# Design and Analysis of Algorithms (203105301)

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# CHAPTER-1

## Introduction

# Outline

- Characteristics of algorithm.
- Analysis of algorithm: Asymptotic analysis of complexity bounds– best, average and worst-case behavior;
- Performance measurements of Algorithm,
- Time and space trade-offs,
- Analysis of recursive algorithms through recurrence relations: Substitution method

# Computational problems?

- **A computational problem specifies an input-output relationship**
  - What does the input look like?
  - What should the output be for each input?
- **Example:**
  - Input: an integer number n
  - Output: Is the number even?
- **Example:**
  - Input: A list of names of people
  - Output: The same list sorted alphabetically

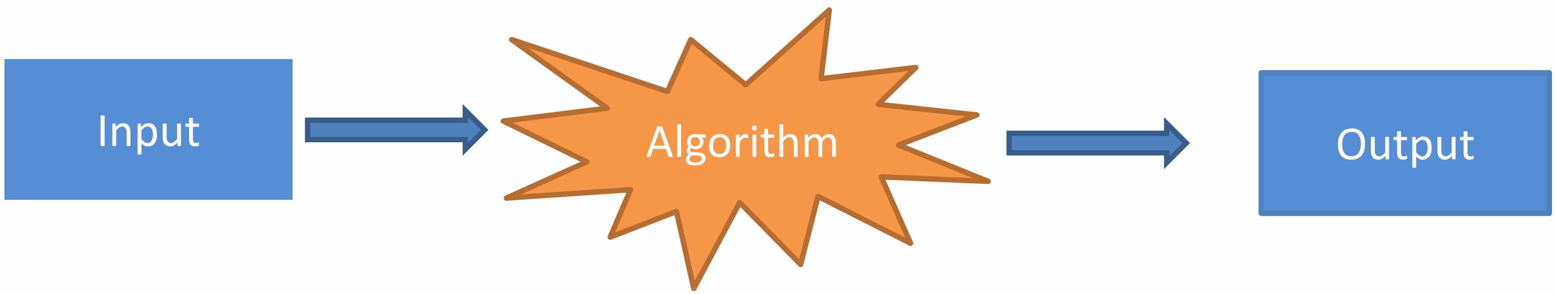
# Problems and Solution as Algorithm

- **For example, we need to solve a computational problem**
  **"Convert a distance in kilometer to meter"**
  An algorithm specifies how to solve it,
  I. 1. Read distance in kilometer
  II. 2. Calculate distance-in-meter = distance-in-kilometer*1000
  III. 3. Print distance-in-meter

# What is Algorithm?

- The word algorithm comes from the name of a Persian mathematician Abu Ja’far Mohammed ibn-i Musa al Khowarizmi.
- Algorithm is a finite set of instructions used to accomplish particular task.
- **An algorithm** takes some value, or set of values, as input and produces some value, or set of values, as output.

# What is Algorithm?




Algorithm

# Characteristics of algorithm

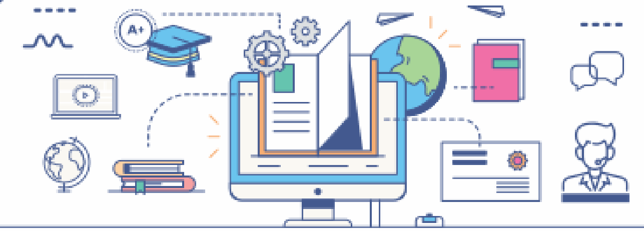
- **Input**
  - Input may be Zero or more
- **Output**
  - At least one output will be produced..
- **Definiteness**
  - Each instruction should be cleared and unambiguous.
- **Finiteness**
  - the steps should be finite.


# Characteristics of algorithm

- **Effectiveness**
  - each step must be performed in a finite amount of time
- **Non – ambiguous**
  - more than one interpretation should not be generated.
- **Correctness**
  - produce an relevant and correct output For each input produce.
- **Generality**
  - applicable to all homogeneous problems.

