

**PARUL UNIVERSITY**  
**FACULTY OF ENGINEERING & TECHNOLOGY**  
**B.Tech. Winter 2021-22 Examination**

**Semester: 1**  
**Subject Code: 03191101**  
**Subject Name: Mathematics - 1**

**Date: 29/10/2021**  
**Time: 02:00pm to 04:30pm**  
**Total Marks: 60**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

**Q.1 Objective Type Questions - (Each of one mark)****(15)**

1. The series  $1 + r + r^2 + r^3 + \dots \infty$  is converges if  
 A.  $|r| < 1$       B.  $|r| \leq -1$       C.  $r \geq 1$       D. not possible.
2. Eigen value of  $\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$   
 A. 0, 0      B. 0, 9      C. 9, 9      D. 0, 9, 0.
3. If  $f(x, y) = c$  then  $\frac{dy}{dx}$  is \_\_\_\_\_.  
 A.  $\frac{f_x}{f_y}$       B.  $-\frac{f_x}{f_y}$       C.  $\frac{f_y}{f_x}$       D.  $-\frac{f_y}{f_x}$
4. If eigen value of a matrix A is  $\lambda$ , then eigen value of  $A^3$  is  
 A.  $3\lambda$       B.  $\frac{3}{\lambda}$       C.  $\lambda^3$       D. None of the given
5. For the function  $z = f(x, y)$ , the point  $(a, b)$  is stationary point if  
 A.  $f_x = 0$       B.  $f_x = 0$  and  $f_y = 0$       C.  $f_y = 0$       D. None of the above
6. Find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  if  $x^3 + y^3 + z^3 + 6xyz = 1$
7. Find the  $\text{Arg}(Z)$ ,  $Z = -1 - i$
8. Check Convergence  $\sum_{n=1}^{\infty} \frac{n+1}{n}$
9. Evaluate  $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$
10. Find the limit  $\lim_{(x,y) \rightarrow (1,2)} \frac{5x^2y}{x^2+y^2}$
11. If  $f'(c) = 0$  and  $f''(c) < 0$ , then  $f$  has a local \_\_\_\_\_.
12. The Jacobian  $\frac{\partial(x, y)}{\partial(r, \theta)}$  where  $x = r \cos \theta$  and  $y = r \sin \theta$ .
13. Find the Rank of the following matrix:  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$
14. Obtain Maclaurin Series of  $e^x$ .
15.  $(2 + 3i)(1 + 2i) =$ \_\_\_\_\_.

**Q.2 Answer the following questions. (Attempt any three)****(15)**

A) Discuss the convergence of  $\sum_{n=1}^{\infty} \frac{|\sin nx|}{n^2}$

B) Find the Taylor's series expansion of  $f(x) = x^3 - 2x + 4$  about  $a = 2$ .

C) If  $u = \cos^{-1} \frac{x+y}{x^2+y^2}$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ .

D) Find inverse of the matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$

**Q.3** A) Test the convergence of  $\sum_{n=1}^{\infty} \frac{n^3 + 2}{2^n + 2}$  **(07)**

B) Investigate for what values of  $\lambda$  and  $\mu$  the equations **(08)**

$x + 2y + z = 8$ ,  $2x + 2y + 2z = 13$ ,  $3x + 4y + \lambda z = \mu$  have (1) no solution, (2) a unique solution and (3) many solutions.

**OR**

B) Solve  $z^4 + 1 = 0$  and locate the roots in the argand diagram. **(08)**

**Q.4** A) Find the area of the region bounded below by  $y = e^x$ , bounded above by  $y = x$ , and bounded on the sides by  $x = 0$  and  $x = 1$  **(07)**

**OR**

A) Find maximum and minimum values of  $2(x^2 - y^2) - x^4 + y^4$ . **(07)**

B) Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$  **(08)**