

Time Series Final Report

Forecasting Zillow Price



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Problem Description

Data Set Description

The modified Zillow dataset recorded the following metrics on California housing:

1. Sale price (median) from January 2004 - December 2015 (monthly)
2. Mortgage rate (median) from January 2004 - August 2017 (monthly)
3. Unemployment rate from January 2004 - August 2017 (monthly)
4. Rental price (median) from January 2010 - August 2017 (monthly)

| Column Name | Non-null Values | Data Type |
|---------------------------------------|-----------------|-----------|
| Date | 164 non-null | object |
| MedianSoldPrice_AllHomes.California | 144 non-null | float64 |
| MedianMortgageRate | 164 non-null | float64 |
| UnemploymentRate | 164 non-null | float64 |
| MedianRentalPrice_AllHomes.California | 92 non-null | float64 |

Total Number of Columns: 5

Total Number of Rows: 164

First five rows:

| | Date | MedianSoldPrice_AllHomes.California | MedianMortgageRate | UnemploymentRate | MedianRentalPrice_AllHomes.California |
|---|------------|-------------------------------------|--------------------|------------------|---------------------------------------|
| 0 | 2004-01-31 | 326752.55 | 5.02 | 7.9 | NaN |
| 1 | 2004-02-29 | 329501.50 | 4.94 | 7.8 | NaN |
| 2 | 2004-03-31 | 340125.45 | 4.74 | 7.8 | NaN |
| 3 | 2004-04-30 | 355329.50 | 5.16 | 7.5 | NaN |
| 4 | 2004-05-31 | 367818.15 | 5.64 | 7.3 | NaN |

Forecasting Goals

We are interested in building a model that can accurately predict the median sale and rental prices of housing in California between January 2016, and August 2017. If we can build a model that can accurately predict prices in this time period, we could potentially deploy the model on a broader scale to aid purchasers in choosing the best time to sign a mortgage or a lease.

Methodology

Data Cleaning

1. We first changed the Date to datetime objects, sorted the data set by Date in descending order from least to most recent, and set the Date as the index.
2. We then truncated the names of the columns associated with price:
 - a. MedianSoldPrice_AllHomes.California changed to MedianPrice
 - b. MedianRentalPrice_AllHomes.California changed to MedianRentalPrice
3. Next, we performed some minor exploration, and discovered a significant amount of NaN values in the MedianPrice (12.2%) and MedianRentalPrice (43.9%) columns. We replaced these missing fields with the median value of their respective columns.

Models

1. SARIMA (seasonal ARIMA) on Median Price
2. Exponential Smoothing on MedianPrice
 - a. ETS(add/add)
 - b. ETS(add/mul)
3. SARIMAX (seasonal ARIMA with exogenous variables) on Mortgage Rate
4. VAR (vector autoregression)
 - a. VAR on Median Mortgage Rate
 - b. VAR on Unemployed Rate Model
 - c. VAR on Median Rental Price Model
 - d. VAR on Unemployment rate and Median Rental Price
 - e. VAR on All Features

Results

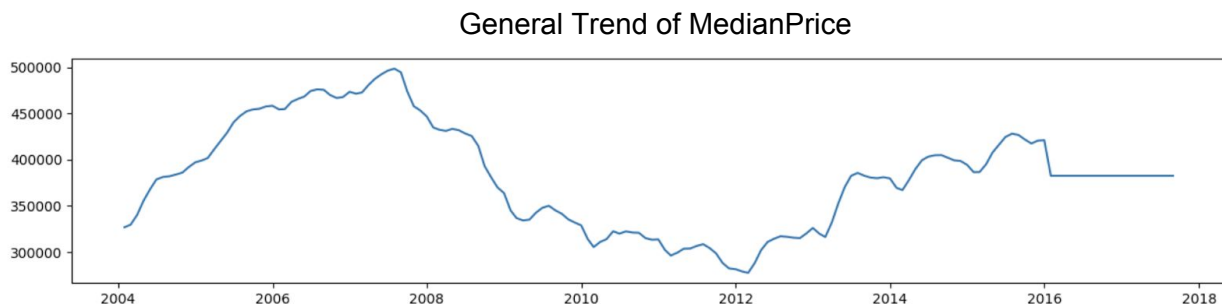
SARIMA Results Summary

We started with a SARIMA model because the graph of the MedianPrice clearly showed both trend and seasonal components. The MedianPrice steadily increased, decreased, and increased again over a period of several years. There is also a notable small decline in the sale price of homes near the end of each year, which rises again after the winter months.

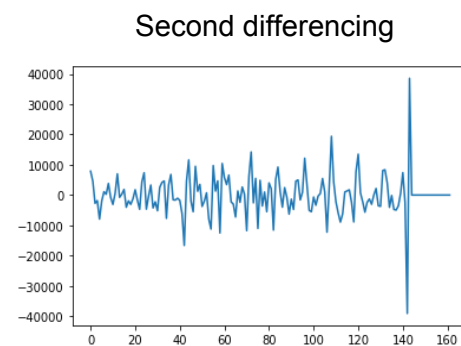
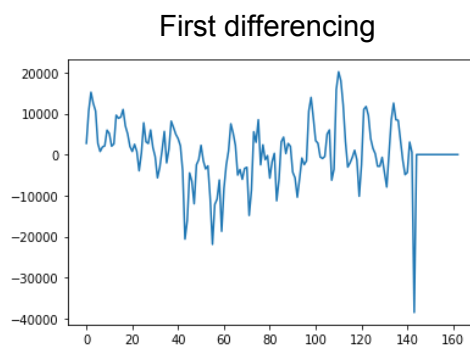
For our first run of SARIMA on the MedianPrice, we determined that $d=2$ for detrending and $D=1$ for removing the seasonality yielded the best ADF test results and the lowest RMSE. The optimal seasonality lag was $m=18$. Using `auto_arima` with AIC and BIC as our metrics, we determined a $\text{SARIMAX}(1, 2, 1) \times (1, 1, 1, 18)$ model to be the best fit. The AIC and BIC scores were $\text{AIC}=2339.16$, $\text{BIC}=2335.41$. The RMSE for this model was 34356.35.

Steps

1. First, we observed both trend and seasonality in MedianPrice.

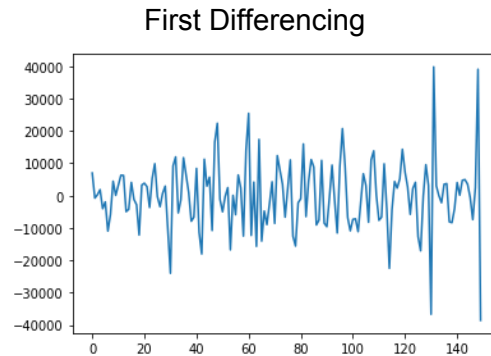


2. We differenced twice to remove the trend component.



Next obtained p-value = 0.000012 from Dickey-Fuller on the second differencing, which is less than 0.05 and therefore acceptable.