# Analysis of algorithm


Image source : Google



# Why algorithm analysis?

- For one problem one or more solutions are available.
- Which one is better ? How can we choose?
- The analysis of an algorithm can help us in understanding of solution in better way.
- Time & Space analysis

# Order of Growth

- Any algorithm is expected to work fast for any input size.
- Smaller input size algorithm will work fine but for higher input size execution time is much higher.
- So how the behavior of algorithm changes with the no. of inputs will give the analysis of the algorithm and is called the Order of Growth.

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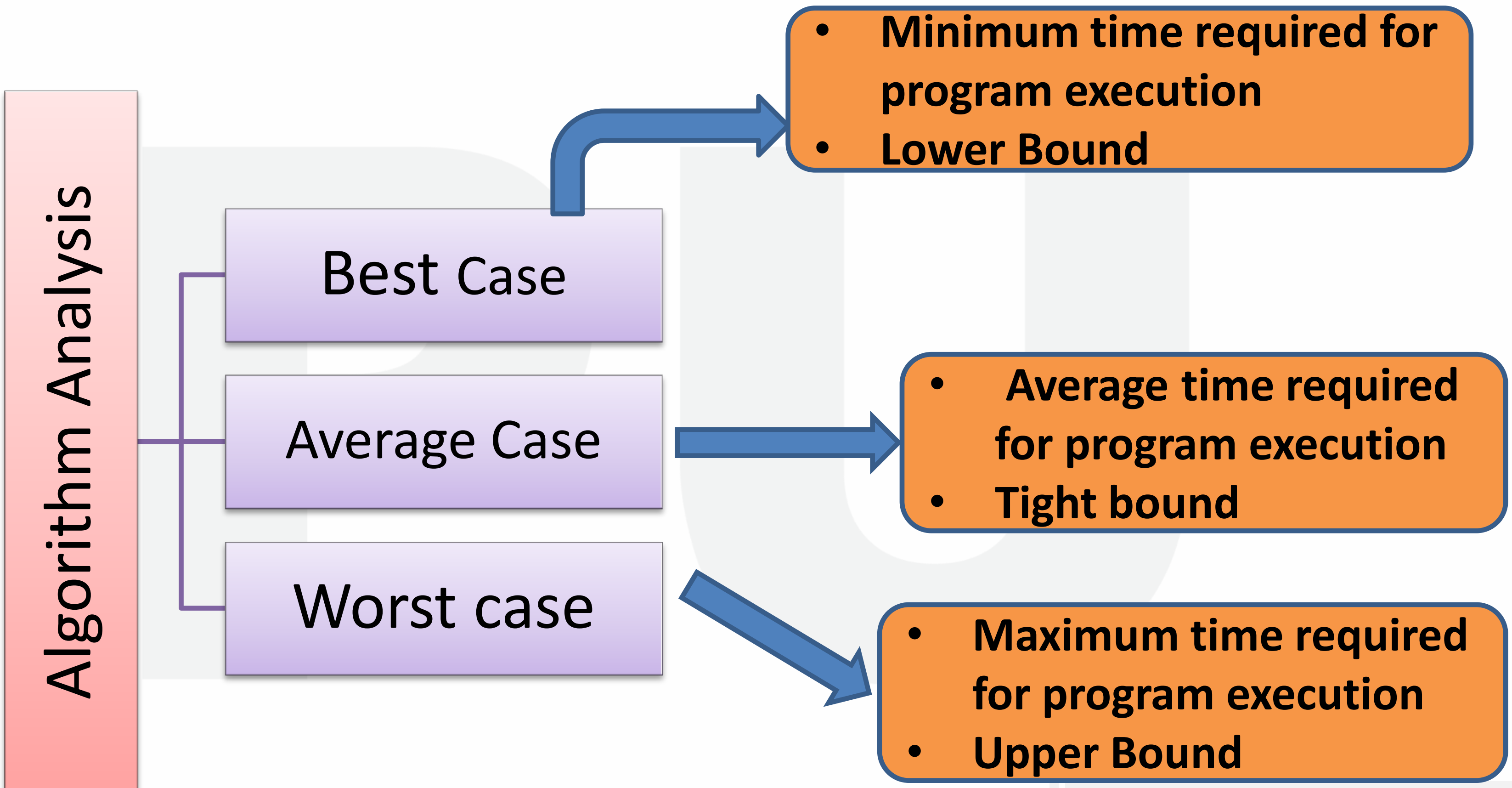
# Algorithm falls under three types



