<u>Testing Hypothesis</u>

<u>Statistical Hypotheses:</u>

A **statistical hypothesis** is an assumption about a population <u>parameter</u>. This assumption may or may not be true. **Hypothesis testing** refers to the formal procedures used by statisticians to accept or reject statistical hypotheses.

The best way to determine whether a statistical hypothesis is true would be to examine the entire population. Since that is often impractical, researchers typically examine a random sample from the population. If sample data are not consistent with the statistical hypothesis, the hypothesis is rejected.

There are two types of statistical hypotheses.

- **Null hypothesis**. The null hypothesis, denoted by H_0, is usually the hypothesis that sample observations result purely from chance.
- **Alternative hypothesis**. The alternative hypothesis, denoted by H_1 or H_a, is the hypothesis that sample observations are influenced by some non-random cause.

For example, suppose we wanted to determine whether a coin was fair and balanced. A null hypothesis might be that half the flips would result in Heads and half, in Tails. The alternative hypothesis might be that the number of Heads and Tails would be very different. Symbolically, these hypotheses would be expressed as

H_0: P = 0.5
H_a: P ≠ 0.5

Suppose we flipped the coin 50 times, resulting in 40 Heads and 10 Tails. Given this result, we would be inclined to reject the null hypothesis. We would conclude, based on the evidence, that the coin was probably not fair and balanced.

<u>The Logic of Hypothesis Tests</u>

⇒ Assume a population distribution with a specified population mean.

⇒ State the hypothesized population mean (this statement is referred to as the null hypothesis). This mean is stated as the *null hypothesis* and is designated H_o.

For example, $\mu = 10$

⇒ State the logical alternative to this hypothesis. This is called the *alternate hypothesis* and is designated H_a.

For example, $\mu \neq 10$.

(Note the alternate hypothesis can have other forms since the concept of not equal can imply $\mu > 10$ or $\mu < 10$.)

Hypothesis Tests

Statisticians follow a formal process to determine whether to reject a null hypothesis, based on sample data. This process, called **hypothesis testing**, consists of four steps.

- State the hypotheses. This involves stating the null and alternative hypotheses. The hypotheses are stated in such a way that they are mutually exclusive. That is, if one is true, the other must be false.
- Formulate an analysis plan. The analysis plan describes how to use sample data to evaluate the null hypothesis. The evaluation often focuses around a single test statistic.
- Analyze sample data. Find the value of the test statistic (mean score, proportion, t statistic, z-score, etc.) described in the analysis plan.
- Interpret results. Apply the decision rule described in the analysis plan. If the value of the test statistic is unlikely, based on the null hypothesis, reject the null hypothesis.

Decision Errors

Two types of errors can result from a hypothesis test.

- **Type I error**. A Type I error occurs when the researcher rejects a null hypothesis when it is true. The probability of committing a Type I error is called the **level of singnificance**. This probability is also called **alpha**, and is often denoted by α.
- **Type II error**. A Type II error occurs when the researcher fails to reject a null hypothesis that is false . The probability of committing a Type II error is called **Beta**, and is often denoted by β. The probability of *not* committing a Type II error is called the **Power** of the test.

TABLE VALUE. (Z – TEST)

%(LEVEL OF SIGNIFICANCE)	C	ALPHA	One tail test	Two tail test
1%	0.99	0.01	2.33	2.575
2%	0.98	0.02	2.05	2.33
5%	0.95	0.05	1.645	1.96
10%	0.90	0.1	1.28	1.645

<u>One Sample / Two Sample Hypothesis Tests</u>

⇒ Applied to determine if the population mean is consistent with a specified value or standard

⇒ Two tests

- the z- test

$$Z = \frac{Difference}{S.E}$$

- the t-test

Assumptions: z-test

- the underlying distribution is normal or the Central Limit Theorem can be assumed to hold
- the sample has been randomly selected
- the population standard deviation is known or the sample size is at least 25.

Assumptions: the t- test

- the underlying distribution is normal or the Central Limit Theorem can be assumed to hold
- the sample has been randomly selected

- **<u>One-Tailed and Two-Tailed Tests</u>**

A test of a statistical hypothesis, where the region of rejection is on only one side of the sampling distribution, is called a **one-tailed test**. For example, suppose the null hypothesis states that the mean is less than or equal to 10. The alternative hypothesis would be that the mean is greater than 10. The region of rejection would consist of a range of numbers located on the right side of sampling distribution; that is, a set of numbers greater than 10.

- A test of a statistical hypothesis, where the region of rejection is on both sides of the sampling distribution, is called a **two-tailed test**. For example, suppose the null hypothesis states that the mean is equal to 10. The alternative hypothesis would be that the mean is less than 10 or greater than 10. The region of rejection would consist of a range of numbers located on both sides of sampling distribution; that is, the region of rejection would consist partly of numbers that were less than 10 and partly of numbers that were greater than 10.

Decision Rules

The analysis plan includes decision rules for rejecting the null hypothesis. In practice, statisticians describe these decision rules in two ways - with reference to a P-value or with reference to a region of acceptance.

Summary of Computational Steps

1. Specify the null hypothesis and an alternative hypothesis.
2. Compute M = ΣX/N.
3. Compute = $\sigma_M = \frac{\sigma}{\sqrt{N}}$.
4. Compute $z = \frac{M - \mu}{\sigma_M}$ where M is the sample mean and μ is the hypothesized value of the population.
5. Write Z_t using the table value.
6. If $Z_c > Z_t$ we reject the null hypothesis. Otherwise we accept null hypothesis.

Testing of Means when the Population Standard Deviation is Known

This section explains how to compute a significance test for the mean of a normally-distributed variable for which the population standard deviation (σ) is known. In practice, the standard deviation is rarely known. However, learning how to compute a significance test when the standard deviation is known is an excellent introduction to how to compute a significance test in the more realistic situation in which the standard deviation has to be estimated.

