ATC-2 Assignment

1DB18IS098
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ISE 5th B

1) Define Regular expression also write regular expression for the following languages ~~over~~ over $\Sigma = \{a, b, c\}$

i) All strings contains exactly one 'a'?

(ii) All strings containing no more than three a's?

(iii) All strings that containing atleast one occurance of each symbol in $\Sigma = \{a, b, c\}$?

→ Regular expression can be formally defined as.

i) ϕ is regular expression denoting an empty langauge

(ii) ε - (Epsilon) is a regular expression indicating the langauges containing empty string.

(iii) a → is regular expression indicating the langauge containing only {a}

(iv) If R is a regular expression denoting the langauge containing only. LR & S is a regular expression denoting the langauge Ls then R+S is a regular expression corresponding to the langauge $L_R \cup L_S$.

i) $\Sigma = \{a, b, c\}$.

exactly one 'a'

a → should be one $b \to b^*$ $c \to c^*$

$\therefore RE = (b+c)^* a (b+c)^*$

(ii) $\Sigma = \{a, b, c\}$

$b \to b^*$ $c \to c^*$ $a \to 3$ a's

$RE = (\varepsilon + a) (\varepsilon + a) \varepsilon + a)$

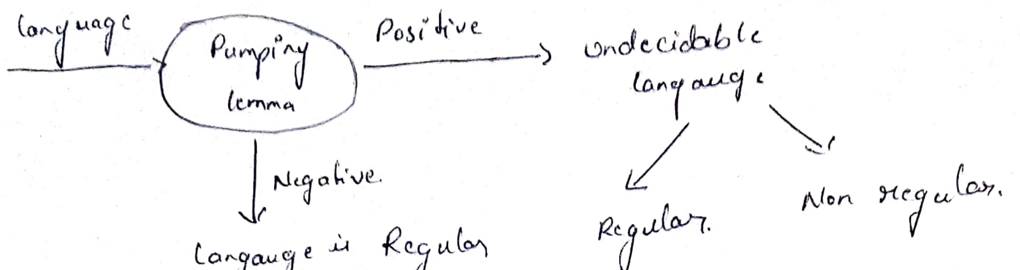
$\therefore RE = (b+c)^* (\varepsilon + a) (b+c)^* (\varepsilon + a) (b+c)^* (\varepsilon + a) (b+c)^*$

(iii) $\Sigma = \{a, b, c\}$

$L = \{a, b, c, aa, bb, cc, abaac, \ldots\} \Rightarrow (a^* + b^* + c^*)$

$\therefore RE = (a+b+c)^*$

(3) State and prove pumping lema for the regular Langauge?



Proof :- If 'L' is a regular Langauge then 'L' has the pumping length 'p' such that any string 's' where $S \geq p$ may be divided into 3 parts. i.e, $S = uvw$ such that the following condition must be true

i) $v \neq \varepsilon$ ie $|v| \geq 1$

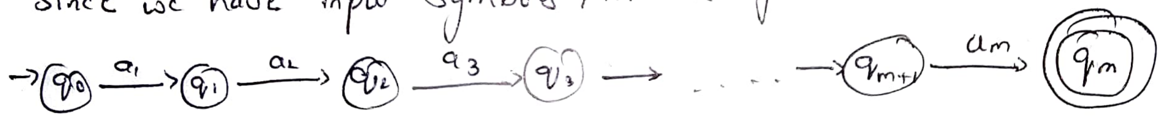
ii) $|uv| \leq n$

iii) then uv^iw is in L for $i \geq 0$

To prove that a langauge is not regular using pumping lemma, follow the steps below, consider $m = (Q, \Sigma, \delta, q_0, F)$ be an FA and if the langauge accepted by the DFA is regular

Let $x = a_1, a_2, a_3 \ldots a_m$ where $m \geq n$ & each a_i in Σ.

Since we have input symbols, m we get



The length of $|x| \geq n$

According to peigon hole principale it is not possible to have distinct transtition from each input, one of the state may have a loop, as shown below. So consider 'x' & divide it into 3 substring

i) First group.
starting prefix from $a_1, a_2, \ldots a_i$
ie $u = a_1, a_2, \ldots a_i$

ii) Second group
Starting loop from
$a_{i+1}, a_{i+2} \ldots a_{j-1}\ a_j$
ie $v = a_{i+1}\ a_{j+2}$

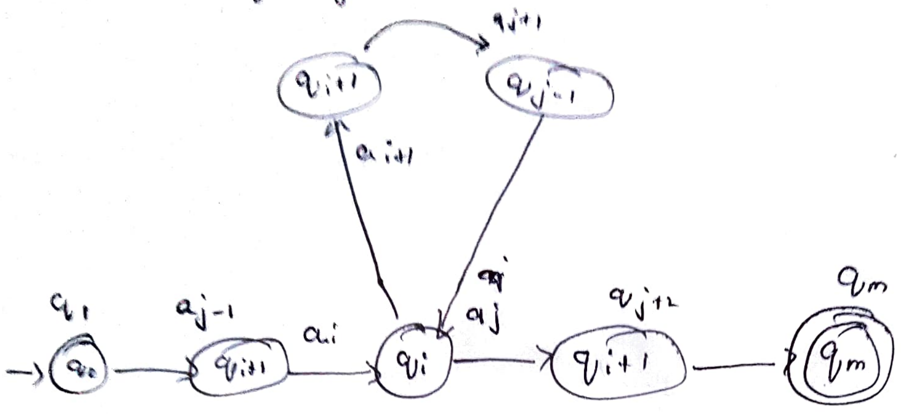
Second group

starting suffix from.

