# Package 'POSetR'

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**Description** Provides a set of basic tools for generating, analyzing, summarizing and visualizing finite partially ordered sets. In particular, it implements flexible and very efficient algorithms for the extraction of linear extensions and for the computation of mutual ranking probabilities and other user-defined functionals, over them. The package is meant as a computationally efficient ``engine", for the implementation of data analysis procedures, on systems of multidimensional ordinal indicators and partially ordered data, in the spirit of Fattore, M. (2016) ``Partially ordered sets and the measurement of multidimensional ordinal deprivation", Social Indicators Research <DOI:10.1007/s11205-015-1059-6>, and Fattore M. and Arcagni, A. (2018) ``A reduced posetic approach to the measurement of multidimensional ordinal deprivation", Social Indicators Research <DOI:10.1007/s11205-016-1501-4>.

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POSetR-package

Partially Ordered Sets in R

## **Description**

Provides a set of basic tools for generating, analyzing, summarizing and visualizing finite partially ordered sets. In particular, it implements flexible and very efficient algorithms for the extraction of linear extensions and for the computation of mutual ranking probabilities and other user-defined functionals, over them. The package is meant as a computationally efficient "engine", for the implementation of data analysis procedures, on systems of multidimensional ordinal indicators and partially ordered data, in the spirit of Fattore, M. (2016) "Partially ordered sets and the measurement of multidimensional ordinal deprivation", Social Indicators Research <DOI:10.1007/s11205-015-1059-6>, and Fattore M. and Arcagni, A. (2018) "A reduced posetic approach to the measurement of multidimensional ordinal deprivation", Social Indicators Research <DOI:10.1007/s11205-016-1501-4>.

#### Author(s)

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antiChain 3

antiChain

Generate an anti-chain from a vector of elements' labels

## **Description**

The function produces an antichain from the vector of elements' labels elements. If elements is a numeric vector, it is first coerced to a character vector.

An antichain is a poset with no comparabilities, therefore the order of the labels in elements does not matter.

#### Usage

```
antiChain(elements)
```

## Arguments

elements

a vector of characters with elements' labels.

#### Value

an environment of class poset.

#### See Also

function poset for more details and to create a generic poset and function chain to create a complete order.

## **Examples**

```
antiChain(LETTERS[1:5])
```

chain

Generate a complete order from a vector of elements' labels

# Description

The function produces a complete order from the vector of elements' labels elements. If elements is a numeric vector, it is first coerced to a character vector.

A complete order is a poset where all of the elements are comparable; the order of the labels in elements defines their position in the chain, from bottom to top.

```
chain(elements)
```

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## **Arguments**

elements

a vector of characters listing the elements' labels.

#### Value

an environment of class poset.

## See Also

function poset for more details and to create a generic poset and function antiChain to create an anti-chain.

# Examples

```
chain(LETTERS[1:5])
```

coverMatrix

Poset cover matrix

## Description

The function returns the cover matrix for objects of class poset.

## Usage

```
coverMatrix(x)
```

# Arguments

Χ

an object of class poset.

## Value

a logical square matrix.

```
dom <- matrix(c(
    "a", "b",
    "c", "b",
    "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
coverMatrix(p)</pre>
```

incidenceMatrix 5

incidenceMatrix

Poset incidence matrix

#### **Description**

The function returns the incidence matrix for objects of class poset.

## Usage

```
incidenceMatrix(x)
```

# Arguments

Х

an object of class poset.

#### Value

a logical square matrix.

## **Examples**

```
dom <- matrix(c(
    "a", "b",
    "c", "b",
    "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
incidenceMatrix(p)</pre>
```

intersection

Intersection of two posets

# Description

Given two posets X and Y on the same set, intersection returns the poset Z defined by a < b in Z if and only if a < b in X and a < b in Y.

# Usage

```
intersection(x, y)
x %it% y
```

# Arguments

```
x an S4 object of class Rcpp_POSet, see poset for details.
y an S4 object of class Rcpp_POSet, see poset for details.
```

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#### Value

an S4 object of class Rcpp\_POSet, see poset for details

#### References

Davey BA, Priestley HA (2002). Introduction to lattices and order. Cambridge university press.

