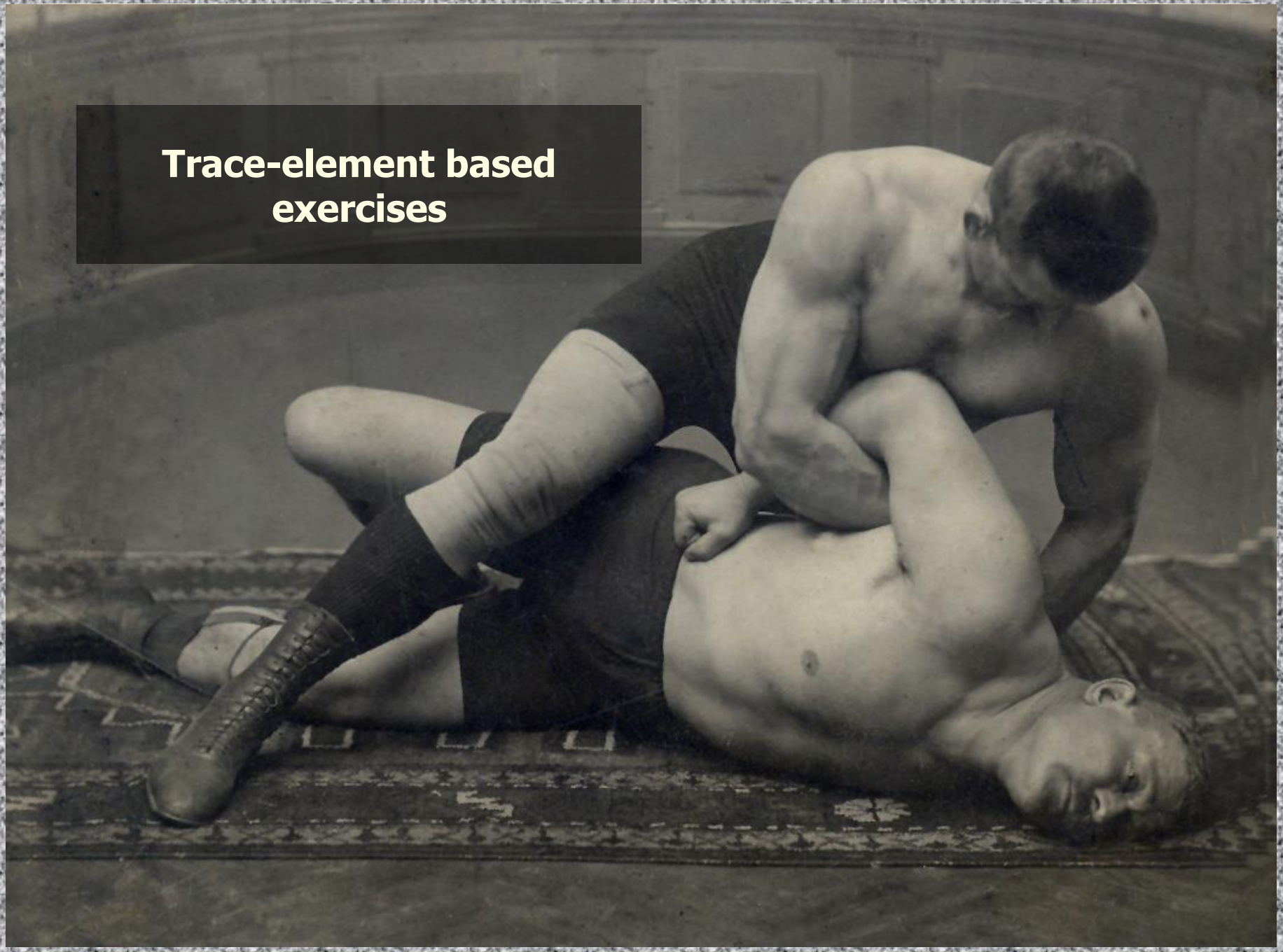

Trace-element based exercises

14.2: Fractional crystallization (direct trc)

The Table shows the REE concentrations in a tonalitic magma (ppm), as well as the partition coefficients for crystallizing mineral phases.

