Date: Thursday, April 4th 2019

Theme: Goal is to find the combination of factor levels that optimize the response.

Question: How do we set up an experiment to do this:

Easy answer: Try all combinations!
↳ Problem: - Often considers too many conditions, and we don't have the resources to run it.
- may not want to try all combinations for fear of losing users due to frustration.

Factorial approach (Try all combinations)

* For a small number of conditions this approach is feasible and should be run.

* Most efficient way to look over all conditions.

One-factor at a time:

- Optimize over one factor (say factor A)
- Hold factor A fixed and optimize over factor B.
- Continue until all factors are analyzed

Advantage: Lot fewer experiments to run than all possible condition.

Disadvantage: May miss the best combination.

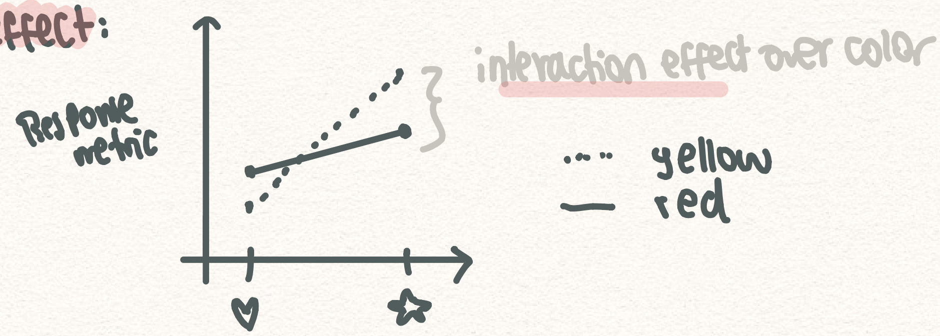
Example:

↳ Main effect and interaction effects

$\underbrace{\{\text{heart, star}\}}_{F_1}$, $\underbrace{\{\text{red, yellow}\}}_{F_2}$

Metric: CTR

Main effect:



* There are k main effect plots that you can draw

↳ one for each one of the k factors

- Let $M \cdot E_z(y)$ = the main effect of changing y and holding z constant.

$$ME_{yellow}(shape) + M \cdot E_{red}(shape)$$

Interaction effects of color on shape =

$$ME_{yellow}(shape) - ME_{red}(shape)$$

Strategy for factorial design:

1) Choose metric of interest (CRT, views, convert, etc.)

2) Choose factors & their levels

   - Keep it simple
   - using redundant factors can be a waste of time.

3) Construct $M = m_1, m_2, \ldots, m_k$ conditions

4) Randomly assign units over these M conditions.

* 5) Identify optimal condition using $\binom{M}{2}$ hypothesis tests using multiple comparison strategy.

   ↳ Comes down to two aims:

   a) Identify which factors are influential (Regression for the win)

   b) Identify optimal condition (testing)

1st step of analysis:

- Plot main effects plots accross factors

  ↓

  "see" if there are any interactions

  ↓

  Test if there are any interactions + test for influence of each factor on metric with regression.

If metric is continous:

- Use linear regression

  binary/multicategorical

  ↳ use logistic or nominal regression

Instagram example:

y_i := session duration of user i

<u>Factors</u>: $\{7:1, \quad 4:1, \quad 1:1, \quad \text{None}\}$ $\quad \{\text{video}, \text{photo}\}$

$\qquad\qquad x_{i1} \qquad x_{i2} \qquad x_{i3} \qquad\qquad\qquad\qquad x_{i4}$

$$x_{i1} = \begin{cases} 1 & \text{if user } i \text{ saw } 7:1 \\ 0 & \text{otherwise} \end{cases}$$

These are main effect terms
↳ coeff. represent main effect of each level

Interaction terms: Take cross-products of main effects terms from different factors.

Freq. Type

$x_{i1}x_{i4}$, $x_{i2}x_{i4}$, $x_{i3}x_{i4}$

The coefficients on these represent interaction.

$$y_i = \underbrace{\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4}}_{\text{main effects}}$$
$$+ \underbrace{\beta_5 x_{i1}x_{i4} + \beta_6 x_{i2}x_{i4} + \beta_7 x_{i3}x_{i4}}_{\text{interaction effects}} + \varepsilon_i$$

↳ Run regression and identify significant coefficients

Test: A sequence of t-tests:

$$H_{0j}: \beta_j = 0 \quad \text{vs} \quad H_{aj}: \beta_j \neq 0.$$

↳ Common strategy: test whether interaction effects are significant first using the full model.

F-test: $H_0: \beta_5 = \beta_6 = \beta_7 = 0$ vs $H_A: \exists j \in \{5,6,7\}: \beta_j \neq 0$.

1) If reject, main effects must be considered in the context of interactions (ie cannot understand the main effect directly),

2) If not rejected, we can re-run the main effects model, which includes no interaction terms & directly assess main effects of each factor. Yay! :)