

STAT333 Applied Probability

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May 25, 2020

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1 Course Information

1.1 Contact

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1.2 Grading Scheme

4 assignments 100%...

1.3 Review of Elementary Probability

Definition 1.3.1 (Probability Function)

For each event A of a sample space S , $P(A)$ is defined as the probability of event A , satisfying 3 conditions:

- $0 \leq P(A) \leq 1$
- $P(S)=1$
- $P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i)$ if the sequence of events A_i are mutually exclusive

Definition 1.3.2 (independent)

X and Y are independent rvs if $f(x, y) = f(x)f(y)$

Definition 1.3.3 (multivariate mgf)

$\phi_{x,y}(a, b) = E(e^{ax+by})$

Theorem 1.3.1

if X_1, X_2, \dots, X_n are independent rvs where $\phi_{X_i}(t)$ is the mgf of $X_i, i = 1, 2, \dots, n$. then $T = \sum_{i=1}^n X_i$ has mgf $\phi_T(t) = \prod_{i=1}^n \phi_{X_i}(t)$

Theorem 1.3.2 (Strong Law of Large Numbers)

If X_1, \dots, X_n is an iid sequence of rvs, each having mean $\mu < \infty$, then with probability 1, as $n \rightarrow \infty$

$$\bar{X}_n = \frac{X_1 + X_2 + \dots + X_n}{n} \rightarrow \mu$$

1.4 Conditional Distributions and Conditional Expectation

Theorem 1.4.1 (Law of total Expectation)

For rvs X and Y ,

$$E[g(X)] = E[E[g(X)|Y]]$$

Theorem 1.4.2

For rvs X and Y ,

$$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}(E[X|Y])$$

1.5 Discrete-time Markov Chain**1.6 The Exponential Distribution and the Poisson Process**