Time Series Analysis of Historical Sunspot Data

Fig. 2 Early sunspot drawings from naked-eye observations. *Left panel*: drawing by John of Worcester, observed in 1128 CE (adapted from Vaquero and Vázquez 2009). *Right panel*: undated drawing from Tiānyuán Yùlì Xiángyìfù, manuscript 305-257 at Naikaku Bunko, Books of Shoheizaka Gakumonjo, in the National Archives of Japan [in Chinese], involved in an imperial manual of Chinese astro-omenological divination compiled in 1424–1425 (adapted from Hayakawa et al. 2017b)

In [2]:

```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import seaborn as sns
```

In [16]:

```
sun = pd.read_csv('SN_m_tot_V2.0.csv', delimiter=';', header=None)
sun = sun.iloc[:, 2:4]
sun.columns = ['year', 'spots']
sun


```

Out[16]:

	year	spots
0	1749.042	96.7
1	1749.123	104.3
2	1749.204	116.7
3	1749.288	92.8
4	1749.371	141.7
...	...	...
3288	2023.042	143.6
3289	2023.122	110.9
3290	2023.204	122.6
3291	2023.286	96.4
3292	2023.371	137.9

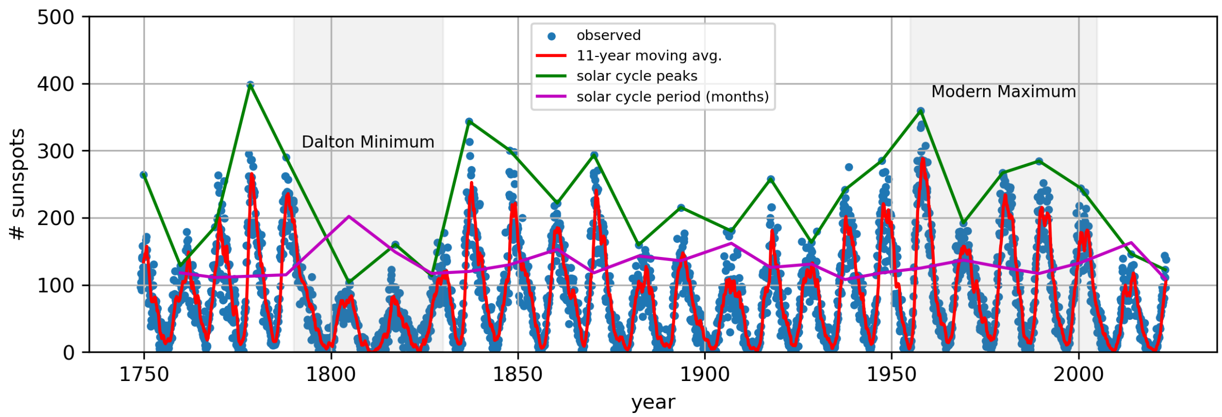
3293 rows × 2 columns

In [30]:

```
# plot the data

sun['ma11'] = sun.spots.rolling(window=11).mean()

plt.figure(figsize=(10,3), dpi=300)
plt.scatter(sun.year, sun.spots, marker='.', label='observed')
plt.plot(sun.year, sun.ma11, c='r' , label='11-year moving avg.')
plt.plot(peak_years, peak_spots, c='g', label='solar cycle peaks')
plt.plot(peak_df.year, peak_df.period*12, c='m', label='solar cycle per
plt.xlabel('year')
plt.ylabel('# sunspots')
plt.ylim(0, 500)
plt.axvspan(1955, 2005, alpha=.1, color='grey')
plt.axvspan(1790, 1830, alpha=.1, color='grey')
plt.grid()
plt.legend(loc='upper center', fontsize='x-small')
plt.text(1810, 305, 'Dalton Minimum', ha='center', fontsize=8)
plt.text(1980, 380, 'Modern Maximum', ha='center', fontsize=8)
plt.show()
```



In [19]:

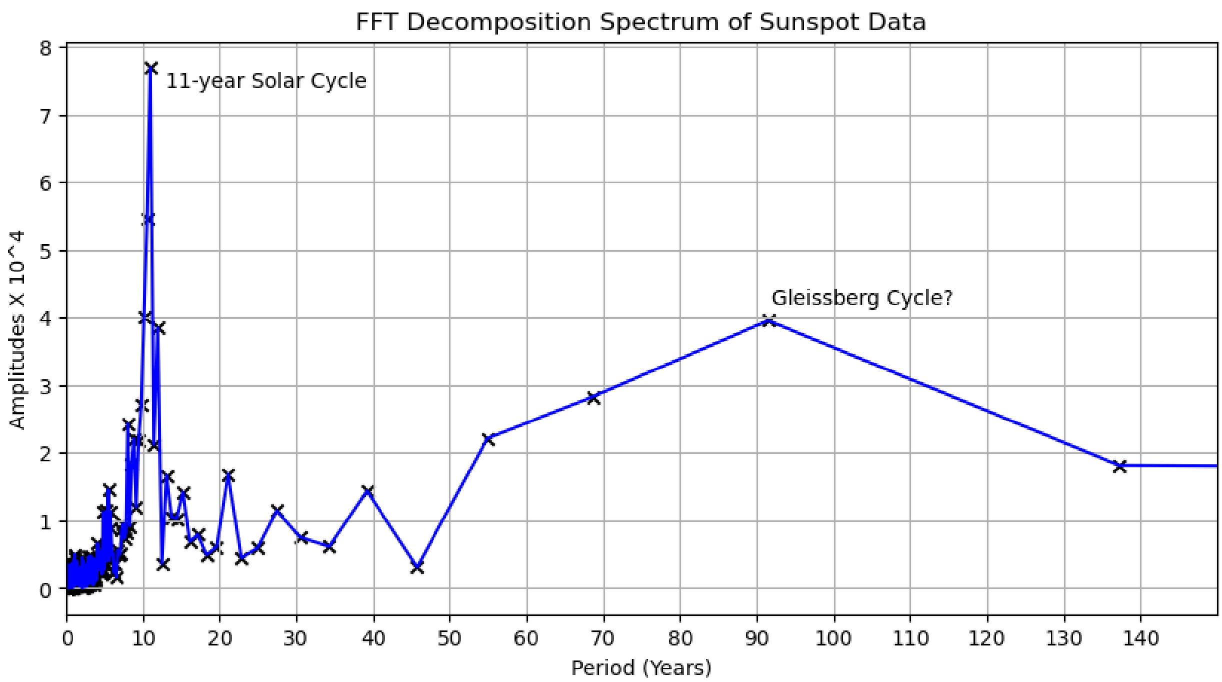
```
# Fast-Fourier Transform Decomposition
N = sun.shape[0] # number of data points
T = 1.0 / 12.0   # monthly observation
xf = np.fft.fftfreq(N,T)
yf = np.fft.fft(sun.spots)

# Keep only positive frequencies
# Remove symmetric values
frequencies = xf[1:N//2]
amplitudes = np.abs(yf[1:N//2]) / 10**4

periods = 1.0 / frequencies   # convert frequences to periods
```

In [20]:

```
# Create a plot
plt.figure(figsize=(10, 5))
plt.scatter(periods, amplitudes, marker = 'x', c='k')
plt.plot(periods, amplitudes, 'b-')
plt.xlabel('Period (Years)')
plt.ylabel('Amplitudes X 10^4')
plt.title('FFT Decomposition Spectrum of Sunspot Data')
plt.grid(True)
plt.xlim(1, 150)  # Limit x range to only show positive period values
plt.xticks(range(0, 150, 10))
plt.text(13, 7.4, '11-year Solar Cycle')
plt.text(92, 4.2, 'Gleissberg Cycle?')
#plt.xscale('log')
plt.show()
```



Wolfgang Gleißberg (March 25, 1903 - April 1, 1985) was a German astronomer known for his work in solar physics and terrestrial magnetism. His most significant contribution to the field was the identification of the long-term solar cycle known as the Gleissberg cycle.

Born in Berlin, Gleißberg began his career studying under the influential astronomer Karl Schwarzschild. He then worked at the Potsdam-Babelsberg Observatory, where he focused on studying solar-terrestrial relationships. During this time, he developed an interest in the cyclical nature of solar phenomena.

Gleißberg is best known for identifying an 80 to 90 year cycle in solar activity, which was later named the Gleissberg cycle. He observed that the sunspot cycle's amplitude and length aren't constant, but instead seem to follow a predictable, longer-term cycle. He postulated this longer cycle could help predict periods of intense solar activity as well as "solar minimums" where activity is lower.

Despite the significance of his work, Gleissberg's ideas were not immediately accepted. It was only later that other scientists came to recognize the importance of his observations and the existence of the cycle bearing his name.

In addition to his work on solar cycles, Gleissberg also contributed to understanding the Earth's magnetic field and its interactions with solar activity. He passed away in 1985, but his work continues to influence the field of solar physics.