3. We differenced once to remove the seasonal component. Assumed lag = 18.



4. We split the data into test and training splits based on the 80-20 rules, and fit `auto_arma` with `d = 2` with seasonality `m = 18`.

Model Parameters

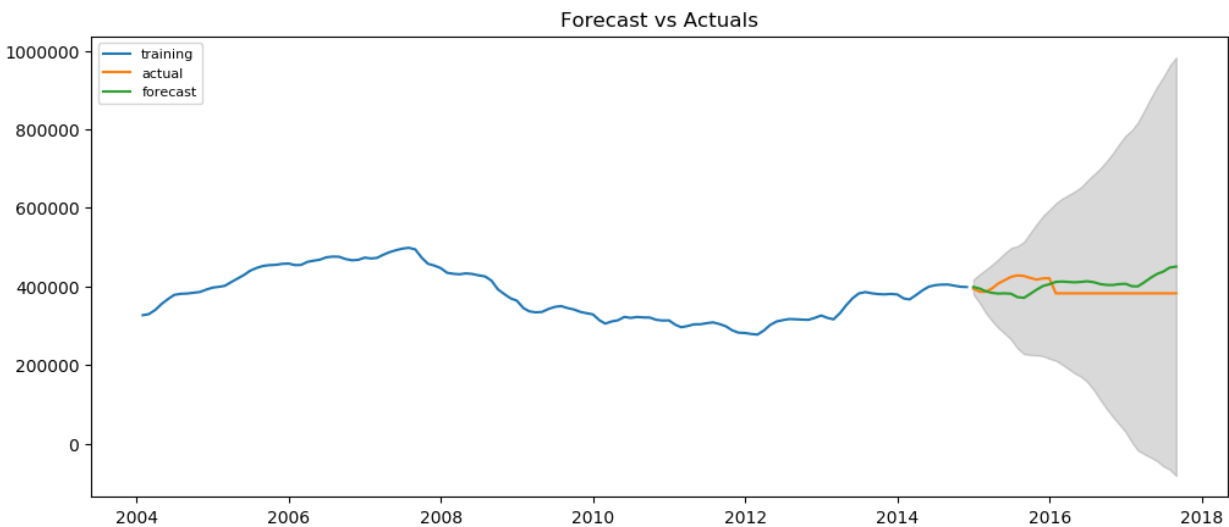
```
model = pm.auto_arma(train, start_p=1, start_q=1,
                    max_p=3, max_q=3, m=18,
                    start_P=0, seasonal=True,
                    d=2, D=1, trace=True,
                    error_action='ignore',
                    suppress_warnings=True,
                    stepwise=True)
```

Summary Table

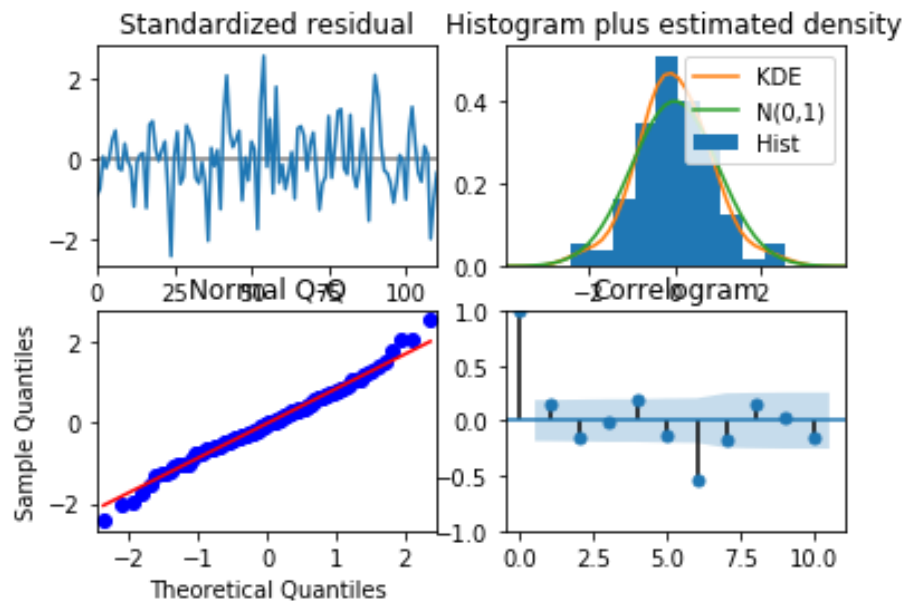
| Statespace Model Results | | | | | | |
|--------------------------|--------------------------------|-------------------|----------|-------|-----------|----------|
| ===== | | | | | | |
| Dep. Variable: | y | No. Observations: | 131 | | | |
| Model: | SARIMAX(1, 2, 1)x(0, 2, 1, 18) | Log Likelihood | -999.179 | | | |
| Date: | Wed, 11 Dec 2019 | AIC | 2008.358 | | | |
| Time: | 11:08:28 | BIC | 2021.021 | | | |
| Sample: | 0 | HQIC | 2013.471 | | | |
| | - 131 | | | | | |
| Covariance Type: | opg | | | | | |
| ===== | | | | | | |
| | coef | std err | z | P> z | [0.025 | 0.975] |
| ----- | | | | | | |
| intercept | 102.1136 | 765.480 | 0.133 | 0.894 | -1398.200 | 1602.428 |
| ar.L1 | -0.9999 | 0.248 | -4.038 | 0.000 | -1.485 | -0.515 |
| ma.L1 | 0.9986 | 0.212 | 4.701 | 0.000 | 0.582 | 1.415 |
| ma.S.L18 | -0.9943 | 0.222 | -4.480 | 0.000 | -1.429 | -0.559 |
| sigma2 | 1.049e+08 | 0.000 | 3.66e+11 | 0.000 | 1.05e+08 | 1.05e+08 |
| ===== | | | | | | |
| Ljung-Box (Q): | 295.60 | Jarque-Bera (JB): | 1.02 | | | |
| Prob(Q): | 0.00 | Prob(JB): | 0.60 | | | |
| Heteroskedasticity (H): | 0.89 | Skew: | 0.14 | | | |
| Prob(H) (two-sided): | 0.75 | Kurtosis: | 2.56 | | | |

Based on AIC and BIC, the best model is SARIMAX(1, 2, 1)x(0, 2, 1, 18). The AIC and BIC scores were AIC=2339.16, BIC=2335.41. The RMSE for this model was 34356.35.

5. We then fit this model with the training set and forecast using the testing set.



6. We then checked the residual diagnostics.



The diagnostics demonstrate White Noise behaviors: zero-mean, constant variance and non-correlation. Normality is also demonstrated.

Exponential Smoothing Results Summary

Using exponential smoothing, the best model chosen from the RMSE evaluation is additive for trend and additive for seasonality with a RMSE of 18358.22. The RMSE for additive trend and multiplicative seasonality was 19220.75.

