a) Fs¿?

Región de entrada hasta 10k

Fs = 20k →

La atenuación debe ser 60db, nos dan el pedo menor lo vamos a ver.

D → Bessel¿? → que gano de poder.

~~B) Butter orden ② con fc = 1k~~

Como mínimo debemos ~~samplear~~ muestrear a Fs = 20KHz para evitar solapamiento.

~~b) El filtro antialias va a ser un butter con a_{min} = 60db @ f_s=10KHz~~

~~Si atenuamos menor que esto vamos a tener ~~señal~~, si atenuamos solapamiento~~
~~si atenuamos más es innecesario porque la señal ya va a estar por debajo~~
~~del piso de ruido del sistema~~

~~a_{max} | 1db~~
~~a_{min} | 60db~~
~~f_s | 10 KHz~~

b) El filtro debe atenuar a $f_s/2 = f_{Nyquist}$

$f_c = 10K$

$f_s = 40K$

$f_{Ny} = 20K$

HOJA 2

2)a) $F_s = 100k$ $\quad \omega_0 = 2\pi f_c$

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}\Bigg|_{s = k\frac{z-1}{z+1}} = \cancel{\frac{\omega_0^2}{s^2+}}$$

$$H(z) = \frac{\omega_0^2}{k^2\frac{(z-1)^2}{(z+1)^2} + \frac{\omega_0}{Q}k\frac{(z-1)}{(z+1)} + \omega_0^2} = \frac{(z+1)^2\omega_0^2}{k^2(z-1)^2 + \frac{\omega_0}{Q}k(z-1)(z+1) + \omega_0^2(z+1)^2}$$

$$H(z) = \frac{(z^2+2z+1)\,\omega_0^2}{k^2(z^2-2z+1) + \frac{\omega_0}{Q}k(z^2-1) + \omega_0^2(z^2+2z+1)}$$

$$H(z) = \frac{(z^2+2z+1)\,\omega_0^2}{z^2\underbrace{\left(k^2+\frac{\omega_0 k}{Q}+\omega_0^2\right)}_{a} + z\underbrace{(\omega_0^2 - k^2)2}_{b} + \underbrace{\omega_0^2 + k^2 - \frac{\omega_0 k}{Q}}_{c}}$$

Todo muy bonito ahora $H(j\Omega)$?

$H(e^{j\Omega})$?

$$H(e^{j\Omega}) = \frac{(e^{j2\Omega} + 2e^{j\Omega} + e^{j0})\,\omega_0^2}{e^{j2\Omega}(a) + e^{j\Omega}(b) + e^{j0}(c)} = \frac{\omega_0^2(2 + e^{j\Omega} + e^{-j\Omega})\,e^{j\Omega}}{(\qquad)}$$

$$H(e^{j\Omega}) = \frac{\cancel{e^{j\Omega}}\,\omega_0^2(2 + e^{j\Omega} + e^{-j\Omega})}{\cancel{e^{j\Omega}}\left[e^{j\Omega}\left(k^2+\frac{\omega_0 k}{Q}+\omega_0^2\right) + (\omega_0^2 - k)2 + e^{-j\Omega}\left(\omega_0^2 + k^2 - \frac{\omega_0 k}{Q}\right)\right]}$$

$$H(e^{j\Omega}) = \frac{\omega_0^2(2 + e^{j\Omega} + e^{-j\Omega})}{(e^{j\Omega}+e^{-j\Omega})(k^2+\omega_0^2) + (\omega_0^2 - k)2 + (e^{j\Omega} - e^{-j\Omega})\left(\frac{\omega_0 k}{Q}\right)}$$

$$H(e^{j\Omega}) = \frac{\omega_0^2\left(2+2\cos(\Omega)\right)}{2(\omega_0^2-k)+2\cos(\Omega)(k^2+\omega_0^2)+2j\,\mathrm{sen}(\Omega)\left(\frac{\omega_0 k}{Q}\right)}$$

y o no quien te grafico ¿?

