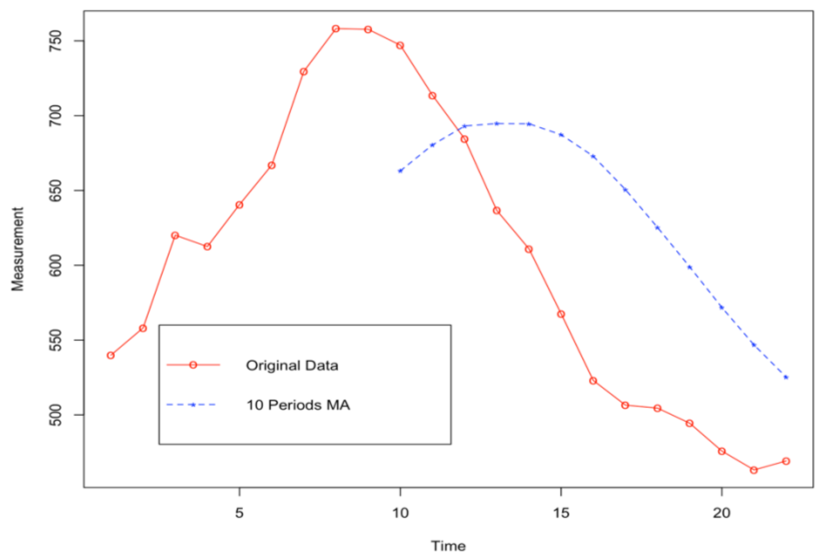
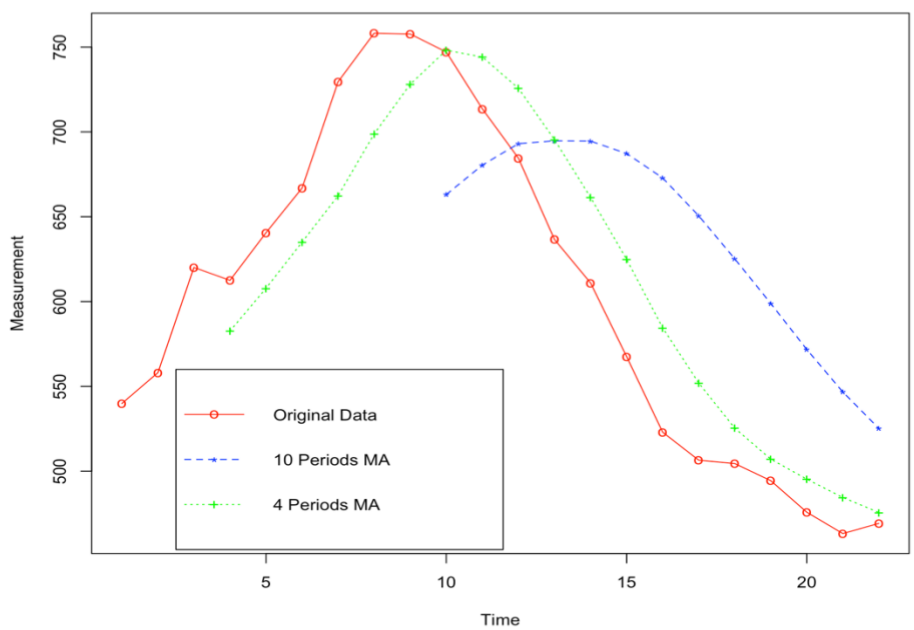
DSC 275/475: Time Series Analysis and Forecasting (Fall 2019)
HW #2 Chunlei Zhou

1. **Consider the measurement data provided in the file: *Measurement_Q1.xls*.**

   a. **Use a 10-period simple moving average to smooth the data. Plot both the smoothed data and the original data values on the same axes.**



   b. **Repeat the procedure with a 4-period simple moving average. Overlay this result on the same plot above.**



c. **What is the effect of changing the span of the simple moving average?**

Changing the span of the simple moving average from 10 to 4 makes the smoothed data captured the trend of the original data better and obtained more available smoothed data.

2. FOR GRADUATE STUDENTS ONLY

a. **Consider the N-span simple moving average applied to data that are uncorrelated with mean μ and variance σ^2. Show that the variance of the weighted moving average is *Var (Mt)* = $\boldsymbol{\sigma^2/N}$.**

The N-span simple moving average M_t can be denoted as follow:

$$\begin{aligned} M_t &= \frac{y_t + y_{T-1} + \cdots + y_{T-N+1}}{N} \\ &= \frac{1}{N}\sum_{t=T-N+1}^{T} y_t \end{aligned}$$

Thus, the variance of M_t can be denoted as:

$$\begin{aligned} Var(M_t) &= Var\left(\frac{1}{N}\sum_{t=T-N+1}^{T} y_t\right) \\ &= \frac{1}{N^2}\sum_{t=T-N+1}^{T} Var(y_t) \end{aligned}$$

