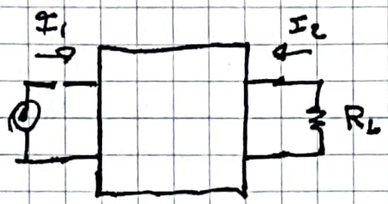
TS13

1)



$$\frac{I_2}{I_1} = H \frac{s^2+5s+4}{s^2+8s+12} \qquad z_{21} = 6H$$

$$-I_2 R_L = V_2 \quad \text{(I)} \qquad V_1 = z_{11} I_1 + z_{12} I_2$$

$$\text{(II)} \quad V_2 = z_{21} I_1 + z_{22} I_2$$

(I) en (II)

$$-I_2 R_L = z_{21} I_1 + I_2 z_{22}$$

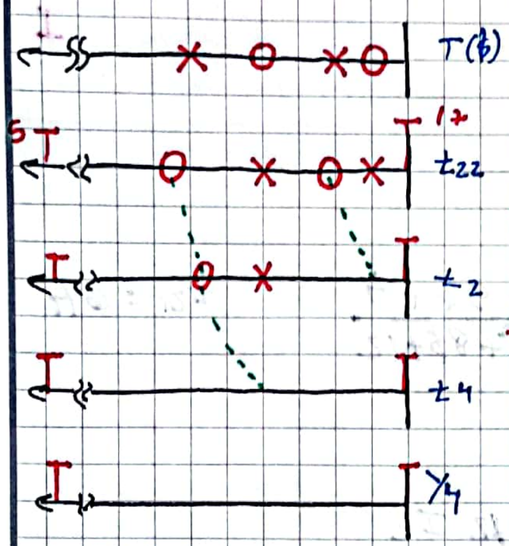
$$-I_2 (R_L + z_{22}) = z_{21} I_1$$

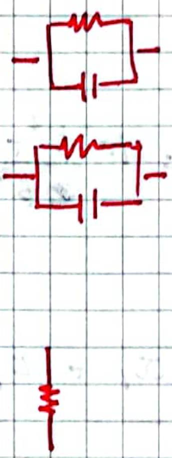
$$-\frac{I_2}{I_1} = \left.\frac{z_{21}}{R_L + z_{22}}\right|_{R_L=1} = \frac{z_{21}}{1 + z_{22}}$$

$$H \frac{s^2+5s+4}{s^2+8s+12} = \frac{6H}{1+z_{22}}$$

$$1 + z_{22} = \frac{(s^2+8s+12)\,6\cancel{H}}{(s^2+5s+4)\,\cancel{H}} \Rightarrow z_{22} = \frac{6s^2+48s+72-s^2-5s-4}{s^2+5s+4}$$

$$z_{22} = \frac{5s^2+43s+68}{s^2+5s+4}$$

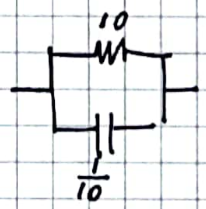




$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$z_{22} = \frac{5s^2+43s+68}{(s+1)(s+4)} \qquad K_2 = \lim_{s \to -1} (s+1)\,\frac{5s^2+43s+68}{(s+1)(s+4)} = \frac{30}{3} = 10$$

$$\frac{10}{s+1} =$$

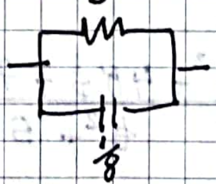


$$z_2 = \frac{5s^2+43s+68}{(s+1)(s+4)} - \frac{10}{(s+1)} = \frac{5s^2+43s+68-10s-40}{(s+1)(s+4)} =$$

