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Image encryption based on Independent Component Analysis and Arnold's Cat Map

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Abstract Security of the multimedia data including image and video is one of the basic requirements for the telecommunications and computer networks. In this paper, a new efficient image encryption technique is presented. It is based on modifying the mixing matrix in Independent Component Analysis (ICA) using the chaotic Arnold's Cat Map (ACM) for encryption. First, the mixing matrix is generated from the ACM by insert square image of any dimension. Second, the mixing process is implemented using the mixing matrix and the image sources the result is the encryption images that depend on the number of sources. Third, images decrypted using ICA algorithms. We use the Joint Approximate Diagonalization of Eigen-matrices (JADE) algorithm as a case study. The results of several experiments, PSNR, SDR and SSIM index tests compared with standard mixing matrix showed that the proposed image encryption system provided effective and safe way for image encryption.

1. Introduction

Over the years, there has been an important development in the field of information security. Yun-Peng et al. [1] presented the encryption scheme that realizes the digital image encryption through the chaos and improving DES. First, encryption scheme uses the logistic chaos sequencer to make the pseudo-random sequence, carries on the RGB with this sequence to the image chaotically, makes double time encryptions with improvement DES, and displays the respective merit. Lin et al. [2] proposed a way to encrypt the image using a linear mixed model of blind source separation (BSS). The encrypted simultaneously multiple images with the same size by mixing it with the same number of key images are statistically independent, the size of which is equal to that of images to be encrypted. Since these multiple images cover mutually through mixing among them while the key images cover them, there are no restrictions on the main space. The proposed method has a high level of security, and the computer simulation results showed its validity.

Ravishankar and Venkateshmurthy [3] proposed several schemes for image encryption. These schemes are produced

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in the form of random so that the content not appears. Encryption and decryption consume large amount of time. Therefore, there is a need for effective algorithm. They proposed a region based selective image encryption technique which provides facilities of selective encryption and selective images of reconstruction. Gao et al. [4] proposed a nonlinear chaotic algorithm (NCA), which uses the power function and the tangent function instead of the linear function.

Yu et al. [5] suggested that the efficiency of image encryption algorithm depends on the reconstruction of the image using some neighboring pixel characteristics. In accordance with the characteristics of different images of various bilateral level slightly, encryption system proposed reconstructs of the bit-level. Since the permutation of sub-images composing high 4-bits of the original image has a relatively high computational complexity, in this scheme the permutation of sub-images is performed with low 4-bits instead, which therefore has a lower computational complexity.

Kamali et al. [6] proposed encryption system that modifies the AES algorithm on the basis of each Shift Row Transformations. If the value in the first row and first column is even, the first and fourth rows are unchanged, and each byte in second and third rows is shifted periodically over different numbers. Last rows' first and third values are unchanged and are periodically turned to leave each byte in the rows. The second and fourth quarters of the state to a different number of bytes. Experimental result showed that MAES gave better results in terms of encryption for security against attacks and increased the statistical performance. Paul et al. [7] proposed encoder to convert bitmaps tricks encryption. Replacement and dissemination processes, based on the matrix, to facilitate fast convert plain text into cipher text, and images encryption. The results of the simulation compared with the results obtained from the Advanced Encryption Standard (AES), showed that the proposed image encryption algorithm was eight times faster than AES. Gautam et al. [8] discussed the use of a block based transformation algorithm, in which image is divided into number of blocks. These blocks are transformed before going through an encryption process. At the receiver side these blocks were retransformed into their original position and decryption process is performed. The advantage of such approach is used mainly on reproducing the original image without loss of information for the encryption and decryption process (based on a blowfish algorithm). Fei and Xiao-cong [9] presented an image encryption algorithm based on two-dimensional (2D) map and complex logistics system Chua's system. It used two successive chaos generated by the MAP logistics 2D (to change the features of color image). Then it used the chaos resulting from the successive models to maximize the benefits of the Chua system on the production of new pixel values. Simulation results showed that the algorithm has good properties of confusion and diffusion, and the big key space.

