Expropriation: A Mechanism Design Approach

Nicolás Figueroa
PUC Chile, ISCI

Vicente Jiménez
PUC Chile, ISCI

Juan-Pablo Montero
PUC Chile, ISCI

PUC Economic Theory Seminar
October 2020

Introduction

- A buyer must purchase a fixed amount of units of a good
- Sellers have market power and convex costs
  - Energy Markets
  - Pollution Permits (e.g. carbon)
  - Conservation Auctions
- What's the optimal (cost-minimizing) way of buying? Is it implementable?
- How do alternative (simpler) mechanism perform?

The Model

- A buyer must purchase $\overline{Q}$ units of a good at the lowest possible cost

- $k > 1$ sellers with cost $C_i(q_i) = \theta_i \dfrac{q_i^2}{2}$

- $\theta_i \sim F_i[a_i, b_i]$ is private information

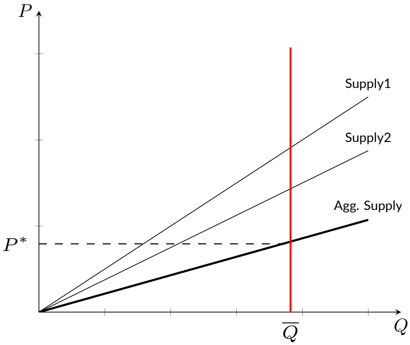
- Sellers have quasilinear utility: $u_i(q_i, t_i) = t_i - \theta_i \frac{q_i^2}{2}$

- Distributions F_i are regular, that is

$$J_i(\theta_i) = \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$$

  is increasing

Uniform Prices: Simple and effective?



- Firms submit supply functions & price is chosen to clear the market
- Widely used
- Not truth-telling and not necessarily optimal
  - Linear costs + capacity constraints $\rightarrow$ optimal
  - No market power $\rightarrow$ optimal

Roadmap

- Optimal Mechanism
- Two Simple(r) Sequential Mechanisms
- Comparing Mechanisms: Numerical Analysis
- A Decomposition Result

Full Information Benchmark

- Optimal buying rule:

$$q_i(\boldsymbol{\theta}) = \left(\frac{\theta_i^{-1}}{\sum_j \theta_j^{-1}} \right) \overline{Q}$$

- All firms sell a positive amount

- More efficient firms sell more

- The exact amount sold by each firm depends on the efficiency of **all** firms

Incomplete Information: Optimal Mechanism

Proposition

Let $J_i(\theta_i) \equiv \theta_i + \dfrac{F_i(\theta)}{f_i(\theta_i)}$ *be the virtual type of firm* i*. The optimal allocation rule is given by*

$$q_i(\boldsymbol{\theta}) = \left(\frac{J_i^{-1}(\theta_i)}{\sum_j J_j^{-1}(\theta_j)} \right) \overline{Q},$$

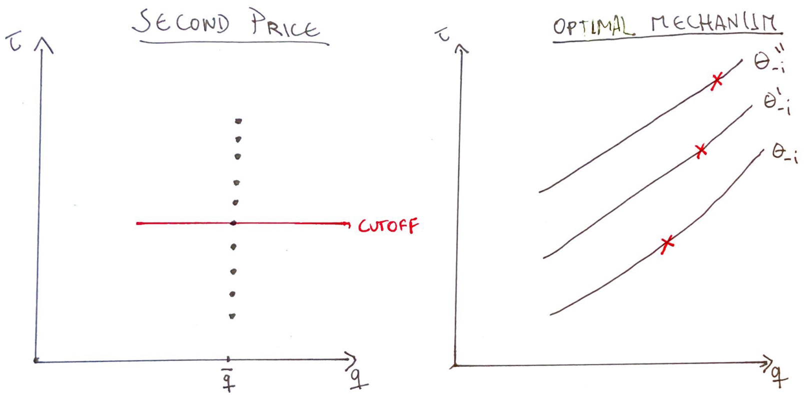
with associated transfers

$$t_i(\boldsymbol{\theta}) = \frac{\theta_i}{2} q_i^2(\boldsymbol{\theta}) + \frac{1}{2} \int_{\theta_i}^{b_i} q_i^2(s_i, \theta_{-i}) ds_i.$$

Implementation of the Optimal Mechanism

- Doesn't correspond to any standard mechanism
- Agents are not offered a (q,t) menu, but a family $\{(q(\theta_{-i}), t(\theta_{-i}))\}_{\theta_{-i}}$ of them
- Moreover, this menus are complex

The Complexity of the Optimal Mechanism



Sequential Mechanisms

- A simpler alternative is offering (q, t) menus sequentially
- This is clearly sub-optimal (less flexibility)
- But how bad is it?

