Optimal pension funding through dynamic simulations: the case of Taiwan public employees retirement system

Shih-Chieh Chang *

Department of Risk Management and Insurance, National Chengchi University, Taipei, Taiwan

Abstract

An approach combining stochastic simulations and dynamic optimization is constructed to decide the optimal funding policy of the defined benefit pension scheme. The results show a significant advantage and flexibility of this approach in projecting the optimal financial status over the traditional deterministic pension valuation. In this study, the optimal contributions are estimated through dynamic programming under the projected workforce and specified constraints. Taiwan public employees retirement system (Tai-PERS) is studied for illustration. This article outlines the procedure of building the proposed dynamic procedure and presents the empirical findings from this study. ©1999 Elsevier Science B.V. All rights reserved.

Keywords: Stochastic simulation; Dynamic optimization; Optimal funding

1. Introduction

Haberman and Sung (1994) proposed a dynamic model of pension funding for a defined benefit plan. They introduced two types of risks concerning the stability and security of funding: the contribution rate risk and the solvency risk. An objective function associated with these two risks was constructed to derive the optimal contributions subject to specific constraints through dynamic optimization. Based on the optimal funding policy, the policy-maker could attain the plan's target financing status by amending the funding schedule.

In this study, the similar methodology is adopted in the planning process of pension funding. However, the turnover workforce and benefit outgo used in the objective function are projected under the assumption of an open group with size constraint. That is, we assume that the overall workforce should be controlled at a constant level, which is required by the government policy in order to reduce the financial burden of this pension system. Furthermore, stochastic simulation is used to estimate explicit financial information of this scheme. The contribution rate risk and the solvency risk are minimized through dynamic optimization. Traditional valuation techniques, however, can merely examine the plan status at a specific valuation date. Stochastic simulation and dynamic optimization are two of the best tools in policy-making, which can help the plan sponsor amend the benefit scheme or funding policy.

---

* E-mail: bchang@nccu.edu.tw

From a literature review, we find similar studies using stochastic simulations. Stochastic simulations using time as the operational parameter in Bacinello (1988) are performed to obtain the best estimates of the projected work-force, while the projected cash flows are scrutinized through dynamic simulations. An extensive review of past pension cost analyses can be found in Shapiro (1985). Early works can be found in Winklevoss (1982) who developed the pension liability and asset simulation model (PLASM) to evaluate the pension financing. The projection techniques can provide valuable inputs for the planning decisions, such as funding policy and investment strategy.

This study emphasizes the importance of pension cost analysis in optimizing the plan financial condition. Similar researches can be found in: Bowers et al. (1982), Winklevoss (1982), McKenna (1982), Shapiro (1977,1985), O'Brien (1986,1987), Bacinello (1988), Dufresne (1988,1989), Haberman (1992,1993,1994), Daykin et al. (1994), and Haberman and Sung (1994). A discussion of a rigorous and tractable stochastic model for pension fund can be found in Janssen and Manca (1997). Our proposed pension financing basically involves projecting a series of discounted cash flows for the future years through stochastic simulations according to the probabilistic experience set of actuarial assumptions. Then the optimal contribution rates are estimated subject to the given performance measure. A brief summary of the advantages of this approach is listed below:

  (i) Detailed demographics of the future workforce can be projected from this model.

  (ii) Through high speed computers, the plan sponsor can forecast the plan's future cash flows, which helps the fund management.

 (iii) The optimal contribution rates can be estimated under various scenarios based on specific plan strategies.

 (iv) The optimal funding and actuarial status of the plan can be estimated under risk measurement implemented through a computerized system.

2. Data description

In this research, the Taiwan public employees retirement system (Tai-PERS) is studied and evaluated through the proposed dynamic optimization procedure. In reality, fluctuations of the inflation rates, rate of returns, rate of increase in salaries and demographic factors subjected to recent economic conditions need to be taken into account when considering cost allocation and projection of the plan. In recent years, with an increase in the percentage of population that comes under pension age in Taiwan, pension related topics have taken on new significance and much attention has been focused on the implementation of a better retirement system for the aging society.

Tai-PERS is a large public retirement system that is designed to provide retirement and ancillary benefits to all government employees. There are 271,215 active members under the current benefit scheme. The present funding policy requires each member to contribute 2.8% of his covered salary while the sponsor of the plan contributes 5.2% of the plan member's covered payroll monthly to a public trust fund. This system is a defined benefit (DB) plan since the participant's retirement benefits are calculated according to the length of his service and his final salary upon retirement. To protect the retirement benefits against post-retirement inflation, the retirees have the option of a lump-sum retirement payment, monthly pension with the cost of living adjustments (COLA), or a mix between lump-sum payment and monthly pension. Since there is no clear relevance between the contribution and the benefit payments, the plan solvency risk is solely the responsibility of the government.

A sample of 3,823 participants is used to evaluate the performance of the proposed approach. A service table is constructed based on the experience data collected from 1 July 1995 to 30 June 1996. Owing to the limitation of the data collection, further updating this table is necessary. In Section 3, the proposed procedure within this framework is formulated. In Section 4, the optimal contributions are estimated under various risk measurements and the associated stochastic cash flows are performed. Section 5 presents the model validation justifications and the final results of optimal plan status of Tai-PERS. Concluding comments are given in Section 6.

3. The dynamic procedure

A simulation-based optimization procedure is proposed to assist policy-makers to evaluate the financing efficiency of the plan. Since management of the dynamic cash flows is critical to financial soundness of the plan, the model has emphasized on two elements that significantly affect the plan's financial balance. One is the annual optimal contribution of the plan members and sponsor, and the other is the fund actuarial status for the active members and pensioners. Every year, the cash inflow from plan members, plan sponsor and the investment returns should sustain the annual normal cost of this plan. Complexity of the actuarial valuation is sometimes viewed as another element of cost. Hence, the proposed simulation-based optimization procedure is built into a user-oriented computerized system to reduce duplication of efforts. This approach is capable of inputting user-specified inflation rates, fund return rates and demographic assumptions to compute the projected cash flows. We use the Visual Basic 5.0 program in the calculation. Implementing the proposed approach into a computerized system can achieve more efficient policy-making.