Ye and Zhao [10] proposed chaos-based image encryption scheme using affine modular maps, which are extensions of linear congruential generators, acting on the unit interval. A permutation process utilized two affine modular maps to get two index order sequences for the shuffling of image pixel positions, while a diffusion process employed another two affine modular maps to produce two pseudo-random gray value sequences for a two-way diffusion of gray values. Liu and Tian [11] proposed algorithm to encrypt images using color map and spatial chaos at the bit level flipping (SBLP). First, the algorithm used the logistic chaos sequence to shuffle the positions of image pixels and then to convert them into a binary matrix component including red, green and blue at one time, rather than the order of the matrix as well as at the level prior to appointment of scrambling bit that has been created by SBLP. Second, the logistics rearranged the chaotic sequence for the position of the current image pixel else. Results of the experiment and security analysis algorithm achieved good results and the complexity of encryption is low, and in addition to that, the key space is large enough to resist against a common attack.

In this paper, a new efficient image encryption technique is presented. It is based on modifying the mixing matrix in Independent Component Analysis (ICA) using the chaotic Arnold's Cat Map (ACM) for encryption. First, the mixing matrix is generated from the ACM by inserting square image of any dimension. Second, the mixing process is implemented using the mixing matrix and the image sources. The result is the encrypted images that depend on the number of sources. Third, images are decrypted using ICA algorithms. The Joint Approximate Diagonalization of Eigen-matrices (JADE) algorithm was tested as a case study. The results of several experiments, PSNR, SDR and SSIM index tests compared with standard mixing matrix showed that the resulted encrypted images from the proposed system provided effective and safe way for image encryption.

2. Arnold's Cat Map (ACM)

According to Arnold's transformation, an image is hit with the transformation that apparently randomizes the original organization of its pixels. However, if iterated enough times, eventually the original image reappears. The number of considered iterations is known as the Arnold's period. The period depends on the image size; i.e., for different size images, Arnold's period will be different [12].

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = A \begin{bmatrix} x_n \\ y_n \end{bmatrix} (\mathrm{mod}\ N) = \begin{bmatrix} 1 & p \\ q & pq+1 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix} (\mathrm{mod}\ N) \tag{1}$$

where N is the size of the image, p and q are positive integer and $det(A) = 1$. $(x_n,\ y_n)$ is the position of samples in the $N \times N$ data such as image, so that

$(x_n, y_n) \in \{0, 1, 2, \ldots, N-1\}$

And $(x_{n+1},\ y_{n+1})$ is the transformed position after cat map, Cat map has two typical factors, which bring chaotic movement: tension (multiply matrix in order to enlarge x, y) and fold (taking mod in order to bring x, y in unit matrix).

Eq. (1) is used to transform each and every pixel coordinates of the image. When all the coordinates are transformed, the image resulted is a scrambled image. At a certain step of iterations, if the resulted image reaches our anticipated target (i.e. up to secret key), we have achieved the requested scrambled image. The decryption of image relies on the transformation periods (i.e. the number of iterations to be followed = Arnold's period − secret key) [13].

3. Independent Component Analysis (ICA)

ICA defines a generative model for observed multivariate data, which usually is given as a large database of samples. In this model, data variable are assumed to be linear or nonlinear mixtures of some unknown latent variables, and mixing system is also unknown. And it is assumed that the underlying variables of non-Gaussian are independent of each other and are called independent components (IC) of the observed data. These independent components are also called sources or factors that can be found by ICA [14,15].

The basic linear model relates the unobservable source image and the observed mixtures:

$$x(t) = As(t) \tag{2}$$

where $s(t) = [s_1(t), \ldots, s_m(t)]^T$ is a $m \times 1$ column vector collecting the source images, similarly vector $x(t)$ collects the n observed signals, A is a $n \times m$ matrix of unknown mixing coefficients, $n \geqslant m$, and t is the time index. This model is instantaneous (or memoryless) because the mixing matrix contains fixed elements, and also noise-free.

