IBP verified acc: 0.0%, 0 verified, time 0.546142578125

CROWN verified acc: 16.6%, 166 verified, time 3.8444900512695312

CROWN verified acc: 16.6%, 166 verified, time 2.686802625656128

alpha-CROWN verified acc: 18.2%, 182 verified, time 42.54144787788391

data size 1000, clean correct 960, robust correct 840

usage: robustness\_verifier.py

[-h]

[--config CONFIG]

[--device {cpu,cuda}]

[--seed SEED]

[--conv\_mode {patches,matrix}]

[--deterministic]

[--double\_fp]

[--loss\_reductio[1]n\_func LOSS\_REDUCTION\_FUNC]

[--record\_lb]

[--load LOAD]

[--start START]

[--end END]

[--num\_outputs NUM\_OUTPUTS]

[--mean MEAN

[MEAN ...]]

[--std STD

[STD ...]]

[--pkl\_path PKL\_PATH]

[--spec\_type {lp,bound}]

[--norm NORM]

[--epsilon EPSILON]

[--lr\_init\_alpha LR\_INIT\_ALPHA]

[--init\_iteration INIT\_ITERATION]

[--share\_slopes]

[--no\_joint\_opt]

[--batch\_size BATCH\_SIZE]

[--lr\_alpha LR\_ALPHA]

[--lr\_beta LR\_BETA]

[--lr\_decay LR\_DECAY]

[--optimizer OPTIMIZER]

[--iteration ITERATION]

[--no\_beta]

[--no\_beta\_warmup]

[--mip\_multi\_proc MIP\_MULTI\_PROC]

[--mip\_threads MIP\_THREADS]

[--mip\_perneuron\_refine\_timeout MIP\_PERNEURON\_REFINE\_TIMEOUT]

[--mip\_refine\_timeout MIP\_REFINE\_TIMEOUT]

[--no\_mip\_early\_stop]

[--max\_domains MAX\_DOMAINS]

[--decision\_thresh DECISION\_THRESH]

[--timeout TIMEOUT]

[--get\_upper\_bound]

[--DFS\_percent DFS\_PERCENT]

[--branching\_method {babsr,fsb,kfsb,sb}]

[--branching\_candidates BRANCHING\_CANDIDATES]

[--branching\_reduceop {min,max,mean,auto}]

[--pgd\_order {before,after,skip}]

[--mode {verified-acc,runnerup,clean-acc,crown-only-verified-acc,alpha-crown-only-verified-acc,ibp-only-verified-acc,attack-only,specify-target}]

[--complete\_verifier {bab,mip,bab-refine,skip}]

[--no\_incomplete]

[--crown]

[--model MODEL]

[--dataset {MNIST,CIFAR,CIFAR\_SDP\_FULL,CIFAR\_RESNET,CIFAR\_SAMPLE,MNIST\_SAMPLE,CIFAR\_ERAN,MNIST\_ERAN,MNIST\_ERAN\_UN,MNIST\_SDP,MNIST\_MADRY\_UN,CIFAR\_SDP,CIFAR\_UN}]

[--filter\_path FILTER\_PATH]

[--data\_idx\_file DATA\_IDX\_FILE]

[--mip\_attack]

[--pgd\_steps PGD\_STEPS]

[--pgd\_restarts PGD\_RESTARTS]

[--no\_pgd\_early\_stop]

[--pgd\_lr\_decay PGD\_LR\_DECAY]

[--pgd\_alpha PGD\_ALPHA]

[--lp\_test {MIP,LP,LP\_intermediate\_refine,MIP\_intermediate\_refine,None}]

number of correctly classified examples: 9

incorrectly classified idx (total 0): []

attack success idx (total 0): []

verification success idx (total 2): [1, 3]

verification failure idx (total 7): [2, 4, 5, 6, 7, 8, 9]

final verified acc: 22.22222222222222%[9]

verifier is called on 9 examples.

total verified: 2

mean time [cnt:9] (excluding attack success): 7.987115038765801

当允许

number of correctly classified examples: 9

incorrectly classified idx (total 0): []

attack success idx (total 0): []

verification success idx (total 2): [1, 3]

verification failure idx (total 7): [2, 4, 5, 6, 7, 8, 9]

final verified acc: 22.22222222222222%[9]

verifier is called on 9 examples.

total verified: 2

mean time [cnt:9] (excluding attack success): 4.643774403466119

general:

device: cuda

seed: 100

conv\_mode: patches

deterministic: false

double\_fp: false

loss\_reduction\_func: sum

record\_bounds: false

mode: verified-acc #使用bab

complete\_verifier: bab

enable\_incomplete\_verification: true

get\_crown\_verified\_acc: false

model:

path: models/eran/mnist\_6\_100\_nat.pth

name: mnist\_6\_100

data:

start: 1

end: 10

num\_outputs: 10

mean: [0.0]

std: [1.0]

pkl\_path: null

dataset: MNIST\_ERAN\_UN

data\_filter\_path: null

data\_idx\_file: null

specification:

type: lp

norm: .inf

epsilon: 0.026

solver:

alpha-crown:

lr\_alpha: 0.1

iteration: 100

share\_slopes: false

no\_joint\_opt: false

beta-crown:

batch\_size: 8

lr\_alpha: 0.01

lr\_beta: 0.05

lr\_decay: 0.98

optimizer: adam

iteration: 20

beta: true

beta\_warmup: true

mip:

parallel\_solvers: null

solver\_threads: 1

refine\_neuron\_timeout: 15

refine\_neuron\_time\_percentage: 0.8

early\_stop: true

bab:

max\_domains: 200000

decision\_thresh: 0

timeout: 10

get\_upper\_bound: false

dfs\_percent: 0.0

branching:

method: babsr

candidates: 3

reduceop: max

attack:

pgd\_order: skip

enable\_mip\_attack: false

pgd\_steps: 100

pgd\_restarts: 30

pgd\_early\_stop: true

pgd\_lr\_decay: 0.99

pgd\_alpha: auto

debug:

lp\_test: null

（一）噪声大小的影响；（20次迭代）

1.无噪声情况

batch: 123, verified acc: 2 / 8, time 0.019946575164794922

batch: 124, verified acc: 3 / 8, time 0.018950939178466797

CROWN verified acc: 16.6%, 166 verified, time 2.723541498184204

specification:

  epsilon: 0.00026

#attack:

#  pgd\_order: after

solver:

  beta-crown:

    batch\_size: 8

    iteration: 20

CROWN verified acc: 96.0%, 960 verified, time 2.702199697494507

2.有噪声

pecification:

  epsilon: 0.0026

#attack:

#  pgd\_order: after

solver:

  beta-crown:

    batch\_size: 8

    iteration: 20

CROWN verified acc: 95.5%, 955 verified, time 2.7482857704162598

3.噪声进一步增大时

specification:

  epsilon: 0.026

#attack:

#  pgd\_order: after

solver:

  beta-crown:

    batch\_size: 8

    iteration: 20

CROWN verified acc: 16.6%, 166 verified, time 2.739769220352173

（二）迭代次数的影响（40次迭代，似乎无影响）

（1）40增大迭代次数

specification:

  epsilon: 0.026

#attack:

#  pgd\_order: after

solver:

  beta-crown:

    batch\_size: 8

    iteration: 40

batch: 124, verified acc: 3 / 8, time 0.019946813583374023

CROWN verified acc: 16.6%, 166 verified, time 2.7019033432006836

（三）增大

Bab-timeout

300->3000

number of correctly classified examples: 9

incorrectly classified idx (total 0): []

attack success idx (total 0): []

verification success idx (total 2): [1, 3]

verification failure idx (total 7): [2, 4, 5, 6, 7, 8, 9]

final verified acc: 22.22222222222222%[9]

verifier is called on 9 examples.

total verified: 2

mean time [cnt:9] (excluding attack success): 7.987115038765801

(nnv) D:\wordspace\python\alpha-beta-CROWN\complete\_verifier>python robustness\_verifier.py --config exp\_configs/cifar\_resnet\_2b.yaml

(nnv) D:\wordspace\python\alpha-beta-CROWN\complete\_verifier>python robustness\_verifier.py --config exp\_configs/cifar\_resnet\_2b D: && cd D:\wordspace\python\alpha-beta-CROWN/complete\_verifier && cmd /C "C:\ProgramData\Anaconda3\envs\nnv\python.exe c:\Users\shaoc\.vscode\extensions\ms-python.python-2022.18.2\pythonFiles\lib\python\debugpy\adapter/../..\debugpy\launcher 59919 -- robustness\_verifier.py --config exp\_configs/mnist\_6\_100\_basic.yaml ".yaml

usage: robustness\_verifier.py [-h] [--config CONFIG] [--device {cpu,cuda}] [--seed SEED] [--conv\_mode {patches,matrix}] [--deterministic] [--double\_fp]

[--loss\_reduction\_func LOSS\_REDUCTION\_FUNC] [--record\_lb] [--load LOAD] [--start START] [--end END] [--num\_outputs NUM\_OUTPUTS]

[--mean MEAN [MEAN ...]] [--std STD [STD ...]] [--pkl\_path PKL\_PATH] [--spec\_type {lp,bound}] [--norm NORM] [--epsilon EPSILON]

[--lr\_init\_alpha LR\_INIT\_ALPHA] [--init\_iteration INIT\_ITERATION] [--share\_slopes] [--no\_joint\_opt] [--batch\_size BATCH\_SIZE]

[--lr\_alpha LR\_ALPHA] [--lr\_beta LR\_BETA] [--lr\_decay LR\_DECAY] [--optimizer OPTIMIZER] [--iteration ITERATION] [--no\_beta]

[--no\_beta\_warmup] [--mip\_multi\_proc MIP\_MULTI\_PROC] [--mip\_threads MIP\_THREADS]

[--mip\_perneuron\_refine\_timeout MIP\_PERNEURON\_REFINE\_TIMEOUT] [--mip\_refine\_timeout MIP\_REFINE\_TIMEOUT] [--no\_mip\_early\_stop]

[--max\_domains MAX\_DOMAINS] [--decision\_thresh DECISION\_THRESH] [--timeout TIMEOUT] [--get\_upper\_bound] [--DFS\_percent DFS\_PERCENT]

[--branching\_method {babsr,fsb,kfsb,sb}] [--branching\_candidates BRANCHING\_CANDIDATES] [--branching\_reduceop {min,max,mean,auto}]

[--pgd\_order {before,after,skip}]

[--mode {verified-acc,runnerup,clean-acc,crown-only-verified-acc,alpha-crown-only-verified-acc,ibp-only-verified-acc,attack-only,specify-target}]

[--complete\_verifier {bab,mip,bab-refine,skip}] [--no\_incomplete] [--crown] [--model MODEL]

[--dataset {MNIST,CIFAR,CIFAR\_SDP\_FULL,CIFAR\_RESNET,CIFAR\_SAMPLE,MNIST\_SAMPLE,CIFAR\_ERAN,MNIST\_ERAN,MNIST\_ERAN\_UN,MNIST\_SDP,MNIST\_MADRY\_UN,CIFAR\_SDP,CIFAR\_UN}]

[--filter\_path FILTER\_PATH] [--data\_idx\_file DATA\_IDX\_FILE] [--mip\_attack] [--pgd\_steps PGD\_STEPS] [--pgd\_restarts PGD\_RESTARTS]

[--no\_pgd\_early\_stop] [--pgd\_lr\_decay PGD\_LR\_DECAY] [--pgd\_alpha PGD\_ALPHA]

[--lp\_test {MIP,LP,LP\_intermediate\_refine,MIP\_intermediate\_refine,None}]

robustness\_verifier.py: error: unrecognized arguments: D:

(nnv) D:\wordspace\python\alpha-beta-CROWN\complete\_verifier> D: && cd D:\wordspace\python\alpha-beta-CROWN/complete\_verifier && cmd /C "C:\ProgramData\Anaconda3\envs\nnv\python.exe c:\Users\shaoc\.vscode\extensions\ms-python.python-2022.18.2\pythonFiles\lib\python\debugpy\adapter/../..\debugpy\launcher 59950 -- robustness\_verifier.py --config exp\_configs/mnist\_6\_100\_basic.yaml "

Configurations:

Experiments at Mon Mar 20 16:50:11 2023 on DESKTOP-IO2LPOE

Sequential(

(0): Flatten()

(1): Linear(in\_features=784, out\_features=100, bias=True)

(2): ReLU()

(3): Linear(in\_features=100, out\_features=100, bias=True)

(4): ReLU()

(5): Linear(in\_features=100, out\_features=100, bias=True)

(6): ReLU()

(7): Linear(in\_features=100, out\_features=100, bias=True)

(8): ReLU()

(9): Linear(in\_features=100, out\_features=100, bias=True)

(10): ReLU()

(11): Linear(in\_features=100, out\_features=10, bias=True)

)

############################

Sampled data loaded. No normalization used!