$\omega_0 = 2\pi f_c \qquad k = \frac{2}{T_s} = 2 f_s = 2\,100k \qquad f_c = 1k$

HOJA 3

3

a1)

$h_1(k) = (1,1) \quad h(0) = 1 \quad h(1) = 1$

~~$h_2(k) = (1,1,1)$~~

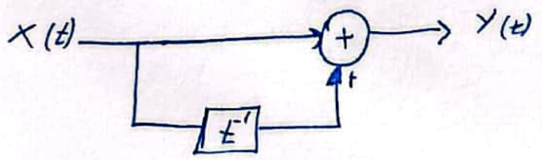
$$y[i] = x[n] * h[i] = \sum_{k=0}^{M-1} x(k)\, h(k-1)$$

$$y[0] = x[0]\, h[0]$$

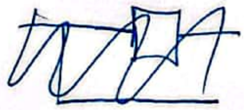
$$y[1] = x[1]\, h[0] + x[0]\, h[1]$$

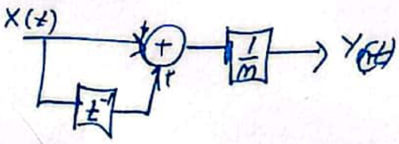
$$y[1] = x[1]\, 1 + x[n-1]\, 1$$

$$Y(z) = X(z) + X(z)\, z^{-1}$$



Para que sea la media aritmetica se lo debe dividir por el total de muestras.





a2) $h_2(k) = (1,1,1)$

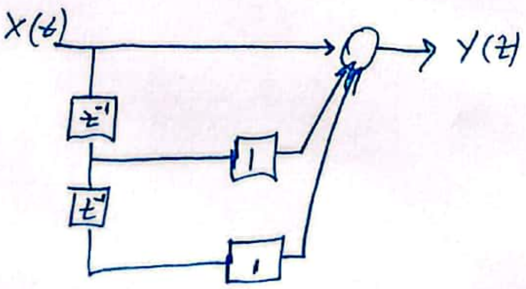
$$y[0] = x[0]\, h[0]$$

$$y[1] = x[1]\, h[0] + x[0]\, h[1]$$

$$y[2] = x[2]\, h[0] + x[1]\, h[1] + x[0]\, h[2]$$

$$y(2) = x(2)\, 1 + x[n-1]\, 1 + x[n-2]\, 1$$

$$Y(z) = X(z) + X(z)\, z^{-1} + X(z)\, z^{-2}$$



~~$\frac{Y(z)}{X(z)} = \frac{}{z^{-2} + z^{-1} +}$~~

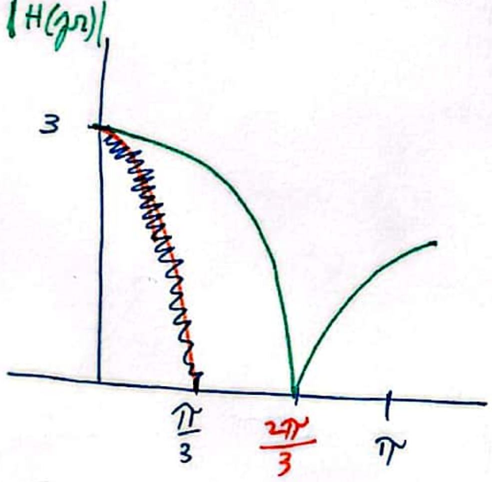
~~$\frac{Y(z)}{X(z)} = \frac{z^2}{z^2 + z + 1}$~~

$$\frac{Y(z)}{X(z)} = z^{-2}+z^{-1}+1 \cdot \frac{z^2}{z^2} = \frac{z^2+z+1}{z^2}$$

$$H(\Omega) = \frac{e^{j2\Omega}+e^{j\Omega}+e^{0}}{e^{j2\Omega}} = \frac{e^{j\Omega}}{e^{j2\Omega}}\left(e^{j\Omega}+e^{0}+e^{-j\Omega}\right) =$$

$$H(j\Omega) = e^{-j\Omega}\left(1+e^{j\Omega}+e^{-j\Omega}\right) = e^{-j\Omega}(1+2\cos(\Omega))$$