1. The first step in hypothesis testing is to specify the null hypothesis and the alternate hypothesis. In testing hypotheses about μ, the null hypothesis is a hypothesized value of μ. Suppose the mean score of all 10-year old children on an anxiety scale were 7. If a researcher were interested in whether 10-year old children with alcoholic parents had a different mean score on the anxiety scale, then the null and alternative hypotheses would be:

$H_0: \mu_{alcoholic} = 7$

H_1: $\mu_{alcoholic} \neq 7$

2. The second step is to choose a significance level. Assume the 0.05 level is chosen.
3. The third step is to compute the mean. Assume M = 8.1.

The fourth step is to compute p, the probability (or probability value) of obtaining a difference between M and the hypothesized value of μ (7.0) as large or larger than the difference obtained in the experiment. Applying the general formula to this problem,

$$z = \frac{M - \mu}{\sigma_M} = \frac{8.1 - 7.0}{\sigma_M}$$

The sample size (N) and the population standard deviation (σ) are needed to calculate σ_M. Assume that N = 16 and σ= 2.0. Then,

$$\sigma_M = \frac{\sigma}{\sqrt{N}} = \frac{2}{\sqrt{16}} = 0.50$$

$$Z = \frac{8.1 - 7}{0.50}$$

= 2.2

At 0.05 level of significance , Z_t = 1.96

5. $Z_c > Z_t$
   we reject the null hypothesis. It is concluded that the mean anxiety score of 10-year-old children with alcoholic parents is higher than the population mean.

<u>Power of a Hypothesis Test</u>

The probability of *not* committing a Type II error is called the **power** of a hypothesis test.

Effect Size

To compute the power of the test, one offers an alternative view about the "true" value of the population parameter, assuming that the null hypothesis is false. The **effect size** is the difference between the true value and the value specified in the null hypothesis.

Effect size = True value - Hypothesized value

For example, suppose the null hypothesis states that a population mean is equal to 100. A researcher might ask: What is the probability of rejecting the null hypothesis if the true population mean is equal to 90? In this example, the effect size would be 90 - 100, which equals -10.

Factors That Affect Power

The power of a hypothesis test is affected by three factors.

- Sample size (*n*). Other things being equal, the greater the sample size, the greater the power of the test.Significnce level (α). The higher the significance level, the higher the power of the test. If you increase the significance level, you reduce the region of acceptance. As a result, you are more likely to reject the null hypothesis. This means you are less likely to accept the null hypothesis when it is false; i.e., less likely to make a Type II error. Hence, the power of the test is increased.
- The "true" value of the parameter being tested. The greater the difference between the "true" value of a parameter and the value specified in the null hypothesis, the greater the power of the test. That is, the greater the effect size, the greater the power of the test.

Problem 1

Other things being equal, which of the following actions will reduce the power of a hypothesis test?

I. Increasing sample size.
II. Increasing significance level.
III. Increasing beta, the probability of a Type II error.

(A) I only
(B) II only
(C) III only
(D) All of the above
(E) None of the above

Solution

The correct answer is (C). Increasing sample size makes the hypothesis test more sensitive - more likely to reject the null hypothesis when it is, in fact, false. Increasing the significance level reduces the region of acceptance, which makes the hypothesis test more likely to reject the null hypothesis, thus increasing the power of the test. Since, by definition, power is equal to one minus beta, the power of a test will get smaller as beta gets bigger.

Problem 2

Suppose a researcher conducts an experiment to test a hypothesis. If she doubles her sample size, which of the following will increase?

I. The power of the hypothesis test.
II. The effect size of the hypothesis test.
III. The probability of making a Type II error.

(A) I only
(B) II only
(C) III only
(D) All of the above
(E) None of the above

Solution

The correct answer is (A). Increasing sample size makes the hypothesis test more sensitive - more likely to reject the null hypothesis when it is, in fact, false. Thus, it increases the power of the test. The effect size is not affected by sample size. And the probability of making a Type II error gets smaller, not bigger, as sample size increases.

Problem 1

In hypothesis testing, which of the following statements is always true?

I. The P-value is greater than the significance level.
II. The P-value is computed from the significance level.
III. The P-value is the parameter in the null hypothesis.
IV. The P-value is a test statistic.
V. The P-value is a probability.

(A) I only
(B) II only
(C) III only
(D) IV only
(E) V only

Solution The correct answer is (E). The P-value is the probability of observing a sample statistic as extreme as the test statistic. It can be greater than the significance level, but it can also be smaller than the significance level. It is not computed from the significance level, it is not the parameter in the null hypothesis, and it is not a test statistic.

Example for Hypothesis Test for a Proportion

1. **In a hospital out of 500 new born babies,280 are boys.Does this information support the hypothesis that the births of boys and girls are in equal proportions?(Take 1% level of significance)**

H_0 : Proportion of boys P=1/2

$H_1 : P \neq \frac{1}{2}$ (two – tailed test)

$$Difference = |p - P|$$

$$= \left|\frac{280}{500} - \frac{1}{2}\right| = 0.06$$

s.E of $p = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{\frac{1}{2}\times\frac{1}{2}}{500}} = 0.02236$

$$Z = \frac{Difference}{S.E} = \frac{0.06}{0.02236} = 2.68 > 258$$

Therefore, H_0 may be rejected at 1% level of significance.

i.e the proportion of births of boys and girls may not be regarded equal.

Interpret Results

If the sample findings are unlikely, given the null hypothesis, the researcher rejects the null hypothesis. Typically, this involves comparing the P-value to the significance level, and rejecting the null hypothesis when the P-value is less than the significance level.

Problem 1

One-Tailed Test

Suppose the previous example is stated a little bit differently. Suppose the CEO claims that *at least* 80 percent of the company's 1,000,000 customers are very satisfied. Again, 100 customers are surveyed using simple random sampling. The result: 73 percent are very satisfied. Based on these results,

should we accept or reject the CEO's hypothesis? Assume a significance level of 0.05. (5%)

Solution: The solution to this problem takes four steps: (1) state the hypotheses, (2) formulate an analysis plan, (3) analyze sample data, and (4) interpret results. We work through those steps below:

State the hypotheses. The first step is to state the null hypothesis and an alternative hypothesis.

Null hypothesis: $P >= 0.80$
Alternative hypothesis: $P < 0.80$(ONE TAILED TEST)

Note that these hypotheses constitute a one-tailed test. The null hypothesis will be rejected only if the sample proportion is too small.