Shape: torch.Size([1000, 1, 28, 28]) torch.Size([1000]) torch.Size([1000])

X range: tensor(1.) tensor(0.) tensor(0.1223)

Note runnerup label is empty here!

############################

epsilon after preprocessing: tensor([[[[0.0260]]]]), data\_max = tensor([[[[1.]]]]), data\_min = tensor([[[[0.]]]])

Task length: 9

saving results to Verified\_ret\_[mnist\_6\_100]\_start=1\_end=10\_iter=20\_b=8\_timeout=10\_branching=babsr-max-3\_lra-init=0.1\_lra=0.01\_lrb=0.05\_PGD=skip.npy

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 0 img ID: 1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 2, correct label 2, image norm 113.13725280761719, logits tensor([-2.0595, 1.7575, 10.0782, 1.0748, -0.9482, -4.0839, -2.3911, 0.3436,

-0.9526, -4.9323], device='cuda:0', grad\_fn=<SelectBackward0>)

D:\wordspace\python\alpha-beta-CROWN\complete\_verifier\auto\_LiRPA\parse\_graph.py:148: FutureWarning: 'torch.onnx.symbolic\_helper.\_set\_opset\_version' is deprecated in version 1.13 and will be removed in version 1.14. Please remove its usage and avoid setting internal variables directly.

\_set\_opset\_version(12)

D:\wordspace\python\alpha-beta-CROWN\complete\_verifier\auto\_LiRPA\parse\_graph.py:45: FutureWarning: 'torch.onnx.\_patch\_torch.\_node\_getitem' is deprecated in version 1.13 and will be removed in version 1.14. Please Internally use '\_node\_get' in symbolic\_helper instead..

attrs = {k: n[k] for k in n.attributeNames()}

Model prediction is: tensor([[-2.0595, 1.7575, 10.0782, 1.0748, -0.9482, -4.0839, -2.3911, 0.3436,

-0.9526, -4.9323]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[0.6301, 0.1344, 0.4976, 2.1355, 2.5526, 2.1028, 1.8815, 1.7729, 3.6696]],

device='cuda:0') None

verified with init bound!

Result: image 1 verification success (with incomplete verifier)!

Wall time: 0.2313833236694336

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 1 img ID: 2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 1, correct label 1, image norm 38.709808349609375, logits tensor([-3.7012, 8.8913, 1.1001, -2.5765, 0.6640, -2.1244, -1.6588, 0.6820,

1.0836, -2.1354], device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[-3.7012, 8.8913, 1.1001, -2.5765, 0.6640, -2.1244, -1.6588, 0.6820,

1.0836, -2.1354]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[ -96.2690, -93.0548, -94.6280, -90.1885, -93.8622, -79.2704,

-82.7055, -85.5262, -107.9016]], device='cuda:0') None

best\_l after optimization: 478.8368835449219 with beta sum per layer: []

alpha/beta optimization time: 6.230117082595825

initial alpha-CROWN bounds: tensor([[-54.9628, -52.4630, -53.5361, -55.5802, -54.9408, -43.2787, -48.4003,

-52.5121, -63.1629]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

##### [1:2] Tested against 9 ######

Model prediction is: tensor([[-3.7012, 8.8913, 1.1001, -2.5765, 0.6640, -2.1244, -1.6588, 0.6820,

1.0836, -2.1354]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

setting alpha for layer /20 start\_node /input.3

setting alpha for layer /20 start\_node /input.7

setting alpha for layer /20 start\_node /input.11

setting alpha for layer /20 start\_node /input.15

not setting layer /20 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /22 start\_node /input.7

setting alpha for layer /22 start\_node /input.11

setting alpha for layer /22 start\_node /input.15

not setting layer /22 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /24 start\_node /input.11

setting alpha for layer /24 start\_node /input.15

not setting layer /24 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /26 start\_node /input.15

not setting layer /26 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

not setting layer /28 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

0 /input torch.Size([1, 100])

1 /input.3 torch.Size([1, 100])

2 /input.7 torch.Size([1, 100])

3 /input.11 torch.Size([1, 100])

4 /input.15 torch.Size([1, 100])

best\_l after optimization: 63.16157150268555 with beta sum per layer: []

alpha/beta optimization time: 1.7557289600372314

alpha-CROWN with fixed intermediate bounds: tensor([[-63.1616]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

-63.16157150268555

第 0 layer size = torch.Size([100]) with unstable 58 nodes

第 1 layer size = torch.Size([100]) with unstable 90 nodes

第 2 layer size = torch.Size([100]) with unstable 100 nodes

第 3 layer size = torch.Size([100]) with unstable 100 nodes

第 4 layer size = torch.Size([100]) with unstable 100 nodes

-----------------

# of total unstable neurons(不定态神经元总数量): 448

-----------------

splitting decisions: [[4, 2]]

best\_l after optimization: 119.68608093261719 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.07429954409599304]

alpha/beta optimization time: 0.4887199401855469

This batch time : update\_bounds func: 0.4917 prepare: 0.0020 bound: 0.4887 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 0.4917 prepare: 0.0020 bound: 0.4887 transfer: 0.0010 finalize: 0.0000

batch bounding time: 0.49268174171447754

Current worst splitting domains [lb, ub] (depth):

[-61.44030, 35.838428] (1), [-58.24578, 35.838428] (1),

length of domains: 2

Total time: 0.4957 pickout: 0.0010 decision: 0.0020 get\_bound: 0.4927 add\_domain: 0.0000

Current lb:-61.44029998779297

2 neurons visited

Global ub: 35.83842849731445, batch ub: inf

Cumulative time: 2.426933765411377

splitting decisions: [[4, 24], [4, 24]]

best\_l after optimization: 224.30345153808594 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 1.5082764625549316]

alpha/beta optimization time: 0.3700113296508789

This batch time : update\_bounds func: 0.3750 prepare: 0.0030 bound: 0.3700 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 0.8667 prepare: 0.0049 bound: 0.8587 transfer: 0.0010 finalize: 0.0010

batch bounding time: 0.374997615814209

Current worst splitting domains [lb, ub] (depth):

[-60.19039, 35.838428] (2), [-57.18229, 35.838428] (2), [-55.28405, 35.838428] (2), [-51.64672, 35.838428] (2),

length of domains: 4

Total time: 0.3790 pickout: 0.0010 decision: 0.0020 get\_bound: 0.3750 add\_domain: 0.0010

Current lb:-60.19038772583008

6 neurons visited

Global ub: 35.83842849731445, batch ub: inf

Cumulative time: 2.80594801902771

splitting decisions: [[4, 58], [4, 58], [4, 58], [4, 58]]

best\_l after optimization: 429.58624267578125 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 3.8753502368927]

alpha/beta optimization time: 0.37203311920166016

This batch time : update\_bounds func: 0.3780 prepare: 0.0040 bound: 0.3730 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 1.2447 prepare: 0.0089 bound: 1.2317 transfer: 0.0010 finalize: 0.0010

batch bounding time: 0.3790154457092285

Current worst splitting domains [lb, ub] (depth):

[-58.75399, 35.838428] (3), [-56.75639, 35.838428] (3), [-55.71530, 35.838428] (3), [-53.84172, 35.838428] (3), [-53.30792, 35.838428] (3), [-52.54908, 35.838428] (3), [-49.65803, 35.838428] (3), [-49.00381, 35.838428] (3),

length of domains: 8

Total time: 0.3850 pickout: 0.0010 decision: 0.0040 get\_bound: 0.3790 add\_domain: 0.0010

Current lb:-58.75398635864258

14 neurons visited

Global ub: 35.83842849731445, batch ub: inf

Cumulative time: 3.1929118633270264

splitting decisions: [[4, 90], [4, 90], [4, 90], [4, 90], [4, 90], [4, 90], [4, 90], [4, 90]]

best\_l after optimization: 825.153076171875 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 11.095968246459961]

alpha/beta optimization time: 0.3670179843902588

This batch time : update\_bounds func: 0.3740 prepare: 0.0040 bound: 0.3670 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 1.6187 prepare: 0.0129 bound: 1.5988 transfer: 0.0010 finalize: 0.0030

batch bounding time: 0.3739964962005615

Current worst splitting domains [lb, ub] (depth):

[-57.71324, 35.838428] (4), [-55.70231, 35.838428] (4), [-55.04212, 35.838428] (4), [-54.56271, 35.838428] (4), [-53.35772, 35.838428] (4), [-52.68240, 35.838428] (4), [-52.63282, 35.838428] (4), [-52.21626, 35.838428] (4), [-51.42300, 35.838428] (4), [-50.88144, 35.838428] (4), [-50.15506, 35.838428] (4), [-49.50262, 35.838428] (4), [-48.61929, 35.838428] (4), [-48.05281, 35.838428] (4), [-46.51925, 35.838428] (4), [-46.09000, 35.838428] (4),

length of domains: 16

Total time: 0.3830 pickout: 0.0020 decision: 0.0050 get\_bound: 0.3740 add\_domain: 0.0020

Current lb:-57.71324157714844

30 neurons visited

Global ub: 35.83842849731445, batch ub: inf

Cumulative time: 3.5758869647979736

splitting decisions: [[4, 61], [4, 61], [4, 61], [4, 61], [4, 61], [4, 61], [4, 61], [4, 61]]

best\_l after optimization: 838.1182250976562 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 10.305689811706543]

alpha/beta optimization time: 0.3790137767791748

This batch time : update\_bounds func: 0.3860 prepare: 0.0040 bound: 0.3800 transfer: 0.0000 finalize: 0.0020

Accumulated time: update\_bounds func: 2.0047 prepare: 0.0168 bound: 1.9787 transfer: 0.0000 finalize: 0.0050

batch bounding time: 0.3869645595550537

Current worst splitting domains [lb, ub] (depth):

[-56.10530, 35.838428] (5), [-55.76483, 35.838428] (5), [-54.08154, 35.838428] (5), [-53.75541, 35.838428] (5), [-53.71981, 35.838428] (5), [-53.17193, 35.838428] (5), [-52.61583, 35.838428] (5), [-52.22053, 35.838428] (5), [-51.91041, 35.838428] (5), [-51.42300, 35.838428] (4), [-51.38409, 35.838428] (5), [-51.34215, 35.838428] (5), [-51.01917, 35.838428] (5), [-50.88144, 35.838428] (4), [-50.40697, 35.838428] (5), [-50.36671, 35.838428] (5), [-50.17279, 35.838428] (5), [-50.15506, 35.838428] (4), [-50.08077, 35.838428] (5), [-49.50262, 35.838428] (4),

length of domains: 24

Total time: 0.3959 pickout: 0.0020 decision: 0.0050 get\_bound: 0.3870 add\_domain: 0.0020

Current lb:-56.10530090332031

46 neurons visited

Global ub: 35.83842849731445, batch ub: inf

Time out!!!!!!!!

Image 2 label 9 verification end, final lower bound -56.10530090332031, upper bound 35.83842849731445, time: 4.013659238815308

2 -56.10530090332031

Result: image 2 verification failure (with branch and bound).

Wall time: 10.347518682479858

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 2 img ID: 3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 0, correct label 0, image norm 145.1529541015625, logits tensor([ 9.1278, -5.8793, 1.7480, -2.8472, -0.8200, 1.4961, -0.1319, -2.7649,

-1.2869, 0.1981], device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[ 9.1278, -5.8793, 1.7480, -2.8472, -0.8200, 1.4961, -0.1319, -2.7649,

-1.2869, 0.1981]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[6.0611, 0.5068, 3.7542, 2.6818, 2.5589, 3.3768, 4.4265, 3.5985, 2.5453]],

device='cuda:0') None

verified with init bound!

Result: image 3 verification success (with incomplete verifier)!

