Characterising the Primordial Cosmic Perturbations Using MAP and PLANCK

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Abstract

The most general homogeneous and isotropic statistical ensemble of linear scalar perturbations which are regular at early times, in a universe with only photons, baryons, neutrinos, and a cold dark matter (CDM) component, is described by a 5x5 symmetric matrix-valued generalization of the power spectrum. This description is complete if the perturbations are Gaussian, and even in the non-Gaussian case describes all observables quadratic in the small perturbations. The matrix valued power spectrum describes the auto- and cross-correlations of the adiabatic, baryon isocurvature, CDM isocurvature, neutrino density isocurvature, and neutrino velocity isocurvature modes. In this paper we examine the prospects for constraining or discovering isocurvature modes using forthcoming MAP and PLANCK measurements of the cosmic microwave background (CMB) anisotropy. We also consider the degradation in estimates of the cosmological parameters resulting from the inclusion of these modes. In the case of MAP measurements of the temperature alone, the degradation is drastic. When isocurvature modes are admitted, uncertainties in the amplitudes of the mode auto- and cross-correlations, and in the cosmological parameters, become of order one. With the inclusion of polarisation (at an optimistic sensitivity) the situation improves for the cosmological parameters but the isocurvature modes are still only weakly constrained. Measurements with PLANCK's estimated errors are far more constraining, especially so with the inclusion of polarisation. If PLANCK operates as planned the amplitudes of isocurvature modes will be constrained to less than ten per cent of the adiabatic mode and simultaneously key cosmological parameters will be estimated to a few per cent or better.

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I. INTRODUCTION

Because the physics by which cosmological perturbations imprint themselves on the cosmic microwave backgroun (CMB) sky is very nearly linear, CMB observations offer a clean and comparatively direct probe of the nature of the primordial perturbations. Starting with COBE, which established a normalization for the perturbation amplitude on large scales [1]. The dramatic recent results of TOCO, BOOMerANG and MAXIMA [2–4] indicating a first Doppler peak, and limiting the amplitude of the second, are beginning to severely constrain cosmological models. With the new forthcoming data from the MAP satellite [5] to be launched in Spring 2001, and from the PLANCK satellite, [6] to be launched in 2007, the situation will greatly improve, and it is hoped that one will be able to determine a host of cosmological parameters with great precision from the CMB alone. [7]

One of the key assumptions underlying this program is that that the primordial fluctuations were adiabatic, that is the relative abundances of different particle species were unperturbed from their thermal equilibrium values. This assumption has the great merit of simplicity and is even justified in many specific models of the origin of the fluctuations. But given its centrality in inferences based on the CMB anisotropy, it seems worthwhile to attempt to check the adiabaticity assumption using CMB data.

From a theoretical point of view, cosmic inflation is the leading contender as an explanation for the origin of the perturbations. Inflation provides an elegant mechanism for simultaneously explaining the large scale structure of the Universe and the perturbations [8,9]. In the simplest inflationary models the latter are indeed of the simplest form, namely adiabatic, Gaussian and with a nearly scale invariant power spectrum. But it is important to note that adiabaticity is not so much a consequence of inflation as of the assumption that no additional information beyond the overall density perturbation survived the inflationary era. If fields exist which were perturbed during inflation, and their excitations survived or decayed in a non-adiabatic manner, then isocurvature perturbations would have generically been concomitantly produced. Many inflationary models have been constructed in which this occurs. Given that there is no currently compelling model, and that most realisations of inflation within unified theories invoke a great number of additional fields, it seems premature to associate the prediction of adiabaticity with inflation. [10]

This situation motivates a more phenomenological approach to analyzing the new data that contemplates a wider range of possibilities for the nature of the primordial fluctuations, and seeks to infer from the data what limits may be set on non-adiabatic perturbations. In a previous paper we examined the most general primordial perturbation possible in a cosmological model with no new physics—with only baryonic matter (and its associated electrons), photons, neutrinos, and a cold dark matter component, which is regular at early times. [11] Primordial here refers to the assumption that the perturbations were generated at a very early time, well before recombination (at $z \approx 1100$), so that any singular (i.e., decaying) modes that may have been produced had ample opportunity to decay away before leaving an imprint on the CMB. In this work we found five regular modes: an adiabatic growing mode, a baryon isocurvature mode, a CDM isocurvature mode, a neutrino density isocurvature mode, and a neutrino velocity isocurvature mode. The adiabatic growing mode assumes a common equation of state, spatially uniform everywhere in the universe. In the baryon isocurvature mode the ratio of baryons to photons varies spatially, [12–14] and

similarly in the CDM isocurvature mode the ratio of CDM to photons varies spatially. [15] For the neutrino isocurvature modes, perturbations in the neutrino energy and momentum densities are balanced by opposing perturbations in the photon-radiation component, so that at early times the total stress-energy perturbation vanishes. At later times, however, the differences in how photons and neutrinos evolve lead to perturbations in the total stress-energy, which generate perturbations in the gravitational potentials, which in turn cause the baryons and CDM to cluster. Upon entering the horizon, the neutrinos free stream behaving as a collisionless fluid, while the photons behave like a perfect fluid because of the strong Thompson scattering off free electrons. The neutrino isocurvature modes, discussed in detail in ref. [11], are implicit in the work of Rebhan and Schwarz [16] and of Challinor and Lasenby [17]; however, these authors do not investigate their implications. If Gaussian perturbations produced by a spatially homogeneous and isotropic random process are assumed, the most general perturbation of these five modes is completely described by the 5x5 symmetric correlation matrix

$$P_{ij}(\mathbf{k}) = \langle A_i(\mathbf{k}) \ A_i(-\mathbf{k}) \rangle \tag{1}$$

where (i, j = 1, ..., 5) labels the modes and $A_i(\mathbf{k})$ indicates the strength of the *i*th mode with wavenumber k. This generalizes the usual scalar power spectrum. In the case of non-Gaussian perturbations, the above matrix suffices to determine the expectation values for all perturbations quadratic in the small perturbation (this includes the CMB power spectrum) if one assumes that the linearized theory is valid, which is a good approximation. The possibility of ascribing cosmological perturbations entirely to isocurvature modes, either to the baryon isocurvature mode or to the CDM isocurvature mode, has previously been considered, and these possibilities were found inconsistent with the existing observational data. [18] – [25] However, apart from the work of Enqvist and Kurki-Suonio [27] and of Pierpaoli, Garcia-Bellido, and Brogani [28] little effort has been devoted to the problem of detecting or constraining admixtures of isocurvature modes observationally. Moreover, when one admits the possibility of more than one mode being excited, the possibility arises of correlations between the several modes. These are characterized by the off-diagonal elements of $P_{ij}(\mathbf{k})$. Linde and Mukhanov [29], Langlois, [30] and Langlois and Riazuelo [31], drawing on the work of Polarski [32] on double inflation, investigated an inflationary model with two scalar fields exciting both the adiabatic and baryon isocurvature modes in varying proportions and with various degrees of correlation between these modes. In an inflationary model with five or more scalar fields (or a single field with the same number of real components), it is generically possible to realize the most general $P_{ij}(\mathbf{k})$ of the form discussed above.

The outline of the paper is as follows. Section II discusses the statistical techniques employed to interpret CMB measurements and reviews the properties of the five regular perturbation modes we allow. Section III gives numerical results in the form of eigenvectors and eigenvalues of the 'Fisher matrix', corresponding to the MAP and PLANCK measurements. The Tables provided allow one to calculate the uncertainty in any cosmological parameter or function thereof, as well as the amplitudes of the primordial perturbation modes. Section IV discusses the significance of the results, and our main conclusion, which is that a high precision measurement of the cosmic polarisation will be essential to an accurate determination of the cosmological parameters and the primordial perturbations.

We should mention three limitations of our analysis. Our main goal is to explore the effect of relaxing the assumption of adiabatic perturbations. Therefore we ignore tensor modes, which would complicate the discussion. But many inflationary models do predict tensor modes and it is important they be included in any complete analysis. The second caveat is that we are only considering very sub-dominant isocurvature perturbations here - our analysis is perturbative around a standard Λ CDM model. Finally we do not vary the spectral indices (or the shape of the spectra) for the isocurvature modes. We consider only 'scale invariant' isocurvature perturbations, as defined below. In inflationary models, isocurvature perturbations (like adiabatic ones) would in general have adjustable power spectra and again that would complicate the discussion.

II. THE GENERAL PRIMORDIAL COSMIC PERTURBATION

Statistical analysis of observational data, be it based on 'classical' or 'frequentist' statistics or on Bayesian statistics, ultimately reduces to considering relative likelihoods of competing theoretical descriptions given the observed data. Because the observational data is not yet available, we must assume a particular underlying theoretical model for computing expectations for relative likelihoods. We assume a statistical model with independent Gaussian distributions for the individual moments of the CMB multipole expansion with variance

$$\langle |a_{\ell m}|^2 \rangle = c_{\ell} + \sigma_{n,\ell}^2 \tag{2}$$

where c_{ℓ} is the variance of the underlying cosmological signal and $\sigma_{n,\ell}^2$ is the variance resulting from detector noise. It follows that the expectation value of the logarithm of the relative likelihood of model B relative to model A under the assumption that the data was produced according the distribution from model A is given by the formula

$$\left\langle \log \left[\frac{p_B}{p_A} \right] \right\rangle_A = \left\langle \log \left[\frac{p(\{a_{\ell m}\}|B)}{p(\{a_{\ell m}\}|A)} \right] \right\rangle_A$$

$$= \frac{f_{sky}}{2} \sum_{l=2}^{\ell_{max}} (2\ell + 1) \left\{ 1 - \frac{c_{\ell,A} + \sigma_{n,\ell}^2}{c_{\ell,B} + \sigma_{n,\ell}^2} + \log \left[\frac{c_{\ell,A} + \sigma_{n,\ell}^2}{c_{\ell,B} + \sigma_{n,\ell}^2} \right] \right\}$$
(3)

where $\{a_{\ell m}\}$ is the observed data, $c_{\ell,A}$ and $c_{\ell,B}$ are the variances of the cosmological signal predicted by models A and B, respectively, and $\sigma_{n,\ell}^2$ is the Gaussian detector noise for each multipole of order ℓ . f_{sky} indicates the fraction of the sky remaining after the galaxy cut.

We consider cosmological models where c_{ℓ} depends on a number of continuous parameters $\alpha_1, \ldots, \alpha_N$. With relatively little detector noise and with changes in the parameters affecting a large number of multipoles, in the region of interest, where the likehood relative to the reference model is comparable, the following quadratic approximation is justified

$$\left\langle \log \left[\frac{p(\alpha_1, \dots, \alpha_N)}{p(\alpha_1^{(0)}, \dots, \alpha_N^{(0)})} \right] \right\rangle_{(0)}$$

$$= -\frac{f_{sky}}{2} \sum_{i,j=1}^{N} \sum_{l=2}^{\ell_{max}} (2\ell + 1) \times \frac{1}{(c_{\ell,B} + \sigma_{n,\ell}^2)^2} \frac{\partial c_{\ell}}{\partial \alpha_i} \frac{\partial c_{\ell}}{\partial \alpha_j} \frac{(\alpha_i - \alpha_i^{(0)})(\alpha_j - \alpha_j^{(0)})}{2}. \tag{4}$$

We write the right hand side as $-\frac{1}{2}F_{ij}(\alpha_i - \alpha_i^{(0)})(\alpha_j - \alpha_j^{(0)})$. In the case where the variations in the logarithm are described by a quadratic form, the matrix F_{ij} is equivalent to the Fisher matrix, and in the sequel we shall refer to it in this way even though the terminology is not completely accurate. The quadratic approximation may break down when considering variations primarily affecting very low- ℓ moments, where a Gaussian approximation to a χ^2 -distribution of low order is inaccurate even near where it is peaked.