In order to recover the original source images from the observed mixtures, we use a simple linear separating system [16]:

$$y(t) = Bx(t) \tag{3}$$

where $y(t) = [y_1(t), \ldots, y_m(t)]^T$ is an estimate $s(t)$, and B is a $n \times n$ (assume $n = m$) separating matrix, as shown in Fig. 1.

There are essentially two distinct, at the expense of the ICA, off-line (batch) processing and on-line algorithms. This paper focuses on batch algorithm using JADE algorithm, and on a common approach for batch ICA algorithms by the following two stages of procedure [17]:

A. Decorrelation or whitening. This stage seeks to diagonalize the covariance matrix of the input signals. This is done through computing the sample covariance matrix, giving the second order statistics of the observed output. From this, a matrix is computed by eigen decomposition which whitens the observed data.
B. Rotation. This stage reduces a measure of the higher order statistics which will ensure that the non-Gaussian output signals are as statistically independent as possible. It is clear that this stage can be carried out by a unitary rotation matrix, to provide the higher order independence. And it is performed by finding a rotation matrix which jointly diagonalize eigenmatrices formed from the fourth order cumulants of the whitened data. The outputs from this stage are the independent components.

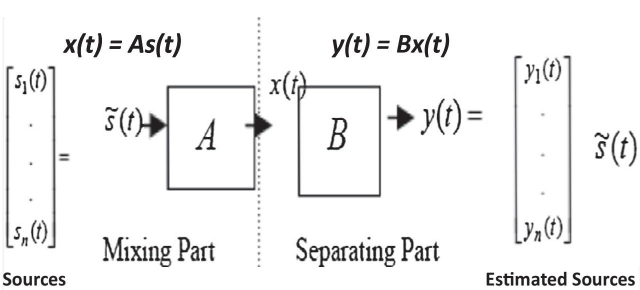


Figure 1 Mixing and separating. Unobserved signals; observations $x(t)$, estimated source images $y(t)$.

Sometimes this approach is referred to as "decorrelation and rotation", and it depends on the measurement of non-Gaussian signals. For Gaussian signals, the higher order statistics are zero already and so no meaningful separation can be achieved by ICA methods. For non-Gaussian random signals the implication is that not only should the signals be uncorrelated, but also the higher order cross-statistics (e.g., moments or cumulants) are zero [18]. The JADE algorithm can be summarized as:

1. *Initialization*. Estimate a whitening matrix $\widehat{W}$ and set $Z = \widehat{W}X$.
2. *Form statistics*. Estimate a maximal set $\{\widehat{Q}_i^Z\}$ of cumulant matrices.
3. *Optimize an orthogonal contrast*. Find the rotation matrix $\widehat{V}$ such that the cumulant matrices are as diagonal as possible, that is, solve (arg min $\sum_i Off(V^+\widehat{Q}_i^Z V)$).
4. *Separate*. Estimate A as

$$\widehat{A} = \widehat{V}\widehat{W}^{-1}. \tag{4}$$

And/or estimate the components as $\widehat{S} = \widehat{A}^{-1}X = \widehat{V}^+Z$.

4. Proposed system

The proposed system has two stages, first we used Arnold's Cat Map (ACM) algorithm, second we used JADE algorithm to recover the original images. Fig. 2, represents the proposed system.