Steps

1. We fit two models, ETS(add/add), ETS(add/mul) with seasonal_periods = 12.
2. We then drew the forecasting plot.



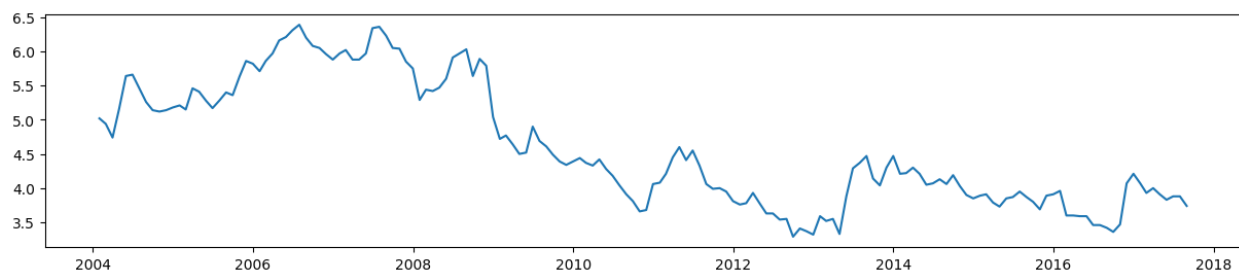
SARIMAX(MortgageRate) Results Summary

For this model, we introduced MortgageRate as an additional feature to predict MedianPrice. We first started with a SARIMA model because the graph of the MortgageRate clearly showed both trend and seasonal components. MortgageRate steadily increased, decreased, and increased again over a period of several years. There is also a notable small decline in MortgageRate near the end of each year, which rises again after the winter months.

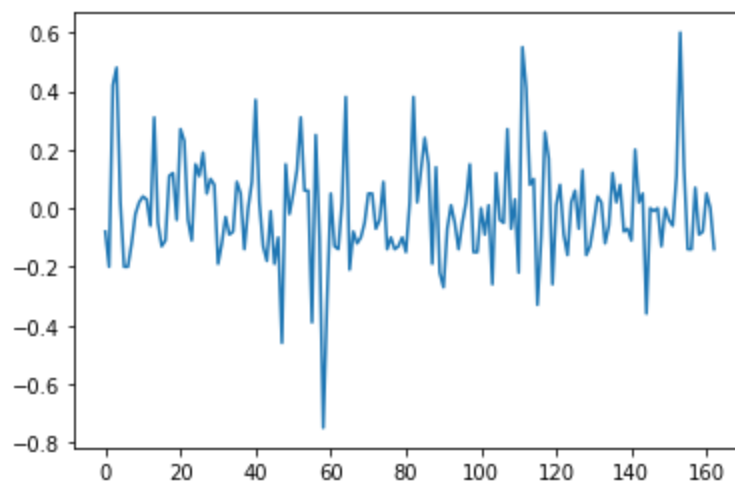
We determined that $d=1$ for trend yielded the best ADF test results and the lowest RMSE. No seasonality lag was required.

Using `auto_arima` with AIC and BIC as our metrics, we determined a SARIMAX(1, 1, 1) model to be the best fit. The AIC and BIC scores were AIC=2655.46, BIC=2666.93. The RMSE for this model was 19150.73.

1. First, we observed both trend and seasonality in MortgageRate.



2. We differenced once to eliminate the trend component.



Dickey-Fuller Test p-value = 1.268969e-08.

- We then fit a SARIMAX(1,1,1) model with train and exog_train.

Summary

```

=====
Statespace Model Results
=====
Dep. Variable:      MedianPrice      No. Observations:      131
Model:              SARIMAX(1, 1, 1)  Log Likelihood         -1323.732
Date:               Wed, 11 Dec 2019  AIC                      2655.464
Time:               11:16:22          BIC                    2666.934
Sample:             01-31-2004        HQIC                   2660.125
                  - 11-30-2014
Covariance Type:    opg
=====

```

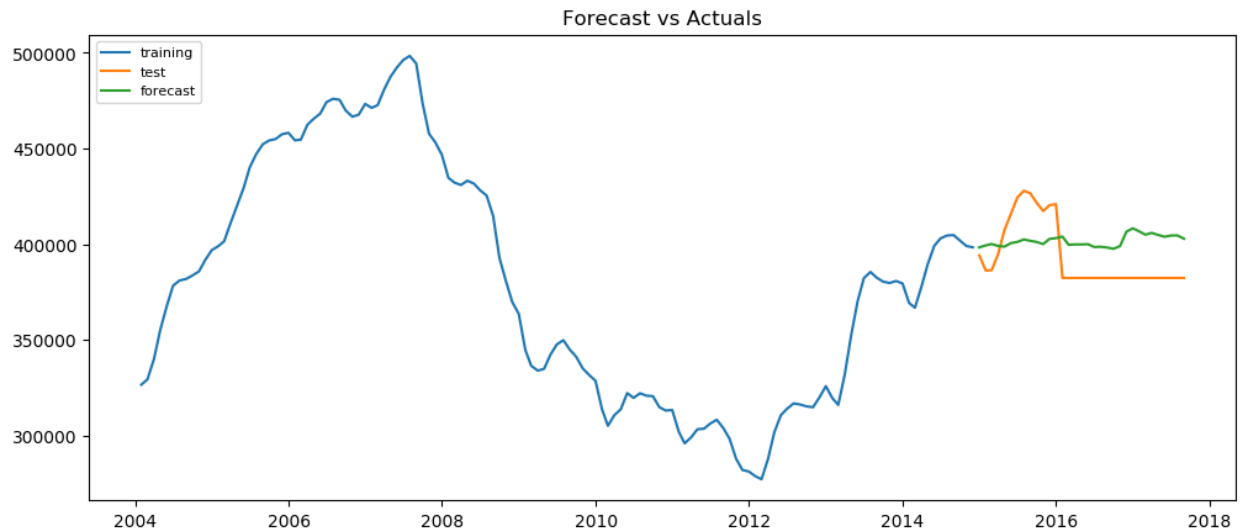
| | coef | std err | z | P> z | [0.025 | 0.975] |
|--------|-----------|----------|----------|-------|----------|----------|
| x1 | 1.232e+04 | 2733.706 | 4.506 | 0.000 | 6960.419 | 1.77e+04 |
| ar.L1 | 0.9151 | 0.058 | 15.874 | 0.000 | 0.802 | 1.028 |
| ma.L1 | -0.8762 | 0.067 | -13.137 | 0.000 | -1.007 | -0.745 |
| sigma2 | 4.088e+07 | 0.053 | 7.71e+08 | 0.000 | 4.09e+07 | 4.09e+07 |

```

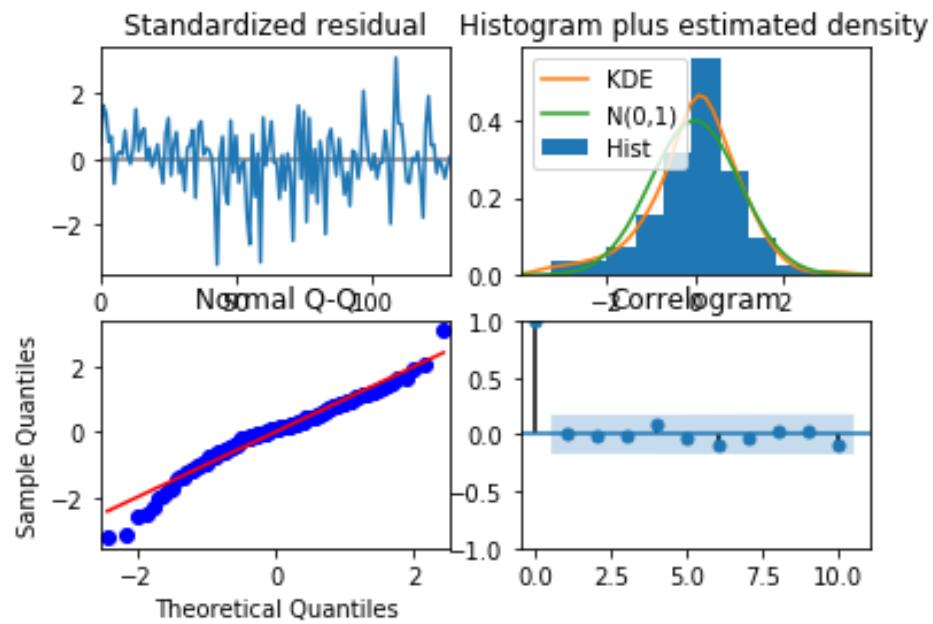
=====
Ljung-Box (Q):      252.36      Jarque-Bera (JB):      9.17
Prob(Q):            0.00        Prob(JB):              0.01
Heteroskedasticity (H): 3.34    Skew:                  0.14
Prob(H) (two-sided):  0.00      Kurtosis:              4.27
=====