When polarization is included, eqn. (2) is modified to become

$$M_{\ell} = \begin{pmatrix} |\langle a_{\ell m} \ a_{\ell m} \rangle| & |\langle a_{\ell m} \ b_{\ell m} \rangle| \\ |\langle b_{\ell m} \ a_{\ell m} \rangle| & |\langle b_{\ell m} \ b_{\ell m} \rangle| \end{pmatrix} = \begin{pmatrix} c_{\ell,T} + \sigma_{n\ell,T}^2 & c_{\ell,C} \\ c_{\ell,C} & c_{\ell,P} + \sigma_{n\ell,P}^2 \end{pmatrix}$$
(5)

and (3) is modified to

$$\left\langle \log \left[\frac{p_B}{p_A} \right] \right\rangle_A = \left\langle \log \left[\frac{p(\{a_{\ell m}\}, \{b_{\ell m}\} | B)}{p(\{a_{\ell m}\}, \{b_{\ell m}\} | A)} \right] \right\rangle_A$$

$$= \frac{f_{sky}}{2} \sum_{l=2}^{\ell_{max}} (2\ell + 1) \left[\operatorname{tr} \left\{ I - M_{\ell A} M_{\ell B}^{-1} \right\} + \ln \left\{ \operatorname{det} \left(M_{\ell A} M_{\ell B}^{-1} \right) \right\} \right]$$
(6)

The second derivative of the above may be expressed as

$$\delta^{2} \left\langle \log \left[\frac{p_{B}}{p_{A}} \right] \right\rangle_{A} = f_{sky} \sum_{l=2}^{\ell_{max}} (2\ell + 1)$$

$$\times \left[\operatorname{tr} \left\{ (\delta M_{\ell}) M_{ref,\ell}^{-1} (\delta M_{\ell}) M_{ref,\ell}^{-1} \right\} + \operatorname{det} \left\{ (\delta M_{\ell}) M_{ref,\ell}^{-1} \right\} - \frac{1}{2} \operatorname{tr}^{2} \left\{ (\delta M_{\ell}) M_{ref,\ell}^{-1} \right\} \right]. (7)$$

Most studies of parameter estimation using future CMB data assume adiabatic perturbations from inflation allowing a set of model parameters, such as H_0 , Ω_b , Ω_{Λ} for example, to be varied. A fiducial, or reference, model is assumed, and the behavior of the relative likelihood in the neighborhood of this reference model is explored.

We extend this approach by including parameters for the strengths of the isocurvature modes and of their cross-correlations, both with respect to one another as well as to the adiabatic mode. In this case the parametric model becomes as follows

$$c_{\ell}(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{\xi}) = c_{\ell}^{adia}(\beta_{1}, \dots, \beta_{N}) + \sum_{A=1}^{4} \gamma_{A} c_{\ell}^{A}(\beta_{1}, \dots, \beta_{N})$$

$$+ \sum_{A=1}^{4} \delta_{A} c_{\ell}^{A-adia}(\beta_{1}, \dots, \beta_{N}) + \sum_{\substack{A,B=1\\A\neq B}}^{4} \xi_{AB} c_{\ell}^{cross,AB}(\beta_{1}, \dots, \beta_{N})$$
(8)

where the indices A, B label the four nonsingular isocurvature modes. Here the vector $\boldsymbol{\beta}$ represents the usual cosmological parameters. The vector $\boldsymbol{\gamma}$ indicates the autocorrelations of the isocurvature modes. The vector $\boldsymbol{\delta}$ indicates the cross-correlation of the isocurvature modes with the adiabatic mode, and symmetric off-diagonal elements ξ_{AB} indicates the cross correlations of the isocurvature modes with each other. The components c_{ℓ}^{A} are computed with a modified version of CMBFAST with the isocurvature mode excited. The cross correlations are computed by running CMBFAST first with two

modes excited and then subtracting the results when each mode individually excited, using $Q(A, B) = \frac{1}{2}[Q(A + B, A + B) - Q(A, A) - Q(B, B)]$ for a quadratic form Q. From the viewpoint of statistical analysis described above, all components of the combined vector $\boldsymbol{\alpha} = (\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{\xi})$ stand on equal footing. The three components $\boldsymbol{\gamma}$, $\boldsymbol{\delta}$, $\boldsymbol{\xi}$ are related to the correlation matrix M given in ref. [11] according to

$$M = \begin{pmatrix} 1 & \delta_1 & \delta_2 & \delta_3 & \delta_4 \\ \delta_1 & \gamma_1 & \xi_{1,2} & \xi_{1,3} & \xi_{1,4} \\ \delta_2 & \xi_{2,1} & \gamma_2 & \xi_{2,3} & \xi_{2,4} \\ \delta_3 & \xi_{3,1} & \xi_{3,2} & \gamma_3 & \xi_{3,4} \\ \delta_4 & \xi_{4,1} & \xi_{4,2} & \xi_{4,3} & \gamma_4 \end{pmatrix}$$
(9)

Some free parameters of our model cannot be treated in the same way as the ones above when the fiducial model is purely adiabatic with no isocurvature component excited. These include the spectral indices for the isocurvature modes. The difficulty arises because near the fiducial model the dependence on these spectral indices vanishes. In this paper we shall simply fix the spectral indices to correspond to 'scale invariant' isocurvature perturbations. For simplicity we also choose the cross correlation power spectra to be just the geometric mean of the autocorrelation power spectra of the same two variables. This assumption is straightforward to generalise, and we shall do so elsewhere.

Figures 1 and 2 show the C_l power spectra for the modes we allow here. We choose our fiducial cosmological model to be h=0.65, $\Omega_b=0.06$, $\Omega_{\Lambda}=0.69$, $\Omega_{cdm}=0.25$, $n_S=1$, and a small amount of reionization with an optical depth to the last scattering surface of $\tau=0.1$. In Figure 1, $l(l+1)C_l$ from adiabatic scale-invariant perturbations is plotted against the same quantity for baryon isocurvature, neutrino isocurvature density and neutrino isocurvature velocity modes. For the baryon isocurvature mode a scale free spectrum of $\delta(\rho_B/\rho_{\gamma})$ is assumed i.e. the variance at early times is a logarithmically divergent integral over wavenumber. For neutrino isocurvature density perturbations a scale free spectrum is assumed for $\delta(\rho_{\nu}/\rho_{\gamma})$ and for neutrino isocurvature velocity perturbations a scale free spectrum is assumed for the bulk neutrino velocity v_{ν} . The CMB spectra for the CDM isocurvature mode agree with those obtained for the baryon isocurvature mode to a fraction of a percent. Therefore, we do not consider the CDM case separately. The plots also show the $l(l+1)C_l$ produced from the cross correlation of the allowed modes.

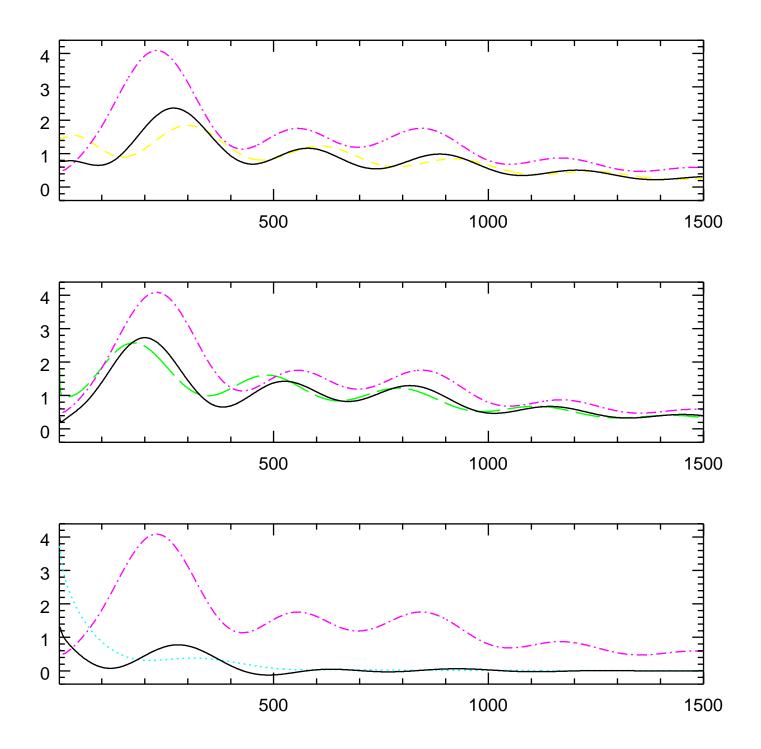


FIG. 1. CMB anisotropy power spectra $l(l+1)C_l$ are plotted versus l. The three plots show cross-correlation power spectra as solid lines: adiabatic-baryon isocurvature density (lower), adiabatic-neutrino isocurvature density (middle) and adiabatic-neutrino isocurvature velocity (upper). The relevant auto-correlation spectra are also shown on each plot as follows: adiabatic perturbations (dot-dashed, magenta), baryon isocurvature (dotted, cyan), neutrino density isocurvature (short dashed, yellow), and neutrino velocity isocurvature (long dashed, green). All are assumed to have scale invariant underlying power spectra.

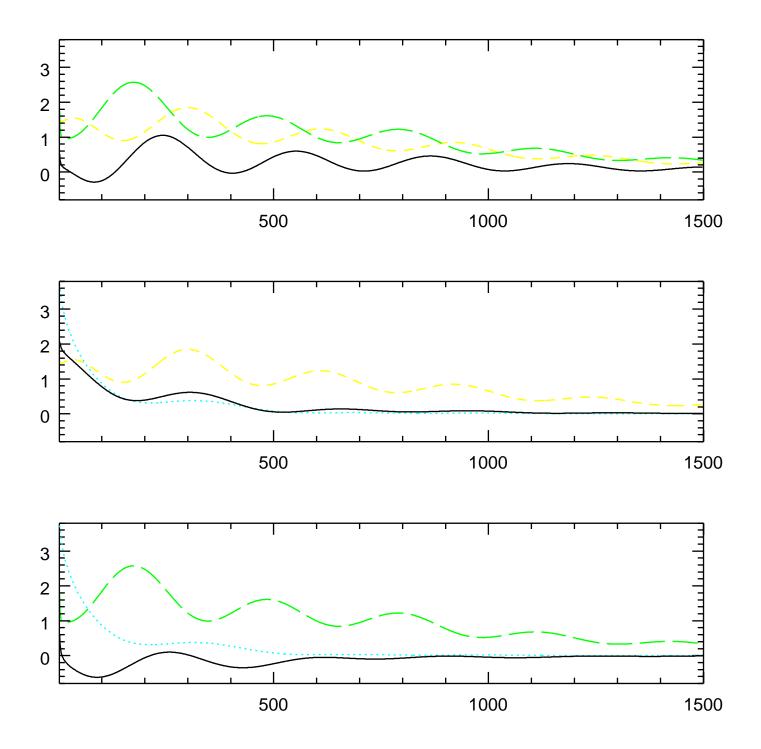


FIG. 2. As in Figure 1, but for the baryon isocurvature -neutrino isocurvature velocity correlation (lower), baryon isocurvature-neutrino isocurvature density correlation (middle) and neutrino isocurvature density-neutrino isocurvature velocity correlation (upper). On each plot the cross-correlation power spectrum is shown as a solid black line, with the corresponding auto-correlation power spectra denoted as in Figure 1.