$$|H(j\Omega)| = |1+2\cos(\Omega)|$$



~~$f = \frac{f_s}{\pi} \cdot (\frac{\Omega}{2})$~~

~~¿f_s?~~

~~$f = 90\,Hz$~~ ~~$\Omega = 2\frac{\pi}{3}$~~

~~$f_s = 50$?~~ maths don't lie

$$1+2\cos(\Omega) = 0$$

$$\Omega = \pi\frac{2}{3}$$

$$\Omega = \frac{\omega}{F_s}$$

$$\Omega = \frac{2}{3}\pi = 2\pi\frac{50}{f_s} \Rightarrow f_s = 150\ Hz$$

HOJA 4

$h_1(k) = 1, -1$

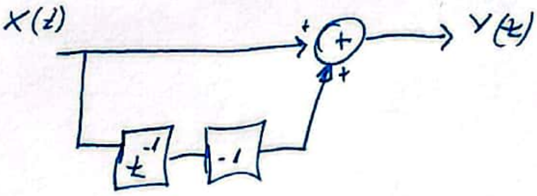
$y[0] = x[0]h(0)$

$y[1] = x[1]h(0) + x[0]h(1)$

$y[2] = x[2]h(0) + x[1]h(1) + x[0]h(2)$

$h_1(k) = (1; -1)$

$y[1] = x[1]\,1 + x[0]\,-1$



$Y(z) = X(z) - X(z)z^{-1}$

$Y(z) = X(z)(1 - z^{-1})$

$H(z) = 1 - z^{-1} = \frac{z-1}{z}$

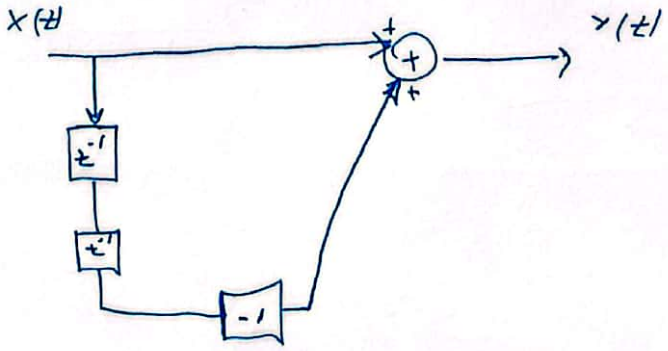
$h_2(k) = (1; 0; -1)$

$y[2] = x[2]h(0) + x[1]h(1) + x[0]h(2)$

$y[2] = x[2]\,1 + x[1]\,0 - x[0]\,1$

$Y(z) = X(z)(1 - z^{-2})$

$H(z) = 1 - z^{-2} = \frac{z^2 - 1}{z^2}$



$$H_1(z) = 1 - z^{-1} \qquad H_2(z) = 1 - z^{-2}$$

$$H_1(j\Omega) = e^{0} - e^{-j\Omega} = e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right) = e^{-j\frac{\Omega}{2}}\, 2j \sin\left(\frac{\Omega}{2}\right)$$

