C, A \oplus B \oplus C$$

$$2^3 - 1 = 7$$

It generates all possible sequences except all 0.

$$y = ax + b$$

affine function

$$F, GF(P), GF(2), 0, 1$$

$$T: U \rightarrow V$$

\odot \oplus
multiplication \rightarrow addition

$$A \odot u_1 \oplus B \odot u_2 = A \odot T(u_1) \oplus B \odot T(u_2)$$

A, B elements of F

$$A, B \in \{0, 1\}$$

$f \rightarrow \oplus$ Linear function

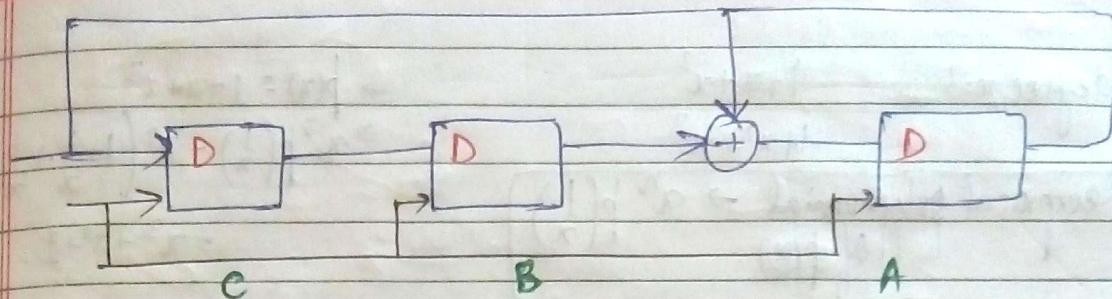
\oplus - modulo 2 addition XOR

\oplus - binary addition (with carry)

$$f = \text{SOP, POS}$$

AND, OR, NOT

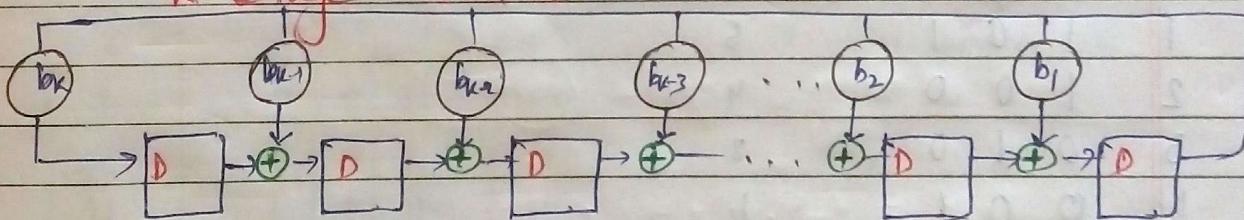
✓ XOR - {AND, OR, NOT}



	C	B	A	
0	1	1	1	7 (Initial seed)
1	1	1	0	6
2	0	1	1	3
3	1	0	0	4
4	0	1	0	2
5	0	0	1	1
6	1	0	1	5
7	1	1	1	7

feedback polynomial
 $f(x) = 1 + x^2 + x^3$.

K-Stage LFSR.



$b_j = 1$, presence of feedback
 $b_j = 0$, absence of feedback.

feedback polynomial of LFSR $p(x) = x^K + b_1x^{K-1} + b_2x^{K-2} + \dots + b_{K-1}x + b_K$.

$$p(x) = 1 + \dots + x^K \quad (b_K=1)$$

K-stage LFSR

ANUBHAV
JAIN
NOTES

Primitive polynomial \rightarrow irreducible, if the polynomial $f(x)$ divides (x^{P^m-1})
 \hookrightarrow generate all elements.

degree, m=3

$$1+x+x^3$$

$$1+x^2+x^3$$

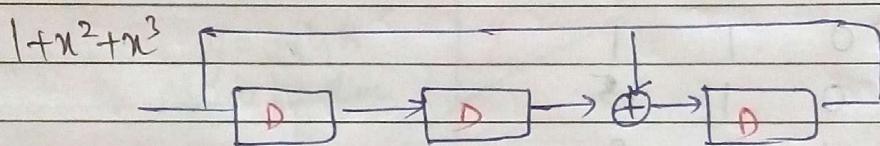
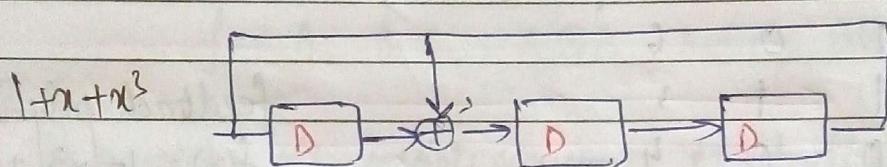
$$p(x) = 1+x+x^3$$

$$\Rightarrow x^3 p\left(\frac{1}{x}\right) = x^3 \left(1 + \frac{1}{x} + \frac{1}{x^3}\right)$$

$$= x^3 + x^2 + 1.$$

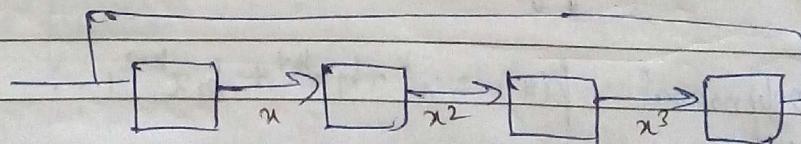
(Reciprocal polynomial $\rightarrow x^n p\left(\frac{1}{x}\right)$)

If $p(x)$ is primitive then reciprocal of $p(x)$ is also primitive.



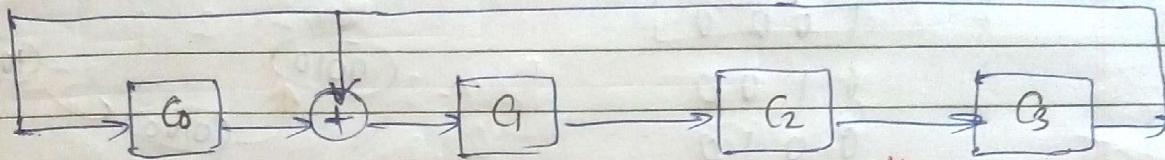
Clk. C B A

0	1	1	1	(IS) ?
1	1	0	1	5
2	1	0	0	4
3	0	1	0	2
4	0	0	1	1
5	1	1	0	6
6	0	1	1	3
7	1	1	1	



$$p(x) = 1+x^4.$$

LFSR LINEAR FEEDBACK SHIFT REGISTER.



feedback polynomial $p(x) = 1 + x + x^4$.

(degree 4 polynomial.)

4-stage LFSR

clk	C ₀	C ₁	C ₂	C ₃	
0	1	0	1	0	10
1	0	1	0	1	5
2	1	1	1	0	14
3	0	1	1	1	7
4	1	1	1	1	15
5	1	0	1	1	11
6	1	0	0	1	9
7	1	0	0	0	8 ✓
8	0	1	0	0	4
9	0	0	1	0	2
10	0	0	0	1	1
11	1	1	0	0	12
12	0	1	1	0	6
13	0	0	1	0	3
14	1	1	0	1	13
15	1	0	1	0	✓ 10

Pseudo Random Pattern generator.
 (Deterministic)

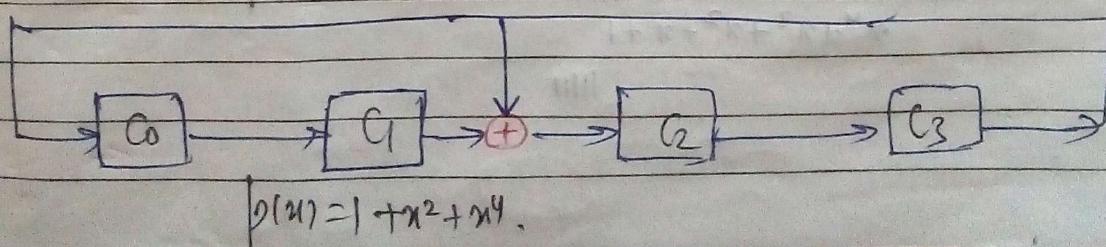
Sequence generated satisfies randomness properties.