Steps of the proposed system

A. Arnold's Cat Map (ACM) algorithm

1. Input any arbitrary image
2. Use num as variable which is represented the No. of Iterations
3. Determine the No. of rows and columns. Which are represented by the variables row and col respectively.
4. for inc = 1 to num
   for row1 = 1 to row
      for col1 = 1 to col
         nrowp = row1
         ncolp = col1
      for ite = 1 to inc
         Shuffle the positions of the pixels of the image using Eq. (1)
      end
   Result the new encryption image
 end
 end
 end

B. Encryption algorithm

The steps are:

1. Input Two images
2. Convert the images to vectors and assign to **S**
3. Mixing matrix **A** which is the result from ACM algorithm
4. Apply Eq. (2) to get the image encryption

C. Decryption algorithm and evaluation process

1. Apply JADE in Eq. (4) to get the decrypted image
2. Evaluation Process

5. Measurements criteria

5.1. Signal to distortion ratio (SDR)

To evaluate the quality of the separated signals by calculating the signal-to-distortion ratio (SDR) between the separated signal $\tilde{s}(t)$ and the original source image $s(t)$ before mixing, a higher value of SDR is good because of the superiority of the signal to that of the distortion. The SDR in dB (decibel) is calculated as follows [19]:

$$\mathrm{SDR} = 10\log_{10}\frac{\sum_t s(t)^2}{\sum_t (s(t) - \tilde{s}(t))^2}\ (\mathrm{dB}) \tag{5}$$

5.2. The Structural SIMilarity (**SSIM**) index

The **structural similarity** (SSIM) index is a method for measuring the similarity between two images. Structural information is the idea that the pixels have strong inter-dependencies especially when they are spatially close. These dependencies carry important information about the structure of the objects in the visual scene. The resultant SSIM index is a decimal value between -1 and 1, and value 1 is only reachable in the case of two identical sets of data.

The SSIM metric is calculated on various windows of an image. The measure between two windows s and y of common size $N \times N$ is [20]:

$$\mathrm{SSIM}(x,y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \tag{6}$$

With

- μ_x the average of x;
- μ_y the average of y;
- σ_x^2 the variance of x;
- σ_y^2 the variance of y;
- σ_{xy} the covariance of x and y;
- $c_1 = (k_1L)^2$, $c_2 = (k_2L)^2$, two variables to stabilize the division with weak denominator;
- L the dynamic range of the pixel-values (typically this is $2^{\#\text{bits per pixel}} - 1$);
- $k_1 = 0.01$ and $k_2 = 0.03$.

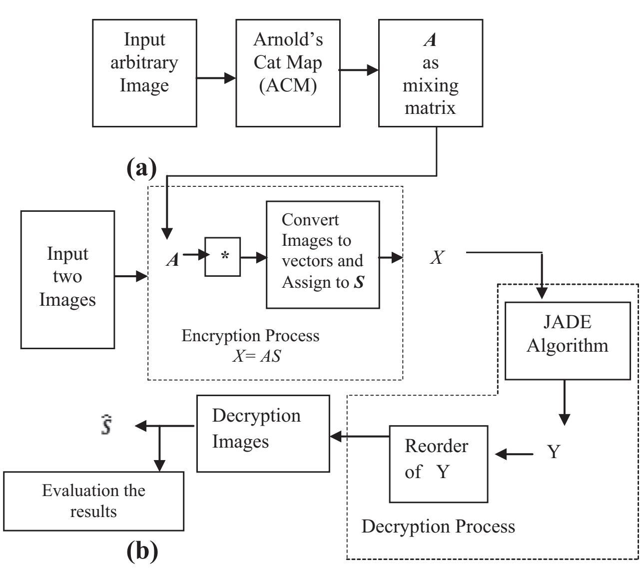


Figure 2 The proposed system, (a) produce the mixing matrix, (b) image encryption and decryption.

5.3. Peak signal to noise ratio (PSNR)

A higher value of PSNR is good because of the superiority of the signal to that of the noise. The larger the PSNR the better the image separation result is.

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{255}{\sqrt{MSE}}\right)\ (\mathrm{dB}) \tag{7}$$

$$\mathrm{MSE} = \frac{1}{MN}\sum_{y=1}^{M}\sum_{x=1}^{N}[I(x,y) - I'(x,y)]^2$$

where $I(x, y)$ is the original image, $I'(x, y)$ is the approximated version and (M, N) are the dimensions of the images. A lower value for Mean Square Error (MSE) means less error, and as seen from the inverse relation between the MSE and PSNR, this translates into a high value of PSNR. Here, the signal is the original image, and the noise is the error in reconstruction.