```

- We then fit this model with the training set and forecast using the testing set.



7. We then checked the residual diagnostics.



The diagnostics demonstrate White Noise behaviors: zero-mean, constant variance and non-correlation. Normality is also demonstrated.

VAR Results Summary

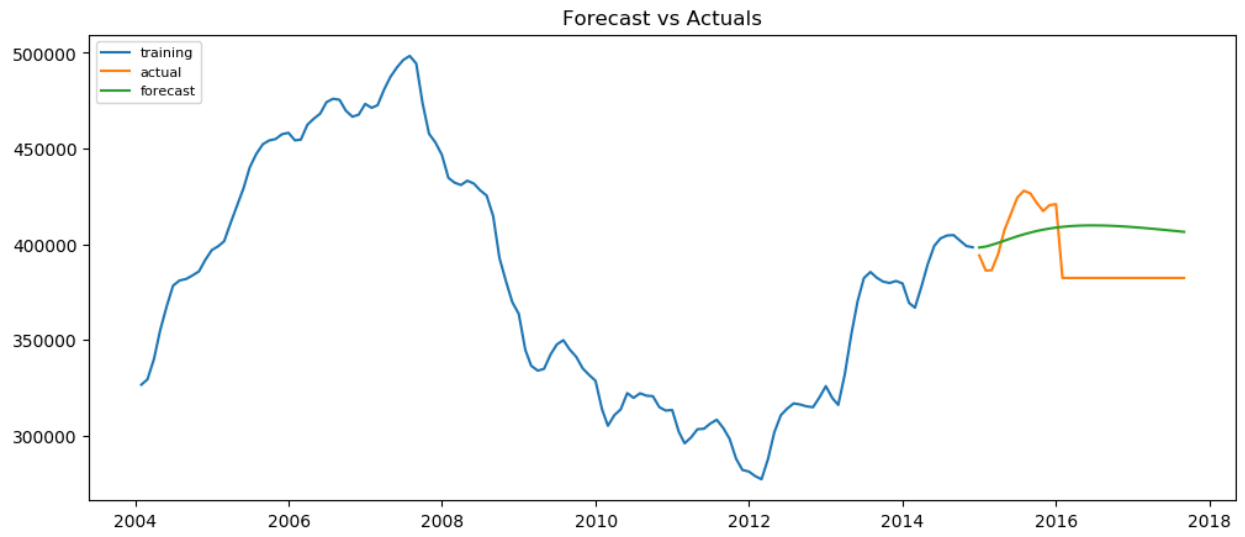
1. VAR (Mortgage Rate)

Using the feature median mortgage rate as an endogenous feature to predict median price, the RMSE was computed to be 22329.18 with p=2.

```
Summary of Regression Results
=====
Model:                                VAR
Method:                               OLS
Date:      Wed, 11, Dec, 2019
Time:      11:18:51
-----
No. of Equations:      2.00000      BIC:                                13.9991
Nobs:                  129.000      HQIC:                               13.8675
Log likelihood:        -1244.73     FPE:                                962718.
AIC:                   13.7774      Det(Omega_mle):                     892214.
-----
Results for equation MedianMortgageRate
=====
               coefficient      std. error      t-stat      prob
-----
const                -0.021004      0.098269      -0.214      0.831
L1.MedianMortgageRate  1.041507      0.091833      11.341      0.000
L1.MedianPrice         0.000005      0.000002       2.084      0.037
L2.MedianMortgageRate  -0.126380      0.091162      -1.386      0.166
L2.MedianPrice         -0.000004      0.000002      -1.548      0.122
=====

Results for equation MedianPrice
=====
               coefficient      std. error      t-stat      prob
-----
const                5046.023219      2941.929625       1.715      0.086
L1.MedianMortgageRate  780.099785      2749.256266       0.284      0.777
L1.MedianPrice         1.666141       0.068942      24.167      0.000
L2.MedianMortgageRate -1281.873974      2729.176325      -0.470      0.639
L2.MedianPrice        -0.672617       0.071364      -9.425      0.000
=====

Correlation matrix of residuals
      MedianMortgageRate  MedianPrice
MedianMortgageRate      1.000000      0.217809
MedianPrice              0.217809      1.000000
```