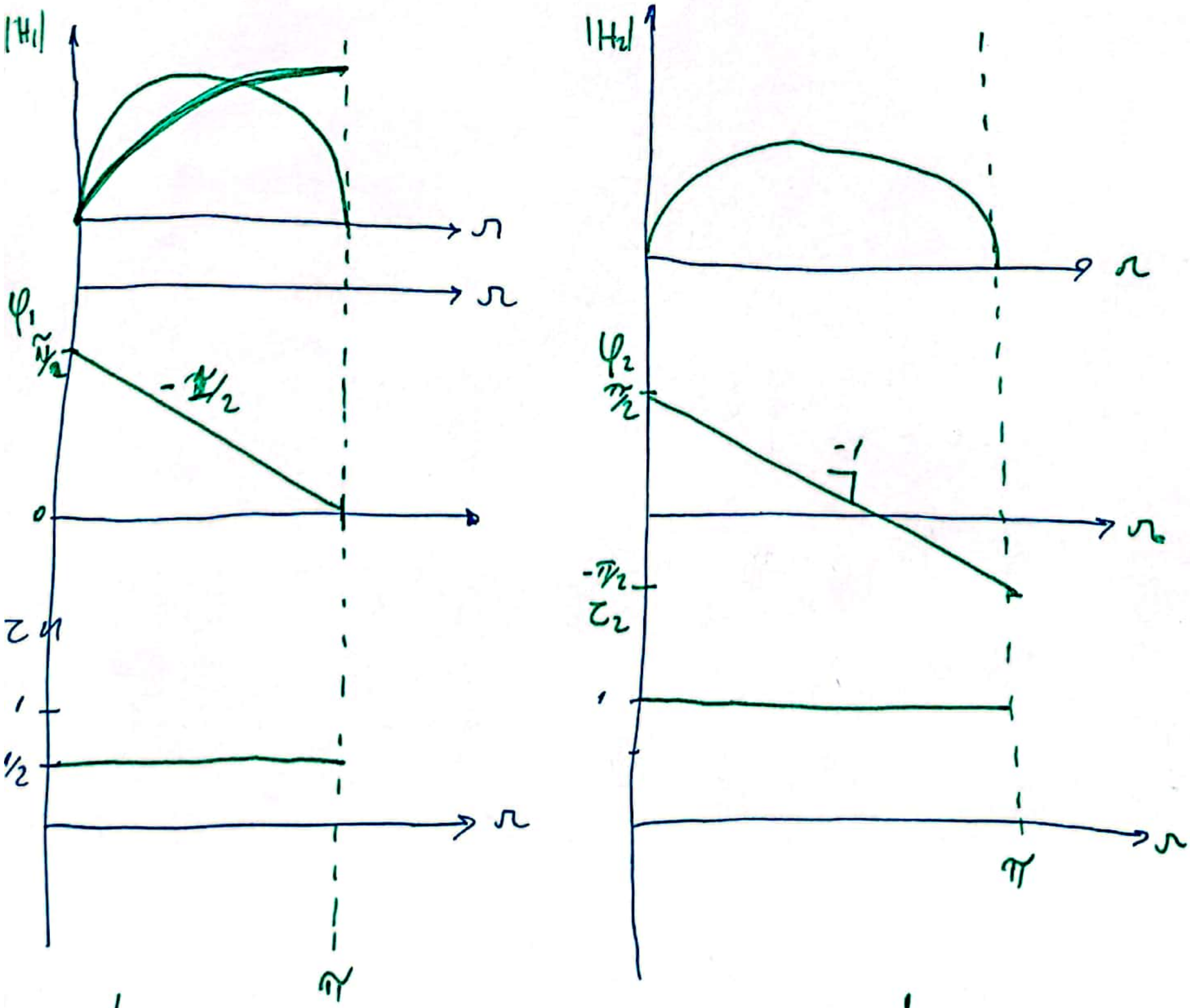
$$H_1(j\Omega) = e^{-j\frac{\Omega}{2}}\, e^{j\frac{\pi}{2}}\, 2 \sin\left(\frac{\Omega}{2}\right) = e^{j\left(\frac{\pi}{2} - \frac{\Omega}{2}\right)}\, 2 \sin\left(\frac{\Omega}{2}\right)$$

$$H_2(j\Omega) = 1 - z^{-2}\Big|_{z = e^{j\Omega}} = e^{0} - e^{-j2\Omega} = e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)$$

$$H_2(j\Omega) = 2j\, e^{-j\Omega} \sin(\Omega) = e^{j\left(\frac{\pi}{2} - \Omega\right)}\, 2 \sin(\Omega)$$

