Complex Systems Boot Camp

ISC6930 | SECTION#4 | DATE: .....

NAME: ..... | SS#: ..... | POINTS: .....

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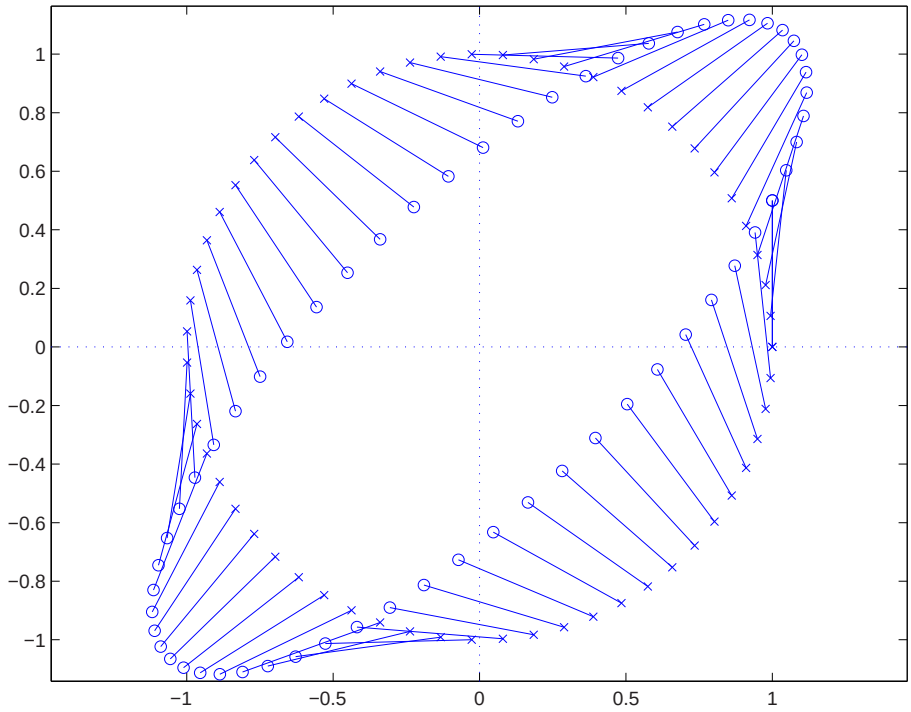
1) A matrix operates upon a vector and produces another vector. The figure below shows the operation of the matrix $\boldsymbol{M} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. Each cross (x) is an input vector to the matrix, and the circle (o) is the corresponding output vector.



a) On the figure, sketch the approximate directions of the eigenvectors of the matrix.
Hint: Imagine a line from the origin to each input vector, and see if the output vector lies on that line.

b) Estimate the eigenvalues corresponding to the approximate eigenvectors.
Hint: Once you determine the direction of an eigenvector, the eigenvalue is simply the ratio of the length of the output vector (i.e. its distance from the origin) to the length of the input vector.

c) Calculate the eigenvalues and eigenvectors of the matrix $\boldsymbol{M}$. Show your work.

d) On the figure, sketch the directions of the eigenvectors you found in Question c), and mark them with an "e" (for "exact").

2) Find the determinants of the following matrices. Show your work.

a) $\begin{pmatrix} 1 & -2 \\ 3 & 7 \end{pmatrix}$ b) $\begin{pmatrix} 2 & -3 \\ -4 & 5 \end{pmatrix}$ c) $\begin{pmatrix} 7 & 2 & 0 \\ 0 & 1 & 6 \\ 11 & 3 & 2 \end{pmatrix}$

3) Convert the following numbers to the exponential form $r\,e^{i\theta}$:

a) 3 b) $4\,i$ c) $3+4\,i$ d) $3-4\,i$

Convert the following numbers to the Cartesian form a+ib:

e) $7\,e^2$ f) e^i g) $e^{i\,\pi}$ h) $e^{-i\,(\pi/2)}$ i) $2\,e^{i\,(\pi/3)}$

4) Calculate the result of the following additions and subtractions. You can express the results in either Cartesian or exponential form. Hint: To add or subtract numbers in exponential form, first convert them to Cartesian notation.

a) $10\,i \quad + \quad 14\,i$ *b)* $6 - i\,\pi \quad + \quad 17$

c) $3-4\,i \quad + \quad -3+4\,i$ *d)* $2\,e^{i\,(\pi/3)} \quad + \quad 3\,e^{i\pi}$

5) Calculate the result of the following operations. You can express the result in either artesian or exponential form. Hint: Some of these operations are much easier if you first convert the numbers to exponential form.

a) $3\,i \quad * \quad -4\,i$ *b)* $5\,i \quad * \quad 12$

c) $5-3\,i \quad * \quad -6\,i$ *d)* $6+2\,i \quad * \quad 7-4\,i$

e) $3-4\,i \quad * \; 3-4\,i$ *f)* $3+4\,i \quad * \quad 3-4\,i$

g) $3+4\,i \quad / \quad 3-4\,i$

5) Compute the complex conjugates of the following numbers. You can express the result in either Cartesian or exponential form.

a) 14 b) $7\,e^{i\,(\pi/4)}$ c) $7+10\,i$ d) $12\,i$ e) $\frac{2}{3}\,e^{i\,(\pi/2)}$