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Real Time DSP
Assignment 1: Audio Crossover

0. Introduction

The following report describe the design of audio crossover system. The mathematical calculations are provided to obtain the coefficients of a differential equations. The main task of the system is transformation of a mono audio signal to stereo signal with two channels, one for high frequencies and the second one for the low frequencies. At the end the frequency response of designed filters is presented.

1. Butterworth Low Pass and High Pass Filter Design [1]

The second-order scaled and normalised analog low-pass filter transfer function is as follow:

$$H(s) = \frac{\omega^2}{s^2+(\omega/Q)s+1}$$

As it is a Butterworth filter the value of Q is:

$$Q=\frac{\sqrt{2}}{2}$$

And w is defined as:

$$\omega = tan(2\pi fc/fs)$$

To derive a formula for digital filter the bilinear transfer is applied to get the equation in a z domain. It is done by substitution.

$$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$$

The analog second order scaled and normalised high pass transfer function is:

$$H(s) = \frac{s^2}{s^2+(\omega/Q)s+\omega^2}$$

The calculations of coefficients of butterworth low pass filter for a differential equation which is defined as

y[n] = a0x[n] + a1x[n-1] + a2x[n-2] + b1y[n-1] + b2y[n-2]

are presented below. In all the calculations 2/T factor is omitted as it cancels out in the process. First of all the equation H(z) is calculated:

$$H(z) \;=\; \frac{(\omega^2)}{(\frac{1-z^{-1}}{1+z^{-1}})^2 + \omega/\frac{\sqrt{2}}{2}(\frac{1-z^{-1}}{1+z^{-1}}) + \omega^2}$$

$$H(z) \;=\; \frac{(\omega^2)}{(\frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}) + \omega\sqrt{2}(\frac{1-z^{-1}}{1+z^{-1}})(\frac{1+z^{-1}}{1+z^{-1}}) + \omega^2}$$

$$H(z) \;=\; \frac{(\omega^2)}{(\frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}) + \omega\sqrt{2}(\frac{1-z^{-2}}{1+2z^{-1}+z^{-2}}) + \omega^2(\frac{1+2z^{-1}z^{-2}}{1+2z^{-1}+z^{-2}})}$$

$$H(z) \;=\; \frac{(\omega^2)}{(\frac{1+\sqrt{2}\,\omega+\omega^2-+2z^{-1}(-1+\omega^2)+z^{-2}(1-\sqrt{2}\,\omega+\omega^2)}{1+2z^{-1}+z^{-2}})}$$

$$H(z) \;=\; \frac{(\omega^2+2\omega^2z^{-1}+\omega^2z^{-2})}{(\frac{1+\sqrt{2}\,\omega+\omega^2-+2z^{-1}(-1+\omega^2)+z^{-2}(1-\sqrt{2}\,\omega+\omega^2)}{1})}$$

$$H(z) = \frac{\frac{\omega^2}{1+\sqrt{2}\,\omega+\omega^2} + \frac{2\omega^2}{1+\sqrt{2}\,\omega+\omega^2}z^{-1} + \frac{\omega^2}{1+\sqrt{2}\,\omega+\omega^2}}{1 + \frac{2(-1+\omega^2)}{1+\sqrt{2}\,\omega+\omega^2}z^{-1} + \frac{1-\sqrt{2}\,\omega+\omega^2}{1+\sqrt{2}\,\omega+\omega^2}z^{-2}}$$

Now having the z transform it is possible to obtain the coefficients for a differential equation:

$$C = \frac{1}{\tan(2\pi fc/fs)}$$

$$a0 = \frac{\omega^2}{1+\sqrt{2}\,\omega+\omega^2} = \frac{tan^2}{1+\sqrt{2}\,tan+tan^2} = \frac{1}{\frac{1}{tan^2}+\frac{1}{tan}\sqrt{2}+1} = \frac{1}{C^2+\sqrt{2}\,C+1}$$

$$a1 = 2\frac{\omega^2}{1+\sqrt{2}\,\omega+\omega^2} = 2a0 \qquad a2 = \frac{\omega^2}{1+\sqrt{2}\,\omega+\omega^2} = a0$$