III. NUMERICAL RESULTS

We now proceed to our numerical results. Both the MAP and PLANCK experiments are considered. Following [34], we assume a fractional sky coverage $f_{sky} = 0.8$ after the galaxy cut and ignore possible foreground contamination. We further assume that the several channels may be combined, using for each multipole the linear combination giving the least variance for the noise. In other words, we assume that linear combinations chosen to project out foreground contamination based on differing spectral properties are not necessary. Under these assumptions, it follows that

$$\frac{1}{\sigma_{n,\ell}^2} = \sum_c \frac{B_{\ell,c}^2}{\sigma_c^2 \theta_{fwhm,c}^2} \tag{10}$$

where c indicates the sum over channels, $B_{\ell,c}^2 = \exp[-(0.425 \ \theta_{fwhm,c} \ \ell)^2]$ is the window function assuming a Gaussian beam, $\theta_{fwhm,c}$ is the full width at half maximum of the cth channel, and $\sigma_c^2 \ \theta_{fwhm,c}^2$ is the mean square noise per multipole. The assumption of no correlations between pixels makes this quantity independent of ℓ . For MAP we assume three channels with $\theta_{fwhm,c} = 0.47^{\circ}$, 0.35° , and 0.21° with $\sigma = 17.2\mu K$, $30\mu K$, and $50\mu K$, respectively, as anticipated after two years of data. For Planck, we use the three lowest frequency channels of the High Frequency Instrument (HFI) with $\theta_{fwhm,c} = 0.18^{\circ}$, 0.13° , and 0.092° with $\sigma_c = 4.5\mu K$, $5.5\mu K$, and $11.8\mu K$, respectively, as expected after 12 months of data (which would correspond to $2.7\mu K$, $2.4\mu K$, and $3.6\mu K$ on a 0.3° square pixel).

We also indicate below the result of including polarization. For the channels for which polarization information is available (all three MAP channels used here and the two higher frequency of the three Planck channels), we assume that $\sigma_{nP}^2 = 2\sigma_{nT}^2$. This assumption is optimistic for MAP. The experiment has not been optimised for a polarisation measurement, and systematics and foreground effects not included here will in all likellihood dominate the polarisation signal. Nevertheless we include calculations making the most optimistic assumption for comparative purposes.

We do not indicate an uncertainty in normalization, because a natural way to compare relative normalizations between spectra of different shapes is lacking. Other authors indicate the uncertainty in the predicted expectation value for the quadrupole. This, however, is a not a very useful quantity given the large uncertainty from cosmic variance in the quadrupole. We instead marginalize over the normalization. In the quadratic approximation this is equivalent to using the best fit normalization for the given values of the parameters. This amounts to replacing the Fisher matrix with the reduced Fisher matrix

$$\hat{F}_{ij} = F_{ij} - \frac{F_{i0}F_{j0}}{F_{00}}. (11)$$

where the index 0 labels the parameter describing the overall normalisation of the power spectrum.

 $^{^{1}\}sigma=27\mu K$, $35\mu K$, and $35\mu K$ are the corresponding temperature errors for a 0.3° square pixel given in the Wang, Spergel, and Strauss paper. [34]

In the following subsections we present tables of the eigenvalues and eigenvectors (spectral decomposition) of the reduced Fisher matrix \hat{F}_{ij} . These contain a wealth of information, which is why we include them in this preprint version of the paper. They enable one to directly determine (in the Gaussian approximation) the uncertainties in any cosmological parameters or perturbation amplitudes. They also clearly illustrate which quantities are and are not accurately determined by the measurements. We hope the reader enjoys perusing these tables.

A. Fiducial Model with No Isocurvature Modes

Table 1-M-T presents the spectral decomposition of the Fisher matrix for adiabatic perturbations in a spatially flat cosmological model with h=0.65, $\Omega_b=0.06$, $\Omega_{\Lambda}=0.69$, $\Omega_{cdm}=0.25$, $n_S=1$, and a small amount of reionization with an optical depth to the last scattering surface of $\tau=0.1$. Variations in the parameters H_0 , Ω_{Λ} , Ω_b , n_S , τ , and Ω_k are considered. Here n_S is the exponent of the power law for the scalar power spectrum and the spatial curvature is characterized in terms of $\Omega_k=(1-\Omega_{\Lambda}-\Omega_{cdm}-\Omega_b)$ Except for the last two of these parameters, all variations are considered fractionally. We give the eigenvectors of the Fisher matrix and the percentage error associated with measuring each eigenvector component (i.e., $100 \times \lambda^{-1/2}$). This table uses the noise parameters for the MAP satellite with only the temperature anisotropy taken into account. The lower table in Table 1 indicates the percentage errors in determining each of the cosmological parameters parameters—that is, $\sqrt{(\hat{F}^{-1})_{ii}}$ for the parameter labeled by i.

Tables 1-M-TP repeat the above analysis but using both temperature and polarization as well as the cross correlation between the temperature and polarization multipole moments. Table 1-P-T, and 1-P-TP are the analogues for the PLANCK satellite.

B. Fiducial Adiabatic Model Plus One Isocurvature Mode

In this subsection we consider models exactly one isocurvature mode excited in addition to the adiabatic mode. We normalize the isocurvature modes so that their mean square power contributed the the CMB temperature anisotropy summed from $\ell = 2$ to $\ell = 1500$ is equivalent to that of the adiabatic mode. For one additional mode, two new parameters are added in addition to those described in the previous section: the isocurvature auto-correlation $\langle II \rangle$ and the cross-correlation with the adiabatic mode $\langle AI \rangle$. The positive definiteness of the matrix valued power spectrum requires that $|\langle AI \rangle| \leq \langle II \rangle$.

1. Baryon Isocurvature Mode

Tables 2-M-T, 2-M-TP, 2-P-T and 2-P-TP perform the above analysis where I indicates the baryon isocurvature mode. A scale-free spectrum for the baryon isocurvature mode is assumed, as defined above, and the cross correlation power spectrum is the geometric mean of the adiabatic and isocurvature spectra.

2. Neutrino Density Isocurvature Mode

Tables 3-M-T, 3-M-TP, 3-P-T and 3-P-TP perform the above analysis where I indicates the neutrino density isocurvature mode, taken to have a scale free spectrum.

3. Neutrino Velocity Isocurvature Mode

Tables 4-M-T, 4-M-TP, 4-P-T and 4-P-TP perform the above analyis where I indicates the neutrino velocity isocurvature mode. Again the velocity mode is taken with a scale free spectrum.

C. Fiducial Adiabatic Model Plus Two Isocurvature Modes

With two isocurvature modes I_1 and I_2 there are five additional parameters: two autocorrelations $\langle I_1I_1\rangle$ and $\langle I_1I_2\rangle$, and three cross correlations $\langle AI_1\rangle$, $\langle AI_2\rangle$, and $\langle I_1I_2\rangle$. Again there is a constraint on these parameters arising from the requirements of positive definiteness of the matrix-valued power spectrum.

For Tables 5-M-T, 5-M-TP, 5-P-T and 5-P-TP, I_1 is the neutrino density isocurvature mode and I_2 is the baryon isocurvature mode. For Tables 6-M-T, 6-M-TP, 6-P-T and 6-P-TP, I_1 is the neutrino velocity isocurvature mode and I_2 is the baryon isocurvature mode. For Tables 7-M-T, 7-M-TP, 7-P-T and 7-P-TP, I_1 is the neutrino density isocurvature mode and I_2 is the neutrino velocity isocurvature mode.

D. Fiducial Adiabatic Model Plus Three Isocurvature Modes

We now include the baryon isocurvature and both neutrino isocurvature modes. There are now three auto-correlations and six cross-correlations. The results are indicated in Tables 8-M-T, 8-M-TP, 8-P-T and 8-P-TP. Given the large uncertainties (of order unity) in all cases except P-TP, the quadratic approximations employed are no longer accurate. Moreover, the large ratios between the largest and smallest eigenvalues make the calculations sensitive to small errors in the computing the Fisher matrix elements, since the smallest eigenvalues control the largest errors. Nevertheless, the result that the errors in the parameters are large (of order unity) is reliable.

IV. DISCUSSION

We have considered the question of to what extent it will be possible to check the assumption that the primordial perturbations were adiabatic. In our view, since the theoretical situation is not clear cut, such a check is probably essential in order for us to reliably interpret the CMB anistropy as a probe of cosmological parameters. Our finding here is that the MAP satellite alone, even with an optimistic assumption about the polarisation measurement, will be unable to set useful limits on the amplitudes of isocurvature modes. The

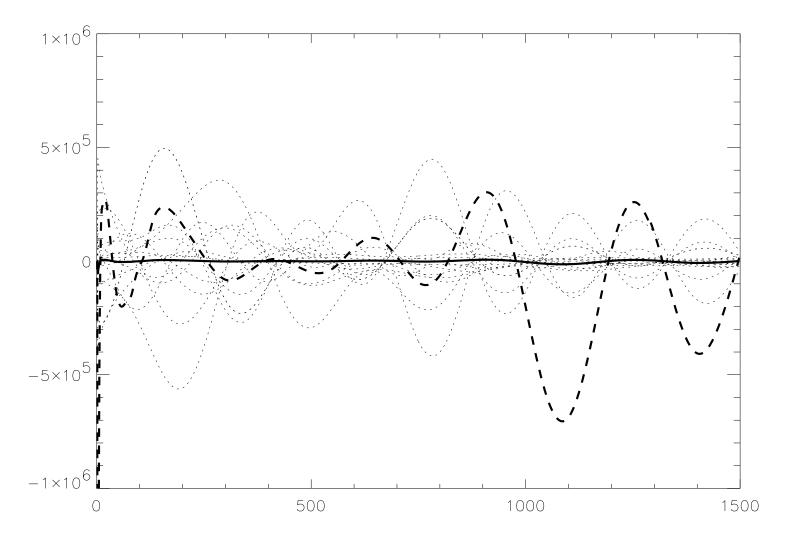


FIG. 3. Illustration of the degeneracy problem. Including isocurvature modes renders the determination of cosmological parameters from the MAP satellite hazardous. Deviations from the fiducial adiabatic model in certain directions in the space of cosmological parameters and density perturbation amplitudes produce nearly degenerate C_l spectra. Here we show the deviations $\delta l(l+1)C_l$ for each parameter change multiplied by the appropriate component of the eigenvector of \hat{F}_{ij} with smallest eigenvalue (given in Table 8-M-TP). The dotted lines show the individual contributions as a function of l, and the solid line the sum after the normalisation has been projected out. The heavy dashed line shows the sum multiplied by 50 to render it more visible. The latter deviation is not measurable with MAP, but is with PLANCK, presumably because the latter is more sensitive to the high l structure.

PLANCK satellite will be much more powerful in this respect, and will be able to limit the amplitudes of isocurvature modes to less than ten per cent of the adiabatic mode.