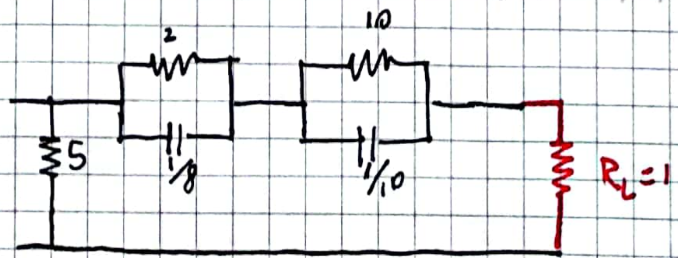
$$z_2 = \frac{5s^2+33s+28}{(s+1)(s+4)} = \frac{5(s+1)(s+5{,}6)}{(s+1)(s+4)} = \frac{5s+28}{s+4}$$

$$K_4 = \lim_{s \to -4} (s+4)\,\frac{5s+28}{s+4} = 8$$

$$\frac{8}{s+4} =$$



$$z_4 = \frac{5s+28}{s+4} - \frac{8}{s+4} = \frac{5s+28-8}{s+4} = \frac{5(s+4)}{s+4} = 5 \qquad Y_4 = \frac{1}{5}$$



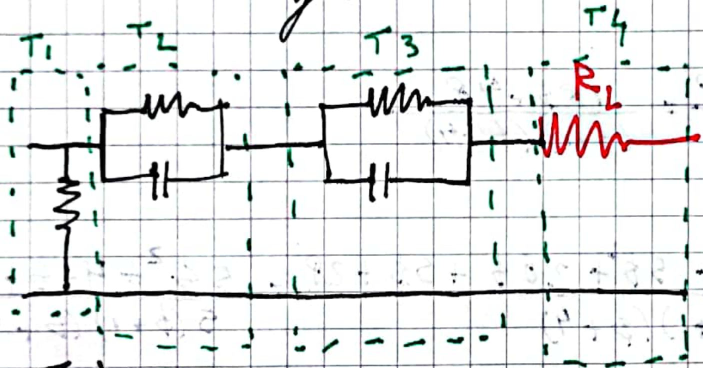
Verificación

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$A = \left.\dfrac{V_1}{V_2}\right|_{-I_2=0}$ $B = \left.\dfrac{V_1}{-I_2}\right|_{V_2=0}$

$C = \left.\dfrac{I_1}{V_2}\right|_{-I_2=0}$ $D = \left.\dfrac{I_1}{-I_2}\right|_{V_2=0}$

Con un leve artilugio



$$\frac{1}{D} = \left.\frac{-V_2}{I}\right|_{V_2=0}$$

La R_L queda indeterminada y en la red original cargada.

$T_1 = \begin{bmatrix} 1 & 0 \\ \frac{1}{5} & 1 \end{bmatrix}$ $T_2 \begin{bmatrix} 1 & \frac{8}{s+4} \\ 0 & 1 \end{bmatrix}$ $T_3 = \begin{bmatrix} 1 & \frac{10}{s+1} \\ 0 & 1 \end{bmatrix}$ $T_4 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$\begin{array}{cc} & \begin{bmatrix} 1 & \frac{8}{s+4} \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ \frac{1}{5} & 1 \end{bmatrix} & \begin{bmatrix} 1 & \frac{8}{s+4} \\ \frac{1}{5} & \frac{5s+28}{5(s+4)} \end{bmatrix} \end{array}$$

$$\frac{8}{s+4}\cdot\frac{1}{5} + 1 = \frac{8 + 5s + 20}{5\,(s+4)} = \frac{5s+28}{5(s+4)}$$

$$\begin{bmatrix} 1 & \frac{10}{s+1} \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{8}{s+4} \\ \frac{1}{5} & \frac{5s+28}{5(s+4)} \end{bmatrix} \begin{bmatrix} 1 & \frac{18s+48}{(s+1)(s+4)} \\ \frac{1}{5} & \frac{5s^2+43s+68}{5(s+1)(s+4)} \end{bmatrix} \begin{bmatrix} 1 & \frac{s^2+23s+52}{s^2+5s+4} \\ \frac{1}{5} & \frac{6}{5}\frac{(s^2+8s+12)}{(s^2+5s+4)} \end{bmatrix}$$

