NOTES: Simons Observatory Isocurvature Forecasts

These notes are for forecasting the Simons Observatory's ability to constrain isocurvature perturbations.

I. FORECASTING METHOD

We begin with an explanation of our Fisher information forecasting method. We calculate the Fisher information matrix via

$$F_{ij} = \sum_{\ell} \left(\frac{\partial C_\ell^\alpha}{\partial \theta_i} \right)^T \cdot (\mathbb{C}_\ell^{-1})_{\alpha\beta} \cdot \frac{\partial C_\ell^\beta}{\partial \theta_j}, \tag{1}$$

where $\alpha, \beta \in \{TT, EE, TE, \phi\phi\}$, θ_i are the model parameters for which we want to estimate the errors. $\mathbb{C}_\ell$ is the covariance matrix, given by

$$\mathbb{C}_\ell = \frac{2}{2\ell+1} \frac{1}{f_{\rm sky}} \begin{pmatrix} (\tilde{C}_\ell^{TT})^2 & (C_\ell^{TE})^2 & \tilde{C}_\ell^{TT} C_\ell^{TE} & (C_\ell^{T\phi})^2 \\ (C_\ell^{TE})^2 & (\tilde{C}_\ell^{EE})^2 & \tilde{C}_\ell^{EE} C_\ell^{TE} & (C_\ell^{E\phi})^2 \\ \tilde{C}_\ell^{TT} C_\ell^{TE} & \tilde{C}_\ell^{EE} C_\ell^{TE} & \frac{1}{2}[(C_\ell^{TE})^2 + \tilde{C}_\ell^{TT} \tilde{C}_\ell^{EE}] & C_\ell^{T\phi} C_\ell^{E\phi} \\ (C_\ell^{T\phi})^2 & (C_\ell^{E\phi})^2 & C_\ell^{T\phi} C_\ell^{E\phi} C_\ell^{TE} & (\tilde{C}_\ell^{\phi\phi})^2 \end{pmatrix}, \tag{2}$$

where $\tilde{C}_\ell^\nu = C_\ell^\nu + N_\ell^\nu$ are the observed power spectra with $\nu \in \{TT, EE, \phi\phi\}$. Here N_ℓ^ν is the residual foreground and noise spectrum, determined by the experimental setup being considered, and $f_{\rm sky}$ is the fraction of the sky observed.

Assuming a forecast over n-parameters the Fisher matrix is an $n \times n$ matrix, the inverse of which gives the parameter covariance matrix:

$$[F]^{-1} = [C] = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \cdots & \sigma_{xn} \\ \sigma_{xy} & \sigma_y^2 & \cdots & \sigma_{yn} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{xn} & \sigma_{yn} & \cdots & \sigma_n^2 \end{bmatrix}, \tag{3}$$

where σ_i are the 1-σ uncertainties in your parameters, marginalizing over the others. The off diagonal terms correspond to the cross-correlation between parameters where $\sigma_{xy} = \rho \sigma_x \sigma_y$, where ρ is the correlation coefficient which varies from zero (independent parameters) to 1 (parameters completely correlated).

A good resource for learning about Fisher forecasts is Dan Coe's guide to Fisher matrices [1].

II. NOISE MODEL

For a ground based experiment like the Simons Observatory, the noise spectra contains a standard white noise component due to the instrument as well as an atmospheric noise component. The combined noise spectrum is modelled as

$$N_\ell = N_{\ell,\rm white} + N_{\ell,\rm atm} \left(\frac{\ell}{\ell_{\rm knee}} \right)^{-\alpha_{\rm knee}}, \tag{4}$$

where the white noise is

$$N_{\ell,\rm white} = (\Delta_X)^2 \exp\left(\frac{\ell(\ell+1)\theta_{\rm FWHM}^2}{8 \ln 2} \right), \tag{5}$$

Δ_X is the map sensitivity for temperature or polarization ($X = T, E$ respectively) in μK-arcmin, and $\theta_{\rm FWHM}$ is the telescope beam width. Note that $\Delta_E = \sqrt{2}\Delta_T$. For the Simons Observatory, $\ell_{\rm knee} = 1000$ and $\alpha_{\rm knee} = 3.5$ and the values for $N_{\ell \rm red}$ in $\mu{\rm K}^2 s{\rm Sr}^{-1}$ are found in Table 2 of [2], recreated here in Table I. $N_{\rm red}$ is related to the atmospheric noise through

$$N_{\ell,\rm atm} = N_{\rm red} \frac{A_{\rm sky}}{t_{\rm obs}}, \tag{6}$$

Freq. [GHz]	θ_{FWHM} [arcmin]	Δ_T (baseline) [μK-arcmin]	Δ_T (goal) [μK-arcmin]	N_{red} [$\mu\mathrm{K}^2$ s]
27	7.4	71	52	100
39	5.1	36	27	39
93	2.2	8	5.8	230
145	1.4	10	6.3	1,500
225	1.0	22	15	17,000
280	0.9	54	37	31,000