Wall time: 0.06080794334411621

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 3 img ID: 4 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 4, correct label 4, image norm 75.439208984375, logits tensor([-1.3820, -1.5346, -0.5229, -2.8616, 6.9759, 0.0258, -2.4641, 0.4381,

-0.5032, 4.0722], device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[-1.3820, -1.5346, -0.5229, -2.8616, 6.9759, 0.0258, -2.4641, 0.4381,

-0.5032, 4.0722]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[-18.1569, -19.6753, -17.4166, -20.6416, -20.8003, -15.6187, -17.5978,

-21.8895, -18.1589]], device='cuda:0') None

best\_l after optimization: 120.32124328613281 with beta sum per layer: []

alpha/beta optimization time: 5.828366994857788

initial alpha-CROWN bounds: tensor([[-13.0582, -13.1080, -12.5255, -14.8746, -14.8996, -10.1389, -12.6094,

-15.5623, -13.5448]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

##### [3:4] Tested against 8 ######

Model prediction is: tensor([[-1.3820, -1.5346, -0.5229, -2.8616, 6.9759, 0.0258, -2.4641, 0.4381,

-0.5032, 4.0722]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

setting alpha for layer /20 start\_node /input.3

setting alpha for layer /20 start\_node /input.7

setting alpha for layer /20 start\_node /input.11

setting alpha for layer /20 start\_node /input.15

not setting layer /20 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /22 start\_node /input.7

setting alpha for layer /22 start\_node /input.11

setting alpha for layer /22 start\_node /input.15

not setting layer /22 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /24 start\_node /input.11

setting alpha for layer /24 start\_node /input.15

not setting layer /24 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /26 start\_node /input.15

not setting layer /26 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

not setting layer /28 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

0 /input torch.Size([1, 100])

1 /input.3 torch.Size([1, 100])

2 /input.7 torch.Size([1, 100])

3 /input.11 torch.Size([1, 100])

4 /input.15 torch.Size([1, 100])

best\_l after optimization: 15.562602996826172 with beta sum per layer: []

alpha/beta optimization time: 1.3418736457824707

alpha-CROWN with fixed intermediate bounds: tensor([[-15.5626]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

-15.562602996826172

第 0 layer size = torch.Size([100]) with unstable 36 nodes

第 1 layer size = torch.Size([100]) with unstable 55 nodes

第 2 layer size = torch.Size([100]) with unstable 78 nodes

第 3 layer size = torch.Size([100]) with unstable 98 nodes

第 4 layer size = torch.Size([100]) with unstable 100 nodes

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# of total unstable neurons(不定态神经元总数量): 367

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splitting decisions: [[4, 13]]

best\_l after optimization: 29.153465270996094 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.6537408232688904]

alpha/beta optimization time: 0.36103391647338867

This batch time : update\_bounds func: 0.3640 prepare: 0.0020 bound: 0.3610 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 2.3687 prepare: 0.0188 bound: 2.3398 transfer: 0.0010 finalize: 0.0050

batch bounding time: 0.3640265464782715

Current worst splitting domains [lb, ub] (depth):

[-15.15863, 83.437393] (1), [-13.99483, 83.437393] (1),

length of domains: 2

Total time: 0.3680 pickout: 0.0010 decision: 0.0020 get\_bound: 0.3650 add\_domain: 0.0000

Current lb:-15.158632278442383

2 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Cumulative time: 1.7398104667663574

splitting decisions: [[4, 64], [4, 64]]

best\_l after optimization: 55.035301208496094 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 1.99711012840271]

alpha/beta optimization time: 0.367018461227417

This batch time : update\_bounds func: 0.3710 prepare: 0.0030 bound: 0.3670 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 2.7397 prepare: 0.0218 bound: 2.7068 transfer: 0.0010 finalize: 0.0050

batch bounding time: 0.37200450897216797

Current worst splitting domains [lb, ub] (depth):

[-14.79041, 83.437393] (2), [-13.74446, 83.437393] (2), [-13.45259, 83.437393] (2), [-13.04784, 83.437393] (2),

length of domains: 4

Total time: 0.3750 pickout: 0.0010 decision: 0.0020 get\_bound: 0.3720 add\_domain: 0.0000

Current lb:-14.790412902832031

6 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Cumulative time: 2.116802453994751

splitting decisions: [[4, 54], [4, 54], [4, 54], [4, 54]]

best\_l after optimization: 105.30148315429688 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 5.983146667480469]

alpha/beta optimization time: 0.36003661155700684

This batch time : update\_bounds func: 0.3651 prepare: 0.0030 bound: 0.3600 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 3.1047 prepare: 0.0248 bound: 3.0668 transfer: 0.0010 finalize: 0.0060

batch bounding time: 0.36602020263671875

Current worst splitting domains [lb, ub] (depth):

[-14.49328, 83.437393] (3), [-13.84091, 83.437393] (3), [-13.33059, 83.437393] (3), [-13.18418, 83.437393] (3), [-13.01536, 83.437393] (3), [-12.68520, 83.437393] (3), [-12.54690, 83.437393] (3), [-12.20507, 83.437393] (3),

length of domains: 8

Total time: 0.3740 pickout: 0.0020 decision: 0.0050 get\_bound: 0.3660 add\_domain: 0.0010

Current lb:-14.49327564239502

14 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Cumulative time: 2.490802049636841

splitting decisions: [[4, 89], [4, 89], [4, 89], [4, 89], [4, 89], [4, 89], [4, 89], [4, 89]]

best\_l after optimization: 200.7020721435547 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 15.489212989807129]

alpha/beta optimization time: 0.37203240394592285

This batch time : update\_bounds func: 0.3790 prepare: 0.0040 bound: 0.3720 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 3.4837 prepare: 0.0288 bound: 3.4389 transfer: 0.0010 finalize: 0.0080

batch bounding time: 0.3789863586425781

Current worst splitting domains [lb, ub] (depth):

[-14.19694, 83.437393] (4), [-13.48542, 83.437393] (4), [-13.22379, 83.437393] (4), [-12.98538, 83.437393] (4), [-12.91641, 83.437393] (4), [-12.86125, 83.437393] (4), [-12.63174, 83.437393] (4), [-12.35676, 83.437393] (4), [-12.35146, 83.437393] (4), [-12.33873, 83.437393] (4), [-12.16544, 83.437393] (4), [-12.12218, 83.437393] (4), [-11.88972, 83.437393] (4), [-11.84174, 83.437393] (4), [-11.79972, 83.437393] (4), [-11.53542, 83.437393] (4),

length of domains: 16

Total time: 0.3890 pickout: 0.0030 decision: 0.0050 get\_bound: 0.3800 add\_domain: 0.0010

Current lb:-14.196941375732422

30 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Cumulative time: 2.880758762359619

splitting decisions: [[4, 69], [4, 69], [4, 69], [4, 69], [4, 69], [4, 69], [4, 69], [4, 69]]

best\_l after optimization: 200.79534912109375 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 16.899250030517578]

alpha/beta optimization time: 0.3680150508880615

This batch time : update\_bounds func: 0.3760 prepare: 0.0040 bound: 0.3690 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 3.8597 prepare: 0.0328 bound: 3.8079 transfer: 0.0010 finalize: 0.0100

batch bounding time: 0.37699198722839355

Current worst splitting domains [lb, ub] (depth):

[-13.88542, 83.437393] (5), [-13.46636, 83.437393] (5), [-13.13112, 83.437393] (5), [-12.92013, 83.437393] (5), [-12.85727, 83.437393] (5), [-12.68525, 83.437393] (5), [-12.56860, 83.437393] (5), [-12.55926, 83.437393] (5), [-12.37739, 83.437393] (5), [-12.35146, 83.437393] (4), [-12.33873, 83.437393] (4), [-12.30814, 83.437393] (5), [-12.17611, 83.437393] (5), [-12.16544, 83.437393] (4), [-12.13702, 83.437393] (5), [-12.12218, 83.437393] (4), [-12.11961, 83.437393] (5), [-12.03171, 83.437393] (5), [-11.96900, 83.437393] (5), [-11.88972, 83.437393] (4),

length of domains: 24

Total time: 0.3890 pickout: 0.0030 decision: 0.0070 get\_bound: 0.3770 add\_domain: 0.0020

Current lb:-13.885416030883789

46 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Cumulative time: 3.2697181701660156

splitting decisions: [[4, 44], [4, 44], [4, 44], [4, 44], [4, 44], [4, 44], [4, 44], [4, 44]]

best\_l after optimization: 201.0857696533203 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 12.530013084411621]

alpha/beta optimization time: 0.37018322944641113

This batch time : update\_bounds func: 0.3772 prepare: 0.0040 bound: 0.3702 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 4.2369 prepare: 0.0368 bound: 4.1781 transfer: 0.0010 finalize: 0.0120

batch bounding time: 0.3771653175354004

Current worst splitting domains [lb, ub] (depth):

[-13.45366, 83.437393] (6), [-13.44821, 83.437393] (6), [-13.01846, 83.437393] (6), [-12.99843, 83.437393] (6), [-12.70684, 83.437393] (6), [-12.69450, 83.437393] (6), [-12.48198, 83.437393] (6), [-12.45936, 83.437393] (6), [-12.43669, 83.437393] (6), [-12.37787, 83.437393] (6), [-12.37739, 83.437393] (5), [-12.35146, 83.437393] (4), [-12.33873, 83.437393] (4), [-12.30814, 83.437393] (5), [-12.29108, 83.437393] (6), [-12.23264, 83.437393] (6), [-12.17611, 83.437393] (5), [-12.16544, 83.437393] (4), [-12.13855, 83.437393] (6), [-12.13702, 83.437393] (5),

length of domains: 32

Total time: 0.3841 pickout: 0.0030 decision: 0.0020 get\_bound: 0.3782 add\_domain: 0.0010

Current lb:-13.453661918640137

62 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Cumulative time: 3.6548633575439453

splitting decisions: [[4, 82], [4, 82], [4, 82], [4, 82], [4, 82], [4, 82], [4, 82], [4, 82]]

best\_l after optimization: 197.89210510253906 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 13.556984901428223]

alpha/beta optimization time: 0.37200450897216797

This batch time : update\_bounds func: 0.3800 prepare: 0.0030 bound: 0.3740 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 4.6169 prepare: 0.0398 bound: 4.5521 transfer: 0.0010 finalize: 0.0140

batch bounding time: 0.37998342514038086

Current worst splitting domains [lb, ub] (depth):

[-13.18823, 83.437393] (7), [-13.18342, 83.437393] (7), [-12.73003, 83.437393] (7), [-12.72277, 83.437393] (7), [-12.71444, 83.437393] (7), [-12.58598, 83.437393] (7), [-12.44523, 83.437393] (7), [-12.44117, 83.437393] (7), [-12.43669, 83.437393] (6), [-12.37787, 83.437393] (6), [-12.37739, 83.437393] (5), [-12.35146, 83.437393] (4), [-12.33873, 83.437393] (4), [-12.30814, 83.437393] (5), [-12.29108, 83.437393] (6), [-12.23264, 83.437393] (6), [-12.21515, 83.437393] (7), [-12.20412, 83.437393] (7), [-12.19370, 83.437393] (7), [-12.18704, 83.437393] (7),

length of domains: 40

Total time: 0.3870 pickout: 0.0020 decision: 0.0030 get\_bound: 0.3800 add\_domain: 0.0020

Current lb:-13.188233375549316

78 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Cumulative time: 4.043821811676025

splitting decisions: [[4, 65], [4, 65], [4, 65], [4, 65], [4, 65], [4, 65], [4, 65], [4, 65]]

best\_l after optimization: 196.9086151123047 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 13.290101051330566]

alpha/beta optimization time: 0.3670177459716797

This batch time : update\_bounds func: 0.3740 prepare: 0.0040 bound: 0.3670 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 4.9909 prepare: 0.0438 bound: 4.9191 transfer: 0.0010 finalize: 0.0160

batch bounding time: 0.37399959564208984

Current worst splitting domains [lb, ub] (depth):

[-12.88626, 83.437393] (8), [-12.88423, 83.437393] (8), [-12.59841, 83.437393] (8), [-12.58934, 83.437393] (8), [-12.44324, 83.437393] (8), [-12.43669, 83.437393] (6), [-12.41866, 83.437393] (8), [-12.40654, 83.437393] (8), [-12.37787, 83.437393] (6), [-12.37739, 83.437393] (5), [-12.35146, 83.437393] (4), [-12.33873, 83.437393] (4), [-12.30814, 83.437393] (5), [-12.30607, 83.437393] (8), [-12.29108, 83.437393] (6), [-12.23264, 83.437393] (6), [-12.21515, 83.437393] (7), [-12.20412, 83.437393] (7), [-12.19370, 83.437393] (7), [-12.18704, 83.437393] (7),

length of domains: 48

Total time: 0.3810 pickout: 0.0030 decision: 0.0020 get\_bound: 0.3750 add\_domain: 0.0010

Current lb:-12.886256217956543

94 neurons visited

Global ub: 83.43739318847656, batch ub: inf

Time out!!!!!!!!

