6CCS3AIN, Tutorial 05 (Version 1.0)

1. For each of the interaction scenarios below:
   (i) Determine which strategies are dominated (and explain why)
   (ii) Use the idea of deleting strongly dominated strategies to simplify the scenario where appropriate.
   (iii) Identify any Nash equilibria.
   (iv) Identify the Pareto optimal outcomes.
   (v) Identify the outcome that maximises social welfare.

(a)

i \ j	L	R
U	3, 3	4, 2
D	1, 1	2, 4

(b)

i \ j	L	R
U	-1, -1	2, 1
D	1, 2	-1, -1

(c)

i \ j	L	R
U	3, 3	1, 1
D	4, 2	2, 4

2. In the lecture we talked about the Prisoner's Dilemma. Another well studied game is the Stag Hunt. Here is the story which describes what it captures:

   > A group of hunters goes stag hunting. If they all stay focussed on the stag, they will catch it and all have a lot of food. If some of them head off to catch rabbits, the stag will escape. In this case the rabbit hunters will have some small amount of food and the (remaining) stag hunters will go hungry. What should each hunter do?

   As a two player game (two hunters, each of which can choose between Rabbit and Stag) this is:

i \ j	R	S
R	2, 2	1, 3
S	3, 1	4, 4

   Note that S is often interpreted as "lie / cooperate" in the Prisoner's dilemma sense, and R as "confess". In that sense, mutual cooperation (both hunters choose S) is the best outcome.

   What are the Nash equilibria and Pareto optimal outcomes?

3. Another canonical 2 player game is "chicken", probably best explained in *Rebel without a Cause*:

   https://www.youtube.com/watch?v=u7hZ9jKrwvo

   Chicken has a payoff matrix like:

i \ j	S	J
S	1, 1	2, 4
J	4, 2	3, 3

where J denotes "jump" (cooperate in Prisoner's dilemma notation) and S denotes "stay in car" (defect in Prisoner's dilemma notation).

What are the Nash equilibria and Pareto optimal outcomes?

How do the outomes of this game compare with those of the Prisoner's dilemma?

4. For the optional computational part of the tutorial.

   On KEATS you can find the file `intro.py`. This is illustrates the cabailities of the Python package called `Nashpy`:

   `https://nashpy.readthedocs.io/en/stable/index.html`

   `Nashpy` provides tools for handling two player games. You can install it using:

   pip install nashpy

   Anaconda includes `pip`, which is a package manager for Python, as well as its own package maanger `conda` so if you have Anaconda installed[1] installation is easy. Otherwise, I'm afraid that you are on your own.

   With `Numpy` installed, you can run `intro.py` with:

   python intro.py

   from the command line. As set up, this loads up *Rock, paper, scissors*, *Matching pennies*, and *Prisoner's dilemma* and shows what `Nashpy` can do, extracting player utilities under specific strategies and searching for Nash equilibria.

   Now try using `Nashpy` to check your answers to part (iii) of Q1 (a)–(e). Note that where there are mixed strategy Nash equilibria, `Nashpy` will find those also.

5. Since `Nashpy` makes it possble to search through the outcomes of the games, you should be able to write code that:

   (a) Identifies dominated strategies.

   (b) Identified Pareto optimal outcomes

   (c) Identifies outcomes that maximise social welfare.

   Use your code to check your answers to parts (i), (iv) and (v) of Q1 (a)–(e).

6. Write code that uses `Nashpy` to search for pure strategy Nash equilibria.

[1] If you don't have Anaconda installed, you can install that from `https://www.anaconda.com/download`. If you are planning to do much with Python, Anaconda is pretty handy to have since it makes it easy to install new packages.

6CCS3AIN, Tutorial 05 Answers (Version 1.0)

1. (a) We have the scenario:

i \ j	L	R
U	3, 3	4, 2
D	1, 1	2, 4

   i. In that initial scenario, R dominates L for j, since j gets a bigger payoff for R, no matter what i plays.

   ii. Since L is dominated, we can delete that option, to give a reduced scenario:

i \ j	R
U	4, 2
D	2, 4

   Given this reduction, D now dominates U for i, so the scenario reduces to:

i \ j	R
D	2, 4

   and we know what each agent will do.

   iii. Consider each of the outcomes in turn.
   Start with (D, R). If j changes strategy, it will get 1 rather than 2. If i changes strategy, it will get 2 rather than 4, so (D, R) is a Nash equilibrium.
   Now consider (U, L). If j changes strategy to R, its payoff will go up to 4. So (U, L) is not a Nash equilibrium.
   Similar reasoning applies to (D, L) and (U, R).

   iv. For Pareto optimality, the only outcome that is **not** a Pareto optimal state is (D, L). For every other state, moving to a different outcome makes one agent worse off.

   v. There are three outcomes with a social welfare of 6: (U, L), (U, R), and (D, R), which is the maximum social welfare of any outcome.

