Homework assignment 7

October 14, 2019

To be handed in by 23:59 on Sunday October 20th. For all your coding exercises, I recommend that you use a Jupyter notebook which you can later export as PDF.

1. Explain what are the problems with high-order methods and hyperbolic PDEs. Refer to shock waves and other discontinuities.

2. What is flux limiting? What are the advantages and drawbacks?

3. Extend your one-dimensional solver to two dimensions. Use *Strang* splitting. Take both the top-hat and Gaussian initial conditions and periodic boundary conditions. Use both equal $u = v = 1$ and unequal velocities $u = \frac{1}{2}v = 1$.

4. (*Exercise 6.2 from book*) Extend your 1-d finite-volume solver for advection to solve Burgers' equation. You will need to change the Riemann solver and use the local velocity in the construction of the interface states. Run the examples shown in Figures 6.4 and 6.5.

5. (*Exercise 6.3 from book*) Using a simple first-order finite-difference method like we described for linear advection, contrast the conservative and non-conservation formulations of Burgers' equation as:

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -\frac{1}{2}\frac{(u_i^n)^2 - (u_{i-1}^n)^2}{\Delta x} \tag{1}$$

and

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -\frac{u_i^n(u_i^n - u_{i-1}^n)}{\Delta x} \tag{2}$$

(Note: these discretizations are upwind so long as $u > 0$).

Run these with the shock initial Riemann conditions:

$$u(x, t = 0) = \begin{cases} 2 & x < 1/2 \\ 1 & x > 1/2 \end{cases} \tag{3}$$

and measure the shock speed from your solution by simply differencing the location of the discontinuity at two different times. Compare to the analytic solution for a shock for the Riemann problem.