Q1]

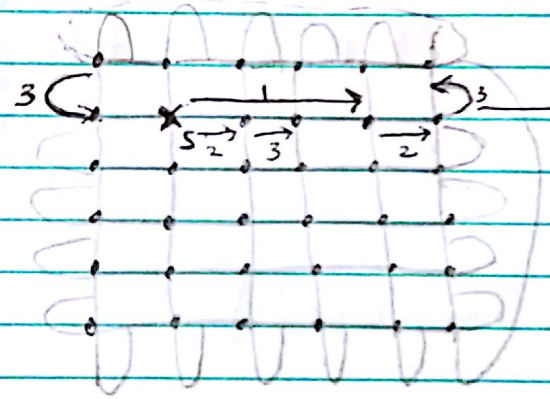
a]

Ans → The formula for calculating the communication time for one to all broadcast in a ring, using cut through routing is →

$$T_c = T_s \log p + T_w m \log p + T_h (p-1)$$

∴ The steps involved in operation of one to all broadcast in a 2-D torus, using cut through routing are →

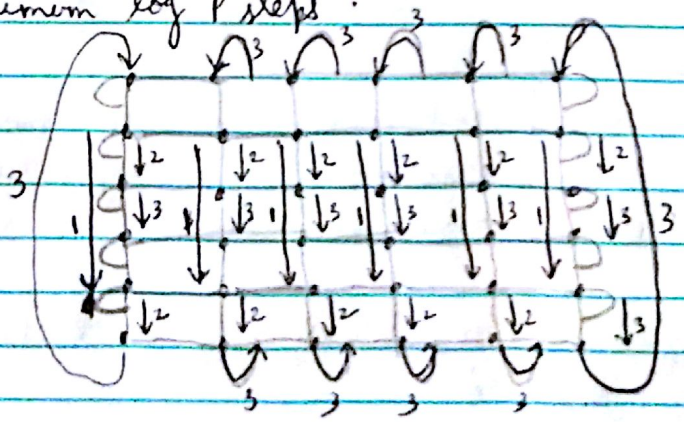
1) Broadcast the message in the row (ring) in which the source element is present.



→ communication (broadcast) will be completed in the entire ring in exactly 3 steps (maximum log p steps).

* The no. above arrow represent the step.

2) Now each element in the 2nd ring will broadcast the message to the elements in their respective columns, again in maximum log p steps.



∴ the cost of communication in 2D torus →

$$T_c = 2\left(T_s \log p + t_w \cdot m \cdot \log p + T_h (p-1)\right)$$

--- here p = no. of processors in one row or column.

here,

$T_s = 15$ micro seconds.
$T_h = 3$ micro seconds
$T_w = 0.1$ micro seconds.
$m = 1000$ bytes.
$p = 6$

$$\therefore T_c = 2\left(15 \times \log 6 + 0.1 \times 1000 \times \log 6 + 3.(6-1)\right)$$

$$= 2\left(15 \times 2.58 + 100 \times 2.58 + 15\right)$$

$$= 2\left(38.7 + 258 + 15\right)$$

$$= 2 \times 311.7.$$

$$= \boxed{623.4 \text{ micro seconds}}$$

b]

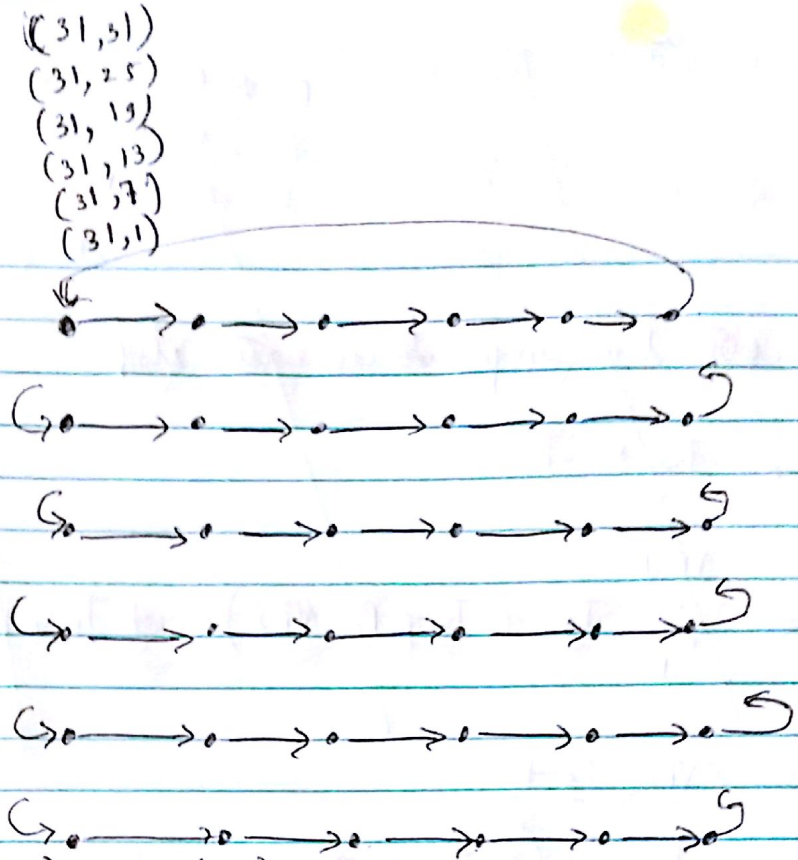
Ans → All to All scatter can be performed in the following steps →

~~1) In the row.~~

1) In all the rows, each node keeps the messages belonging to the nodes in its column, and transfers rest of all messages to other processors in nodes.

no. of steps in distributing message across row = $(\sqrt{p}-1)$

message size = $\sum_{i=1}^{\sqrt{p}-1} \sqrt{p} - \sqrt{i}$



(1,1) (2,2)
(1,7) (2,8)
(1,13) (2,14)
(1,19) (2,20)
(1,25) (2,26)
(1,31) (2,32)

(source, dest).

as we can see, all the processors keep the messages meant for others in some column.
The above process is repeated $(P-1)$ times.

at the end of it, each processor has the processor meant for it, and the messages meant for others in some column.

2) In the second step, the some operation is carried out along the colums. The message size is reduced in following way →

$(\sqrt{P}-1), (\sqrt{P}-2) \cdots 1$

Total cost $= \left(2T_s + T_w \cdot m P + T_h\right)(NP-1)$

$= \left(2 \times 15 + 0.1 \times 1000 \times 36 + 3\right)(6-1)$

$= \left(30 + 3600 + 3\right) 5$

$= \left(3633\right) 5$

$= \boxed{18165 \text{ micro seconds.}}$