#### See Also

poset

#### **Examples**

```
dom <- matrix(c(
   "a", "b",
   "c", "b",
   "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
plot(p)
dom <- matrix(c(
   "a", "b",
   "c", "b",
   "d", "b"
), ncol = 2, byrow = TRUE)
q <- poset(x = dom)
plot(q)
plot(p %it% q)</pre>
```

LEapply

Applies scalar functions over the set of linear extensions of a poset and returns the corresponding average values

## **Description**

"LEapply" is composed of three main elements: (i) the linear extensions generator, (ii) the application of the argument functions to the linear extensions and (iii) the computation of the averages of the results, for each function separately; see Fattore M (2016). "Partially ordered sets and the measurement of multidimensional ordinal deprivation." *Social Indicators Research*, **128**(2), 835–858...

```
LEapply(x, ...)
## S3 method for class 'poset'
LEapply(
   x,
```

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```
FUN = "MutualRankingProbability",
  generator = c("AllLE", "BubleyDyer"),
 bubleydyer.precision = 10,
 bubleydyer.nit = NULL,
 bubleydyer.progressBar = TRUE,
 degrees = NULL
)
```

#### **Arguments**

an environment of class poset, see poset for details. Х

optional arguments to FUN.

FUN the function, or a list of functions, to be applied to each linear extension: see

'Details'.

a string specifying the method used to generate the linear extensions. The default generator

value is "AllLE". See section 'Details' below.

bubleydyer.precision

considered only if "BubleyDyer" generator is selected. It corresponds to the number of digit precision of the frequencies in the sampling distributions of

linear extensions.

bubleydyer.nit considered only if "BubleyDyer" generator is selected. Number of iterations in

the Bubley-Dyer algorithm, if NULL (default) it is set as indicated in Bubley and Dyer (1999) depending on the value of bubleydyer.precision and the

number of elements of the poset.

bubleydyer.progressBar

logical that indicates whether to show a text progress bar or not

degrees

to generate the lexicographic linear extensions of a product order, the poset x describes the dominance (e.g. relative importance) between ordinal variables and degrees is a numerical vector specifying the number of degrees of each

variable, represented by in the poset.

#### **Details**

Argument FUN must be either a function or a list of functions, each one depending on a vector of characters representing the names of the elements of the poset.

If degrees is not NULL but a numerical vector as long as the number of elements in the poset, the poset elements are considered as ordinal variables. Therefore degrees represents their number of degrees that are represented as integer numbers starting from 0. In this case, LEapply generates the lexicographical linear extensions of the product order of the ordeinal variables. Its elements are called profiles and they are obtained by the combination of the degrees of variables separated by a dash. For details about lexicographical linear extensions and profiles see Fattore M, Arcagni A (2018). "A reduced posetic approach to the measurement of multidimensional ordinal deprivation." Social Indicators Research, 136(3), 1053–1070...

Some functions are already implemented in the C++ library and they can be called by their names. Currently, such functions are "MutualRankingProbability", "Separation", and "AverageHeight". 8 LEapply

Each function in FUN must return a numerical or logical matrix. Each function can depend on additional arguments that can be passed through . . .; such additional arguments must be the same for all the functions in the list.

Argument generator specifies the linear extension generation algorithm. The available generators are "AllLE", that produces all of the linear extensions of the input poset, and "BubleyDyer", which samples uniformly from the set of linear extensions, through an MCMC algorithm (Bubley and Dyer 1999).

#### Value

The average values of the argument functions FUN over the set of linear extensions (or lexicographic ones if degrees argument is not NULL).

#### References

Bubley R, Dyer M (1999). "Faster random generation of linear extensions." *Discrete mathematics*, **201**(1-3), 81–88.

Fattore M (2016). "Partially ordered sets and the measurement of multidimensional ordinal deprivation." *Social Indicators Research*, **128**(2), 835–858.

Fattore M, Arcagni A (2018). "A reduced posetic approach to the measurement of multidimensional ordinal deprivation." *Social Indicators Research*, **136**(3), 1053–1070.

Habib M, Medina R, Nourine L, Steiner G (2001). "Efficient algorithms on distributive lattices." *Discrete Applied Mathematics*, **110**(2-3), 169–187.

#### See Also

poset

```
dom <- matrix(c(</pre>
"a", "b",
"c", "b"
"b", "d"
), ncol = 2, byrow = TRUE)
p \leftarrow poset(x = dom)
## Not run:
LEapply(
  x = p,
  FUN = "MutualRankingProbability",
  generator = "AllLE",
  degrees = c(3, 2, 3, 2)
a_rank_dist <- function(le) {
  return(matrix(le == "a"))
LEapply(x = p, FUN = a\_rank\_dist)
## End(Not run)
```

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plot.poset

Plotting the Hasse diagram of a poset

#### Description

plot produces an igraph object and shows the Hasse diagram.

# Usage

```
## S3 method for class 'poset'
plot(
    x,
    vertex.color = rgb(1, 1, 1, 1),
    vertex.label = x$pointer$elements(),
    vertex.label.color = rgb(0, 0, 0, 1),
    vertex.label.family = "sans",
    edge.color = rgb(0, 0, 0, 1),
    edge.label = NA,
    edge.arrow.mode = "-",
    asp = 0,
    ...,
    equispaced = FALSE,
    show = TRUE
)
```

#### **Arguments**