A second issue is the extent to which the errors in determining cosmological parameters from the CMB alone degrade as an increasing number of isocurvature modes is admitted. When in addition to the adiabatic mode only one isocurvature mode is allowed, MAP (with polarisation) can still place quite stringent constraints on the possible contribution of the isocurvature modes (Tables 2-4), and the increase in the errors in determining the other cosmological parameters is quite modest. However, when more than one isocurvature mode and the corresponding correlations are considered, the errors increase quite dramatically, and worse, there are only very weak limits on the contamination of the adiabatic mode with isocurvature modes. In the case of all four isocurvature modes the fractional errors become of order one, even when polarization information (using the optimistic estimate) is taken into account. Without a polarisation measurement, admitting isocurvature modes ruins MAP's ability to measure cosmological parameters. The behaviour observed is likely to be simply a result of the model possessing too many degrees of freedom. For all models considered the CMB moments are rather smooth, slowly varying functions of ℓ , so that a spline passing through a rather modest number of points would quite accurately characterize any of the theoretical models considered. Hence in regard to parameter estimation the CMB data in practice contains much less useful information than one might naively conclude if one argued that all of the C_{ℓ} 's are independent. Instead of giving us $l_{max} \sim 1000$ numbers, we obtain more like 10.

The situation is improved if we consider the PLANCK's estimated sensitivities. PLANCK has been designed to accurately measure polarisation and so the estimated errors here may be more realistic than those for MAP. Our results demonstrate there is a high payoff for an accurate measurement of the polarisation. Table 8-P-TP shows that even when we allow all possible isocurvature modes, with arbitrary cross correlations, then PLANCK can set upper limits of less than ten per cent on the isocurvature mode auto- and cross-correlation power relative to the adiabatic power. Simultaneously, PLANCK can constrain the most interesting cosmological parameters to a few per cent or better.

In conclusion this study has focussed on one difficulty in interpreting the CMB anisotropy data, namely in checking the assumption that the primordial perturbations were adiabatic. We have made severe idealisations in other respects, namely assuming Gaussianity and uniform noise, and in ignoring foreground contamination. Dealing with these issues will be a massive challenge for the real experiments. Nevertheless the calculations reported do offer a clear goal and a lesson, namely that high precision measurements of the polarisation, as well as the temperature of the cosmic microwave sky, will likely be essential to a conclusive understanding of it.

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	1	2	3	4	5	6
% error	0.15	0.40	1.30	2.98	26.38	48.81
$\delta h/h$	17.85	-69.72	-51.48	-2.55	-41.04	-21.90
$\delta\Omega_b/\Omega_b$	-12.13	-36.20	-31.41	9.60	64.16	57.86
$\delta\Omega_k$	-89.17	-8.12	-8.25	-38.79	-1.46	-20.20
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-39.76	-4.37	8.07	82.30	-34.96	18.43
$\delta n_s/n_s$	-1.47	61.08	-76.84	1.16	-14.57	12.16
$ au_{reion}$	0.58	-3.56	18.02	-40.27	-52.56	72.65

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$
15.23	32.93	9.94	13.12	7.15	38.09

Table 1-M-T

	1	2	3	4	5	6
% error	0.14	0.40	1.23	1.76	3.17	20.22
$\delta h/h$	17.64	-69.86	-47.80	-19.59	2.25	-46.20
$\delta\Omega_b/\Omega_b$	-12.17	-36.19	-28.46	-11.32	18.48	85.22
$\delta\Omega_k$	-89.28	-8.07	-6.81	-12.74	-40.30	-11.40
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-39.58	-4.07	6.09	23.20	86.05	-20.92
$\delta n_s/n_s$	-1.41	60.91	-69.11	-35.32	15.32	-5.43
$ au_{reion}$	0.71	-4.20	45.23	-86.83	19.75	-2.39

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$
9.37	17.25	2.65	5.05	1.61	1.81

Table 1-M-TP

	1	2	3	4	5	6
% error	0.03	0.23	0.30	1.81	5.79	28.28
$\delta h/h$	-26.45	-24.44	-80.07	15.36	12.68	43.54
$\delta\Omega_b/\Omega_b$	7.36	-11.91	-45.38	13.05	-16.75	-85.40
$\delta\Omega_k$	87.22	4.52	-28.26	-35.55	-3.73	17.20
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	40.46	-20.69	16.16	85.09	19.64	6.93
$\delta n_s/n_s$	-1.24	93.84	-21.68	25.55	8.37	0.59
$ au_{reion}$	0.00	-2.64	0.15	-20.91	95.34	-21.60

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$
12.34	24.18	4.91	2.74	0.73	8.24

Table 1-P-T

	1	2	3	4	5	6
% error	0.02	0.17	0.21	0.39	0.85	3.54
$\delta h/h$	25.69	79.23	28.41	2.28	11.84	45.93
$\delta\Omega_b/\Omega_b$	-7.20	42.53	14.76	0.94	25.82	-85.17
$\delta\Omega_k$	-88.15	28.76	-2.41	-4.21	-35.59	10.56
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-38.95	-20.43	20.50	10.74	83.89	22.19
$\delta n_s/n_s$	0.41	25.76	-92.22	-3.76	28.08	5.32
$ au_{reion}$	0.03	-2.19	6.60	-99.23	10.04	2.00

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$
1.64	3.02	0.48	1.06	0.36	0.40

Table 1-P-TP

	1	2	3	4	5	6	7	8
% error	0.14	0.40	1.06	2.72	3.70	9.93	27.37	50.58
$\delta h/h$	17.60	69.74	-35.52	18.12	-29.12	-11.82	43.39	-19.12
$\delta\Omega_b/\Omega_b$	-12.17	35.94	-21.20	2.49	-26.09	-5.31	-65.74	55.44
$\delta\Omega_k$	-88.78	6.76	-0.81	35.71	9.93	-17.69	2.88	-19.39
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-39.59	3.67	7.18	-70.50	-37.94	20.41	34.13	19.42
$\delta n_s/n_s$	-1.40	-59.71	-66.91	18.11	-33.90	-7.53	15.38	13.56
$ au_{reion}$	0.58	3.34	15.22	27.92	31.54	-2.40	48.69	74.87
$\langle II \rangle$	7.22	-13.28	58.78	38.50	-69.19	-6.02	2.93	1.28
$\langle AI \rangle$	6.17	-5.67	8.50	-28.81	1.36	-94.89	2.44	4.02

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
15.41	33.34	10.05	13.91	8.23	40.17	3.08	9.70

Table 2-M-T

	1	2	3	4	5	6	7	8
% error	0.14	0.40	1.02	1.72	2.61	3.56	8.25	21.00
$\delta h/h$	17.40	69.88	-34.11	6.52	-26.43	-26.21	14.75	-44.77
$\delta\Omega_b/\Omega_b$	-12.20	35.93	-20.12	3.64	-5.94	-28.81	-8.08	84.92
$\delta\Omega_k$	-88.90	6.70	-0.41	8.12	-32.71	21.21	19.50	-9.29
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-39.40	3.37	6.78	-20.14	57.57	-60.94	-20.46	-23.21
$\delta n_s/n_s$	-1.35	-59.51	-63.75	20.41	-22.81	-37.54	4.71	-4.87
$ au_{reion}$	0.71	3.90	29.89	93.20	13.90	-14.27	1.53	-2.19
$\langle II \rangle$	7.23	-13.38	57.93	-16.72	-60.28	-49.99	-1.04	-0.13
$\langle AI \rangle$	6.20	-5.75	8.51	-9.47	23.13	-14.41	94.30	11.44

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
9.56	17.88	2.78	5.81	1.99	1.81	2.47	8.19

Table 2-M-TP

	1	2	3	4	5	6	7	8
% error	0.03	0.23	0.29	1.28	1.86	2.77	6.54	29.55
$\delta h/h$	26.44	23.89	-76.50	-19.39	-20.05	-8.96	-10.93	43.59
$\delta\Omega_b/\Omega_b$	-7.36	11.60	-43.18	-9.20	-14.94	-6.04	21.28	-84.55
$\delta\Omega_k$	-87.19	-4.70	-25.98	-17.99	32.60	-1.64	3.31	17.36
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-40.44	20.77	15.31	22.59	-82.49	-5.26	-17.66	6.60
$\delta n_s/n_s$	1.24	-93.89	-19.61	-3.17	-27.24	-0.58	-6.72	0.54
$ au_{reion}$	0.00	2.65	0.31	-14.37	15.24	-2.62	-94.58	-24.54
$\langle II \rangle$	2.17	2.76	29.50	-92.08	-22.51	5.45	10.06	1.26
$\langle AI \rangle$	1.76	-3.27	10.90	-3.25	5.08	-99.08	3.67	1.29

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
12.91	25.02	5.17	2.76	0.73	9.54	1.47	2.78

Table 2-P-T

	1	2	3	4	5	6	7	8
% error	0.02	0.16	0.21	0.34	0.41	0.81	1.25	3.66
$\delta h/h$	25.66	74.17	-30.82	-20.87	14.52	-13.44	2.38	45.36
$\delta\Omega_b/\Omega_b$	-7.20	39.74	-16.02	-9.95	6.21	-25.02	-8.31	-85.32
$\delta\Omega_k$	-88.10	25.90	1.36	-13.06	2.37	31.41	17.40	10.03
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-38.92	-19.98	-19.72	10.69	7.26	-77.12	-32.64	22.75
$\delta n_s/n_s$	0.41	25.57	91.10	-14.87	3.45	-24.71	-13.22	5.29
$ au_{reion}$	0.03	-3.61	-7.02	-49.24	-85.86	-11.58	0.29	2.50
$\langle II \rangle$	3.01	-32.08	-5.96	-80.21	46.69	7.69	-15.81	-2.04
$\langle AI \rangle$	2.15	-12.39	4.27	-10.06	11.33	-38.16	90.18	-2.36

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
1.67	3.13	0.50	1.12	0.38	0.41	0.40	1.17

Table 2-P-TP

	1	2	3	4	5	6	7	8
% error	0.15	0.40	0.94	1.80	8.29	16.18	49.96	56.13
$\delta h/h$	-17.82	-68.13	-40.87	-31.79	1.82	-11.28	21.87	41.84
$\delta\Omega_b/\Omega_b$	12.13	-35.23	-25.70	-14.28	2.56	48.34	-58.33	-44.74
$\delta\Omega_k$	89.13	-7.95	-0.15	-24.34	-25.29	-15.77	21.21	-7.86
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.74	-3.87	-8.11	47.89	50.40	18.36	-20.36	52.48
$\delta n_s/n_s$	1.46	61.11	-40.22	-58.10	14.39	13.87	-12.88	26.54
$ au_{reion}$	-0.58	-3.93	16.82	-7.20	-28.62	-54.35	-71.30	28.16
$\langle II \rangle$	-0.89	-13.88	73.20	-40.45	2.66	43.80	-0.57	29.77
$\langle AI \rangle$	2.86	-9.82	18.57	-28.91	76.00	-43.41	-2.39	-32.38

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
25.98	39.26	11.95	31.59	16.46	40.02	18.18	20.52

Table 3-M-T

	1	2	3	4	5	6	7	8
% error	0.14	0.39	0.91	1.69	1.91	7.07	10.07	24.56
$\delta h/h$	-17.62	-68.22	-39.88	-31.83	8.53	1.24	-1.92	-48.52
$\delta\Omega_b/\Omega_b$	12.17	-35.19	-24.59	-15.84	-0.89	-24.01	-36.80	76.33
$\delta\Omega_k$	89.25	-7.89	-0.22	-20.90	13.77	33.47	14.28	-4.71
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.56	-3.56	-7.68	40.21	-29.47	-62.54	-27.05	-35.13
$\delta n_s/n_s$	1.40	60.94	-37.49	-57.91	14.83	-24.36	-22.39	-14.51
$ au_{reion}$	-0.71	-4.81	30.81	-47.84	-81.96	4.18	1.02	-1.26
$\langle II \rangle$	-0.82	-14.05	71.08	-23.02	39.94	-13.38	-47.21	-14.73
$\langle AI \rangle$	2.88	-9.97	18.42	-22.58	18.30	-60.00	70.54	11.53