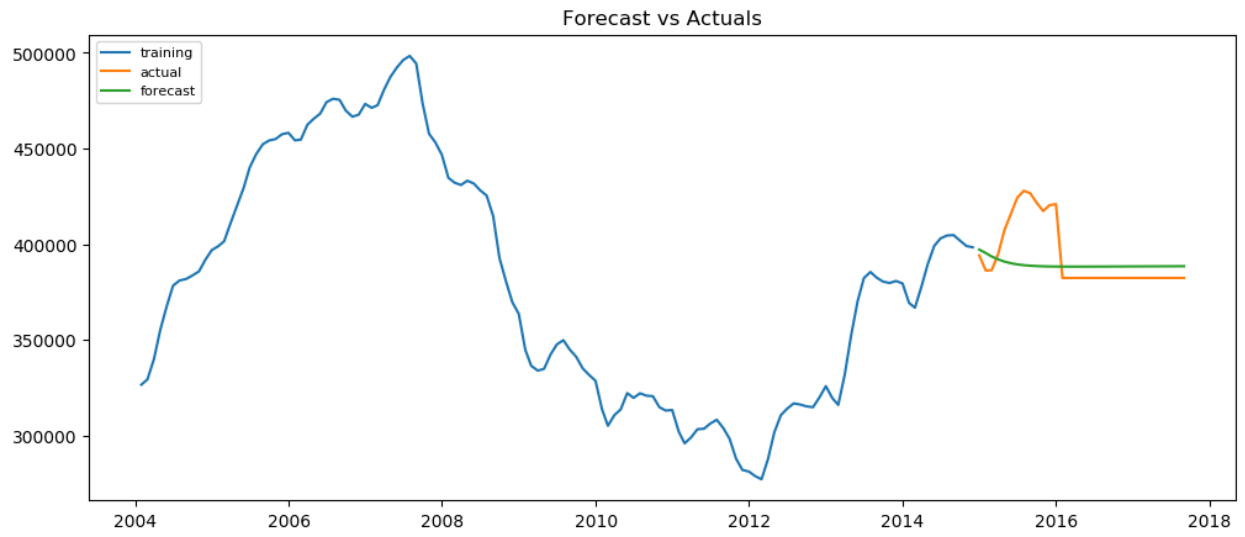
2. VAR (Unemployed Rate)

Using the feature of unemployment rate as an endogenous feature to predict median price, the RMSE was computed to be 17390.46 with $p=2$.

```
Summary of Regression Results
=====
Model:                                VAR
Method:                               OLS
Date:      Wed, 11, Dec, 2019
Time:      11:21:42
-----
No. of Equations:      2.00000      BIC:                                16.1327
Nobs:                  128.000      HQIC:                               15.9475
Log likelihood:        -1361.78      FPE:                                7.42943e+06
AIC:                   15.8207      Det(Omega_mle):                     6.67895e+06
-----
Results for equation UnemploymentRate
=====
              coefficient      std. error      t-stat      prob
-----
const              3.967600      0.989520      4.010      0.000
L1.UnemploymentRate  0.640480      0.090069      7.111      0.000
L1.MedianPrice      0.000005      0.000008      0.643      0.520
L2.UnemploymentRate -0.121786      0.106489     -1.144      0.253
L2.MedianPrice      -0.000021      0.000015     -1.349      0.177
L3.UnemploymentRate  0.107360      0.091033      1.179      0.238
L3.MedianPrice      0.000012      0.000008      1.375      0.169
=====

Results for equation MedianPrice
=====
              coefficient      std. error      t-stat      prob
-----
const     -14182.148118     10360.955059     -1.369      0.171
L1.UnemploymentRate    709.035749      943.083412      0.752      0.452
L1.MedianPrice         1.802578      0.088135     20.452      0.000
L2.UnemploymentRate    804.626088     1115.009805      0.722      0.471
L2.MedianPrice        -1.019221      0.159543     -6.388      0.000
L3.UnemploymentRate    173.487860      953.174878      0.182      0.856
L3.MedianPrice         0.223085      0.088287      2.527      0.012
=====

Correlation matrix of residuals
      UnemploymentRate  MedianPrice
UnemploymentRate      1.000000     -0.103813
MedianPrice           -0.103813      1.000000
```



3. VAR (Median Rental Price)

Using the feature of rate as an endogenous feature to predict median price, the RMSE was computed to be 15423.91 with $p=2$.

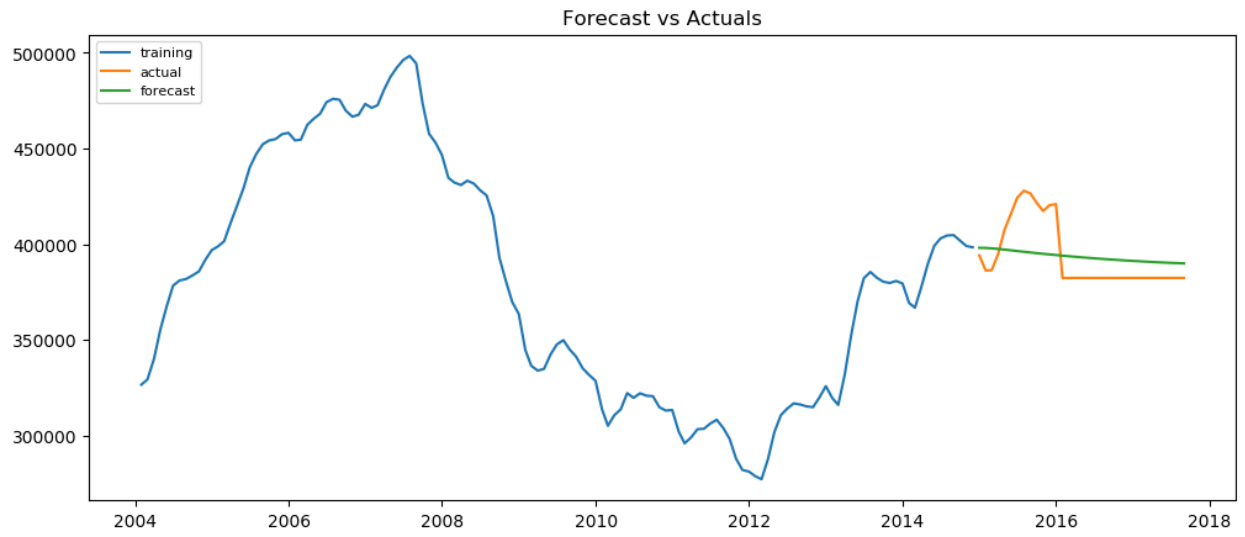
```

Summary of Regression Results
=====
Model:                                VAR
Method:                               OLS
Date:      Wed, 11, Dec, 2019
Time:      11:22:05
-----
No. of Equations:      2.00000      BIC:                                25.5800
Nobs:                  128.000      HQIC:                               25.3948
Log likelihood:        -1966.40      FPE:                                9.41577e+10
AIC:                   25.2680      Det(Omega_mle):                     8.46464e+10
-----
Results for equation MedianRentalPrice
=====
               coefficient      std. error      t-stat      prob
-----
const                64.913265      63.664866      1.020      0.308
L1.MedianRentalPrice    0.810301      0.090550      8.949      0.000
L1.MedianPrice          0.000344      0.000933      0.368      0.713
L2.MedianRentalPrice    0.240874      0.114621      2.101      0.036
L2.MedianPrice          -0.000465      0.001702     -0.273      0.785
L3.MedianRentalPrice   -0.100882      0.090188     -1.119      0.263
L3.MedianPrice          0.000219      0.000934      0.234      0.815
=====

Results for equation MedianPrice
=====
               coefficient      std. error      t-stat      prob
-----
const             10549.994884      5970.335881      1.767      0.077
L1.MedianRentalPrice    0.930753      8.491581      0.110      0.913
L1.MedianPrice          1.811016      0.087477     20.703      0.000
L2.MedianRentalPrice    6.702343     10.748906      0.624      0.533
L2.MedianPrice         -1.034985      0.159586     -6.485      0.000
L3.MedianRentalPrice   -11.081181      8.457627     -1.310      0.190
L3.MedianPrice          0.215195      0.087573      2.457      0.014
=====

Correlation matrix of residuals
      MedianRentalPrice  MedianPrice
MedianRentalPrice      1.000000    -0.070086
MedianPrice            -0.070086     1.000000