1. Balanced

2. Correlation is less

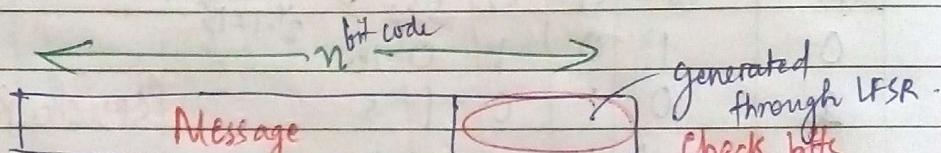
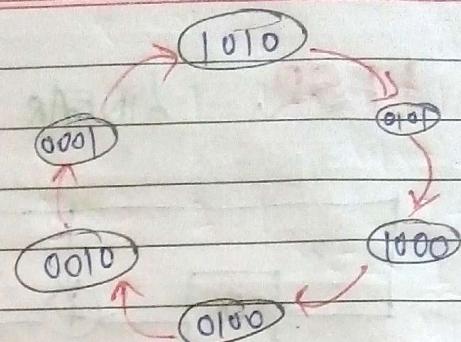
ANUBHAV
 JAIN
 NOTES

n stage LFSR with primitive polynomial will generate $2^n - 1$ sequences. (max. length sequence).



$$p(x) = 1 + x^2 + x^4$$

clock	C_0	C_1	C_2	C_3
0	1	0	1	0
1	0	1	0	1
2	1	0	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1
6	1	0	1	0



Message - $\begin{smallmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{smallmatrix}$

$$m(x) = 1 \cdot x^0 + 0 \cdot x^1 + 1 \cdot x^2 + \dots + 1 \cdot x^2 + x^3 + x^7 + x^9.$$

$$p(x) = 1 + x^2 + x^4 + x^5 \quad (110101)$$

$$\frac{m(x)}{p(x)} = q(x) + \frac{r(x)}{p(x)}$$

$$\text{Code} = x^5 \cdot m(x) + r(x) \\ (x^{14} + x^{12} + x^8 + x^7 + x^5) + \underbrace{\dots}_{\text{FCS}}$$

VAHINI
INIA
TEST

$$\begin{array}{r} x^5 + x^4 + x^2 + 1 \\ \times x^9 + x^7 + x^3 + x^2 + x^0 \\ \hline x^9 + x^8 + x^6 + x^4 \\ + x^8 + x^7 + x^6 + x^4 + x^3 + x^2 + 1 \\ \hline x^8 + x^7 + x^5 + x^3 \end{array}$$

Modulo 2 addition \equiv subtraction

$$\begin{array}{r} x^6 + x^5 + x^4 + x^2 + 1 \\ - x^6 + x^5 + x^3 + x \\ \hline x^4 + x^3 + x^2 + x + 1 \end{array}$$

1111

$m(x)$ = dividend
 $p(x)$ = divisor.

CRC → Cyclic Redundancy Code

PRP → Pseudo Random Pattern generator.

Orion

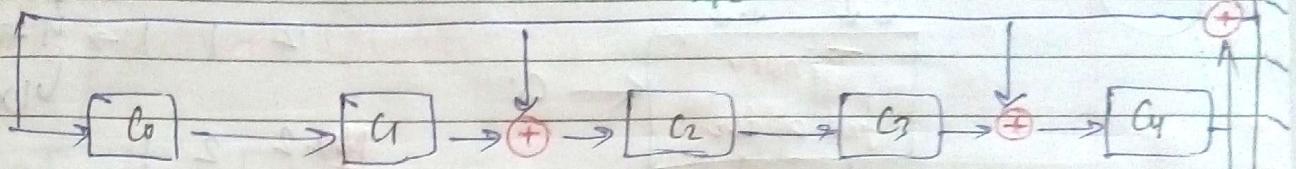
PAGE:

DATE:

1 / 1

$C_4 \oplus I$

I



$$1+x^2+x^4+x^5 \leftarrow \text{Remainder}$$

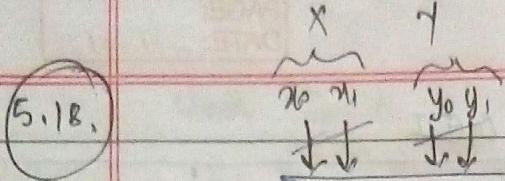
Input

	$C_{1,1}$	$C_{3,1}$	$C_{2,1}$	$C_{1,2}$	$C_{3,2}$	$C_{2,2}$	$C_{0,1}$	$C_{4} \oplus G \oplus I$	$G \oplus G \oplus I$	$G \oplus I$	$I(\text{input})$
0	0	0	0	0	0	1	1	1	1	1	1
1	1	0	1	0	1	1	1	1	1	0	1
2	1	1	1	1	1	1	1	0	1	1	0
3	1	1	1	1	0	0	0	1	0	0	0
4	0	1	0	0	1	1	0	0	0	0	0
5	1	0	0	0	1	0	0	0	1	0	0
6	1	0	0	0	1	0	0	0	0	1	0
7	0	0	0	1	0	0	0	1	0	1	0
8	1	0	0	0	1	1	0	1	1	0	0
9	0	1	0	1	0	0	0	1	0	0	1

$$m^k \cdot m(u) + r(u)$$

$$\begin{array}{r}
 x^5 + x^4 + x^2 + 1 \\
 \times x^{14} + x^{12} + x^8 + x^7 + x^5 \\
 \hline
 x^{19} + x^{18} + x^6 + x^4 + x^2 + x \\
 + x^{13} + x^{11} + x^9 \\
 \hline
 x^{15} + x^{12} + x^{11} + x^9 + x^8 + x^7 + x^5 \\
 + x^{13} + x^{10} + x^8 \\
 \hline
 x^{11} + x^{10} + x^9 + x^7 + x^5 \\
 x^{11} + x^{10} + x^8 + x^6 \\
 \hline
 x^9 + x^8 + x^7 + x^6 + x^5 \\
 x^9 + x^8 + x^6 + x^4 \\
 \hline
 x^7 + x^5 + x^4 \\
 x^7 + x^6 + x^4 + x^2 \\
 \hline
 x^6 + x^5 + x^2 \\
 x^6 + x^5 + x^3 + 1 \\
 \hline
 x^3 + x^2 + 1
 \end{array}$$

