MECE 5397

Project B – Diffusion Equation

Write a computer code to solve the two-dimensional diffusion equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \;=\; \frac{\partial u}{\partial t} \tag{1}$$

The domain of interest is the rectangle

$$a_x < x < b_x\,, \qquad a_y < y < b_y \tag{2}$$

and the boundary conditions

$$u(x, y = b_y) \;=\; f_b(x)\,, \qquad u(x, y = a_y) \;=\; g_b(x)\,, \tag{3}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=b_x} \;=\; 0\,, \qquad u(x = a_x, y) \;=\; g_b(a_x) + \frac{y - a_y}{b_y - a_y}[f_b(a_x) - g_b(a_x)] \tag{4}$$

$$a_x \;= a_y \;=\; -\pi\,, \qquad b_x \;=\; b_y \;=\; \pi \tag{5}$$

$$g_b(x) \;=\; (b_x - x)^2 \cos\frac{\pi x}{b_x}\,, \qquad f_b(x) \;=\; x(b_x - x)^2 \tag{6}$$

Use ghost node(s) for Neumann condition(s).
Carry out the time integration to steady state, i.e., until the result becomes independent of time.