```

4. VAR (Unemployment rate and Median Rental Price)

Using the feature of rate as an endogenous feature to predict median price, the RMSE was computed to be 19263.65 with $p=3$.

```

Summary of Regression Results
=====
Model:                VAR
Method:               OLS
Date:                 Tue, 10, Dec, 2019
Time:                 23:28:08
=====
No. of Equations:      3.00000    BIC:                24.5524
Nobs:                  128.000    HQIC:               24.1555
Log likelihood:        -2043.45    FPE:                2.36094e+10
AIC:                   23.8840    Det(Omega_mle):     1.88399e+10
=====

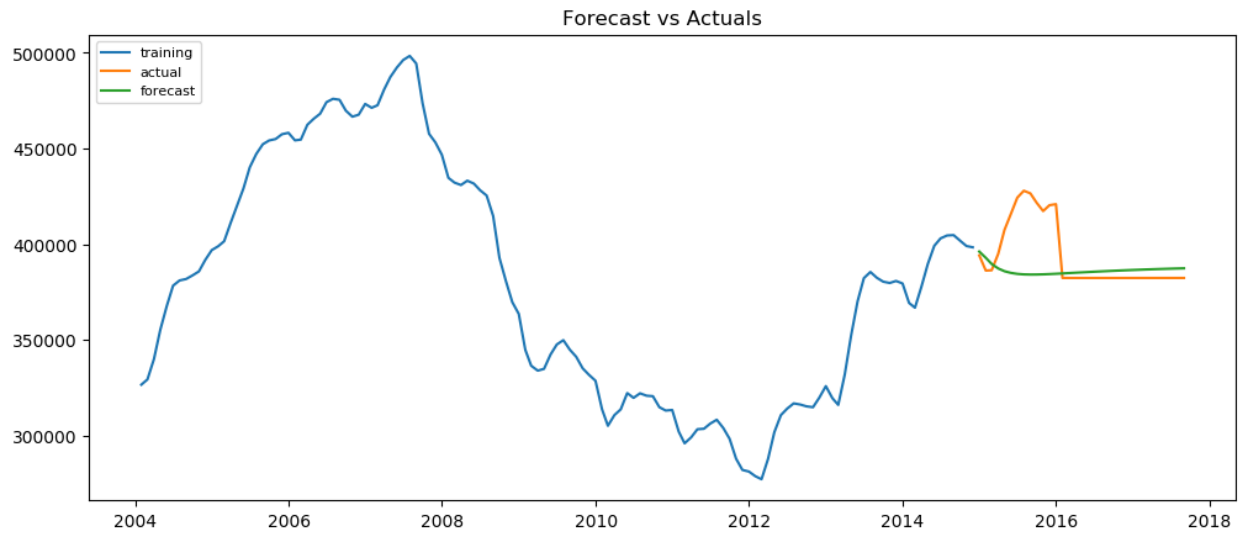
Results for equation UnemploymentRate
=====
              coefficient      std. error      t-stat      prob
-----
const          4.308374         1.016944         4.237       0.000
L1.UnemploymentRate  0.605508         0.091528         6.616       0.000
L1.MedianRentalPrice 0.001148         0.000837         1.372       0.170
L1.MedianPrice       0.000008         0.000009         0.970       0.332
L2.UnemploymentRate -0.140065         0.106573        -1.314       0.189
L2.MedianRentalPrice 0.000160         0.001022         0.157       0.876
L2.MedianPrice       -0.000022         0.000015        -1.455       0.146
L3.UnemploymentRate  0.048865         0.095817         0.510       0.610
L3.MedianRentalPrice -0.000849         0.000811        -1.047       0.295
L3.MedianPrice       0.000009         0.000009         1.048       0.295
=====

Results for equation MedianRentalPrice
=====
              coefficient      std. error      t-stat      prob
-----
const        -189.087773         112.153249        -1.686       0.092
L1.UnemploymentRate  16.376584         10.094150         1.622       0.105
L1.MedianRentalPrice  0.743884         0.092306         8.059       0.000
L1.MedianPrice       -0.000052         0.000939        -0.056       0.956
L2.UnemploymentRate  7.478008         11.753393         0.636       0.525
L2.MedianRentalPrice  0.231547         0.112720         2.054       0.040
L2.MedianPrice       -0.000292         0.001676        -0.174       0.862
L3.UnemploymentRate  7.937336         10.567095         0.751       0.453
L3.MedianRentalPrice -0.068504         0.089398        -0.766       0.444
L3.MedianPrice       0.000757         0.000946         0.800       0.424
=====

Results for equation MedianPrice
=====
              coefficient      std. error      t-stat      prob
-----
const        -12486.437952        10543.955185        -1.184       0.236
L1.UnemploymentRate  1082.653859         948.989560         1.141       0.254
L1.MedianRentalPrice -5.342086         8.678006         -0.616       0.538
L1.MedianPrice       1.769153         0.088249        20.047       0.000
L2.UnemploymentRate  1020.185341        1104.981308         0.923       0.356
L2.MedianRentalPrice  5.973705         10.597283         0.564       0.573
L2.MedianPrice       -1.013191         0.157607        -6.429       0.000
L3.UnemploymentRate  804.224998         993.452935         0.810       0.418
L3.MedianRentalPrice -8.117699         8.404664         -0.966       0.334
L3.MedianPrice       0.263939         0.088949         2.967       0.003
=====

Correlation matrix of residuals
UnemploymentRate  MedianRentalPrice  MedianPrice
UnemploymentRate    1.000000      0.066751     -0.084340
MedianRentalPrice    0.066751      1.000000     -0.137841
MedianPrice          -0.084340     -0.137841      1.000000

