23/10/2019

Probability

Ex. A Small Community Consist of 12 women each of whom has two children. If 1 women & 1 of her children has to be choosen then How many different choices are possible.

12

Answer → 24

If there are two experiments to be performed. experiment 1 results in m possible outcomes. for each outcome of experiment-1 there are n possible outcomes of experiment-2 then together there are mn possible outcomes of the two experiments.

r experiment

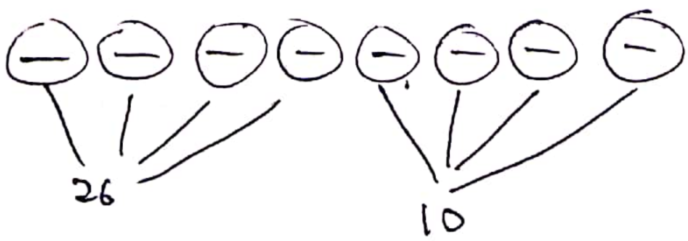
first exp. → n_1 possible outcome

for each outcome of 1^{st} exp. — n_2 possible outcomes in 2^{nd} exp.

" " " $r-1$ exp. → n_r " " " r^{th} exp.

Total possible outcomes $= n_1 \times n_2 - - - n_r$

Eg How many 8 ~~place~~ no. License plate are possible if first 4 places are occupied by letters and Last 4 places by number



$= 26^4 \times 10^4$

$=$

(21)

...der Collection of 3 novels by authors A, B & C. 2 mathematics ... by authors D & E and 1 physics book by author P. ...o many argements are possible if books are to be distinguish by authors.

A B C D E P

novels, mathematic, physic

$6!$

$6 \times 5 \times 4 \times 3 \times 2 \times 1$

$^3P_2 = \frac{3!}{2!}$ $\quad$ $^2P_1 = \frac{2!}{1!}$ $\quad$ $^1P_1 = \frac{1!}{1!}$

$3 \times 2 \times 1$

Ans $\frac{6!}{3!\,2!}$

$\frac{6 \times 5 \times \cancel{4}\,2}{\cancel{2}}$

$= 60$

Que In General arrangement of n-object where n_1 are alike ~~alive~~ and n_2 are a like ~~fine~~ till n_n are alkike then total no. of arrangement are equal to

Sol^n $\frac{n!}{n_1!\, n_2! \cdots n_n!}$

Que, A Tennis Tournament has 9 compitors 3 from India, 2 from Japan & 4 from Malaysia Results of the tournament are ananced by Nationality of the players in the order in which they are play. How many such list are possible.

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$$\frac{9!}{3!\,2!\,4!}$$

$$= \frac{9 \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6} \times 5}{\cancel{6} \times \cancel{2}}$$

$$= 9 \times 4 \times 7 \times 5$$

→ In a collection of n-objects in how many ways we can select r objects. w

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

→ A ~~Theory~~ Jury of 7 is to be formed from a group of 30 people. How many jury can be formed.

$$^{30}C_{7}$$

$$\frac{30!}{23!\,7!} = \frac{\overset{5}{\cancel{30}} \times 29 \times \overset{\overset{2}{\cancel{4}}}{\cancel{28}} \times \overset{9}{\cancel{27}} \times 26 \times \overset{5}{\cancel{25}} \times \overset{6}{\cancel{24}}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 29 \times 26 \times 5 \times 2 \times 9 \times 5 \times 6$$

$$=$$

→ In Case ~~of~~ the group of 30 consist of 10 women, 20 men and it is required that 2 women and 5 men should form the jury. How many possible Juries are formed