Table I. Band dependent parameters for the LAT noise models for SO.

where the sky area is $A_{\mathrm{sky}} = 4\pi f_{\mathrm{sky}}$ [Sr] and $f_{\mathrm{sky}} = 0.4$, and the total observation time is $t_{\mathrm{obs}} = N_{\mathrm{total}}\eta_{\text{obs-eff}}\eta_{\text{map-cuts}}$, where $N_{\mathrm{total}} = 5$ years is the survey period, $\eta_{\text{obs-eff}} = 0.2$ is the observing efficiency, and $\eta_{\text{map-cuts}}$ represents the 15% data cuts due to noisy map edges.

To reduce the impact of noise and foregrounds we use the internal linear combination (ILC) technique to optimally combine data from multiple frequency bands, reducing the overall impact of noise and foregrounds. The minimum variance CMB map (MV-ILC) is

$$S_{\ell m} = \sum_{i=1}^{N_{\mathrm{bands}}} w_\ell^i M_{\ell m}^i, \tag{7}$$

where $M_{\ell m}^i$ is the spherical harmonic transform of the map from the ith frequency band and the multipole dependent weights w_ℓ^i are tuned in order to minimize the overall variance, and are derived as

$$w_\ell = \frac{\mathbf{C}_\ell^{-1} A_s}{A_s^\dagger \mathbf{C}_\ell^{-1} A_s}, \tag{8}$$

where $\mathbf{C}_\ell$ is a $N_{\mathrm{bands}} \times N_{\mathrm{bands}}$ matrix containing the covariance of foregrounds and beam-deconvolved noise across different frequencies, A_s is a N_{bands} length vector giving the frequency response of the CMB in different bands. The sum of the residual noise and foregrounds power in the MV-ILC map is then given as

$$N_\ell = \frac{1}{A_s^T \mathbf{C}_\ell^{-1} A_s}, \tag{9}$$

which we use for forecasting. We do not calculate these noise curves ourselves but rather use those generated by the SO noise code outlined in Sec. 2.5 of [2]. More details of the ILC method can be found in [3–6].

III. ISOCURVATURE PERTURBATIONS

Initial conditions in the early Universe generated by inflation are constrained to be almost entirely adiabatic, i.e. with equal number density fluctuations for all species. However, non-adiabatic contributions to the observed temperature variance are still allowed up to about 2%. Isocurvature perturbations are defined as variations in the relative densities of different species without changing the overall energy density, which lead to fluctuations in the ratios of the different species. For two species a, and b, the density isocurvature perturbation is defined by a non-zero

$$S_{ab} \equiv \frac{\delta_a}{1+w_a} - \frac{\delta_b}{1+w_b}, \tag{10}$$

where $\delta = \delta\rho/\rho$ is the dimensionless density perturbation, and w is the equation of state. You can similarly define isocurvature modes in velocity perturbations θ. One can form arbitrary linear combinations of the different isocurvature modes and correlate them with the adiabatic mode. In general, the isocurvature modes are defined with respect to the photons. The full basis of non-decaying isocurvature perturbations are defined in [7] as the baryon density (BDI), cold dark matter density (CDI), neutrino density (NDI), and neutrino velocity (NVI) isocurvature modes. Another mode exists where the baryon and dark matter density perturbations are equal and opposite, producing a vanishing total matter perturbation, known as the compensated isocurvature perturbation (CIP) [8–10]. However the CIP has no effect on the CMB or matter power spectrum at linear order, making it harder to constrain. Note that the baryon and CDM density isocurvature modes are indistinguishable from each other in the CMB, and therefore are generally studied together, effectively reducing the overall parameter space to constrain.

While single field inflation cannot produce isocurvature perturbations, multi-field inflation models generically produce both correlated and un-correlated isocurvature as the different fields generate different density variations [11, 12], making the number densities of different components spatially vary. It should be noted that there is no known generation mechanism for the NVI mode, although most analyses still include it for completeness. Planck provides the most stringent constraints on isocurvature to date [13–15], constraining them to contribute less than $\sim 1.5\%$ to the temperature variance of the CMB.