In [21]:

```
# Phase diagram

sun['delta'] = sun.spots.diff()
sun['delta2'] = sun.delta.diff()
sun
```

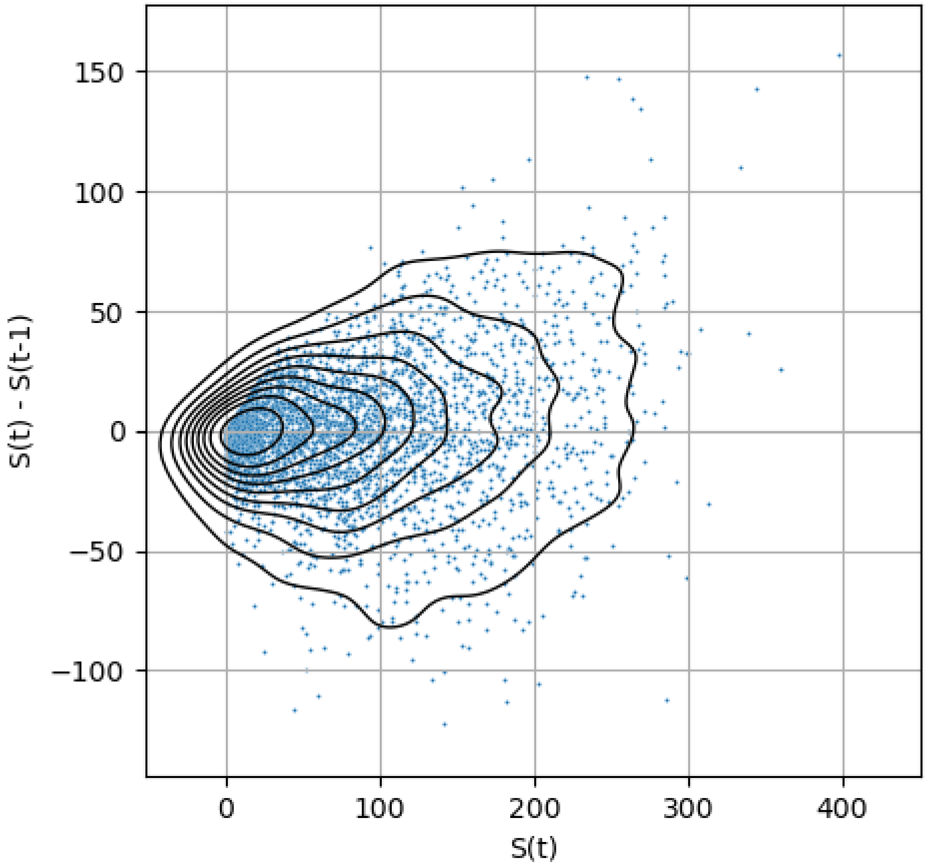
Out[21]:

	year	spots	ma11	delta	delta2
0	1749.042	96.7	NaN	NaN	NaN
1	1749.123	104.3	NaN	7.6	NaN
2	1749.204	116.7	NaN	12.4	4.8
3	1749.288	92.8	NaN	-23.9	-36.3
4	1749.371	141.7	NaN	48.9	72.8
...	...	...	...	...	...
3288	2023.042	143.6	93.072727	30.8	-1.5
3289	2023.122	110.9	96.009091	-32.7	-63.5
3290	2023.204	122.6	99.518182	11.7	44.4
3291	2023.286	96.4	99.509091	-26.2	-37.9
3292	2023.371	137.9	105.654545	41.5	67.7

3293 rows × 5 columns

In [22]:

```
plt.figure(figsize=(5,5), dpi=100)
sns.scatterplot(x=sun.spots, y=sun.delta, s=2)
#sns.histplot(x=sun.delta, y=sun.delta2, bins=100, pthresh=.2, cmap='vi
sns.kdeplot(x=sun.spots, y=sun.delta, levels=10, color='k', linewidths=
plt.grid()
plt.xlabel('S(t)')
plt.ylabel('S(t) - S(t-1)')
plt.show()
```