1101010110



Ternary
full adder

C_i

$\frac{X}{3}$

* Carry cannot become 2.
Quotient

C_0

S_0

Base-3

0, 1, 2

X Y Ci S
2 1 1 → 4 → 1 1

Max. 2 2 1

$1 \times 3^1 + 1 \times 3^0$

$3+1=4$

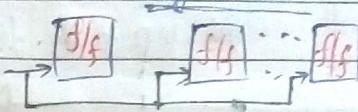
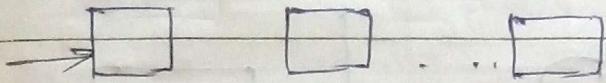
$\frac{5}{3} \rightarrow 1 \frac{2}{3}$
Max. Max.

$$(2)_3 + (2)_3 + (1)_3 = (12)_3$$

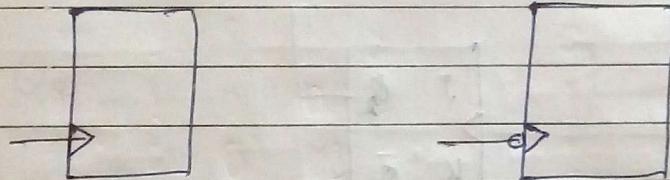
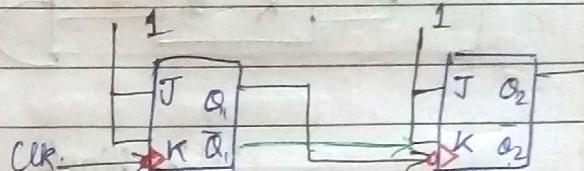
Reminder

C_i	$x_{m_1 m_0}$	$y_{y_1 y_0}$	C_0	$S_1 S_0$
0 0	0 0	0 0	0 0	0 0
0 0	0 1	1 0	0 1	
0 0	1 0	2 0	1 0	
0 1	0 0	1 0	0 1	
0 0 1	0 1	2 0	1 0	
0 0 1	1 0	3 1	0 0	
1 0	0 0	0 0	0 0	1 0
1 0	0 1	3 1	0 0	
1 0	1 0	4 1	0 1	
0 0	0 0	1 0	0 1	
0 0	0 1	2 0	1 0	
0 0	1 0	3 1	0 0	
0 1	0 0	2 0	1 0	
1 0 1	0 1	3 1	0 0	
0 1	1 0	4 1	0 1	
1 0	0 0	3 1	0 0	
1 0	0 1	4 1	0 1	
1 0	1 0	5 1	1 0	

Asynchronous Counter



0 00
1 01
2 10
3 11
00



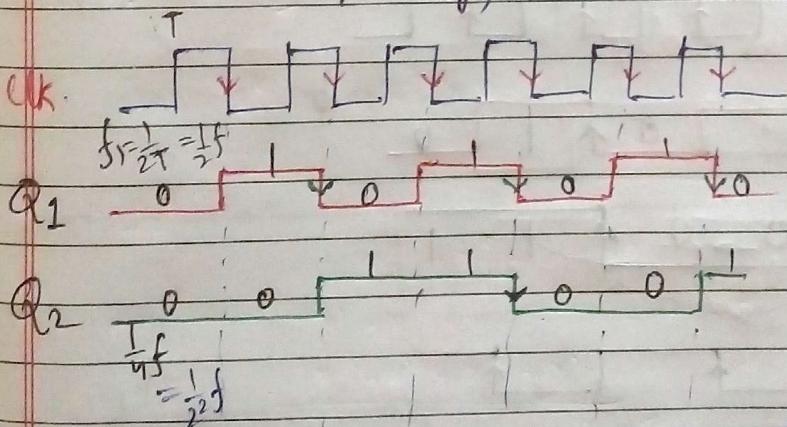
Positive edge triggered.

Negative Edge of the clock.

Ripple Counter

(Asynchronous counter,
Previous output feeds the current input).

initially, it is reset $Q_1 = 0, Q_2 = 0$



Up counter.

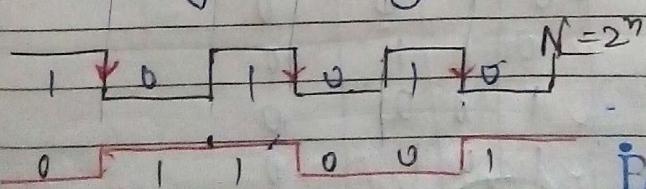
Q_2	Q_1
0	0
0	1
1	0
1	1

Asynchronous design is usually simpler than synchronous design.

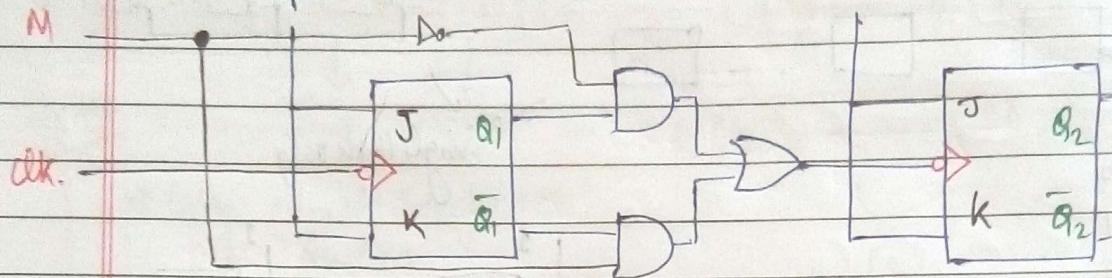
But: Problems:-

* Propagation delay. (If \bar{Q}_1 is connected to K_2)

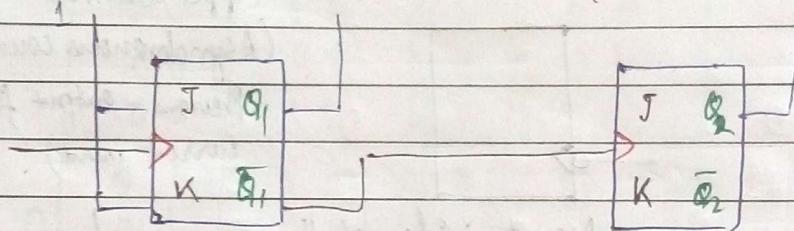
Down counter:-



Mode M_1^{up}
 M_2^{down}
 $M_1^{\text{up}} + M_2^{\text{down}}$



2-bit Ripple up/down counter.



positive-edge clock - 2bit up counter

clk.

Q_1

Q_2

\bar{Q}_1

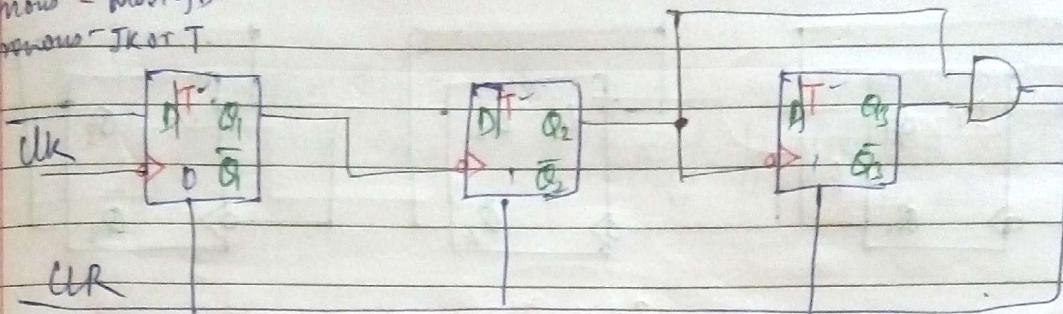
Modulo-6 counter:

$$Q_3 Q_2 \bar{Q}_1 + Q_3 \bar{Q}_2 Q_1$$

Q_3	Q_2	Q_1	CIR
✓ 0	0	0	0
✓ 0	0	1	0
✓ 0	1	0	0
✓ 0	1	1	0
✓ 1	0	0	0
✓ 1	0	1	0
✓ 1	1	0	1
✓ 0	0	0	111-X

VAHESLA
T
SETON

Synchronous - Mostly D
Asynchronous - JK or T



clk

 Q_1 Q_2 Q_3

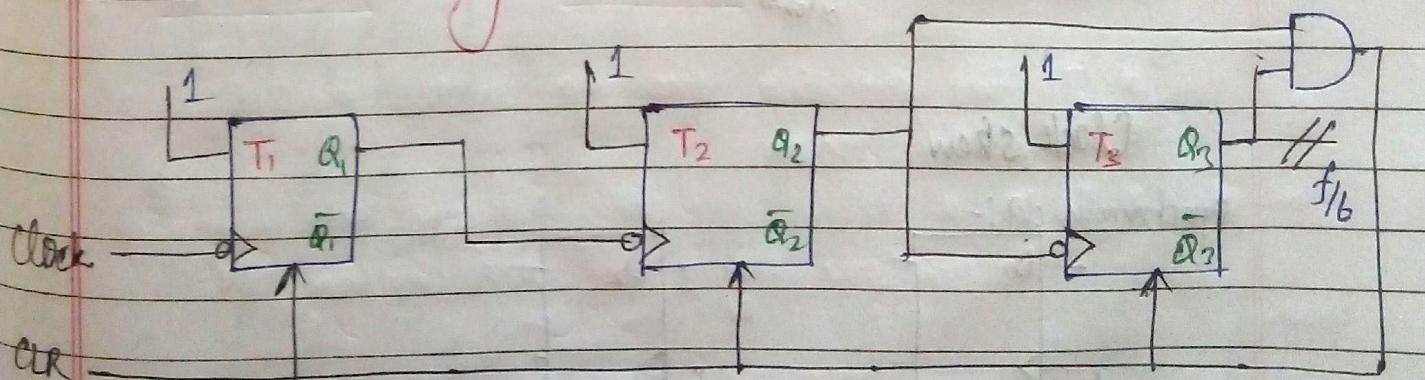
$$Q_1 = Q_2 = Q_3 = 0$$

t = propagation delay of 2-input AND.

$$f \rightarrow f/6$$

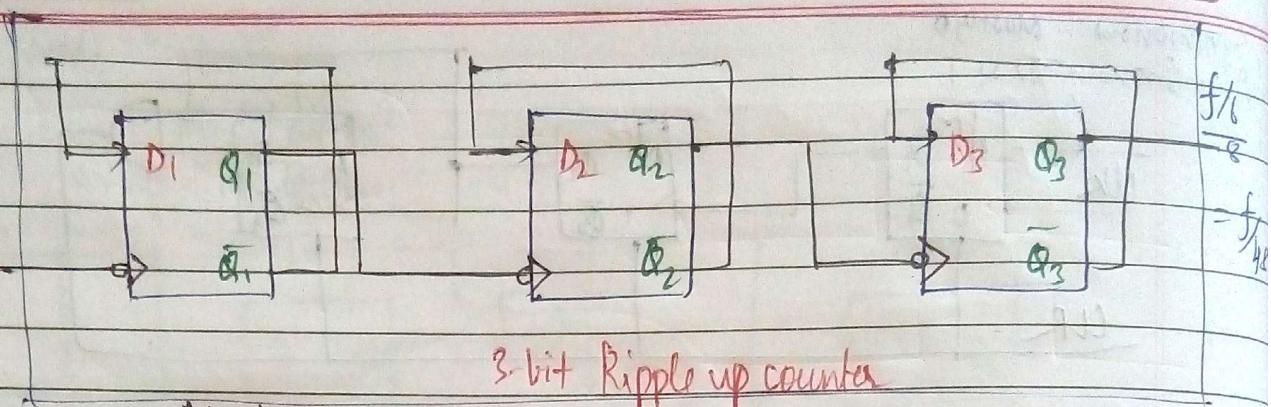
30/03/17

Asynchronous Counter

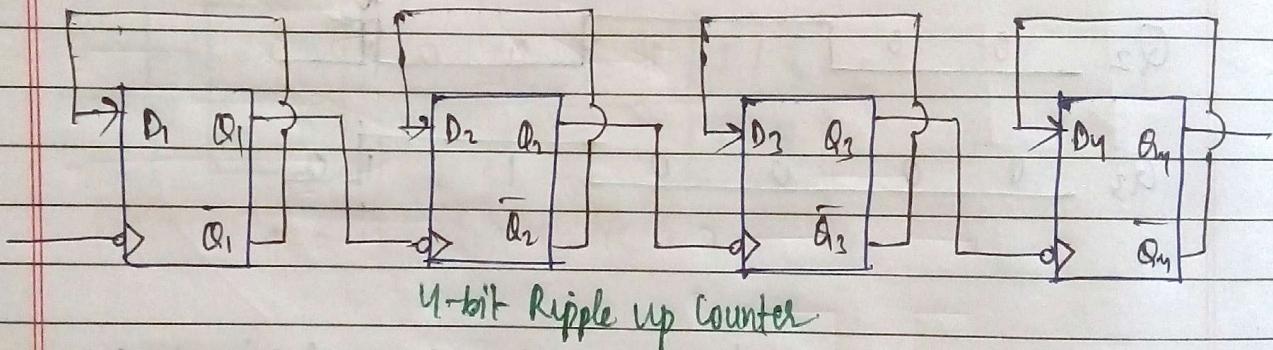


Modulo-6 ($T = f/6$)

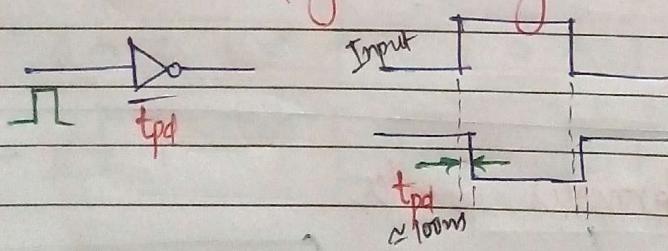
translated
from
page
no. 11
of
f/6



Cascading Ripple Counter
Modulo M — Modulo N = Modulo MN Counter.

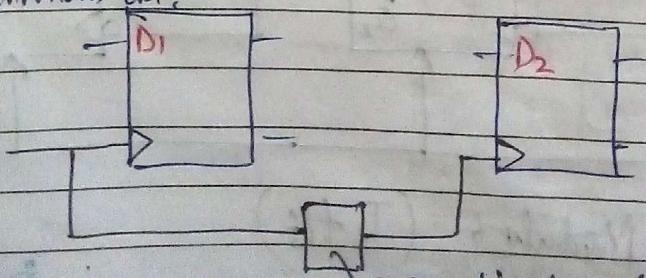


Effect of Propagation Delay



Clock skew

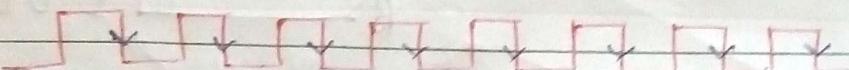
Synchronous ckt:-



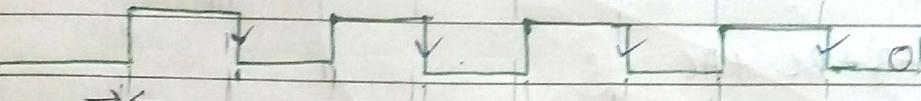
combinational \rightarrow Propagation delay

In asynchronous ckts, even if there is no combinational ch, there is a process time delay through flipflops.