6. Results and discussions

The proposed system was done under Dell Laptop, with O.S. Window system, Processor Core 2 Duo and RAM 2.00 GB, which is implemented using MATLAB. The experimental results are shown in Table 1 which shows the comparison between standard mixing matrix (random matrix) and Arnold Cat Map matrix using the three parameters. This comparison was carried out with different images and different dimensions.

In order to provide quantitative measures on the performance of the objective quality assessment models, we use the performance evaluation criteria (SDR, SSIM and PSNR).

We observed the SDR from Figs. 3–5 using 7 images with (120×120), (128×128) and (512×512) dimensions. The results are irregular, and sometimes the result is likely to favor random matrix than the Arnold Cat Map matrix and vice versa.

For using the SSIM as comparison criterion, the result is shown in Figs. 6–8 also we used 7 images with (120×120), (128×128) and (512×512) dimensions. The results of Arnold Cat Map matrix are better than random matrix.

The PSNR results from Figs. 9–11 shown with those of Arnold Cat Map matrix are better than random matrix.

Beside of the objective test, we clarify the subjective test as shown in Table 2; we chose samples of images to show the difference between the Arnold Cat Map matrix and random matrix.

Table 1 Comparison between random matrix and Arnold Cat Map matrix using PSNR, SDR and SSIM index.

Image dimension	Image no.	Image name		PSNR		SDR				SSIM index			
		1	2	Random matrix	Arnold matrix	1		2		1		2	
						Random matrix	Arnold matrix	Random matrix	Arnold matrix	Random matrix	Arnold matrix	Random matrix	Arnold matrix
120 × 120	1.	Avatar1	Avatar2	5.7039	10.8314	−1.6357	6.0227	5.9344	5.9344	−0.1950	0.323	0.6165	0.616
	2.	Avatar1	Cabpic	6.3340	13.9242	7.9761	7.9761	8.4852	8.4852	−0.0693	0.6904	0.1169	0.7218
	3.	Cus4	Xxx	4.7103	10.8878	2.2885	2.2885	2.1789	2.1789	−0.0226	0.4933	−0.0226	0.6597
	4.	Avatar2	Cus2	5.3303	9.7250	4.6796	4.6796	4.5871	4.5871	−0.0583	0.5151	0.3420	0.2924
	5.	debbie	barbara	5.4368	9.8059	−2.2127	6.9680	1.9642	1.9642	−0.0427	0.2465	0.6116	0.6116
	6.	18	debbie	5.8150	10.1908	2.7486	2.7486	6.6115	6.6115	−0.1099	0.6179	0.2280	0.2289
	7.	Cus4	barbara	5.4777	10.0865	−4.4679	4.7917	2.3372	2.3372	−0.1114	0.3412	0.5844	0.5844
128 × 128	1.	1	2	4.5697	10.8155	14.4711	14.471	2.3848	2.3848	−0.1343	0.8642	0.1419	0.2147
	2.	3	4	5.3127	9.0138	1.4780	1.4780	3.2239	3.2239	−0.1734	0.4850	0.2737	0.3493
	3.	5	6	5.4490	7.8741	−0.0898	3.9366	2.5312	2.5312	−0.1646	0.2454	0.4780	0.4780
	4.	7	baboon	4.9310	11.4455	5.4544	5.4544	4.7586	4.7586	−0.0711	0.6080	−0.0875	0.6984
	5.	lenna	na2	6.1625	9.1758	0.7354	0.7354	4.4537	4.4537	−0.1408	0.5312	0.2294	0.2647
	6.	mandel	1	6.1551	11.2806	−1.0602	4.8670	12.335	12.335	−0.3731	0.4757	0.6630	0.6630
	7.	lenna	2	6.0286	9.0229	−2.3487	3.7098	1.7356	1.7356	−0.2001	0.3148	0.5241	0.5241
512 × 512	1.	1	2	5.1543	9.2045	2.6092	2.6092	−0.818	−0.818	−0.0916	0.6356	0.0735	0.2372
	2.	14	15	5.8254	9.2619	3.4320	3.4320	3.6300	3.6300	−0.0559	0.6108	0.2668	0.2536
	3.	16	debbie	5.4594	10.5912	3.9158	3.9158	7.3022	7.3022	−0.0208	0.6907	0.3866	0.2143
	4.	barbara	18	5.9035	8.3079	1.2493	3.3956	3.4543	1.6724	−0.1004	0.2165	0.4498	0.4498
	5.	barbara	house	6.5413	10.5930	−1.5690	4.1637	7.5353	7.5353	−0.1110	0.2356	0.7005	0.7005
	6.	12	11	5.9476	7.9630	1.1416	1.1416	1.8317	1.8317	−0.0372	0.6949	0.2711	0.1576
	7.	17	4	5.7275	8.6466	4.4850	4.0636	3.8439	3.2631	0.1140	0.1722	0.6822	0.6822