# Rate of Growth

- **Rate at which the running time increases as a function of input is called rate of growth.**

| Time Complexity | Name | Performance |
|---|---|---|
| 1 | Constant | Best |
| logn | Logarithmic | Very good |
| n | Linear | Good |
| nlogn | Linear Logarithmic | Fair |
| $n^2$ | Quadratic | Acceptable |
| $n^3$ | Cubic | Poor |
| $2^n$ | Exponential | Bad |

# Asymptotic Complexity

- Refers to defining the mathematical bound of its run-time performance.
- Running time of an algorithm as a function of input size n for large n.
- asymptotic means approaching a value or curve arbitrarily.

**Asymptotic Notations**

**O Notation**

- express the tight upper bound of an algorithm
- f(n)=O(g(n)) implies: O(g(n)) = { f(n) : there exists positive constants c>0 and n0 such that $f(n) \le c.g(n)$ for all n > n0. }

# Asymptotic analysis

**Ω Notation**

- express the lower bound of an algorithm
- *$f(n) = \Omega(g(n))$ implies: $\Omega(g(n)) = \{ f(n)$ : there exists positive constants $c>0$ and $n_0$ such that $f(n) \geq g(n)$ for all $n > n_0$. $\}$*

**θ Notation**

- express both the lower bound and the upper bound (tight bound)”
- *$f(n) = \Theta(g(n))$ implies: $\Theta(g(n)) = \{ f(n)$ : there exists positive constants $c_1>0$, $c_2>0$ and $n_0$ such that $c_1.g(n) \leq f(n) \leq c_2.g(n)$ for all $n > n_0$. $\}$*

