Homework assignment 6

October 7, 2019

To be handed in by 23:59 on Sunday October 13th. For all your coding exercises, I recommend that you use a Jupyter notebook which you can later export as PDF.

1. Explain the connection between characteristics and hyperbolic PDEs.

2. Explain why the Burgers equation can be seen as a hyperbolic equation. What are the characteristics, and how do they map on to fluid particles?

3. *(Exercise 4.3 in book)* Write a code to solve the 1-d linear advection equation using the upwind discretization (Eq. 4.2) on the domain $[0, 1]$ with $u = 1$ and periodic boundary conditions. For initial conditions, try both a Gaussian profile and a top-hat:

$$a = \begin{cases} 0 & \textit{if} \quad & x & < 1/3 \\ 1 & \textit{if} \quad 1/3 \leq & x & < 2/3 \\ 0 & \textit{if} \quad 2/3 \leq & x & \end{cases} \tag{1}$$

4. Repeat exercise 3, but using an implicit upwind scheme. Choose different values for $\mathcal{C}$ condition, and describe the difference between *stability* and *accuracy*.

5. *(Exercise 5.1 in the book)* Write a second-order solver for the linear advection equation. To mimic a real hydrodynamics code, your code should have routines for initializing the state, filling boundary conditions, computing the timestep, constructing the interface states, solving the Riemann problem, and doing the update. The problem flow should look like:

- set initial conditions
- main evolution loop—loop until final time reached
  - fill boundary conditions
  - get timestep (Eq. **??**)
  - compute interface states (Eqs. **??** and **??**)
  - solve Riemann problem at all interfaces (Eq. **??**)
  - do conservative update (Eqs. **??** and **??**)

Use both the top-hat and Gaussian initial conditions and periodic boundary conditions and compare to the first-order method.