- **Formulate an analysis plan**. For this analysis, the significance level is 0.05. The test method, shown in the next section, is a one-sample z-test.
- **Analyze sample data**. Using sample data, we calculate the standard deviation (σ) and compute the z-score test statistic (z).

$$\sigma = \text{sqrt}[P * (1 - P) / n] = \text{sqrt} [(0.8 * 0.2) / 100] = \text{sqrt}(0.0016) = 0.04$$
$$z = (p - P) / \sigma = (.73 - .80)/0.04 = -1.75$$

where P is the hypothesized value of population proportion in the null hypothesis, p is the sample proportion, and n is the sample size.

Since we have a one-tailed test, the P-value is the probability that the z-score is less than -1.75. We use the Normal Distribution Calculator to find $P(z < -1.75) = 0.04$. Thus, the P-value = 0.04.

- **Interpret results**. Since the P-value (0.04) is less than the significance level (0.05), we cannot accept the null hypothesis. (NULL HYPOTHESES IS REJECTED)

Note: If you use this approach on an exam, you may also want to mention why this approach is appropriate. Specifically, the approach is appropriate because the sampling method was simple random sampling, the sample included at least 10 successes and 10 failures, and the population size was at least 10 times the sample size.

Example: One Sample Hypothesis Test

Large Sample: Sample size: n > 30

1. The scores on an aptitude test required for entry into a certain job position have a mean of 500 and a standard deviation of 120. If a random sample of 36 applicants has a mean of 546, is there evidence that their mean score is different from the mean that is expected from all applicants?

Ans: Null and Alternative Hypothesis

H_0: $\mu = 500$

Ha: $\mu \neq 500$

Convert 546 to a z-score to compare it to the assumed population mean.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{546 - 500}{\frac{120}{\sqrt{36}}} = \frac{46}{20} = 2.3$$

$Z_t = 1.96$(5% level of significance)

$Z_c > Z_t$

we reject the null hypothesis.

Thus, we conclude that the population mean is not 500; that is we reject the null hypothesis and accept the alternate, concluding that the mean is not 500.

Let's construct a 95% confidence interval estimate of the population mean.

$546 \pm 1.96 * (\frac{120}{\sqrt{36}}) = 546 \pm 39.2$

The lower limit of the interval is 546 - 39.2 = 506.8

The upper limit of the interval is 546 + 39.2 = 585.2

Thus, we conclude that the actual mean score for the population from which this sample was drawn falls between 507 and 585.

2. Do problem number 1 assuming that the sample size is 16.

Approach the problem the same way as in 1, using the t-distribution.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{546 - 500}{\frac{120}{\sqrt{16}}} = \frac{46}{30} = 1.53$$

The degrees of freedom is 16-1=15

Using the t-table with 15 degrees of freedom, we find the closest t-value to 1.53 is 1.753

4.**A sample of 400 students has a mean height of 171.38 cms. Can it be reasonably regarded as a random sample from a large population with mean height 171.17 and standard deviation 3.3 cms ?**

Ans:

$$H_0 : \mu = 171.17$$
$$H_1 : \mu \neq 171.17$$

$$Difference = |\bar{x} - \mu|$$

$$= |171.38 - 171.17| = 0.21$$

$$S.E of \bar{x} = \frac{\sigma}{\sqrt{n}} = \frac{3.3}{\sqrt{400}} 0.165$$

$$Z = \frac{Diff.}{S.E.} = \frac{0.21}{0.165} = 1.27 < 1.96$$

Therefore, H_0 may be accepted at 5% level of significance.

Therefore, the sample may be regarded as a random sample from a population with mean 171.17

4. **A random sample of 100 students from a college of 1200 students gave mean and S.D of heights as 66 inches and 1.2 inches respectively. Test the hypothesis that the average height of all the students of the college is 65.8**

   Ans:

$$H_0 : \mu = 65.8$$
$$H_1 : \mu \neq 65.8$$

$Difference = |\bar{x} - \mu|$

$$= |66 - 65.8| = 0.2$$

As S.D of the population is not Known and sampling fraction $\frac{n}{N} = \frac{100}{1200} = 0.08$ is more than 0.05,we use the following formula for S.E.

$$S.E of \bar{x} = \frac{S}{\sqrt{n-1}} \times \sqrt{\frac{N-n}{N-1}} = \frac{1.2}{\sqrt{100-1}} \times \sqrt{\frac{1200-100}{1200-1}} = 0.12$$

$$Z = \frac{Diff.}{S.E.} = \frac{0.2}{0.12} = 1.67 < 1.96$$

Therefore, H_0 may be accepted. i.e. average height of all students of the college may be regarded as 65 inches.

Testing Hypothesis: Two –Sample test

Hypothesis Testing for Differences between means and proportions

In many decision-making situations, people need to determine whether the parameters of two populations are alike or different. For example (i) A company may want to test whether its female employees receive lower salaries than its male employees for the same work. (ii) A drug manufacturer may need to know whether a new drug causes one reaction in one group of experimental animals but a different reaction in another group. Hence, decision makers are concerned with the parameters of two populations and applying hypothesis testing procedure for their needs.

Difference between means

Suppose we take a random sample from the distribution of population 1 and another population 2.If we then subtract the two sample means, we get $\bar{x}_1 - \bar{x}_2$ .This difference will be positive if $\bar{x}_1$ is larger than $\bar{x}_2$ and negative if $\bar{x}_2$ is larger than $\bar{x}_1$.

The mean of the sampling distribution of the difference between sample means is symbolized as $\mu_{\bar{x}_1 - \bar{x}_2}$ or $\mu_{\bar{x}_1} - \mu_{\bar{x}_2}$ or simply $\mu_1 - \mu_2$.

If $\mu_1 = \mu_2$ then, $\mu_{\bar{x}_1} - \mu_{\bar{x}_2} = 0$.