A dynamic approach is proposed to decide the pension funding subject to specific constraints. Under certain risk criteria given in the estimation, an optimal contribution can be obtained. In our study, the size constrained population assumption is used to project the plan workforce. The procedure of our approach is constructed as follows:

Step 1. The future information of the active and inactive members in the system is simulated through a series of dynamic processes using Bernoulli trials. The working status of each member is simulated according to the decrement probabilities from the service table. Let $E_{x,t}$ denote the working status of the employee age x between t and $t + 1$ year in the future. The process is shown as follows:

(i) Simulate $E_{x,t}$ by generating a pseudorandom number from Bernoulli ($p_x^{(\tau)}$), where $p_x^{(\tau)}$ is obtained from the service table.
(ii) If $E_{x,t} = 1$ (i.e., this member is in working status), then go to step (v).
(iii) If $E_{x,t} = 0$ (i.e., this member is not in working status), then a pseudorandom number of multinomial distribution is generated from

$$\text{Multi} \left(\frac{q_x^{(d)}}{q_x^{(\tau)}}, \frac{q_x^{(i)}}{q_x^{(\tau)}}, \frac{q_x^{(l)}}{q_x^{(\tau)}}, \frac{q_x^{(r)}}{q_x^{(\tau)}} \right)$$

where $q_x^{(\tau)} = q_x^{(d)} + q_x^{(i)} + q_x^{(l)} + q_x^{(r)}$. The superscript (i) stands for disability; (d) for death; (l) for layoff and withdrawal and (r) for retirement. Let $R_{x,t}$ denote the living status of the retiree aged x at time t and $B_{x,t}$ denote the option of his retirement benefits. If $R_{x,t} = 1$ (i.e., the member is retiring), a pseudorandom number $B_{x,t}$ is generated from Bernoulli ($b_{x,t}$) to simulate the chosen benefit program where $b_{x,t}$ is estimated from past experience.
(iv) If $B_{x,t} = 1$ (i.e., the retiree chooses the monthly annuity), the living status $P_{x,t}$ is generated from Bernoulli (q_{x+t}) where q_{x+t} is estimated from the annuity table of the retiree. If $B_{x,t} = 0$ (i.e., the retiree choose the lump-sum payment), the benefit payment is then computed.
(v) Let $t = t + 1$.

Step 2. The contributions are assumed to be paid at the beginning of each year in proportion to the covered payroll of each active employee. A constant rate is paid by the active employees, whereas the remaining part is paid by the government. The major cash flows are defined as following:

(i) Benefit functions B_t including withdrawal, disabled, death, layoff and retirement benefits.
(ii) Contributions C_t from active employees and new entrants.
(iii) Returns R_t from pension fund and the accumulated random fund asset F_t.

The process is shown as follows:

(i) Set time $t = 0$ at the plan valuation date.
(ii) Use the simulated database from Step 1.
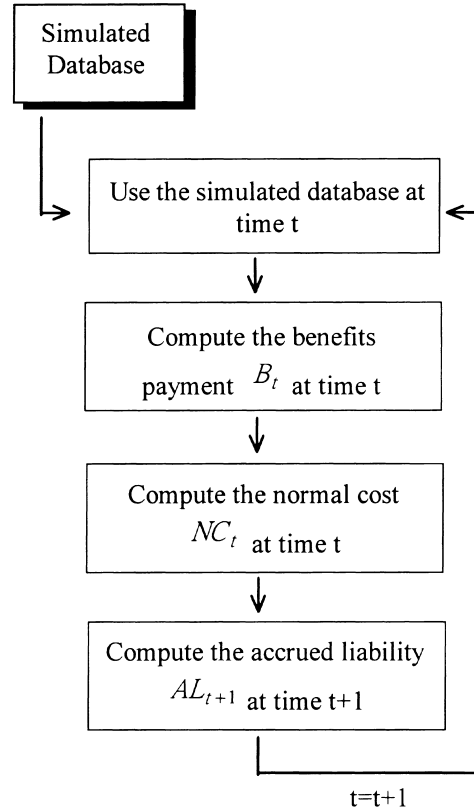(iii) Compute the benefits payment B_t at time t.

Fig. 1. Diagram of the procedures in Step 2.

(iv) Compute the normal cost NC_t at time t.
 (v) Compute the accrued liability AL_{t+1} at time $t + 1$.
(vi) Let $t = t + 1$.
A simple line diagram is plotted in Fig. 1 for clarification.

   The Monte Carlo simulations can be carried out by repeating Steps 1 and 2. Tables 1–3 provide the detailed procedure of this algorithm in Steps 1 and 2.

   Step 3. Based on the projected benefits, C_t is estimated through minimizing the performance measure defined in Eq. (1) from present to time $T - 1$. Detailed clarification and explanation of this performance measure will be discussed in Section 4.

$$\Gamma(C_0, \dots, C_{T-1}) = E\left\{\sum_{t=0}^{T-1}\left[v_t\left(1 - \frac{C_t}{NC_t}\right)^2 + v_{t+1}\beta_{t+1}\left(1 - \frac{F_{t+1}}{\eta AL_{t+1}}\right)^2\right]\right\}. \tag{1}$$

The actuarial notations and fund recursive relationship are given as follows:

N	the set of chosen time span
$\{i_t\}_{t \in N}$	return rate of pension fund at time t
$\{C_t\}_{t \in N}$	contribution at time t
$\{v_t\}_{t \in N}$	discount factor at time t
$\{\beta_t\}_{t \in N}$	risk weighted ratio at time t
$\{NC_t\}_{t \in N}$	normal cost at time t
$\{F_t\}_{t \in N}$	pension fund asset at time t