$$^{10}C_{2} + {}^{20}C_{5}$$

t is a collection of object from a points can be finite or infinite

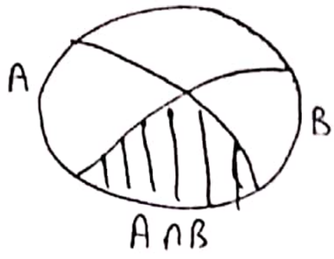
Ω or S

or two set A and B Belonging to universal set there sum or a joint or union is

$$A \cup B = \{w \in \Omega : w \in A \text{ or } w \in B\}$$

Intersection or product of A & B

$$A \cap B = \{w \in \Omega : w \in A \,\&\, w \in B\}$$

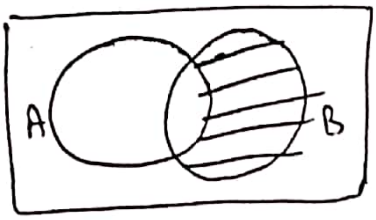


A^c or $\bar{A}$

$$A \cap B = \{w \in \Omega : w \notin A\}$$

$$A - B = \{w \in \Omega : w \in A \,\&\, w \notin B\}$$

or $\bar{A} \cap B$. $\longrightarrow$



- **Proper Subset**

If $B \subseteq A$ but $B \neq A$

If A & B are said to be mutually exclusive or disjoint If $A \cap B =$ (

- **LAWS :**

If A, B or C are subset of universal set

1) Idemptent Law

$$A \cap A = A \cup A$$

2) Commutative Law

$$A \cup B = B \cup A \quad , \quad A \cap B = B \cap A$$

3) Associative

$$A \cap (B \cap C) = (A \cap B) \cap C \quad ; \quad A \cup (B \cup C) = (A \cup B) \cup C$$

4) distributive Law

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

5) Demogan's Law

(i) $(A \cap B)^c = A^c \cup B'$

(ii) $(A \cup B)^c = A^c \cap B^c$

(iii) $(A^c)^c = A$

(iv) $B \subseteq A$ iff $A \cap B = B$

(v) $B \subseteq A$ iff $A \cup B = A$

Extended Union, Intersection or Complement of sets

Let A, $i \in I$ be subset of Ω

$$\bigcap_{i=I} A = \{ w \in \Omega \quad w \in A \quad \forall \; i \in I \}$$

$$\bigcup_{i=I} A = \{ w \in \Omega \quad w \in A \text{ for atleast one } i \in I \}$$

$$\left(\bigcap_{i=I} A_i\right)^c = \bigcup_{i \in I} A^c \;\&\; \left(\bigcup_{i=I} A_i\right)^c = \bigcap A_i^c \quad i \in I$$

...mple space.

...e set containing all possible outcome of an experiment is the ...mple space corresponding to experiments.

- Any subset E of samplespace S is known as Event. if E & F is a subset of S then ~~EA~~ $E \cup F$, $E \cap F$, E^c & allother operations define for subset of S are events.

Axioms of Probability

1) $0 \leq P(E) \leq 1$, $E \subseteq \Omega$

2) $P(\Omega) = 1$

3) For any sequence of mutually exclusive events $\{E_i\}$ $i \in I$

4) $E_i \cap E_j = \phi$, $i, j \in I$

5) $P\left(\bigcup_{i \in I} E_i\right) = \sum_{i \in I} P(E_i)$ where $P(E)$ is ~~Probibili~~ Probability of occurence of E

Ans A commity of 5 is to be selected from a group 6 men and 9 women if selection is made Randomly what is the probability that the commity will have 3 men & 2 women.

$$= \cancel{\frac{{}^6C_3 \times {}^9C_2}{{}^{15}C_5}}$$

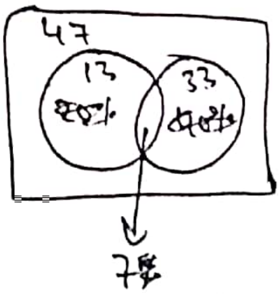
$$\cancel{\frac{6 \times 5 \times 4 \times \cancel{3} \; 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3}{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}}$$

$$= \frac{{}^6C_3 \times {}^9C_2}{{}^{15}C_5}$$

$P(\phi) = 0$ $\quad$ $P(E^c) = 1 - P(E)$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Ques 20% of Indians smoke cigrate, 40% smoke ~~weeds~~ Bidis and 7% smoke both. What percent of people do not smoke.