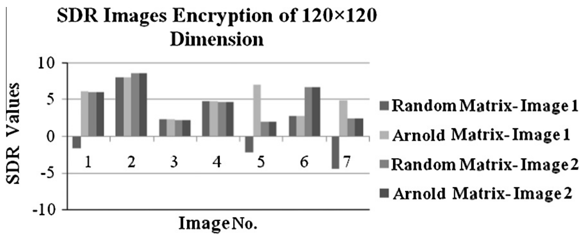


Figure 3 SDR images encryption of 120 × 120 dimension.

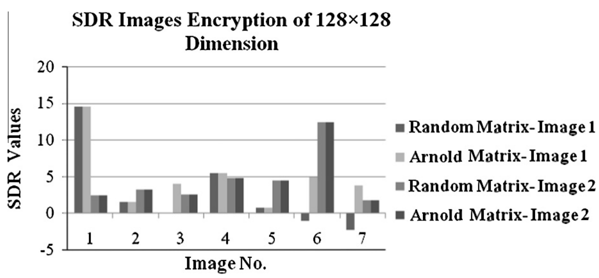


Figure 4 SDR images encryption of 128 × 128 dimension.

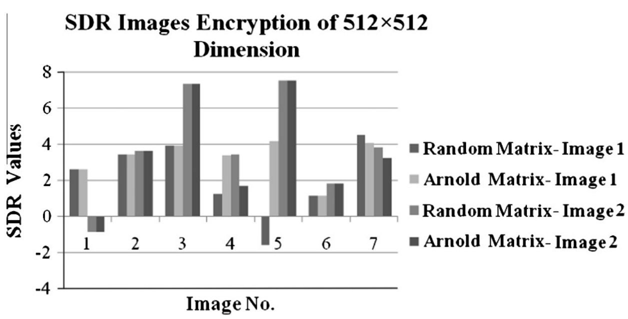


Figure 5 SDR images encryption of 512 × 512 dimension.

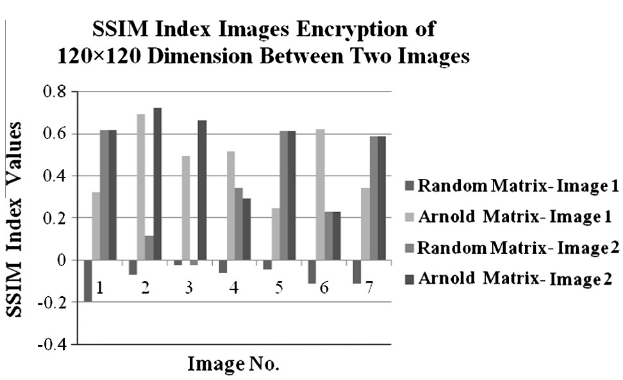


Figure 6 SSIM images encryption of 120 × 120 dimension.