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
11.94	19.19	3.04	10.11	4.69	1.84	6.14	8.76

Table 3-M-TP

	1	2	3	4	5	6	7	8
% error	0.03	0.23	0.30	1.07	2.20	4.70	15.88	36.36
$\delta h/h$	-26.45	-24.52	79.75	7.96	14.23	7.83	21.64	-39.29
$\delta\Omega_b/\Omega_b$	7.36	-11.94	45.21	8.80	6.53	5.67	-44.41	75.07
$\delta\Omega_k$	87.22	4.35	28.21	-14.14	-32.60	-4.24	8.22	-15.17
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	40.46	-20.25	-16.28	34.56	78.86	14.46	4.41	-6.77
$\delta n_s/n_s$	-1.24	92.82	22.31	-2.39	29.49	2.93	0.91	-0.52
$ au_{reion}$	0.00	-2.73	-0.17	-19.68	3.10	23.78	81.83	48.31
$\langle II \rangle$	0.09	-11.93	-1.54	-86.07	29.39	29.44	-24.70	-10.28
$\langle AI \rangle$	0.55	-7.88	5.97	-25.75	27.02	-90.77	12.87	10.23

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
14.70	28.20	5.72	3.19	0.74	21.88	5.71	6.05

Table 3-P-T

	1	2	3	4	5	6	7	8
% error	0.02	0.17	0.21	0.38	0.65	0.98	1.58	3.80
$\delta h/h$	-25.68	-79.31	26.65	4.34	-11.77	-8.63	-11.10	-44.56
$\delta\Omega_b/\Omega_b$	7.20	-42.56	13.74	2.43	-15.08	-20.48	18.62	83.36
$\delta\Omega_k$	88.15	-28.65	-2.55	-5.14	18.08	27.66	-14.63	-8.34
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	38.95	20.27	19.65	14.04	-46.59	-64.98	20.85	-25.72
$\delta n_s/n_s$	-0.41	-24.84	-90.82	-8.41	-1.55	-29.35	11.91	-7.58
$ au_{reion}$	-0.03	2.12	7.78	-95.41	-28.52	3.88	1.00	-1.85
$\langle II \rangle$	0.02	1.82	16.35	-22.82	71.93	-59.25	-22.77	2.44
$\langle AI \rangle$	0.71	-6.55	11.46	-7.60	33.87	11.48	90.67	-16.38

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
1.71	3.19	0.49	1.25	0.49	0.41	0.84	1.58

Table 3-P-TP

	1	2	3	4	5	6	7	8
% error	0.15	0.40	1.00	1.76	6.28	7.68	39.52	58.91
$\delta h/h$	-17.90	69.48	-27.55	-40.79	2.87	5.80	-25.11	41.91
$\delta\Omega_b/\Omega_b$	12.06	36.29	-19.39	-19.54	-4.06	33.22	20.01	-79.11
$\delta\Omega_k$	89.00	8.64	-1.92	-22.19	-10.59	-31.96	13.35	13.98
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.67	4.86	-8.73	46.61	17.99	58.83	-47.48	10.68
$\delta n_s/n_s$	1.50	-60.14	-56.64	-48.59	-4.20	17.37	-22.16	0.43
$ au_{reion}$	-0.56	3.27	18.46	-6.96	-2.61	-45.67	-76.92	-39.89
$\langle II \rangle$	3.81	-9.74	69.87	-46.76	-26.05	44.60	-9.31	8.25
$\langle AI \rangle$	4.61	-5.81	18.81	-26.34	94.00	-1.74	6.51	-3.97

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
26.63	47.35	10.12	20.35	8.93	38.58	7.26	6.87

Table 4-M-T

	1	2	3	4	5	6	7	8
% error	0.14	0.40	0.97	1.67	1.84	5.54	6.40	29.31
$\delta h/h$	-17.69	69.64	-26.23	-40.98	7.05	0.75	5.14	48.92
$\delta\Omega_b/\Omega_b$	12.10	36.29	-18.13	-20.06	-0.27	1.04	41.24	-78.12
$\delta\Omega_k$	89.12	8.58	-1.66	-21.47	5.62	9.02	-37.29	4.10
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.49	4.57	-8.94	44.18	-15.85	-12.40	68.87	35.22
$\delta n_s/n_s$	1.43	-59.99	-52.99	-52.93	-5.16	6.30	24.43	11.25
$ au_{reion}$	-0.69	3.80	33.16	-27.55	-90.08	2.88	0.95	1.89
$\langle II \rangle$	3.93	-9.66	68.07	-36.92	37.48	28.92	39.23	10.77
$\langle AI \rangle$	4.61	-5.68	18.97	-24.12	11.08	-94.23	1.56	-0.60

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
14.37	23.06	2.75	11.28	3.81	1.84	4.49	5.25

Table 4-M-TP

	1	2	3	4	5	6	7	8
% error	0.03	0.23	0.30	0.92	1.93	2.96	6.55	30.58
$\delta h/h$	-26.44	-24.37	-78.56	-12.35	18.58	3.04	-11.67	-43.49
$\delta\Omega_b/\Omega_b$	7.35	-11.87	-44.68	-4.50	12.84	6.49	20.29	84.67
$\delta\Omega_k$	87.18	4.57	-28.10	-10.81	-33.33	-8.57	1.34	-16.94
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	40.44	-20.69	15.33	22.72	79.94	22.68	-15.04	-7.37
$\delta n_s/n_s$	-1.24	93.85	-21.22	1.12	25.78	5.67	-6.49	-0.67
$ au_{reion}$	0.00	-2.64	0.35	-11.33	-12.08	-4.15	-95.47	24.24
$\langle II \rangle$	2.42	-0.03	18.27	-92.87	15.18	27.29	7.55	-1.71
$\langle AI \rangle$	2.11	-1.27	4.72	-20.96	30.78	-92.56	3.34	2.64

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
13.33	25.93	5.23	2.99	0.74	9.70	1.41	2.93

Table 4-P-T

	1	2	3	4	5	6	7	8
% error	0.02	0.17	0.21	0.30	0.39	0.89	1.11	3.72
$\delta h/h$	-25.67	-78.84	29.26	3.95	1.64	12.67	0.37	45.72
$\delta\Omega_b/\Omega_b$	7.20	-42.40	14.77	5.85	-0.45	19.31	19.51	-84.52
$\delta\Omega_k$	88.10	-28.78	-1.86	-5.32	-2.71	-29.14	-20.63	9.84
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	38.92	20.38	17.97	21.07	5.98	72.01	39.29	23.18
$\delta n_s/n_s$	-0.41	-26.72	-89.92	-16.88	-2.55	24.16	17.04	5.95
$ au_{reion}$	-0.03	2.35	7.16	-12.93	-98.54	7.42	2.69	2.02
$\langle II \rangle$	2.69	4.88	19.82	-92.45	15.21	25.38	-12.19	-2.25
$\langle AI \rangle$	2.33	-0.34	8.37	-21.89	2.49	-46.35	84.88	9.31

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle II \rangle$	$\langle AI \rangle$
1.71	3.16	0.51	1.16	0.41	0.40	0.40	1.09

Table 4-P-TP

	1	2	3	4	5	6	7	8	9	10	11
% error	0.15	0.39	0.67	1.64	4.75	6.53	16.27	29.15	41.78	128.96	142.34
$\delta h/h$	-17.86	-66.89	-27.94	-43.27	6.66	-4.95	-3.99	3.57	28.55	-39.20	10.24
$\delta\Omega_b/\Omega_b$	12.07	-34.63	-18.23	-22.19	14.46	-19.72	21.74	-19.11	-48.63	61.11	17.17
$\delta\Omega_k$	88.97	-8.20	-1.50	-20.02	3.67	28.27	-14.75	3.95	-3.35	-12.19	-20.22
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.65	-3.88	-10.47	40.09	-6.91	-47.61	29.77	-4.27	35.72	-8.76	46.09
$\delta n_s/n_s$	1.47	60.62	-21.85	-65.50	0.49	-2.25	24.63	-9.14	19.57	0.74	21.82
$ au_{reion}$	-0.56	-4.02	12.03	-0.24	-2.70	28.40	-58.84	-31.87	32.02	33.89	48.71
$\langle I_1I_1\rangle$	-0.82	-14.32	53.52	-10.44	-30.99	36.78	40.85	-2.22	-23.99	-22.64	41.78
$\langle I_2I_2 \rangle$	3.82	6.29	50.34	-18.60	59.47	-35.46	-22.34	33.72	-10.28	-9.97	20.77
$\langle I_1 A \rangle$	2.87	-9.87	14.34	-19.13	-50.19	-13.78	-4.43	64.83	24.31	41.82	-7.18
$\langle I_2 A \rangle$	4.61	4.65	15.71	-19.24	-50.07	-52.92	-36.20	-36.66	-27.26	-23.61	-8.81
$\langle I_1 I_2 \rangle$	0.95	-13.79	47.99	-12.08	12.45	-10.40	28.99	-42.43	45.92	20.87	-43.73

		$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2 I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
53.97	85.25	32.98	68.48	32.52	84.13	67.40	34.36	58.99	37.16	71.71

Table 5-M-T

	1	2	3	4	5	6	7	8	9	10	11
% error	0.14	0.39	0.65	1.58	1.80	4.42	5.75	8.54	26.58	29.25	52.80
$\delta h/h$	-17.65	-66.93	-27.92	-43.39	-2.31	-7.79	2.86	-2.26	33.79	-23.10	28.16
$\delta\Omega_b/\Omega_b$	12.11	-34.56	-17.96	-21.61	-3.91	-11.49	29.66	-23.91	-65.37	36.63	-25.60
$\delta\Omega_k$	89.08	-8.11	-1.64	-20.12	-0.78	-4.48	-31.48	23.01	2.78	-6.46	-0.95
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.46	-3.55	-10.38	40.88	-0.87	5.29	52.17	-51.21	26.92	-7.93	22.17
$\delta n_s/n_s$	1.40	60.48	-20.41	-64.74	-11.01	-4.33	5.85	-37.91	8.29	-6.00	4.04
$ au_{reion}$	-0.68	-4.98	18.72	4.59	-97.60	-6.98	2.45	4.26	1.17	-1.92	0.56
$\langle I_1 I_1 \rangle$	-0.75	-14.58	52.90	-8.65	5.52	22.84	-39.91	-48.75	-19.52	11.11	43.45
$\langle I_2I_2 angle$	3.94	5.98	50.11	-20.31	13.97	-51.68	40.98	26.80	12.31	28.34	28.40
$\langle I_1 A \rangle$	2.89	-10.03	14.40	-18.62	-1.75	52.19	9.74	2.08	47.24	56.11	-33.63
$\langle I_2 A \rangle$	4.61	4.45	15.72	-19.72	0.57	59.92	43.82	33.48	-30.68	-36.58	20.32
$\langle I_1 I_2 \rangle$	1.04	-14.02	47.50	-11.82	10.13	-13.59	8.52	-24.42	8.80	-50.67	-61.68

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2 I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
18.66	24.64	3.42			1.96	24.24	17.91	27.35	17.82	35.93