$$\varphi_1 = \frac{\pi}{2} - \frac{\Omega}{2} \qquad \tau_1 = \frac{1}{2}$$

$$\varphi_2 = \frac{\pi}{2} - \Omega \qquad \tau_2 = 1$$



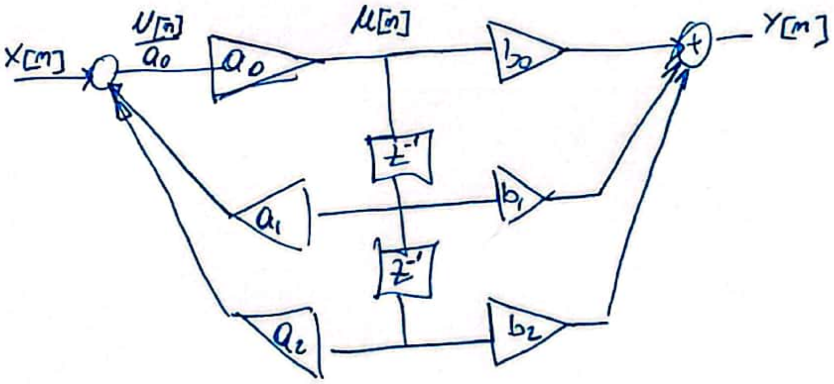
$\frac{1}{2}$ $\qquad$ 1

#4

a)

Conf: $x(t) + x(t) z^{-N} (-c_1) = y_1(t)$

$$\boxed{C(t) = 1 - C_1 z^{-N}}$$



$$X[n] + U[n-1]\,a_1 + U[n-2]\,a_2 = U[n]\,\frac{1}{a_0}$$

$$U[n]\,b_0 + U[n-1]\,b_1 + U[n-2]\,b_2 = Y[n]$$

$$X[n] = U[n]\,\frac{1}{a_0} - U[n-1]\,a_1 - U[n-2]\,a_2$$

$$X(t) = U(t)\left(\frac{1}{a_0} - a_1 z^{-1} - a_2 z^{-2}\right)$$

$$Y(t) = U(t)\left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right)$$

$$\frac{Y(t)}{X(t)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{\frac{1}{a_0} - a_1 z^{-1} - a_2 z^{-2}}$$

b) $a_0 = 1 \quad a_1 = 1 \quad b_0 = \frac{1}{N} \qquad c_1 = 1 \quad N = 3, 4 \text{ y } 5$

$$H(z) = (1 - c_1 z^{-N}) \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{\frac{1}{a_0} + a_1 z^{-1} + a_2 z^{-2}}$$

$$H(z) = (1 - z^{-N}) \frac{\frac{1}{N}}{1 + z^{-1}} = \frac{1 - z^{-N}}{1 - z^{-1}} \frac{1}{N}$$

$N = 3$

$$H(z) = \frac{1}{3} \frac{1 - z^{-3}}{1 - z^{-1}}$$

$$\frac{Y(z)}{X(z)} = \frac{1}{3} \frac{1 - z^{-3}}{1 - z^{-1}} \rightarrow Y(z)(1 - z^{-1}) = X(z) \frac{1}{3} (1 - z^{-3})$$

Es un filtro del tipo IIR, es de naturaleza recursiva. Se resultado de valores pasados de la salida, si quita la entrada sigue habiendo salida.

Ventajas ¿?

N	3	4	5
$H(z)$	$\frac{1}{3} \frac{1 - z^{-3}}{1 - z^{-1}}$	$\frac{1}{4} \frac{1 - z^{-4}}{1 - z^{-1}}$	$\frac{1}{5} \frac{1 - z^{-5}}{1 - z^{-1}}$

HOJA 6

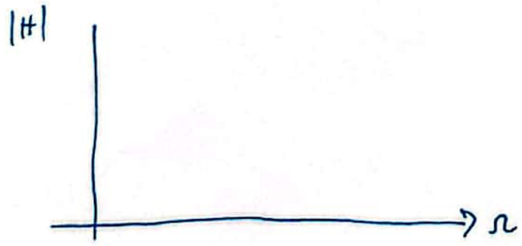
$$H(z) = \frac{1}{3}\,\frac{1 - z^{-3}}{1 - z^{-1}} =$$