Image 4 label 8 verification end, final lower bound -12.886256217956543, upper bound 83.43739318847656, time: 4.4666526317596436

4 -12.886256217956543

Result: image 4 verification failure (with branch and bound).

Wall time: 10.400773525238037

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 4 img ID: 5 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 1, correct label 1, image norm 54.333335876464844, logits tensor([-4.7250, 11.5023, 1.0272, -3.2290, 0.6359, -2.8726, -2.2377, 1.1579,

1.3752, -2.3811], device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[-4.7250, 11.5023, 1.0272, -3.2290, 0.6359, -2.8726, -2.2377, 1.1579,

1.3752, -2.3811]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[-48.4515, -49.2391, -48.2552, -47.3599, -48.1597, -39.1093, -43.1400,

-44.7019, -54.4206]], device='cuda:0') None

best\_l after optimization: 271.88824462890625 with beta sum per layer: []

alpha/beta optimization time: 5.743069887161255

initial alpha-CROWN bounds: tensor([[-30.6538, -30.0703, -30.4463, -31.6674, -31.0793, -24.1114, -28.0108,

-30.0567, -35.7923]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

##### [4:5] Tested against 9 ######

Model prediction is: tensor([[-4.7250, 11.5023, 1.0272, -3.2290, 0.6359, -2.8726, -2.2377, 1.1579,

1.3752, -2.3811]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

setting alpha for layer /20 start\_node /input.3

setting alpha for layer /20 start\_node /input.7

setting alpha for layer /20 start\_node /input.11

setting alpha for layer /20 start\_node /input.15

not setting layer /20 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /22 start\_node /input.7

setting alpha for layer /22 start\_node /input.11

setting alpha for layer /22 start\_node /input.15

not setting layer /22 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /24 start\_node /input.11

setting alpha for layer /24 start\_node /input.15

not setting layer /24 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /26 start\_node /input.15

not setting layer /26 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

not setting layer /28 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

0 /input torch.Size([1, 100])

1 /input.3 torch.Size([1, 100])

2 /input.7 torch.Size([1, 100])

3 /input.11 torch.Size([1, 100])

4 /input.15 torch.Size([1, 100])

best\_l after optimization: 35.79090118408203 with beta sum per layer: []

alpha/beta optimization time: 1.5129516124725342

alpha-CROWN with fixed intermediate bounds: tensor([[-35.7909]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

-35.79090118408203

第 0 layer size = torch.Size([100]) with unstable 45 nodes

第 1 layer size = torch.Size([100]) with unstable 65 nodes

第 2 layer size = torch.Size([100]) with unstable 95 nodes

第 3 layer size = torch.Size([100]) with unstable 100 nodes

第 4 layer size = torch.Size([100]) with unstable 100 nodes

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# of total unstable neurons(不定态神经元总数量): 405

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splitting decisions: [[4, 2]]

best\_l after optimization: 67.89137268066406 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.04423461854457855]

alpha/beta optimization time: 0.35804200172424316

This batch time : update\_bounds func: 0.3630 prepare: 0.0040 bound: 0.3580 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 5.3539 prepare: 0.0477 bound: 5.2771 transfer: 0.0010 finalize: 0.0160

batch bounding time: 0.364025354385376

Current worst splitting domains [lb, ub] (depth):

[-34.76644, 63.209099] (1), [-33.12493, 63.209099] (1),

length of domains: 2

Total time: 0.3670 pickout: 0.0010 decision: 0.0020 get\_bound: 0.3640 add\_domain: 0.0000

Current lb:-34.76643753051758

2 neurons visited

Global ub: 63.20909881591797, batch ub: inf

Cumulative time: 1.9138789176940918

splitting decisions: [[4, 24], [4, 24]]

best\_l after optimization: 127.78738403320312 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 1.384743094444275]

alpha/beta optimization time: 0.36983609199523926

This batch time : update\_bounds func: 0.3739 prepare: 0.0030 bound: 0.3698 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 5.7278 prepare: 0.0507 bound: 5.6469 transfer: 0.0010 finalize: 0.0160

batch bounding time: 0.3738541603088379

Current worst splitting domains [lb, ub] (depth):

[-34.02255, 63.209099] (2), [-32.40486, 63.209099] (2), [-31.59008, 63.209099] (2), [-29.76989, 63.209099] (2),

length of domains: 4

Total time: 0.3798 pickout: 0.0010 decision: 0.0040 get\_bound: 0.3748 add\_domain: 0.0000

Current lb:-34.02254867553711

6 neurons visited

Global ub: 63.20909881591797, batch ub: inf

Cumulative time: 2.2946877479553223

splitting decisions: [[4, 58], [4, 58], [4, 58], [4, 58]]

best\_l after optimization: 244.7890625 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 3.7275991439819336]

alpha/beta optimization time: 0.3640024662017822

This batch time : update\_bounds func: 0.3690 prepare: 0.0030 bound: 0.3640 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 6.0967 prepare: 0.0537 bound: 6.0110 transfer: 0.0010 finalize: 0.0170

batch bounding time: 0.368990421295166

Current worst splitting domains [lb, ub] (depth):

[-33.17487, 63.209099] (3), [-32.11957, 63.209099] (3), [-31.51657, 63.209099] (3), [-30.54670, 63.209099] (3), [-30.43368, 63.209099] (3), [-30.06165, 63.209099] (3), [-28.64788, 63.209099] (3), [-28.28815, 63.209099] (3),

length of domains: 8

Total time: 0.3760 pickout: 0.0020 decision: 0.0040 get\_bound: 0.3690 add\_domain: 0.0010

Current lb:-33.174869537353516

14 neurons visited

Global ub: 63.20909881591797, batch ub: inf

Cumulative time: 2.671657085418701

splitting decisions: [[4, 90], [4, 90], [4, 90], [4, 90], [4, 90], [4, 90], [4, 90], [4, 90]]

best\_l after optimization: 470.364990234375 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 10.351826667785645]

alpha/beta optimization time: 0.37699151039123535

This batch time : update\_bounds func: 0.3840 prepare: 0.0040 bound: 0.3770 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 6.4807 prepare: 0.0577 bound: 6.3879 transfer: 0.0010 finalize: 0.0190

batch bounding time: 0.38417983055114746

Current worst splitting domains [lb, ub] (depth):

[-32.44807, 63.209099] (4), [-31.50339, 63.209099] (4), [-31.30844, 63.209099] (4), [-30.75695, 63.209099] (4), [-30.20854, 63.209099] (4), [-29.98326, 63.209099] (4), [-29.82348, 63.209099] (4), [-29.81833, 63.209099] (4), [-29.39127, 63.209099] (4), [-28.97404, 63.209099] (4), [-28.58831, 63.209099] (4), [-28.29732, 63.209099] (4), [-28.01979, 63.209099] (4), [-27.69089, 63.209099] (4), [-26.90876, 63.209099] (4), [-26.64412, 63.209099] (4),

length of domains: 16

Total time: 0.3922 pickout: 0.0030 decision: 0.0040 get\_bound: 0.3842 add\_domain: 0.0010

Current lb:-32.44807052612305

30 neurons visited

Global ub: 63.20909881591797, batch ub: inf

Cumulative time: 3.064814329147339

splitting decisions: [[4, 61], [4, 61], [4, 61], [4, 61], [4, 61], [4, 61], [4, 61], [4, 61]]

best\_l after optimization: 474.6843566894531 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 9.070026397705078]

alpha/beta optimization time: 0.37702012062072754

This batch time : update\_bounds func: 0.3840 prepare: 0.0040 bound: 0.3770 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 6.8647 prepare: 0.0617 bound: 6.7650 transfer: 0.0010 finalize: 0.0210

batch bounding time: 0.3839731216430664

Current worst splitting domains [lb, ub] (depth):

[-31.49261, 63.209099] (5), [-31.34230, 63.209099] (5), [-30.46697, 63.209099] (5), [-30.43908, 63.209099] (5), [-30.43104, 63.209099] (5), [-29.95318, 63.209099] (5), [-29.87797, 63.209099] (5), [-29.55053, 63.209099] (5), [-29.39127, 63.209099] (4), [-29.24949, 63.209099] (5), [-29.18721, 63.209099] (5), [-29.04380, 63.209099] (5), [-28.98732, 63.209099] (5), [-28.97404, 63.209099] (4), [-28.77992, 63.209099] (5), [-28.65605, 63.209099] (5), [-28.65086, 63.209099] (5), [-28.58831, 63.209099] (4), [-28.57604, 63.209099] (5), [-28.29732, 63.209099] (4),

length of domains: 24

Total time: 0.3920 pickout: 0.0030 decision: 0.0030 get\_bound: 0.3840 add\_domain: 0.0020

Current lb:-31.492612838745117

46 neurons visited

Global ub: 63.20909881591797, batch ub: inf

Cumulative time: 3.4577624797821045

splitting decisions: [[3, 72], [4, 31], [4, 31], [4, 31], [4, 31], [4, 31], [4, 31], [4, 31]]

best\_l after optimization: 466.75115966796875 with beta sum per layer: [0.0, 0.0, 0.0, 0.2493797093629837, 11.556482315063477]

alpha/beta optimization time: 0.39594030380249023

This batch time : update\_bounds func: 0.4029 prepare: 0.0040 bound: 0.3959 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 7.2676 prepare: 0.0657 bound: 7.1609 transfer: 0.0010 finalize: 0.0230

batch bounding time: 0.402923583984375

Current worst splitting domains [lb, ub] (depth):

[-30.94421, 63.209099] (6), [-30.85146, 63.209099] (6), [-29.89647, 63.209099] (6), [-29.89061, 63.209099] (6), [-29.82014, 63.209099] (6), [-29.81422, 63.209099] (6), [-29.46419, 63.209099] (6), [-29.39127, 63.209099] (4), [-29.38085, 63.209099] (6), [-29.24949, 63.209099] (5), [-29.18721, 63.209099] (5), [-29.16188, 63.209099] (6), [-29.05634, 63.209099] (6), [-29.04380, 63.209099] (5), [-28.98732, 63.209099] (5), [-28.97404, 63.209099] (4), [-28.92068, 63.209099] (6), [-28.77992, 63.209099] (5), [-28.65605, 63.209099] (5), [-28.65086, 63.209099] (5),

length of domains: 32

Total time: 0.4149 pickout: 0.0040 decision: 0.0050 get\_bound: 0.4039 add\_domain: 0.0020

Current lb:-30.944210052490234

62 neurons visited

Global ub: 63.20909881591797, batch ub: inf

Cumulative time: 3.872652769088745

splitting decisions: [[4, 31], [3, 72], [4, 31], [4, 87], [4, 87], [4, 87], [4, 87], [4, 61]]

best\_l after optimization: 463.4593505859375 with beta sum per layer: [0.0, 0.0, 0.0, 0.93647301197052, 7.277864456176758]

alpha/beta optimization time: 0.42386651039123535

This batch time : update\_bounds func: 0.4328 prepare: 0.0040 bound: 0.4259 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 7.7005 prepare: 0.0697 bound: 7.5868 transfer: 0.0010 finalize: 0.0250

batch bounding time: 0.4338388442993164

Current worst splitting domains [lb, ub] (depth):

[-30.35400, 63.209099] (7), [-30.30550, 63.209099] (7), [-29.38085, 63.209099] (6), [-29.31907, 63.209099] (7), [-29.24949, 63.209099] (5), [-29.23215, 63.209099] (7), [-29.18721, 63.209099] (5), [-29.16188, 63.209099] (6), [-29.15280, 63.209099] (7), [-29.05634, 63.209099] (6), [-29.04380, 63.209099] (5), [-28.98732, 63.209099] (5), [-28.97404, 63.209099] (4), [-28.92549, 63.209099] (7), [-28.92068, 63.209099] (6), [-28.87885, 63.209099] (7), [-28.87646, 63.209099] (7), [-28.86424, 63.209099] (7), [-28.84718, 63.209099] (7), [-28.83303, 63.209099] (7),

length of domains: 40

Total time: 0.4428 pickout: 0.0030 decision: 0.0050 get\_bound: 0.4338 add\_domain: 0.0010

Current lb:-30.35400390625

78 neurons visited

Global ub: 63.20909881591797, batch ub: inf

Time out!!!!!!!!

