STA490 Analysis

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Introduction

The objective of this analysis is to investigate the diurnal pattern of reaction time. In particular, the analysis is in search of any association between the measurement of the day and reaction time. The data being analyzed was collected from 39 students from the 2018-2019 cohort of students from STA490 at the University of Toronto.

Data Collection and Preparation

Protocol

40 students from the 2018-2019 cohort of STA490 students were asked to perform the reaction time test from https://questionnaire.censusatschool.ca/practice/reaction 4 times each day for two days.The two days were to differ by business, as determined by each individual student. As well, the tests were to be taken at four to six hour intervals, with the first test within an hour of waking up. Data collected are listed later in this analysis.

Data Cleaning

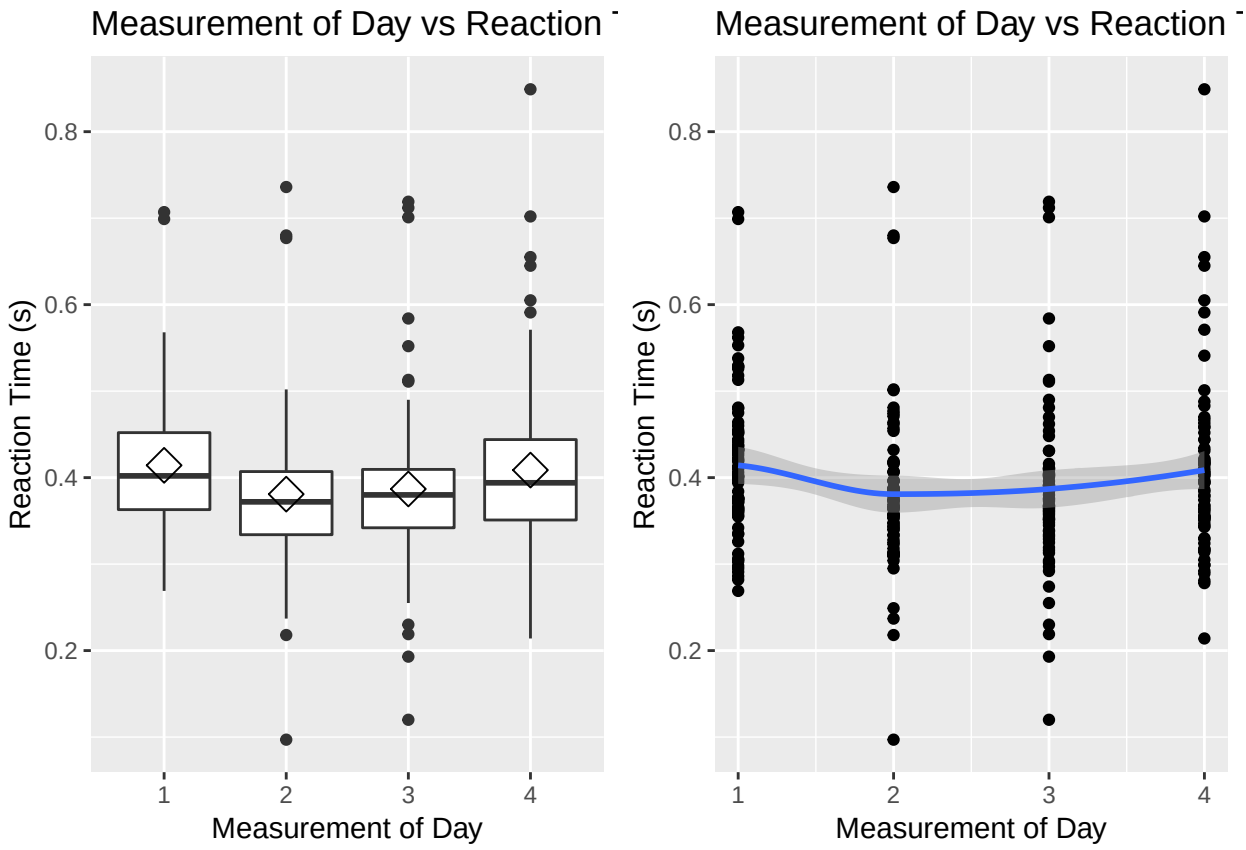
Before any analysis was performed, steps were taken to make the data consistent and clean. In particular, if an MEQ was reported as one of the categorical names instead of a number then it was replaced by the average number in that category. Another variable was also created for MEQ, called "MEQ Category" and is a categorical variable representing MEQ, as opposed to the numerical value that was reported. Also, any illness that was reported, regardless of severity, was recorded as an illness. The table below describes the variables that are included in the analysis.