Table 1
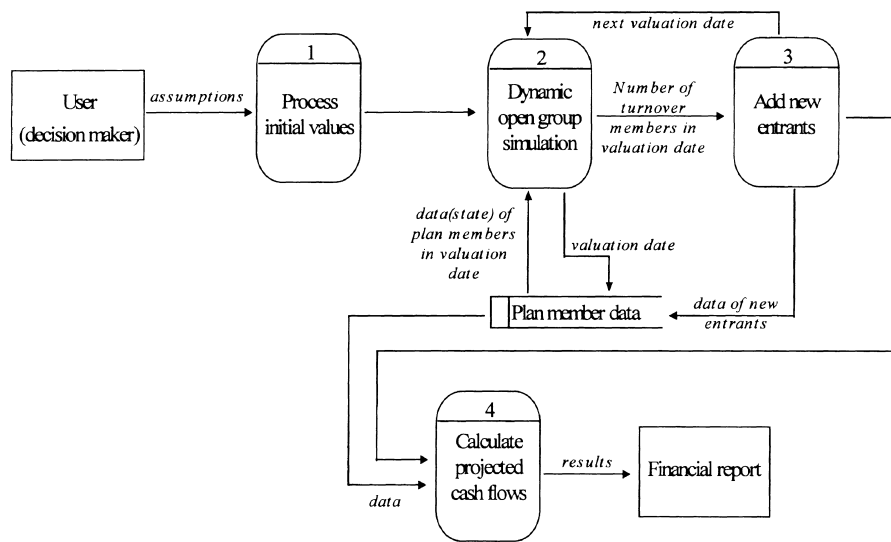Flow chart of the open group simulations

Table 2
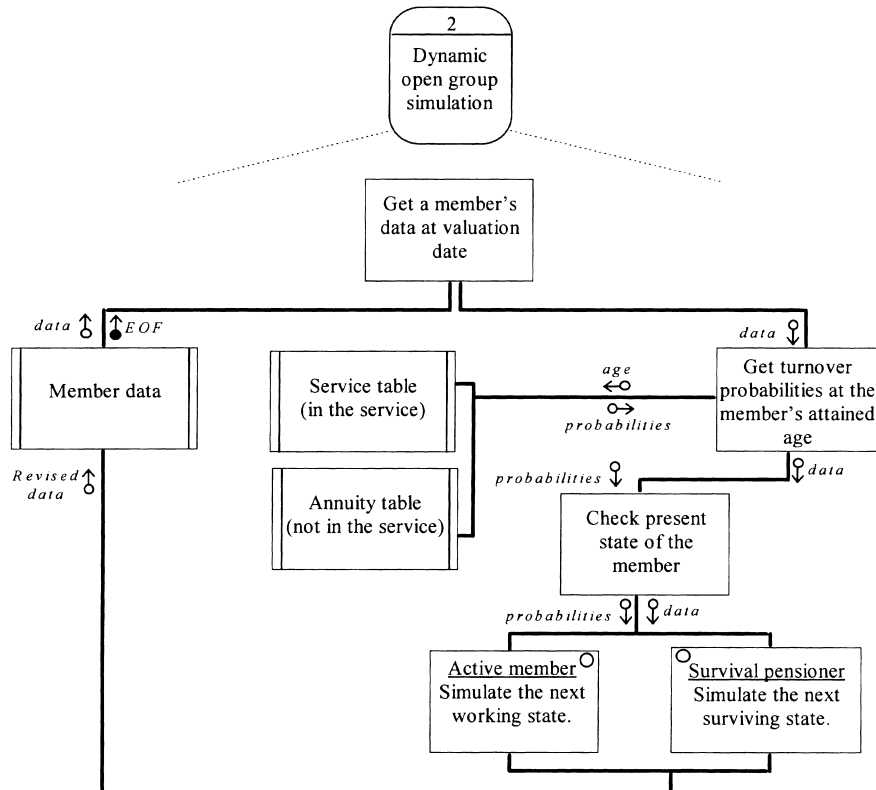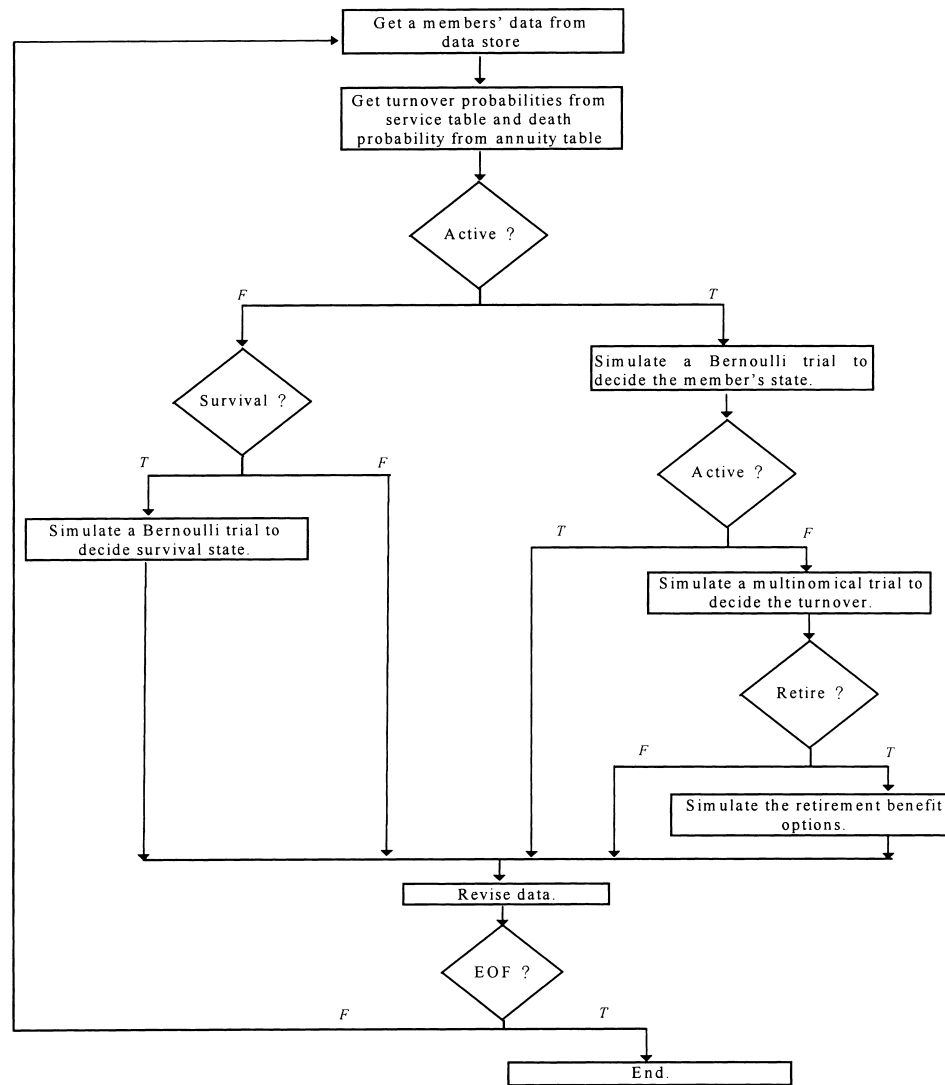Algorithms of the open group simulations in (2)