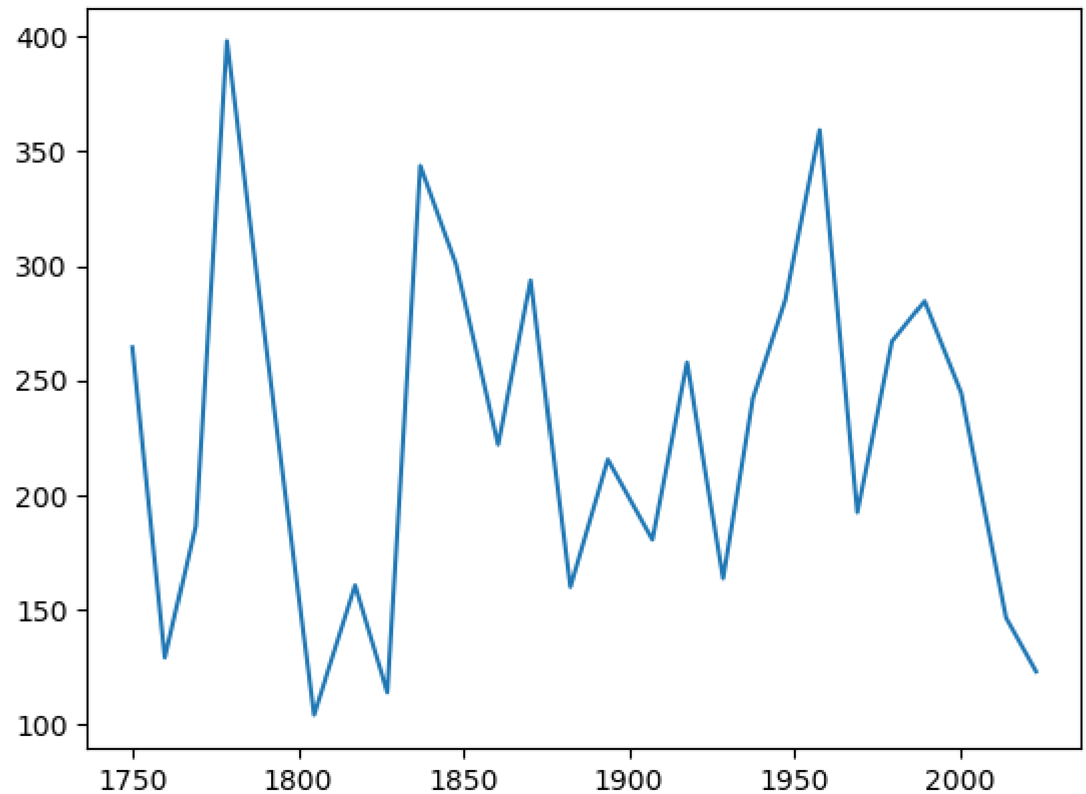


In [23]:

```python
# Finding peak values
from scipy.signal import find_peaks
peaks, _ = find_peaks(sun.spots, distance=9*12)
peak_years = sun.year[peaks]
peak_spots = sun.spots[peaks]
plt.plot(peak_years, peak_spots)
```

Out[23]:

```
[<matplotlib.lines.Line2D at 0x1643692b0>]
```



In [24]:

```
peak_df = pd.DataFrame({
    'year': peak_years,
    'spots': peak_spots
})

peak_df['period'] = peak_df.year.diff()
peak_df
```

Out[24]:

	year	spots	period
10	1749.874	264.3	NaN
128	1759.707	128.7	9.833
239	1768.958	186.3	9.251
352	1778.371	398.2	9.413
467	1787.958	290.0	9.587
669	1804.791	103.8	16.833
818	1817.204	160.3	12.413
935	1826.958	113.6	9.754
1055	1836.958	343.8	10.000
1185	1847.790	300.6	10.832
1338	1860.540	221.9	12.750
1456	1870.371	293.6	9.831
1599	1882.288	159.6	11.917
1735	1893.623	215.4	11.335
1897	1907.123	180.3	13.500
2023	1917.623	257.7	10.500
2154	1928.540	163.4	10.917
2262	1937.538	241.8	8.998
2380	1947.371	285.0	9.833
2505	1957.790	359.4	10.419
2642	1969.204	192.3	11.414
2768	1979.707	266.9	10.503
2885	1989.455	284.5	9.748
3018	2000.540	244.3	11.085
3181	2014.123	146.1	13.583
3290	2023.204	122.6	9.081

In [25]:
```
peak_df.mean()
```

Out[25]:
```
year      1886.831115
spots      227.861538
period      10.933200
dtype: float64
```