$$\frac{10}{s+1} + \frac{8}{s+4} = \frac{10s+40+8s+8}{(s+1)(s+4)} = \frac{18s+48}{(s+1)(s+4)}$$

$$\frac{10}{5(s+1)} + \frac{5s+28}{5(s+4)} = \frac{10s+40+5s^2+28s+5s+28}{5(s+1)(s+4)} = \frac{5s^2+43s+68}{5(s+1)(s+4)}$$

$$1 + \frac{18s+48}{s^2+5s+4} = \frac{s^2+23s+52}{s^2+5s+4}$$

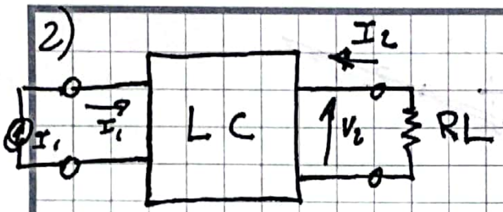
$$D = \frac{1}{5} + \frac{5s^2+43s+68}{5(s^2+5s+4)} = \frac{5s^2+43s+68+s^2+5s+4}{5(s^2+5s+4)}$$

$$\frac{6s^2+48s+72}{5(s^2+5s+4)} = \frac{6}{5}\frac{(s^2+8s+12)}{(s^2+5s+4)}$$

$$\frac{1}{D} = \frac{5}{6}\frac{s^2+5s+4}{s^2+8s+12} = H\frac{s^2+5s+4}{s^2+8s+12}$$

$$\boxed{H = \frac{5}{6}}$$

2)



$$-V_2 \frac{1}{R_L} = I_2 \quad \text{I}$$

$$T(s) = \frac{V_2}{I_1} = \frac{K \; s}{s^3+2s^2+2s+1}$$

$Y_{11} V_1 + Y_{12} V_2 = I_1$

$Y_{21} V_1 + Y_{22} V_2 = I_2$

o

$z_{11} I_1 + z_{12} I_2 = V_1$

$$\boxed{z_{21} I_1 + z_{22} I_2 = V_2} \quad \text{II}$$

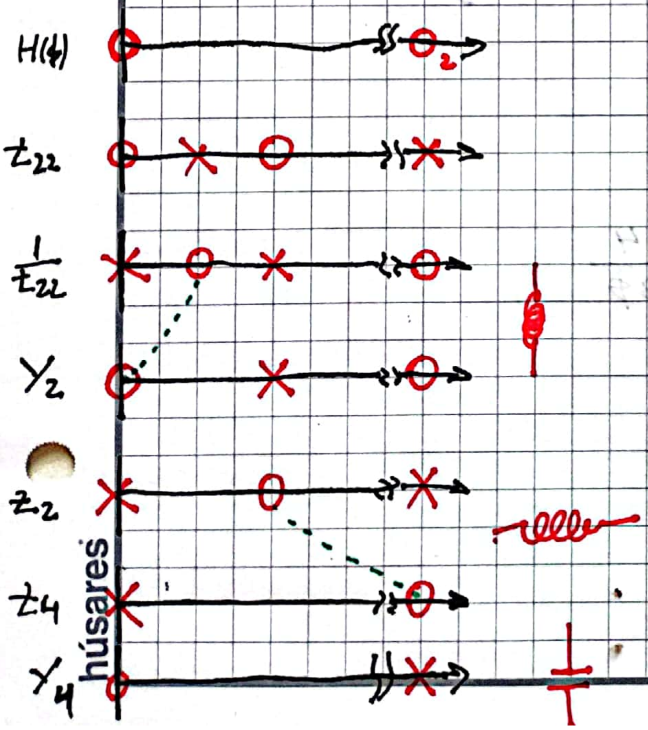
I en II

$$z_{21} I_1 - \frac{V_2}{R_L} z_{22} = V_2 \Rightarrow \frac{V_2}{I_1} = \frac{z_{21}}{1+\frac{z_{22}}{R_L}}\Bigg|_{R_L=1} = \frac{z_{21}}{1+z_{22}}$$