The standard deviation of the distribution of the difference between the sample means is called the standard error of the difference between two means and is calculated by using this formula:

$$\sigma_d = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$$

where, ${\sigma_1}^2$ =variance of population 1

${\sigma_2}^2$ =variance of population 2

n_1 =size of sample from population 1

n_2 =size of sample from population 2

d= $\overline{x_1} - \overline{x_2}$

If two population standard deviations are not known, we can estimate the standard error of the difference between two means by using the formula

$$\hat{\sigma}_d = \sqrt{\frac{\hat{\sigma}_1^2}{n_1} + \frac{\hat{\sigma}_2^2}{n_2}}$$

where, $\hat{\sigma}_1^2$ =estimated variance of population 1

$\hat{\sigma}_2^2$ = estimated variance of population 2

<u>Tests for difference between means: Large sample sizes</u>

When both sample sizes are greater than 30 we have to do two-tailed test of a hypothesis about the difference between two means.

Steps:

1. State your hypothesis, type of test and significance level.

(Often, researchers choose significance levels equal to 0.01, 0.05, or 0.10; but any value between 0 and 1 can be used)

2. Choose the appropriate distribution and the critical value.

3. Compute the standard error and standardize the sample statistic.

4. Sketch the distribution and mark the sample value and critical values.

5. Interpret the result by testing the difference between means when $\mu_1 - \mu_2 \neq 0$.

Example 1 The mean height of 50 male students who showed above average participation in college athletics was 68.2 inches with a standard deviation of 2.5 inches; while 50 male students who showed no interest in such participation had a mean height of 67.5 inches with a standard deviation of 2.8 inches.

(i)Test the hypothesis that male students who participate in college athletics are taller than other male students.

(ii)By how much should the sample size of each of the two groups be increased in order that the observed difference of 0.7 inches in mean heights be significant at the 5% level of significance.

Solution. Let X_1 and X_2 denote the height (in inches) of athletic participants and non-athletic participants respectively. In the usual notations, we are given:

$s_1 = 2.5$, $n_1 = 50, \overline{x_1} = 68.2, s_2 = 2.8$, $n_2 = 50$, $\overline{x_2} = 67.5$

$$\hat{\sigma}_d = \sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}$$

$$= \sqrt{\frac{6.25}{50} + \frac{7.84}{50}}$$

$$= 0.53$$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

$\alpha = 0.05$ (level of significance)

$$z = \frac{\overline{x_1} - \overline{x_2}}{\hat{\sigma}_d}$$

$$= \frac{68.2 - 67.5}{0.53}$$

$$= 1.32$$

For a right –tailed test, the critical value of z at 5% level of significance is 1.645.

(i)Since, the calculated value of z (=1.32) is less than the critical value (=1.645),it is not significant at 5% level of significance. Hence, the null hypothesis is accepted and we conclude that the college athletes are not taller than other male students.

(ii)The difference between the mean heights of two groups, each of size n will be significant at 5% level of significance if $z \geq 1.645$

=> $\frac{68.2-67.5}{\sqrt{\frac{6.25}{n}+\frac{7.84}{n}}} \geq 1.645$

Or $\frac{0.7}{\sqrt{\frac{14.09}{n}}} \geq 1.645$

$$n \geq \left(\frac{1.645 \times 3.754}{0.7}\right)^2 \approx 78$$

Hence the sample size of each of the two groups should be increased by atleast 78-50=28,in order that the difference between the mean heights of two groups is significant.

Example 2 Two independent samples of observations were collected. For the first sample of 60 elements, the mean was 86 and the standard deviation 6.The second sample of 75 elements had a mean of 82 and a standard deviation of 9.

(a)Compute the estimation standard error of the difference between the two means.

(b)Using $\alpha = 0.01$, test whether the two samples can reasonably be considered to have come from populations with the same mean.

Solution. $s_1 = 6$, $n_1 = 60, \overline{x_1} = 86$, $s_2 = 9$, $n_2 = 75$, $\overline{x_2} = 82$

(a) $\hat{\sigma}_d = \sqrt{\frac{s_1^{\,2}}{n_1}+\frac{s_2^{\,2}}{n_2}}$

$$= \sqrt{\frac{36}{60}+\frac{81}{75}}$$

$$= 1.296$$

(b)$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$\alpha = 0.01$ (level of significance)

The limits of the acceptance region are $z = \pm 2.58$ or $\overline{x_1} - \overline{x_2} = 0 \pm z\hat{\sigma}_d$

$=$

$\pm 2.58(1.296)$

$$= \pm 3.344$$

Because the observed z value $= \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)_{H_0}}{\hat{\sigma}_d}$

$$= \frac{(86-82)-0}{1.296}$$

$$= 3.09 > 2.58$$

Hence, we reject H_0.

It is reasonable to conclude that the two samples come from different populations.

<u>(t TEST) Test for Differences between Means: Small Sample Sizes</u>

Suppose two independent small samples of size n_1 and n_2 are drawn from two normal populations and the means of the samples are $\overline{x_1}$ and $\overline{x_2}$ respectively. If we want to test the hypothesis that population means are equal we can apply *t* test in the following way.

Steps:

1. State your hypothesis, type of test and significance level.

The table below shows three sets of null and alternative hypotheses. Each makes a statement about the difference *d* between the mean of one population μ_1 and the mean of another population μ_2. (In the table, the symbol ≠ means " not equal to ".)

Set	Null hypothesis	Alternative hypothesis	Number of tails
1	$\mu_1 - \mu_2 = d$	$\mu_1 - \mu_2 \neq d$	2
2	$\mu_1 - \mu_2 \geq d$	$\mu_1 - \mu_2 < d$	1
3	$\mu_1 - \mu_2 \leq d$	$\mu_1 - \mu_2 > d$	1

The first set of hypotheses (Set 1) is an example of a two-tailed test, since an extreme value on either side of the sampling distribution would cause a researcher to reject the null hypothesis. The other two sets of hypotheses (Sets 2 and 3) are one-tailed tests, since an extreme value on only one side of the sampling distribution would cause a researcher to reject the null hypothesis.