$$b1 = \frac{2(-1+\omega^2)}{1+\sqrt{2}\,\omega+\omega^2} = \frac{-2+tan^2}{1+\sqrt{2}\,tan+tan^2} = 2\left(\frac{-1}{tan^2}\,\frac{1}{\frac{1}{tan^2}+\frac{\sqrt{2}}{tan}+1}+\frac{1}{\frac{1}{tan^2}+\frac{\sqrt{2}}{tan}+1}\right) = 2a0\left(1-\frac{1}{tan^2}\right) = 2a0(1-C^2)$$

$$b2 = \frac{1-\sqrt{2}\,\omega+\omega^2}{1+\sqrt{2}\,\omega+\omega^2} = \frac{1-\sqrt{2}\,tan+tan^2}{1+\sqrt{2}\,tan+tan^2} = \frac{\frac{1}{tan^2}-\frac{\sqrt{2}}{tan}+1}{\frac{1}{tan^2}+\frac{\sqrt{2}}{tan}+1} = a0\left(1-\frac{\sqrt{2}}{tan}+\frac{1}{tan^2}\right) = a0(1-\sqrt{2}\,C+C^2$$

The calculations of differential equation coefficients for butterworth high pass filter are presented below:

$$H(z) = \frac{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\omega\sqrt{2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+\omega^2}$$

$$H(z) = \frac{\left(\frac{1-2z^{-1}+z^{-1}}{1+2z^{-1}+z^{-1}}\right)}{\left(\frac{1+\sqrt{2}\,\omega+\omega^2+2z^{-1}(-1+\omega^2)+(1-\sqrt{2}\,\omega+\omega^2)z^{-2}}{1+2z^{-1}+z^{-2}}\right)}$$

$$H(z)=\frac{(\frac{1-2z^{-1}+z^{-1}}{1+2z^{-1}+z^{-1}})(1+2z^{-1}+z^{-2})}{(\frac{1+\sqrt{2}\ \omega+\omega^2+2z^{-1}(-1+\omega^2)+(1-\sqrt{2}\ \omega+\omega^2)z^{-2}}{1})}$$

$$H(z)=\frac{(1-2z^{-1}+z^{-2})}{(\frac{1+\sqrt{2}\ \omega+\omega^2+2z^{-1}(-1+\omega^2)+(1-\sqrt{2}\ \omega+\omega^2)z^{-2}}{1})}$$

$$H(z)=\frac{\frac{1}{(1+\sqrt{2}\ \omega+\omega^2)}-\frac{2}{(1+\sqrt{2}\ \omega+\omega^2)}z^{-1}+\frac{1}{(1+\sqrt{2}\ \omega+\omega^2)}}{1+\frac{2(-1+\omega^2)}{(1+\sqrt{2}\ \omega+\omega^2)}z^{-1}+\frac{(1-\sqrt{2}\ \omega+\omega^2)}{(1+\sqrt{2}\ \omega+\omega^2)}z^{-2}}$$

$$a0=\frac{1}{1+\sqrt{2}\ \omega+\omega^2}=\frac{1}{1+tan\sqrt{2}\ +tan^2}=\frac{1}{1+\sqrt{2}\ C+C^2}$$

$$a1=-2(\frac{1}{1+\sqrt{2}\ \omega+\omega^2})=-2a0 \qquad a2=(\frac{1}{1+\sqrt{2}\ \omega+\omega^2})=a0$$

$$b1=\frac{-2+2\omega^2}{1+\sqrt{2}\ \omega+\omega^2}=\frac{-2+2tan^2}{1+\sqrt{2}\ tan+tan^2}=a0(-2+2tan^2)=2a0(C^2-1)$$

$$b2=\frac{1-\sqrt{2}\ \omega+\omega^2}{1+\sqrt{2}\ \omega+\omega^2}=\frac{1-\sqrt{2}\ tan+tan^2}{1+\sqrt{2}\ tan+tan^2}=a0(1-\sqrt{2}\ C+C^2)$$

$$C = \tan(2\pi fc/fs)$$

Results of measurements from oscilloscope with sinusoidal input signal

Frequency(Hz)	Max HPF (mVolts)	Max LPF (mVolts)	Magnitude(dB) HPF	Magnitude(dB) LPF
0	0	0	0	0
100	104	584	-51.3269	0
200	128	584	-45.3357	0
300	168	584	-37.4893	0
400	192	584	-33.6364	0
500	224	584	-29.1886	0
600	272	584	-23.586	0
700	340	550	-17.1479	-1.7307
800	384	536	-13.6364	-2.4747
850	400	528	-12.4586	-2.908
900	432	520	-10.2379	-3.349
950	464	512	-8.176	-3.796
1000	488	480	-6.7209	-5.6586
1500	500	328	-6.0200	-16.6454
2000	504	168	-5.7901	-35.9501
3000	576	160	-1.9372	-37.3579
4000	576	112	-1.9372	-47.6493
5000	576	104	-1.9372	-49.7876
6000	576	104	-1.9372	49.7876
7000	600	104	-0.759	49.7876
8000	616	104	0	49.7876
9000	616	104	0	49.7876
10000	616	104	0	49.7876

