Name: Vivian Lin
ID: 6043749830
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Dr. Mohammad Rajati

Homework 6 ISLR Questions

6.6.3 In Jupyter Notebook

6.6.5 a) Ridge = $\sum_{i=1}^{n}(y_i - \hat{\beta}_0 - \sum_{j=1}^{p}(\hat{\beta}_j x_{ij}))^2 + \lambda\sum_{j=1}^{p}\hat{\beta}_j^2$

For this problem, $x_{11} = x_{12}$, $x_{21} = x_{22}$, $\hat{\beta}_0 = 0$

$(y_1 - \hat{\beta}_1 x_{11} - \hat{\beta}_2 x_{12})^2 + (y_2 - \hat{\beta}_1 x_{21} - \hat{\beta}_2 x_{22})^2 + \lambda(\hat{\beta}_1^2 + \hat{\beta}_2^2)$

$(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_1)^2 + (y_2 - \hat{\beta}_1 x_2 - \hat{\beta}_2 x_2)^2 + \lambda(\hat{\beta}_1^2 + \hat{\beta}_2^2)$

b) Take partial derivative of ↑

$$\begin{cases} \hat{\beta}_1(x_1^2 + x_2^2 + \lambda) + \hat{\beta}_2(x_1^2 + x_2^2) - y_1 x_1 - y_2 x_2 = 0 \\ \hat{\beta}_1(x_1^2 + x_2^2) + \hat{\beta}_2(x_1^2 + x_2^2 + \lambda) - y_1 x_1 - y_2 x_2 = 0 \end{cases}$$

Rearrange and cancel:

$$\hat{\beta}_1(x_1^2 + x_2^2 + \lambda) + \hat{\beta}_2(x_1^2 + x_2^2) = \hat{\beta}_1(x_1^2 + x_2^2) + \hat{\beta}_2(x_1^2 + x_2^2 + \lambda)$$

$$\hat{\beta}_1 \lambda = \hat{\beta}_2 \lambda \longrightarrow \hat{\beta}_1 = \hat{\beta}_2$$

c) Lasso: $\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}|\beta_j|$

$(y_1 - \hat{\beta}_1 x_{11} - \hat{\beta}_2 x_{12})^2 + (y_2 - \hat{\beta}_1 x_{21} - \hat{\beta}_2 x_{22})^2 + \lambda(|\hat{\beta}_1| + |\hat{\beta}_2|)$

$(y_1 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_2)^2 + (y_2 - \hat{\beta}_1 x_1 - \hat{\beta}_2 x_2)^2 + \lambda(|\hat{\beta}_1| + |\hat{\beta}_2|)$

d) Since problem given us $x_{11} + x_{21} = 0$ $x_{12} + x_{22} = 0$

Another way to minimize is to look at by the squared terms subjected to $|\hat{\beta}_1| + |\hat{\beta}_2| \le s$. ⇒ Simplify the squared terms:

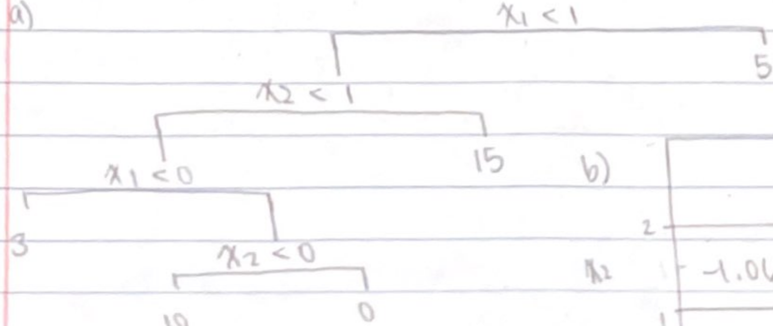
$$2[y_1 + \underbrace{(\hat{\beta}_1 + \hat{\beta}_2)}x_1]^2$$

There are many combinations of $\hat{\beta}_1 + \hat{\beta}_2$ that will yield same solution

Hence there are multiple solutions for Lasso with general form of solution as:

$$\begin{cases} \hat{\beta}_1 + \hat{\beta}_2 = s & \hat{\beta}_1 \ge 0 \;\; \hat{\beta}_2 \ge 0 \\ \hat{\beta}_1 + \hat{\beta}_2 = -s & \hat{\beta}_1 \le 0 \;\; \hat{\beta}_2 \le 0 \end{cases}$$

8.4.5 a)



b)