Since the variance of an individual observation y_t is σ^2, then the variance of the moving average is:

$$\begin{aligned} Var(M_t) &= \frac{1}{N^2} \times N \times \sigma^2 \\ &= \frac{\sigma^2}{N} \end{aligned}$$

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b. **Consider an N-span moving average where each observation is weighted by a constant, say, $a_j > 0$. Therefore, the weighted moving average at the end of period T is, $M_T^w = \sum_{t=T-N+1}^{t=T} a_{T+1-t} y_t$. The variance of the original time series y_t is σ². Show that *Var (Mt)* = $\sigma^2 \sum_{j=1}^{N} a_j{}^2$.**

The N-span simple moving average M_T^W can be denoted as follow:

$$M_T^W = \sum_{t=T-N+1}^{t=T} a_{T+1-t} y_t$$

Thus, the variance of M_T^W can be denoted as:

$$Var(M_T^W) = Var\left(\sum_{t=T-N+1}^{t=T} a_{T+1-t} y_t\right)$$

$$= \sum_{t=T-N+1}^{t=T} a_{T+1-t}{}^2 Var(y_t)$$

Since the variance of the original time series y_t is σ^2, we have:

$$Var(M_T^W) = \sum_{j=1}^{N} a_j^2 \, Var(y_t)$$

$$= \sigma^2 \sum_{j=1}^{N} a_j^2$$

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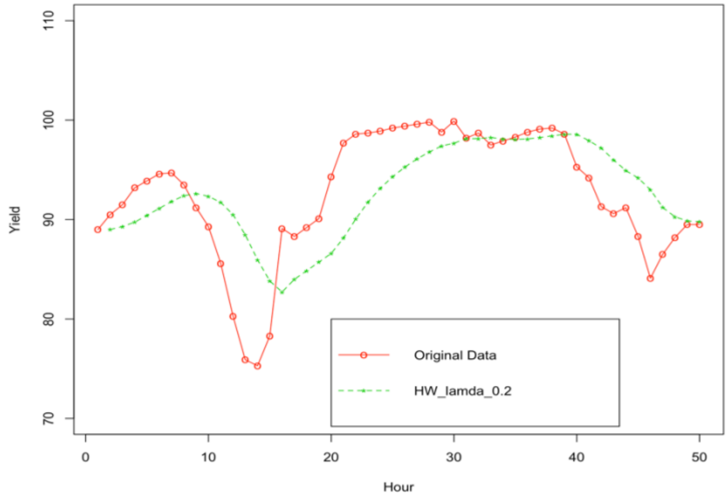
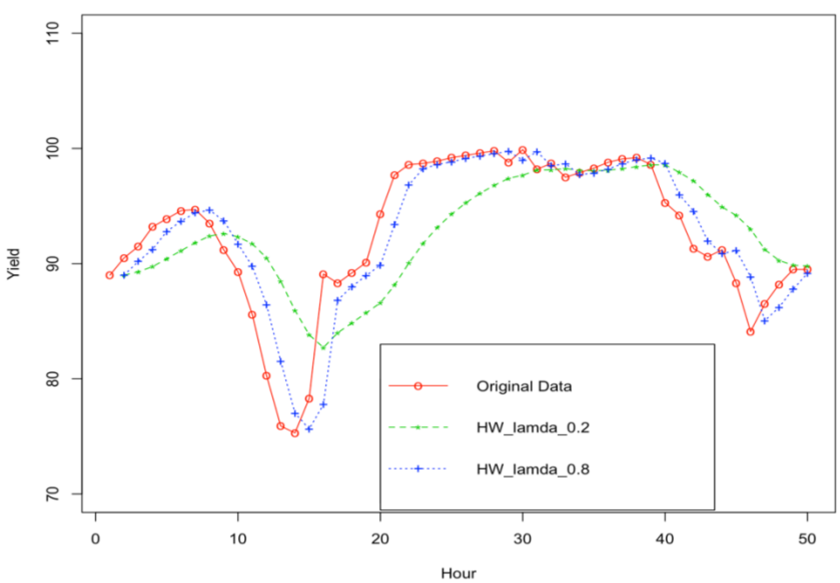
3. ***Yield_Data.xls* presents data on the hourly yield from a chemical process.**

   a. **Use simple (first order) exponential smoothing with λ = 0.2 to smooth the data. Overlay the smoothed plot on the original time series.**



   b. **Change the smoothing constant (λ) to λ = 0.8. Smooth the data with this new value of λ. Overlay the smoothed plot on the same plot made in (a).**



   c. **Compute the Mean Square Difference between the original data and the smoothed data for λ = 0.2 and λ = 0.8.**

   For λ = 0.2, the Mean Square Difference is 15.98977.
   For λ = 0.8, the Mean Square Difference is 0.3009854.