When the null hypothesis states that there is no difference between the two population means (i.e., d = 0), the null and alternative hypothesis are often stated in the following form.

$$H_0: \mu_1 = \mu_2$$
$$H_a: \mu_1 \neq \mu_2$$

2. Choose the appropriate distribution and the critical value.

3. Compute the standard error and standardize the sample statistic.

Under the assumption that both the population have the same variance.

$$t = \frac{|\overline{x_1} - \overline{x_2}|}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{|\overline{x_1} - \overline{x_2}|}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

where, $S^2 = \frac{1}{n_1+n_2-2}\{\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2\}$

$$= \frac{1}{n_1+n_2-2}\{n_1{S_1}^2 + n_2{S_2}^2\}$$

where, ${S_1}^2 = \frac{1}{n_1}\sum(x_1 - \overline{x_1})^2$ and ${S_2}^2 = \frac{1}{n_2}\sum(x_2 - \overline{x_2})^2$

t is based on $\mathrm{n}_1 + \mathrm{n}_2 - 2$ degrees of freedom. For testing the null hypothesis H_0: $\mu_1 = \mu_2$

against H_1: $\mu_1 \neq \mu_2$ the value of *t* is computed from the given data and it is compared with the table value of *t* on appropriate degrees of freedom and at a required level of significance. The decision regarding acceptance or rejection of the hypothesis is then taken.

4. Sketch the distribution and mark the sample value and critical values.

5. Interpret the result by testing the difference between means when $\mu_1 - \mu_2 \neq 0$.

<u>Example 1</u>. Samples of two types of electric bulbs were tested for length of life and following data were obtained.

	Type I	Type II
Number of Units	8	7
Mean (in hours)	1134	1024
S.D.(in hours)	35	40

Test at 5% level whether the difference in the sample means is significant.

Solution. Here, $n_1 = 8$, $\overline{x_1} = 1134, S_1 = 35$

$n_2 = 7$, $\overline{x_2} = 1024, S_2 = 40$

$$S^2 = \frac{1}{n_1+n_2-2}\{n_1{S_1}^2 + n_2{S_2}^2\}$$

$$= \frac{1}{8+7-2}\{8(35)^2 + 7(40)^2\}$$

$$= 1615.38$$

Therefore, $S = \sqrt{1615.38} = 40.192$

$$t = \frac{|\overline{x_1} - \overline{x_2}|}{S} \times \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$= \frac{|1134 - 1024|}{40.192} \times \sqrt{\frac{8 \times 7}{8+7}}$$

$$= 5.288$$

D.f= $n_1 + n_2 - 2 = 13$

Table value of *t* on 13 d.f and at 5% level of significance =2.16

As $t_{cal} > t_{tab}$

Therefore, H_0 is rejected.

Hence,the two types of bulbs differ significantly so far as their mean lives are concerned.

Example 2. Below are given the gain in weights (in lbs) of cows fed on two diets X and Y.

Diet X	25	32	30	32	24	14	32			
Diet Y	24	34	22	30	42	31	40	30	32	35

Test at 5% level whether the two diets differ as regard their effects on mean increase in weight.

Solution. H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 \neq \mu_2$

x_1	x_2	$x_1 - \overline{x_1}$	$(x_1 - \overline{x_1})^2$	$x_2 - \overline{x_2}$	$(x_2 - \overline{x_2})^2$
25	24	-2	4	-8	64
32	34	5	25	2	4
30	22	3	9	-10	100
32	30	5	25	-2	4
24	42	-3	9	10	100
14	31	-13	169	-1	1
32	40	5	25	8	64
	30			-2	4
	32			0	0
	35			3	9
189	320	0	266	0	350

$$\overline{x_1} = \frac{\sum \mathrm{x_1}}{\mathrm{n_1}} = \frac{189}{7} = 27\ ,\ \overline{x_2} = \frac{\sum \mathrm{x_2}}{\mathrm{n_2}} = \frac{320}{10} = 32$$

$$S^2 = \frac{1}{n_1+n_2-2}\{n_1{S_1}^2 + n_2{S_2}^2\}$$

$$= \frac{1}{7+10-2}\{266 + 350\}$$

$$= 41.067$$

Therefore, $S = \sqrt{41.067} = 6.41$

$$t = \frac{|\overline{x_1} - \overline{x_2}|}{S} \times \sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$= \frac{|37-32|}{6.41} \times \sqrt{\frac{7\times10}{7+10}}$$

$$= 1.58$$

D.f= $n_1 + n_2 - 2 = 15$

Table value of *t* on 15 d.f and at 5% level of significance =2.131

As $t_{cal} < t_{tab}$

Therefore, H_0 is accepted.

Hence, diets do not differ significantly.

Testing Differences between Means with Dependent Samples

Sometimes, however, it makes sense to take samples that are not independent of each other. Often, the use of such dependent (or paired) samples enables us perform more precise analysis, because they will allow us to control for extraneous factors. With dependent samples, we still follow the same basic procedure of hypothesis testing. The only difference is the use of different formula for the estimated standard error of the sample differences and that we will require that both samples to be of the same size.

We will compute by following steps:

1) $\bar{x} = \frac{\sum x}{n}$

where, $\sum x$ is the sum of corresponding differences of the two samples.

n is the sample size

2) $s^2 = \frac{1}{n-1}(\sum x^2 - n\bar{x}^2)$

$$\hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{n}}$$

3)By considering the hypothesis and given level of significance compute the value of *t* according to acceptance region.

4) Compute the observed *t* value by the formula $\frac{\bar{x}-\mu_{H_0}}{\hat{\sigma}_{\bar{x}}}$.

5) Interpret the result.

<u>Example 1</u> Sherri Welch is a quality control engineer with the windshield wiper manufacturing division of Emsco, Inc. Emsco is currently considering two new synthetic rubbers for its wiper blades, and Sherri was charges with seeing whether blades made with the two compounds wear equally well. She equipped 12 cars belonging to other Emsco employees with one blade made of each of the two compounds. On cars 1 to 6, the right blade was made of compound A and the left blade was made of compound B; on cars 7 to 12, compound A was used for the left blade. The cars were driven under normal operating conditions until the blades no longer did a satisfactory job of clearing the windshield of rain. The data below give the usable life (in days) of the blades. At $\alpha = 0.05$, do the two compounds wear equally well?