Planck parameterizes isocurvature perturbations using a "two-scales" model, where the primordial power spectrum is

$$\mathcal{P}_{ab}(k) = \exp\left[\left(\frac{\ln(k)-\ln(k_2)}{\ln(k_1)-\ln(k_2)}\right)\ln\left(\mathcal{P}_{ab}^{(1)}\right) + \left(\frac{\ln(k)-\ln(k_1)}{\ln(k_2)-\ln(k_1)}\right)\ln\left(\mathcal{P}_{ab}^{(2)}\right)\right], \tag{11}$$

where a, b $= \mathcal{I}$, $\mathcal{R}$ and $\mathcal{I} = \mathcal{I}_{BDI/CDI}$, $\mathcal{I}_{NDI}$, or $\mathcal{I}_{NVI}$. Here $\mathcal{R}$ refers to the standard curvature perturbations and $\mathcal{I}$ refers to the entropy or isocurvature perturbations. The scales are set to $k_1 = 2\times10^{-3}$ Mpc^{-1} and $k_2 = 0.1$ Mpc^{-1} so that $[k_1, k_2]$ spans most of the range constrained by Planck data. The amplitudes $\mathcal{P}_{ab}^{(1)}$ and $\mathcal{P}_{ab}^{(2)}$ are constants defined at k_1 and k_2, respectively.

Alternatively, you could define the isocurvature and correlated primordial power spectra in the standard "one-scale" power-law formulation where

$$\mathcal{P}_{\mathcal{R}\mathcal{R}}(k) = A_{\mathcal{R}\mathcal{R}}\left(\frac{k}{k_{\rm piv}}\right)^{n_{\mathcal{R}\mathcal{R}}-1}, \tag{12}$$

$$\mathcal{P}_{\mathcal{R}\mathcal{I}}(k) = A_{\mathcal{R}\mathcal{I}}\left(\frac{k}{k_{\rm piv}}\right)^{n_{\mathcal{R}\mathcal{I}}-1}, \tag{13}$$

$$\mathcal{P}_{\mathcal{I}\mathcal{I}}(k) = A_{\mathcal{I}\mathcal{I}}\left(\frac{k}{k_{\rm piv}}\right)^{n_{\mathcal{I}\mathcal{I}}-1}, \tag{14}$$

where for the curvature perturbations $A_{\mathcal{R}\mathcal{R}} = A_s$ is just the ΛCDM model scalar spectral amplitude and $n_{\mathcal{R}\mathcal{R}} = n_s$ is the scalar spectral index. The correlation spectral amplitude is assumed to be $n_{\mathcal{R}\mathcal{I}} = (n_{\mathcal{R}\mathcal{R}} + n_{\mathcal{I}\mathcal{I}})/2$, making $\mathcal{P}_{\mathcal{R}\mathcal{I}}{}^(2)$ not an independent parameter. You can translate between these two parameterizations using

$$n_{ab} = \frac{\ln\left(\mathcal{P}_{ab}^{(2)}/\mathcal{P}_{ab}^{(1)}\right)}{\ln(k_2/k_1)} + 1, \tag{15}$$

and

$$A_{ab} = \mathcal{P}_{ab}^{(1)}\exp[(n_{ab}-1)\ln(k_{\rm piv}/k_1)]. \tag{16}$$

Under either parameterization, for physical intuition, the more useful quantities to consider are the primordial isocurvature fraction, defined as

$$\beta_{\rm iso} = \frac{\mathcal{P}_{\mathcal{I}\mathcal{I}}(k)}{\mathcal{P}_{\mathcal{R}\mathcal{R}}(k)+\mathcal{P}_{\mathcal{I}\mathcal{I}}(k)}, \tag{17}$$

and the correlation fraction

$$\cos\Delta = \frac{\mathcal{P}_{\mathcal{R}\mathcal{I}}(k)}{(\mathcal{P}_{\mathcal{R}\mathcal{R}}(k)\mathcal{P}_{\mathcal{I}\mathcal{I}}(k))^{1/2}}, \tag{18}$$

which stays inside the interval (-1,1), corresponding to fully anti-correlated and fully correlated modes, at either end. Also the fraction of total power in each of the curvature ($\mathcal{R}\mathcal{R}$), isocurvature ($\mathcal{I}\mathcal{I}$), and correlated ($\mathcal{R}\mathcal{I}$) modes:

$$\alpha_{ab}(\ell_{\rm min},\ell_{\rm max}) = \frac{(\Delta T)^2_{ab}(\ell_{\rm min},\ell_{\rm max})}{(\Delta T)^2_{\rm tot}(\ell_{\rm min},\ell_{\rm max})}, \tag{19}$$

where

$$(\Delta T)^2_{ab}(\ell_{\rm min},\ell_{\rm max}) = \sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}}(2\ell+1)C^{TT}_{ab,\ell}. \tag{20}$$