```



5. VAR (All Features)

Using the feature of rate as an endogenous feature to predict median price, the RMSE was computed to be 28470.47 with $p=4$.

```

Summary of Regression Results
=====
Model:                VAR
Method:               OLS
Date:                 Tue, 10, Dec, 2019
Time:                 23:28:08
=====
No. of Equations:     4.00000    BIC:                21.3454
Nobs:                 130.000    HQIC:              21.0835
Log likelihood:       -2076.63    FPE:                1.19860e+09
AIC:                  20.9043    Det(Omega_mle):    1.03065e+09
=====
Results for equation MedianMortgageRate
=====
              coefficient      std. error      t-stat      prob
-----
const          -0.267206         0.309395       -0.864       0.388
L1.MedianMortgageRate  0.901033         0.037537       24.004       0.000
L1.MedianPrice    0.000002         0.000001        2.777       0.005
L1.MedianRentalPrice 0.000009         0.000106        0.089       0.929
L1.UnemploymentRate 0.020337         0.026989        0.754       0.451
=====

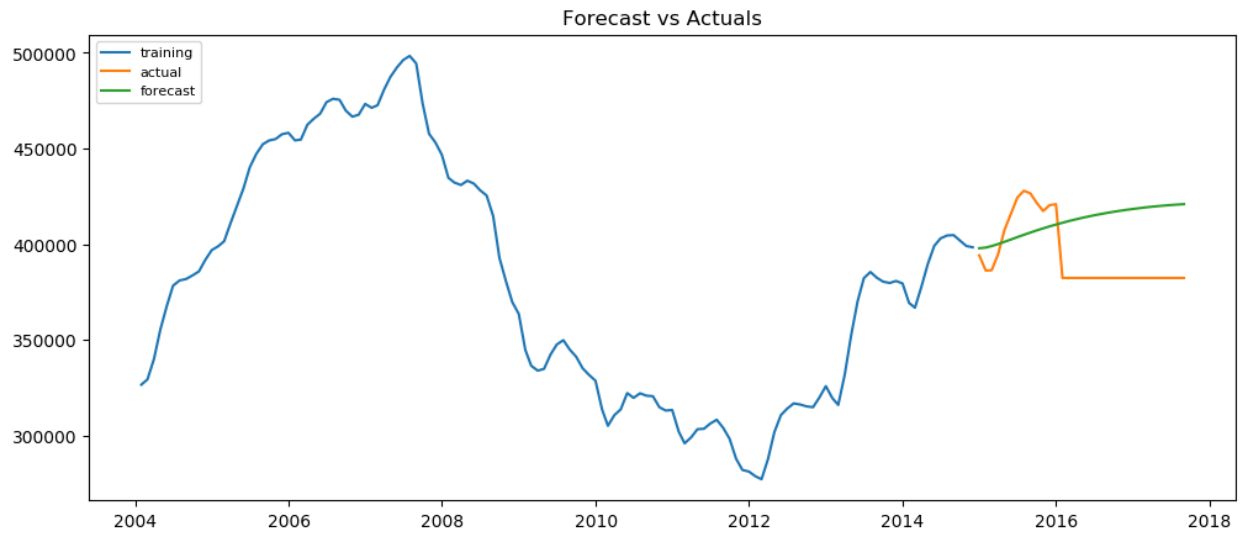
Results for equation MedianPrice
=====
              coefficient      std. error      t-stat      prob
-----
const        -16547.423050       11576.353908       -1.429       0.153
L1.MedianMortgageRate -1793.048907       1404.497039       -1.277       0.202
L1.MedianPrice   1.050804         0.020353       51.630       0.000
L1.MedianRentalPrice -9.976747         3.971683        -2.512       0.012
L1.UnemploymentRate 3824.911829       1009.807022        3.788       0.000
=====

Results for equation MedianRentalPrice
=====
              coefficient      std. error      t-stat      prob
-----
const        -78.873573         92.370912        -0.854       0.393
L1.MedianMortgageRate  3.992310         11.206868        0.356       0.722
L1.MedianPrice    0.000229         0.000162        1.408       0.159
L1.MedianRentalPrice 0.922195         0.031691       29.099       0.000
L1.UnemploymentRate 18.812985         8.057528        2.335       0.020
=====

Results for equation UnemploymentRate
=====
              coefficient      std. error      t-stat      prob
-----
const          3.925163         0.828940        4.735       0.000
L1.MedianMortgageRate  0.060170         0.100571        0.598       0.550
L1.MedianPrice   -0.000005         0.000001       -3.435       0.001
L1.MedianRentalPrice 0.000348         0.000284        1.224       0.221
L1.UnemploymentRate 0.568192         0.072309        7.858       0.000
=====

Correlation matrix of residuals
MedianMortgageRate  MedianPrice  MedianRentalPrice  UnemploymentRate
MedianMortgageRate    1.000000    0.288388    0.000990    -0.021122
MedianPrice            0.288388    1.000000   -0.101017    0.018170
MedianRentalPrice      0.000990   -0.101017    1.000000    0.053939
UnemploymentRate      -0.021122    0.018170    0.053939    1.000000

```



Model Summary & Conclusion

| Model | RMSE |
|--|-----------------|
| SARIMA | 34356.35 |
| ETS (add/add) | 18358.22 |
| ETS (add/mul) | 19220.75 |
| SARIMAX | 19150.73 |
| VAR on Median Mortgage Rate | 22329.19 |
| VAR on Unemployed Rate Model | 17390.69 |
| VAR on Median Rental Price Model | 15423.91 |
| VAR on Unemployment rate and Median Rental Price | 19263.65 |
| VAR on All Features | 28470.47 |

We determined VAR (Median Rental Price) was our best model, with an RMSE of 15423.91.

This is likely because VAR is optimized for multiple time series that influence each other.

Both Median Rental Price and Median Sale Price are subject to many of the same exogenous forces which determine the cost of housing, such as the supply of vacant units and the income of the people living in the area.

These variables also endogenously influence each other, since if rental prices get high enough that it becomes economical to purchase a home, the increased demand on home purchases will drive up home purchase prices. Similarly, increases in the price of housing can make it more economical to rent, which will drive up rent prices.