***OR***, *if & only if $f(n)=O(g(n))$ and $f(n)= \Omega(g(n))$ for all $n > n_0$*

# Asymptotic analysis

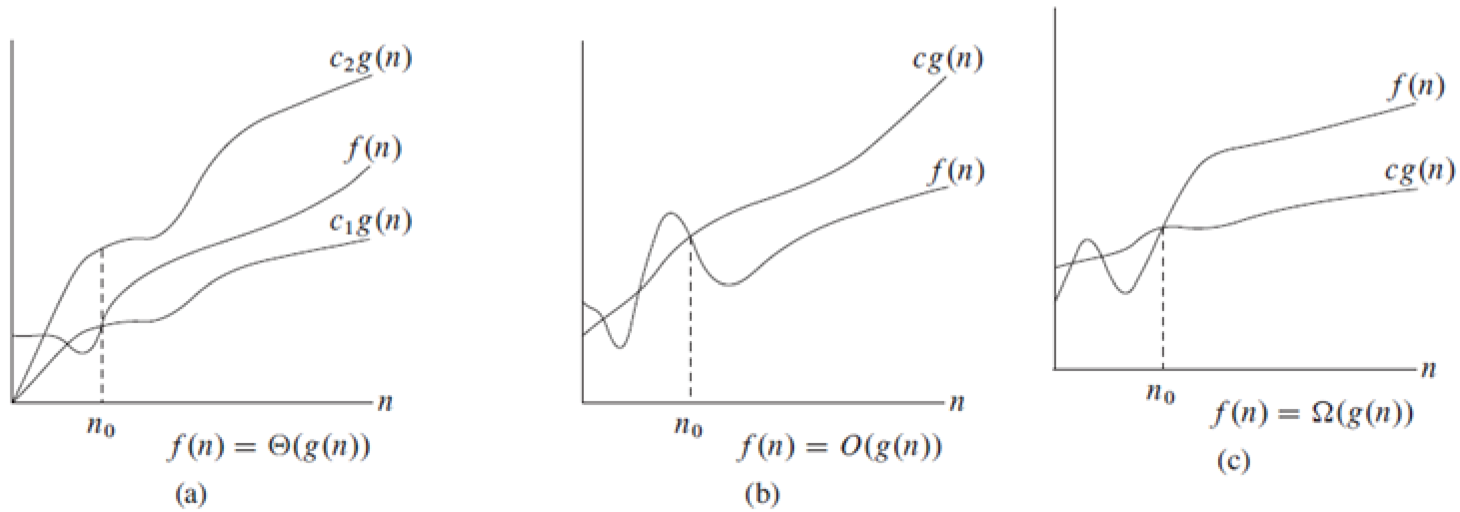



Fig: Asymptotic average bound (a), upper bound (b)and lower bound(c)
Image source:http://staff.ustc.edu.cn/~csli/graduate/algorithms/book6/chap02.htm

# Asymptotic analysis

**Examples on Big-O Notation(Upper bound/Worst case)**

1)find upper bound for f(n)=2n+2

solution:2n+2<=4n, for all n>=1

so 2n+2=O(n) with c=4 and n0 =1

2)find upper bound for f(n)=2n2+1

solution:2n2+1<=3n2, for all n>=1

so 2n2+1=O(n2) with c=3 and n0 =1

# Asymptotic analysis

**Examples on Big-O Notation(Upper bound/Worst case)**

*3)Find upper bound for f(n)=5n4+4n+1*

*Solution : 5n4+4n+1 <=7n4, for all n>=2*

*so 5n4+4n+1 =O(n4) with c=7 and n0 =2*

# Now Try to solve below Questions

I. Find upper bound for f(n)=200

II. Find upper bound for f(n)=n3+n2

III. Show that 20n3=O(n4) for appropriate c and n0.

# Asymptotic analysis

**Examples on Big- Ω Notation(Lower bound/ Best case)**

1)find lower bound for f(n)=2n+2

solution:

2n<=2n+2, for all n>=1

so 2n+2= Ω(n) with c=2 and n0 =1

2)find lower bound for f(n)=2n2+1

solution:

n2<=2n2+1, for all n>=1

so 2n2+1= Ω(n2) with c=1 and n0 =1

# Asymptotic analysis

**Examples on Big- Ω Notation(Lower bound/ Best case)**

3)Find lower bound for f(n)=5n4+4n+1

Solution :

$4n^4 <= 5n^4+4n+1$, for all $n>=1$

so $5n^4+4n+1 = \Omega(n^4)$ with $c=4$ and $n_0 =1$

# Now Try to solve below Questions

I. For $\sqrt{n}+54 = \Omega(\lg n)$. Choose $c$ and $n_0$

II. Any linear *function an* + *b* is in $\Omega(n)$. **How?**

# Asymptotic analysis

**Examples on Big- θ Notation(Tight bound/ Average case)**

1)find tight bound for f(n)=2n+2

solution:

2n<=2n+2<=4n, for all n>=1

so 2n+2= θ(n) with c1 =2, c2 =4 and n0 =1

2)find lower bound for f(n)=2n2+1

solution:

n2<=2n2+1<=3n2, for all n>=1

so 2n2+1= θ(n2) with c1 =1, c2 =3 and n0 =1

# Now Try to solve below Questions

I. Is $5n^3 \in Q(n^4)$ ??

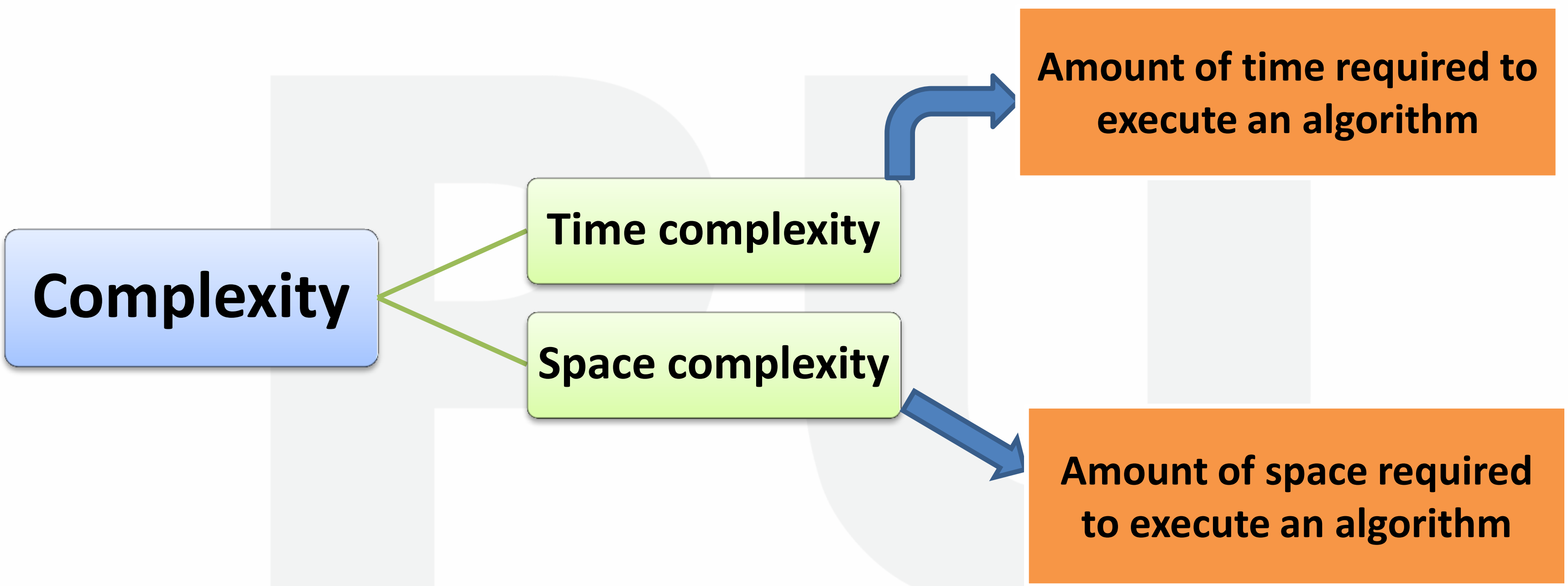
II. How about $3^{2n} \in Q(3^n)$??

# Performance measurements of Algorithm

- Measure of the amount of time and/or space required by an algorithm for an input of a given size (n).
- **What effects run time of an algorithm?**
  I. computer used, hardware platform
  II. representation of abstract data types (ADT's)
  III. efficiency of compiler
  IV. competence of implementer (programming skills)
  V. complexity of underlying algorithm
  VI. size of the input