Table 1: Variable Descriptions

Variable	Description
ID	The unique ID number for every student
Measurement Number	Number of the measurement. Ranges from 1 to 8
Raw Time	The 24 hour format time when the data was recorded
Day Type	1 indicates busy day, 0 indicates light day
Stimulant	1 indicates a stimulant was taken, 0 indicates otherwise
Fatigue	Fatigue measured on the Samn-Perelli 7-pt scale. Ranges from 1 to 7
Hunger	Hunger measured on the hunger scale, value decreasing with hunger level
Illness	1 indicates illness, 0 indicates otherwise
Sleep	The hours of sleep that was had the night before the observation
MEQ	The Morningness Eveningness Questionnaire score, ranging from 16 to 86
Reaction Time	The reaction time that was measured (in seconds)
Break Protocol	1 indicates protocol was broken, 0 indicates otherwise
MEQ Category	Classfication of MEQ score into Evening, Morning, or Neither.
Measurement of the Day	If the observation was first, second, third, or fourth observation of the day

Analysis

An initial look at Reaction Time vs Measurement of the Day



An initial look at the reaction time vs measurement of the day suggested that there may be some differences between the mean reaction time when the test is the first, second, third, or fourth of the day. For example, the reaction time for the first measurement appears to be generally higher than that of the second measurement of the day.

Linear Mixed Models

Because of the nature of the data, it was also reasonable to do a linear mixed model. For the mixed models, I used subject as a random effect since each individual was expected to perform the reaction test 8 times so there were multiple observations per person.

Since each individual was asked to perform the tests on a busy and a light day, I treated those measurements as repliates, as opposed to variables of interest. Thus, the response variable that I will be looking at will be the mean of the replicates. In addition, within-subject error is bound to occur since individuals were asked to perform the test multiple times. So, ID is used as a random effect.

Model 1:

This is the simplest model where Reaction time is a function of measurement of the day, the random effect of each individual, and a random error.

$ReactionTime_{ij} = B_0 + B_1 * I_{ij}(Measurement2) + B_2 * I_{ij}(Measurement3) + B_3 I_{ij}(Measurement4) + U_i + \epsilon_{ij}$

where:

$ReactionTime_{ij}$ is the mean reaction time for subject i at the jth measurement of the day

$I(Measurementk)$ is equal to 1 if the ij^{th} observation is the kth measurement of the day, and 0 otherwise

The ANOVA table:

```
##                                   numDF denDF   F-value p-value
## (Intercept)                           1   113 1009.0113  <.0001
## as.factor(Measurement.of.the.Day)     3   113    4.8732  0.0032
```

The p-value from the ANOVA table above is 0.0032, which means that at the 0.05 significance level, there is enough evidence to reject the null hypothesis that the measurement of the day is not associated with reaction time. Thus, measurement of the day is indeed associated with reaction time and on average, reaction time does differ depending on whether the test was the first, second, third, or fourth test of the day.

Model 2 - Controlling for MEQ

$ReactionTime_{ij} = B_0 + B_1 * I(Measurement2) + B_2 * I(Measurement3) + B_3 I(Measurement4) + B_4 * I_{ij}(MorningMEQ) + B_4 * I_{ij}(NeitherMEQ) + U_i + \epsilon_{ij}$

where:

$ReactionTime_{ij}$ is the mean reaction time for subject i at the jth measurement of the day

$I(Measurementk)$ is equal to 1 if the ij^{th} observation is the kth measurement of the day, and 0 otherwise

$I_{ij}(NeitherMEQ)$ is equal to 1 if the MEQ category for subject i is Neither, 0 otherwise

$I_{ij}(MorningMEQ)$ is equal to 1 if the MEQ category for subject i is Morning, 0 otherwise

The ANOVA table:

```
##                                   numDF denDF  F-value p-value
## (Intercept)                           1   107 945.4229  <.0001
## as.factor(Measurement.of.the.Day)     3   107   4.8585  0.0033
## MEQ.Category                          2    34   0.5034  0.6089
```

According to this ANOVA test, the p-value is 0.0033 which is statistically significant. Thus, controlling for MEQ Category, Measurement of the Day still has an effect on the reaction time on average. However, MEQ is not a significant predictor since its p-value is large (0.6089).