The Optimal Sequential Mechanism

Proposition

The optimal sequential allocation rule is given by

$$q_i(\theta_{\leq i}) = \left(\frac{J_i^{-1}(\theta_i)}{J_i^{-1}(\theta_i) + (A_{i+1})^{-1}} \right) \overline{Q}_i(\theta_{<i}),$$

where the sequence $\{A_j\}_{j=1}^{k}$ *is defined recursively as*

$$\begin{aligned} A_k &= b_k, \\ A_j &= \mathbb{E}_{\theta_j} \left(\frac{1}{J_j^{-1}(\theta_j) + (A_{j+1})^{-1}} \right) \quad \textit{for all } j < k. \end{aligned}$$

The Optimal Sequential Mechanism

- Agent i is offered a menu which would be optimal if there was another seller of “artificial type” A_{i+1}
- A_i can be computed ex-ante
- Simple (q, t) menu that depends only on the remaining quantity to be bought
- We couldn’t stablish the optimal order

Going even simpler: Linear prices

- Simple menus *par excellence*: posted prices
- Sellers are faced sequentially and offered a price-per-unit
- This is equivalent to constrain (q, t) so that $t = t(q)$ is linear

Sequential Posted Prices

Proposition

Define $\mu_{i,1} \equiv \mathbb{E}_{\theta_i}(1/\theta_i)$ *and* $\mu_{i,2} \equiv \mathbb{E}_{\theta_i}(1/\theta_i^2)$. *Also, define recursively*

$$\begin{aligned} B_k &= b_k \\ B_i &= B_{i+1} - \frac{(B_{i+1})^2 \mu_{i,1}^2}{2\mu_{i,1} + B_{i+1}\mu_{i,2}} \end{aligned}$$

then the price offered to seller i *and its corresponding allocation rule are given by*

$$P_i(\theta_{<i}) = \frac{B_{i+1}\mu_{i,1}}{2\mu_{i,1} + B_{i+1}\mu_{i,2}} \bar{Q}_i(\theta_{<i}) \qquad q_i(\theta_{\leq i}) = \frac{P_i}{\theta_i}$$

Moreover, the expected cost of the mechanism is

$$C(\overline{Q}) = B_1 \frac{\overline{Q}^2}{2}.$$

Sequential Posted Prices: A parametric condition

- The above proposition requires that for every $i < k$,

$$\frac{B_{i+1}}{2 + B_{i+1}\dfrac{\mu_{i,2}}{\mu_{i,1}}} \leq a_i.$$

- $\frac{\mu_{i,2}}{\mu_{i,1}}$ must be big $\rightarrow$ high heterogeneity of types

- Also, B_{i+1} must be small $\rightarrow$ low costs ahead

- High prices are ineffective, since there is a high likelihood of inefficient types that would sell too much

- Optimal mechanism never induces foreclosure of future sellers

Numerical Analysis: Comparing mechanisms

Uniform Distribution

	Optimal Sequential (%)	Sequential Posted Prices (%)
$[1, 2]$	2.49	44.37
$[1, 4]$	7.08	37.59
$[50, 51]$	0.00	65.59
$[90, 100]$	0.09	61.37

Numerical Analysis: Comparing mechanisms

Parabolic Distribution

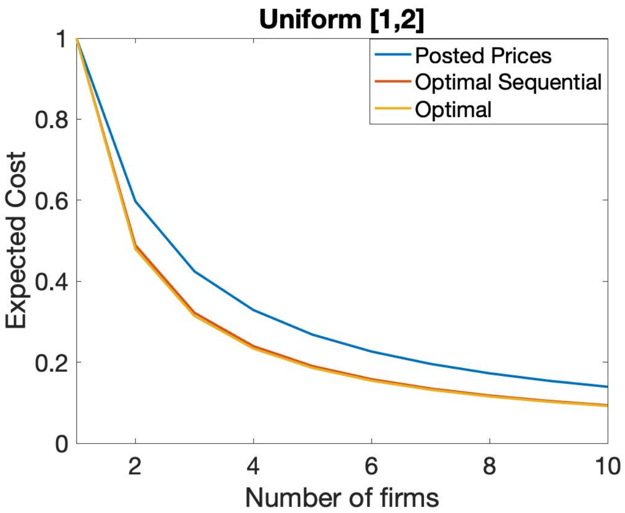
	Optimal Sequential (%)	Sequential Posted Prices (%)
$[1,2]$	2.82	51.74
$[1,4]$	10.79	49.36
$[50,51]$	0.00	64.10
$[90,100]$	0.32	61.94

Numerical Analysis: Comparing mechanisms

Inverse Parabolic Distribution

	Optimal Sequential ($\%$)	Sequential Posted Prices ($\%$)
$[1, 2]$	1.93	46.27
$[1, 4]$	6.11	40.13
$[50, 51]$	0.00	66.13
$[90, 100]$	0.70	61.57