Image 5 label 9 verification end, final lower bound -30.35400390625, upper bound 63.20909881591797, time: 4.355332136154175

5 -30.35400390625

Result: image 5 verification failure (with branch and bound).

Wall time: 10.202157020568848

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 5 img ID: 6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 4, correct label 4, image norm 83.07451629638672, logits tensor([-1.5976, -0.7424, -0.5628, -2.3540, 4.8149, 0.1136, -1.5458, -0.1815,

1.7611, 2.0121], device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[-1.5976, -0.7424, -0.5628, -2.3540, 4.8149, 0.1136, -1.5458, -0.1815,

1.7611, 2.0121]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[-26.0873, -28.5947, -23.5834, -30.2691, -30.2925, -24.7699, -23.7624,

-32.5792, -23.6095]], device='cuda:0') None

best\_l after optimization: 151.32089233398438 with beta sum per layer: []

alpha/beta optimization time: 5.834349155426025

initial alpha-CROWN bounds: tensor([[-15.8625, -17.1918, -15.5900, -18.7306, -19.3684, -13.9732, -15.0824,

-19.6682, -15.8538]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

##### [5:6] Tested against 8 ######

Model prediction is: tensor([[-1.5976, -0.7424, -0.5628, -2.3540, 4.8149, 0.1136, -1.5458, -0.1815,

1.7611, 2.0121]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

setting alpha for layer /20 start\_node /input.3

setting alpha for layer /20 start\_node /input.7

setting alpha for layer /20 start\_node /input.11

setting alpha for layer /20 start\_node /input.15

not setting layer /20 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /22 start\_node /input.7

setting alpha for layer /22 start\_node /input.11

setting alpha for layer /22 start\_node /input.15

not setting layer /22 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /24 start\_node /input.11

setting alpha for layer /24 start\_node /input.15

not setting layer /24 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /26 start\_node /input.15

not setting layer /26 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

not setting layer /28 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

0 /input torch.Size([1, 100])

1 /input.3 torch.Size([1, 100])

2 /input.7 torch.Size([1, 100])

3 /input.11 torch.Size([1, 100])

4 /input.15 torch.Size([1, 100])

best\_l after optimization: 19.668052673339844 with beta sum per layer: []

alpha/beta optimization time: 1.51987624168396

alpha-CROWN with fixed intermediate bounds: tensor([[-19.6681]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

-19.668052673339844

第 0 layer size = torch.Size([100]) with unstable 32 nodes

第 1 layer size = torch.Size([100]) with unstable 51 nodes

第 2 layer size = torch.Size([100]) with unstable 78 nodes

第 3 layer size = torch.Size([100]) with unstable 100 nodes

第 4 layer size = torch.Size([100]) with unstable 100 nodes

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# of total unstable neurons(不定态神经元总数量): 361

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splitting decisions: [[4, 13]]

best\_l after optimization: 37.20012664794922 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.5398327708244324]

alpha/beta optimization time: 0.36901235580444336

This batch time : update\_bounds func: 0.3760 prepare: 0.0050 bound: 0.3700 transfer: 0.0000 finalize: 0.0010

Accumulated time: update\_bounds func: 8.0765 prepare: 0.0747 bound: 7.9568 transfer: 0.0000 finalize: 0.0260

batch bounding time: 0.3759951591491699

Current worst splitting domains [lb, ub] (depth):

[-19.28759, 79.331947] (1), [-17.91253, 79.331947] (1),

length of domains: 2

Total time: 0.3790 pickout: 0.0010 decision: 0.0010 get\_bound: 0.3760 add\_domain: 0.0010

Current lb:-19.28759002685547

2 neurons visited

Global ub: 79.33194732666016, batch ub: inf

Cumulative time: 1.926788568496704

splitting decisions: [[4, 64], [4, 64]]

best\_l after optimization: 70.58253479003906 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 1.9100773334503174]

alpha/beta optimization time: 0.36276745796203613

This batch time : update\_bounds func: 0.3678 prepare: 0.0020 bound: 0.3638 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 8.4442 prepare: 0.0767 bound: 8.3205 transfer: 0.0010 finalize: 0.0270

batch bounding time: 0.3677551746368408

Current worst splitting domains [lb, ub] (depth):

[-18.88631, 79.331947] (2), [-17.62530, 79.331947] (2), [-17.27613, 79.331947] (2), [-16.79478, 79.331947] (2),

length of domains: 4

Total time: 0.3727 pickout: 0.0010 decision: 0.0030 get\_bound: 0.3678 add\_domain: 0.0010

Current lb:-18.886314392089844

6 neurons visited

Global ub: 79.33194732666016, batch ub: inf

Cumulative time: 2.3004980087280273

splitting decisions: [[4, 54], [4, 54], [4, 54], [4, 54]]

best\_l after optimization: 134.45538330078125 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 6.380160331726074]

alpha/beta optimization time: 0.3899567127227783

This batch time : update\_bounds func: 0.3969 prepare: 0.0050 bound: 0.3900 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 8.8411 prepare: 0.0817 bound: 8.7105 transfer: 0.0010 finalize: 0.0280

batch bounding time: 0.3979353904724121

Current worst splitting domains [lb, ub] (depth):

[-18.55884, 79.331947] (3), [-17.50156, 79.331947] (3), [-17.20585, 79.331947] (3), [-17.00843, 79.331947] (3), [-16.45130, 79.331947] (3), [-16.43619, 79.331947] (3), [-15.87786, 79.331947] (3), [-15.41536, 79.331947] (3),

length of domains: 8

Total time: 0.4019 pickout: 0.0010 decision: 0.0020 get\_bound: 0.3979 add\_domain: 0.0010

Current lb:-18.5588436126709

14 neurons visited

Global ub: 79.33194732666016, batch ub: inf

Cumulative time: 2.703420877456665

splitting decisions: [[4, 89], [4, 89], [4, 89], [4, 89], [4, 89], [4, 89], [4, 89], [4, 89]]

best\_l after optimization: 255.44825744628906 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 16.47698211669922]

alpha/beta optimization time: 0.382976770401001

This batch time : update\_bounds func: 0.3890 prepare: 0.0040 bound: 0.3830 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 9.2301 prepare: 0.0856 bound: 9.0935 transfer: 0.0010 finalize: 0.0290

batch bounding time: 0.3899848461151123

Current worst splitting domains [lb, ub] (depth):

[-18.25859, 79.331947] (4), [-17.10005, 79.331947] (4), [-16.84949, 79.331947] (4), [-16.66643, 79.331947] (4), [-16.64463, 79.331947] (4), [-16.05919, 79.331947] (4), [-16.02812, 79.331947] (4), [-16.00509, 79.331947] (4), [-15.79701, 79.331947] (4), [-15.78467, 79.331947] (4), [-15.38563, 79.331947] (4), [-15.29379, 79.331947] (4), [-15.24713, 79.331947] (4), [-14.92300, 79.331947] (4), [-14.90246, 79.331947] (4), [-14.50297, 79.331947] (4),

length of domains: 16

Total time: 0.3969 pickout: 0.0020 decision: 0.0040 get\_bound: 0.3900 add\_domain: 0.0010

Current lb:-18.258586883544922

30 neurons visited

Global ub: 79.33194732666016, batch ub: inf

Cumulative time: 3.102353096008301

splitting decisions: [[4, 69], [4, 69], [4, 69], [4, 69], [4, 69], [4, 69], [4, 69], [4, 69]]

best\_l after optimization: 253.74932861328125 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 19.51340103149414]

alpha/beta optimization time: 0.3730006217956543

This batch time : update\_bounds func: 0.3800 prepare: 0.0040 bound: 0.3730 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 9.6101 prepare: 0.0896 bound: 9.4665 transfer: 0.0010 finalize: 0.0310

batch bounding time: 0.37998318672180176

Current worst splitting domains [lb, ub] (depth):

[-17.93480, 79.331947] (5), [-16.89339, 79.331947] (5), [-16.65575, 79.331947] (5), [-16.55066, 79.331947] (5), [-16.37143, 79.331947] (5), [-16.14088, 79.331947] (5), [-16.13498, 79.331947] (5), [-15.81139, 79.331947] (5), [-15.79701, 79.331947] (4), [-15.78467, 79.331947] (4), [-15.67044, 79.331947] (5), [-15.53106, 79.331947] (5), [-15.46912, 79.331947] (5), [-15.38563, 79.331947] (4), [-15.29379, 79.331947] (4), [-15.24713, 79.331947] (4), [-15.20555, 79.331947] (5), [-15.10936, 79.331947] (5), [-15.05797, 79.331947] (5), [-14.92300, 79.331947] (4),

length of domains: 24

Total time: 0.3870 pickout: 0.0030 decision: 0.0020 get\_bound: 0.3810 add\_domain: 0.0010

Current lb:-17.934804916381836

46 neurons visited

Global ub: 79.33194732666016, batch ub: inf

Cumulative time: 3.4903149604797363

splitting decisions: [[4, 82], [4, 82], [4, 82], [4, 82], [4, 82], [4, 82], [4, 82], [4, 82]]

best\_l after optimization: 253.22686767578125 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 19.3476505279541]

alpha/beta optimization time: 0.3596808910369873

This batch time : update\_bounds func: 0.3667 prepare: 0.0040 bound: 0.3597 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 9.9768 prepare: 0.0936 bound: 9.8262 transfer: 0.0010 finalize: 0.0330

batch bounding time: 0.36666274070739746

Current worst splitting domains [lb, ub] (depth):

[-17.63144, 79.331947] (6), [-16.79069, 79.331947] (6), [-16.51171, 79.331947] (6), [-16.28215, 79.331947] (6), [-16.22462, 79.331947] (6), [-16.07800, 79.331947] (6), [-15.79701, 79.331947] (4), [-15.78467, 79.331947] (4), [-15.73976, 79.331947] (6), [-15.73115, 79.331947] (6), [-15.70032, 79.331947] (6), [-15.67044, 79.331947] (5), [-15.54664, 79.331947] (6), [-15.53106, 79.331947] (5), [-15.52215, 79.331947] (6), [-15.46912, 79.331947] (5), [-15.40231, 79.331947] (6), [-15.38563, 79.331947] (4), [-15.29379, 79.331947] (4), [-15.24713, 79.331947] (4),

length of domains: 32

Total time: 0.3736 pickout: 0.0030 decision: 0.0020 get\_bound: 0.3667 add\_domain: 0.0020

Current lb:-17.631441116333008

62 neurons visited

Global ub: 79.33194732666016, batch ub: inf

Cumulative time: 3.864957094192505

splitting decisions: [[4, 44], [4, 44], [4, 44], [4, 44], [4, 44], [4, 44], [4, 69], [4, 69]]

best\_l after optimization: 252.43301391601562 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 16.318099975585938]

alpha/beta optimization time: 0.3759951591491699

This batch time : update\_bounds func: 0.3830 prepare: 0.0040 bound: 0.3760 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 10.3598 prepare: 0.0976 bound: 10.2021 transfer: 0.0010 finalize: 0.0350

batch bounding time: 0.3829765319824219

Current worst splitting domains [lb, ub] (depth):

[-17.13279, 79.331947] (7), [-17.11442, 79.331947] (7), [-16.21863, 79.331947] (7), [-16.20850, 79.331947] (7), [-15.98577, 79.331947] (7), [-15.97882, 79.331947] (7), [-15.81034, 79.331947] (7), [-15.75763, 79.331947] (7), [-15.74938, 79.331947] (7), [-15.73976, 79.331947] (6), [-15.73115, 79.331947] (6), [-15.72575, 79.331947] (7), [-15.70032, 79.331947] (6), [-15.67044, 79.331947] (5), [-15.60849, 79.331947] (7), [-15.54664, 79.331947] (6), [-15.53106, 79.331947] (5), [-15.52215, 79.331947] (6), [-15.51779, 79.331947] (7), [-15.47206, 79.331947] (5),

length of domains: 40

Total time: 0.3920 pickout: 0.0030 decision: 0.0040 get\_bound: 0.3830 add\_domain: 0.0020

Current lb:-17.13279151916504

78 neurons visited

Global ub: 79.33194732666016, batch ub: inf

Time out!!!!!!!!