Model 3: Controlling for Fatigue and MEQ

$ReactionTime_{ij} = B_0 + B_1 * I(Measurement2) + B_2 * I(Measurement3) + B_3 I(Measurement4) + B_4 * I_{ij}(MorningMEQ) + B_4 * I_{ij}(NeitherMEQ) + Fatigue_{ij} + U_i + \epsilon_{ij}$

where:

$ReactionTime_{ij}$ is the mean reaction time for subject i at the jth measurement of the day $I(Measurementk)$ is equal to 1 if the ij^{th} observation is the kth measurement of the day, and 0 otherwise $I_{ij}(NeitherMEQ)$ is equal to 1 if the MEQ category for subject i is Neither, 0 otherwise $I_{ij}(MorningMEQ)$ is equal to 1 if the MEQ category for subject i is Morning, 0 otherwise $Fatigue_{ij}$ is the mean fatigue level for subject i at the jth measurement of the day

The ANOVA table:

```
##                                   numDF denDF  F-value p-value
## (Intercept)                           1   106 905.9953  <.0001
## as.factor(Measurement.of.the.Day)     3   106   5.9606  0.0009
## MEQ.Category                          2    34   0.4821  0.6216
## Fatigue                               1   106  23.8317  <.0001
```

According to this ANOVA test, the p-value associated with Measurement of the Day is 0.0009 which is statistically significant. Thus, controlling for MEQ Category and Fatigue, Measurement of the Day still has an effect on the reaction time on average. Notice that MEQ does not play a significant role as a predictor of reaction time since its p-value is very large (0.6216).

Model 4: Controlling for Fatigue

The ANOVA table:

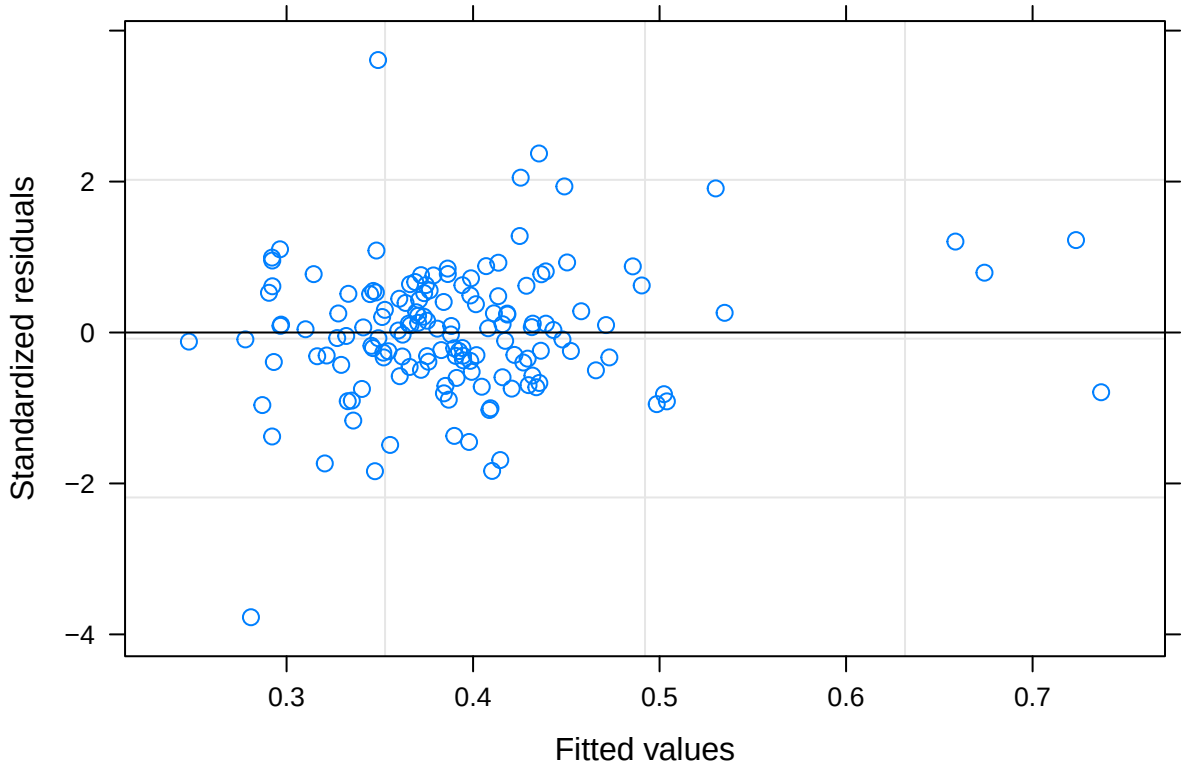
```
##                                    numDF denDF  F-value p-value
## (Intercept)                            1   106 927.6022  <.0001
## as.factor(Measurement.of.the.Day)      3   106   5.9601   9e-04
## Fatigue                                1   106  23.5867  <.0001
```

