

Prefix Sum

What is Prefix Sum?



$$arr = \begin{cases} 0 & i & 2 & 3 & 4 & 5 & 6 \\ 5, & i, & k, & 2, & 4, & 3, & 2 \end{cases}$$

$$pre = \begin{cases} 5, 6, 14, 16, 20, 23, 25 \end{cases}$$

$$pre[i] = arr[i] + pre[i-1]$$



Q1: Running sum of 1D Array



Q2: Range Sum Query ∰Mutable

$$arr = \{5, 1, 8, 2, 4, 3, 2\}$$
 Length = n

If we calculate the sum from left to right using for loop, then $T \cdot C \cdot = O(n)$ for one query

$$T \cdot C = O(m^*n)$$

[Leetcode 303]



Q2: Range Sum Query - Mutable

$$arr = \begin{cases} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 2 & 2 & 4 & 3 & 2 \end{cases}$$

[Leetcode 303]

Homework:



ivot Index
$$0 \mid 2 \mid 3 \mid 4 \mid 5$$
 gr = $\{1, 7, 3, 6, 5, 6\}$

$$ans = 3$$

$$\alpha m = \{10, 1, 2, 3, -6\}$$

arr =
$$\{6, -3, -1, -2, 10\}$$

Homework:



Q: Find Pivot Index

```
public int pivotIndex(int[] nums) {
   int n = nums.length;
   for(int i=0;i<n;i++){
      int leftSum = 0, rightSum = 0;
      for(int j=0;j<i;j++){
        leftSum += nums[j];
      }
      for(int j=i+1;j<n;j++){
        rightSum += nums[j];
      }
      if(leftSum==rightSum) return i;
   }
   return -1;
}</pre>
```

```
Time Complexity: O(n²)
Extra Space: O(1)
```

Classwork

Homework:



Q: Find Pivot Index

Index
$$0 \mid 2 \quad 3 \quad 4 \quad 5$$

$$0 \mid -2 \quad 3 \quad 4 \quad 5$$

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$$0 \mid -2 \quad 4 \quad 5$$

$$\Rightarrow$$
 rightsum = Sum(i+1, n-1) = Sum(0,n-1) - sum(0,i)
pre[n-1] - pre[i]

Homework:



Q: Find Pivot Index

```
public int pivotIndex(int[] nums) {
    int n = nums.length;
    // make nums prefix sum of nums
    for(int i=1;i<n;i++){
        nums[i] += nums[i-1];
    }
    for(int i=0;i<n;i++){
        int leftSum = 0;
        if(i>0) leftSum = nums[i-1];
        int rightSum = nums[n-1] - nums[i];
        if(leftSum==rightSum) return i;
    }
    return -1;
}
```

```
Time Complexity: O(n)
Extra Space: O(1)
```

Homework:



Q: Find the Score of all Prefixes of an Array

$$arr 2 = \begin{cases} 2, 7, 2, 10, 5, 3 \end{cases}$$
 $arr 2 = \begin{cases} 14, 9, 20, 15, 13 \end{cases}$
 $arr 3 = \begin{cases} 14, 23, 43, 58, 71 \end{cases}$

$$\alpha m^2[i] = am[i] + max(0,i)$$



Q3: Product of array except self [Pure suterview]

$$arr = \{2, 2, 4, 3\}$$



$$arr = \{7, 2, 4, 3\}$$



Q3: Product of array except self

and =
$$\begin{cases} 24, 56, 84, 42 \end{cases}$$

arr = $\begin{cases} 7, 3, 2, 4 \end{cases}$

$$arr = \{7, 3, 2, 4\}$$
 $pre = \{1, 7, 21, 42\}$
 $suf = \{24, 8, 4, 1\}$

[Leetcode 238]



```
public int[] productExceptSelf(int[] arr) {
    int n = arr.length;
    int[] pre = new int[n];
    pre[0] = 1;
    for(int i=1;i<n;i++){
        pre[i] = arr[i-1] * pre[i-1];
    int[] suf = new int[n];
    suf[n-1] = 1;
    for(int i=n-2; i>=0; i--){
        suf[i] = arr[i+1] * suf[i+1];
    int[] ans = new int[n];
    for(int i=0;i<n;i++){
        ans[i] = pre[i] * suf[i];
    return ans;
```

```
T \cdot C \cdot = O(n)

R \cdot S \cdot = O(n)
```



$$arr = \{7, 3, 2, 4\}$$
 $pre = \{1, 7, 21, 42\}$
 $suf = \{24, 8, 4, 1\}$
 $ans = \{24, 86, 84, 42\}$

arr =
$$\{7, 3, 2, 4\}$$

pre = $\{1, 7, 21, 42\}$
suf = $L;$
pre(i) += (suf + arr[i+1])





```
public int[] productExceptSelf(int[] arr) {
    int n = arr.length;
    int[] pre = new int[n];
    pre[0] = 1;
    for(int i=1;i<n;i++){
        pre[i] = arr[i-1] * pre[i-1];
    int suf = 1;
    for(int i=n-2;i>=0;i--){
        suf *= arr[i+1];
        pre[i] *= (suf);
    return pre;
```

```
Best Method

L

T \cdot C \cdot = O(n)

\xi \cdot S \cdot = O(n)
```