Table 5-M-TP

	1	2	3	4	5	6	7	8	9	10	11
% error	0.03	0.23	0.30	0.67	1.70	2.13	2.95	9.35	12.27	20.47	57.63
$\delta h/h$	-26.44	-23.83	-78.51	-5.81	-17.40	-6.33	8.72	6.24	-10.14	-16.19	-41.39
$\delta\Omega_b/\Omega_b$	7.35	-11.52	-44.65	-0.96	-13.62	-0.09	5.85	-25.70	3.35	36.95	74.58
$\delta\Omega_k$	87.17	4.50	-27.97	-9.38	18.95	24.38	-16.10	3.90	-2.05	-6.40	-15.41
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	40.44	-19.95	15.13	21.57	-45.51	-57.17	43.18	1.22	-1.29	-3.77	-7.15
$\delta n_s/n_s$	-1.24	92.03	-20.72	-13.06	-9.67	-24.70	14.46	0.58	3.29	-2.39	0.06
$ au_{reion}$	0.00	-2.81	0.38	-10.41	15.53	-5.60	11.71	12.90	-58.43	-65.00	40.84
$\langle I_1I_1 \rangle$	0.09	-12.31	2.58	-48.40	49.39	-37.29	21.20	12.15	-27.29	46.86	-11.48
$\langle I_2I_2 angle$	2.42	-1.08	18.39	-62.18	-59.02	40.59	12.86	4.97	-18.70	9.92	-4.31
$\langle I_1 A \rangle$	0.55	-7.94	-5.41	-18.77	-11.25	-21.81	-31.33	77.80	36.47	-5.66	23.59
$\langle I_2 A \rangle$	2.11	-1.54	4.72	-15.34	-21.53	-41.48	-75.19	-34.59	-25.70	-0.40	-6.47
$\langle I_1 I_2 \rangle$	0.41	-13.65	2.39	-48.33	15.33	-14.86	13.06	-41.18	58.05	-41.96	4.41

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1 \rangle$	$\langle I_2 I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
24.13	43.71	9.02	4.62		28.01	12.25	4.22	16.13	6.34	12.09

Table 5-P-T

	1	2	3	4	5	6	7	8	9	10	11
% error	0.02	0.17	0.19	0.27	0.39	0.80	1.00	1.18	3.47	4.84	5.62
$\delta h/h$	-25.67	78.65	25.86	12.84	1.72	-11.00	-8.96	8.86	-37.95	18.08	16.50
$\delta\Omega_b/\Omega_b$	7.20	42.30	12.51	9.46	-0.54	-11.73	-21.12	-15.57	64.05	-42.47	-34.35
$\delta\Omega_k$	88.10	28.67	-0.93	-5.08	-2.61	16.07	29.60	12.41	-8.54	2.17	4.16
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	38.92	-20.34	12.47	24.03	5.58	-45.42	-65.17	-20.01	-16.47	16.05	8.26
$\delta n_s/n_s$	-0.41	26.68	-77.89	-45.41	-3.09	-16.56	-19.65	-20.21	-2.86	7.24	4.54
$ au_{reion}$	-0.03	-2.42	9.07	-9.57	-97.64	-0.71	-12.18	11.19	0.40	-0.73	3.53
$\langle I_1 I_1 \rangle$	0.03	-2.82	22.43	-22.67	-8.12	-26.97	34.18	-56.12	-34.85	-1.06	-51.47
$\langle I_2 I_2 \rangle$	2.69	-5.16	33.05	-70.27	18.15	-8.49	-25.85	48.80	-1.87	1.38	-21.92
$\langle I_1 A \rangle$	0.71	6.02	13.97	-8.44	-0.81	40.67	-13.29	-27.95	33.42	75.92	-14.26
$\langle I_2 A \rangle$	2.33	0.40	10.95	-14.73	3.37	65.04	-37.74	-33.91	-31.30	-41.83	10.58
$\langle I_1 I_2 \rangle$	0.43	-1.21	30.98	-35.35	2.97	-21.50	19.45	-33.61	28.09	-6.06	70.60

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
1.85	3.60	0.53	1.33	0.59	0.47	3.23	1.40	3.96	2.49	4.12

Table 5-P-TP

	1	2	3	4	5	6	7	8	9	10	11
% error	0.14	0.39	0.82	1.65	2.93	7.31	8.18	16.75	58.28	67.13	389.21
$\delta h/h$	-17.57	-68.38	32.07	-15.03	-33.67	-1.01	17.24	0.21	-21.97	-30.74	-30.02
$\delta\Omega_b/\Omega_b$	12.17	-35.10	19.42	-3.63	-22.58	0.73	-13.10	-35.81	1.86	46.21	64.23
$\delta\Omega_k$	88.74	-6.64	-2.79	-17.80	-14.83	24.12	17.04	11.18	15.83	0.23	-17.01
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.56	-3.24	2.84	45.56	15.82	-45.38	-15.02	-21.84	-50.34	-27.78	4.74
$\delta n_s/n_s$	1.39	59.82	41.96	-44.33	-35.05	-8.63	1.58	-19.39	-30.22	-9.69	2.36
$ au_{reion}$	-0.58	-3.69	-14.25	-10.57	5.39	24.15	8.65	59.71	-65.86	16.96	28.18
$\langle I_1I_1 angle$	-7.23	12.30	-44.72	28.01	-61.74	-21.73	50.90	-0.71	2.56	0.29	11.45
$\langle I_2I_2 angle$	-0.92	-12.86	-61.79	-53.61	11.78	3.13	-9.39	-36.74	-11.91	-34.26	15.50
$\langle I_1 A \rangle$	-6.17	5.62	-3.54	19.46	17.24	49.00	33.50	-53.08	-32.68	29.44	-31.08
$\langle I_2 A \rangle$	2.86	-9.55	-14.65	-31.77	9.30	-57.68	3.15	2.56	-10.51	59.65	-39.73
$\langle I_1 I_2 \rangle$	1.46	-2.57	23.88	-15.65	47.90	-22.14	71.60	-0.23	12.29	-12.96	30.74

	$\delta\Omega_b/\Omega_b$									
119.3	5 252.00	66.92	39.69	21.20	117.20	44.85	65.23	124.46	159.92	120.35

Table 6-M-T

	1	2	3	4	5	6	7	8	9	10	11
% error	0.14	0.39	0.80	1.52	1.88	2.61	5.75	6.61	9.66	26.39	37.23
$\delta h/h$	-17.37	-68.47	31.55	-11.53	-10.42	35.39	13.69	3.46	7.17	-47.45	4.31
$\delta\Omega_b/\Omega_b$	12.21	-35.06	18.93	-3.57	-0.39	20.74	-5.67	-18.60	37.35	77.76	-0.44
$\delta\Omega_k$	88.85	-6.56	-2.78	-13.31	-13.52	15.67	8.73	32.63	-16.91	-4.52	1.51
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.38	-2.92	2.71	37.18	29.88	-15.70	-2.89	-56.37	39.07	-34.56	2.00
$\delta n_s/n_s$	1.34	59.62	40.36	-41.21	-17.45	36.17	4.59	-16.85	31.22	-13.44	6.56
$ au_{reion}$	-0.71	-4.48	-23.73	-49.18	80.51	20.43	-8.59	3.03	-2.70	-2.03	1.91
$\langle I_1I_1 angle$	-7.24	12.38	-44.42	32.68	-1.32	60.00	53.69	-12.79	-3.78	5.65	-7.63
$\langle I_2I_2 angle$	-0.85	-13.00	-61.09	-39.67	-37.32	-16.78	-4.84	-4.04	47.39	-12.59	-20.15
$\langle I_1 A \rangle$	-6.20	5.70	-3.59	17.04	13.20	-11.67	22.48	52.46	49.45	-0.85	60.10
$\langle I_2 A \rangle$	2.88	-9.70	-14.49	-27.06	-17.01	-13.01	17.12	-46.55	-32.44	6.94	70.26
$\langle I_1 I_2 \rangle$	1.41	-2.79	22.62	-22.64	13.04	-43.72	76.79	-2.76	1.09	7.95	-30.27

	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1 angle$	$\langle I_2I_2 angle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
12.71	20.88	3.10	10.62	5.52	2.08	4.85	9.47	23.18	26.61	12.35

Table 6-M-TP

	1	2	3	4	5	6	7	8	9	10	11
% error	0.03	0.23	0.28	0.92	1.53	2.11	2.74	5.71	16.09	34.79	46.10
$\delta h/h$	-26.44	23.26	76.15	9.42	-26.21	2.62	12.98	-6.75	13.84	30.40	29.16
$\delta\Omega_b/\Omega_b$	7.36	11.23	42.95	2.68	-18.94	0.11	3.36	9.58	-27.28	-61.58	-54.49
$\delta\Omega_k$	87.19	-4.73	25.67	16.18	7.93	-28.72	-15.28	-5.85	3.36	14.15	9.37
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	40.44	20.44	-14.99	-26.46	-35.55	60.98	43.65	4.94	6.13	0.99	8.41
$\delta n_s/n_s$	-1.24	-92.92	18.67	10.32	-12.86	23.14	14.05	2.96	0.59	1.26	0.10
$ au_{reion}$	0.00	2.75	-0.32	15.48	10.38	-2.01	16.25	1.36	84.49	3.19	-47.19
$\langle I_1 I_1 \rangle$	-2.17	3.19	-29.51	52.85	-64.24	-9.04	-12.90	-43.20	2.36	-8.41	1.94
$\langle I_2I_2 angle$	0.09	12.04	-2.16	68.91	42.33	19.74	37.63	13.54	-9.90	-23.58	25.81
$\langle I_1 A \rangle$	-1.76	-3.22	-10.87	-2.55	-8.63	-51.21	70.25	-1.76	-27.35	30.48	-23.44
$\langle I_2 A \rangle$	0.55	7.80	5.74	18.89	22.50	42.60	-15.65	-31.77	-31.24	50.91	-49.05
$\langle I_1 I_2 \rangle$	0.19	-4.99	9.88	-26.29	29.23	-1.06	21.37	-82.04	7.04	-30.37	13.04

$\delta h/$	$h \mid \delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2 I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
17.2	6 33.31	6.62	4.43	0.85	25.68	4.12	14.63	15.92	29.24	13.10

Table 6-P-T

	1	2	3	4	5	6	7	8	9	10	11
% error	0.02	0.16	0.20	0.31	0.40	0.61	0.85	1.16	1.37	3.78	6.90
$\delta h/h$	-25.66	-72.88	-31.23	22.24	-14.57	-16.36	-5.07	-1.03	-3.29	-44.77	-6.14
$\delta\Omega_b/\Omega_b$	7.20	-39.06	-16.14	10.52	-6.66	-15.34	-14.59	-19.08	8.33	83.64	11.38
$\delta\Omega_k$	88.09	-25.37	0.60	14.29	-0.62	11.94	24.35	19.70	-13.79	-8.63	-1.69
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	38.92	19.83	-18.19	-12.79	-10.34	-38.69	-53.44	-43.06	25.33	-25.13	-3.29
$\delta n_s/n_s$	-0.41	-25.52	88.53	22.07	-0.21	-0.15	-18.42	-22.77	10.44	-6.52	-3.97
$ au_{reion}$	-0.03	3.77	-8.29	36.56	88.78	-22.80	-10.74	7.55	-0.43	-1.40	-1.86
$\langle I_1 I_1 \rangle$	-3.01	33.14	-7.53	75.14	-32.47	-14.22	28.96	-4.23	30.74	-0.89	11.46
$\langle I_2 I_2 \rangle$	0.02	3.99	-17.00	21.78	9.13	60.86	-9.15	-59.84	-33.04	-7.61	24.88
$\langle I_1 A \rangle$	-2.15	12.29	4.80	6.80	-8.96	-39.87	23.30	-30.01	-64.04	8.35	-49.91
$\langle I_2 A \rangle$	0.71	-6.51	-11.66	-2.48	11.77	35.00	16.55	-20.95	48.01	5.65	-73.50
$\langle I_1 I_2 \rangle$	0.43	-12.35	4.75	-32.10	19.93	-24.84	64.18	-43.35	23.17	-9.44	34.01

