PARUL UNIVERSITY - FACULTY OF ENGINEERING & TECHNOLOGY
Department of Applied Science & Humanities
3rd Semester B. Tech (CSE, IT)
Discrete Mathematics (303191202)

Assignment :2

Q.1 Define the following terms:
- (a) Proposition
- (b) Truth value of proposition
- (c) Tautology and Contradiction
- (d) Converse, contrapositive and Inverse of propositions.
- (e) Universal quantifier.
- (f) Logically equivalent propositions.
- (g) Propositional satisfiability

Q.2 Construct a truth table for each of these compound propositions
a) $(p \rightarrow q) \rightarrow (\neg q \rightarrow p)$
b) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow r)$
c) $(p \rightarrow q) \vee (\neg p \rightarrow \neg r)$

Q.3 Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by using the laws of logical equivalences.

Q.4 State the converse, contrapositive and Inverse of propositions of each of the following statements:
a) If you eat too much junk food, then you might gain weight.
b) If the power goes out, then the computer will shut down.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Q.5 Let p and q be the propositions
p : The program is readable.
q : The program is well structured.
Express each of these propositions as an English sentence.
a) $\neg p$ b) $p \vee q$ c) $p \rightarrow q$ d) $p \wedge q$
e) $p \leftrightarrow q$ f) $\neg p \rightarrow \neg q$ g) $\neg p \wedge \neg q$ h) $\neg p \vee (p \wedge q)$

Q.6 Construct a truth table for each of following compound proposition:
(a) $p \Rightarrow \sim p$, (b) $p \oplus (p \vee q)$,(c) $(p \vee q) \Rightarrow (p \wedge q)$

Q.7 Find the bitwise OR, bitwise And & bitwise XOR of each of the following pairs of bit strings:
(a) 1011110100, 1110001110
(b) 110010000, 111110001

Q.9 State the following Laws for mathematical logic and prove them using truth table:
- (a) Associative laws.
- (b) Distributive laws.
- (c) De Morgan’s laws.

Q.10 Show that each of these conditional statements is a tautology by using truth tables.

a) $[(p \wedge q) \rightarrow (p \vee q)]$

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Q.11 Determine whether the given compound propositions is satisfiable.

$(p \leftrightarrow q) \wedge (p \leftrightarrow \neg q)$

Q.12 Prove that if *n* is an integer and n^2 is odd, then *n* is odd.

Q.13 Use the method of contradiction, prove that $\sqrt{p}$, p is prime is irrational number

Q.14 Use a direct proof to show that the sum of two odd integers is even.

Q.15 Give a direct proof that if *m* and *n* are both perfect squares, then *nm* is also a perfect square.

Q.16 Show that if *n* is an integer and $n^3 + 5$ is odd, then *n* is even using

a) a proof by contraposition.

b) a proof by contradiction.

Q.17 Prove that For all integers a, b and c , if a divides b and b divides c, then a divides c.

Q.18 Identify the identity element in $\mathbb{Z}$ under the operation $*$ given as $a * b = a + b - 3$, for any $a, b \in \mathbb{Z}$. Also identify the inverse element of any member $a \in \mathbb{Z}$

Q.19 Check if the set of all non-negative integers is an abelian group under usual addition of integers.

Q.20 Show that the binary operation $*$ defined on the set of positive rational numbers Q^+, by

$a * b = \frac{ab}{4}$ forms an abelian group.