CS185c: HW #2

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1) - Line segment (0,0,1), (0, 0, -1)
 - Rotate by 45° ($\pi/4$) counter-clockwise about the x-axis
 - Move it (3, -2, 4) from the origin

HOMOGENEOUS TRANSFORMATION MATRIX

$$\begin{matrix} x' \\ y' \\ z \end{matrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{matrix} x_t \\ y_t \\ z_t \end{matrix} \qquad \left[\begin{array}{ccc|c} & R & & \begin{matrix} x_t \\ y_t \\ z_t \end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

4×4 4×1

For (0,0,1)

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \cos\theta & -\sin\theta & -2 \\ 0 & \sin\theta & \cos\theta & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -\sin\theta - 2 \\ \cos\theta + 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -\frac{1}{\sqrt{2}} - 2 \\ \frac{1}{\sqrt{2}} + 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2.71 \\ 4.71 \\ 1 \end{bmatrix}$$

rotation about x-axis

For (0,0,-1)

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & \cos\theta & -\sin\theta & -2 \\ 0 & \sin\theta & \cos\theta & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ \sin\theta - 2 \\ -\cos\theta + 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{1}{\sqrt{2}} - 2 \\ -\frac{1}{\sqrt{2}} + 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1.29 \\ 3.29 \\ 1 \end{bmatrix}$$

For scaling by a factor of 2

$$2 \begin{bmatrix} 3 \\ -2.71 \\ 4.71 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -5.41 \\ 9.42 \\ 2 \end{bmatrix} \quad \& \quad 2 \begin{bmatrix} 3 \\ -1.29 \\ 3.29 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -2.58 \\ 6.58 \\ 2 \end{bmatrix}$$

2) Perform a CC rotation of $\theta = \pi/3$ about vector $(-1,3,5)$

- compute the quaternion needed to do this
- Let $\vec{v} = (1,0,0)$ show your quat. works by computing $q\vec{v}q^{-1}$ & verifying it makes sense

$$\begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2)\,\hat{k} \end{bmatrix}$$ so long as $\hat{k}$ is a unit vector

make $(-1,3,5)$ a unit vector

$$\vec{u}_{\vec{v}} = \frac{\vec{v}}{|\vec{v}|} = \frac{(-1,3,5)}{\sqrt{(-1)^2+3^2+5^2}} = \frac{(-1,3,5)}{\sqrt{35}}$$

$$\begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2)\left(\frac{1}{\sqrt{35}}\begin{pmatrix}-1\\3\\5\end{pmatrix}\right) \end{bmatrix} = \begin{bmatrix} \cos \pi/6 \\ \sin \pi/6 \left(\frac{1}{\sqrt{35}}\begin{bmatrix}-1\\3\\5\end{bmatrix}\right) \end{bmatrix}$$

$$q\vec{v}q^{-1} = \begin{bmatrix} \cos(\pi/6) \\ \sin(\pi/6)\left(\frac{-1}{\sqrt{35}}\right) \\ \sin(\pi/6)\left(\frac{3}{\sqrt{35}}\right) \\ \sin(\pi/6)\left(\frac{5}{\sqrt{35}}\right) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \cos(\pi/6) \\ -\sin(\pi/6)\left(\frac{-1}{\sqrt{35}}\right) \\ -\sin(\pi/6)\left(\frac{3}{\sqrt{35}}\right) \\ -\sin(\pi/6)\left(\frac{5}{\sqrt{35}}\right) \end{bmatrix} = \begin{bmatrix} .866 \\ -.085 \\ .254 \\ .423 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} .866 \\ .085 \\ -.254 \\ -.423 \end{bmatrix}$$

.866 .5 .5

$$= \begin{bmatrix} -.085 \\ .866 \\ .423 \\ .254 \end{bmatrix} \begin{bmatrix} .866 \\ .085 \\ -.254 \\ -.423 \end{bmatrix} = \begin{bmatrix} -.3598 \\ -.1143 \\ .2392 \\ -.2553 \end{bmatrix}$$

3) Suppose distances in our virtual world are measured in meters. Let's assume peoples eyes are 6cm apart. We want (0.6m) to view a scene from pt (5,0,5) looking @ the (0,0,0) origin. What is the matrix of each eye?

center of screen $\hat{c}$ (unit vector)

point, eye position

$$\hat{c} = \frac{\vec{p} - \vec{e}}{\|\vec{p} - \vec{e}\|} = \frac{(0,0,0) - (5,0,5)}{\sqrt{(-5)^2 + 0^2 + (-5)^2}} = \boxed{\frac{(-5,0,-5)}{\sqrt{50}} = \hat{c}}$$

up direction of eye (unit vector)

$$\boxed{\hat{u} = (0,1,0)}$$

we know $\hat{z} = -\hat{c}$

$\hat{x} = \hat{u} \times \hat{z}$

$\hat{y} = \hat{z} \times \hat{x}$

$$\boxed{\hat{z} = \frac{(5,0,5)}{\sqrt{50}}}$$

$$\hat{x} = \hat{u} \times \hat{z} = \begin{matrix} 0 & 1 & 0 \\ 5/\sqrt{50} & 0 & 5/\sqrt{50} \end{matrix} = \left(\tfrac{5}{\sqrt{50}} - 0,\ 0 - 0,\ 0 - 5/\sqrt{50}\right)$$

$$\boxed{\hat{x} = \frac{(5,0,-5)}{\sqrt{50}}}$$

$$\hat{y} = \hat{z} \times \hat{x} = \begin{matrix} 5/\sqrt{50} & 0 & 5/\sqrt{50} \\ 5/\sqrt{50} & 0 & -5/\sqrt{50} \end{matrix} = \left(0-0,\ \tfrac{25}{50} + \tfrac{25}{50},\ 0-0\right)$$

$$\boxed{\hat{y} = (0,1,0)}$$

$$T_{eye} = \begin{bmatrix} \hat{x}_1 & \hat{x}_2 & \hat{x}_3 & 0 \\ \hat{y}_1 & \hat{y}_2 & \hat{y}_3 & 0 \\ \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_1 \\ 0 & 1 & 0 & -e_2 \\ 0 & 0 & 1 & -e_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$T_{eye} = \begin{bmatrix} -5/\sqrt{50} & 0 & 5/\sqrt{50} & 0 \\ 0 & 1 & 0 & 0 \\ 5/\sqrt{50} & 0 & 5/\sqrt{50} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5/\sqrt{50} & 0 & 5/\sqrt{50} & 0 \\ 0 & 1 & 0 & 0 \\ 5/\sqrt{50} & 0 & 5/\sqrt{50} & -50/\sqrt{50} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For right eye $T = T_{eye} T_{right} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -t/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ where $t = .06$ for 6cm

$\to .03$

$$= \begin{bmatrix} -.707 & 0 & .707 & .021 \\ 0 & 1 & 0 & 0 \\ .707 & 0 & .707 & 7.092 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For left eye $= T = T_{eye} T_{left} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & +.03 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -.707 & 0 & .707 & -.021 \\ 0 & 1 & 0 & 0 \\ .707 & 0 & .707 & 7.049 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$