Complex Systems Boot Camp

ISC6930 | SECTION#3 | DATE: ....

NAME: .... | SS#: .... | POINTS: ......

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1) Let $\vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{y} = \begin{pmatrix} 0.5 \\ 2 \end{pmatrix}$

a) Draw the two vectors and evaluate their sum $\vec{x} + \vec{y}$ and difference $\vec{x} + \vec{y}$ graphically.

b) Calculate the magnitude and direction of the vector x.

2) What is the sum of the vectors in the following diagram.

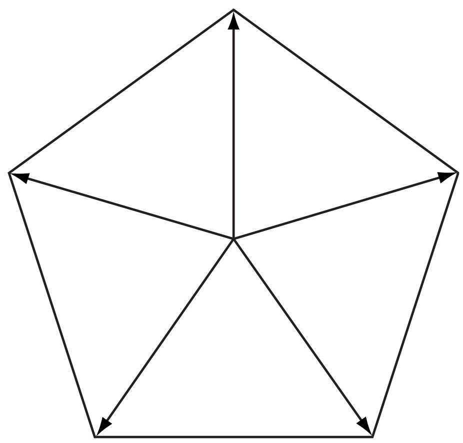

Figure 1: Vector sum

The vectors are of equal length originating at the center of a regular pentagon and ending at the vertices.

3) Let $\vec{a} = 2\,\vec{e_1} + 3\,\vec{e_2}$ and $\vec{b} = \vec{e_1} + 4\,\vec{e_2}$, $\vec{e_1}$ and $\vec{e_2}$ being unit vectors along the x and y-axis, respectively.

a) Find the magnitude of $\vec{b}$ and its angle with the y-axis.

b) Find the vector perpendicular to $\vec{a}$.

c) Evaluate the projection of the vector $\vec{a}$ onto the vector $\vec{b}$.

4) Let $\vec{x} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\vec{y} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $\vec{z} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$. Evaluate $\vec{x} \cdot (\vec{y} + \vec{z})$ and $\vec{x} \times (\vec{y} + \vec{z})$.

5) Use the cross product to determine the area of the triangle with the vertices $\vec{a} = (1, 0, 0)$, $\vec{b} = (0, 1, 0)$; and $c = (0, 0, 0)$.

6) Let matrix $\boldsymbol{A} = \begin{pmatrix} 9 & -5 & 0 \\ 0 & 6 & 0 \\ 4 & 7 & 2 \end{pmatrix}$ and matrix $\boldsymbol{B} = \begin{pmatrix} 2 & 0 & -6 \\ 8 & 0 & 3 \\ 1 & -3 & 0 \end{pmatrix}$.

Compute the following quantities:

a) $\boldsymbol{A} + \boldsymbol{B}$ b) $\boldsymbol{A} - \boldsymbol{B}$ c) $\boldsymbol{A}\,\boldsymbol{B}$ e) $\boldsymbol{B}\,\boldsymbol{A}$

7) Let matrix $\boldsymbol{A} = \begin{pmatrix} 1 & 4 & 5 \\ 0 & 2 & -6 \end{pmatrix}$, and matrix $\boldsymbol{B} = \begin{pmatrix} -2 & 0 & 2 \\ 9 & -5 & 12 \end{pmatrix}$.

Compute the following quantities:

a) $\boldsymbol{A} + \boldsymbol{B}$ b) $\boldsymbol{A} - \boldsymbol{B}$ c) $\boldsymbol{A}\,\boldsymbol{A}^{\boldsymbol{T}}$ d) $\boldsymbol{A}^{\boldsymbol{T}}\,\boldsymbol{B}$ e) $\boldsymbol{A}\,\boldsymbol{B}^{\boldsymbol{T}}$

8) Express the following system of equations as a vector equation with matrix $\boldsymbol{A}$ such that

$$\boldsymbol{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad \text{where} \qquad \begin{aligned} a &= -4\,x + c \\ b &= 17\,y - 2\,z + 3\,x \\ c &= -2\,y - x \end{aligned}$$

Express the following system of equations as a vector equation with matrix $\boldsymbol{B}$ such that

$$\boldsymbol{B} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} \qquad \text{where} \qquad \begin{aligned} p &= 2c + 11a - 4b \\ q &= -a - b + c \end{aligned}$$

Specify the matrix $\boldsymbol{C}$ which combines the operation of $\boldsymbol{A}$ and $\boldsymbol{B}$ such that $\boldsymbol{C} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$

9) For of the following operations, specify the 2×2 matrix that takes a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ and

a) Stretches x by a factor of 3 and shrinks y by a factor of 2.

b) Mirrors x around the y-axis,while leaving y unchanged.

c) Replaces y by the sum of x and y.

d) Combines the operations (a)-(c), as performed one after another.

