Assignment 2

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Due: 18 February.

1   Data

(*20 Points*) Together with this assignment, you will find a .cvs file called Data.cvs. The data was taken from the Carbon Dioxide Information Analysis Center, Environmental Sciences Division, Oak Ridge National Laboratory, Tennessee, United States. It gives the CO_2 emissions (*metric tons per capita*)[1] of several countries. Choose your favourite country and

1. (*5 Points*) Fit a straight line passing through the endpoints;

2. (*15 Points*) Fit a linear least-squares model. Find the maximum error in magnitude in the first case. Predict the year in which the CO_2 level may exceed 3 metric tons per capita.

2   Mathematical Preliminaries

- (*5 Points*) Is the following time-varying function positive definite?

$$V(t, x_1, x_2) = t\left(x_1^2 + x_2^2\right) + 4x_1 x_2 \sin t.$$

- (*10 Points*) Test the stability of the zero solution for the following system

$$
\begin{aligned}
x_1' &= -x_1^3 + x_1^4 \\
x_2' &= x_1^4 + x_2^3
\end{aligned}
$$

- (*15 Points*) Test the linear stability of the zero solution $x_1(t) = 0, x_2(t) = 0$ in the Lotka-Volterra population model, i.e.,

$$
\begin{aligned}
x_1' &= ax_1 - x_1 x_2 \\
x_2' &= -bx_2 + x_1 x_2,
\end{aligned}
$$

for your favourite pair of integers (a, b). *Suggestion:* For one of the two critial points will find, try to shift the origin to that critial point, by a simple coordinate transformation, and construct a Lyapunov function.

Remark: Please upload in a single PDF file your solutions. You can use LaTeX or scan hand-written work or images for parts of solutions only if they are extremely *clean* and *legible*[2].

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[1] For more information see http://data.worldbank.org/indicator/EN.ATM.CO2E.PC?locations=CO

[2] You will be constantly dealing with mathematical notation and LaTeX is the best for that. If you do not know how to use it, I will be more than happy to help you. For more information visit http://www.ctan.org/tex-archive/info/lshort/. I personally use LyX, which combines the power and flexibility of TeX/LaTeX with the ease of use of a graphical interface. See http://www.lyx.org.