Car	1	2	3	4	5	6	7	8	9	10	11	12
Left blade	162	323	220	274	165	271	233	156	238	211	241	154
Right blade	183	347	247	269	189	257	224	178	263	199	263	148

<u>Solution.</u>

Car	1	2	3	4	5	6	7	8	9	10	11	12
Left blade	162	323	220	274	165	271	233	156	238	211	241	154
Right blade	183	347	247	269	189	257	224	178	263	199	263	148
Difference	21	24	27	-5	24	-14	9	-22	-25	12	-22	6

$$\bar{x} = \frac{\sum x}{n} = \frac{35}{12} = 2.9167\ days$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right) = \frac{1}{11}(4397 - 12(2.9167)^2) = 390.45, s = \sqrt{s^2} = 19.76\ days$$

$$\hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{19.76}{\sqrt{12}} = 5.7042\ days$$

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$

$\alpha = 0.05$

The limits of the acceptance region are $t = \pm 2.201$,or

$$\bar{x} = 0 \pm t \hat{\sigma}_{\bar{x}} = \pm 2.201(5.7042) = \pm 12.55\ days$$

Because the observed *t* value $= \frac{\bar{x} - \mu_{H_0}}{\hat{\sigma}_{\bar{x}}} = \frac{2.9167-0}{5.7042} = 0.511 < 2.201$

(or $\bar{x} = 2.9167 < 12.55$),we do not reject H_0.The two compounds are not significantly different with respect to usable life.

Example.2 Nine computer-components dealers in major metropolitan areas were asked for their prices on two similar color inkjet printers. The results of this survey are given below. At $= 0.05$, it is reasonable to assert that, on average, the Apson printer is less expensive than the Okaydata printer?

Dealer	1	2	3	4	5	6	7	8	9
Apson price(in dollars)	250	319	285	260	305	295	289	309	275
Okaydata price(in dollars)	270	325	269	275	289	285	295	325	300

Solution.

Dealer	1	2	3	4	5	6	7	8	9
Apson price(in dollars)	250	319	285	260	305	295	289	309	275
Okaydata price(in dollars)	270	325	269	275	289	285	295	325	300
Difference	20	6	-16	15	-16	-10	6	16	25

$$\bar{x} = \frac{\sum x}{n} = \frac{46}{9} = 5.1111\ dollars$$

$$s^2 = \frac{1}{n-1}\left(\sum x^2 - n\bar{x}^2\right) = \frac{1}{8}(2190 - 9(5.1111)^2) = 244.36, s = \sqrt{s^2} = 15.63\ dollars$$

$$\hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{15.63}{\sqrt{9}} = 5.21\ dollars$$

$H_0 : \mu_0 = \mu_A$

$H_1 : \mu_0 > \mu_A$

$\alpha = 0.05$

The upper limit of the acceptance region is $t = 1.860$,or

$$\bar{x} = 0 \pm t\hat{\sigma}_{\bar{x}} = 1.860(5.21) = 9.69\ dollars$$

Because the observed *t* value $= \frac{\bar{x} - \mu_{H_0}}{\hat{\sigma}_{\bar{x}}} = \frac{5.1111 - 0}{5.21} = 0.981 < 1.860$

(or $\bar{x} = 5.1111 < 9.69$),we do not reject H_0.On average, the Apson inkjet printer is not significantly less expensive than the Okaydata inkjet printer.

<u>Tests for difference between Proportions: Large sample sizes</u>

This approach consists of four steps:

(1) State the hypothesis

(2) Formulate an analysis plan

(3) Analyze sample data

(4) Interpret results.

<u>State the hypothesis</u>

Every hypothesis test requires the analyst to state a null hypothesis and an alternative hypothesis. The table below shows three sets of hypothesis. Each makes a statement about the difference *d* between two population proportions, P_1 and P_2.

Set	**Null hypothesis**	**Alternative hypothesis**	**Number of tails**
1	$P_1 - P_2 = 0$	$P_1 - P_2 \neq 0$	2
2	$P_1 - P_2 \geq 0$	$P_1 - P_2 < 0$	1
3	$P_1 - P_2 \leq 0$	$P_1 - P_2 > 0$	1

The first set of hypotheses (Set 1) is an example of a two-tailed test, since an extreme value on either side of the sampling distribution would cause a researcher to reject the null hypothesis. The other two sets of hypotheses (Sets 2 and 3) are one-tailed tests, since an extreme value on only one side of the sampling distribution would cause a researcher to reject the null hypothesis.

When the null hypothesis states that there is no difference between the two population proportions (i.e., d = 0), the null and alternative hypothesis for a two-tailed test are often stated in the following form.

$$H_0: P_1 = P_2$$
$$H_1: P_1 \neq P_2$$

Formulate an Analysis Plan

The analysis plan describes how to use sample data to accept or reject the null hypothesis. It should specify the following elements.

- Significance level. Often, researchers choose significance levels equal to 0.01, 0.05, or 0.10; but any value between 0 and 1 can be used.
- Test method. Use the two-proportion z-test to determine whether the hypothesized difference between population proportions differs significantly from the observed sample difference.

Analyze Sample Data

Using sample data, complete the following computations to find the test statistic and its associated P-Value.

- **Pooled sample proportion.** Since the null hypothesis states that $P_1=P_2$, we use a pooled sample proportion (P) to compute the standard error of the sampling distribution.

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

where, P_1 is the sample proportion from population 1

P_2 is the sample proportion from population 2

n_1 is the size of sample 1

n_2 is the size of sample 2.

- **Standard error.** Compute the standard error (SE) of the sampling distribution difference between two proportions.

$$\mathrm{SE} = \sqrt{\mathrm{PQ}\left(\frac{1}{\mathrm{n}_1} + \frac{1}{\mathrm{n}_2}\right)}$$

where, P is the pooled sample proportion,

Q=1-P

n_1 is the size of sample 1

n_2 is the size of sample 2.

- **Test statistic.** The test statistic is a z-score (z) defined by the following equation.

$$z = (P_1 - P_2) / SE$$

where, P_1 is the proportion from sample 1

P_2 is the proportion from sample 2

SE is the standard error of the sampling distribution.

Example 1 In a year there are 956 births in a town A, of which 52.5% were males, while in towns A and B combined, this proportion in a total of 1,406 births was 0.496.Is there any significant difference in the proportion of male births in the two towns? Take 5% level of significance.