Image 6 label 8 verification end, final lower bound -17.13279151916504, upper bound 79.33194732666016, time: 4.296802282333374

6 -17.13279151916504

Result: image 6 verification failure (with branch and bound).

Wall time: 10.238861799240112

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 6 img ID: 7 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 9, correct label 9, image norm 82.59607696533203, logits tensor([-1.5174e+00, -2.0436e+00, -1.7616e+00, 1.6516e+00, 8.5354e-01,

-2.9163e-03, -4.1906e+00, 5.4028e-01, 7.4383e-01, 6.1605e+00],

device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[-1.5174e+00, -2.0436e+00, -1.7616e+00, 1.6516e+00, 8.5354e-01,

-2.9163e-03, -4.1906e+00, 5.4028e-01, 7.4383e-01, 6.1605e+00]],

device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[-50.3112, -52.4911, -59.5219, -50.7275, -36.3782, -48.3131, -54.5636,

-41.1229, -51.3289]], device='cuda:0') None

best\_l after optimization: 248.64524841308594 with beta sum per layer: []

alpha/beta optimization time: 5.853922605514526

initial alpha-CROWN bounds: tensor([[-26.8971, -30.9152, -34.5254, -28.4662, -21.0301, -27.0514, -27.8475,

-23.7006, -28.2117]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

##### [6:7] Tested against 2 ######

Model prediction is: tensor([[-1.5174e+00, -2.0436e+00, -1.7616e+00, 1.6516e+00, 8.5354e-01,

-2.9163e-03, -4.1906e+00, 5.4028e-01, 7.4383e-01, 6.1605e+00]],

device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

setting alpha for layer /20 start\_node /input.3

setting alpha for layer /20 start\_node /input.7

setting alpha for layer /20 start\_node /input.11

setting alpha for layer /20 start\_node /input.15

not setting layer /20 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /22 start\_node /input.7

setting alpha for layer /22 start\_node /input.11

setting alpha for layer /22 start\_node /input.15

not setting layer /22 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /24 start\_node /input.11

setting alpha for layer /24 start\_node /input.15

not setting layer /24 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /26 start\_node /input.15

not setting layer /26 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

not setting layer /28 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

0 /input torch.Size([1, 100])

1 /input.3 torch.Size([1, 100])

2 /input.7 torch.Size([1, 100])

3 /input.11 torch.Size([1, 100])

4 /input.15 torch.Size([1, 100])

best\_l after optimization: 34.523948669433594 with beta sum per layer: []

alpha/beta optimization time: 1.5167183876037598

alpha-CROWN with fixed intermediate bounds: tensor([[-34.5239]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

-34.523948669433594

第 0 layer size = torch.Size([100]) with unstable 41 nodes

第 1 layer size = torch.Size([100]) with unstable 81 nodes

第 2 layer size = torch.Size([100]) with unstable 100 nodes

第 3 layer size = torch.Size([100]) with unstable 100 nodes

第 4 layer size = torch.Size([100]) with unstable 100 nodes

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# of total unstable neurons(不定态神经元总数量): 422

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splitting decisions: [[4, 1]]

best\_l after optimization: 64.55699157714844 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.6513972878456116]

alpha/beta optimization time: 0.3726208209991455

This batch time : update\_bounds func: 0.3756 prepare: 0.0020 bound: 0.3726 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 10.7354 prepare: 0.0996 bound: 10.5748 transfer: 0.0010 finalize: 0.0350

batch bounding time: 0.3766462802886963

Current worst splitting domains [lb, ub] (depth):

[-33.94632, 64.476051] (1), [-30.61068, 64.476051] (1),

length of domains: 2

Total time: 0.3816 pickout: 0.0020 decision: 0.0030 get\_bound: 0.3766 add\_domain: 0.0000

Current lb:-33.94631576538086

2 neurons visited

Global ub: 64.4760513305664, batch ub: inf

Cumulative time: 1.9312269687652588

splitting decisions: [[4, 21], [4, 21]]

best\_l after optimization: 120.22540283203125 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 2.034317970275879]

alpha/beta optimization time: 0.3640263080596924

This batch time : update\_bounds func: 0.3680 prepare: 0.0030 bound: 0.3640 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 11.1034 prepare: 0.1026 bound: 10.9388 transfer: 0.0010 finalize: 0.0350

batch bounding time: 0.36901307106018066

Current worst splitting domains [lb, ub] (depth):

[-33.41204, 64.476051] (2), [-29.94351, 64.476051] (2), [-29.62714, 64.476051] (2), [-27.24271, 64.476051] (2),

length of domains: 4

Total time: 0.3730 pickout: 0.0010 decision: 0.0030 get\_bound: 0.3690 add\_domain: 0.0000

Current lb:-33.41203689575195

6 neurons visited

Global ub: 64.4760513305664, batch ub: inf

Cumulative time: 2.305227041244507

splitting decisions: [[4, 72], [4, 72], [4, 72], [4, 72]]

best\_l after optimization: 227.289306640625 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 5.59809684753418]

alpha/beta optimization time: 0.3809804916381836

This batch time : update\_bounds func: 0.3870 prepare: 0.0040 bound: 0.3810 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 11.4904 prepare: 0.1066 bound: 11.3198 transfer: 0.0010 finalize: 0.0360

batch bounding time: 0.38796186447143555

Current worst splitting domains [lb, ub] (depth):

[-32.56546, 64.476051] (3), [-30.94352, 64.476051] (3), [-29.11716, 64.476051] (3), [-28.89373, 64.476051] (3), [-27.28386, 64.476051] (3), [-27.16874, 64.476051] (3), [-26.49351, 64.476051] (3), [-24.82331, 64.476051] (3),

length of domains: 8

Total time: 0.3920 pickout: 0.0020 decision: 0.0020 get\_bound: 0.3880 add\_domain: 0.0000

Current lb:-32.56545639038086

14 neurons visited

Global ub: 64.4760513305664, batch ub: inf

Cumulative time: 2.6981751918792725

splitting decisions: [[4, 23], [4, 23], [4, 23], [4, 23], [4, 23], [4, 23], [4, 23], [4, 23]]

best\_l after optimization: 431.5682678222656 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 14.575346946716309]

alpha/beta optimization time: 0.3759944438934326

This batch time : update\_bounds func: 0.3840 prepare: 0.0040 bound: 0.3770 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 11.8743 prepare: 0.1106 bound: 11.6968 transfer: 0.0010 finalize: 0.0380

batch bounding time: 0.3839726448059082

Current worst splitting domains [lb, ub] (depth):

[-32.00429, 64.476051] (4), [-30.32777, 64.476051] (4), [-29.70337, 64.476051] (4), [-28.23670, 64.476051] (4), [-28.23511, 64.476051] (4), [-28.13900, 64.476051] (4), [-27.07051, 64.476051] (4), [-26.75191, 64.476051] (4), [-26.64159, 64.476051] (4), [-26.32930, 64.476051] (4), [-25.65240, 64.476051] (4), [-25.31517, 64.476051] (4), [-25.05051, 64.476051] (4), [-24.88947, 64.476051] (4), [-24.16729, 64.476051] (4), [-23.05388, 64.476051] (4),

length of domains: 16

Total time: 0.3920 pickout: 0.0020 decision: 0.0040 get\_bound: 0.3850 add\_domain: 0.0010

Current lb:-32.00428771972656

30 neurons visited

Global ub: 64.4760513305664, batch ub: inf

Cumulative time: 3.0911242961883545

splitting decisions: [[4, 8], [4, 8], [4, 8], [4, 8], [4, 8], [4, 8], [4, 8], [4, 8]]

best\_l after optimization: 437.3499755859375 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 16.328035354614258]

alpha/beta optimization time: 0.37636351585388184

This batch time : update\_bounds func: 0.3834 prepare: 0.0030 bound: 0.3774 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 12.2577 prepare: 0.1136 bound: 12.0741 transfer: 0.0010 finalize: 0.0400

batch bounding time: 0.38337135314941406

Current worst splitting domains [lb, ub] (depth):

[-31.38698, 64.476051] (5), [-29.57269, 64.476051] (5), [-29.30737, 64.476051] (5), [-28.93887, 64.476051] (5), [-28.25664, 64.476051] (5), [-27.63426, 64.476051] (5), [-27.47122, 64.476051] (5), [-27.36300, 64.476051] (5), [-27.13729, 64.476051] (5), [-26.64159, 64.476051] (4), [-26.32930, 64.476051] (4), [-26.27142, 64.476051] (5), [-26.21298, 64.476051] (5), [-26.20259, 64.476051] (5), [-26.06601, 64.476051] (5), [-25.94464, 64.476051] (5), [-25.65240, 64.476051] (4), [-25.31517, 64.476051] (4), [-25.22456, 64.476051] (5), [-25.05051, 64.476051] (4),

length of domains: 24

Total time: 0.3923 pickout: 0.0020 decision: 0.0050 get\_bound: 0.3834 add\_domain: 0.0020

Current lb:-31.38697624206543

46 neurons visited

Global ub: 64.4760513305664, batch ub: inf

Cumulative time: 3.4844412803649902

splitting decisions: [[4, 11], [4, 11], [4, 11], [4, 11], [4, 11], [4, 11], [4, 11], [4, 11]]

best\_l after optimization: 442.35455322265625 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 14.043636322021484]

alpha/beta optimization time: 0.35764384269714355

This batch time : update\_bounds func: 0.3657 prepare: 0.0040 bound: 0.3576 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 12.6234 prepare: 0.1176 bound: 12.4318 transfer: 0.0010 finalize: 0.0419

batch bounding time: 0.36565327644348145

Current worst splitting domains [lb, ub] (depth):

[-30.48368, 64.476051] (6), [-30.28904, 64.476051] (6), [-28.56264, 64.476051] (6), [-28.48797, 64.476051] (6), [-28.19463, 64.476051] (6), [-27.95500, 64.476051] (6), [-27.93343, 64.476051] (6), [-27.80269, 64.476051] (6), [-27.13729, 64.476051] (5), [-27.10417, 64.476051] (6), [-27.07113, 64.476051] (6), [-26.86672, 64.476051] (6), [-26.64159, 64.476051] (4), [-26.50441, 64.476051] (6), [-26.32930, 64.476051] (4), [-26.31246, 64.476051] (6), [-26.30234, 64.476051] (6), [-26.29932, 64.476051] (6), [-26.27142, 64.476051] (5), [-26.21298, 64.476051] (5),

length of domains: 32

Total time: 0.3736 pickout: 0.0020 decision: 0.0040 get\_bound: 0.3657 add\_domain: 0.0020

Current lb:-30.483680725097656

62 neurons visited

Global ub: 64.4760513305664, batch ub: inf

Cumulative time: 3.859041929244995

splitting decisions: [[4, 86], [4, 86], [4, 86], [4, 86], [4, 86], [4, 86], [3, 57], [3, 57]]

best\_l after optimization: 443.2215576171875 with beta sum per layer: [0.0, 0.0, 0.0, 0.36882105469703674, 13.242822647094727]

alpha/beta optimization time: 0.3779895305633545

This batch time : update\_bounds func: 0.3850 prepare: 0.0040 bound: 0.3780 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 13.0083 prepare: 0.1216 bound: 12.8098 transfer: 0.0010 finalize: 0.0439

batch bounding time: 0.38496947288513184

Current worst splitting domains [lb, ub] (depth):

[-29.75048, 64.476051] (7), [-29.51950, 64.476051] (7), [-29.23537, 64.476051] (7), [-29.09604, 64.476051] (7), [-27.76698, 64.476051] (7), [-27.68220, 64.476051] (7), [-27.47651, 64.476051] (7), [-27.40018, 64.476051] (7), [-27.37982, 64.476051] (7), [-27.34093, 64.476051] (7), [-27.32958, 64.476051] (7), [-27.18107, 64.476051] (7), [-27.13729, 64.476051] (5), [-27.10417, 64.476051] (6), [-27.07113, 64.476051] (6), [-26.87147, 64.476051] (7), [-26.86672, 64.476051] (6), [-26.72924, 64.476051] (7), [-26.64159, 64.476051] (4), [-26.50441, 64.476051] (6),

length of domains: 40

Total time: 0.3929 pickout: 0.0030 decision: 0.0030 get\_bound: 0.3860 add\_domain: 0.0010

Current lb:-29.75048065185547

78 neurons visited

Global ub: 64.4760513305664, batch ub: inf

Time out!!!!!!!!