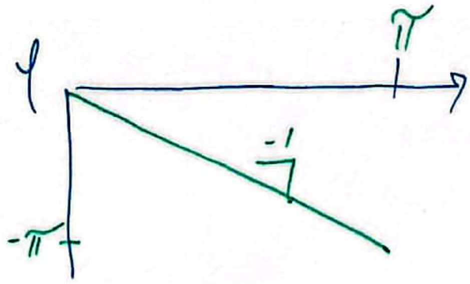
$$H(j\Omega) = \frac{1}{3}\,\frac{e^{0} - e^{-j3\Omega}}{e^{0} - e^{-j\Omega}} = \frac{1}{3}\,\frac{e^{-j\frac{3}{2}\Omega}\left(e^{j\frac{3}{2}\Omega} - e^{-j\frac{3}{2}\Omega}\right)}{e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right)}$$

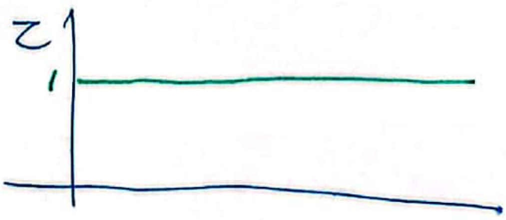
$$H(j\Omega) = \frac{1}{3}\, e^{j\left(-\frac{3}{2}\Omega + \frac{\Omega}{2}\right)}\,\frac{\cancel{2j}\,\mathrm{sen}\left(\Omega\frac{3}{2}\right)}{\cancel{2j}\,\mathrm{sen}\left(\frac{\Omega}{2}\right)} = \frac{1}{3}\, e^{-j\Omega}\,\frac{\mathrm{sen}\left(\Omega\frac{3}{2}\right)}{\mathrm{sen}\left(\Omega/2\right)}$$

$$H(j\Omega) = e^{-j\Omega}\,\frac{1}{3}\,\frac{\mathrm{sen}(\Omega 3/2)}{\mathrm{sen}\left(\frac{\Omega}{2}\right)}$$

$$\varphi = e^{-j\Omega}$$







$$H(z) = \frac{1}{4}\,\frac{1 - z^{-4}}{1 - z^{-1}} \Rightarrow \frac{1}{4}\,\frac{e^{0} - e^{-j4\Omega}}{e^{0} - e^{-j\Omega}} = \frac{1}{4}\,\frac{e^{-j2\Omega}}{e^{-j\frac{\Omega}{2}}}\,\frac{\left(e^{j2\Omega} - e^{-j2\Omega}\right)}{e^{j\frac{\Omega}{2}} - e^{-j\Omega/2}} = \frac{1}{4}\,\frac{\mathrm{sen}(2\Omega)}{\mathrm{sen}(\Omega/2)}\, e^{-j\frac{3}{2}\Omega}$$

$h_6(k) = (1,1,1,1,1,1)$

$Y[6] = X[6]\,1 + X[5]\,1 + X[4]\,1 + X_3[1] + X[2]\,1 + X[1]\,1$

$Y(z) = X(z)\,(1 + z^{-1} + z^{-2} + z^{3} + z^{-4} + z^{-5})$

No se puede implementar $h_6(k)$ con la red propuesta.

D) $a_0 = 1$ $a_1 = 1-\alpha$ $b = \alpha$ $\alpha = 0{,}9$

$$H(z) = (1-0). \frac{\alpha}{1+(1-\alpha)z^{-1}} = \frac{\alpha}{1+(1-\alpha)z^{-1}}$$

$$H(z) = \frac{\alpha}{1+z^{-1}-\alpha z^{-1}} =$$

$$\cancel{H(j\Omega) = \frac{\alpha}{e^{0} + e^{j\Omega} - \alpha e^{-j\Omega}} = \frac{\alpha}{e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}} + \alpha e^{j\frac{\Omega}{2}}\right)}}$$

$$\cancel{\frac{A}{k} = \frac{1-\alpha}{k}}$$

$$H(z) = \frac{\alpha}{1-\alpha+\alpha + (1-\alpha)z^{-1}} = \frac{\alpha}{\alpha + (1-\alpha) + (1-\alpha)z^{-1}}$$