Sol. $\mathrm{n}_1 = 956, \mathrm{n}_1 + \mathrm{n}_2 = 1{,}406 \;=> \mathrm{n}_2 = 450$

Let P_1 be the proportion of males in the sample of town A =0.525

and P_2 be the proportion of males in the sample of town B

Combined proportion $\mathrm{P} = 0.496$(given)

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$0.496 = \frac{956 \times 0.525 + 450 \times P_2}{1{,}406}$$

$$=> P_2 = 0.434$$

Let H_0: $P_1 = P_2$
H_1: $P_1 \neq P_2$

$$Q = 1 - P = 0.504$$

$$= \sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \sqrt{0.496 \times 0.504 \left(\frac{1}{956} + \frac{1}{450}\right)}$$

$$= 0.027$$

z = (P_1 - P_2) / SE

$$= \frac{0.091}{0.027}$$

$$= 3.368$$

Since, z>1.96, the null hypothesis is rejected at 5% level of significance, i.e. the data are inconsistent with the hypothesis

$P_1 = P_2$ and we conclude that there is significant difference in the proportion of male births in the towns A and B.

Example 2. In two large populations, there are 30 and 25 percent respectively of blue-eyed people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations? Take 5% level of significance.

Solution. . $n_1 = 1200, n_2 = 900$

Let P_1 be the proportion of blue –eyed people in the first population =0.30

and P_2 be the proportion of blue –eyed people in the second population=0.25

Combined proportion

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$= \frac{1200 \times 0.30 + 900 \times 0.25}{2100}$$

$$= 0.279$$

Let H_0: $P_1 = P_2$
H_1: $P_1 \neq P_2$

$$Q = 1 - P = 0.721$$

SE$= \sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$$= \sqrt{0.279 \times 0.721 \left(\frac{1}{1200} + \frac{1}{900}\right)}$$

$$= 0.0197$$

$z = (P_1 - P_2) / SE$

$$= \frac{0.05}{0.0197}$$

$$= 2.538$$

Since, z>1.96, the null hypothesis is rejected at , i.e. the data are inconsistent with the hypothesis

$P_1 = P_2$ and we conclude that the difference in population proportions is unlikely to be hidden in sampling.

<u>Probability Values:Another way to look at Testing Hypothesis</u>

P-value. The P-value is the probability of observing a sample statistic as extreme as the test statistic. Since the test statistic is a z-score, use the Normal Distribution Calculator to assess the probability associated with the z-score. The analysis is a two-proportion z-test.

Steps

1.If σ is known ,and if we are doing a one-tailed test,,we will compute the probability value

from the normal-distribution table directly .

2.Calculate standard error of the mean

$$\sigma_{\bar{x}} = \frac{\hat{\sigma}}{\sqrt{n}}$$

3.Calculate z-score by using the formula $z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}}$

Note:If σ is unknown,we will use t-distribution with n-1 degrees of freedom.

<u>Interpret Results</u>

If the sample findings are unlikely, given the null hypothesis, the researcher rejects the null hypothesis. Typically, this involves comparing the P-value to the significance level, and rejecting the null hypothesis when the P-value is less than the significance level.

Example 1. The coffee Institute has claimed that more than 40% of the adults regularly have a cup of coffee with breakfast. A random sample of 450 individuals revealed that 200 of them were regular coffee drinker at breakfast.What is the probability value for a test of hypothesis seeking to show that the Coffee Institute's claim was correct?

Sol. n=450, $\bar{P} = \frac{200}{450} = 0.4444$

$H_0: P = 0.4$

$H_1: P > 0.4$

The probability value is the probability that $\bar{P} \geq 0.4444$,that is,

$P\left(z \geq \frac{0.4444-0.4}{\sqrt{0.4(0.6)/450}}\right) = P(z \geq 1.92) = 0.5 - 0.4726 = 0.0274.$

1. **Chi-square as a Test of Independence:**
   When the data are classified according to two attributes, Chi-square (χ^2) can also be used to test the hypotheses that the two attributes are independent.

   Suppose the data are classified into r classes $A_1, A_2, \ldots, A_r$ according to attribute A and into c classes $B_1, B_2, \ldots, B_c$ according to attribute B. The representation of the data in a cross-classified table known as acontigency table is given below. In the $r \times c$ contingency table the observed frequencies of different cells are shown below:

	B_1	B_2	B_3	$\cdots$	B_c
A_1	O_{11}	O_{12}	O_{13}	$\cdots$	O_{1c}
A_2	O_{21}	O_{22}	O_{23}	$\cdots$	O_{2c}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_r	O_{r1}	O_{r2}	O_{r3}	$\cdots$	O_{rc}

A_i = Total of i^{th} row and B_j = total of j^{th} column
O_{ij} = frequency of cell ij (i^{th} row and j^{th} column)
$N = \sum A_i = \sum B_j$= total frequency

Under the null hypothesis that the two attributes A and B are independent, we shall find the expected frequency of $(i, j)^{th}$ cell.
The probability that any observation will fall in the i^{th} row = $\frac{A_i}{N}$
Similarly the probability that any observation will fall in the j^{th} column = $\frac{B_i}{N}$

And probability that any observation will fall in the i^{th} row and j^{th} column = $\frac{A_i}{N} \times \frac{B_i}{N}$

$\therefore$ Expected frequency of $(i,j)^{th}$ cell = $e_{ij} = \frac{A_i}{N} \times \frac{B_i}{N} \times N = \frac{A_i B_i}{N}$

Thus we can find expected frequencies of all the cells. From observed frequencies O_{ij} and expected frequency e_{ij}; the value of χ^2 can be obtained by following formula:

$$\chi^2 = \sum_i \sum_j \frac{\left(O_{ij} - e_{ij}\right)^2}{e_{ij}}$$

The number of independent cells in a $r \times c$ contingency table is $(r-1)(c-1)$. Hence the degrees of freedom in a $r \times c$ table is $(r-1)(c-1)$.

For testing the hypothesis of independence of two attributes A and B, the value of χ^2 is found out and is compared with the table value of χ^2 on $(r-1)(c-1)$ d.f. and at a required level of significance. If calculated value χ^2 is less than the table of χ^2, the hypothesis that the attributes are independent may be accepted.