Forecasting Results

Prediction Table

| Date | ARIMA | ETS(add/add) | ETS(add/mul) | SARIMAX(Mortgage Rate) | VAR(Mortgage Rate) | VAR(Unemployed Rate) | VAR(Median Rental Price) |
|------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|--------------------------|
| 2014-12-31 | 391436.63 513036900 | 395992.8106 8571600 | 396820.874717 6020 | 398440.057956 2530 | 398363.045057 54800 | 397275.290556 4390 | 398208.897685 7380 |
| 2015-01-31 | 374593.99 521364 | 386499.0552 4032600 | 386710.747723 0130 | 399464.459613 7580 | 398852.500173 1120 | 395485.114774 57800 | 398130.677671 0070 |
| 2015-02-28 | 350937.52 674160200 | 382691.0816 0577900 | 382351.871243 4190 | 400197.367915 21000 | 399724.353696 95800 | 393681.946850 51600 | 397927.596071 0930 |
| 2015-03-31 | 328657.46 70212170 | 392555.7128 2353100 | 393646.187742 68100 | 399164.406719 30800 | 400799.629395 71300 | 392171.594557 40000 | 397640.563722 1780 |
| 2015-04-30 | 308130.33 594193500 | 403051.5864 186010 | 405244.296299 8200 | 398832.758061 2030 | 401955.338692 5180 | 391006.930715 7570 | 397286.085762 5200 |
| 2015-05-31 | 289764.42 83758740 | 410019.9408 788870 | 412204.665348 09700 | 400683.835671 6060 | 403110.736317 7880 | 390154.144078 1190 | 396907.794882 3540 |
| 2015-06-30 | 268988.77 944083700 | 411598.5583 221230 | 413522.806267 30400 | 401271.426968 534 | 404214.553103 86600 | 389553.815809 0040 | 396524.502149 21400 |
| 2015-07-31 | 238454.80 783249100 | 411092.0908 7429400 | 413474.553770 07200 | 402569.159716 1480 | 405235.722994 04400 | 389141.877351 23100 | 396148.209559 8790 |
| 2015-08-31 | 216818.98 229275000 | 408684.8854 163860 | 411256.236955 3260 | 401869.447321 61100 | 406156.904156 2010 | 388863.776813 5590 | 395782.458363 7180 |
| 2015-09-30 | 210592.42 87015930 | 405064.2727 3872200 | 407286.669950 4690 | 401268.664775 30400 | 406969.988606 4580 | 388678.843905 22900 | 395428.414258 5650 |
| 2015-10-31 | 205387.64 295960200 | 400996.0336 460640 | 401982.189466 3950 | 400152.951398 5940 | 407672.998985 8830 | 388558.159190 7790 | 395085.859029 4880 |
| 2015-11-30 | 197960.40 83358520 | 397824.2566 207290 | 398013.788824 11300 | 402835.625861 45300 | 408267.949218 52400 | 388481.480220 1470 | 394754.459413 69000 |
| 2015-12-31 | 184405.41 621656500 | 396029.7819 2192100 | 396916.803934 67700 | 403282.403387 62900 | 408759.374241 53700 | 388434.835104 87400 | 394433.895270 01300 |
| 2016-01-31 | 168104.06 98505950 | 386505.2978 7645600 | 386730.425051 93100 | 404081.722504 8290 | 409153.323880 2750 | 388408.661237 8670 | 394123.961902 8320 |
| 2016-02-29 | 150950.62 036698800 | 382692.1356 821320 | 382355.966980 4730 | 399814.937469 3960 | 409456.678480 0790 | 388396.414668 19800 | 393824.512950 9180 |
| 2016-03-31 | 130647.83 031248500 | 392555.8908 0554600 | 393647.075465 9730 | 399968.526068 5170 | 409676.687375 02100 | 388393.607629 2890 | 393535.433110 9770 |
| 2016-04-30 | 110600.09 196832500 | 403051.6164 7107000 | 405244.488694 9590 | 399985.894844 9740 | 409820.661481 2460 | 388397.159534 90000 | 393256.610272 21700 |

| | | | | | | | |
|------------|-------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| 2016-05-31 | 93222.256 91163660 | 410019.9459 532820 | 412204.706548 0120 | 400114.517942 17400 | 409895.772293 7390 | 388404.954507 8420 | 392987.925498 9390 |
| 2016-06-30 | 69033.961 48455320 | 411598.5591 789400 | 413522.814968 70600 | 398630.833964 45500 | 409908.924151 7860 | 388415.538439 60700 | 392729.248468 72400 |
| 2016-07-31 | 34207.964 374855000 | 411092.0910 189690 | 413474.555601 7330 | 398738.549649 2650 | 409866.676774 27800 | 388427.912151 346 | 392480.437390 0820 |
| 2016-08-31 | -7929.4205 11092050 | 408684.8854 4081400 | 411256.237338 8700 | 398344.387460 94400 | 409775.202109 05600 | 388441.389823 6700 | 392241.339895 2010 |
| 2016-09-30 | -48288.139 447656600 | 405064.2727 4284700 | 407286.670030 436 | 397695.491078 1530 | 409640.264434 27000 | 388455.501636 3240 | 392011.794330 5690 |
| 2016-10-31 | -86439.731 8021373 | 400996.0336 4676000 | 401982.189483 0110 | 399133.063798 7730 | 409467.216049 97700 | 388469.926680 407 | 391791.630895 521 |
| 2016-11-30 | -121943.02 86496130 | 397824.2566 2084700 | 398013.788827 57700 | 406599.638722 72300 | 409261.003260 53100 | 388484.446714 0240 | 391580.672640 05700 |
| 2016-12-31 | -159864.53 896964800 | 396029.7819 219410 | 396916.803935 4050 | 408393.344613 8830 | 409026.178989 30600 | 388498.914266 0700 | 391378.736323 5110 |
| 2017-01-31 | -208376.84 61212520 | 386505.2978 7645900 | 386730.425052 08100 | 406732.035016 3710 | 408766.919507 0860 | 388513.230624 8040 | 391185.633188 23100 |
| 2017-02-28 | -246741.59 190473500 | 382692.1356 821330 | 382355.966980 50400 | 405065.355801 9830 | 408487.043546 85300 | 388527.330647 5860 | 391001.169673 1690 |
| 2017-03-31 | -267736.94 524152500 | 392555.8908 0554600 | 393647.075465 97900 | 405980.623609 84300 | 408190.032627 0140 | 388541.172279 70600 | 390825.148085 1830 |
| 2017-04-30 | -287337.49 368354800 | 403051.6164 7107000 | 405244.488694 96000 | 404920.452991 702 | 407879.051786 16000 | 388554.729323 301 | 390657.367234 4130 |
| 2017-05-31 | -309157.09 130508800 | 410019.9459 532820 | 412204.706548 012 | 403979.351098 06300 | 407556.970196 59100 | 388567.986449 1960 | 390497.623036 80400 |
| 2017-06-30 | -337530.93 47557370 | 411598.5591 789400 | 413522.814968 7060 | 404635.873425 8120 | 407226.381306 67300 | 388580.935756 7120 | 390345.709084 7290 |
| 2017-07-31 | -368946.63 80812870 | 411092.0910 189690 | 413474.555601 73300 | 404673.030272 4310 | 406889.622288 4980 | 388593.574401 6900 | 390201.417186 3970 |
| 2017-08-31 | -400715.27 85146090 | 408684.8854 4081400 | 411256.237338 8700 | 402982.459472 8200 | 406548.792654 411 | 388605.902961 5880 | 390064.537874 7060 |

Responsibility Table

| | | | | |
|--------------------|---|---|---|---|
| Group Members | Vivian Chu | Ding Ding | Max Calehuff | Dillon Quan |
| Proportion of Work | 25% | 25% | 25% | 25% |
| List of Work | <p>Initial check for the data(summary tables and graphs)</p> <p>Discussion of problems and chosen methods</p> <p>Data Cleaning and Modeling</p> <p>Forecasting and analyzing the forecasting results</p> <p>Discussion of model selection Results</p> | <p>Initial check for the data(summary tables and graphs)</p> <p>Discussion of problems and chosen methods</p> <p>Final Discussion</p> <p>Write the Report</p> | <p>Initial check for the data(summary tables and graphs)</p> <p>Discussion of problems and chosen methods</p> <p>Final Discussion</p> <p>Write the Report</p> | <p>Initial check for the data(summary tables and graphs)</p> <p>Discussion of problems and chosen methods</p> <p>Model Fitting</p> <p>Forecasting and analyzing the forecasting results</p> <p>Final Discussion</p> <p>Prediction Table</p> |