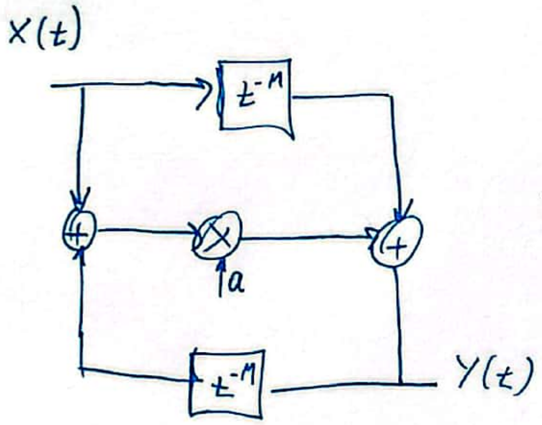
$$H(z) = \frac{0{,}9}{1+0{,}1z^{-1}} = 0{,}9$$

$$H(j\Omega) = \frac{0{,}9}{0{,}9e^{j0} + 0{,}1e^{j0} + e^{j\Omega}\,0{,}1} = \frac{0{,}9}{0{,}9e^{j0} + e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}}\right)0{,}1}$$

$$H(j\Omega) = \frac{0,9}{0,9 + e^{-j\frac{\Omega}{2}}\, 0,1\, 2\cos\left(\frac{\Omega}{2}\right)}$$

Blablablablablablabla ....

$\cancel{+\frac{10}{10} \quad 0,1 \quad \frac{10}{10}}$



$$Y(t) = X(t)\, z^{-M} + [X(t) + Y(t)\, z^{-M}]\, a$$

$$Y(t) = X(t)\, z^{-M} + X(t)\, a + Y(t)\, z^{-M} a$$

$$Y(t)(1 - a z^{-M}) = X(t)(a + z^{-M})$$

$$\frac{Y(t)}{X(t)} = H(t) = \frac{a + z^{-M}}{1 - a z^{-M}}$$

$\cancel{X(t)\, z^{-M} + Y(t)\, z\, t}$

$$X(t)\, z^{-M} + (X(t) - Y(t)\, z^{-M})\, a = Y(t)$$

$$X(t)\, z^{-M} + X(t)\, a = Y(t) + Y(t)\, z^{-M} a$$

$\cancel{\frac{Y(t)}{X(t)}}$ $Y(t)\,(1 + z^{-M} a) = X(t)(z^{-M} + a)$

$$\frac{a + z^{-M}}{1 + a z^{-M}} = \frac{0,8 + z^{-2}}{1 + 0,8 z^{-2}} = \frac{z^2 0,8 + 0z + 1}{z^2 + 0z + 0,8}$$

2) C

HOJA 8

$f_c = 6k$ HP $F_{s1} = 100\,k$ $F_{s2} = 10\,k$

$$LP = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad 2HP \Rightarrow s = \frac{1}{s} \Rightarrow \frac{\omega_0^2}{\left(\frac{1}{s^2}\right) + \frac{\omega_0}{Q}\frac{1}{s} + \omega_0^2} = \frac{\omega_0^2 s^2}{1 + \frac{\omega_0}{Q}s + \omega_0^2 s^2}$$

$$H(s) = \frac{s^2}{s^2 + \frac{s}{Q\omega_0} + \frac{1}{\omega_0^2}} \qquad s = k\,\frac{z-1}{z+1} \wedge k = 2/s$$

$$H(z) = \frac{k^2\left(\frac{z-1}{z+1}\right)^2}{k^2\left(\frac{z-1}{z+1}\right)^2 + k\,\frac{z-1}{z+1}\,\frac{1}{\omega_0 Q} + \frac{1}{\omega_0^2}} = \frac{k^2(z-1)^2}{k^2(z-1)^2 + \frac{k(z-1)(z+1)}{\omega_0 Q} + \frac{(z+1)^2}{\omega_0^2}}$$

$$H(z) = \frac{k^2(z^2 - 2z + 1)}{k^2(z^2 - 2z + 1) + \frac{k}{\omega_0 Q}(z^2 - 1) + \frac{z^2 + 2z + 1}{\omega_0^2}} =$$

$$\boxed{H(z) = \frac{k^2(z^2 - 2z + 1)}{z^2\left(k^2 + \frac{k}{\omega_0 Q} + \frac{1}{\omega_0^2}\right) + z\left(\frac{1}{\omega_0^2} - k^2\right)2 + \frac{1}{\omega_0^2} + k^2 - \frac{k}{\omega_0 Q}}}$$

Esto es lo que voy a emular.