10) Figure 2 below shows 4 vectors, each plotted as filled circles, in a Cartesian coordinate space.

The four vectors are: $\overrightarrow{v_1} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$, $\overrightarrow{v_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\overrightarrow{v_3} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$.

The dashed line is the x-axis, and the vertical dark line is the y-axis.

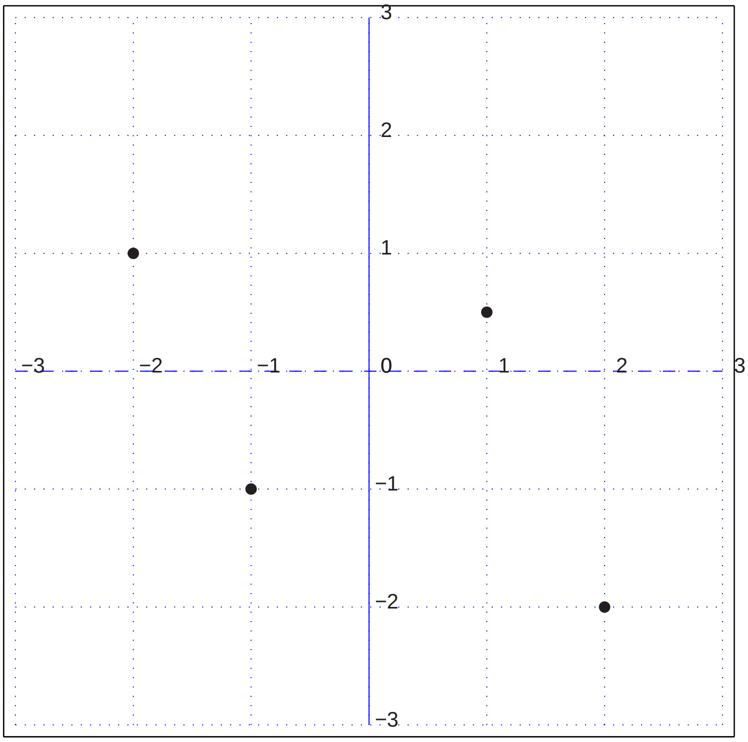

Figure 2: Cartesian coordinates

Figure 3 shows the coordinate space as a result of the transformation $\boldsymbol{M} = \begin{pmatrix} 0.7 & -0.7 \\ 0.7 & 0.7 \end{pmatrix}$. Note that the dashed line is the "x" coordinate (i.e. first component of the vector) in the new coordinate space.

a) Using the grid lines in the figure, estimate the new coordinates of $\overrightarrow{v_2}$ and $\overrightarrow{v_3}$.

b) Using the transformation $\overrightarrow{v_{new}} = \boldsymbol{M}\,\overrightarrow{v_{old}}$, calculate the new coordinates of $\overrightarrow{v_2}$ and $\overrightarrow{v_3}$.

Figure 4 shows the result of transforming the Cartesian coordinate space of Figure 2 into polar coordinates.

c) Using the grid lines in the figure, estimate the new coordinates of each of the four vectors. To convert the angle from degrees to radians, multiply the angle in degrees by $\pi/180^0$

d) Calculate the new coordinates of $\overrightarrow{v_2}$ and $\overrightarrow{v_3}$.

