



Heaps → priority queue

Lecture 1

What and Why?

Array

LL



Stack

Queue

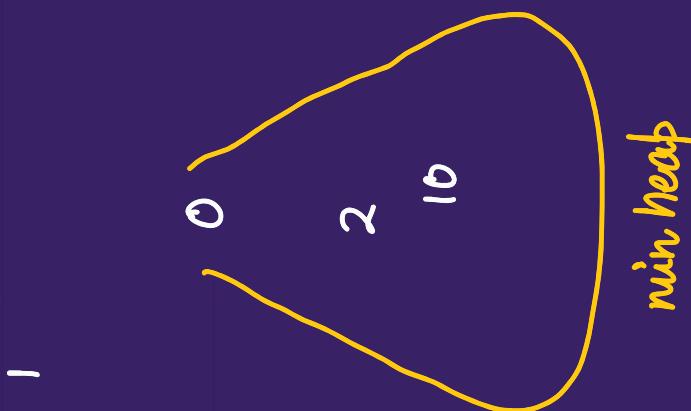
HashMap

Heap

push $\rightarrow O(1)$ (add)
pop $\rightarrow O(1)$ (remove)
peek $\rightarrow O(1)$ (get)



add (2)
add (10)
add (1)
remove()
add (0)



What and Why?



heap \rightarrow add : the element is added to heap & the minimum element is at top T.C. = $O(\log n)$

remove : the top (\min^m) ele is removed & the minimum element is at top T.C. = $O(\log n)$

peek : returns the top (\min^m) element T.C. = $O(1)$

size : returns the size

2 Types of heaps



- Minheap : Java Collection Framework (Builtin)

If we are adding n elements one by one in a heap then →

$$\begin{aligned} \text{no. of ops} &\approx \log(1) + \log(2) + \log(3) + \dots \log(n) = \log(n!) \\ &\approx n \log n \end{aligned}$$

$$\text{T.C.} = O(n \log n)$$

2 Types of heaps

Minheap vs Arraylist \rightarrow Vala heap time add me
T.C. = $O(n \log n)$

add(2)

add(10)

add(1)

remove()

add(0)

add(20)

arr = {2}

arr = {2, 10} \rightarrow {10, 2}

arr = {10, 2, 1} \rightarrow {10, 2, 1}

arr = {10, 2}

arr = {10, 2, 0} \rightarrow {10, 2, 0}

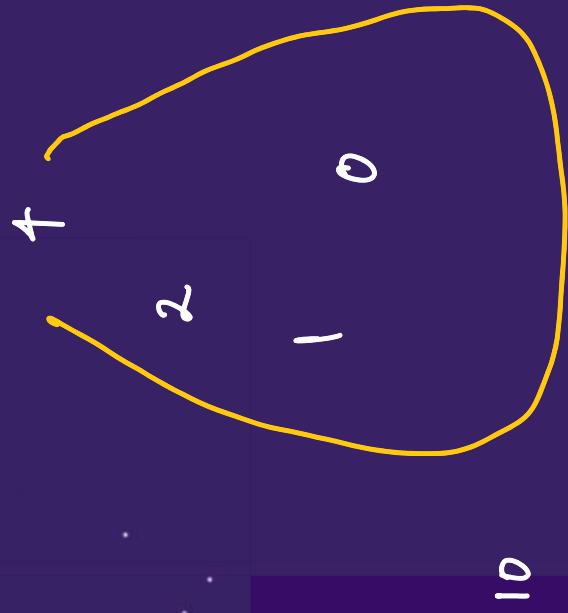
arr = {10, 2, 0, 20} \rightarrow {20, 10, 2, 0}

Priority Queue STL



2) Maxheap

✓ add(2)
✓ add(10)
✓ add(1)
✓ remove()
✓ add(0)
✓ add(4)



Priority Queue < Integer > pq = new Priority Queue < > (Collections.reverseOrder());

Problem Identification



- 1) k^m smallest / largest / closest / frequent / k-sorted array
Out of n , if we are working on ' k ' elements
- 2) $O(n \log n) < O(n \log k) < O(n)$
- 3) Minimise , Maximise , Continuous sorting
- 3) For k smallest \rightarrow we use MaxHeap .

