Assignment 1

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Problem 1

Part 1:

$$
\begin{array}{rcl}
\textsf{Property } p ::= & & \varepsilon \\
& | & \textsf{n} = \textsf{val} \mid \textsf{n} \neq \textsf{val} \mid \textsf{n} < \textsf{val} \mid \textsf{n} > \textsf{val} \\
& | & \textsf{s} = \textsf{str} \mid \textsf{s} \neq \textsf{str} \\
& | & \textsf{p1} \vee \textsf{p2} \\
& | & \textsf{p1} \wedge \textsf{p2} \\
\\
\textsf{bool} ::= & & true \mid false \\
\textsf{Schema } \tau ::= & & \textsf{number}\langle p \rangle \\
& | & \textsf{string}\langle p \rangle \\
& | & \textsf{bool} \\
& | & [\,\tau^{k\langle p \rangle}] \\
& | & \{(\,\textsf{string}\langle p \rangle : \tau)^{\,k\langle p \rangle}\}
\end{array}
$$

Part 2:

$$\frac{}{\textsf{false} \sim \textsf{bool}} \text{ (S-BOOL-FALSE)} \qquad \frac{}{\textsf{true} \sim \textsf{bool}} \text{ (S-BOOL-TRUE)}$$

$$\frac{}{n \sim \textsf{number}} \text{ (S-NUM)} \qquad \frac{\textsf{seq is \textbackslash n terminated sequence of characters}}{seq \sim \textsf{string}} \text{ (S-STRING)}$$

$$\frac{n \sim properties}{n \sim \tau} \text{ (S-NUM-PROPERTY)} \qquad \frac{seq \sim properties}{seq \sim \tau} \text{ (S-STRING-PROPERTY)}$$

$$\frac{}{\{\} \sim \tau} \text{ (S-EMPTY-OBJECT)} \qquad \frac{}{[\,] \sim \tau} \text{ (S-EMPTY-ARRAY)}$$

$$\frac{k \sim properties}{\{(\ s\langle p\rangle : \tau)\ ^k\} \sim \{s_1\langle p\rangle : \tau, ..., s_k\langle p\rangle : \tau\}} \text{ (S-CARDINALITY-OBJECT)}$$

$$\frac{k \sim properties}{[\tau^k]\ \sim [\tau_1, ..., \tau_k]} \text{ (S-CARDINALITY-ARRAY)}$$

$$\frac{\textsf{all the elements of array have same type}}{[\,j_1, ..., j_n]\ \sim \tau} \text{ (S-ARRAY)}$$

$$\frac{\forall s \in \{s_1, ..., s_n\}\ s \sim properties}{\{s_1\langle p\rangle : \tau, ..., s_k\langle p\rangle : \tau\} \sim \tau} \text{ (S-OBJECT)}$$

Problem 2

Part 1:

$$\frac{}{(\,\varepsilon a, j)\ \mapsto (\,a : j)} \text{ (EPSILON-ACCESSOR)} \qquad \frac{s \sim \textsf{string}}{(\,.sa, \{s : j\})\ \mapsto (\,a : j)} \text{ (KEY-ACCESSOR)}$$

$$\frac{0 \leq k \leq n}{(\,[\,k]a,\ [\,j_0, j_1, j_2, ..., j_k, ..., j_n]\,)\ \mapsto (\,a, j_k)} \text{ (ARRAY-ACCESSOR)}$$

$$\frac{}{(\,|a, [\,j_1, ..., j_k, ..., j_n]\,)\ \mapsto (\,[\,(a, j_1), ..., (a, j_k), ...(a, j_n)]\,)\,]} \text{ (MAPS-ACCESSOR)}$$

Part 2:

$$\frac{}{\varepsilon \sim \tau}\ (\textsc{Valid-Epsilon-Accessor}) \qquad \frac{\{(s:j)\} = \tau \quad s \sim \mathsf{string}}{.s \sim \tau}\ (\textsc{Valid-Key-Accessor})$$

$$\frac{[j] = \tau \quad n \leq size\,of\,array}{[n] \sim \tau}\ (\textsc{Valid-Array-Accessor}) \qquad \frac{[j] = \tau}{| \sim \tau}\ (\textsc{Valid-Maps-Accessor})$$

Accessor safety: for all a, j, τ, if $a \sim \tau$ and $j \sim \tau$, then there exists a j' such that $(a, j) \overset{*}{\mapsto} \varepsilon, j'$.

Proof. • Induction hypothesis: Let $A = a_1 a_2 ... a_n$ is series of accessor for which ***Accessor safety*** holds true.

$(A, j) \overset{*}{\mapsto} (\varepsilon, j')$

• Let say $(.sA, j_0) \mapsto (A, j)$ then by inversion lemma of VALID-KEY-ACCESSOR $j_0 \sim \{(s:j)\}$ and due to induction hypothesis following holds true $(.sA, j_0) \overset{*}{\mapsto} (\varepsilon, j')$ thus accessor safety holds true.

• Let say $([n]A, j_0) \mapsto (A, j)$ then by inversion lemma of VALID-ARRAY-ACCESSOR $j_0 \sim [j]$ and due to induction hypothesis following holds true $([n]A, j_0) \overset{*}{\mapsto} (\varepsilon, j')$ thus accessor safety holds true.

• Let say $(|A, j_0) \mapsto (A, j)$ then by inversion lemma of VALID-MAPS-ACCESSOR $j_0 \sim [j]$ and due to induction hypothesis holds true for all values in $[j]$, folloding holds true $(|A, j_0) \overset{*}{\mapsto} (\varepsilon, j')$ thus accessor safety holds true.

Thus with structural induction we have proved that if ***Accessor safety*** holds on sub-part then it holds for valid composition of accesor sequence.

□