$$T(s) = K \frac{s \; \cancel{2s^2+1}}{\underset{2s^2+1}{s^3+2s} + \underset{2s^2+1}{2s^2+1}} = K \frac{\frac{s}{2s^2+1}}{1+\frac{s^3+2s}{2s^2+1}}$$

$$z_{22} = \frac{s\,(s^2+2)}{(2s^2+1)}$$

$$z_{22} = \frac{V_2}{I_2}\Bigg|_{I_1=0}$$



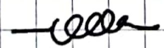
$$z_{22} = \frac{s(s^2+2)}{2s^2+1}$$

$$\frac{1}{z_{22}} = \frac{2s^2+1}{s(s^2+2)} \qquad K_{02} = \lim_{s\to 0} s\,\frac{2s^2+1}{s(s^2+2)} = \frac{1}{2}$$

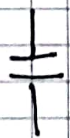
$$\frac{1}{2s} =$$  2

$$Y_2 = \frac{2s^2+1}{s(s^2+2)} - \frac{1}{2s} = \frac{2s^2+1-\frac{1}{2}s^2-1}{s(s^2+2)} = \frac{\frac{3}{2}s^2}{s(s^2+2)} = \frac{3}{2}\,\frac{s}{s^2+2}$$

$$Z_2 = \frac{2(s^2+2)}{3s} \qquad K_{\infty 4} = \lim_{s\to\infty} \frac{1}{s}\,\frac{2(s^2+2)}{3s} = \frac{2}{3}$$

$$\frac{2}{3}s =$$  $\frac{2}{3}$

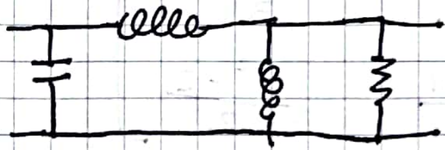
$$Z_4 = \frac{2s^2+4}{3s} - \frac{2s}{3} = \frac{2s^2+4-2s^2}{3s} = \frac{4}{3s}$$

$$Y_4 = \frac{3s}{4}$$  $\frac{3}{4}$

IV

Verificación

$T(s) = \frac{V_2}{I_1}$ $\qquad \frac{1}{C} = \frac{V_2}{I_1}\Big|_{I_2=0}$



$T_1 = \begin{bmatrix} 1 & 0 \\ \frac{3s}{4} & 1 \end{bmatrix} \quad T_2 \begin{bmatrix} 1 & \frac{2}{3}s \\ 0 & 1 \end{bmatrix} \quad T_3 \begin{bmatrix} 1 & 0 \\ \frac{1}{2s} & 1 \end{bmatrix} \quad T_4 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & \frac{2}{3}s \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ \frac{3s}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{2s}{3} \\ \frac{3s}{4} & \frac{s^2+2}{2} \end{bmatrix}$$

$$\frac{\cancel{3}s}{4} \frac{2}{\cancel{3}} s + 1 = \frac{s^2}{2} + 1 = \frac{s^2+2}{2}$$

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{2s} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{2s}{3} \\ \frac{3s}{4} & \frac{s^2+2}{2} \end{bmatrix} \begin{bmatrix} \frac{4}{3} & \frac{2s}{3} \\ \frac{4s^2+2}{4s} & \frac{s^2+2}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{3} & \frac{2s}{3} \\ \frac{4s^2+2}{4s} & \frac{s^2+2}{2} \end{bmatrix} \begin{bmatrix} \frac{4+2s}{3} & \frac{2s}{3} \\ \frac{s^3+2s^2+2s+1}{2s} & \frac{s^2+2}{2} \end{bmatrix}$$

$$T(s) = \frac{1}{C} = 2\,\frac{s}{s^3+2s^2+2s+1} = K\,\frac{s}{s^3+2s^2+2s+1}$$

$$\boxed{K = 2}$$