		$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2 I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
1.76	3.27	0.51	1.26	0.54	0.44	0.97	1.97	3.60	5.13	2.51

Table 6-P-TP

	1	2	3	4	5	6	7	8	9	10	11
% error	0.14	0.39	0.87	1.65	2.42	6.18	6.82	19.15	51.30	140.60	451.42
$\delta h/h$	17.65	69.39	20.75	29.55	-23.01	-12.27	15.26	-5.27	2.37	47.70	18.64
$\delta\Omega_b/\Omega_b$	-12.10	35.97	14.08	12.08	-18.04	-7.20	-15.46	-39.29	-35.72	-51.67	-45.94
$\delta\Omega_k$	-88.63	7.25	-1.08	16.78	-6.53	16.22	33.18	1.00	13.98	-3.32	13.97
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-39.50	4.16	3.38	-45.98	0.71	-33.13	-48.09	-2.36	-23.27	47.41	-9.52
$\delta n_s/n_s$	-1.43	-58.68	54.27	36.48	-35.61	-9.33	2.45	-10.50	-16.97	21.71	-6.96
$ au_{reion}$	0.57	3.05	-15.26	10.03	11.54	18.43	28.36	51.77	-69.42	15.96	-25.01
$\langle I_1 I_1 \rangle$	7.16	-13.76	-46.94	-23.99	-44.80	-38.78	51.14	-23.55	-0.73	8.10	-15.29
$\langle I_2I_2 angle$	-3.74	-10.37	-57.42	56.15	7.83	14.19	-33.69	-25.66	6.80	29.85	-20.85
$\langle I_1 A \rangle$	6.16	-5.62	-2.72	-22.36	5.93	47.19	9.31	-57.98	-41.52	11.82	42.99
$\langle I_2 A \rangle$	-4.58	-5.91	-12.90	30.41	28.32	-61.39	-2.27	-2.00	-31.33	-23.47	52.35
$\langle I_1 I_2 \rangle$	0.33	-2.44	22.52	-0.28	69.25	-17.59	38.23	-32.44	11.01	20.55	-36.40

			$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$							
107.64	220.62	63.70	80.32	44.74	120.93	70.24	103.29	196.28	239.18	167.06

Table 7-M-T

	1	2	3	4	5	6	7	8	9	10	11
% error	0.14	0.39	0.85	1.53	1.77	2.29	4.98	5.81	10.55	26.75	53.08
$\delta h/h$	17.45	69.55	20.11	-24.73	-17.72	23.02	19.62	3.76	14.00	34.83	34.49
$\delta\Omega_b/\Omega_b$	-12.14	35.97	13.57	-10.19	-6.64	16.63	-9.54	22.44	38.00	-60.46	-47.49
$\delta\Omega_k$	-88.74	7.18	-1.21	-13.91	-8.85	7.90	24.06	-31.86	-11.14	1.32	4.28
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	-39.31	3.87	3.61	41.26	19.73	-1.57	-31.95	55.98	29.60	28.87	21.65
$\delta n_s/n_s$	-1.37	-58.49	52.49	-34.86	-13.30	38.96	4.24	13.48	23.97	8.11	8.41
$ au_{reion}$	0.70	3.48	-25.12	-38.54	81.65	34.13	-5.09	-0.19	-2.64	0.47	2.32
$\langle I_1 I_1 \rangle$	7.18	-13.86	-46.38	31.74	-11.73	42.58	61.68	16.87	20.24	5.40	-10.94
$\langle I_2I_2 angle$	-3.85	-10.30	-56.78	-47.26	-32.63	-17.37	-30.27	-0.69	40.27	22.22	-4.73
$\langle I_1 A \rangle$	6.19	-5.68	-2.56	21.15	10.57	-2.65	-2.54	-45.64	52.46	-37.83	55.53
$\langle I_2 A \rangle$	-4.58	-5.76	-12.79	-30.07	-5.13	-29.93	29.06	52.74	-20.87	-42.20	45.97
$\langle I_1 I_2 \rangle$	0.41	-2.29	21.56	-9.76	31.46	-58.55	48.05	-1.51	39.48	22.22	-25.55

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
	30.25			5.75	2.18			31.77	27.20	15.64

Table 7-M-TP

	1	2	3	4	5	6	7	8	9	10	11
% error	0.03	0.23	0.28	0.75	1.30	1.84	2.36	3.68	12.90	29.46	63.43
$\delta h/h$	-26.43	23.69	75.04	17.42	-10.53	11.20	21.40	-12.42	-13.94	-25.50	33.35
$\delta\Omega_b/\Omega_b$	7.35	11.48	42.49	8.58	-7.86	10.36	5.56	-5.26	44.16	48.09	-58.69
$\delta\Omega_k$	87.14	-4.79	25.79	11.05	-4.35	-34.43	0.63	10.25	-3.30	-10.09	13.17
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	40.42	20.79	-14.44	-16.86	-6.15	78.74	11.84	-30.52	-10.00	-3.88	4.08
$\delta n_s/n_s$	-1.24	-93.92	18.88	4.14	-5.42	24.16	10.57	-8.27	-3.09	-0.58	-0.33
$ au_{reion}$	0.00	2.66	-0.48	7.69	1.43	-18.99	24.18	-24.30	-79.69	33.79	-30.05
$\langle I_1 I_1 \rangle$	-2.17	2.88	-29.53	33.12	-63.69	-2.27	55.65	20.57	10.50	-15.39	-9.40
$\langle I_2I_2 angle$	2.41	0.33	-18.07	69.29	46.08	-2.54	7.74	-43.49	18.12	-20.06	-7.39
$\langle I_1 A \rangle$	-1.76	-3.26	-10.34	-15.06	-28.16	-27.86	6.82	-62.02	25.05	38.92	45.26
$\langle I_2 A \rangle$	2.11	1.32	-4.24	8.45	42.76	12.92	46.36	41.02	5.69	49.07	40.45
$\langle I_1 I_2 \rangle$	0.35	-1.50	3.03	-54.26	30.63	-21.98	57.63	-16.69	16.90	-34.87	-22.87

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2 I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
22.53	40.24	8.91	3.62		23.86	7.81	8.10	31.17	29.52	17.98

Table 7-P-T

	1	2	3	4	5	6	7	8	9	10	11
% error	0.02	0.16	0.21	0.26	0.36	0.39	0.66	0.97	1.29	3.72	5.80
$\delta h/h$	-25.65	-72.99	-33.74	9.53	-19.66	10.57	7.95	-10.97	3.27	45.33	6.18
$\delta\Omega_b/\Omega_b$	7.20	-39.26	-16.97	2.29	-12.49	5.24	2.79	-26.51	-4.56	-83.19	-16.66
$\delta\Omega_k$	88.04	-25.66	-0.37	10.30	-6.09	-1.77	10.78	34.58	6.02	9.80	0.50
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	38.90	19.68	-15.68	-18.33	-8.16	14.83	-18.78	-79.15	-7.84	22.78	5.83
$\delta n_s/n_s$	-0.41	-26.71	86.00	30.47	-3.89	-2.24	3.19	-29.34	-5.87	5.70	1.95
$ au_{reion}$	-0.03	3.89	-7.83	9.34	-41.30	-87.71	-18.89	-7.21	4.39	2.50	-1.87
$\langle I_1I_1\rangle$	-3.01	33.41	-10.08	43.92	-59.41	23.99	42.46	2.04	-26.60	-3.03	14.64
$\langle I_2I_2 angle$	2.69	8.69	-24.40	64.28	52.02	-16.11	19.01	-22.47	33.06	-3.16	15.97
$\langle I_1 A \rangle$	-2.15	11.93	6.31	-4.97	-22.35	10.34	24.08	-8.09	64.68	9.68	-65.27
$\langle I_2 A \rangle$	2.33	-1.01	-8.43	5.43	28.96	-19.41	29.01	-6.16	-61.39	15.60	-61.53
$\langle I_1 I_2 \rangle$	0.36	-4.68	7.91	-48.28	7.65	-24.92	74.21	-13.83	8.09	-3.69	33.57

$\delta h/h$		$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1 angle$	$\langle I_2I_2 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A \rangle$	$\langle I_1 I_2 \rangle$
1.74	3.26	0.51	1.21	0.43	0.43	1.00	1.09	3.90	3.71	2.03

Table 7-P-TP

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
% error	0.14	0.39	0.62	1.41	2.16	5.38	5.74	14.96	18.25	32.36	105.10	169.59	508.44	593.36	2048.04
$\delta h/h$	-17.61	-67.20	-23.63	-23.60	30.85	15.00	1.84	-1.28	3.29	-12.83	12.85	-35.16	10.58	34.46	-1.11
$\delta\Omega_b/\Omega_b$	12.11	-34.55	-15.02	-9.50	20.75	-4.41	9.56	-32.05	-18.82	-19.45	-32.43	23.68	-38.91	-53.32	8.53
$\delta\Omega_k$	88.58	-6.97	0.84	-11.08	11.33	12.67	-27.24	6.97	10.99	-8.26	20.98	12.36	2.32	10.64	-1.20
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.47	-3.32	-7.04	36.42	-15.50	-8.83	44.35	-16.07	-18.05	11.69	-31.97	-53.23	10.63	8.95	-0.74
$\delta n_s/n_s$	1.40	59.40	-25.46	-49.54	37.47	17.87	1.85	-25.58	-2.07	0.17	-18.29	-24.45	5.24	3.19	-3.42
$ au_{reion}$	-0.56	-3.77	11.15	-3.68	-5.73	1.64	-23.21	36.97	55.84	-23.00	-59.05	-24.86	-6.14	-10.82	0.40
$\langle I_1 I_1 angle$	-7.17	11.77	31.90	41.75	29.45	65.89	6.43	3.94	-9.95	-32.14	13.71	-13.28	-12.05	-11.57	-3.00
$\langle I_2 I_2 angle$	-0.85	-13.22	49.31	-23.21	-19.20	25.10	-22.71	-31.26	-3.80	46.98	-10.59	-16.40	-29.61	13.03	26.70
$\langle I_3I_3 angle$	3.73	7.19	46.02	-23.16	18.36	-48.96	6.03	24.52	-33.58	-27.91	10.04	-24.27	-31.32	10.53	-14.85
$\langle I_1 A \rangle$	-6.16	5.55	1.07	17.50	-13.39	-9.46	-42.59	-36.12	-27.08	-48.80	-27.10	12.74	18.82	39.73	17.10
$\langle I_2 A angle$	2.87	-9.56	12.72	-21.90	-10.81	30.77	26.14	18.91	-23.42	5.75	-39.95	40.38	7.51	28.71	-49.66
$\langle I_3 A \rangle$	4.58	4.85	13.09	-23.92	-12.48	6.42	55.93	5.93	18.54	-29.46	4.21	19.96	4.76	16.61	62.65
$\langle I_1 I_2 angle$	1.43	-2.32	-17.62	-22.22	-30.93	22.17	-19.39	43.99	-53.86	-3.25	-0.65	-19.47	20.07	-31.36	27.44
$\langle I_1 I_3 angle$	-0.33	3.08	-13.43	-19.88	-61.88	14.43	6.32	-25.23	15.18	-36.30	26.95	-19.12	-27.47	-6.35	-35.57
$\langle I_2 I_3 angle$	0.92	-12.77	44.73	-19.26	0.92	-7.05	4.24	-27.94	8.32	-9.67	4.96	-5.49	67.80	-39.15	-16.38