$$\begin{array}{r} 33 \\ 7 \\ \hline 13 \\ \hline 53 \end{array} \qquad \begin{array}{r} 100 \\ -53 \\ \hline 47 \end{array}$$

Conditional Probability

$P(A) > 0$, $\quad P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

Consider the example in which two fair dices are roled so event A is first die shows ~~6~~ 2 and B is the sum of 2 no. are six

36

$\cancel{A = \frac{2}{6}}$

$= (2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$

$A) = \dfrac{6}{36}$

$B = (1,5)\ (2,4)\ (3,3)\ (4,2)\ (5,1)$

$B) = \dfrac{5}{36}$

$A \cap B = (4,2)$

(24)

$P(A \cap B) = \frac{1}{36}$.

$P(A) = \frac{6}{36}$

$$P(B|A) = \frac{1/36}{6/36} = 1/6$$

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$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$ Baye's Theorem

Naive Baye's Classification

Fruit	Yellow	Sweet	Long	Total
Mango	350	450	0	~~800~~ 650
Banana	400	300	350	400
other	50	100	50	100
Total	800	850	400	1200

P(mango/yellow)

$$P(Y/M) = \frac{350}{650} = 0.53$$

$$P(S|M) = \frac{450}{650} = 0.69$$

$$P(L|M) = 0$$

$$P(Y/B) = \frac{400}{400} = 1$$

$$P(S/B) = \frac{300}{400} = \frac{3}{4} = 0.7$$

$$P(L/B) = \frac{350}{400} = \frac{35}{40}$$

$$= 0.875$$

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$= P(Y/B) \times P(S/B) \times P(L/B)$

$= 1 \times 0.75 \times 0.875$

$= 0.65$

$P(Y/O) = 0.33$

$P(S/O) = 0.66$

$P(L/O) = 0.33$

$= P(Y/O) \times P(S/O) \times P(L/O)$

$= 0.33 \times 0.66 \times 0.33$

$= 0.074$

$$P(\text{Mango}/\text{Long, sweet, yellow}) = \frac{P(L/M)\ P(S/M)\ P(Y/M)}{P(L)\ P(S)\ P(Y)} \cdot P(M)$$

$$P(B \mid L, S, Y) = \frac{P(L/B)\ P(S/B)\ P(Y/B)}{P(L)\ P(S)\ P(Y)} \cdot P(B)$$

$= \cancel{0.875 \times 0.75 \times}$

$$\frac{0.65}{\frac{\cancel{350}}{\cancel{400}} \times \frac{\cancel{300}}{\cancel{850}} \times \frac{\cancel{350}}{\cancel{800}}} \times \frac{\cancel{400}}{\cancel{1200}}$$

$$= \frac{0.65}{0.875 \times 0.35 \times 0.43} \times \frac{1}{3}$$

No.	Color	Type	Origin	Stolen ~~Status~~
1	Red	Sports	Dom	yes
2	Red	"	"	No
3	Red	"	"	yes
4	Yellow	"	"	No
5	Yellow	"	Imp	yes
6	Yellow	SUV	"	No
7	Yellow	SUV	"	yes
8	Yellow	"	Dom	NO
9	Red	"	Imp	No
10	Red	sports	Imp	yes

$P(\text{Red}; SUV, \text{Domestic}) =$

$P(\text{Red}) = 5/10 = 0.5$

$P(SUV) = 4/10 = 0.4$

$P(\text{Domestic}) = 5/10 = 0.5$

$P(R) \times P(SUV) \times P(Dom) = 0.1$

$$P\left(\frac{S}{R, SUV, Dom}\right) = \frac{P\left(\frac{R, SUV, Dom}{S}\right) \times P(S)}{P(R)\,P(SUV)\,P(Dom)}$$

$P(R/S) = 2/5 =$

$P(SUV/S) = 4/5 =$

$P(Dom/S) = 5/5) =$

$P\left(\frac{R, SUV, Dom}{S}\right) =$

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Law of Total Probability

Let S be a sample space & Let $E_1, E_2, \ldots E_n$ be a mutually exclusive events associated with a random experiment. If A is any event which occurs with E_1 or $E_2 \ldots$ or E_n then

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + \cdots + P(E_n)\,P(A/E_n)$$

Ques A Bag contains 4 Red and 3 Black balls. A second Bag contains 2 Red & 4 Black balls. One bag is selected at Random. from the selected bag 1 ball is drawn find the Prob. that Ball drawn is Red.

