# Homework assignment 3

September 11, 2019

*To be handed in by 23:59 on Sunday September 15th. For all your coding exercises, I recommend that you use a Jupyter notebook which you can later export as PDF.*

1. Briefly describe the difference in implementing Dirichlet and Neumann boundary conditions in finite difference and volume codes.

2. Exercise 10.2 in the book. Write a one-dimensional explicit diffusion solver for the domain $[0, 1]$ with Neumann boundary conditions at each end and $k = 1$, using a second order discretization in space and a first order explicit Euler in time.

If we begin with a Gaussian, the resulting solution is also a Gaussian, giving a solution[1]:

$$\phi(x,t) = (\phi_2 - \phi_1)\sqrt{\frac{t_0}{t+t_0}}e^{-\frac{1}{4}(x-x_c)^2/k(t+t_0)} + \phi_1 \tag{1}$$

Initialize our problem with $t = 0$, and take $t_0 = 0.001$, $\phi_1 = 1$, and $\phi_2 = 2$, and $x_c$ is the coordinate of the center of the domain. Run until $t = 0.01$ and compare to the analytic solution above.

3. Exercise 10.3 in the book. Write a one-dimensional implicit diffusion solver for the domain $[0, 1]$ with Neumann boundary conditions at each end and $k = 1$. Your solver should use a tridiagonal solver, which can be either implicit Euler or Crank-Nicholson and initialize a matrix like that above. Use a timestep close to the explicit step, a grid with N = 128 zones. Use a Gaussian for your initial conditions (as you did for the explicit problem).

4. Describe in what circumstances are finite volume and finite difference discretizations of the diffusion equation are equal (with, or without a constant diffusivity). Use Figure 3.1 from the book in your answer. Why is it desirable for them to be equal?

[1] Note: the 2- and 3-d solutions are slightly different than this 1-d solution