The ANOVA table above shows that Fatigue and Measurement of the day both have a significant effect on reaction time. However, I must make sure that the data satisfies the model assumptions.

Model Diagnostics:

Assumptions for the Linear Mixed Model are:

- The explanatory variables are related linearly to the response.
- The errors have constant variance.
- The errors are independent.
- The errors are Normally distributed.

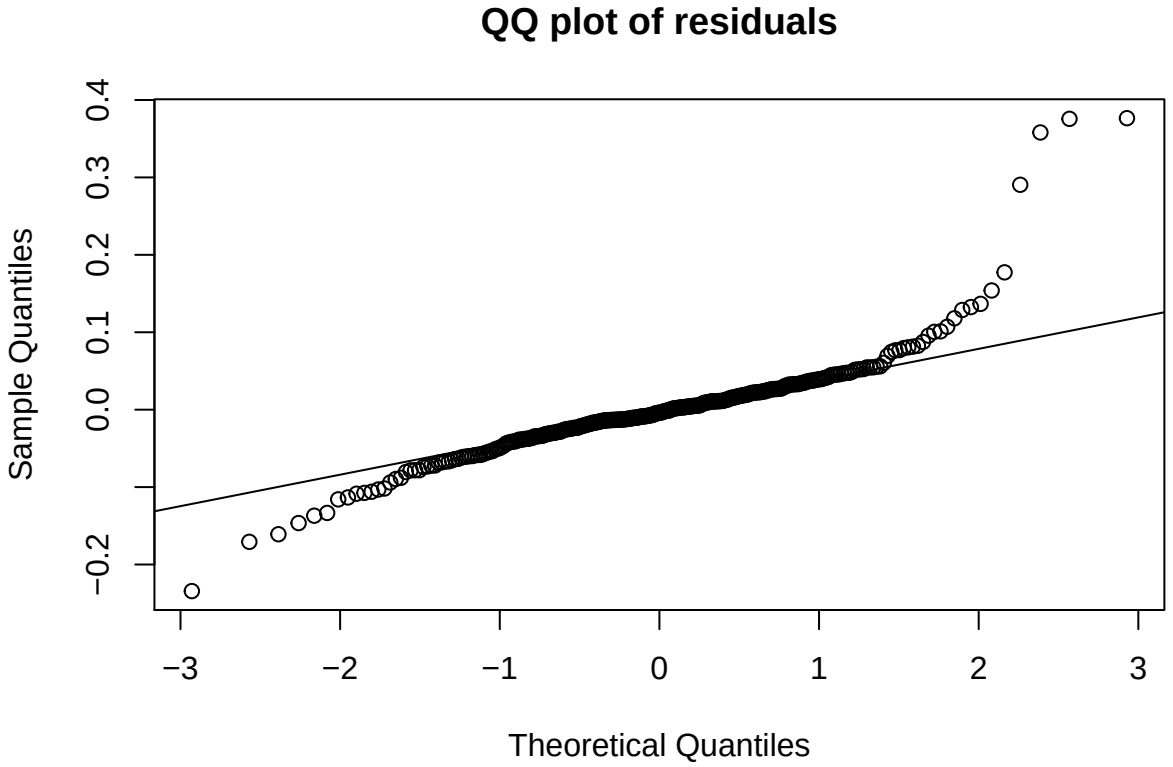


According to the plot above, the residuals seem evenly scattered and have constant variance.

```
##
## Call:
## lm(formula = res[-n] ~ res[-1])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.16639 -0.03125 -0.00101  0.02483  0.42095
```

```
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0001988  0.0033554  -0.059    0.953
## res[-1]      0.4945132  0.0509075   9.714   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05744 on 291 degrees of freedom
## Multiple R-squared:  0.2449, Adjusted R-squared:  0.2423
## F-statistic: 94.36 on 1 and 291 DF,  p-value: < 2.2e-16
```

The model above is a linear regression on the residuals and it yields p-value <2e-16, suggesting that there is autocorrelation amongst the errors.