Table 3
Flow chart of the open group simulations in (2)

```
                    ┌─────────────────────────┐
              ┌────▶│  Get a members' data from│
              │     │       data store         │
              │     └─────────────┬───────────┘
              │                   ▼
              │     ┌─────────────────────────┐
              │     │ Get turnover probabilities from │
              │     │   service table and death      │
              │     │ probability from annuity table │
              │     └─────────────┬───────────┘
              │                   ▼
              │               ◇ Active ? ◇
              │           F  ╱         ╲  T
              │       ┌──────┘           └──────┐
              │       ▼                         ▼
              │   ◇ Survival ? ◇         ┌─────────────────────┐
              │  T ╱        ╲ F          │ Simulate a Bernoulli trial to │
              │ ┌──┘          └──┐       │ decide the member's state.    │
              │ ▼                │       └──────────┬──────────┘
              │ ┌──────────────┐ │                  ▼
              │ │Simulate a Bernoulli trial to│  ◇ Active ? ◇
              │ │ decide survival state.│    T ╱       ╲ F
              │ └──────────────┘        ┌────┘         └────┐
              │                         │                   ▼
              │                         │      ┌─────────────────────┐
              │                         │      │Simulate a multinomical trial to│
              │                         │      │   decide the turnover.│
              │                         │      └──────────┬──────────┘
              │                         │                 ▼
              │                         │             ◇ Retire ? ◇
              │                         │          F ╱        ╲ T
              │                         │      ┌─────┘         └──┐
              │                         │      │                  ▼
              │                         │      │   ┌─────────────────────┐
              │                         │      │   │Simulate the retirement benefit│
              │                         │      │   │      options.         │
              │                         │      │   └──────────┬──────────┘
              │                         ▼      ▼              │
              │           ┌──────────────────────────┐
              │           │       Revise data.        │
              │           └─────────────┬────────────┘
              │                         ▼
              │                     ◇ EOF ? ◇
              │                 F ╱        ╲ T
              └──────────────────┘         └──┐
                                               ▼
                                ┌──────────────────────────┐
                                │          End.             │
                                └──────────────────────────┘
```

$\{AL_t\}_{t \in N}$ accrued liability at time t

η target fund ratio

During the year between time t and $t + 1$ the fund balance will increase by the contribution C_t and decrease by the benefit outgo B_t at the beginning of the year simulated from Step 2 plus the investment return

$$F_{t+1} = (F_t + C_t - B_t)(1 + i_t), \quad i_t \sim \text{IIDN}(\theta, \sigma^2) \tag{2}$$

where the real return i_t between time t and $t + 1$ are assumed to be a sequence of independent and identically distributed normal (IIDN) variables with mean θ and variance σ^2.

4. The proposed model

After the results from these simulations are obtained, the benefit payments are estimated and recorded in the database. In our approach, time is assumed to be the operational parameter and the above steps are repeated as the fund development in time. A simpler procedure focusing on individual proposed by Bacinello (1988) is not used since we intend to forecast dynamically the overall financial condition of the plan each year. Since bias may exist owing to variation in various realizations, we summarize the statistics of these estimates based on the simulations. For a particular jth simulation, the projected benefit at time $t + 1$ is denoted by $_jB_{t+1}$. From these simulations, a median estimate $_j\hat{B}_{t+1}$ that satisfies $\Pr(_jB_{t+1} \leq_j \hat{B}_{t+1}) \leq 50\%$, $J =\{$set of all simulations$\}$ is used to estimate the contribution at time $t + 1$. The cash flows of the benefit payments, normal costs and the accrued liability in 20 years are projected. Then the optimal contributions are computed through the optimization procedure.

In order to determine the parameters that best fit the data, a performance measure, $_0\Gamma_{T-1}$, is defined to measure the adequacy of the estimation between time 0 and time $T - 1$. The performance measure is a nonnegative function based on the sum of discounted squared deviations. The parameter estimates are determined by minimizing the combined performance measure in Eq. (1).

Similar to Haberman and Sung (1994), $v_t(1 - C_t/NC_t)^2$ is used to denote the contribution rate risk, while $v_{t+1}\beta_{t+1}(1 - F_{t+1}/AL_{t+1})^2$ is used to denote the solvency risk where β_{t+1} is the relative weight in measuring the fund financial stability at time t. The relative ratios are used in measuring the discounted quadratic deviation over the chosen time horizon. The advantage of using this criterion is in decision making process, which is clearly requisite in predicting the performance ratio by years. Then the optimization could be formulated as

$$\min_{C_t,...,C_{T-1}_t} \Gamma_{T-1} = \min_{C_t,...,C_{T-1}} E\sum_{s=t}^{T-1}\left[v_s\left(1 - \frac{C_s}{NC_s}\right)^2 + v_{s+1}\beta_{s+1}\left(1 - \frac{F_{s+1}}{\eta AL_{s+1}}\right)^2 \middle| H_t\right], \tag{3}$$

where H_t is the σ-field generated by $\{F_0, \dots, F_t\}$. $\{F_t\}_{t\in N}$ are assumed to follow a first-order Markov process, which is the case that Tai-PERS internally evaluate its financial status annually. Then

$$E\sum_{s=t}^{T-1}\left[v_s\left(1 - \frac{C_s}{NC_s}\right)^2 + v_{s+1}\beta_{s+1}\left(1 - \frac{F_{s+1}}{\eta AL_{s+1}}\right)^2 \middle| H_t\right]$$
$$= E\sum_{s=t}^{T-1}\left[v_s\left(1 - \frac{C_s}{NC_s}\right)^2 + v_{s+1}\beta_{s+1}\left(1 - \frac{F_{s+1}}{\eta AL_{s+1}}\right)^2 \middle| F_t\right]. \tag{4}$$