Prob. of section a bage $= \frac{1}{2} = P(E_1) = P(E_2)$

$\frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \times \frac{2}{6}$ A = Red Ball

$$P(A/E_1) = \frac{P(A \cap E_1)}{P(E_1)} = \cancel{P(A) + P(E_1) - P(} \; \frac{4/7}{1/2} = 8$$

$$= 8/7$$

$$P(A/E_2) = \frac{P(A \cap E_2)}{P(E_2)} = \frac{2/6}{1/2} = 2/3$$

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) = \frac{1}{2} \times \frac{8}{7} + \frac{1}{2} \times \frac{2}{3} =$$

Ques In a bolt factory machinie A, B & C Manfacture 25%, 35%, 40% of the total bolts of their output 5%, 4% & 2% are respectively defective bolts. A bolt is drawn random from the Product. what is the probability the bolt drawn is defective.

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= bolt is Manufactured by M/c A

E_2 = " " " " " B

E_3 = " " " " " C

A = bolt is defective

$P(E_1) = \frac{25}{100}$ $P(E_2) = 35/100$ $P(E_3) = 40/100$

$P(A/E_1) = \frac{5}{100}$ $P(A/E_2) = \frac{4}{100}$ $P(A/E_3) = 2/100$

P(A)

Que There are 3 bags each containing 5 white Balls & 3 Black Balls also there are two bags each containing ~~two~~ 2 white Balls and 4 Black Balls. A white Ball is drawn at Random find the prob. that this white Ball is from a Bag of 1^{st} group.

selecting Bag from 1^{st} Group = 3/5

" " " 2^{nd} " = 2/5

selecting white Ball from 1^{st} Group = 5/8

" " " " 2^{nd} " = 2/6

$$= \frac{3/5 \times 5/8}{3/5 \times 5/8 + 2/5 \times 2/5} = \frac{45}{61}$$

Random Variables

Suppose we have 2 dice

$x = \{\text{sum of 2 no. that show up}\}$

$2 \leq x \leq 12$ x is a random Variable

$x = 2$	$\{(1,1)$	$= 1/36 \rightarrow$ Prob. distribution
$x = 3$	$(2,1)\ (1,2)$	$= 2/36$
$x = 4$	$((1,3)\ (2,2)\ (3,1)\}$	$= 3/36$

x is a real value function mapping the subset of sample space is 2 real no. ~~Small~~ S. is called a random Variable. Since the Values of Random variable are determined by the outcome of the experiment we can assign Prob. to the value it takes.

Ans 3 coins are tossed together $x(w)$ = no. of Heads in $w \in S$ is a Random Variable. find out the Prob. distribution.

H H H
H H T
H T H
H T T
T H H
T H T
T T H
T T T

$\cancel{= 7/8}$

$w = 0 \quad P(w) = 1/8$

$w = 1 \quad P(w) = 3/8$

$w = 2 \quad P(w) = \cancel{2/8}\ 3/8$

$w = 3 \quad P(w) = 1/8$

An Urn Contains 4 white balls & 6 Red balls. Four balls are drawn at Random from the Urn. Find probability distribution no. of White balls.