	Concentration	Partition coefficients		
	Source H34	Pl	Amp	Ilm
La	32.04	0.4	0.74	0.005
Ce	61.9	0.27	1.52	0.006
Nd	25.75	0.21	4.26	0.0075
Sm	3.97	0.13	7.77	0.01
Eu	0.966	2.15	5.14	0.007
Gd	2.3	0.097	10	0.017
Dy	1.16	0.064	13	0.028
Er	0.46	0.055	12	0.035
Yb	0.37	0.049	8.4	0.075
Lu	0.057	0.046	6	0.1

ttg_a.data

A differentiated magma is generated by 30% fractional crystallization of a parental magma (`H34`); the cumulate consists of plagioclase, hornblende and ilmenite.

- Calculate the differentiated magma composition for a cumulate made up of 49 % plagioclase, 49 % hornblende, and 2 % ilmenite.
- By means of the *GCDkit* function *spider*, plot chondrite-normalized REE patterns (using normalization values after *Boynton 1984*).

14.2: Fractional crystallization (direct trc)

Rayleigh equation:

$$C_L = C_0 F^{(D-1)}$$ Eq. [11.5]

where bulk distribution coefficient *D* for element α:

$$D_\alpha = \sum_{i=1}^{n} m_i K_{D\alpha}^{i/\mathrm{L}}$$ Eq. [10.4]

From definition of *D*, instantaneous solid:

$$C_{S.inst} = DC_L$$ Eq. [11.8],
CORRECTED FROM THE BOOK

The bulk solid:

$$C_{S.bulk} = C_0 \frac{1-F^D}{1-F}$$ Eq. [11.9]

15.1: Fractional crystallization (reversed Ex. 14.2)

Table contains trace-element compositions of two tonalites, a primitive WR1 *and a differentiated* WR2*, together with corresponding partition coefficients for the relevant rock-forming minerals.*

	WR1	WR2	Pl	Amp	Ilm
La	32.04	1.224	0.4	0.74	0.005
Ce	61.9	2.270	0.27	1.52	0.006
Nd	25.75	1.170	0.21	4.26	0.0075
Sm	3.97	0.158	0.13	7.77	0.01
Eu	0.966	0.320	2.15	5.14	0.007
Gd	2.3	0.119	0.097	10	0.017
Dy	1.16	0.085	0.064	13	0.028
Er	0.46	0.057	0.055	12	0.035
Yb	0.37	0.101	0.049	8.4	0.075
Lu	0.057	0.025	0.046	6	0.1

`ttg_a2.data`

Let's assume that `WR1` and `WR2` represent pure compositions of a tonalitic melt differentiating by Rayleigh-type fractional crystallization.

- Using the least-square method, estimate the modal composition of the cumulate and the degree of fractional crystallization.

15.1: Fractional crystallization (reversed Ex. 14.2)

From Rayleigh equation [11.5]:

$$C_L = C_0 F^{(D-1)}$$

$$\ln\left(\frac{C_L}{C_0}\right)_j = \left(D_j - 1\right)\ln F = \sum_{i=1}^{n}\left(\left(K_{D_j}^{i/L} - 1\right)m_i\right)\ln F \qquad \text{Eq. [12.1]}$$

Defining the unknown as a vector of length *n:*

$$\overrightarrow{\boldsymbol{M}} = \begin{pmatrix} m_1 \ln F \\ m_2 \ln F \\ \vdots \\ m_n \ln F \end{pmatrix} \qquad \text{Eq. [12.2]}$$

Matrix of partition coefficients (less one):

$$\overline{\overline{\boldsymbol{D}}} = \begin{pmatrix} K_{D_1}^{1/L} - 1 & \cdots & K_{D_1}^{n/L} - 1 \\ \vdots & \ddots & \vdots \\ K_{D_p}^{1/L} - 1 & \cdots & K_{D_p}^{n/L} - 1 \end{pmatrix} \qquad \text{Eq. [12.3]}$$