Example: In an industry, 200 workers employed for a specific job were classified according to their performance and training received / not received. Test independence of training and performance. The data are summarized as follows:

	Performance		
	Good	Not good	Total
Trained	100	50	150
Untrained	20	30	50
	120	80	200

Solution: H_0 : Performance is independent of training.

	Performance		
	Good	Not good	Total
Trained	100 (90)	50 (60)	150
Untrained	20 (30)	30 (20)	50
	120	80	200

Expected frequency of cell (1,1) = $\frac{(150)(120)}{200} = 90$

The expected frequencies of different cells are indicated in brackets in above table.

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$

$$= \frac{(100-90)^2}{90} + \frac{(50-60)^2}{60} + \frac{(20-30)^2}{30} + \frac{(30-20)^2}{20}$$
$$= 1.11 + 1.67 + 3.33 + 5 = 11.11$$
$$d.f. = (r-1)(c-1) = (2-1)(2-1) = 1$$
On 1 d.f. and at 5% significance level, table value of $\chi^2 = 3.84$

i.e. $\chi^2_{cal} > \chi^2_{tab}$

HenceH_0 is rejected

Thus performance depends upon training.

Example: The result in the last exam of a sample of 100 students is given below:

	1st class	2nd class	3rd class	Total
Boys	10	28	12	50
Girls	20	22	2	50
Total	30	50	20	100

Can it be said that the performance in the exam depends upon gender.

Solution:H_0 : Gender and performance in the exam are independent.

	1st class	2nd class	3rd class	Total
Boys	10 (15)	28 (25)	12 (10)	50
Girls	20 (15)	22 (25)	2 (10)	50
Total	30	50	20	100

Expected frequency of cell (1,1) = $\frac{(50)(30)}{100} = 15$

The expected frequencies of different cells are indicated in brackets in above table.

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$
$$= \frac{(10-15)^2}{15} + \frac{(28-25)^2}{25} + \frac{(12-10)^2}{10} + \frac{(20-15)^2}{15} + \frac{(22-25)^2}{25} + \frac{(8-10)^2}{10}$$
$$= 1.67 + 0.3 + 0.4 + 1.67 + 0.36 + 0.4 = 4.86$$
$$d.f. = (r-1)(c-1) = (2-1)(3-1) = 2$$
On 2 d.f. and at 5% significance level, table value of $\chi^2 = 5.99$

i.e. $\chi^2_{cal} < \chi^2_{tab}$

hence H_0 is accepted.

Thus performance does not depend upon gender.

Exercise: In a certain sample of 2000 families, 1400 families are consumers of tea. Out of 1800 Hindu families, 1236 families consume tea. Use χ^2 test and state whether there is any significant difference between consumption of tea among Hindu and non-Hindu families.

2. Chi-square as a Test of Goodness of Fit: Testing the appropriateness of a distribution

Under the null hypothesis that there is no significant difference between observed and expected frequencies, the value of χ^2 is calculated by the formula:

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$

If all observed frequencies and expected frequencies are equal, the value of χ^2 will be zero. This will signify a perfect agreement of observations with expectations. More the value of χ^2, more is the divergence between the observed and expected frequencies.

The value of χ^2 is calculated from the given data and it is compared with the table value of χ^2 on $n-1$ degrees of freedom (d.f.) and at arequired significance level. If calculated value of χ^2 i.e. χ^2_{cal}<χ^2_{tab} i.e. table value of χ^2 , the null hypothesis may be accepted and it can be concluded that the given frequency fits the hypothesis. And if χ^2_{cal}>χ^2_{tab}, the null hypothesis may be rejected and it can concluded that the observed frequency distribution does not fit the hypothesis.

Note: Here, d.f. = $-k-1$, where k is the number of parameters estimated.

Example: A die is thrown for 300 times and the following distribution is obtained. Can the die be regarded as unbiased.

Number on the die	1	2	3	4	5	6
Frequency	41	44	49	53	57	56

Solution:H_0 : Die is unbiased i.e. the probability of getting any number on die is $\frac{1}{6}$.

Number on die	Observed frequency o_i	Expected frequency e_i	$\frac{(o_i - e_i)^2}{e_i}$
1	41	50	$\frac{81}{50} = 1.62$
2	44	50	0.72
3	49	50	0.02
4	53	50	0.18
5	57	50	0.98
6	56	50	0.72
Total	300	300	4.24

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i} = 4.24$$
$$d.f. = n - 1 = 6 - 1 = 5$$
The table value χ^2_{tab} on 5 d.f. and 5% significance level is
$$\chi^2_{tab} = 11.07$$
Hence $\chi^2_{cal} < \chi^2_{tab}$
Thus H_0 may be accepted.
Therefore die may be regarded as unbiased.

Example: The number of road accidents on a highway during a week is given below. Can it be considered that the proportion of accidents are equal for all days?

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Number of accidents	14	16	8	12	11	9	14

Solution:H_0 : the proportion of accidents is the same for all the days i.e. probability of an accident on any day is $\frac{1}{7}$.

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun	Total
Observed frequency	14	16	8	12	11	9	14	84
Expected frequency	12	12	12	12	12	12	12	84

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$
$$= \frac{(14-12)^2}{12} + \frac{(16-12)^2}{12} + \frac{(8-12)^2}{12} + \frac{(12-12)^2}{12} + \frac{(11-12)^2}{12} + \frac{(9-12)^2}{12}$$
$$+ \frac{(14-12)^2}{12} = \frac{4+6+16+0+1+9+4}{12} = \frac{50}{12}$$
$$= 4.17$$
$$d.f. = n - 1 = 7 - 1 = 6$$
Table value χ^2 on 6 d.f. and at 5% significance level χ^2_{tab}=12.59
$$\chi^2_{cal} < \chi^2_{tab}$$
Hence H_0 may be accepted. Thus proportions of accidnts is same for all days.

Exercise: the units produced by a plant are classified into four grades. The past performance of the plant shows that the respective proportions are 8:4:2:1. To check the run of the plant 600 parts are examined and classified as follows. Is there any evidence of a change in production standards?

Grades	1st	2nd	3rd	4th	Total
Units	340	130	100	30	600