Ques:

Q1 : Find the kth smallest element in a given array.

$$\text{arr} = \{ 10, 2, 3, 4, -4, -2, 6 \} \quad k=3$$

M-1 : Brute Force
Selection Sort (k passes) then $\text{arr}[k-1]$
 $T.C. = O(n^2)$

M-2 : Built-in Sort \rightarrow Quick Sort / Merge Sort
 $T.C. = O(n \log n)$
 $S.C. = O(\log n) / O(n)$
 $\text{arr}[k-1]$

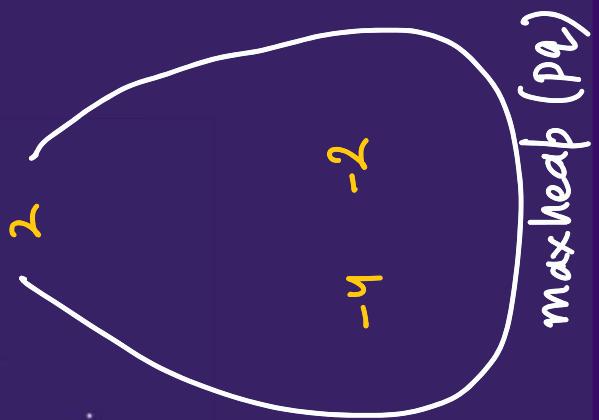
Ques:

Q1 : Find the kth smallest element in a given array.

$$\text{arr} = \{ 10, 2, 3, 8, -4, -2, 6 \} \quad K = 3$$

Restrict the size of maxheap to 'K'

if ($\text{pq}.\text{size}() > K$) $\text{pq}.\text{remove}()$



Ques:

Q1 : Find the k th smallest element in a given array.

```
for (int ele : arr) {  
    pq.add(ele);  
    if (pq.size() > k) pq.remove();  
}  
total = (2n-k) log k  
T.C. = O(n log k)
```

Ques:

Q1 : Find the kth smallest element in a given array.

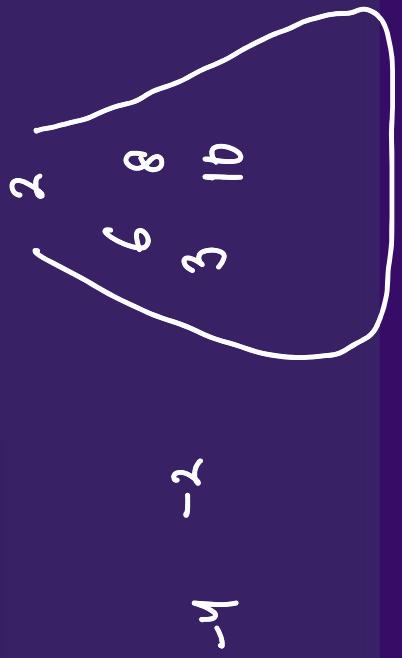
$$\text{arr} = \{ 10, 2, 3, 8, -4, -2, 6 \} \quad k = 3$$

M-3 : Using heap / PQ But which heap ?

If we use minheap, then

add $\rightarrow n \log n$
remove $\rightarrow k \log n$

$$\begin{array}{l} T.C. = O(n \log n) \\ S.C. = O(n) \end{array}$$



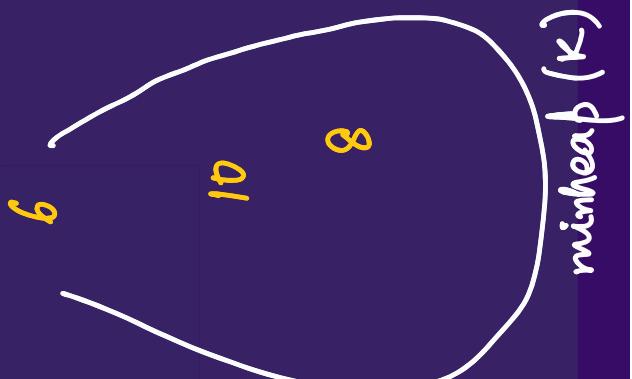
Ques:

Q2 : Find the k th largest element in a given array.

$$\text{arr} = \{ 10, 2, 3, 8, -4, -2, 6 \} \quad k = 3$$

Time Complexity $\rightarrow O(n \log k)$

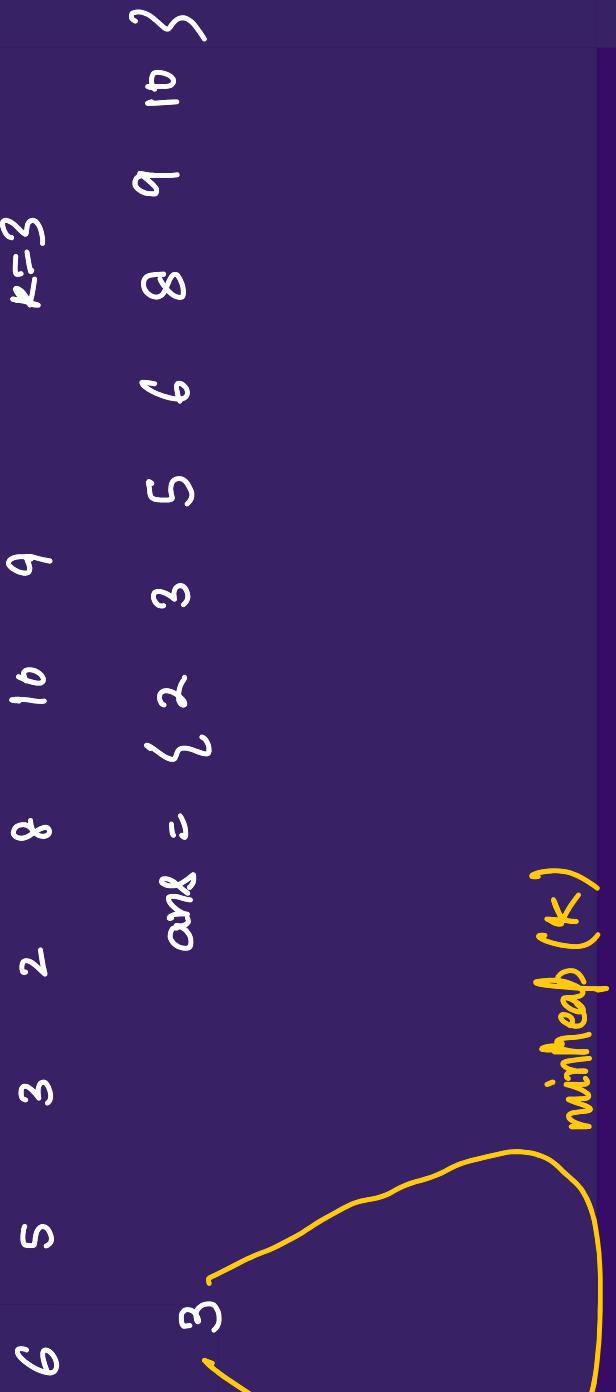
Auxiliary Space $\rightarrow O(k)$



Ques: [Good Interview Question]

Q3 : Sort a 'k' sorted array (sort a nearly sorted array).

arr = {6,5,3,2,8,10,9} k=3 | arr = {3,1,4,2,5} k=2



Ques:

Q4 : K Closest Points to Origin

$$arr = \left\{ \begin{array}{l} \{3, 3\}, \{5, -1\}, \{-2, 4\}, \{1, 0\}, \{3, 2\} \end{array} \right\} \quad k=2$$
$$\sqrt{18} \quad \sqrt{26} \quad \sqrt{20} \quad \sqrt{1} \quad \sqrt{13}$$

\rightarrow k smallest distances \rightarrow Maxheap

$$ans = \left\{ \begin{array}{l} \{1, 0\}, \{3, 2\} \end{array} \right\}$$

[Leetcode 973]



Ques:

Q4 : K Closest Points to Origin

arr = { {3,3}, {5,-1}, {-2,4}, {1,0}, {3,2} } $K=2$
 d 18 26 20 13

ans = { {3,2}, {1,0} }
13,{3,2}
1,{1,0}
20,{2,1}
26,{5,-1}
18,{3,3}



maxheap(K)

map < Distance, Coordinates >
arr

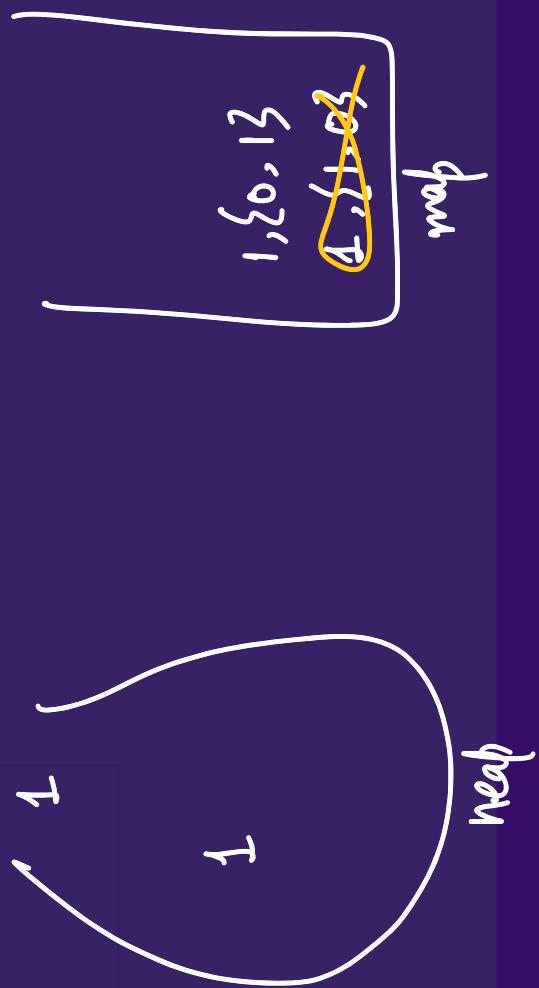
[Leetcode 973]

Ques:

Q4 : K Closest Points to Origin

$$\text{arr} = \begin{Bmatrix} \{1, 0\}, \{0, 1\} \\ 1 \\ 1 \end{Bmatrix}$$

$K=2$



[Leetcode 973]



Ques: Custom Comparator

Q4 : K Closest Points to Origin

pg < triplet>



M-2

arr = { {3,3}, {5,-1}, {-2,4}, {1,0}, {3,2} }
k=3
18 26 20 1 13

(13, 3, 2)

If I am inserting an object in PQ,
pls use Comparable

(18, 3, 3)
(1, 1, 0)

heap

[Leetcode 973]

Ques:

Q5 : Find K Closest Elements → Comparable

arr = { 4, 8, 18, 4, 9, 6 }
 | 0 10 4 1 2

K=3 X=8

arr = { 4, 5, 6, 9, 8, 9 }
 | 2 3 4 5 6 7

[Leetcode 658]

Ques:

Q5 : Find K Closest Elements → Comparable

array = { 1, 2, 3, 4, 5, 6, 7 } x = 4, k = 4
 3 2 1 0 1 2 3

Closeness → a, b, x

a is closer to x if $|x-a| < |x-b|$
2. if $(x-a) == (x-b)$ then if $a < b$ then
a is closer

[Leetcode 658]



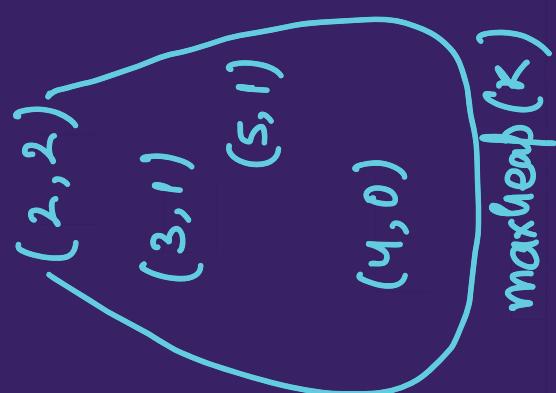
Ques:

