Parul University

Faculty of Engineering & Technology

Department of Applied Sciences and Humanities

1st Year B.Tech Programme (All Branches)

Mathematics – 1 (303191101)

Unit – 6 Multivariable Calculus

Tutorial 2

1.	Evaluate the following limits, if exists: a) $\lim\limits_{(x,y)\to(0,0)} \frac{xycoscos\, y}{3x^2+y^2}$ b) $\frac{x^2+y^2+1}{3+x^2+3y^2}$ c) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ d) $\lim\limits_{(x,y)\to(0,1)} e^{\frac{-1}{x^2(y-1)^2}}$
2.	Check whether the given function is continuous or not, if yes then find point of continuity. a) $f(x,y)=\begin{cases} \frac{x^2y^2}{2x^2+y^2} & ,if\ \ (x,y)\neq(0,0) \\ 1 & ,if\ \ (x,y)=(0,0) \end{cases}$ b) $f(x,y)=\begin{cases} \frac{xy}{x^2+xy+y^2} & ,if\ \ (x,y)\neq(0,0) \\ 0 & ,\ if\ \ (x,y)=(0,0) \end{cases}$ c) $f(x,y)=\begin{cases} (x^2+y^2)\, sin \sin\left(\frac{1}{x^2+y^2}\right), & (x,y)\neq(0,0) \\ 0 & ,(x,y)=(0,0) \end{cases}$ d) $f(x,y)=\begin{cases} (2x^2+y), & (x,y)\neq(1,2) \\ 0 & (x,y)=(1,2) \end{cases}$
3.	Do as directed: a) For $u = x^3y+e^{xy^2}$ show that ; show that $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$ b) Find the first order partial derivatives at a given point when $f(x,y)=y\sin(xy)$ at $\left(0,\frac{\pi}{2}\right)$ c) Find all second order partial derivatives for x^2ysinx d) Find $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}$ for $u=e^{5xyz}$ e) Find indicated partial derivatives for $f(r,s,t)=r\ln(rs^2t^3)$; f_{rss} , f_{rst}

4.	Using Euler's theorems , a) Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=4u$ for $u=x^2yz-4y^2z^2+2xz^3$ b)Show that $xu_x+yu_y=6u$ if $u=x^4y^2\left(\frac{y}{x}\right)$ c)Show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$ for $u=\left(\frac{x}{y}\right)^{\frac{y}{x}}$. d)If $u=logx+logy$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2$. e)If $u=\frac{1}{3}log\ log\ \left(\frac{x^3+y^3}{x^2+y^2}\right)$, find the value of (i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$
5.	a)If $y\ log\ (cosx)=xlog(siny)$, find $\frac{dy}{dx}$. b)If $x^3+y^3=3axy$,find $\frac{dy}{dx}$.
6.	a)Find the equation of the tangent plane and normal line to the surface $z+8=xe^ycosz$ at the point (8,0,0). b)Find the equation of the normal line of the sphere $x^2+y^2+z^2=6$ at the point (a,b,c). Show that the normal line passes through the origin.
7.	a) Find the stationary value of $x^3+y^3-3axy, a>0$. b) Examine the function $x^3y^2(12-3x-4y)$ for extreme values.
8.	a)Find the minimum values of x^2yz^3, subject to the condition $2x+y+3z=a$ b)Find the minimum values of x^2+y^2, subject to the condition $ax+by=c$
9.	a)Find the $Jacobian$ $\frac{\partial(u,v)}{\partial(x,y)}$ for the following functions: (i)$u=x^2-y^2, v=2xy$ (ii)$u=\frac{y-x}{1+xy}, v=tan^{-1}y-tan^{-1}x$ b)For the transformations $x=e^vsecu, y=e^vtanu$,prove that $\frac{\partial(x,y)\partial(u,v)}{\partial(u,v)\partial(x,y)}=1$
10.	Expand x^y near the point (1,1) upto the first-degree terms.