15.1: Fractional crystallization (reversed Ex. 14.2)

Defining vector of left hand sides in Eq. 12.1:

$$\overrightarrow{C_v} = \begin{pmatrix} \ln\left(\frac{C_L^1}{C_0^1}\right) \\ \vdots \\ \ln\left(\frac{C_L^p}{C_0^p}\right) \end{pmatrix}$$

Eq. [12.4]

We can rewrite the Eq. 12.1 in a matrix form and solve by least squares:

Then: $$\sum \overrightarrow{M} = \sum_{i=1}^{n} m_i \ln F = \ln F$$

$$\overrightarrow{C_v} = \overline{\overline{D}} \times \overrightarrow{M}$$

Eq. [12.5]

$$F = e^{\sum \overrightarrow{M}}$$

Eq. [12.7]

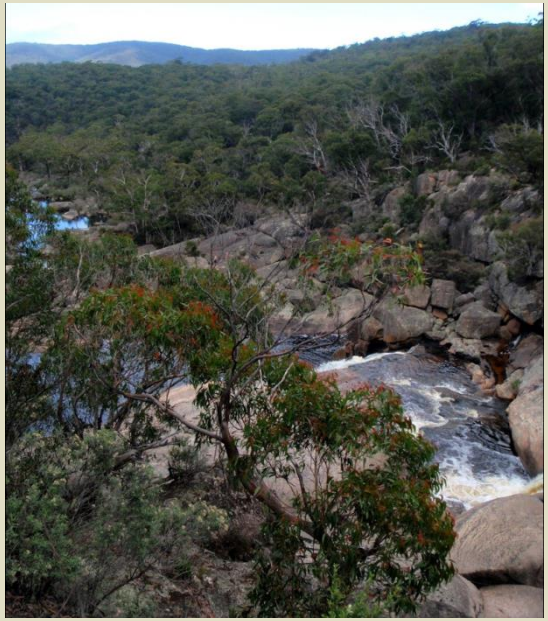
Least-squares method is in R implemented by the function `lsfit` setting `intercept = FALSE`, so that the model passes through the origin.

$$\begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{pmatrix} = \frac{\overrightarrow{M}}{\ln F}$$

Eq. [12.8]

Accessories and saturation exercises

14.5 (completed): Saturation models for accessory minerals



The file contains major-element and Zr contents in Boggy Plain Suite of the Lachlan Fold Belt (New South Wales, Australia) [Wyborn, 1983; OzChem compilation http://www.ga.gov.au]

boggy_plain_data.data

Based on the Watson and Harrison's (1983) zircon saturation model, in *GCDkit*:

- Plot a binary graph SiO_2 vs. Zr for the Boggy Plain Suite showing the inflected trend characteristic of the saturation in zircon
- Plot a binary graph of M vs. Zr (ppm) with superimposed isotherms of zircon saturation temperatures in ºC [Eq. (13.3)].
- Plot a binary plot SiO_2 vs. Zr saturation temperature in ºC for the same

14.5 (completed): Saturation models for accessory minerals

Saturation equation (Watson & Harrison 1983):

$$ln\left(\frac{c_{Zrn}^{Zr}}{\left(C_L^{Zr}\right)_{sat}}\right) = -3.80 - 0.85(M-1) + \frac{12900}{T}$$

Eq. [13.1]

Which can be transformed to

c_{Zrn}^{Zr} = 497644 ppm (49.7 wt. %)

$$\left(C_L^{Zr}\right)_{sat} = \frac{c_{Zrn}^{Zr}}{e^{\left(-3.80-0.85(M-1)+\frac{12900}{T}\right)}}$$

GCDkit saturation plugin:

```
zrSaturation(T=…)
```

Returns a matrix with columns:

```
M, Zr, Zr.sat, TZr.sat.C
```

$$M = \frac{Na + K + 2Ca}{Al \times Si}$$