Image 7 label 2 verification end, final lower bound -29.75048065185547, upper bound 64.4760513305664, time: 4.290856838226318

7 -29.75048065185547

Result: image 7 verification failure (with branch and bound).

Wall time: 10.251465559005737

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 7 img ID: 8 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 5, correct label 5, image norm 120.5254898071289, logits tensor([ 0.7643, -1.3512, -2.0357, -0.1822, -1.6505, 4.3357, 3.0884, -1.6486,

0.5165, -0.8307], device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[ 0.7643, -1.3512, -2.0357, -0.1822, -1.6505, 4.3357, 3.0884, -1.6486,

0.5165, -0.8307]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[-42.6692, -52.5425, -58.9182, -42.4024, -55.1606, -43.0846, -51.2213,

-52.1039, -55.6594]], device='cuda:0') None

best\_l after optimization: 275.42901611328125 with beta sum per layer: []

alpha/beta optimization time: 5.883232593536377

initial alpha-CROWN bounds: tensor([[-23.7620, -32.4844, -34.9677, -26.3610, -33.9083, -24.6528, -31.3695,

-32.5286, -35.3947]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

##### [7:8] Tested against 9 ######

Model prediction is: tensor([[ 0.7643, -1.3512, -2.0357, -0.1822, -1.6505, 4.3357, 3.0884, -1.6486,

0.5165, -0.8307]], device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

setting alpha for layer /20 start\_node /input.3

setting alpha for layer /20 start\_node /input.7

setting alpha for layer /20 start\_node /input.11

setting alpha for layer /20 start\_node /input.15

not setting layer /20 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /22 start\_node /input.7

setting alpha for layer /22 start\_node /input.11

setting alpha for layer /22 start\_node /input.15

not setting layer /22 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /24 start\_node /input.11

setting alpha for layer /24 start\_node /input.15

not setting layer /24 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /26 start\_node /input.15

not setting layer /26 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

not setting layer /28 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

0 /input torch.Size([1, 100])

1 /input.3 torch.Size([1, 100])

2 /input.7 torch.Size([1, 100])

3 /input.11 torch.Size([1, 100])

4 /input.15 torch.Size([1, 100])

best\_l after optimization: 35.39437484741211 with beta sum per layer: []

alpha/beta optimization time: 1.465064287185669

alpha-CROWN with fixed intermediate bounds: tensor([[-35.3944]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

-35.39437484741211

第 0 layer size = torch.Size([100]) with unstable 29 nodes

第 1 layer size = torch.Size([100]) with unstable 75 nodes

第 2 layer size = torch.Size([100]) with unstable 100 nodes

第 3 layer size = torch.Size([100]) with unstable 100 nodes

第 4 layer size = torch.Size([100]) with unstable 100 nodes

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# of total unstable neurons(不定态神经元总数量): 404

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splitting decisions: [[4, 78]]

best\_l after optimization: 67.49420166015625 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.437787801027298]

alpha/beta optimization time: 0.37086057662963867

This batch time : update\_bounds func: 0.3739 prepare: 0.0020 bound: 0.3709 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 13.3822 prepare: 0.1235 bound: 13.1806 transfer: 0.0010 finalize: 0.0439

batch bounding time: 0.3738546371459961

Current worst splitting domains [lb, ub] (depth):

[-34.58923, 63.605625] (1), [-32.90496, 63.605625] (1),

length of domains: 2

Total time: 0.3798 pickout: 0.0010 decision: 0.0040 get\_bound: 0.3749 add\_domain: 0.0000

Current lb:-34.5892333984375

2 neurons visited

Global ub: 63.60562515258789, batch ub: inf

Cumulative time: 1.8748266696929932

splitting decisions: [[4, 44], [4, 44]]

best\_l after optimization: 129.35812377929688 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.6079138517379761]

alpha/beta optimization time: 0.386965274810791

This batch time : update\_bounds func: 0.3920 prepare: 0.0030 bound: 0.3870 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 13.7741 prepare: 0.1265 bound: 13.5676 transfer: 0.0010 finalize: 0.0449

batch bounding time: 0.391951322555542

Current worst splitting domains [lb, ub] (depth):

[-33.41069, 63.605625] (2), [-32.97772, 63.605625] (2), [-31.73568, 63.605625] (2), [-31.23403, 63.605625] (2),

length of domains: 4

Total time: 0.3999 pickout: 0.0020 decision: 0.0050 get\_bound: 0.3929 add\_domain: 0.0000

Current lb:-33.41069412231445

6 neurons visited

Global ub: 63.60562515258789, batch ub: inf

Cumulative time: 2.276749610900879

splitting decisions: [[4, 66], [4, 66], [4, 66], [4, 66]]

best\_l after optimization: 243.8265380859375 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 3.7267253398895264]

alpha/beta optimization time: 0.38198041915893555

This batch time : update\_bounds func: 0.3890 prepare: 0.0040 bound: 0.3820 transfer: 0.0020 finalize: 0.0010

Accumulated time: update\_bounds func: 14.1631 prepare: 0.1305 bound: 13.9496 transfer: 0.0020 finalize: 0.0459

batch bounding time: 0.3899562358856201

Current worst splitting domains [lb, ub] (depth):

[-32.83566, 63.605625] (3), [-32.51377, 63.605625] (3), [-31.08816, 63.605625] (3), [-30.63335, 63.605625] (3), [-29.98893, 63.605625] (3), [-29.51292, 63.605625] (3), [-28.87794, 63.605625] (3), [-28.37582, 63.605625] (3),

length of domains: 8

Total time: 0.3959 pickout: 0.0010 decision: 0.0040 get\_bound: 0.3900 add\_domain: 0.0010

Current lb:-32.835655212402344

14 neurons visited

Global ub: 63.60562515258789, batch ub: inf

Cumulative time: 2.6736881732940674

splitting decisions: [[4, 38], [4, 38], [4, 38], [4, 38], [4, 38], [4, 38], [4, 38], [4, 38]]

best\_l after optimization: 464.8771057128906 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 9.503913879394531]

alpha/beta optimization time: 0.3779890537261963

This batch time : update\_bounds func: 0.3850 prepare: 0.0040 bound: 0.3780 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 14.5481 prepare: 0.1345 bound: 14.3276 transfer: 0.0010 finalize: 0.0479

batch bounding time: 0.38497066497802734

Current worst splitting domains [lb, ub] (depth):

[-32.06139, 63.605625] (4), [-31.76446, 63.605625] (4), [-30.82367, 63.605625] (4), [-30.44298, 63.605625] (4), [-30.32353, 63.605625] (4), [-29.99384, 63.605625] (4), [-28.78935, 63.605625] (4), [-28.77325, 63.605625] (4), [-28.49706, 63.605625] (4), [-28.39109, 63.605625] (4), [-28.25060, 63.605625] (4), [-27.87665, 63.605625] (4), [-27.87585, 63.605625] (4), [-27.44844, 63.605625] (4), [-27.12342, 63.605625] (4), [-26.44153, 63.605625] (4),

length of domains: 16

Total time: 0.3920 pickout: 0.0020 decision: 0.0030 get\_bound: 0.3860 add\_domain: 0.0010

Current lb:-32.0613899230957

30 neurons visited

Global ub: 63.60562515258789, batch ub: inf

Cumulative time: 3.0666377544403076

splitting decisions: [[4, 71], [4, 71], [4, 71], [4, 71], [4, 71], [4, 71], [4, 71], [4, 71]]

best\_l after optimization: 465.73699951171875 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 7.947667598724365]

alpha/beta optimization time: 0.37400054931640625

This batch time : update\_bounds func: 0.3810 prepare: 0.0040 bound: 0.3740 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 14.9290 prepare: 0.1385 bound: 14.7016 transfer: 0.0010 finalize: 0.0499

batch bounding time: 0.3809800148010254

Current worst splitting domains [lb, ub] (depth):

[-31.22081, 63.605625] (5), [-30.97714, 63.605625] (5), [-30.42433, 63.605625] (5), [-29.94880, 63.605625] (5), [-29.76006, 63.605625] (5), [-29.61675, 63.605625] (5), [-29.37940, 63.605625] (5), [-29.33292, 63.605625] (5), [-29.20996, 63.605625] (5), [-28.78965, 63.605625] (5), [-28.77274, 63.605625] (5), [-28.49706, 63.605625] (4), [-28.39109, 63.605625] (4), [-28.25060, 63.605625] (4), [-28.17334, 63.605625] (5), [-27.90813, 63.605625] (5), [-27.87665, 63.605625] (4), [-27.87585, 63.605625] (4), [-27.71655, 63.605625] (5), [-27.44844, 63.605625] (4),

length of domains: 24

Total time: 0.3890 pickout: 0.0020 decision: 0.0040 get\_bound: 0.3820 add\_domain: 0.0010

Current lb:-31.220813751220703

46 neurons visited

Global ub: 63.60562515258789, batch ub: inf

Cumulative time: 3.4575912952423096

splitting decisions: [[4, 33], [4, 33], [4, 33], [4, 33], [4, 33], [4, 33], [4, 33], [4, 33]]

best\_l after optimization: 452.5462646484375 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 9.763423919677734]

alpha/beta optimization time: 0.38499879837036133

This batch time : update\_bounds func: 0.3950 prepare: 0.0050 bound: 0.3860 transfer: 0.0020 finalize: 0.0020

Accumulated time: update\_bounds func: 15.3240 prepare: 0.1435 bound: 15.0876 transfer: 0.0020 finalize: 0.0519

batch bounding time: 0.3949716091156006

Current worst splitting domains [lb, ub] (depth):

[-30.69759, 63.605625] (6), [-30.40984, 63.605625] (6), [-29.79871, 63.605625] (6), [-29.31603, 63.605625] (6), [-29.20996, 63.605625] (5), [-29.11717, 63.605625] (6), [-29.09739, 63.605625] (6), [-28.78965, 63.605625] (5), [-28.77274, 63.605625] (5), [-28.72199, 63.605625] (6), [-28.61462, 63.605625] (6), [-28.49706, 63.605625] (4), [-28.39109, 63.605625] (4), [-28.25060, 63.605625] (4), [-28.17334, 63.605625] (5), [-27.90813, 63.605625] (5), [-27.87665, 63.605625] (4), [-27.87585, 63.605625] (4), [-27.78193, 63.605625] (6), [-27.77098, 63.605625] (6),

length of domains: 32

Total time: 0.4049 pickout: 0.0040 decision: 0.0040 get\_bound: 0.3950 add\_domain: 0.0020

Current lb:-30.697593688964844

62 neurons visited

Global ub: 63.60562515258789, batch ub: inf

Cumulative time: 3.8635060787200928

splitting decisions: [[4, 90], [4, 90], [4, 90], [4, 90], [4, 33], [4, 90], [4, 90], [4, 33]]

best\_l after optimization: 451.1412658691406 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 9.022809982299805]

alpha/beta optimization time: 0.3871912956237793

This batch time : update\_bounds func: 0.3942 prepare: 0.0040 bound: 0.3872 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 15.7182 prepare: 0.1475 bound: 15.4747 transfer: 0.0010 finalize: 0.0539

batch bounding time: 0.39417243003845215

Current worst splitting domains [lb, ub] (depth):

[-30.06997, 63.605625] (7), [-29.78023, 63.605625] (7), [-29.10007, 63.605625] (7), [-28.77274, 63.605625] (5), [-28.72199, 63.605625] (6), [-28.71247, 63.605625] (7), [-28.70982, 63.605625] (6), [-28.62337, 63.605625] (7), [-28.61462, 63.605625] (6), [-28.60633, 63.605625] (7), [-28.52803, 63.605625] (7), [-28.49706, 63.605625] (4), [-28.48195, 63.605625] (7), [-28.39109, 63.605625] (4), [-28.25060, 63.605625] (4), [-28.17334, 63.605625] (5), [-28.08605, 63.605625] (6), [-27.96148, 63.605625] (7), [-27.90813, 63.605625] (5), [-27.87665, 63.605625] (4),

length of domains: 40

Total time: 0.4022 pickout: 0.0020 decision: 0.0040 get\_bound: 0.3952 add\_domain: 0.0010

Current lb:-30.069971084594727

78 neurons visited

Global ub: 63.60562515258789, batch ub: inf

Time out!!!!!!!!

Image 8 label 9 verification end, final lower bound -30.069971084594727, upper bound 63.60562515258789, time: 4.311533451080322

8 -30.069971084594727

Result: image 8 verification failure (with branch and bound).