The residuals also appear to stray from being normal, according to the QQ-plot. However, mixed models are often prone to these assumption violations and the mixed model could still be valid. To investigate this further, it may be useful to employ a a nonparametric test.

Non-Parametric Method: Wilcoxon Signed Rank Test

About the Wilcoxon Signed-Rank Test

To dive deeper into my analysis, I decided to proceed with a Wilcoxon Signed Rank Test to find an association between Reaction Time and every pair of Measurement-of-the-Day levels. There are several reasons why I decided to proceed with this test.

In general, the Wilcoxon Signed Rank Test is a nonparametric test that is equivalent to the dependent t-test. It is used to compare data between two sets of observations that are collected from the same participants.

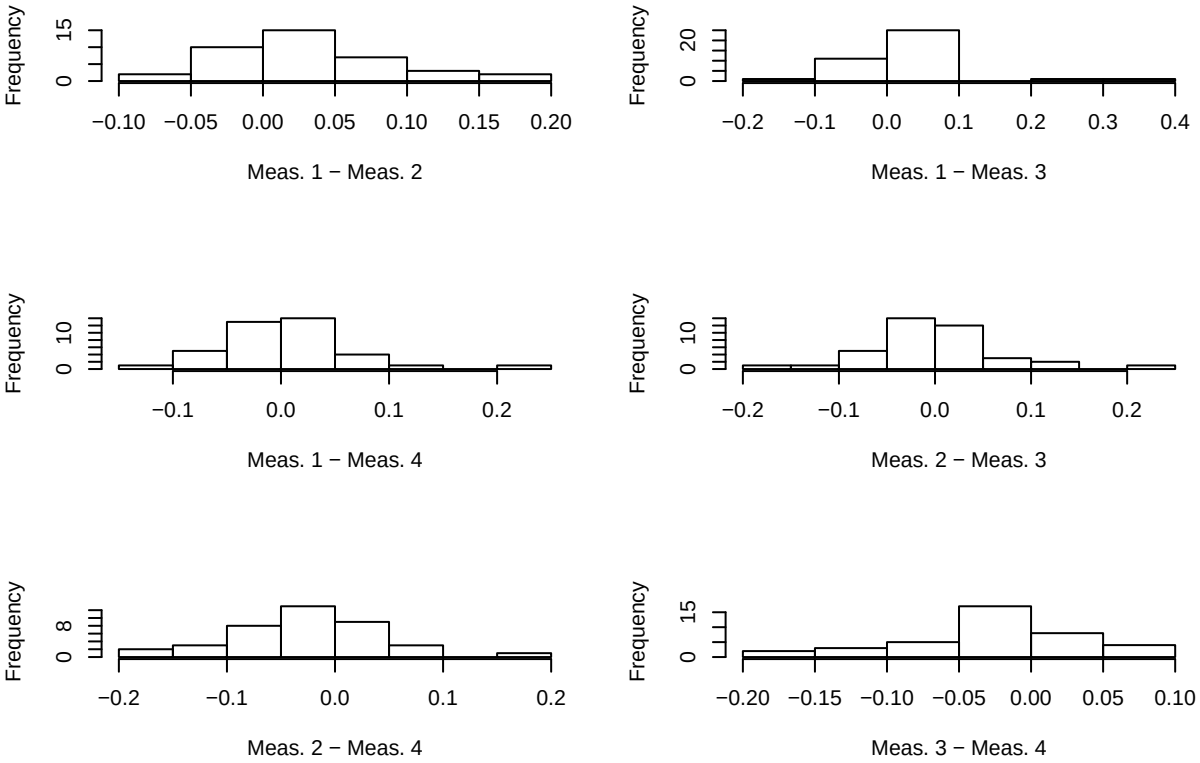
The Wilcoxon Signed Rank Test assumes:

- The dependent variable is continuous
- The independent variable is composed of two categorical groups where the same subjects are present in both groups
- The differences between every pair of groups above have a symmetric distribution

In this analysis, the Wilcoxon Signed Rank Test was used to compare the reaction time data between different groups of Measurement of the Day. The Wilcoxon Signed Rank Test is appropriate because of several reasons:

- My objective is to compare the data between pairs of Measurement-of-the-day levels (i.e. the reaction time for the first measurement of the day vs the reaction time for the second measurement of the day)
- Every level of Measurement-of-the-day has data from the same pool of individuals
- Reaction time (the dependent variable) is a continuous variable
- Measurement of the day (the independent variable) is composed of two groups since I will be comparing each pair of levels
- The differences between the two groups generally have a symmetric distribution. Aside from Measurement 1 - Measurement 3, most of the differences seem to be fairly symmetrically distributed.

Histograms of the Differences in Measurement Levels:



For each individual, I took the average of their reaction time for each measurement of the day. For example, if reaction time was A for the first observation on a busy day and B in the first observation on the light day for individual X, then I recorded X's Reaction Time for Measurement of the Day #1 as mean(A+B). The reason for doing this is so that the observations within one group of Measurement-of-the-Day level are independent of each other and this would eliminate the random effect of each individual on repeated samples within one group.

Note: One individual did not have any information recorded for Measurement #4 on either days. Since the Wilcoxon signed rank test requires equal sample sizes, I took the reaction time at Measurement #4 for all other individuals and used the average of these values in place of the missing data. I did this because the objective is to compare means and I believe that appending this mean value will not affect the results too greatly.

Results of the Wilcoxon Signed-Rank Test

P-values from Wilcoxon Test:

Measurement 1 and 2: 0.001014328

Measurement 1 and 3: 0.006783241

Measurement 1 and 4: 0.5765988

Measurement 2 and 3: 0.5672076

Measurement 2 and 4: 0.007849126

Measurement 3 and 4: 0.02292273

Significant Difference Between Measurement Times:

Measurement 1 and 2: TRUE

Measurement 1 and 3: TRUE

Measurement 1 and 4: FALSE

Measurement 2 and 3: FALSE

Measurement 2 and 4: TRUE

Measurement 3 and 4: FALSE

The p-values for testing if the means between the groups are the same. However, because I am doing 6 comparisons at once, I need to correct for multiple comparisons. Thus, the alpha rate is changed from 0.05 to 0.05/6.

The values below show which pairs of Measurement Numbers had mean reaction times that were statistically significantly different. From the Wilcoxon Signed-Rank test, it appears that on average, the first measurement of reaction time is significantly different from the second measurement, the first measurement of reaction time is significantly different from the third measurement, and the second measurement of reaction time is significantly different from the fourth measurement.

These results are consistent with the results from the linear mixed model.

Results

To recap, this analysis was designed to explore the diurnal patterns of reaction time. In particular, I was interested in associations between the measurement of the day and reaction time. One of the assumptions in this analysis was that fatigue can account for variations in sleep, hunger, and illness. I also treated the measurements from the busy and light days as replicates, as opposed to variables of interest. Thus, my main focus was on MEQ category and fatigue, and how they might affect the diurnal pattern of reaction time.