0 — $^4C_0 \times {^6C_4}$ = - - - -

1 — $^4C_1 \times {^6C_3}$

2 — $^4C_2 \times {^6C_2}$

3 — $^4C_3 \times {^6C_1}$

4 — $^4C_4 \times {^6C_0}$

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1) A probability function $p(x)$ is Said to be pdf if $\Sigma p(x_i) = 1$ for $i = 1, 2, 3, \ldots$

Mean of Discrete Random Variable

$$\mu_i = E(x) = \Sigma x_i \, p(x_i)$$

Variance $V = E(x^2) - [E(x)]^2 = \Sigma x_i^2 \, p(x_i) - [\Sigma x_i \, p(x_i)]^2$

Standard Deviation (σ)

$$\sigma = \sqrt{V}$$

- CDF (Cummulative Distribution func.) (F)

If x is discrete Random Variable then Distribution func. of x is given by $F(x) = P(X \leq x)$

$$= \sum_{x \leq x_i} p(x_i), \quad x_i \in X$$

If x is discrete Random Variable having a pdf $p(x)$ then the expected Value $E(x)$ is defined $E(x) = \Sigma x_i \, p(x_i)$

E(x) can also interpreted as Centre of Gravity of masses

$E(x) = \{P(x_i\}_{i=1}^{n}$ located at p.ts $x_i = 1, 2, \ldots n$

1st Moment

Variance $= E\left((X - E(X))^2\right)$

2nd Moment

$= E\left[X^2 - 2X\,E(X) + (E(X))^2\right]$

$= E(X^2) - 2E(X)\,E(X) + [E(x)]^2$

Properties

$= E(x^2) - [E(x)]^2$

F is a non decreasing function i.e $a<b$ then $f(a) < f(b)$

Ques The ~~medium~~ pdf of Random Variable given by

$$P(x) = \begin{cases} k & \text{if } x=0 \\ 2k & \text{if } x=1 \\ 3k & \text{if } x=2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find value of k

(ii) Evaluate $P(x<2)$, $P(x \le 2)$, $P(0<x<2)$

(iii) Find CDF of x

(i) PDF = $k + 2k + 3k + 0 = 1$

$6k = 1$

$\boxed{k = 1/6}$

(ii) $P(x<2) = k + 2k = 3k$

$= 3(1/6) = 1/2$

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$P(x \le 2) = k + 2k + 3k = 6k$

$= 6\left(\frac{1}{6}\right) = 1$

$P(0 < x < 2) = 2k = 2\left(\frac{1}{6}\right) = \frac{1}{3}$

(iii) $x \le a$

CDF

x	0	1	2	> 2
$F(x)$	k	$3k$	$6k$	$6k$
CDF	$\frac{1}{6}$	$\frac{1}{2}$	1	1

* ~~Standard Deviation~~ (σ)

Ans find Mean, Variance & S.D from Discrete Random Variable x having following pdf

$$P(x) = \begin{cases} \frac{1}{3} & x = 0 \\ \frac{1}{3} & x = 1 \\ \frac{1}{3} & x = 2 \\ 0 & \text{otherwise.} \end{cases}$$

$\text{Mean} = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 2 \times \frac{1}{3}$

$= 0 + \frac{1}{3} + \frac{2}{3}$

(i) Mean $= 1$.

(ii) Variance $= E(x^2) - [E(x)]^2$

$= \left[(0)^2 \frac{1}{3} + (1)^2 \frac{1}{3} + 4 \times \frac{1}{3}\right] - [1]^2$

$= \frac{1}{3} + \frac{4}{3} - (1)^2$

$= \frac{5}{2} - 1 = \frac{2}{2}$

(iii) standard deviation $= \sqrt{V} = \sqrt{2/3}$

Ques If $P(x)$ is pdf defined as

$$P(x) = \begin{cases} 1/4 & x = -2 \\ 1/4 & x = 3 \\ 1/4 & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

Determine

(i) $P(1 < x \leq 4)$

(ii) $P(x > 5)$

(iii) $P(2x - 3 > 1)$

$P(1 < x \leq 4) = 1/4$

$P(x > 5) = 1/4$

$P(2x-3 > 1) = \cancel{1/4\ P(2(-2)-3>1) + P(x}$

$P(2(3)-3 > 1) + P(2(6)-3 > 1)$

$\underbrace{P(3 > 1)}_{\text{True}} + \underbrace{P(9 > 1)}_{\text{True}} = 1/4 + 1/4 = 1/2$