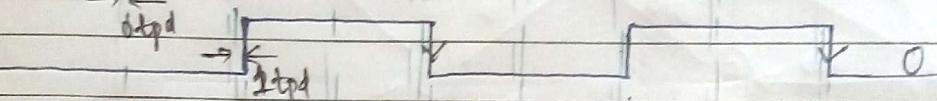
Clock



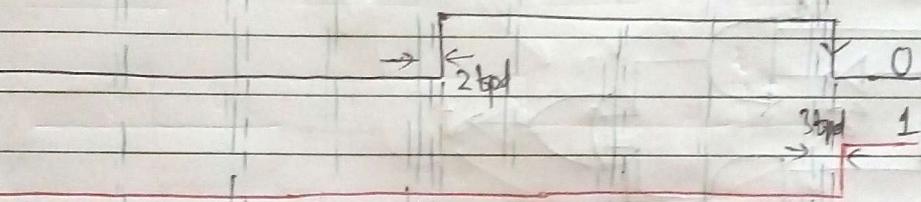
Q_1 .



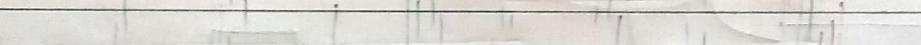
Q_2 .



Q_3 .



Q_4 .

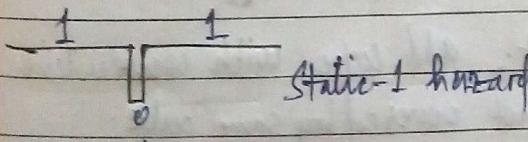


0 1 1 1
1 0 0 0
1 0 0 0 1

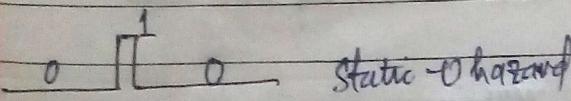
if $(3 t_{fpd} > T_{clk})$

Hazard.

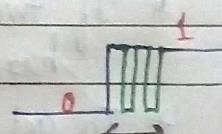
- 0 hazard
- 1 hazard



Static-1 hazard



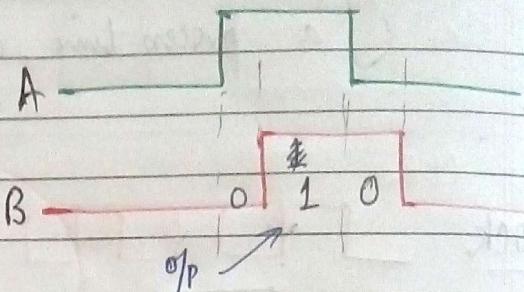
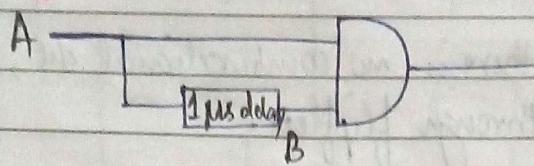
Static-0 hazard



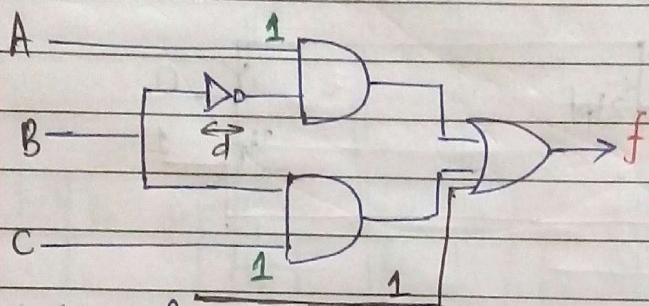
transient

dynamical hazard

Combinational Ckt.

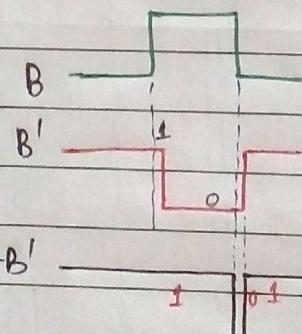


$$f = AB' + BC + AC$$

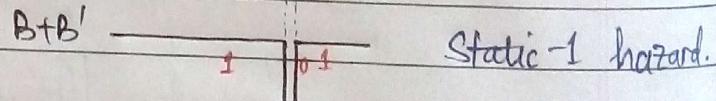


$$f = B + B' = 1, \text{ if } A=1, C=1$$

if static 1 is added,
no hazard.



	A	B	C	f
BC	00	01	10	0
	01	10	11	1
AC	11	10	11	1



* If two consecutive ones are covered by 2 diff. cubes then hazard occurs.

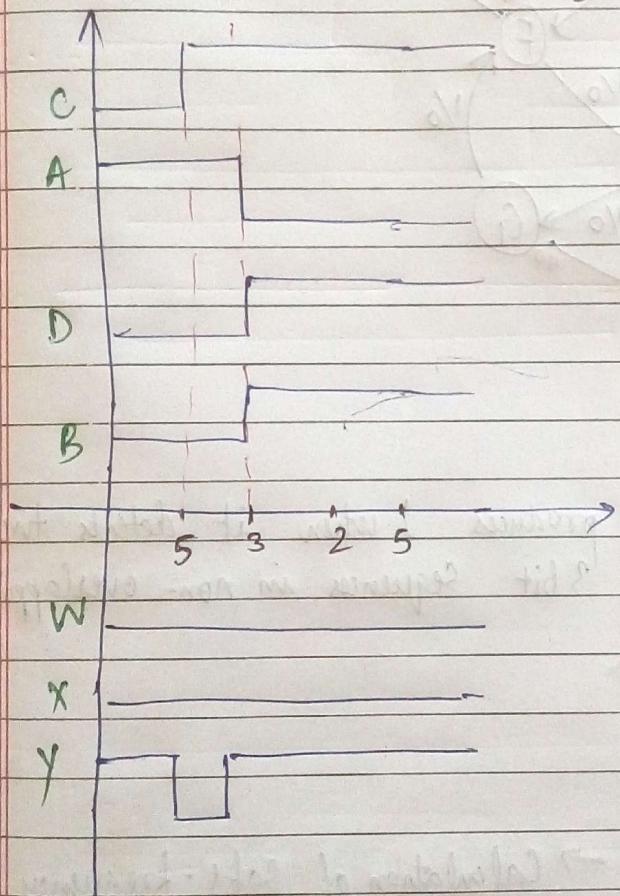
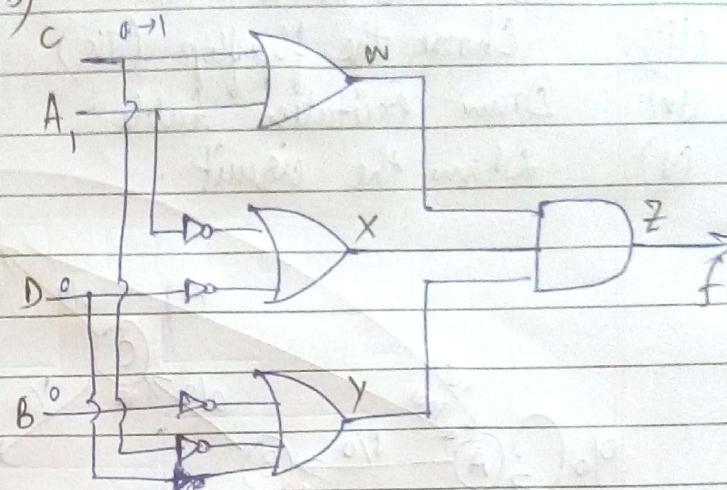
* To get a hazard free ckt; add cube with consecutive 1's which were earlier not covered by same cube.
or

try to change mapping.