According to the linear mixed model, it appears that, controlling for fatigue and MEQ category, measurement of the day is still a useful predictor for reaction time. Further, The Wilcoxon signed-rank test paired with bonferroni correction showed that there is significant difference between the average reaction times for the first and second measurement, first and third measurement, and second and fourth measurement of the day. In conclusion, measurement of the day is a useful predictor on the reaction time.

```
## Linear mixed-effects model fit by REML
##  Data: temp3
##         AIC       BIC   logLik
##   -375.2468 -357.1432 193.6234
##
## Random effects:
##  Formula: ~1 | ID
##         (Intercept)   Residual
## StdDev:  0.07418435 0.04745818
##
## Fixed effects: Reaction.Time ~ as.factor(Measurement.of.the.Day)
##                                       Value  Std.Error  DF   t-value
## (Intercept)                          0.4143590 0.01410183 113 29.383359
## as.factor(Measurement.of.the.Day)2 -0.0332821 0.01074716 113 -3.096822
## as.factor(Measurement.of.the.Day)3 -0.0312436 0.01074716 113 -2.907148
## as.factor(Measurement.of.the.Day)4 -0.0072687 0.01083822 113 -0.670653
##                                     p-value
## (Intercept)                          0.0000
## as.factor(Measurement.of.the.Day)2  0.0025
## as.factor(Measurement.of.the.Day)3  0.0044
## as.factor(Measurement.of.the.Day)4  0.5038
##  Correlation:
##                                     (Intr) a.(M...D)2 a.(M...D)3
## as.factor(Measurement.of.the.Day)2 -0.381
## as.factor(Measurement.of.the.Day)3 -0.381  0.500
## as.factor(Measurement.of.the.Day)4 -0.378  0.496      0.496
##
## Standardized Within-Group Residuals:
##          Min           Q1          Med           Q3          Max
## -3.980791839 -0.414980270 -0.009239032  0.393657350  3.431131529
##
## Number of Observations: 155
## Number of Groups: 39
```

```
## Linear mixed-effects model fit by REML
##  Data: temp4
##         AIC       BIC   logLik
##   -344.7515 -321.1614 180.3758
##
## Random effects:
##  Formula: ~1 | ID
```

```
##         (Intercept)  Residual
## StdDev:  0.07396228 0.0472283
##
## Fixed effects: Reaction.Time ~ as.factor(Measurement.of.the.Day) + MEQ.Category
##                                       Value  Std.Error  DF   t-value
## (Intercept)                          0.4271093 0.02181567 107 19.578099
## as.factor(Measurement.of.the.Day)2 -0.0350405 0.01098036 107 -3.191202
## as.factor(Measurement.of.the.Day)3 -0.0322973 0.01098036 107 -2.941371
## as.factor(Measurement.of.the.Day)4 -0.0097064 0.01107860 107 -0.876141
## MEQ.CategoryMorning                  0.0050268 0.08036555  34  0.062549
## MEQ.CategoryNeither                 -0.0257077 0.02655108  34 -0.968237
##                                      p-value
## (Intercept)                           0.0000
## as.factor(Measurement.of.the.Day)2  0.0019
## as.factor(Measurement.of.the.Day)3  0.0040
## as.factor(Measurement.of.the.Day)4  0.3829
## MEQ.CategoryMorning                   0.9505
## MEQ.CategoryNeither                   0.3398
##  Correlation:
##                                      (Intr) a.(M...D)2 a.(M...D)3 a.(M...D)4
## as.factor(Measurement.of.the.Day)2 -0.252
## as.factor(Measurement.of.the.Day)3 -0.252  0.500
## as.factor(Measurement.of.the.Day)4 -0.252  0.496      0.496
## MEQ.CategoryMorning                -0.246  0.000      0.000      0.000
## MEQ.CategoryNeither                -0.744  0.000      0.000      0.003
##                                      MEQ.CM
## as.factor(Measurement.of.the.Day)2
## as.factor(Measurement.of.the.Day)3
## as.factor(Measurement.of.the.Day)4
## MEQ.CategoryMorning
## MEQ.CategoryNeither                   0.202
##
## Standardized Within-Group Residuals:
##           Min            Q1           Med            Q3           Max
## -3.9771562593 -0.3923662276  0.0001348963  0.3788062385  3.4485335907
##
## Number of Observations: 147
## Number of Groups: 37
```

```
## Linear mixed-effects model fit by REML
##  Data: temp5
##         AIC       BIC   logLik
##   -355.0106 -328.5358 186.5053
##
## Random effects:
##  Formula: ~1 | ID
##         (Intercept)   Residual
## StdDev:  0.07639972 0.04264303
##
## Fixed effects: Reaction.Time ~ as.factor(Measurement.of.the.Day) + MEQ.Category +      Fatigue
##                                       Value  Std.Error  DF   t-value
## (Intercept)                          0.3599019 0.02599753 106 13.843697
## as.factor(Measurement.of.the.Day)2 -0.0225545 0.01023891 106 -2.202820
## as.factor(Measurement.of.the.Day)3 -0.0282258 0.00994932 106 -2.836952
```