Sort, Prefix Sum, B.S.



n subset

Q4: Longest subsequence with limited sum

arr =
$$\{4, 5, 2, 1, 2, 7\}$$

queries = $\{5, 10, 11\}$

and = $\{3, 4, 4\}$

1
2
1, 2
2, 2
1, 2, 2 $\Rightarrow enft = 3$ 4
5
4, 1

[Leetcode 2389]



[Leetcode 2389]

Q4: Longest subsequence with limited sum

arr =
$$\{4, 5, 2, 1, 2, 4\}$$
 cont arr = $\{1, 2, 2, 4, 5, 4\}$

| pre |
| queries = $\{5, 10, 11\}$

ans = $\{3, 2, 3\}$
| u |
| if (arr(mid) > que(i)) hi = mid - 1
| else {
| ans (i] = max (ans (i), mid + 1);
| lo = mid + 1;
| Leetcode 23



Q4: Longest subsequence with limited sum

```
public int[] answerQueries(int[] arr, int[] queries) {
    Arrays.sort(arr);
    int n = arr.length, m = queries.length;
    for(int i=1;i<n;i++){
        arr[i] += arr[i-1];
    int[] ans = new int[m];
    for(int i=0;i<m;i++){
        int lo = 0, hi = n-1;
        while(lo<=hi){</pre>
            int mid = lo + (hi-lo)/2;
            if(arr[mid]>queries[i]) hi = mid - 1;
            else{
                ans[i] = Math.max(ans[i],mid+1);
                lo = mid + 1;
    return ans;
```

```
Sort \rightarrow nlogn

pre \rightarrow n

8.S \rightarrow m.logn

T.C. = O((m+n) \log n)
```

[Leetcode 2389]



Q5: Corporate Flight Bookings

$$arr = \{\{1,4,10\},\{2,3,20\},\{2,5,25\}\}\}$$
 $n=5$

[Leetcode 1109]





Q5: Corporate Flight Bookings

```
public int[] corpFlightBookings(int[][] arr, int n) {
    int[] ans = new int[n];
    for(int i=0;i<arr.length;i++){</pre>
        int first = arr[i][0];
                                                           T \cdot C \cdot = O(m+n)
        int last = arr[i][1];
        int seats = arr[i][2];
        ans[first-1] += seats;
        if(last<n) ans[last] -= seats;</pre>
    for(int i=1;i<n;i++){
        ans[i] += ans[i-1];
    return ans; // 3ms
```

Optimized via Prefix Sum





```
Q5: Corporate Flight Bookings
                                         s length = m
public int[] corpFlightBookings(int[][] arr, int n) {
    int[] ans = new int[n];
    for(int i=0;i<arr.length;i++){</pre>
                                                       T.C. = O(m*n)
        int first = arr[i][0];
        int last = arr[i][1];
        int seats = arr[i][2];
        for(int j=first-1;j<=last-1;j++){
    ans[j] += seats;
    → ()(n)
    return ans; // 748ms
```

Brute Force





Q6: Subarray sum equals K

$$arr = \{1, 2, 4, 3, 7, 8, -13\}$$
 $K = 7$

$$\rightarrow$$
 $\{1,2,43, \{4,3\}, \{73, \{8,-1\}\}$ ans = 4



Q6: Subarray sum equals K

Generating all subarrays
$$\rightarrow T \cdot C = O(n^2)$$

 $\xi \cdot \zeta = O(1)$

SKILLS

Q6: Subarray sum equals K

$$ar = \{1, 2, 4, 3, -3, 0, 7, 8, -13\}$$

pre =
$$\{1, 3, 7, 10, 7, 7, 14, 12, 21\}$$

$$\begin{array}{c|c}
(23,1) \\
(14,1) \\
(21,1) \\
(3,3)
\end{array}$$

$$\begin{array}{c}
(3,1) \\
(1,1)
\end{array}$$

$$\Gamma \cdot C \cdot = O(n)$$

$$E \cdot S \cdot = O(n)$$

Court & X X X X X X

K = 7

Ques: T.C. =
$$O(n)$$

A.S. = $O(n)$

Q7: Minimum Penalty for a shop

[Leetcode 2483]

Homework:

SKILLS

Q: Reducing Dishes

THANKYOU