Product of Sums

$$\text{Ex: } (A+C) \cdot (A'+D') \cdot (B'+C'+D)$$

CD \ AB	00 01	11 10
00	0	0
01	0	0
11	0	0
10	0	0



Assume propagation delay (t_{pd})

\rightarrow 3 μs

\rightarrow 5 μs

$$(C+D') \cdot (A+A'+D) \cdot (A'+B'+C)$$

Hazard free chart

Q

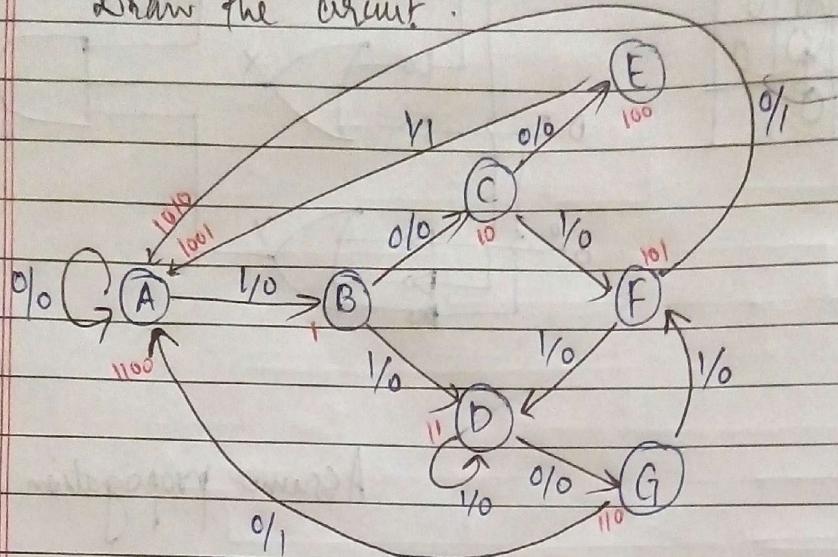
Design a (2 input, 2 output) circuit that produces a output 1 whenever it detects the sequence 1100 or 1010 or 1001.

The circuit goes back to its initial state after giving an output 1.

n no. of 1's (0's) in preceding m sequences $m \geq n$ Kohavi

Input: 0 1 0 1 0 1 1 0 1
Output: 0 0 0 0 0 0 1 1 1

- Steps:-
- (i) Draw the state transition diagram.
 - (ii) Draw state table.
 - (iii) Choose the flip flop (JK)
 - (iv) Draw excitation table.
 - (v) Draw the circuit.



Q. Design a circuit which produces 1 when it detects two 1's or three 1's in the preceding 3 bit sequences in non-overlapping condition.

Q. Asynchronous Counter design \rightarrow Calculation of Safe-frequency.

Q. For what value of propagation delay a 10 bit counter misses a count with the clock frequency of 10MHz.

(10 MHz)

$$f_c < \frac{1}{n t_{pd}}$$

$$t_{pd} = \frac{1}{10 f_c} = \frac{1}{10 \times 10^7} = 10 \text{ ns}$$

VAHATI
2023

Capabilities and Limitations of Sequential Machines

Defn

Sequential Machines: It is a quintuple $\langle I, O, S, \delta, \lambda \rangle$

I : Input

O : Output

S : State

δ : Output function

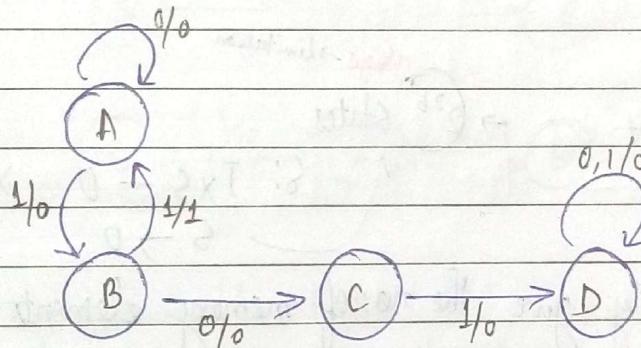
λ : State-transition function

$$\lambda: S(t+1) = \lambda \{ S(t), x(t) \}$$

$I \times S \rightarrow S$, \times -cartesian product

$Z(t) = \delta(S(t))$, Moore M/c

$= \delta(S(t), x(t))$, Mealy M/c



Input: 100, 101

Sink

D: terminal/graveyard state

Source, no other state exists for where the M/c can enter into that state.

Input	Output
00	00
01	00
10	00
11	01
000	000
001	000
010	000
011	001
100	-
110	010
111	010

Input: 1 1 1 1 1 1 1 ...

1 $\frac{1 \times 2}{2}$ ← $K(K+1)/2^{\text{th}}$ 1 at I/P \Rightarrow 1 at O/P

3 $\frac{2 \times 3}{2}$
6 $\frac{3 \times 6}{2}$
10 $\frac{5 \times 10}{2}$

Output: 1 0 1 0 0 1 0 0 0 1 ...

It is difficult to design M/c.

Serial Adder

Carry - 0/1



Serial Multiplier

b - input

b - input

2b-bit output

Huge \rightarrow Limitation
 $\rightarrow 2^{2b}$ States

$S: I \times S = 0 \quad X$
 $S \rightarrow 0$

No. of state directly give the no. of memory elements.

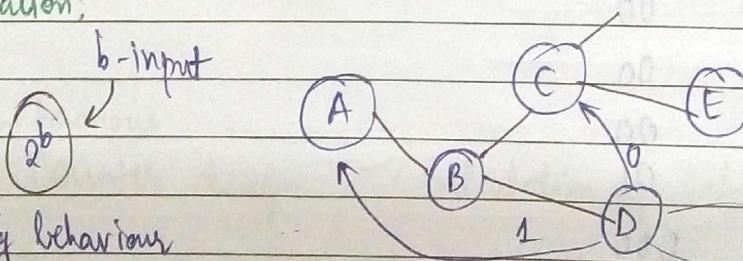
Minimization.

Limitation is due to the no. of memory elements.

\rightarrow Whether we can reduce the no. of memory elements (f/f_s).

Observation:

There exist redundancy which eventually results the repeating behaviour of the m/c.



Testing & verification of ckt.

Redundancy is not wanted.

Minimization of Sequential ckt.