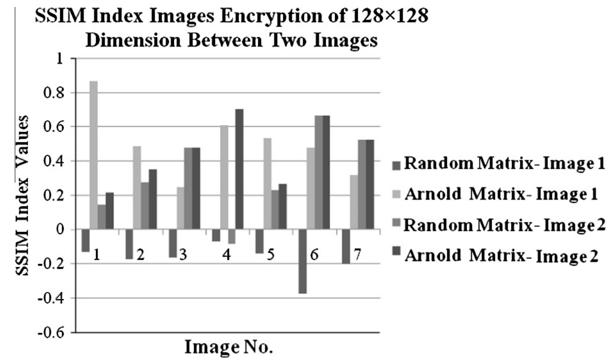


Figure 7 SSIM images encryption of 128 × 128 dimension.

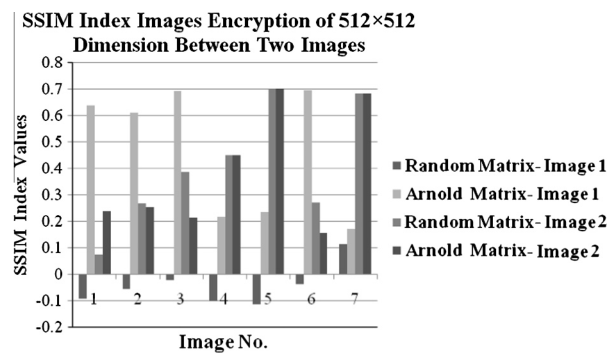


Figure 8 SSIM images encryption of 512 × 512 dimension.

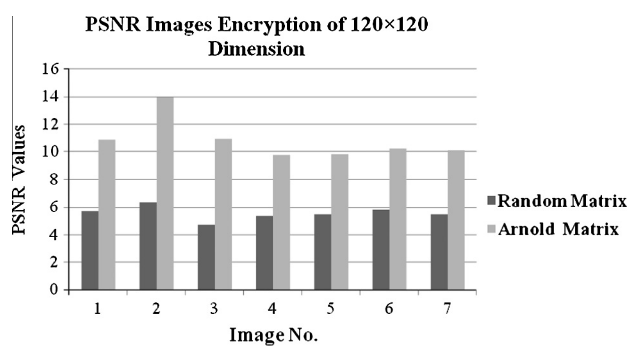


Figure 9 PSNR images encryption of 120 × 120 dimension.