~~$H(z) = k^2(z^2 - z + 1) + \frac{k}{\omega_0 Q}(z^2 -$~~

$$H(\Omega) = \frac{k^2\left(e^{j2\Omega} - 2e^{j\Omega} + e^{0}\right) = e^{j\Omega}\big(-2 + \underbrace{e^{-j\Omega} + e^{j\Omega}}_{2\cos(\Omega)}\big)}{k^2\left(e^{j2\Omega} - 2e^{j\Omega}\right.}$$

$$H(j\Omega) = \frac{k^2\left(e^{j2\Omega} - 2e^{j\Omega} + e^{0}\right)}{k^2\left(e^{j2\Omega} - 2e^{j\Omega} + e^{0}\right) + \frac{k}{\omega_0 Q}\left(e^{j2\Omega} - e^{0}\right) + \frac{1}{\omega_0^2}\left(e^{j2\Omega} + 2e^{j\Omega} + e^{0}\right)}$$

$$H(j\Omega) = \frac{k^2 e^{j\Omega}\left(e^{j\Omega} - 2 + e^{-j\Omega}\right)}{k^2 e^{j\Omega}\left(e^{j\Omega} - 2 + e^{-j\Omega}\right) + \frac{k}{\omega_0 Q} e^{j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right) + \frac{e^{j\Omega}}{\omega_0^2}\left(e^{j\Omega} + 2 + e^{-j\Omega}\right)}$$

$$H(j\Omega) = \frac{e^{j\Omega}\, k^2\left(-2 + 2\cos(\Omega)\right)}{e^{j\Omega}\left[k^2\left(-2+2\cos(\Omega)\right) + \frac{k}{\omega_0 Q} j2\sin(\Omega) + \frac{1}{\omega_0^2}\left(2 + 2\cos(\Omega)\right)\right]}$$

Como llevo eso, no podemos graficar ni la fase.

HOJA 9

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \qquad s = \frac{1}{s}$$

$$\frac{\omega_0^2}{\left(\frac{1}{s}\right)^2 + \frac{\omega_0}{Q}\frac{1}{s} + \omega_0^2} \cdot \frac{s^2}{s^2} = \frac{s^2\omega_0^2}{1 + \frac{\omega_0}{Q}s + \omega_0^2 s^2}$$

$$s = k\,\frac{z-1}{z+1}$$

$$\frac{k^2\left(\frac{z-1}{z+1}\right)^2\omega_0^2}{1 + \frac{\omega_0}{Q}k\frac{z-1}{z+1} + \omega_0^2\left(\frac{z-1}{z+1}\right)^2 k^2}$$

$$k' = k\omega_0 = \frac{k}{2\pi f_0}$$

$$\frac{k'^2\,\frac{(z-1)^2}{(z+1)^2}}{1 + \frac{\cancel{\omega_0}k'}{Q}\cdot\frac{z-1}{z+1} + k'^2\frac{(z-1)^2}{z+1}} \cdot \frac{(z+1)^2}{(z+1)^2}$$

$$\frac{k'^2(z^2-2z+1)}{(z^2+2z+1) + \frac{k'}{Q}\underbrace{(z-1)(z+1)}_{z^2-1} + k'^2(z^2-2z+1)} = \boxed{\frac{k'^2(z^2-2z+1)}{z^2\left(1+k'^2+\frac{k'}{Q}\right) + z(2-2k'^2) + 1 - \frac{k'}{Q} + k'^2}}$$