By defining $V_t(F_t)$ as

$$V_t(F_t) = \min_{C_t,...,C_{T-1}} E\sum_{s=t}^{T-1}\left[v_s\left(1 - \frac{C_s}{NC_s}\right)^2 + V_{s+1}\beta_{s+1}\left(1 - \frac{F_{s+1}}{\eta AL_{s+1}}\right)^2 \middle| F_t\right], \tag{5}$$

we have

$$V_t(F_t) = \min_{C_t} E\left\{ v_t\left(1 - \frac{C_t}{NC_t}\right)^2 + v_{t+1}\beta_{t+1}\left(1 - \frac{F_{t+1}}{\eta AL_{t+1}}\right)^2 |F_t + V_{t+1}(F_{t+1})| F_t\right\} \tag{6}$$

then C_t can be estimated by induction. To solve the Bellman equation (Aström, 1970), we assume that $V_t(F_t) = a_1(t)F_t^2 + a_2(t)F_t + a_3(t)$ for all t, $0 \leq t \leq T$. Eq. (6) could be written as $\Gamma(C_t)$ which is a second-order function of C_t. Then the optimal contribution $\hat{C}_t$ satisfying the unique solution is obtained in Eq. (7).

$$\hat{C}_t = \frac{2v_t/NC_t + 2v_{t+1}\beta_{t+1}H/\eta AL_{t+1} - 2(v_{t+1}\beta_{t+1}/\eta^2 AL_{t+1}^2 + a_1(t+1))K(F_t - B_t) - a_2(t+1)H}{2vt/NC_t + 2v_{t+1}\beta_{t+1}K/\eta^2 AL_{t+1}^2 + 2a_1(t+1)K}, \tag{7}$$

where

$$H = 1 + \theta, \qquad K = H + \sigma^2.$$

We set $a_1(T) = a_2(T) = a_3(T) = 0$ as the boundary condition in the above equation which means no bias at time $t = T$. That is, the long term financial status of this pension system is assumed to have no risk incurred. Then we could solve the recursive relationship for $a_1(t)$ and $a_2(t)$ for $0 \leq t \leq T$. Given

$$D_t = \frac{2v_t}{NC_t} + \frac{2v_{t+1}\beta_{t+1}H}{\eta AL_{t+1}} + \frac{2v_{t+1}\beta_{t+1}K}{\eta^2 AL_{t+1}^2}B_t + 2a_1(t+1)KB_t - a_2(t+1)H,$$

$$E_t = -\frac{2v_{t+1}\beta_{t+1}K}{\eta^2 AL_{t+1}^2} - 2a_1(t+1)K,$$

$$G_t = \frac{2v_t}{NC_t} + \frac{2v_{t+1}\beta_{t+1}K}{\eta^2 AL_{t+1}^2} + 2a_1(t+1)K,$$

the recursive relationship for $a_t(t)$ and $a_2(t)$ are solved by the following equations:

$$a_1(t) = \frac{v_t E_t^2}{G_t^2 NC_t^2} + v_{t+1}\beta_{t+1}\frac{K}{\eta^2 AL_{t+1}^2}\left(1 + \frac{E_t}{G_t}\right)^2 + a_1(t+1)K\left(1 + \frac{E_t}{G_t}\right)^2, \tag{8}$$

$$\begin{aligned}
a_2(t) = &\frac{2E_t}{G_t NC_t}\left(\frac{D_t}{G_t NC_t} - 1\right) - 2v_{t+1}\beta_{t+1}\frac{H}{\eta AL_{t+1}}\left(1 + \frac{E_t}{G_t}\right) \\
&+ 2v_{t+1}\beta_{t+1}\frac{K}{\eta^2 AL_{t+1}^2}\left(1 + \frac{E_t}{G_t}\right)\left(\frac{D_t}{G_t} - B_t\right) + 2a_1(t+1)K\left(1 + \frac{E_t}{G_t}\right)\left(\frac{D_t}{G_t} - B_t\right) \\
&+ a_2(t+1)H\left(1 + \frac{E_t}{G_t}\right). \tag{9}
\end{aligned}$$

5. Results and analysis

We now use Tai-PERS to illustrate these results. 3,823 samples are collected for the calculations. The actuarial assumptions are:

– *Population:* Tai-PERS service table based on 1995–1996; 1989 Taiwan Standard Ordinary life table (1989 TSO) for the retiree's annuity table.
– *Actuarial cost method:* Entry age normal (EAN) cost method.
– *Salary scale and inflation rate:* 3.5% for annual salary increase and 3% for annual inflation rate.
– *Interest rate:* 6% for pension valuation, i.e., we assume $v_t = v^t = (1.06)^{-t}$, i.e., a constant discount rate assumption.
– *Target fund ratio:* $\eta = 75\%$ for every year.
– *Risk measurement weight:* $\beta_t = 60\%$ for every year.
– *Fund return rate:* $\theta = 10\%$ and $\sigma^2 = 0.04\%$.

With the above assumptions, the estimated actuarial accrued liabilities, normal costs and benefit payments are simulated based on Steps 1 and 2. The numerical values of the optimal contributions are listed in Table 4.

