Pascal Matrix Discussion

Error Discussion

Summary of findings:

Our group chose to analyze the error generated by the LU factorization function. We found that there is a converse relationship between error and the dimensions of a matrix. Furthermore, as the dimensions of the matrix P increases the error compounds. Thus, there is a clear increase of error at an exponential rate.

It is also important to note that this graph is plotting how during LU factorization, the actual L and U matrices can differ from the initial matrix. Thus the solution (x) is not completely accurate and there is an error component that needs to be accounted for.

Summary of findings:

Ideally Px = b should be completely true in order for x to be the accurate solution to the function. However, this is not the case with QR and LU decomposition, which give us x values that are slightly different and thus have an error component to them that needs to be accounted for.

This graph is showing how as the dimensions of the matrix, P in this situation, affects the error involved. There is definitely a converse relationship as it was the case with the first graph. However, this graph differs in comparison, since the error does not exponentially increase in this case. As the column dimensions increase the rate of change of error slows and the increase of error plateaus to a certain degree.

This indicates that while LU factorization can be a great method while solving for the vector solution x for large matrices. Since the error would not have increase very rapidly. And the results would be fairly accurate.

Follow up Discussion

Answer the following questions in the associated written component for this part of the project:

1. Why is it justified to use the LU or QR−factorizations as opposed of calculating an inverse matrix?

As we calculate Ax = b for the value of x. One way of solving for x, is to find the inverse of A and then find x = A-1 b. This way of finding x results in the most accurate value of x. However, finding the inverse of a large matrix takes a lot of mathematical operations.

It is still justified to use LU or QR factorization because these methods calculate x with far fewer operations. This means that x can be found much faster. Thus, we are trading accuracy for efficiency. It’s also a better to use LU and QR because sometimes the inverse of A might not exist.

1. What is the benefit of using LU or QR−factorizations in this way? (Your answer should consider the benefit in terms of conditioning error.)

When we are working with very large matrices, finding the inverse can take up very many operations and it can be very time consuming to find the inverse. So, LU and QR factorization are good replacement methods for solving the value of x. The x value computed by LU and QR factorization is not completely accurate. There is an error involved now.