Ques Find probability function for the following CDF

$$F(x) = \begin{cases} 1/8 & \text{if } x = 0 \\ 1/4 & \text{if } x = 2 \\ 5/8 & \text{if } x = 3 \\ 1 & \text{if } x = 4 \end{cases}$$

x	0	2	3	4
F(x)	1/8	1/4	5/8	1
CDF	1/8	1/4	5/8	1

Answer

0	2	3	4
1/8	1/8	3/8	3/8

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Bernoulli Random Variable

A experiment in which the outcome is either a success or failure

If x is corresponding R.V then

for success $\to X = 1$ or $X = 0 \leftarrow$ for failure

Let $P(X=1) = p$, $P(X=0) = 1-p$

$E(x) = 1 \cdot p + 0 \cdot (1-p) = p$

$$Var(x) = 1 \cdot p + 0\,(1-p) - [E(x)]^2$$
$$= p - p^2$$
$$= p(1-p)$$

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Continuous Random Variable

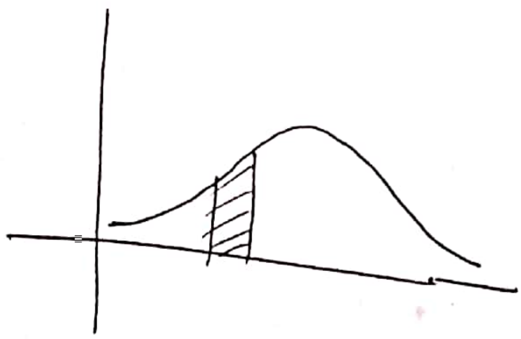
We say that x is a continuous R.V if there exist a non-negative function $f(x)$ defined for all real $x \in (-\infty, \infty)$ having property that for set B of real no.

$$P(X \in B) = \int_B f(x)\,dx$$

If function $f(x)$ is pdf of x

Let $B = [a, b]$

$$P[a \le X \le b] = \int_a^b f(x)\,dx$$

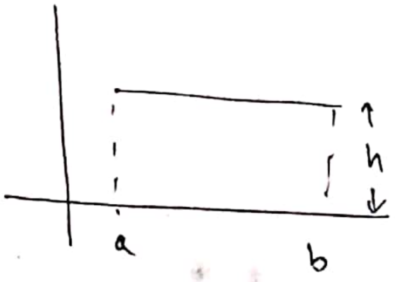


If $a = b$

$$P(X=a) = \int_a^a f(x)\,dx = 0$$

∘ Uniform Random Variable

a R.V is said to be uniformly distributed over interval (a, b) if its pdf is given by
$$f_x(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

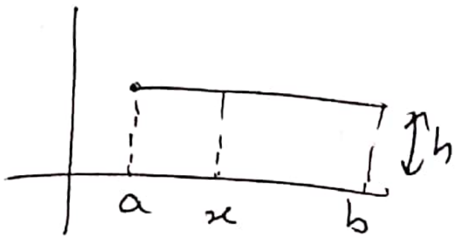


$$(b-a)h = 1$$
$$h = \frac{1}{b-a}$$

CDF $\quad F(a) = \int_{-\infty}^{\infty} f(x)\,dx$

CDF

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \le x \le b \\ 1 & \text{if } x > b \end{cases}$$



$$E(x) = \frac{b+a}{2}$$
$$Var(x) = \frac{(b-a)^2}{12}$$

CDF $\quad F_x(x) = P(x \le x)$

where $x \in [a, b]$

$$F_x(x) = \int_{-\infty}^{x} f(x)\,dx$$
$$= \int_{a}^{x} f(x)\,dx$$
$$= \int_{a}^{x} \frac{1}{b-a}\,dx$$