Q5 : Find K Closest Elements \rightarrow Comparable

$$\text{arr} = \{ 1, 2, 3, 4, 5, 6, 7 \} \quad x = 4, \quad k = 4$$
$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ & 3 & 2 & 1 & 0 & 1 & 2 & 3 \end{matrix}$$

Maxheap

k closest /
smallest



T.C. = $O(n \log k)$

A.S. = $O(k)$

ele, $|x - ele|$

$pq < pair \downarrow$

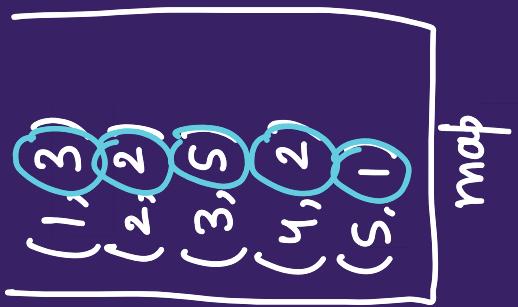
Ques:

Q6: Top K Frequent Elements \rightarrow K largest elements

arr = { 1, 2, 1, 3, 3, 2, 3, 4, 5, 4 } K = 3
return those 'K' elements which occur the most in given array

In the ques:

It is given, each distinct ele has a unique frequency
 $\&$ Space \rightarrow n, K, R \rightarrow O(n)
Time \rightarrow O(n log K)



[Leetcode 347]

Homework:

Q : Sort Array by Increasing Frequency.



[Leetcode 1636]

Ques:

Q7 : Last Stone Weight

$$\downarrow$$
$$arr = \{ 2, 7, 4, 1, 8, 1 \}$$

At Every Step
we are taking

2 largest stones

max heap

$$\downarrow$$
$$\{ 2, 4, 1, 1, 1 \}$$

$$\{ 1, 1, 1 \}$$

$$\{ 1, 1 \}$$

$$\{ 1 \}$$

$$\{ \}$$

Brute Force

↓
use ArrayList & Sorting

Time $\rightarrow n \log n + (n-1) \log(n-1)$

$+ (n-2) \log(n-2)$

\dots

$\sum_{r=n}^1 r \log r \approx \boxed{n^2 \log n}$

$r \rightarrow n \rightarrow 1$

[Leetcode 1046]

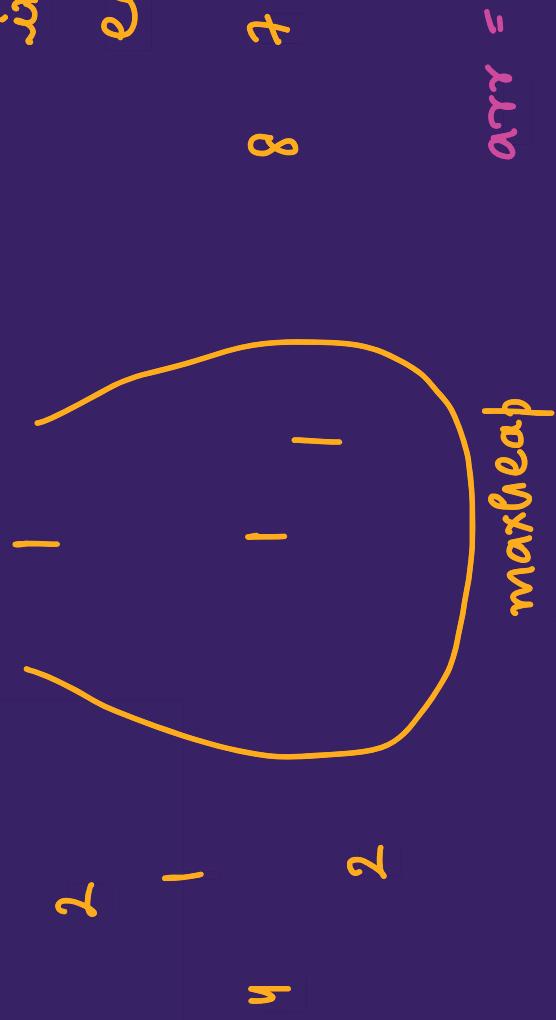
Ques:

Q7 : Last Stone Weight

arr = { 2, 7, 4, 1, 8, 1 }

here , we have at max 'n' elements in heap . So insertion, removal of each element will take $\log n$

T.C. = $O(n \log n)$



ans = { 4, 10, 20 }

[Leetcode 1046]

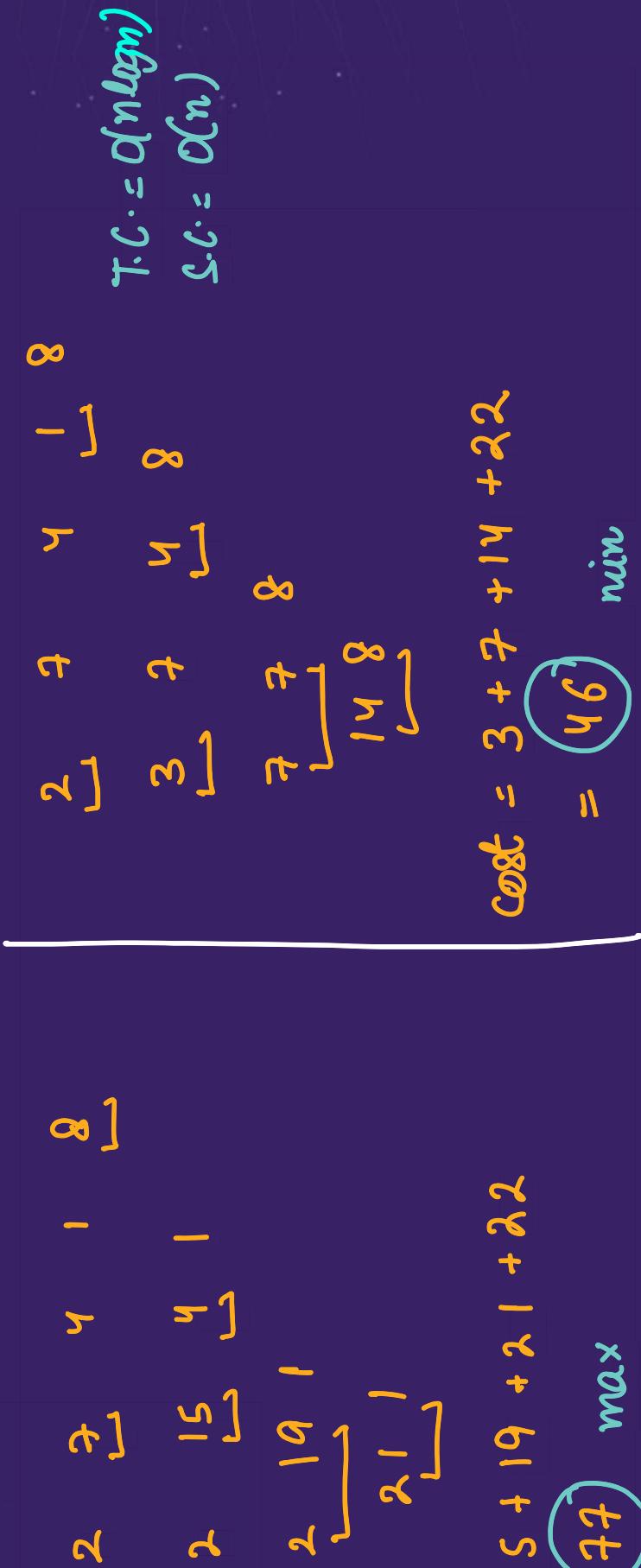
 SKILLS

Ques: I can connect 2 ropes at a time & the cost of joining is the sum of length of ropes

Q8 : Minimum Cost

$$\text{arr} = \{2, 7, 4, 1, 8\}$$

I have 'n' ropes of some lengths



$$\text{Cost} = 15 + 19 + 21 + 22 = 77 \text{ max}$$

$$\text{Cost} = 3 + 7 + 14 + 22 = 46 \text{ min}$$