We assume that the fund provides benefits for 3,823 employees in Tai-PERS and the initial fund is set to be 373,211,585 NT dollars. This corresponds to 1.41% of the 271,215 plan participants in 1996. The future optimal fund status is forecasted through the recursive relation in Eqs. (8) and (9). With the boundary condition being to have no risk at the end of the forecast period, the explicit optimal contributions could be projected by recursively

Table 4
Optimal contributions of Tai-PERS

	1997	1998	1999	2000	2001	2002	2003	2004
Fund asset	373,211,585	600,381,624	861,349,176	1,120,815,583	1,339,070,206	1,765,948,737	2,092,758,922	2,476,541,889
Optimal contributions	275,496,575	256,618,424	253,850,803	246,475,587	238,192,682	277,859,062	224,117,339	219,592,961
Benefits outgo	106,636,560	48,948,364	89,903,272	95,409,192	67,699,656	76,361,992	65,474,592	102,758,288
Accrued liability	585,530,240	851,652,800	1,152,353,024	1,461,198,464	1,775,253,248	2,101,083,136	2,454,753,024	2,813,157,888
Normal cost	264,658,176	254,203,072	250,750,880	247,360,208	243,759,120	240,339,776	238,243,632	235,766,864
Covered payroll	1,091,617,528	1,136,869,006	1,166,532,793	1,195,472,961	1,224,048,954	1,260,401,090	1,291,833,377	1,324,803,499
Fund return rate	10.76%	6.60%	9.32%	10.00%	12.51%	9.14%	10.00%	10.00%
$F_t/\eta AL_t$	0.850	0.940	0.997	1.023	1.051	1.121	1.137	1.174
C_t/NC_t	1.041	1.010	1.012	0.996	0.977	0.948	0.941	0.931
Optimal contribution rate	25.24%	22.57%	21.76%	20.62%	19.46%	18.08%	17.35%	16.58%

	2005	2006	2007	2008	2009	2010	2011	2012
Fund asset	2,852,714,280	3,293,112,124	3,641,364,756	4,073,949,110	4,563,110,444	5,013,138,724	5,476,497,663	6,060,102,605
Optimal contributions	215,734,838	210,697,528	208,076,022	206,097,227	203,120,348	203,325,805	203,156,242	202,318,111
Benefits outgo	134,871,760	193,478,128	161,202,912	209,269,152	186,510,336	222,970,272	225,303,168	255,283,040
Accrued liability	3,183,145,472	3,578,989,056	3,953,183,488	4,343,038,464	4,712,074,752	5,146,928,640	5,565,712,384	5,987,062,784
Normal cost	233,146,752	231,159,440	227,970,288	225,249,024	221,934,336	219,645,904	216,970,176	214,121,360
Covered payroll	1,358,531,004	1,384,319,519	1,413,676,091	1,435,214,137	1,472,414,452	1,496,865,737	1,521,840,935	1,549,543,771
Fund return rate	12.26%	10.00%	10.46%	12.09%	9.46%	9.67%	11.11%	10.00%
$F_t/\eta AL_t$	1.195	1.227	1.228	1.251	11.291	1.299	1.312	1.350
C_t/NC_t	0.925	0.911	0.913	0.915	0.915	0.926	0.936	0.945
Optimal contribution rate	15.88%	15.22%	14.72%	14.36%	13.80%	13.58%	13.35%	13.06%

	2013	2014	2015	2016	2017	
Fund asset	6,607,851,586	6,985,743,560	7,400,130,518	7,998,144,864	8,309,446,988	
Optimal contributions	202,729,988	202,641,694	202,119,295	200,590,284		
Benefits outgo	340,608,224	384,232,000	422,253,632	510,401,472	564,830,528	
Accrued liability	6,427,823,616	6,821,858,816	7,183,346,176	7,431,084,544	7,703,323,648	
Normal cost	212,082,800	209,096,448	206,210,160	202,645,072	199,309,184	
Covered payroll	1,570,998,111	1,584,815,608	1,584,733,735	1,598,323,058	1,607,790,314	
Fund return rate	7.97%	8.76%	11.39%	8.08%		
$F_t/\eta AL_t$	1.371	1.365	1.374	1.435	1.438	Fund return mean:10%
						Fund return variance:0.04%
C_t/NC_t	0.956	0.969	0.980	0.990		Discount_rate:6%
						Risk_weight:0.6
Optimal contribution rate	12.90%	12.79%	12.75%	12.55%		Fund_ratio:75%

computing estimates C_t using Eq. (7). Fig. 2 shows the simulated benefit payments, normal costs, accrued liabilities between 1998 and 2017 based on 50 dynamic simulations. The benefit payments and accrued liabilities are increasing by years while the normal costs are decreasing. In Table 2, the results show that the contribution ratios are increasing through the years (i.e., 1.041% in 1997) and finally approach the target fund ratio at 0.99% in 2017. The contributed rates that are obtained by dividing the total contribution by covered payroll are highlighted by the dotted line. It shows that contribution rates are decreasing by years from 25.24% in 1997 to 12.55% in 2017.
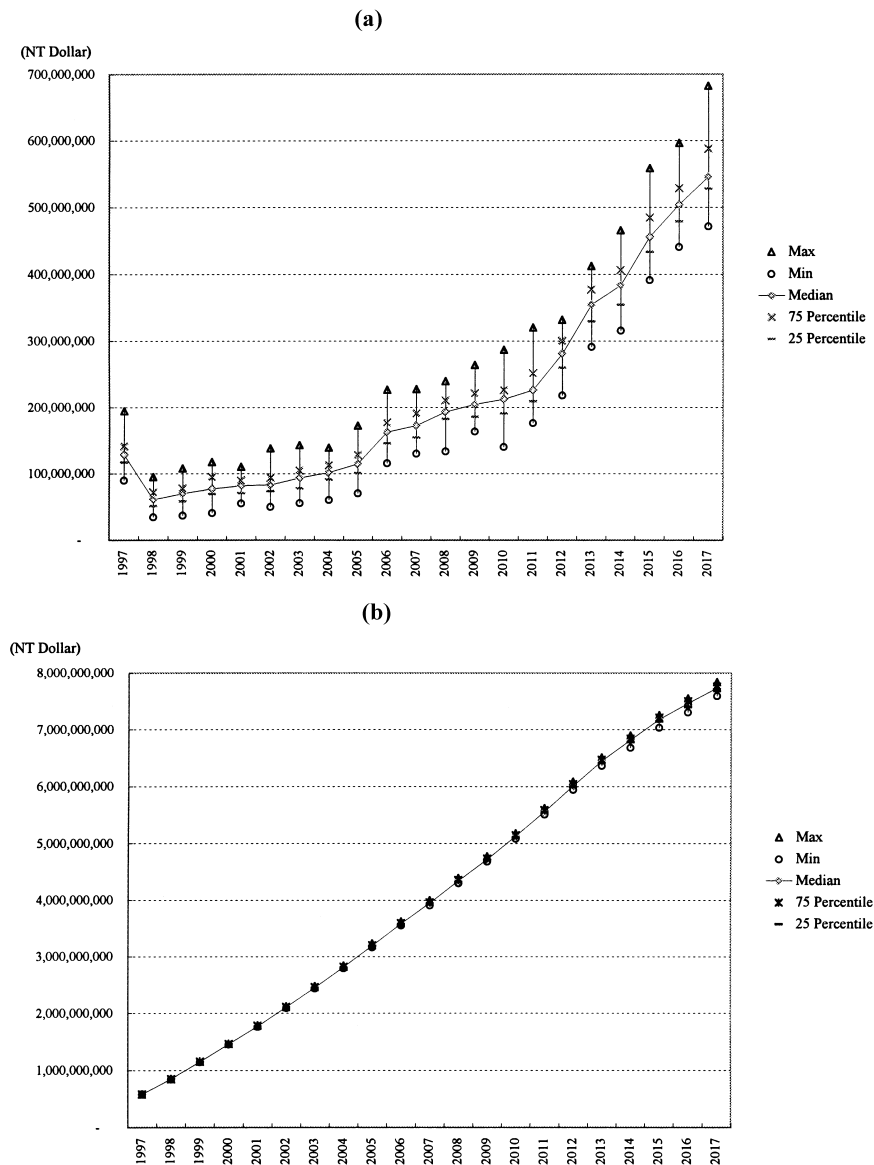
(a)

