Department of Computer Science

University of Delhi

MCAC 103: Mathematical Techniques for Computer Science Applications (CIA-I)

Time: 1 hour | February 10, 2021 | Maximum Marks: 15

1. Can we say that the performance of PCA will be outstanding when eigenvalues are nearly equal? Justify your answer. [01]

2. Which of the following can be the first 2 principal components after applying PCA?

   (a) $(0.5, -0.5, 0.5, 0.5)$ and $(0.61, 0.61, 0, 0)$
   (b) $(0.5, 0.5, 0.5, 0.5)$ and $(0, 0, -0.71, -0.71)$
   (c) $(0.5, 0.5, 0.5, 0.5)$ and $(0.5, 0.5, -0.5, -0.5)$
   (d) $(0.5, 0.5, 0.5, 0.5)$ and $(1, 0, 0, 0)$ [01]

3. Given below is an outline of algorithm **X** where a few of segments $(\cdots?\cdots)$ are intentionally left blank. Identify the algorithm and then fill the segments marked with $\cdots?\cdots$. [2.5]

Algorithm 1: X(A, τ, k)

input : Data Matrix: $A \in \mathbb{R}^{n\times n}$, Threshold: τ
output: $\alpha_1 \geq \alpha_2 \cdots \geq \alpha_k, k \leq n$

for i = *1 to k* **do**
  $t \leftarrow 0$
  $\cdots?\cdots$
  repeat
    $x^{(t+1)} \leftarrow \frac{\cdots?\cdots}{\|Ax^{(t)}\|_F}$
    $t \leftarrow t+1$
  until $(\|x^{(t+1)} - x^{(t)}\|_F \leq \cdots?\cdots)$;
  Let x be $x^{(t)}$ at which convergence is obtained
  $\alpha_i \leftarrow \cdots?\cdots$
  $A \leftarrow A - \alpha_i \cdots?\cdots$
end

4. The table below displays the daily closing prices of three stocks **X**,**Y** and **Z** for 5 days. Compute the covariance between the stocks. [2.5]

Day	**X**	**Y**	**Z**
1	90	60	90
2	90	90	30
3	60	60	60
4	60	60	90
5	30	30	30

5. Using row reductions show $\begin{bmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{bmatrix} = \begin{bmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{bmatrix}$ [04]

6. The system of linear equations $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 14 \end{bmatrix}$ has [04]

   (a) infinitely many solutions
   (b) no solution
   (c) exactly two solutions
   (d) a unique solution