Wall time: 10.297507047653198

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% idx: 8 img ID: 9 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

predicted label 9, correct label 9, image norm 122.94117736816406, logits tensor([-1.6307e+00, -1.3142e+00, -2.0152e+00, -7.5960e-03, 2.4411e+00,

-6.7647e-01, -4.0144e+00, 1.0917e+00, -2.4220e-01, 7.7717e+00],

device='cuda:0', grad\_fn=<SelectBackward0>)

Model prediction is: tensor([[-1.6307e+00, -1.3142e+00, -2.0152e+00, -7.5960e-03, 2.4411e+00,

-6.7647e-01, -4.0144e+00, 1.0917e+00, -2.4220e-01, 7.7717e+00]],

device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

initial CROWN bounds: tensor([[-33.5607, -33.9962, -38.6643, -32.2348, -25.1611, -30.0958, -36.0461,

-29.4656, -33.6112]], device='cuda:0') None

best\_l after optimization: 134.45924377441406 with beta sum per layer: []

alpha/beta optimization time: 5.9705164432525635

initial alpha-CROWN bounds: tensor([[-14.9086, -15.9242, -18.3287, -15.0857, -11.8783, -14.4171, -15.0131,

-13.3890, -15.5144]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

##### [8:9] Tested against 2 ######

Model prediction is: tensor([[-1.6307e+00, -1.3142e+00, -2.0152e+00, -7.5960e-03, 2.4411e+00,

-6.7647e-01, -4.0144e+00, 1.0917e+00, -2.4220e-01, 7.7717e+00]],

device='cuda:0', grad\_fn=<AddBackward0>)

alpha-CROWN optimizable variables initialized.

setting alpha for layer /20 start\_node /input.3

setting alpha for layer /20 start\_node /input.7

setting alpha for layer /20 start\_node /input.11

setting alpha for layer /20 start\_node /input.15

not setting layer /20 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /22 start\_node /input.7

setting alpha for layer /22 start\_node /input.11

setting alpha for layer /22 start\_node /input.15

not setting layer /22 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /24 start\_node /input.11

setting alpha for layer /24 start\_node /input.15

not setting layer /24 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

setting alpha for layer /26 start\_node /input.15

not setting layer /26 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

not setting layer /28 start\_node /29 because shape mismatch (torch.Size([2, 1, 1, 100]) != torch.Size([2, 9, 1, 100]))

0 /input torch.Size([1, 100])

1 /input.3 torch.Size([1, 100])

2 /input.7 torch.Size([1, 100])

3 /input.11 torch.Size([1, 100])

4 /input.15 torch.Size([1, 100])

best\_l after optimization: 18.32622718811035 with beta sum per layer: []

alpha/beta optimization time: 1.6007168292999268

alpha-CROWN with fixed intermediate bounds: tensor([[-18.3262]], device='cuda:0', grad\_fn=<AsStridedBackward0>) None

-18.32622718811035

第 0 layer size = torch.Size([100]) with unstable 29 nodes

第 1 layer size = torch.Size([100]) with unstable 51 nodes

第 2 layer size = torch.Size([100]) with unstable 88 nodes

第 3 layer size = torch.Size([100]) with unstable 100 nodes

第 4 layer size = torch.Size([100]) with unstable 100 nodes

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# of total unstable neurons(不定态神经元总数量): 368

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splitting decisions: [[4, 1]]

best\_l after optimization: 34.61310577392578 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 0.41060397028923035]

alpha/beta optimization time: 0.3898186683654785

This batch time : update\_bounds func: 0.3938 prepare: 0.0030 bound: 0.3898 transfer: 0.0010 finalize: 0.0000

Accumulated time: update\_bounds func: 16.1120 prepare: 0.1505 bound: 15.8646 transfer: 0.0010 finalize: 0.0539

batch bounding time: 0.39484286308288574

Current worst splitting domains [lb, ub] (depth):

[-17.91185, 80.673775] (1), [-16.70126, 80.673775] (1),

length of domains: 2

Total time: 0.3988 pickout: 0.0010 decision: 0.0030 get\_bound: 0.3948 add\_domain: 0.0000

Current lb:-17.91185188293457

2 neurons visited

Global ub: 80.67377471923828, batch ub: inf

Cumulative time: 2.028437614440918

splitting decisions: [[4, 21], [4, 21]]

best\_l after optimization: 64.50080871582031 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 1.729874849319458]

alpha/beta optimization time: 0.3889601230621338

This batch time : update\_bounds func: 0.3929 prepare: 0.0020 bound: 0.3890 transfer: 0.0010 finalize: 0.0010

Accumulated time: update\_bounds func: 16.5049 prepare: 0.1525 bound: 16.2535 transfer: 0.0010 finalize: 0.0549

batch bounding time: 0.39294886589050293

Current worst splitting domains [lb, ub] (depth):

[-17.55648, 80.673775] (2), [-16.20294, 80.673775] (2), [-15.86013, 80.673775] (2), [-14.88127, 80.673775] (2),

length of domains: 4

Total time: 0.4019 pickout: 0.0020 decision: 0.0070 get\_bound: 0.3929 add\_domain: 0.0000

Current lb:-17.556476593017578

6 neurons visited

Global ub: 80.67377471923828, batch ub: inf

Cumulative time: 2.4313595294952393

splitting decisions: [[4, 72], [4, 72], [4, 72], [4, 72]]

best\_l after optimization: 122.15568542480469 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 4.991086006164551]

alpha/beta optimization time: 0.365023136138916

This batch time : update\_bounds func: 0.3710 prepare: 0.0030 bound: 0.3660 transfer: 0.0000 finalize: 0.0020

Accumulated time: update\_bounds func: 16.8759 prepare: 0.1555 bound: 16.6195 transfer: 0.0000 finalize: 0.0569

batch bounding time: 0.37100744247436523

Current worst splitting domains [lb, ub] (depth):

[-17.11901, 80.673775] (3), [-16.41096, 80.673775] (3), [-15.72831, 80.673775] (3), [-15.41730, 80.673775] (3), [-14.98207, 80.673775] (3), [-14.48324, 80.673775] (3), [-14.44547, 80.673775] (3), [-13.56932, 80.673775] (3),

length of domains: 8

Total time: 0.3780 pickout: 0.0020 decision: 0.0040 get\_bound: 0.3710 add\_domain: 0.0010

Current lb:-17.119014739990234

14 neurons visited

Global ub: 80.67377471923828, batch ub: inf

Cumulative time: 2.8103461265563965

splitting decisions: [[4, 8], [4, 8], [4, 8], [4, 8], [4, 8], [4, 8], [4, 8], [4, 8]]

best\_l after optimization: 231.36952209472656 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 12.427556991577148]

alpha/beta optimization time: 0.36405491828918457

This batch time : update\_bounds func: 0.3730 prepare: 0.0060 bound: 0.3650 transfer: 0.0000 finalize: 0.0020

Accumulated time: update\_bounds func: 17.2489 prepare: 0.1614 bound: 16.9846 transfer: 0.0000 finalize: 0.0589

batch bounding time: 0.37400054931640625

Current worst splitting domains [lb, ub] (depth):

[-16.78015, 80.673775] (4), [-15.93672, 80.673775] (4), [-15.62316, 80.673775] (4), [-15.35595, 80.673775] (4), [-15.14459, 80.673775] (4), [-14.96783, 80.673775] (4), [-14.57879, 80.673775] (4), [-14.47201, 80.673775] (4), [-14.19757, 80.673775] (4), [-13.97986, 80.673775] (4), [-13.96137, 80.673775] (4), [-13.87980, 80.673775] (4), [-13.43355, 80.673775] (4), [-13.36173, 80.673775] (4), [-13.03246, 80.673775] (4), [-12.66397, 80.673775] (4),

length of domains: 16

Total time: 0.3810 pickout: 0.0030 decision: 0.0030 get\_bound: 0.3740 add\_domain: 0.0010

Current lb:-16.7801513671875

30 neurons visited

Global ub: 80.67377471923828, batch ub: inf

Cumulative time: 3.1923232078552246

splitting decisions: [[4, 23], [4, 23], [4, 23], [4, 23], [4, 23], [4, 23], [4, 23], [4, 23]]

best\_l after optimization: 232.43280029296875 with beta sum per layer: [0.0, 0.0, 0.0, 0.0, 15.338359832763672]

alpha/beta optimization time: 0.3869655132293701

This batch time : update\_bounds func: 0.3939 prepare: 0.0040 bound: 0.3870 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 17.6429 prepare: 0.1654 bound: 17.3715 transfer: 0.0010 finalize: 0.0609

batch bounding time: 0.3949434757232666

Current worst splitting domains [lb, ub] (depth):

[-16.42583, 80.673775] (5), [-15.57802, 80.673775] (5), [-15.37281, 80.673775] (5), [-15.09077, 80.673775] (5), [-14.97040, 80.673775] (5), [-14.74358, 80.673775] (5), [-14.67278, 80.673775] (5), [-14.47177, 80.673775] (5), [-14.23257, 80.673775] (5), [-14.22653, 80.673775] (5), [-14.21817, 80.673775] (5), [-14.19757, 80.673775] (4), [-14.09940, 80.673775] (5), [-13.97986, 80.673775] (4), [-13.97787, 80.673775] (5), [-13.96137, 80.673775] (4), [-13.87980, 80.673775] (4), [-13.76404, 80.673775] (5), [-13.45164, 80.673775] (5), [-13.43355, 80.673775] (4),

length of domains: 24

Total time: 0.4019 pickout: 0.0020 decision: 0.0030 get\_bound: 0.3949 add\_domain: 0.0020

Current lb:-16.425825119018555

46 neurons visited

Global ub: 80.67377471923828, batch ub: inf

Cumulative time: 3.5952463150024414

splitting decisions: [[3, 57], [4, 11], [3, 57], [4, 11], [4, 11], [4, 11], [4, 11], [4, 11]]

best\_l after optimization: 232.64431762695312 with beta sum per layer: [0.0, 0.0, 0.0, 0.009671268984675407, 13.093461990356445]

alpha/beta optimization time: 0.3905966281890869

This batch time : update\_bounds func: 0.3996 prepare: 0.0060 bound: 0.3906 transfer: 0.0010 finalize: 0.0020

Accumulated time: update\_bounds func: 18.0425 prepare: 0.1714 bound: 17.7621 transfer: 0.0010 finalize: 0.0629

batch bounding time: 0.4005703926086426

Current worst splitting domains [lb, ub] (depth):

[-15.97240, 80.673775] (6), [-15.65916, 80.673775] (6), [-15.02492, 80.673775] (6), [-14.94612, 80.673775] (6), [-14.81881, 80.673775] (6), [-14.53129, 80.673775] (6), [-14.52296, 80.673775] (6), [-14.48905, 80.673775] (6), [-14.37508, 80.673775] (6), [-14.23257, 80.673775] (5), [-14.22653, 80.673775] (5), [-14.21817, 80.673775] (5), [-14.19757, 80.673775] (4), [-14.19297, 80.673775] (6), [-14.18324, 80.673775] (6), [-14.09940, 80.673775] (5), [-14.07859, 80.673775] (6), [-14.05970, 80.673775] (6), [-14.03799, 80.673775] (6), [-13.97986, 80.673775] (4),

length of domains: 32

Total time: 0.4076 pickout: 0.0020 decision: 0.0030 get\_bound: 0.4006 add\_domain: 0.0020

Current lb:-15.972399711608887

62 neurons visited

Global ub: 80.67377471923828, batch ub: inf

Time out!!!!!!!!

Image 9 label 2 verification end, final lower bound -15.972399711608887, upper bound 80.67377471923828, time: 4.040697336196899

9 -15.972399711608887

Result: image 9 verification failure (with branch and bound).

Wall time: 10.114939451217651

number of correctly classified examples: 9

incorrectly classified idx (total 0): []

attack success idx (total 0): []

verification success idx (total 2): [1, 3]

verification failure idx (total 7): [2, 4, 5, 6, 7, 8, 9]

final verified acc: 22.22222222222222%[9]

verifier is called on 9 examples.

total verified: 2[1]

mean time [cnt:9] (e[1]xcluding attack success): 7.987115038765801

(nnv) D:\wordspace\python\alpha-beta-CROWN\complete\_verifier>^A

[1] K. He, X. Zhang, S. Ren, and J. Sun, "Deep Residual Learning for Image Recognition." pp. 770-778.