(b)

Fig. 2. Continued....

Fig. 3 compares the contribution ratios and optimal fund ratios from 1997 to 2016. It shows that the estimated contribution ratios vary by years, suggesting that the variable contribution ratios are influenced by the demographic assumptions based on the simulations. The optimal fund ratios are gradually increasing by years from 85% in 1998 to 143.5% in 2017. The optimal fund ratios significantly deviating from 1 between 1997 and 2017 might be partially due to the assigned risk measurement weight $\beta = 0.6$ since the weight assigned in contribution rate risk is larger than that in solvency risk. [1] In this study, a set of given actuarial assumptions are chosen to illustrate the optimal procedure. By varying the assumptions, the plan manager could foresee the optimal plan financial status.

---

[1] This pattern may be caused by the given risk measurement ratio. Further analysis is needed to explore this.
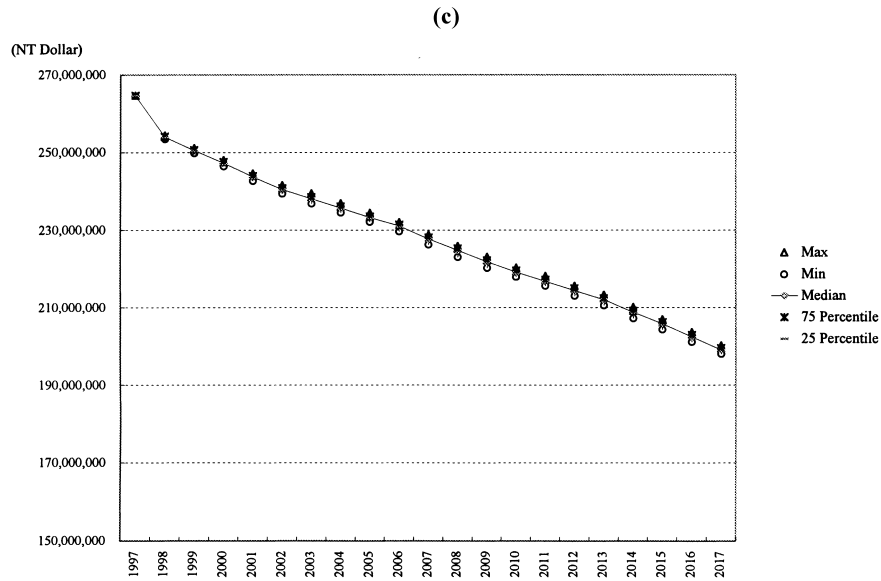
(c)

Fig. 2. (a) Simulated benefit in 1998–2017 (open dynamical model, number of simulations= 50); interest rate= 6%; salary increase rate= 3.5%. (b) Simulated accrued liability in 1998–2017 (open dynamical model, number of simulations= 50); interest rate= 6%; cost method: EAN. (c) Simulated normal cost in 1998–2017 (open dynamical model, number of simulations= 50); interest rate= 6%; cost method: EAN.
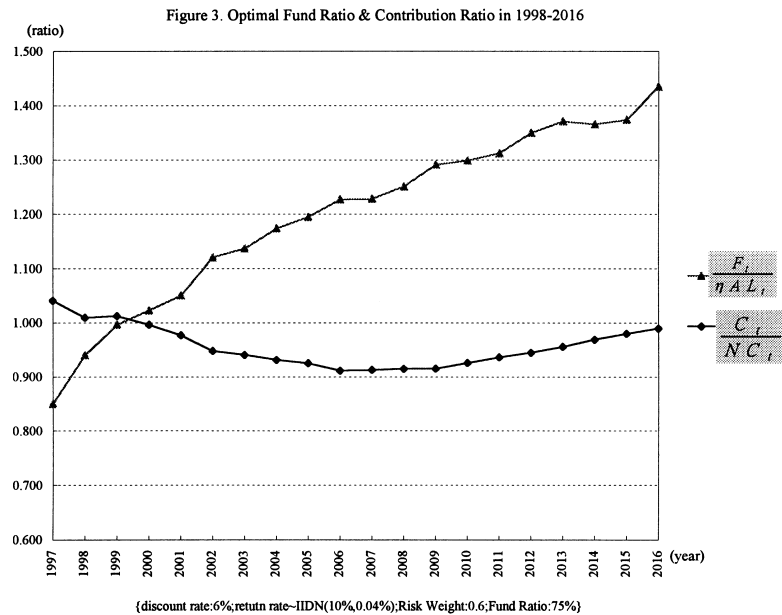

Fig. 3. Optimal fund ratio and contribution ratio in 1998–2016.