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$$\left[\frac{1}{b-a}x\right]_a^x$$

$$\frac{x}{b-a} - \frac{a}{b-a} = \frac{x-a}{b-a}$$

where $x > b$

$$F_x(b) = \frac{b-a}{b-a} = 1$$

Expected Value

$$E(x) = \int_a^b x f(x)\,dx$$

$$= \int_a^b \frac{x}{b-a}\,dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{2(b-a)}(b^2-a^2) = \frac{b+a}{2}$$

Variance

$$Var(x) = E[x^2] - [E(x)]^2$$

$$E[x^2] = \int_a^b x^2 f(x)\,dx$$

$$= \int_a^b \frac{x^2}{b-a}\,dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{3(b-a)}(b^3-a^3)$$

$$= \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$$

$$= b^2+a^2+ab$$

$$Var(x) = \frac{b^2+a^2+ab}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{b^2+a^2+ab}{3} - \frac{a^2+b^2+2ab}{4}$$

$$= \frac{4b^2+4a^2+4ab-5a^2-3b^2-6ab}{12}$$

$$= \frac{(a-b)^2}{12}$$

Que If x is uniformaly distributed over interval [0, 10]. Compute Probability

(a) $2 < x < 9$ Answer $7/10$

(b) $1 < x < 4$ $3/10$

(c) $x < 5$ $5/10$

(d) $x > 6$ $4/10$

Exponential Distribution

Continuous R.V whose pdf is given for some $\lambda > 0$ by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

is said to be exponential R.V with permater λ

$$C.D.F = F(x) = P[X \leq x]$$

$$= \int_a^x \lambda e^{-\lambda y}\, dy$$

$$= 1 - e^{-\lambda x} \quad x \geq 0$$

$$\int_0^\infty f(x)dx = 1$$

Proof $\int_0^\infty \lambda e^{-\lambda x} dx = \lambda \int_0^\infty e^{-\lambda x} dx = \left[\lambda e^{-\lambda x}\left(\frac{1}{-\lambda}\right)\right]_0^\infty$

$$= -\left[e^{-\lambda x}\right]_0^\infty = -\left[e^{-\infty} - e^0\right]$$

$$= -(0-1) = 1$$

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~~Cont~~ Moment Generating Function

MGF = $\phi(x)$

$$\phi(t) = E[e^{tx}] = \begin{cases} \sum_x e^{tx} p(x) & \text{if } x \text{ discrete} \\ \int_{-\infty}^{\infty} e^{tx} f(x)\,dx & \text{~~otherwise~~ Continuous} \end{cases}$$

$$\phi'(t) = \frac{d}{dx} E[e^{tx}] = E\left[\frac{d}{dt} e^{tx}\right] = E[x e^{tx}]$$

$$\phi'(0) = E[x \cancel{e^{tx}}]$$

$$\phi''(t) = \frac{d}{dt}\phi'(t) = \frac{d}{dt} E[xe^{tx}]$$

$$= E\left[\frac{d}{dt}(xe^{tx})\right] = E[x^2 e^{tx}]$$

$$\phi''(0) = E[x^2]$$

$$\phi(t) = E[e^{tx}]$$

$$= \int_0^{\infty} e^{tx} \lambda e^{-\lambda x} dx$$

$$= \lambda \int_0^{\infty} e^{-(\lambda - t)x} dx$$

$$= \frac{\lambda}{\lambda - t} \; ; \; t < \lambda$$

$$\phi'(t) = \frac{\lambda}{(\lambda - t)^2} \qquad \phi''(t) = \frac{2\lambda}{(\lambda - t)^3}$$

$$E[x] = \phi'(0) = \frac{\lambda}{\lambda^2} = 1/\lambda \qquad Var[x] = \phi''(0) - \left(\frac{1}{\lambda}\right)^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

