Problem 1

Because

$$\mathbf{x}_i \sim \mathcal{N}(\mathbf{x}; \mu, \Sigma)$$

According to the rules of normal distribution standardization

$$\frac{\mathbf{x} - \mu}{\sigma} \sim \mathcal{N}(0, \mathbf{I}_N)$$

$$\mathcal{N}(\frac{\mathbf{x}_i - \mu}{U\Lambda^{\frac{1}{2}}}; 0, \mathbf{I}_N) = \mathcal{N}(\mathbf{x}; \mu, \Sigma)$$

Since we already know

$$\mathbf{h}_i \sim \mathcal{N}(0, \mathbf{I}_N)$$

we can conclude

$$\mathbf{h}_i = \frac{\mathbf{x}_i - \mu}{U\Lambda^{\frac{1}{2}}}$$

$$\mathbf{x}_i = \mu + U\Lambda^{\frac{1}{2}}\mathbf{h}_i$$