```
an S4 object of class Rcpp_POSet, see poset for details.
                  argument of the plot. igraph function, see igraph.plotting for details.
vertex.color
vertex.label
                  argument of the plot.igraph function, see igraph.plotting for details.
vertex.label.color
                  argument of the plot.igraph function, see igraph.plotting for details.
vertex.label.family
                  argument of the plot.igraph function, see igraph.plotting for details.
                  argument of the plot. igraph function, see igraph.plotting for details.
edge.color
edge.label
                  argument of the plot.igraph function, see igraph.plotting for details.
edge.arrow.mode
                  argument\ of\ the\ \verb|plot.igrap| function, see \ \verb|igraph.plotting| for\ details.
                  argument of the plot.igraph function, see igraph.plotting for details.
asp
                  additional plotting parameters, see igraph.plotting for details.
                  logical, if TRUE the nodes on the same level of the Hasse diagram are horizontally
equispaced
                  equispaced.
show
                  logical, if TRUE (default) the Hasse diagram is plotted.
```

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#### **Details**

plot.poset computes the cover relation and produces the corresponding Directed Acyclic Graph (DAG), as an igraph object, returned as invisible output. Function layout\_with\_sugiyama generates the DAG layout with edges oriented from top to bottom. When equispaced=TRUE, nodes on the same Hasse diagram level are horizontally equispaced.

The Hasse diagram is displayed by a call to plot.igraph (some default argument values are set to get a cleaner plot, by exploiting Hasse diagram properties.

Setting show = FALSE produces the igraph object, without showing the Hasse diagram.

Note that

#### Value

```
an igraph object.
```

#### See Also

```
poset, igraph, igraph.plotting
```

## **Examples**

```
dom <- matrix(c(
    "a", "b",
    "c", "b",
    "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
hasse <- plot(p)
class(hasse)</pre>
```

poset

Generates a Partially Ordered SET from the list of dominances

#### **Description**

Function poset creates a poset from a dominance list. x argument is a two-column matrix, each row defines a pair of comparable elements, where the element in the first column is dominated by or coincide with the element in the second column. If the elements of x are numeric, they are first coerced to character and used as elements labels.

#### **Usage**

```
poset(x = NULL, elements = unique(as.character(x)))
```

#### **Arguments**

x an object of class matrix with two columns, listing the dominances, by rows. elements a vector of characters listing all the labels of the elements.

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#### **Details**

A partial order relation is reflexive, transitive and anti-symmetric. Given the dominance list provided by the user, the function produces the smallest poset comprising them (reflexive and transitive closure); in case the dominances provided by the user imply non-trivial cicles, violating anti-simmetry, the function returns an error.

By default elements is equal to all the different labels available in x. If some elements are incomparable, list all of the elements in elements or include self-comparabilities in x. Notice that antichains can be created in a simpler way, by function antiChain.

#### Value

an environment of class poset; this class contains a pointer with different C++ methods used by other functions of the package.

#### See Also

in the package are available functions that simply the creation of particular posets: antiChain to create a poset without comparabilities, chain to create a complete order, poset\_from\_dataframe to create a poset from a statistical population.

## **Examples**

```
dom <- matrix(c(
    "a", "b",
    "c", "b",
    "b", "d"
), ncol = 2, byrow = TRUE)
poset(x = dom)
poset(x = dom, elements = letters[1:5])</pre>
```

poset\_from\_dataframe

Generates a Partially Ordered SET from a data frame through the component-wise comparability relation

#### **Description**

This function generates a poset from an object x of class data. frame that lists by rows all the observations in a statistical population and by columns the values of different orderable variables (i.e. vectors of class ordered or numeric).

```
poset_from_dataframe(x, collapse = "-")
```

poset\_from\_igraph

#### **Arguments**

x an object of class data.frame.

collapse an object of class character that defines the separator between observations

with the same profile, by default "-". For more information see the details section.

#### **Details**

The poset is generated through the component-wise comparability relation (Davey and Priestley 2002). Note that some observations may have the same profile (i.e. the same values for all the variables) therefore they have to be included into the same element of the poset, than the label of such element correspond to the labels of the statistical units separated by the collapse character.

#### Value

an S4 object of class Rcpp\_POSet; this class contains different C++ methods used by other functions of the package.

#### References

Davey BA, Priestley HA (2002). Introduction to lattices and order. Cambridge university press.

#### See Also

poset.

## **Examples**