State Minimization

s_i, s_j are two states of M

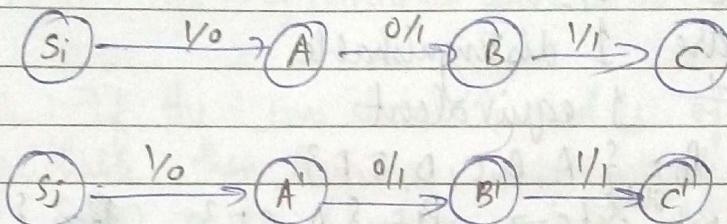
If we apply the same input sequence X to s_i and s_j , then their behaviour is same i.e. they produce same output sequences.

$$s_i = s_j, \quad s_j = s_k$$

$$\Rightarrow s_i = s_k$$

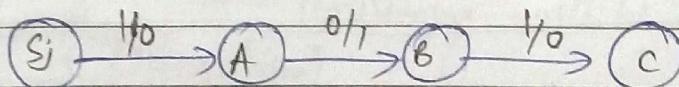
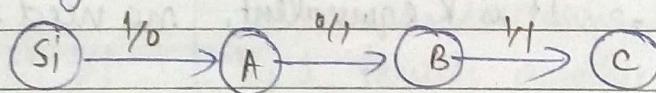
$M_{\text{redundant}} \rightarrow M_1$
 M/ϵ with redundant states $\rightarrow M_1$ without redundant states.

$X = 101$



k -distinguishable states

If s_i and s_j are the state of a machine M , if for a minimum value of K applying the k -sequences show a different output sequence then s_i and s_j are k -distinguishable.



$K=3 ; 101$

Outputs
 $\begin{cases} s_i : 011 \\ s_j : 010 \end{cases}$

imp 1010

1011

(K=3)

$M - n$ states ; max^m input sequence $\Rightarrow n-1$
 (to check)

PS

NS, Z

	$a=0$	$a=1$
A	E, 0	D, 1
B	F, 0	D, 0
C	E, 0	B, 1
D	F, 0	B, 0
E	C, 0	F, 1
F	B, 0	C, 0

1-sequence 0, 1

A, B are 1 distinguishable

A, C $\xrightarrow{0,1}$ 1 equivalent

$$P_0 = \{A, B, C, D, E, F\}$$

1 equivalent sets $\xrightarrow{\text{states}} P_1 = \{A, C, E\}, \{B, D, F\}$

$$P_2 = \{A, C, E\}, \{B, D\}, \{F\}$$

$$P_3 = \{A, C\}, \{E\}, \{B, D\}, \{F\}$$

$$P_4 = \{A, C\}, \{E\}, \{B, D\}, \{F\}$$

$$P_5 = \{A, C\}, \{E\}, \{B, D\}, \{F\}.$$

If $P_k = P_{k+1}$, It is k equivalent, no need to check for $k+2$ or further.

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Theorem 1 : The equivalence partition is unique.

$$P_0 = \{A, B, C, D, E, F\}$$

$$\{A, C\}, \{E\}, \{B, D\}, \{F\}.$$

 $M \rightarrow M_1$ M_1 - minimizedlet P_a and P_b be two partition.There exist atleast two states s_i & s_j whose position in a block is different in P_a & P_b .

$$P_a = \{s_i, s_j, \dots\} \{ \dots \}$$

$$P_b = \{s_i, \dots\} \{s_i, \dots\}$$

Now, say s_i and s_j are in two different blocks in partition P_b , whereas they are in same block in partition P_a .

Since s_i and s_j are in 2 diff. blocks in P_b . So, there exists atleast one input sequence which produces diffⁿ output when applied to s_i and s_j . So, s_i and s_j are not equivalent states.

So, s_i and s_j cannot be in same block of P_a .

Hence, our assumption was wrong. s_i & s_j cannot have diff positions in partitions \Rightarrow Eq. Partition is unique.

Theorem 2: If the two states s_i and s_j of machine M are distinguishable then they are distinguishable by a sequence of length $(n-1)$ or less where n is the no. of states in M.

$$P_0 = \{A, B, C, D, E, F\}$$

$$P_1 = \{A\} \{B, C, D, E, F\}$$

$P_i < P_j$, if each block of P_i is contained in P_j .

$$P_2 < P_1.$$

$P_{k+1} < P_k -$ { P_k is the partition
K is restricted by $(n-1)$ } when input sequence of
length K is applied.

that means increasing length of input sequence
by 1, results another partition.

\rightarrow Partitioning will be stopped when $P_{k+1} = P_k$

$$P_3 = \{A, C, \{B\}, \{E\}\}, \{B, D\}, \{F\}$$

$$\begin{array}{ccccc} \alpha & A & \beta & B & \gamma \\ \alpha & C & \beta & D & \gamma \\ \alpha & \delta & \delta & \epsilon & \delta \\ \alpha = A & \beta = C & \gamma = D & \delta = B & \epsilon = E \end{array} \quad (\text{Isomorphism})$$

$$PS \quad x=0 \quad x=1$$

$$A \quad E, 0 \quad D, 1$$

$$B \quad F, 0 \quad D, 0$$

$$C \quad E, 0 \quad B, 1$$

$$D \quad F, 0 \quad B, 0$$

$$E \quad C, 0 \quad F, 1$$

$$F \quad B, 0 \quad C, 0$$

$$PS \quad x=0 \quad x=1$$

$$\alpha \quad \beta, 0 \quad \gamma, 1$$

$$\beta \quad \alpha, 0 \quad \delta, 1$$

$$\gamma \quad \delta, 0 \quad \gamma, 0$$

$$\delta \quad \gamma, 0 \quad \alpha, 0$$

6 states \rightarrow 4 states.

Complete Machine

Transitions (next states) and the behaviour (output sequence) of machine M is completely specified.

Incomplete Specified

Transitions and/or behaviour of machine M is not specified (defined).

		NS, Z		NS, Z	
PS	$x=0$	$x=1$	PS	$x=0$	$x=1$
A	B, 1	-	A	B, 1	T, 0
B	-, 0	C, 0	B	T, 0	C, 0
C	A, 1	B, 0	C	A, 1	B, 0
			T	T, -	T, -

Incompletely specified

We don't care about behaviour of state T as it is not defined in our machine.
 \Rightarrow It is completely specified.

		NS, Z		NS, Z	
PS	$x=0$	$x=1$	PS	$x=0$	$x=1$
A	C, 1	E, -	a	B, 1	d, 0
B	C, -	E, 1	b	B, 0	d, 1
C	B, 0	A, 1	c	B, 1	y, 1
D	D, 0	E, 1	b	d, 0	d, 1
E	D, 1	A, 0	y	B, 1	a, 0

$$\text{Output}=0 \quad P_1 = \{A, E\} \setminus \{B, C, D\}$$

$$\text{Output}=1 \quad P_1 = \{A, B\} \setminus \{C, D\} \cup \{E\}$$

State Splitting

$$B \rightarrow \{B^1, B^{11}\}$$

$$B^+ = B^1 \\ = B^{11}$$

		NS	
PS	$x=0$	$x=1$	NS
A	A, 0	C, 0	
B	B, 0	B, -	
C	B, 0	A, 1	
A'	A, 0	C, 0	
B'	B, 0	B, -	
B''	B, 0	B, -	
C	B^+, 0	A, 1	

$\{A, B'\}$	$\{B'', C\}$	α	β	d	$d, 0 \quad \beta, 0$
$\{A, B'\}$	$\{B'', C\}$	α	β	d	$d, 0 \quad d, 1$
$\{A, B'\}$	$\{B'', C\}$	α	β	d	$d, 0 \quad \beta, 0$
$\{A, B'\}$	$\{B'', C\}$	α	β	d	$\beta, 0 \quad d, 1$

Example:

PS

Ns

 $x=0 \quad x=1$

A

E, 0

C, 0

Find the partitions and
minimize the machine.