```
## as.factor(Measurement.of.the.Day)4 -0.0336211 0.01114019 106 -3.018002
## MEQ.CategoryMorning                -0.0112933 0.08217113  34 -0.137437
## MEQ.CategoryNeither                -0.0292576 0.02713334  34 -1.078288
## Fatigue                             0.0200863 0.00411455 106  4.881776
##                                    p-value
## (Intercept)                         0.0000
## as.factor(Measurement.of.the.Day)2  0.0298
## as.factor(Measurement.of.the.Day)3  0.0055
## as.factor(Measurement.of.the.Day)4  0.0032
## MEQ.CategoryMorning                 0.8915
## MEQ.CategoryNeither                 0.2885
## Fatigue                             0.0000
##  Correlation:
##                                    (Intr) a.(M...D)2 a.(M...D)3 a.(M...D)4
## as.factor(Measurement.of.the.Day)2 -0.317
## as.factor(Measurement.of.the.Day)3 -0.234  0.503
## as.factor(Measurement.of.the.Day)4  0.062  0.321      0.407
## MEQ.CategoryMorning                -0.189 -0.010     -0.003      0.018
## MEQ.CategoryNeither                -0.623 -0.007     -0.002      0.014
## Fatigue                            -0.530  0.250      0.084     -0.440
##                                    MEQ.CM MEQ.CN
## as.factor(Measurement.of.the.Day)2
## as.factor(Measurement.of.the.Day)3
## as.factor(Measurement.of.the.Day)4
## MEQ.CategoryMorning
## MEQ.CategoryNeither                 0.203
## Fatigue                            -0.041 -0.027
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -3.74629731 -0.42423058  0.04752563  0.52846874  3.63126640
##
## Number of Observations: 147
## Number of Groups: 37

## Linear mixed-effects model fit by REML
##  Data: temp5
##        AIC       BIC   logLik
##   -366.429 -345.7382 190.2145
##
## Random effects:
##  Formula: ~1 | ID
##         (Intercept)   Residual
## StdDev:  0.07543368 0.04264452
##
## Fixed effects: Reaction.Time ~ as.factor(Measurement.of.the.Day) + Fatigue
##                                        Value   Std.Error  DF   t-value
## (Intercept)                         0.3426533 0.020164006 106 16.993314
## as.factor(Measurement.of.the.Day)2 -0.0226355 0.010238382 106 -2.210852
## as.factor(Measurement.of.the.Day)3 -0.0282522 0.009949573 106 -2.839538
## as.factor(Measurement.of.the.Day)4 -0.0334378 0.011137424 106 -3.002295
## Fatigue                             0.0199559 0.004109011 106  4.856610
##                                    p-value
## (Intercept)                         0.0000
```

```
## as.factor(Measurement.of.the.Day)2  0.0292
## as.factor(Measurement.of.the.Day)3  0.0054
## as.factor(Measurement.of.the.Day)4  0.0033
## Fatigue                             0.0000
##  Correlation:
##                                    (Intr) a.(M...D)2 a.(M...D)3 a.(M...D)4
## as.factor(Measurement.of.the.Day)2 -0.415
## as.factor(Measurement.of.the.Day)3 -0.304  0.503
## as.factor(Measurement.of.the.Day)4  0.092  0.321      0.407
## Fatigue                            -0.708  0.249      0.084     -0.440
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -3.77286696 -0.41338311  0.03344304  0.51689843  3.60993624
##
## Number of Observations: 147
## Number of Groups: 37
```

```
## Linear mixed-effects model fit by REML
##  Data: temp5
##         AIC       BIC   logLik
##   -371.2092 -353.4322 191.6046
##
## Random effects:
##  Formula: ~1 | ID
##         (Intercept)   Residual
## StdDev:  0.07585836 0.04451826
##
## Fixed effects: Reaction.Time ~ Fatigue + MEQ.Category
##                            Value  Std.Error  DF   t-value p-value
## (Intercept)           0.3478682 0.02411616 109 14.424688  0.0000
## Fatigue               0.0174487 0.00338211 109  5.159135  0.0000
## MEQ.CategoryMorning  -0.0091503 0.08187757  34 -0.111756  0.9117
## MEQ.CategoryNeither  -0.0286530 0.02703967  34 -1.059663  0.2968
##  Correlation:
##                      (Intr) Fatigu MEQ.CM
## Fatigue              -0.482
## MEQ.CategoryMorning  -0.210 -0.034
## MEQ.CategoryNeither  -0.675 -0.020  0.202
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -3.85569828 -0.43810011 -0.06265343  0.49045882  3.96362080
##
## Number of Observations: 147
## Number of Groups: 37
```