```
set <- ordered(letters[1:5])
set.seed(0)
n <- 30
x <- data.frame(
    A = sample(set, n, replace = TRUE),
    B = sample(set, n, replace = TRUE),
    C = sample(set, n, replace = TRUE)
)

p <- poset_from_dataframe(x, collapse = ",")

plot(p, equispaced = TRUE, vertex.frame.color = "lightgray", vertex.color = "lightgray")</pre>
```

#### **Description**

This function generates a poset from an objectg. The function checks if the graph is directed and cyclic in oreder it represent the cover-relation.

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#### Usage

```
poset_from_igraph(g)
```

#### **Arguments**

```
g an igraph object
```

#### **Details**

Note that the Hasse diagram represents the cover relation from the top to the bottom, therefore its representation is the opposite of the plot of the graph with Sugiyama layout.

#### Value

an environment of class poset.

#### References

Davey BA, Priestley HA (2002). Introduction to lattices and order. Cambridge university press.

#### See Also

```
poset.
```

## **Examples**

```
library(igraph)
g <- make_tree(10)
ly <- igraph::layout_with_sugiyama(g)$layout
g$layout <- ly
plot(g)
p <- poset_from_igraph(g)
plot(p)</pre>
```

## Description

This function generates a poset from an incidence matrix x. Such matrix is a named and logical that is TRUE if the row element is dominated by the column one, FALSE otherwise. Such incidence relation should be reflexive, anti-symmetric, and transitive..

```
poset_from_incidence(x)
```

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#### **Arguments**

x a logical matrix.

## Value

an environment of class poset.

#### References

Davey BA, Priestley HA (2002). Introduction to lattices and order. Cambridge university press.

#### See Also

poset.

#### **Examples**

```
x <- c(
  TRUE, FALSE, FALSE, FALSE, FALSE, FALSE,
  TRUE, TRUE, FALSE, FALSE, FALSE, FALSE,
  TRUE, TRUE, TRUE, FALSE, FALSE, FALSE,
  TRUE, FALSE, FALSE, TRUE, FALSE,
  TRUE, TRUE, FALSE, TRUE, TRUE, FALSE,
  TRUE, TRUE, TRUE, TRUE, TRUE, TRUE
)
x <- matrix(x, nrow = 6, ncol = 6)
rownames(x) <- colnames(x) <- LETTERS[1:6]
p <- poset_from_incidence(x)
plot(p)</pre>
```

print.poset

Method for the print function that shows the poset elements and comparabilities

## **Description**

print prints the list of poset elements and all of the strict dominances in it.

# Usage

```
## S3 method for class 'poset'
print(x, max = NULL, ...)
```

#### **Arguments**

x an object of class Rcpp\_POSet.

max a non-null value for max specifies the approximate maximum number of entries to be printed. The default, NULL, uses getOption("max.print"): see that help page for more details.

... further arguments passed to or from other methods.

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## Value

nothing

## **Examples**

```
dom <- matrix(c(
   "a", "b",
   "c", "b",
   "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
print(p)</pre>
```

print.summary\_poset

Method for the print function that shows the poset summary

# Description

print prints the poset summary.

## Usage

```
## S3 method for class 'summary_poset'
print(x, ...)
```

## **Arguments**

x an object of class summary\_poset.

... further arguments passed to or from other methods.

#### Value

nothing

#### See Also

```
summary.poset
```

```
dom <- matrix(c(
   "a", "b",
   "c", "b",
   "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
summary(p)
summary(chain(1:4))
summary(antiChain(LETTERS[1:5]))</pre>
```

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productOrder

Product order between two posets

# Description

The function returns the product poset of two posets X and Y.

#### Usage

```
productOrder(x, y, sep = "-")
x %po% y
```

## Arguments

```
    x an S4 object of class poset, see poset for details.
    y an S4 object of class poset, see poset for details.
    sep a character object indicating the separator to be used to paste profiles names.
```

#### Value

an environment of class poset, see poset for details

#### References

Davey BA, Priestley HA (2002). Introduction to lattices and order. Cambridge university press.

# See Also

poset

```
dom <- matrix(c(
   "a", "b",
   "c", "b",
   "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
q <- chain(1:3)
plot(p %po% q)</pre>
```

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summary.poset

Poset summary

## **Description**

Method of the summary function for objects of class poset.

# Usage

```
## S3 method for class 'poset'
summary(object, ...)
```

## **Arguments**

object an object of class poset.

... further arguments passed to or from other methods.

#### Value

An S3 object of class summary\_poset listing and counting the poset elements, their strict dominances and their incomparabilities.

```
dom <- matrix(c(
   "a", "b",
   "c", "b",
   "b", "d"
), ncol = 2, byrow = TRUE)
p <- poset(x = dom)
summary(p)

summary(chain(1:4))
summary(antiChain(LETTERS[1:5]))</pre>
```

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