B

C, 0

A, 0

C

B, 0

G, 0

D

G, 0

A, 0

E

F, 1

B, 0

F

E, 0

D, 0

G

D, 0

G, 0

Ns

 $x=0 \quad x=1$

d

Y, 1, S, 0

B

d, 0 E, 0

Y

d, 0 S, 0

S

E, 0 B, 0

E

S, 0 E, 0

$$P_1 = \{A, B, C, D, F, G\} \{E\}$$

$$P_2 = \{A, F\}, \{B, C, D, G\} \{E\}$$

$$P_3 = \{A, F\}, \{B, D\}, \{C, G\}, \{E\}$$

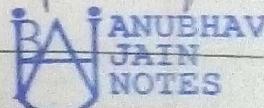
$$P_4 = \{A\}, \{F\}, \{B, D\}, \{C, G\}, \{E\}$$

$$P_5 = \{A\}, \{F\}, \{B, D\}, \{C, G\}, \{E\} = P_6$$

 $\beta \quad Y \quad S \quad E \quad d$ Defⁿ

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Cover / Contain - State S_i of m/c M_1 is said to cover or contain state S_j of $M_1 \cup M_2$, iff the input sequence which is applicable to S_j is also applicable to S_i and when then input sequence is applied to both M_1 and M_2 , with the initial state S_i & S_j respectively they produce same output sequences provided output sequences are specified.



Compatible - Two states S_i & S_j of a m/c M are compatible iff every input sequence applicable to $S_i \& S_j$, the same

Output sequence will be produced whenever both output symbols are specified.

$$\left. \begin{array}{l} \{s_i, s_j\} \\ \{s_j, s_k\} \\ \{s_i, s_k\} \end{array} \right\} \xrightarrow{\alpha} \{s_i, s_j, s_k, s_\ell\}$$

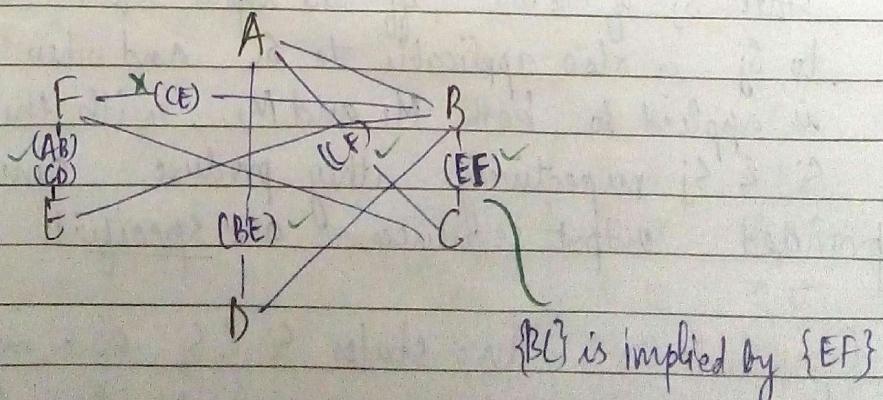
M	PS	NS, Z	I ₁	I ₂	I ₃	I ₄	— input sequences
A	—		C, 1	E, 1	B, 1		
B	E, 0	—	—	—			
C	F, 0	F, 1	—	—			
D	—	—	B, 1	—			
E	—	F, 0	A, 0	D, 1			
F	C, 0	—	B, 0	C, 1			

Merger Graph

It is a graph with n vertices, where n is the no. of states of m/c M

If the states s_i & s_j are not conflicting both next state as well as output then put an undirected arc between s_i and s_j .

If s_p and s_q are not conflicting for output but conflict for next state, put an undirected interrupt arc between s_p & s_q .



$\checkmark AB, CD, BD, CF, BE, EF$
 $\checkmark AD, AC, BC$

To get a maximum compatible state. (to get a complete polygon)

$$\{ABC\} \{ACD\} = \{AB\} \{CD\} \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$\alpha \quad \beta \quad \gamma \quad \delta$

Complete polygon (each node is connected to other node).

$$\{ABC\} \{ACD\} \{BDC\} \{BCF\}$$

$\alpha \quad \beta \quad \gamma \quad \delta$

Maximal compatible states give the bound of minimization.

		NS, Z			
PS		I ₁	I ₂	I ₃	I ₄
{AB}	α	$\gamma, 0$	$\beta, 1$	$\gamma, 1$	$\alpha, 1$
{CD}	β	$\gamma, 0$	$\gamma, 1$	$\alpha, 1$	-
{EF}	γ	$\beta, 0$	$\gamma, 0$	$\alpha, 0$	$\beta, 1$

MERGER TABLE

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PS	I ₁	I ₂	NS, Z
A	E, 0	B, 0	$\{S_i, S_j\}$
B	F, 0	A, 0	S_i
C	E, -	G, 0	S_j
D	F, 1	D, 0	I_k sequence to both S_i & S_j
E	C, 1	G, 0	$S_p - S_q$
F	D, -	B, 0	$\{S_p, S_q\}$

✓: Compatible
✗: Not compatible

B	AF EF			
C	BC	AC EF		
D	✗	✗	CD EF	
E	✗	✗	✓	CF CD
F	DE	AB	BC	DF CD BC

A B C D E

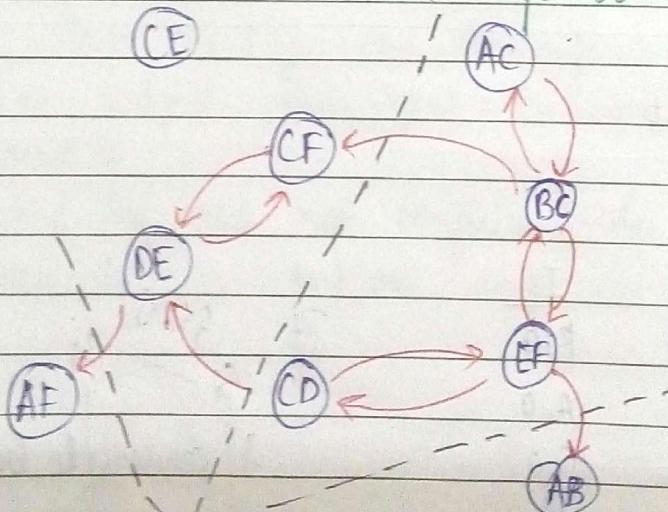
Merge table to compatibility graph.

$$\{EF\} \quad \{DE\} \quad \{CF\} \quad \{CE\} \quad \{CD\} \quad \{BC\} \quad \{AF\} \quad \{AC\} \quad \{AB\}$$

Column

	Compatible sets
E	{EF}
D	{EF} {DE}
C	{CEF} {CDE}
B	{CEF} {CDE} {BC}
A	{CEF} {DE} {ABC} {ACF}

Maximal Compatible sets.



Set of compatible sets of two nodes.

$$\{AC\} \{BC\} \{EF\} \{CD\} + \{AB\} \\ \{ABC\} \{CD\} \{EF\}$$

		NS, Z
PS	I ₁ I ₂	
{ABC}	α	Y _{1,0} $\alpha, 0$
{CD}	β	Y _{1,1} $\beta, 0$

{EF}	γ	B, L $\alpha, 0$
------	----------	------------------

A	000	
B	001	} 2
C	010	
D	011	} 3 Hamming distance.
E	100	

\Rightarrow State Assignment can be changed.