$a_{j+1}\ a_{j+2} \cdots a_m$

i.e.,

$w = a_{j+1}\ a_{j+2} \cdots a_m$



$\longleftarrow$ prefix (u) $\longrightarrow$ loop (v) $\longrightarrow$ suffix (w)

- u takes machine from q_0 to q_i
- v takes machine from q_i to q_i
- w takes machine from q_i to q_m

→ The minimum string that can be accepted by finite automata is

uw if $i = 0$.

$\boxed{uv^i w \in L}$ //

③ Define context free gramer and obtain CFG to generate $L = \{ w \in \{0,b\}^* / N_a(w) = N_b(w) \}$?

Sol :- context free grammar 'G' (CFG) is a 4 tuple denoted by

$G = (V, T, P, S)$ where $V \rightarrow$ set of variables

$T \rightarrow$ set of terminals

$P \rightarrow$ set of productions

$S \rightarrow$ set of start symbol.

In context free grammar all the productions are of the following $A \rightarrow \alpha$ where $\alpha \in (V \cup T)^*$ & A is non terminal.

Given

$L = \{ w \in \{a,b\}^* / N_a(w) = N_b(w) \}$

$S \to \varepsilon$

$S \to aSb$

$S \to bSa.$

hence, $G = (V, T, P, S)$

$V = \{S\}.$

$T = \{a, b\}$

production, $P = \{ S \to \varepsilon$

$S \to aSb$

$S \to bSa$

$\}$

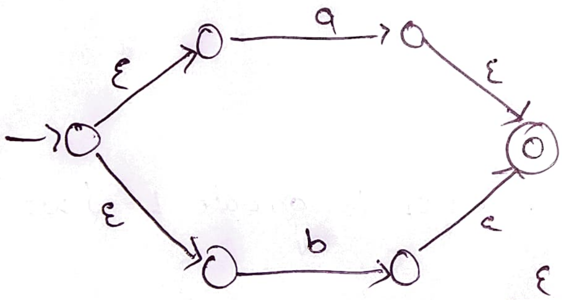
where $S \to$ start symbol

(ii) write the ε-NFA for regular expression $(a+b)^* ab (a+b)^*$?