  Out of these above (V) and (VI) are generally most significant

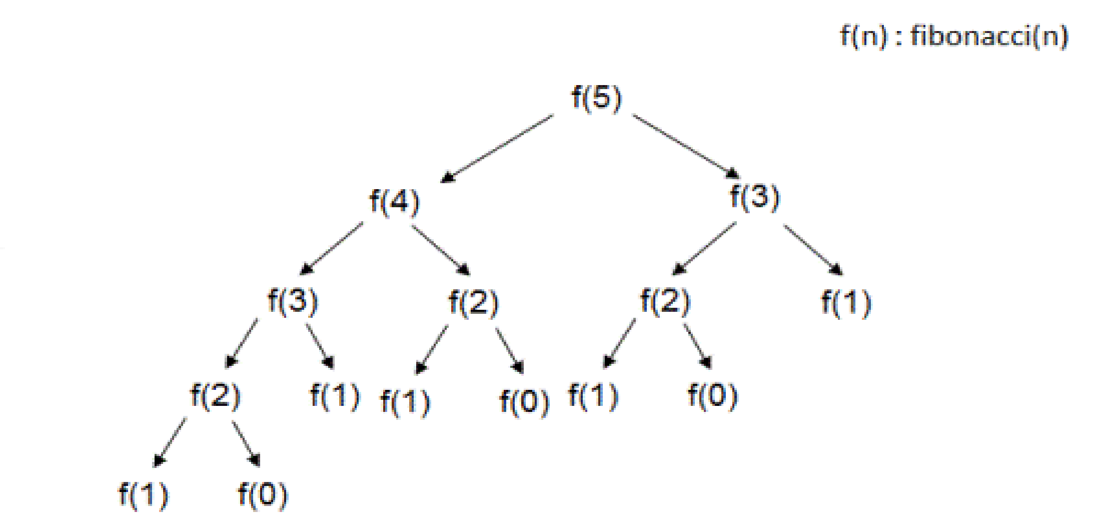
# Time complexity and Space complexity



# Analysis of recursive algorithms through recurrence relations

**Recurrence relations:**

- Refer as an equation that recursively defines a sequence where the next term is a function of the previous terms
- Can easily describe the runtime of recursive algorithms.




Recursion tree generated for computing 5th number of fibonacci sequence

# Let’s create recurrence relation

**Example:**

```
f(n)
   {
      if (n == 0)
                  return 1;
      else
             return  f(n - 1);
   }
```

# Recurrence relation

- The base case(termination condition) is reached when n == 0. The method performs one comparison. Thus, the number of operations when n == 0, T(0), is some constant c.
- When n !=0, the method calls itself,
- using ONE recursive call, with a parameter n – 1.

# Recurrence relation

Therefore the recurrence relation is:

**Recursive Case:** T(n)=T(n-1),for n>0

**Base Case:** T(n)=c , for n=0 (Here T(n) is running time and n is input size)

Recurrence relation for factorial is T(n)=T(n-1) , for n>0

T(n)=c, for n=0

# Exercise

**Find out recurrence relation for following algorithm.**

I. 
```
int fib(int n)
 {
  if (n <= 1)
    return n;
   else
     return fib(n-1) + fib(n-2);
  }
```

# Exercise

**Find out recurrence relation for following algorithm.**

ii.)
```
int  A(int n) {
        if (n == 1)
         return 2;
         else
        return  A (n / 2) + A( n / 2) + 5;
}
```

# Solving Recurrence Relation

I. Substitution Method
II. Iterative Method
III. Recurrence Tree Method
IV. Master’s Method

# Solving Recurrence Relations- Substitution Method

I. **$T(n)=T(n-1)+1$, for $n>0$**

**$T(n)=1$, for $n=0$**

Solution:

$T(n)=T(n-1)+1$ ..........................1)

find $T(n-1)$ and put value in equation 1)

$T(n-1)=T(n-2)+1$ ........................2)

$T(n-2)=T(n-3)+1$ ........................3)

put the value of equation 2) and 3) in 1)

$T(n)=T(n-3)+3$

# Solving Recurrence Relations- Substitution Method

continue for k times .now equation look like,

$T(n)=T(n-k)+k$

to reach base case ,n-k=0 so k=n

$T(n)=T(n-n)+n$

$T(n)=1+n$

**so ,T(n)=O(n)**

# Solving Recurrence Relations- Substitution Method

**II. T(n)=T(n-1)+n, for n>1**

**T(n)=1, for n=1**

Solution:

T(n)=T(n-1)+n ......................1)

T(n-1)=T(n-2)+(n-1)

T(n-2)=T(n-3)+(n-2)

Put the value in equation 1)

T(n)=T(n-2)+(n-1)+n

T(n)=T(n-3)+(n-2)+(n-1)+n ......................2)

# Solving Recurrence Relations- Substitution Method

continue for k times .now equation look like,

**$T(n)=T(n-(k+1))+(n-k)+......(n-2)+(n-1)+n$** ....................3)

To reach base case ,$n-(k+1)=1$ so $k=n-2$

Put the value in equation 2)

$T(n)=T(n-(n-2+1))+(n-(n-2))+......(n-2)+(n-1)+n$

$T(n)=T(1)+2+......(n-2)+(n-1)+n$

$T(n)=1+2+......(n-2)+(n-1)+n$

$T(n)=(n(n+1))/2$

**So $T(n)=O(n^2)$**

# Exercise

Find the complexity of the below recurrence:

$$T(n) = \begin{cases} 3T(n-1), if\ n > 0, \\ 1, \quad\quad\quad\quad otherwise \end{cases}$$

# DIGITAL LEARNING CONTENT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