1. Implement Gibbs sampling in Restrictive Boltzmann Machine (RBM) and update weights using contrastive divergence. Use MNIST as the dataset and the model should be implemented using only Numpy library (i.e., do not use TensorFlow, Keras, Theano, PyTorch etc.). [10 marks]

   (a) Implement an RBM and Gibbs sampling. Keep your input size, hidden layer size, and number of steps (k) in the Gibbs chain to be variables, so that these can be easily changed. [4]

   (b) Use L2 weight decay and momentum in the implementation of training using contrastive divergence. [3]

   (c) Visualize the evolution of MNIST digits in RBM after some k steps. That is, after training, produce some figures such that first figure in the row is a test image, and then the next three images in the row are fantasy images for for $k = 3$ starting with the given test image. [2]

2. Compare direct classfication of pixels and learned RBM features. [5]

   (a) Implement and train a softmax regression classifier with L2 regularization and cross-entropy loss to classify the flattened image directly using pixel values [3]

   (b) Train a softmax classifier with L2 regularization and cross entropy loss to classify the images by taking their RBM features (hidden variables) as inputs instead of the pixels. Compare results with the previous sub-question. [3]