IV. IMPLEMENTATION

In `CLASS`, you can include isocurvature perturbations by setting the initial conditions parameter `ic` to include any of the four standard isocurvature modes, as well as adiabatic perturbations. You set the parameterization of the primordial spectrum via the `Pk_ini_type` parameter. Setting it to `analytic_Pk` gives the standard power-law parameterization, whereas setting it to `two_scales` gives the Planck parameterization.

For the `analytic_Pk` parameterization, you can set the isocurvature-to-curvature ratio defined in the code as `f_x`, where `x` specifies each isocurvature mode, the tilt in each mode `n_x`, and the running of the tilt `alpha_x`, all of which are defined at the pivot scale. You can similarly define the correlation fraction $\cos\Delta$, defined in the code as `c_x_y` with `x,y` being the two correlated modes.

Alternatively, under the `two_scales` parameterization, you must define the two scales k_1, and k_2, and the amplitudes at each scale in the curvature, isocurvature and correlated power spectra.

For each isocurvature mode, and the special cases for specific inflationary models laid out in [15], or other generating mechanisms, one can derive constraints on either the parameters defining the isocurvature power spectra, or the derived parameters $\beta_{\rm iso}$, $\cos\Delta$, and α_{ab}. The Simons Observatory laid out the methods for their constraints in [2], and Planck laid out the different models worth constraining in [15].

Note the Simons Observatory ran the forecasts in [2] assuming a combination of Planck and SO data, i.e. Planck+SO noise curves, but this is not necessary for these forecasts. To add in any other experiments, like Planck or CMB-S4, all you would need to do is add in the noise curve to the provided Fisher code.

[1] D. Coe, "Fisher matrices and confidence ellipses: A quick-start guide and software," (2009), arXiv:0906.4123 [astro-ph.IM].
[2] P. Ade *et al.* (Simons Observatory), JCAP **02**, 056 (2019), arXiv:1808.07445 [astro-ph.CO].
[3] P. A. R. Ade *et al.* (Planck), Astron. Astrophys. **571**, A12 (2014), arXiv:1303.5072 [astro-ph.CO].
[4] J.-F. Cardoso, M. Martin, J. Delabrouille, M. Betoule, and G. Patanchon, (2008), arXiv:0803.1814 [astro-ph].
[5] E. Hughes, F. Ge, F.-Y. Cyr-Racine, L. Knox, and S. Raghunathan, Phys. Rev. D **109**, 103516 (2024), arXiv:2311.05678 [astro-ph.CO].
[6] S. Raghunathan, Astrophys. J. **928**, 16 (2022), arXiv:2112.07656 [astro-ph.CO].
[7] M. Bucher, K. Moodley, and N. Turok, Phys. Rev. D **62**, 083508 (2000), arXiv:astro-ph/9904231.
[8] D. Grin, O. Dore, and M. Kamionkowski, Phys. Rev. D **84**, 123003 (2011), arXiv:1107.5047 [astro-ph.CO].
[9] D. Grin, O. Dore, and M. Kamionkowski, Phys. Rev. Lett. **107**, 261301 (2011), arXiv:1107.1716 [astro-ph.CO].
[10] D. Grin, D. Hanson, G. P. Holder, O. Doré, and M. Kamionkowski, Phys. Rev. D **89**, 023006 (2014), arXiv:1306.4319 [astro-ph.CO].
[11] S. Weinberg, Phys. Rev. D **70**, 043541 (2004), arXiv:astro-ph/0401313.
[12] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, Phys. Rev. D **62**, 043527 (2000), arXiv:astro-ph/0003278.
[13] P. A. R. Ade *et al.* (Planck), Astron. Astrophys. **571**, A22 (2014), arXiv:1303.5082 [astro-ph.CO].
[14] P. A. R. Ade *et al.* (Planck), Astron. Astrophys. **594**, A20 (2016), arXiv:1502.02114 [astro-ph.CO].
[15] Y. Akrami *et al.* (Planck), Astron. Astrophys. **641**, A10 (2020), arXiv:1807.06211 [astro-ph.CO].