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1 I_1 \rangle$	$\langle I_2 I_2 angle$	$\langle I_3I_3 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A angle$	$\langle I_3 A \rangle$	$\langle I_1 I_2 angle$	$\langle I_1I_3 \rangle$	$\langle I_2 I_3 \rangle$
221.30	415.45	75.32	123.63	89.89	104.81	114.34	573.46	351.79	434.70	1035.02	1287.60	601.70	744.00	534.32

Table 8-M-T

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
% error	0.14	0.39	0.61	1.33	1.71	2.09	4.36	5.35	8.13	8.90	14.64	27.23	49.09	55.69	109.92
$\delta h/h$	-17.40	-67.27	-23.62	-20.12	19.93	27.17	-17.76	2.59	5.65	3.23	13.14	-37.13	-30.01	-1.62	-15.97
$\delta\Omega_b/\Omega_b$	12.15	-34.50	-14.90	-8.18	8.85	17.87	5.56	14.54	33.73	-5.86	26.30	62.98	32.37	13.74	25.80
$\delta\Omega_k$	88.69	-6.87	0.71	-9.14	8.18	11.05	-20.11	-25.60	-23.75	11.65	3.76	-5.10	0.75	-2.84	-2.25
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	39.29	-2.99	-7.11	33.89	-16.82	-13.33	21.72	42.01	50.86	-25.67	-2.24	-23.61	-25.55	-0.03	-9.55
$\delta n_s/n_s$	1.34	59.24	-24.30	-46.28	21.39	36.63	-14.83	4.68	28.21	-25.59	10.12	-8.99	-7.90	-4.26	-1.16
$ au_{reion}$	-0.69	-4.61	16.67	-16.36	-81.61	51.50	8.26	1.09	-6.00	-0.28	2.86	-1.60	-1.57	-1.15	-0.13
$\langle I_1 I_1 angle$	-7.19	11.79	31.55	45.05	6.60	27.31	-67.09	20.66	15.17	22.35	11.60	-5.32	5.57	9.11	7.86
$\langle I_2 I_2 angle$	-0.78	-13.44	49.15	-20.51	-0.13	-17.05	-23.25	-23.80	12.38	-52.51	-12.61	15.70	-21.33	40.32	-13.72
$\langle I_3I_3 angle$	3.86	6.91	45.99	-22.08	20.71	6.45	39.92	1.27	21.01	52.49	13.64	-5.15	-32.86	19.31	20.18
$\langle I_1 A \rangle$	-6.19	5.62	0.94	17.31	-10.16	-9.69	-4.72	-44.84	23.63	5.32	44.60	28.70	-32.22	-47.52	-26.53
$\langle I_2 A angle$	2.89	-9.73	12.73	-21.90	-0.85	-10.75	-22.16	30.06	-7.14	-4.13	-34.23	22.68	-33.76	-55.84	41.62
$\langle I_3 A \rangle$	4.58	4.64	13.00	-24.76	0.74	-14.85	-5.44	57.31	-27.71	11.08	26.04	22.54	-0.20	-0.50	-59.69
$\langle I_1 I_2 angle$	1.38	-2.51	-16.88	-26.17	-21.21	-23.67	-24.42	-11.13	46.81	45.95	-44.27	4.43	16.44	5.24	-26.68
$\langle I_1 I_3 \rangle$	-0.41	2.94	-13.12	-23.94	-30.41	-50.58	-25.44	4.42	-1.36	3.05	51.07	-25.40	1.32	15.73	39.79
$\langle I_2 I_3 angle$	1.01	-12.97	44.56	-17.29	10.26	-4.65	8.27	-1.47	21.28	-14.32	10.91	-34.29	57.60	-45.25	-5.47

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1\rangle$	$\langle I_2I_2 \rangle$	$\langle I_3I_3\rangle$	$\langle I_1 A \rangle$	$\langle I_2 A angle$	$\langle I_3 A \rangle$	$\langle I_1 I_2 angle$	$\langle I_1 I_3 \rangle$	$\langle I_2 I_3 \rangle$
25.17	37.86	4.31	18.37	6.52	2.22	11.35	29.81	30.03	43.75	58.33	66.13	31.79	45.79	39.57

Table 8-M-TP

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
% error	0.03	0.23	0.28	0.61	1.16	1.35	1.94	3.31	5.20	8.84	16.61	28.77	36.84	67.74	244.05
$\delta h/h$	-26.43	-22.24	-74.97	-13.40	-15.01	-3.58	18.59	-9.23	2.21	14.48	-17.68	0.77	-11.82	-37.20	16.97
$\delta\Omega_b/\Omega_b$	7.35	-10.63	-42.40	-6.00	-11.85	0.26	10.33	-3.08	17.06	12.02	33.61	-9.54	27.80	66.58	-29.66
$\delta\Omega_k$	87.14	4.98	-25.35	-11.49	-0.80	-13.53	-27.52	15.41	-2.21	10.45	-5.92	-0.09	-6.90	-13.25	7.48
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	40.42	-20.16	13.70	19.90	-12.43	19.32	69.77	-40.53	1.54	-15.84	-4.07	2.17	1.30	-7.86	0.01
$\delta n_s/n_s$	-1.24	92.11	-16.34	-16.31	-1.48	-3.42	28.35	-9.98	0.37	-7.43	0.38	0.09	-2.21	-0.84	-1.51
$ au_{reion}$	0.00	-2.83	0.48	-8.48	7.43	-13.07	-2.54	-15.00	0.10	-6.41	-81.42	-26.54	-15.21	42.82	-4.76
$\langle I_1 I_1 angle$	-2.17	-3.79	29.55	-24.89	-49.27	-40.84	26.40	20.35	-26.04	46.96	-3.56	-6.30	2.54	-5.38	-17.62
$\langle I_2 I_2 angle$	0.09	-12.46	2.99	-39.96	33.80	-44.40	8.29	-20.25	-12.14	-17.86	-1.07	45.28	41.13	5.45	20.17
$\langle I_3I_3 angle$	2.41	-1.66	18.25	-55.40	11.23	47.90	-9.98	-30.90	27.34	45.47	-2.45	12.48	-8.25	-3.97	-7.31
$\langle I_1 A \rangle$	-1.76	3.17	10.32	10.34	-6.20	-37.40	-14.62	-49.00	20.68	18.37	28.41	-37.67	-15.14	7.26	49.48
$\langle I_2 A angle$	0.55	-7.85	-5.27	-15.52	18.82	8.81	16.03	8.91	-49.28	0.09	24.07	13.57	-64.80	33.44	19.78
$\langle I_3 A \rangle$	2.11	-1.63	4.21	-9.95	32.88	19.74	32.18	45.83	3.65	17.92	-3.26	-41.95	31.73	3.05	46.34
$\langle I_1 I_2 angle$	0.19	5.21	-9.97	13.78	20.31	15.29	-16.75	-36.39	-68.83	22.22	1.17	-27.68	31.17	-11.77	-17.97
$\langle I_1 I_3 \rangle$	0.35	1.99	-3.22	35.27	58.15	-30.60	20.36	3.89	21.61	43.50	-1.36	12.07	-18.87	-8.68	-32.49
$\langle I_2 I_3 angle$	0.41	-13.82	2.75	-41.87	21.40	-16.68	6.78	-0.06	6.97	-39.40	21.37	-51.76	-16.50	-25.35	-40.18

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1 angle$	$\langle I_2 I_2 \rangle$	$\langle I_3I_3 \rangle$	$\langle I_1 A \rangle$	$\langle I_2 A angle$	$\langle I_3 A \rangle$	$\langle I_1 I_2 angle$	$\langle I_1 I_3 \rangle$	$\overline{\langle I_2 I_3 \rangle}$
48.78	86.15	20.56	5.93	3.92	35.35	43.45	53.29	19.18	121.59	58.75	114.39	46.91	80.01	100.97

Table 8-P-T

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
% error	0.02	0.16	0.19	0.24	0.34	0.38	0.62	0.96	0.99	1.16	2.18	3.75	5.24	6.89	11.07
$\delta h/h$	-25.64	69.28	37.95	-0.13	-27.51	4.24	8.46	-7.98	-8.76	-3.10	-13.80	39.04	18.49	-6.05	3.69
$\delta\Omega_b/\Omega_b$	7.20	37.42	18.77	1.83	-15.85	1.01	3.23	-25.31	-7.48	7.77	12.65	-72.23	-39.22	11.64	-11.04
$\delta\Omega_k$	88.04	24.63	5.04	-10.43	-6.43	-3.65	9.57	30.92	15.32	-6.51	-7.42	7.27	4.19	-1.75	0.75
$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	38.89	-19.45	6.36	23.43	-12.13	10.96	-14.22	-70.94	-35.52	8.92	7.01	22.39	10.06	-3.15	3.88
$\delta n_s/n_s$	-0.41	29.49	-64.23	-60.96	4.73	-6.31	4.62	-30.85	-2.33	11.44	6.08	5.91	4.47	-3.00	3.99
$ au_{reion}$	-0.03	-4.62	9.00	-4.77	-11.69	-94.40	-23.16	-2.19	-13.51	-7.14	-6.31	-1.15	1.73	-3.02	0.29
$\langle I_1 I_1 angle$	-3.01	-35.51	12.56	-30.30	-59.93	-2.17	53.39	10.41	-14.59	18.28	18.73	-0.23	9.00	10.21	-3.92
$\langle I_2 I_2 angle$	0.02	-6.99	23.30	-13.07	8.49	-10.53	1.97	-26.68	59.48	10.49	18.76	37.18	-39.16	32.76	-18.71
$\langle I_3 I_3 \rangle$	2.69	-12.66	37.61	-47.00	43.03	6.40	14.74	-7.88	-35.64	-32.78	-28.76	-1.92	-7.11	26.60	10.20
$\langle I_1 A \rangle$	-2.15	-11.15	-8.83	3.38	-19.21	-0.78	28.11	-19.83	18.81	-46.81	-27.89	3.31	-40.52	-55.35	13.82
$\langle I_2 A angle$	0.71	5.04	14.01	0.34	18.91	-6.93	8.19	8.28	-5.36	26.27	38.46	6.08	-20.68	-16.64	79.35
$\langle I_3 A \rangle$	2.33	0.49	10.21	-2.07	30.13	-6.44	18.58	9.02	-24.08	56.31	-17.09	12.55	-25.35	-44.30	-41.29
$\langle I_1 I_2 angle$	0.43	14.12	-9.44	18.77	24.47	-11.53	32.93	4.44	-23.62	-42.73	63.58	13.05	-0.52	-3.71	-30.20
$\langle I_1 I_3 angle$	0.36	7.63	-16.49	38.76	22.37	-22.61	60.72	-19.25	14.80	15.17	-31.82	-10.05	21.96	28.96	14.67
$\langle I_2 I_3 angle$	0.43	-6.88	32.87	-20.78	19.90	-2.45	4.59	-23.12	37.72	-0.12	18.28	-28.37	55.85	-41.92	-6.63

$\delta h/h$	$\delta\Omega_b/\Omega_b$	$\delta\Omega_k$	$\delta\Omega_{\Lambda}/\Omega_{\Lambda}$	$\delta n_s/n_s$	$ au_{reion}$	$\langle I_1I_1 \rangle$	$\langle I_2 I_2 \rangle$	$\langle I_3I_3\rangle$	$\langle I_1 A angle$					
1.89	3.72	0.54	1.36	0.71	0.50	1.14	4.02	2.34	4.71	8.97	5.73	3.71	2.97	4.35

Table 8-P-TP