Numerical Analysis: Comparing mechanisms



Numerical Analysis: Takeaways

- Optimal sequential mechanism performs well, linear pricing doesn't
  - Simplicity can be bought at a small cost
  - Linearity is expensive
- Little sellers heteoregenity $\rightarrow$ Optimal sequential mechanism $\sim$ Optimal mechanism
- The difference in performance remains significant as the number of firms grows

Allocations v. Rents: A Decomposition Result

- What explains the difference in performance?
- From mechanism design
  - Losses due to misallocation
  - Losses due to rents to the worst type (zero in the optimal mechanism)
- A simple expression to compute losses due to misallocation
- An application to our mechanisms: source of bad performance of posted prices

A Decomposition Result

Proposition

Let (q^*, t^*) denote the optimal mechanism and (q^0, t^0) any direct mechanism. The difference in expected costs between these mechanisms can be written as

$$
\begin{aligned}
D_0 = \frac{1}{2}\mathbb{E}_\theta \Bigg\{ & \sum_{i=1}^{k-1} (J_i(\theta_i) + J_k(\theta_k))(q_i^0(\theta) - q_i^*(\theta))^2 \\
& + J_k(\theta_k) \sum_{j \neq i}^{k-1} \sum_{i=1}^{k-1} (q_i^0(\theta) - q_i^*(\theta))(q_j^0(\theta) - q_j^*(\theta)) \Bigg\} \\
& + \sum_{i=1}^{k} U_i(b_i; (q^0, t^0)) - \underbrace{\sum_{i=1}^{k} U_i(b_i; (q^*, t^*))}_{=0},
\end{aligned}
$$

where

$$
U_i(b_i; (q^0, t^0)) \equiv \mathbb{E}_{\theta_{-i}}\{u_i(b_i, \theta_{-i}; (q^0, t^0))\}.
$$

Numerical Analysis: Comparing Mechanisms

- A simple way of improving linear pricing: entrance fee
- Does it solve the problem?

Uniform Distribution

	Optimal Sequential ($\%$)	Sequential Posted Prices ($\%$)	SPP + Fee ($\%$)
$[1, 2]$	2.49	44.37	6.68
$[1, 4]$	7.08	37.59	10.22
$[50, 51]$	0.00	65.59	10.76
$[90, 100]$	0.09	61.37	9.44

Numerical Analysis: Comparing Mechanisms

Parabolic Distribution

	Optimal Sequential (%)	Sequential Posted Prices (%)	SPP + Fee (%)
$[1, 2]$	2.82	51.74	14.03
$[1, 4]$	10.79	49.36	25.25
$[50, 51]$	0.00	64.10	10.79
$[90, 100]$	0.32	61.94	10.16

Numerical Analysis: Comparing Mechanisms

Inverse Parabolic Distribution

	Optimal Sequential ($\%$)	Sequential Posted Prices ($\%$)	SPP + Fee ($\%$)
$[1, 2]$	1.93	46.27	8.63
$[1, 4]$	6.11	40.13	10.84
$[50, 51]$	0.00	66.13	10.88
$[90, 100]$	0.70	61.57	9.79

Decomposition: Takeaways

- The bulk of the bad performance of posted prices lies in the rents left to the least efficient firms
- Good news: fees are an easy fix
- Nevertheless, misallocation also plays a non-negligible role
- This doesn't have a fix, as it is inherent to the mechanism

Back to Uniform Prices

- This mechanism is optimal if linear costs or no market power
- We'll see that in our setting this mechanism is far away from the optimal
- For analytical tractability, we consider a simpler but similar setting: one strategic player (dominant firm) and a competitive fringe

Uniform Prices: Performance

Uniform Distribution, $\theta_F = b$

$[\mathbf{a}, \mathbf{b}]$	**Uniform Prices ($\%$)**	**UP + Rebate ($\%$)**
$[1, 2]$	160.34	92.16
$[1, 4]$	161.66	138.23
$[50, 51]$	166.23	99.57
$[90, 100]$	173.90	167.23

- Uniform prices perform very badly, even when charging an entrance fee to the large firm
- Why? Misallocation + Fringe Rents (“unextractable”)

Conclusions

- Three buying mechanisms
  - **Optimal**: Best we can do, but complex and difficult to implement
  - **Optimal Sequential**: Simple (q,t) menus and a pretty good approximation of the optimal cost-wise
  - **Sequential Posted Prices**: Extremely simple but very expensive. Can be considerably improved adding entrance fees
- Prices are quite expensive when marginal costs are increasing
- If feasible, entrance fees can help a lot
- Mechanism design can make a sizable difference without giving up simplicity