   (b) This time the game is

i \ j	L	R
U	-1, -1	2, 1
D	1, 2	-1, -1

   i. There are no dominated strategies — no strategy is better for either i or j no matter what the other does.

   ii. No simplification is possible.

   iii. Consider each outcome in turn.
   Start with (U, R). If j changes strategy, it gets -1 rather than 2. If i changes strategy, it gets -1 rather than 1. So (U, R) is a Nash equilibrium.
   Similarly, (D, L) is a Nash equilibrium.
   Now consider (D, R). Both i and j can individually improve their payoff to 1 if they change strategy. So (D, R) is not a Nash equilibrium.
   Similarly for (U, L).

   iv. Both (U, R) and (D, L) are Pareto optimal since moving away from either will make at least one agent worse off.

   v. Both (U, R) and (D, L) maximise social welfare.

(c) We have:

i \ j	L	R
U	3, 3	1, 1
D	4, 2	2, 4

i. In this scenario, L dominates R for j.

ii. We can reduce the scenario to:

i \ j	L
U	3, 3
D	4, 2

and U now dominates for i, so the scenario reduces to:

i \ j	L
U	3, 3

iii. (U, L) is a Nash equilibrium since if j changes strategy its payoff will drop from 3 to 1 and if i changes strategy, its payoff will change from 3 to 2.
(D, R) is not a Nash equilbrium because j can change strategy and increase its payoff to 4.
Similarly, (U, R) and (D, R) are not Nash equilibria.

iv. The only outcome that is not Pareto optimal is (U, R). For all the others, there is no way to improve the payoff to one agent without reducing it for another.

v. Every outcome except (U, R) maximises social welfare: all the other outcomes have a social welfare of 6.

2. The Stag Hunt scenario is:

i \ j	R	S
R	2, 2	1, 3
S	3, 1	4, 4

What are the Nash equilibria?

Well, (R, R) is a Nash equilibrium. Even though both agents can get a better payoff than the 2 they get in (R, R), they cannot get that payoff by changing strategy on their own. In other words, if i switches to S while j plays R, then the stag will escape, and i's payoff will drop to 1. Similarly, if j plays S while i sticks to R.

The reasoning about (R, R) tells us that (R, S) is not a Nash equilibrium because i can improve its payoff by changing to S, and j can improve its payoff by changing to R. Similarly (S, R) is not a Nash equilibrium.

Finally, (S, S) is a Nash equilibrium because neither agent can do better by playing R.

(S, S) is the only Pareto optimal solution. For every other outcome, switching to (S, S) will improve the outcome for both agents.

3. Chicken has as normal-form game is:

i \ j	S	J
S	1, 1	2, 4
J	4, 2	3, 3

(S, S) is not a Nash equilibrium — j can improve its payoff to 2 by swicthing to J, and so can i. So both drivers staying in the car is not a Nash eqilibrium (as we might hope).

(S, J) is a Nash eqilibrium. If i tries to move away, it will get a ayoff of 3 rather than 4, and if j tries ot move away it will get a payoff of 1 rather than 2. Similarly, $J, S)$ is a Nash equilibrim. This is the essence of why Chicken is a hard game to play — assuming you buy into the fundamentally stupid idea of the game in the

first place — your best strategy of playing is: if I think you will stay in the car, I should jump out, while if I think you will jump out of the car, I should stay in.

(J, J) is not a Nash equilibroum since eithe rplayer can improve its payoff by switching to S.

Every outcome except (S, S) is Pareto optimal.

Compared with Prisoner's dilemma, the maian difference is that the worst payout for both players occurs when they both play S ("defect" in Prisoner's dillema). That makes the outcomes in which players pick different strategies Nash equilibria.

4. No solution will be provided for the computational part, but you can check your solution against the answers above.

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