Fig. 4 compares the optimal contribution rates under various risk measurements. The risk ratios are modified to reflect the user's subjective risk measurement in performing the optimization. It shows that the contribution rates increase with an increase in risk measurement. Through minimizing the performance measure, the explicit fund information could be obtained from Eq. (1).
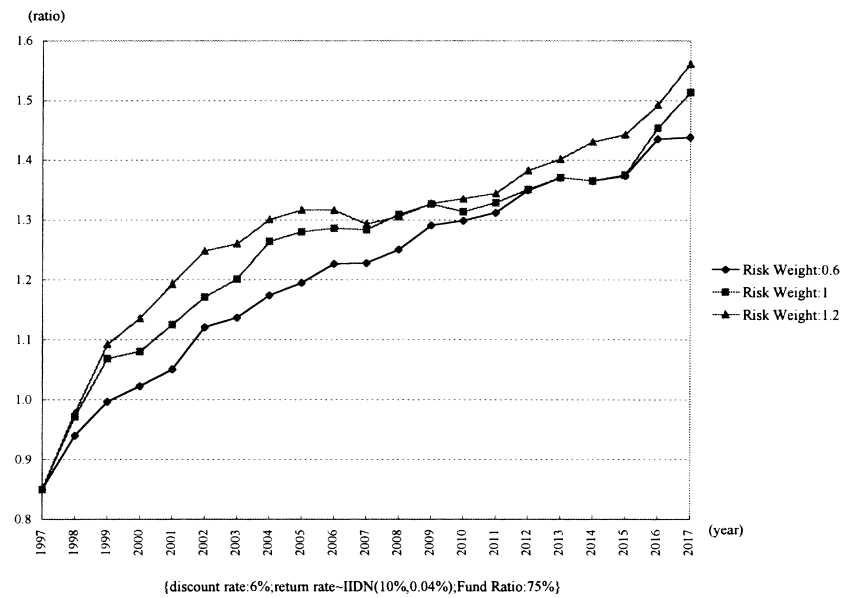
{discount rate:6%;return rate~IIDN(10%,0.04%);Fund Ratio:75%}

Fig. 4. Optimal fund ratio in 1998–2017 under various risk weight.

{discount rate:6%;return rate~IIDN(10%,0.04%);Risk Weight:0.6}

Fig. 5. Optimal fund ratio in 1998–2017 under various target fund ratio.

Fig. 5 compares the optimal contribution rates under various target fund ratios.The target fund ratios are given at various levels with respect to the attained accrued liabilities to reflect the user's subjective management requirement. It shows that the fund ratios are increasing at higher target fund ratios, while the trend of increases in fund ratios have shown a similar pattern both in Figs. 4 and 5. This occurs because we set a constant mean of the return rate at 10% for the fund in 1997–2017.

6. Conclusions

The purpose of this paper is to provide a flexible method which allows the plan manager to incorporate his own risk measurement in funding policy. The approach in this model has integrated two relatively recent innovations in pension financing: the stochastic simulations in estimating the factors in the objective function and the dynamic programming in calculating the optimal contributions. This study could be adopted in assisting policy making and as a benchmark to compare the results from the many actuarial cost methods used in plan valuation.

The approach proposed in this paper has linked the possible scenarios and equation-type optimal solutions together, which provide an alternative way to decide funding policy. The stochastic modeling of the liabilities may provide helpful guidance on investment strategy for proper assessment of the trade-off between various risks. Hence the future fund financing could be explored in detail using this approach. Since controlling the pension fund solvency and keeping contribution at a steady level are two main goals in pension financing, the proposed method proves helpful in achieving these goals.

References

Aström, K.J., 1970. Introduction to Stochastic Control Theory. Academic Press, New York.

Bacinello, A.R., 1988. A stochastic simulation procedure for pension scheme. Insurance: Mathematics and Economics 7, 153–161.

Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A., Nesbitt, C.J., 1982. Notes on the dynamics of pension funding. Insurance: Mathematics and Economics 1, 261–270.

Daykin, C.D., Pentikainen, T., Pesonen, M., 1994. Practical Risk Theory for Actuaries. Monographs on Statistics and Applied Probability, vol. 53. (Chapman & Hall, London).

Dufresne, D., 1988. Moments of pension fund contributions and fund levels when rates of return are random. Journal of the Institute of Actuaries 115, 535–544.

Dufresne, D., 1989. Stability of pension systems when rates of return are random. Insurance: Mathematics and Economics 6, 129–134.

Haberman, S., 1992. Pension funding with time delays: a stochastic approach. Insurance: Mathematics and Economics 11, 179–189.

Haberman, S., 1993. Pension funding with time delays and autoregressive rates of investment return. Insurance: Mathematics and Economics 13, 45–56.

Haberman, S., 1994. Autoregressive rates of return and the variability of pension contributions and fund levels for a defined benefit pension scheme. Insurance: Mathematics and Economics 14, 219–240.

Haberman, S., Sung, J.H., 1994. Dynamic approaches to pension funding. Insurance: Mathematics and Economics 15, 151–162.

Janssen, J., Manca, R., 1997. A realistic non-homogeneous stochastic pension fund model on scenario basis. Scandinavian Actuarial Journal 2, 113–137.

McKenna, F.W., 1982. Pension plan cost risk. Journal of Risk and Insurance 49, 193–217.

O'Brien, T., 1986. A stochastic-dynamic approach to pension funding. Insurance: Mathematics and Economics 5, 141–146.

O'Brien, T., 1987. A two-parameter family of pension contribution functions and stochastic optimization. Insurance: Mathematics and Economics 6, 129–134.

Shapiro, A.F., 1977. The relevance of expected persistency rates when projecting pension costs. Journal of Risk and Insurance XLIV, 623–638.

Shapiro, A.F., 1985. Contributions to the evolution of pension cost analysis. Journal of Risk and Insurance 52, 81–99.

Winklevoss, H.E., 1982. Plasm: pension liability and asset simulation model. Journal of Finance XXXVII (2) 585-594.