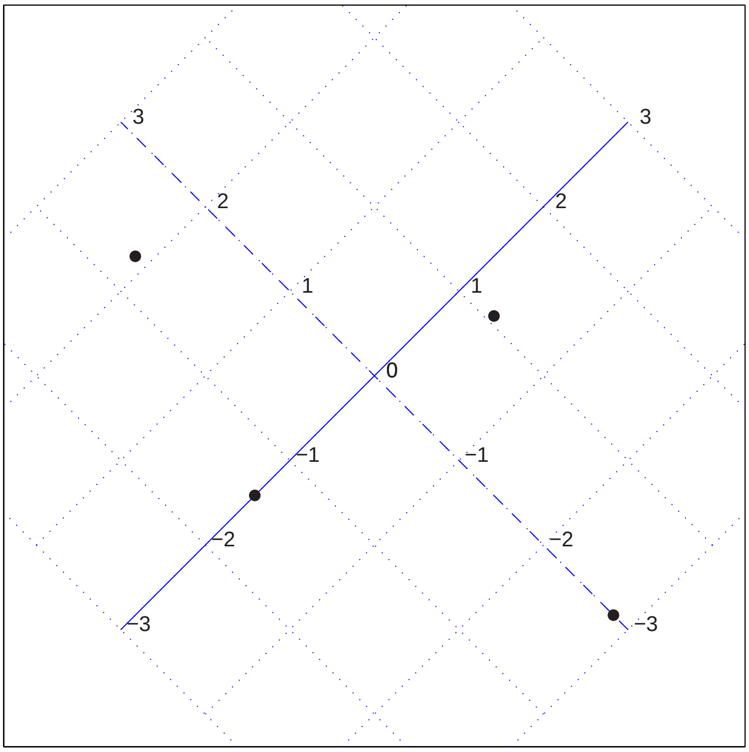

Figure 3: Rotated cartesian coordinates

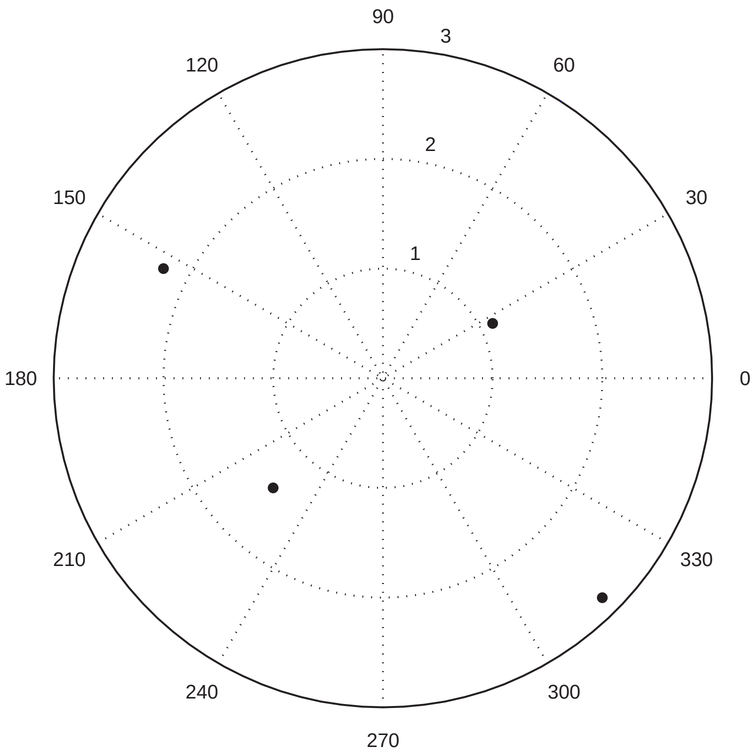

Figure 4: Polar coordinates

13) Mark the following points in the polar coordinate system below and convert them to cartesian coordinates.

a) $r = 2$ $\theta = \pi$ b) $r = -3$ $\theta = 3\pi$

c) $r = 1$ $\theta = -\pi/2$ d) $r = 1/2$ $\theta = \pi/4$

e) $r = 0$ $\theta = \pi/6$

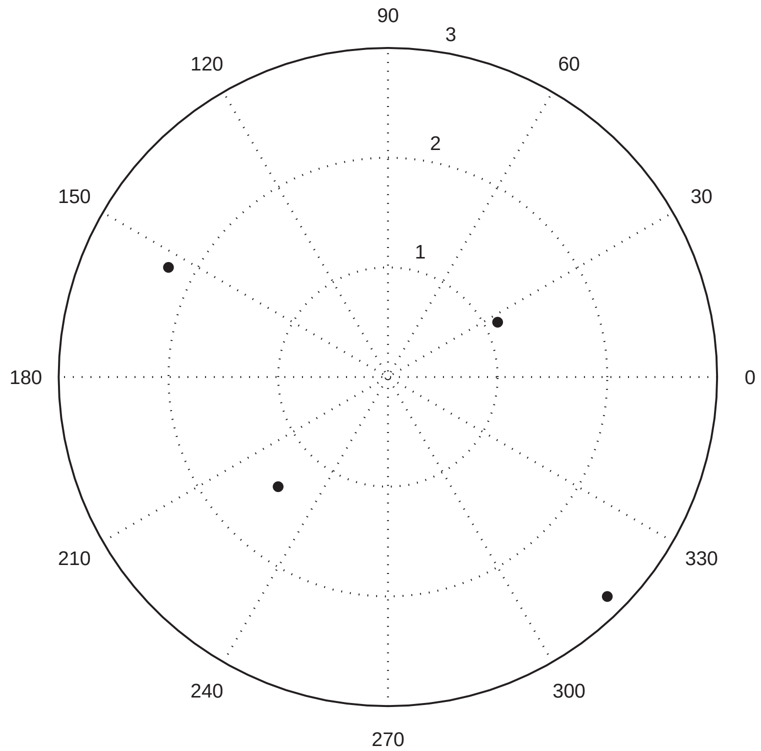

Figure 5: Polar coordinates