



Parul University

Faculty of Engineering & Technology

Department of Applied Sciences and Humanities

1st Year B.Tech Programme (All Branches)

Mathematics – 1 (303191101)

Assignment -2

Q-1	<p>Do as directed:</p> <p>1 .When we differentiate an expression with respect to one of a number of independent variables, we are engaged in:</p> <p>(a)Partial differentiation (b)Integration</p> <p>(c)Finding definite integrals (d)Total differentiation</p> <p>2.For $f(x, y, z) = xe^y \cos z - z - 8$, $f_x(3, 0, 0) = \underline{\hspace{2cm}}$</p> <p>3.For function $f(x, y) = x^2 e^y$, find all second order partial derivatives.</p> <p>4.If $z = x \sin y + y \cos x$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$</p> <p>5.Evaluate f_{xxy} for $f = x^3 y^2$ at $(-1, 2)$</p> <p>6.Evaluate $\lim_{(x,y) \rightarrow (1,2)} \frac{x^2 + y}{3x + y^2}$.</p> <p>7 Find $f_{xyyx} = 2x^3 y^3 - 3x^2 y$</p>
Q-2	<p>1. If $u = y^2 e^{\frac{y}{x}} + x^2 \tan^{-1} \left(\frac{x}{y} \right)$, show that</p> <p>(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$</p> <p>(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u$</p> <p>2. If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, then prove that</p> <p>(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$</p> <p>(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4} (\tan^3 u - \tan u)$</p>

Q-3	<p>(a) If $x^y + y^x = c$ then, find $\frac{dy}{dx}$.</p> <p>(b) If $y \sin x = x \cos y$, find $\frac{dy}{dx}$.</p>
Q-4	<p>(a) If $u = \tan^{-1}\left(\frac{y}{x}\right)$, $x = e^t - e^{-t}$, $y = e^t + e^{-t}$, find $\frac{du}{dt}$.</p> <p>(b) If $z = e^{xy}$, $x = t \cos t$, $y = t \sin t$, find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$</p>
Q-5	If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$
Q-6	<p>Find the equations of tangent plane and normal line for the following functions.</p> <p>(a) Ellipsoid $\frac{x^2}{4} + \frac{y^2}{1} + \frac{z^2}{9} = 3$ at the point $(-2, 1, -3)$</p> <p>(b) Surface $z = 2x^2 + y^2$ at the point $(1, 1, 3)$</p>
Q-7	Find the extreme values of the function $x^3 + y^3 - 63(x + y) + 12xy$.
Q-8	<p>(a) Find the minimum values of $x^2 y z^3$, subject to the condition $2x + y + 3z = a$</p> <p>(b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin.</p>
Q-9	<p>(a) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = x^2 - y^2$, $v = 2xy$.</p> <p>(b) If $u = 2xy$, $v = x^2 - y^2$ and $x = r \cos \theta$, $y = r \sin \theta$ then, evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$.</p>
Q-10	Expand $e^x \log(1 + y)$ in powers of x and y upto second degree.