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- ## Gamma Distribution

A R.V is said to have gamma distribution with parameters (α, λ), $\lambda > 0$, $\alpha > 0$, if its density function is given by

$$f(x) = \begin{cases} \dfrac{\lambda e^{-\lambda x} (\lambda x)^{\alpha-1}}{\Gamma(\alpha)} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

where $\Gamma(\alpha) = \int_0^\infty \lambda e^{-\lambda x} (\lambda x)^{\alpha-1} dx$

$$= \int_0^\infty e^{-y} y^{\alpha-1} dy \qquad [\text{Let } y = \lambda x)$$

Apply integration by parts

$uv - \int v\,du$ yields with $u = y^{\alpha-1}$, $dv = e^{-y} dy$

$v = e^{-y}$ for $\alpha > 1$

$$\int_0^\infty e^{-y} y^{\alpha-1} dy = \left[-e^{-y} y^{\alpha-1}\right]_{y=0}^{\infty} + \int_0^\infty e^{-y} (\alpha-1) y^{\alpha-2} dy$$

$$= (\alpha-1) \int_0^\infty e^{-y} y^{\alpha-2} dy$$

or

$$\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$$

When α is an integer say $\alpha = n$

$$\Gamma(n) = (n-1)\Gamma(n-1) = (n-1)(n-2)\Gamma(n-2) = \cdots = (n-1)!\,\Gamma(1)$$

Because $\Gamma(1) = \int_0^\infty e^{-y} dy = 1$ we see that $\Gamma(n) = (n-1)!$

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When $\alpha = 1$ a Gamma distribution approximate to exponential will Means $\frac{1}{\lambda}$.

$$E[X] = \frac{\alpha}{\lambda}$$

$$Var[X] = \frac{\alpha}{\lambda^2}$$

$$\phi(t) = E[e^{tX}]$$

$$= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty e^{tx} e^{-\lambda x} \cdot x^{\alpha-1} dx$$

$$= \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^\infty e^{-(\lambda - t)x} x^{\alpha-1} dx$$

$$= \left(\frac{\lambda}{\lambda - t}\right)^\alpha$$

$$\phi'(t) = \frac{\alpha \lambda^\alpha}{(\lambda - t)^{\alpha+1}}$$

$$\phi''(t) = \frac{\alpha(\alpha+1)\lambda^\alpha}{(\lambda - t)^{\alpha+2}}$$

$$E[X] = \phi'(0) = \frac{\alpha}{\lambda}$$

$$Var(X) = E[X^2] - [E(X)]^2$$

$$= \phi''(0) - (\phi'(0))^2$$

$$= \frac{\alpha}{\lambda^2}$$

Normal or Gaussian Distribution

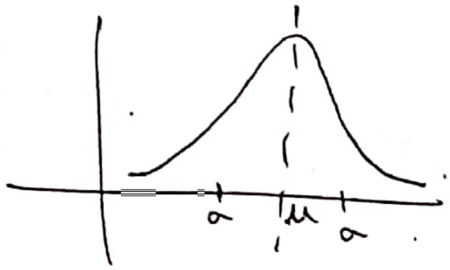
$$X \sim N(\mu, \sigma^2)$$

μ = Mean $\quad \sigma^2$ = Variance

$$f_x(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty \le x < \infty$$

Mean $= \mu$

Variance $= \sigma^2$



Markov Inequality

Let x be a the ~~Vooro~~ Valued random Variable

$$P(x \ge a) \le \frac{E[x]}{a}$$

Proof - Let I be an ~~Rando~~ indicator R.V

$$I = \begin{cases} 1 & \text{if } x \ge a \\ 0 & \text{otherwise} \end{cases}$$

	I	~~X/a~~ X/a
If $x \ge a$	1	≥ 1 or $\ge I$
If $x < a$	0	≥ 0 or $\ge I$

X/a is always greater then I

so, $E[I] \le E[X/a]$

or

$$\frac{1}{a} E[x]$$

$E[I]$ is $P(x \ge a)$

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Ques: Tosse an unbiased coin n-times. Compute the probability that for R.V x where x is the no. of Heades. $x \geq \frac{3n}{4}$

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