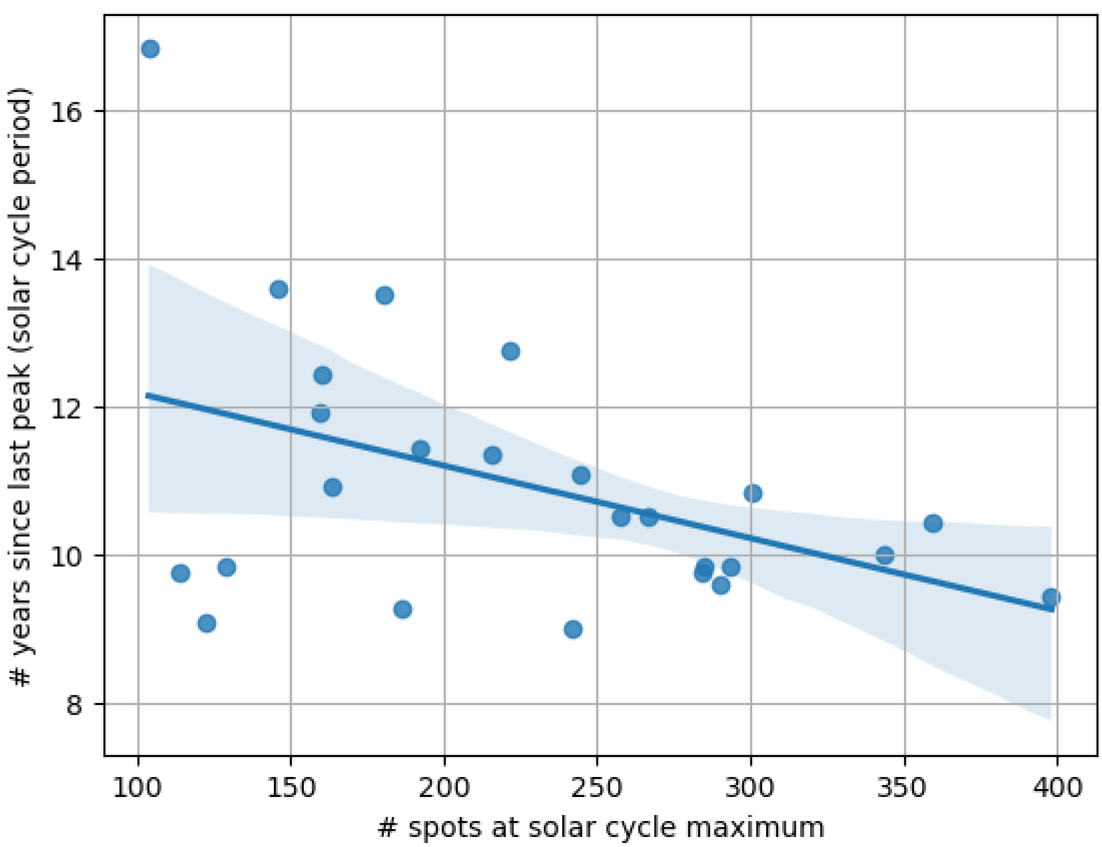
In [26]:
```
peak_df.max()
```

Out[26]:
```
year      2023.204
spots      398.200
period      16.833
dtype: float64
```

In [27]:
```
peak_df.min()
```

Out[27]:
```
year      1749.874
spots      103.800
period       8.998
dtype: float64
```

In [28]:
```
# Is period correlated with the amplitude of the solar cycle (# of spot
sns.regplot(peak_df, x='spots', y='period', ci=95)
plt.xlabel('# spots at solar cycle maximum')
plt.ylabel('# years since last peak (solar cycle period)')
plt.grid()
plt.show()
```



In [29]:
```
# Moderate correlation
peak_df.spots.corr(peak_df.period)
```

Out[29]:
```
-0.4422776998276138
```

It is indeed observed that the period of the solar cycle can vary, and some research suggests that it tends to be longer during periods of low solar activity. The 11-year cycle is an average, and individual cycles can range from about 9 to 14 years.

The Dalton Minimum, a period of low solar activity that occurred from about 1790 to 1830, is a good example. During this time, there were fewer sunspots and the period of the solar cycle was observed to be slightly longer than the average 11 years.

However, the relationship between solar cycle length and solar activity is complex and not yet fully understood. While there is some evidence of a correlation, there are also many other factors at play and exceptions to this rule.

Remember that our understanding of these phenomena is based on a limited historical record, and new observations or analyses could lead to revisions or refinements of these ideas. Solar physics is a complex field and many aspects of the sun's behavior are still topics of ongoing research.

In [ ]:
```

```