cs467

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1 introduction and background

The non-intuitive behavior of photon-mirror-beamsplitter system results from features of quantum mechanics called *superposition and interference*

the second beam aplitter has caused two paths (in superposition) to interfere, resulting in cancelation of the 0 path

2 Qubits and the framework of quantum mechanics

Quantum information is the result of reformulating information theory in this quantum framework

2.1 State of a Quantum System

the light-splitter example is an example of a 2-state quantum system:

a photon that follow one of two paths, which we identify by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

and noted a path state of the photo can be described as $\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$ with $|\alpha_0|^2 + |\alpha_1|^2 = 1$

Definition 2.1.1 (State Space Postulate)
the state of a system is described by a unit vector in a Hilbert Space $\mathbb{H}$

$\mathbb{H}$ can be infinite dimensional.

In practice, we cannot distinguish a continous state from a discrete state having small spacing

we label one basis vector with $|0>$, one with $|1>$, they are orthogonal to each other

we represent general state by $\alpha_0|0> + \alpha_1|1>$ with $|\alpha_0|^2 + |\alpha_1|^2 = 1$

α_0 and α_1 are complex coefficient, often called *amplitude* of basis state $|0>$ and $|1>$

amplitude α can be decomposed uniquely as a product $e^{i\theta}|\alpha|$ where $|\alpha|$ is non-negative corresponding to magnitude of α, $e^{i\theta}$ has norm 1.

value θ is *phase*, $e^{i\theta}$ is *phase vector*

the state is described by unit vector means $|\alpha_0|^2 + |\alpha_1|^2 = 1$

this is called *normalization constriant*

vector $e^{i\theta}|\phi>$ is equivalent to the state described by $|\phi>$

example of ϕ would be $|0> + |1>$

question: is this example not considering it has 100% appearing in 0 and 1 state

On the other hand, relative phase factors between 2 orthogonal states in superposition are physically significant, and the state described by the vector

$|0> + |1>$ is physically different from $|0> + e^{i\theta}|1>$

Theorem 2.1.1 (State SpacePostulate)
we can describe the most general state $|\phi>$ of a single qubit by a vector of form

$$|\phi> = cos(\theta/2)|0> + e^{i*k} sin(\theta/2)|1>$$

on a classical computer, a *classical bit* could be represented by a 0/1,

there is also probablitistic classical bit $\begin{pmatrix} p_0 \\ p_1 \end{pmatrix}$
we can represent two probabilities by 2-dimensional unit vector
now we go back to quantum bit, which is described by a complex unit vector $|\phi>$ in 2 dimentional Hilbert space. Up to a (physically insignificant) flobal phase factor, such a vector can be written in the form

$$|\phi >= cos(\theta/2)|0> + e^{i*k} sin(\theta/2)|1>$$

this state vector is often depicted as a point on a 3-dimentional sphere, the *Bloch Sphere*
points on the sphere can be expressed in Cartesian corrdinates as

$$(x, y, z) = (sin\theta cos\phi, sin\theta sin\phi, cos\theta)$$

2.2 Time Evolution of a Closed System

Theorem 2.2.1 (Evolution Postulate)
The time-Evolution of state of a *closed* quantum system is described by a unitary operator. that is for any evolution of the closed system there exsist a unitary operator U s.t. if initial state is $|\psi_i>$ then after evolution the state will be

$$|\psi_2 >= U|\psi_1>$$

2.3 Composite Systems

Theorem 2.3.1 (Composition of Systems Postulate)
When two physical systems are treated as one combined system. the state space of combined physical system is product space $H_1 \otimes H_2$ of the state space H_1, H_2 of the component subsystems.
If first system is in state $|\psi_1>$, second system in state $|\psi_2>$, the state of combined system is

$$|\psi_1> \otimes |\psi_2>$$

2.4 Measurement

state of a single-qubit system is represented as a vector in Hilbert space
The evolution of state of a system during a Measurement is not unitary
if the state $\sum_i \alpha_i |i>$ is provided as input. it will output i with probability $|\alpha_i|^2$ and leave the system in state $|i>$

Theorem 2.4.1
For a given orthonormal basis $B = \{|\varphi_i\}$ of a state space H_A for a system A, it's possible to perform a Von Neumann measurement on system H_A with respect to basis B that, given a state

$$|\psi\rangle = \sum_i \alpha|\varphi_i\rangle$$

outputs label i with probablity $|\alpha_i|^2$ and leaves the system in state $|\varphi_i\rangle$
For state $|\psi\rangle = \sum_i \alpha_i|\varphi_i\rangle$, note that $\alpha_i = \langle\varphi_i|\psi\rangle$ and thus

$$|\alpha_i|^2 = \alpha_i * \alpha_i = \langle\psi|\varphi_i\rangle\langle\varphi_i|\psi\rangle$$

question: do we usually know state of a quantum before we do the measure

One slight generalization of Von Neumann measurements:

A Von Neumann Measurement is special kind of projective Measurement

Recall orthogonal projection is operator P with

Note: $A^\dagger$ stands for congugate transpose of A

$$P^2 = P, P^\dagger = P$$

For any decomposition of identy operator $I = \sum_i P_i$ into orthogonal projectors P_i, there exisits projective Measurement that outputs i with probablity $p(i) = \langle\psi|P_i|\psi\rangle$ and leaves the system in renormalized state $\frac{P_i|\psi\rangle}{\sqrt{p(i)}}$.

In other words, this measurement projects the input state $|\psi\rangle$ into one of the orthogonal subspaces corresponding to the projection operators P_i, with probablity of sqaure of amplitude of component of $|\psi|\rangle$ in that subspace

Von Neumann measurement is special case of a projective measurement where all projectors P_i has rank one (in other words, are of $|\psi_i\rangle\langle\psi_i|$)

The simplest example of a Von Neumann measurement is a complete measurement in the computational basis. This can be viewed as following decompositon

$$I = \sum_{i\in\{0,1\}^n} P_i$$

where $P_i = |i\rangle\langle i|$

projective measurements are often described as *observable* An observable is a Hermitean operator M acting on state space of the system it has decomposition

$$M = \sum_i m_i P_i$$

where P_i is orthogonal projector on eigenspace of M with real eigenvalue m_i

Measuring the observable corresponds to performing a projective measurement with respect to the decomposition $I = \sum_i P_i$ where the measurement outcome i corresponds to eigenvalue m_i