## योगः कर्ममु कौशलम् PARUL UNIVERSITY

## **Parul University**

Faculty of Engineering & Technology

Department of Applied Sciences and Humanities

1st Year B.Tech Programme (All Branches)

Mathematics – 1 (303191101)

**Assignment -3** 

## Q.1 Short questions.

- 1. Give the definition of 'order' and 'degree'.
- 2. Find order and degree of the given equations.

$$(i)(\frac{dy}{dx})^2 + \frac{d^2y}{dx^2}, \quad (ii)\frac{dy}{dx} + (\frac{d^2y}{dx^2})^3 + 3(\frac{dy}{dx})^3$$

- 3. How many variables have/has in ordinary differential equation. And in partial differential equation.
- 4. Give the examples of ordinary differential equation and partial differential equation.
- 5. What is the sufficient condition for Exact differential equation?
- 6. Integrating factor of non-exact homogeneous differential equation with  $Mx + Ny \neq 0$  is \_\_\_\_\_.
- 7. Let  $\{a_n\}$  be a sequence, if for every  $\varepsilon > 0$  there exist an integer N such that  $n \ge N \Longrightarrow |a_n l| < \varepsilon$  if such a number exist then we write  $\lim_{n \to \infty} a_n = \underline{\hspace{1cm}}$ .
- 8. A sequence is said to be convergent if the sequence is has \_\_\_\_\_ limit.
- 9. A sequence is  $\{a_n\}$  is said to be \_\_\_\_\_ if  $a_n < a_{n+1}$  for each value of n.
- 10. A sequence  $\{a_n\}$  is bounded above and bounded below both the it is \_\_\_\_\_.
- 11. A series is said to be \_\_\_\_\_\_ if while writing the n<sup>th</sup> partial sum all terms except first and last vanish.
- 12. The series  $\sum_{n=0}^{\infty} \frac{1}{n^p}$  converges if p \_\_\_ 1 and diverges if p \_\_\_ 1.
- 13. "If  $\lim_{n\to\infty}\frac{a_n}{b_n}=c>0$ , then  $\sum a_n$  and  $\sum b_n$  both converges or both diverges."

Above statement is true or false?

- 14. A series in which the terms are alternatively positive and negative is called a/an\_\_\_\_.
- 15.  $\lim_{n\to\infty} |a_n|^{1/n} = L$  shows the \_\_\_\_\_ test.

## Solve examples.

1. Find solution of non-linear differential equation

(i) 
$$x \frac{dy}{dx} + y = x^3 y^6$$
, (ii)  $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$ .

2. Solve (i) 
$$2xy dx + (1 + x^2) dy = 0$$
 (ii)  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + \cos y + x} = 0$ .

3. Find solution of linear differential equation.

(i) 
$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$
, (ii)  $y' + y \tan x = \sin 2x$ 

4. Check the exactness and solve:

$$(i)(x^4 + y^4)dx - xy^3dy = 0$$
,  $(ii)(2x\log x - xy)dy + 2ydx = 0$ .

5. Verify that  $y = e^{-x}(a\cos x + b\sin x)$  is a solution of y'' + 2y' + 2y = 0, where a and b are constants.

6. Form the differential equation of  $y = (C_1 + C_2 x)e^{2x}$ 

7. Find the sum of the series  $\log 2 + \log \frac{3}{2} + \log \frac{4}{3} + \cdots + \infty$ .

8. Test the convergence of the following series using suitable test.

(i) 
$$\sum_{n=1}^{\infty} n^{-\pi}$$
, (ii)  $\sum_{n=1}^{\infty} \sqrt[4]{(n)^2}$ , (iii)  $\sum_{n=1}^{\infty} \frac{2^n}{7^n + 8}$ ,

$$(iv)\sum_{n=1}^{\infty}\frac{1}{4+\sqrt[3]{n}}, \qquad (v)\sum_{n=1}^{\infty}\frac{3^n}{2^{n+3}}, \qquad (vi)\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2},$$

$$(vii)$$
  $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$ ,  $(viii)$   $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ ,

$$(ix) 1 + \frac{2}{3} + \frac{4}{9} + \dots + \infty$$
,  $(x) 1 + \frac{2^2}{2^2} + \frac{3^2}{2^2} + \dots + \frac{n^2}{2^2}$