Table 1: Measurements of the frequency response for Low Pass and High Pass Filters , for the sinusoidal input signal with frequency range between 0 to 10000Hz, and cut-off frequency of 1000Hz.

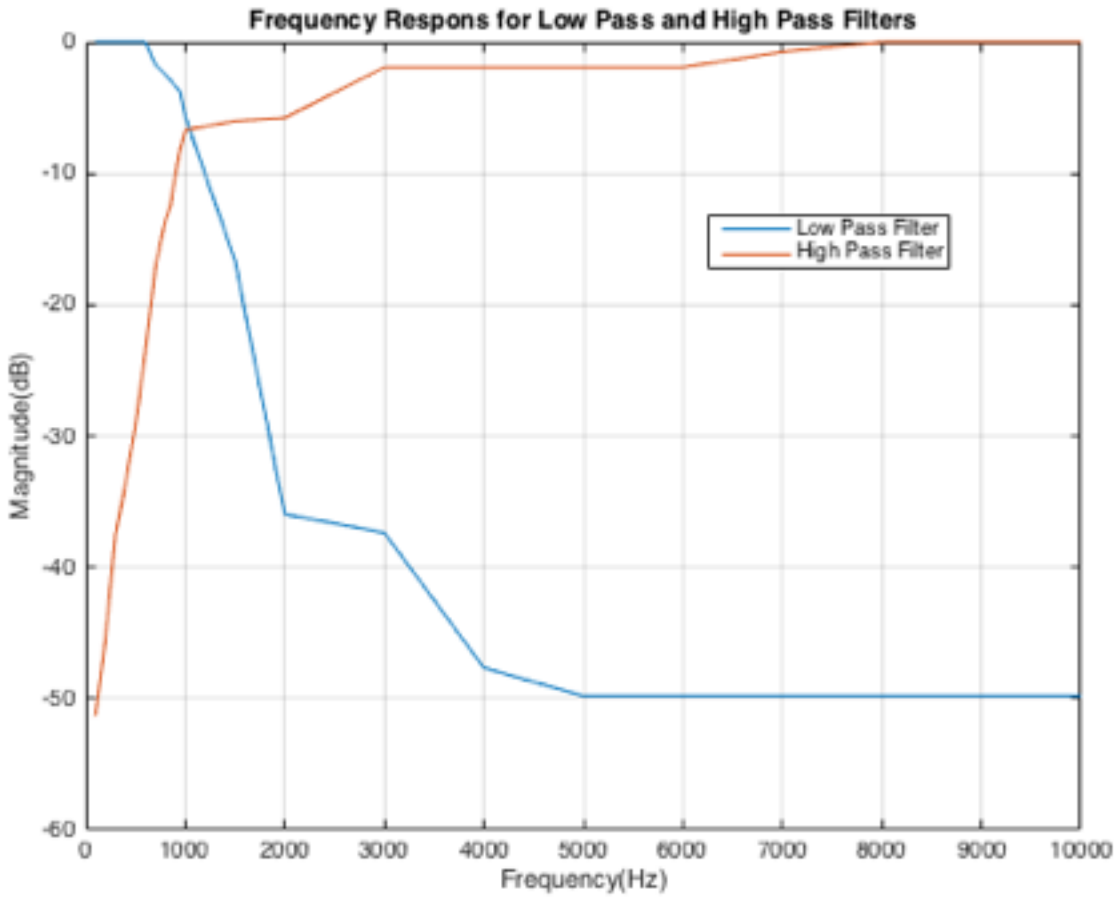

Figure1: Frequency response of Low Pass and High Pass Filters with cut off frequency set to 1kHz.

As it is visible on the plot the system works as it was expected. The low pass filter is attenuating the high frequencies and is passing the low ones, and the high pass filter do the opposite.

References

[1] PIRKLE, W.(2013) *Designing Audio Effect Plug-Ins in C++ with digital audio signal processing theory.* London:Focal Press Taylor and Francis Group.