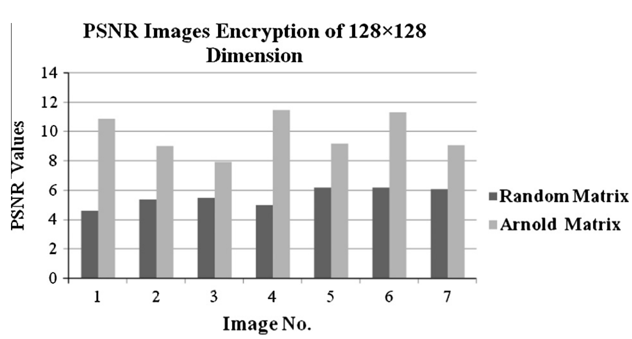
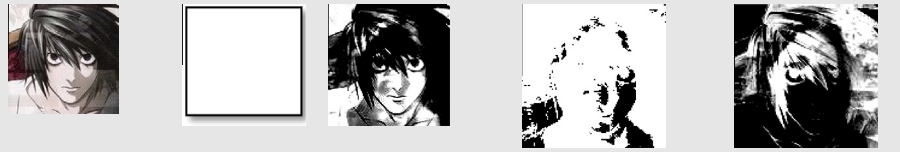
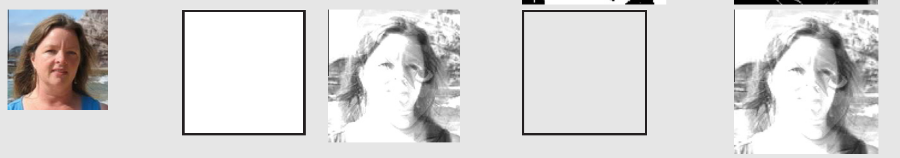
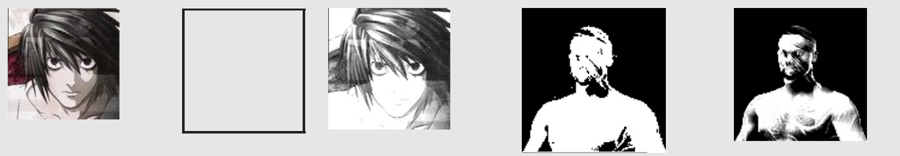
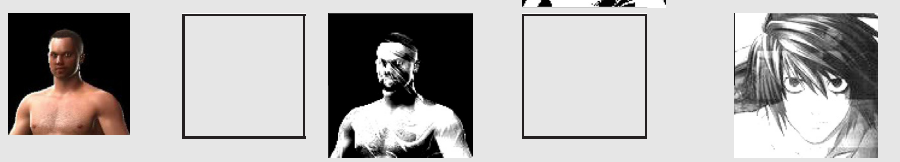
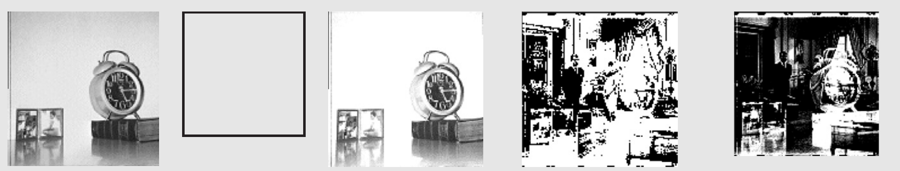
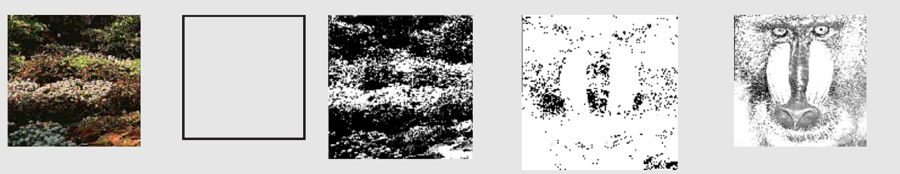


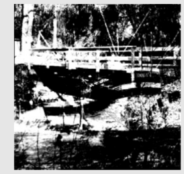
Figure 10 PSNR images encryption of 128 × 128 dimension.

Table 2 Samples of subjective test of random matrix and Arnold Cat Map matrix.

Dimension	Mixed images	Original image	Arnold Cat Map matrix		Random matrix	
			Encrypted	Decrypted	Encrypted	Decrypted
120 × 120	Two original images mixed 1 and 2					
						
	Two original images mixed 1 and 2					
						
128 × 128	Two original images mixed 1 and 2					
						
	Two original images mixed 1 and 2					
						
512 × 512	Two original images mixed 1 and 2					
						

(*continued on next page*)

Table 2 (*continued*)

Dimension	Mixed images	Original image	Arnold Cat Map matrix		Random matrix	
			Encrypted	Decrypted	Encrypted	Decrypted
	Two original images mixed 1 and 2					
						

7. Conclusions

In this paper, we proposed a new Encryption method consists of a combination of Independent Component Analysis and Arnold Cat Map matrix. The experimental results showed that the proposed approach yields a better encryption performance compared to random matrix using JADE algorithm. The results have shown the unintelligibility of the tested images using Arnold Cat Map matrix in comparison with random matrix. Also it can be estimated that the cryptanalytic efforts that need to analyze this proposed system will be very high in comparison with the efforts paid to the system based on ICA alone.

For future works we can study in detail the strength of the ACM generator matrix and check its effort on the final results (in encryption and decryption).

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