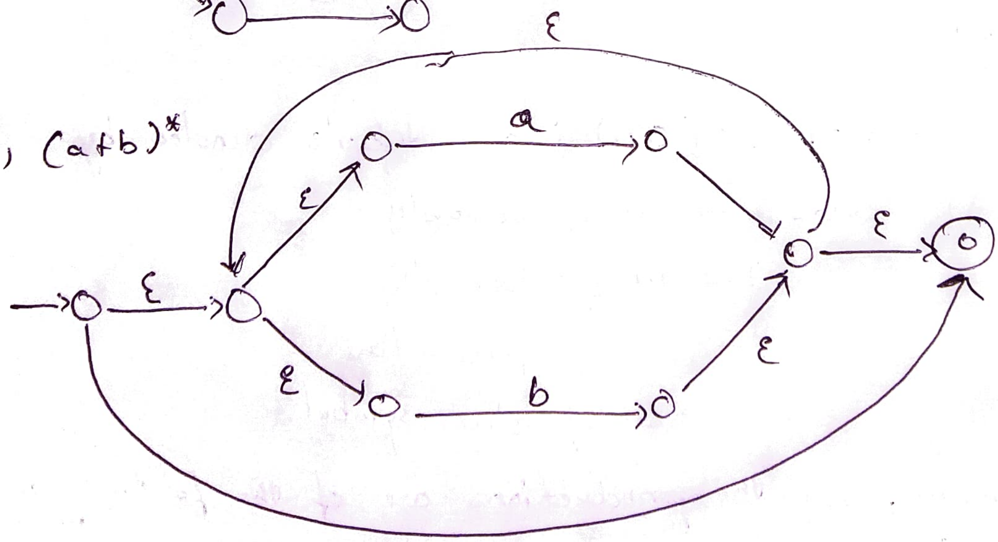
Sol :- Given, RE = $(a+b)^* ab (a+b)^*$

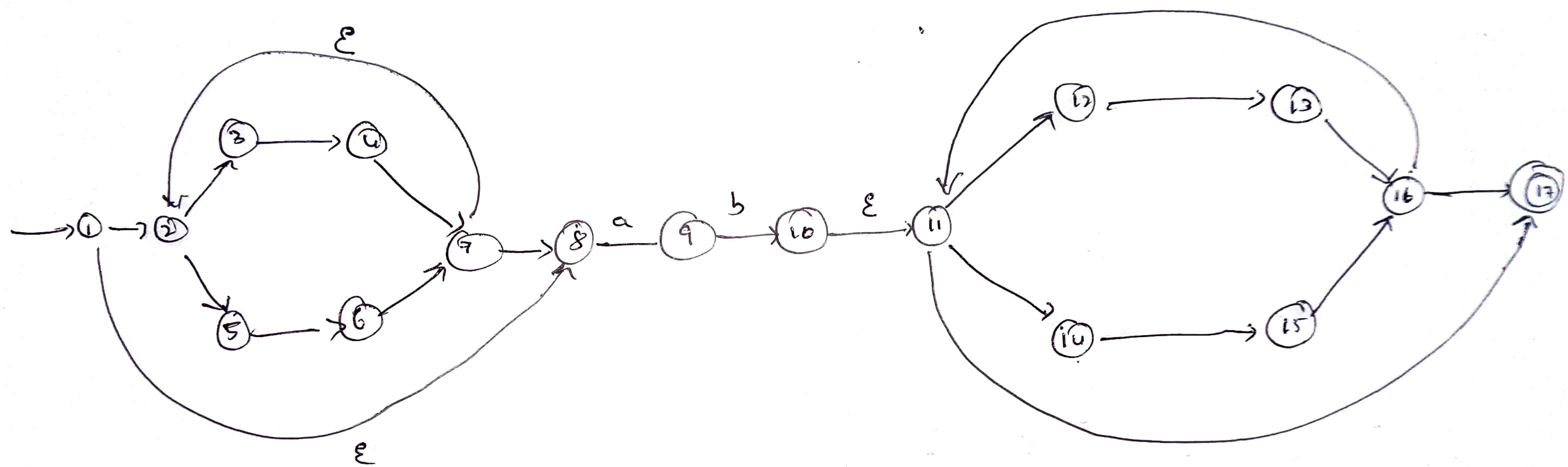
i) It is in the form, $R = R_1 + R_2$ $(a+b)^*$

$\Rightarrow (a+b)$



ii) $(a+b)^*$

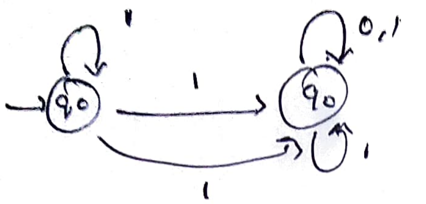




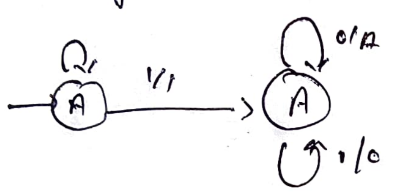
$(a+b)^* ab (a+b)^*$ is obtained //

(5) Design a mealy machine that takes binary as input & produse 2's complement of that number as output. Assume that string is read from LSB to MSB & carry is discarded?

→ Step (i) construct the FSM



Step (ii) chang FSM into mealy machine.

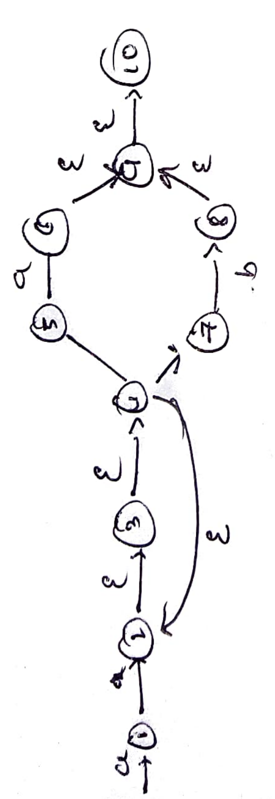


(6) Design NDFSM for RE which accepts the langauge L(aa* (a+b))?

→ Given L = { aa* (a+b)) }.

$\Sigma = \{a, b\}$.

This is in the form $L(R) = (R_1)^*$



(7)

Show that $L = \{a^n b^n \mid n \geq 0\}$ is not regular?

→ step (i) Let 'L' be regular & n be no of states in FA.

consider $x = a^n b^n$

Step (ii) since $|x| \geq = 2n \geq n$, we can split 'x' into uvw such that $|uv| \leq n$

& $x = \underbrace{aaaaa}_{u}\ \underbrace{a}_{v}\ \underbrace{bbbbbb}_{w}$

where $|u| = n-1$ & $|v| = 1$ & $|uv| = n-1+1$ & $|w| = n$

According to pumping lemma $uv^iw \in L$ for $i = 0, 1, 2 \ldots$

(iii) if $i = 0$, the starting v doesnot appear & so the number of a's will be less than no of b's.

i.e n, number of a's should be followed by n number of b's which is not true when $i = 0$.

∴ Hence language $L = \{a^n b^n \mid n \geq 0\}$ is not regular.