Package 'NPP'

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Description Posterior sampling in several commonly used distributions using normalized power prior as described in Duan, Ye and Smith (2006) <doi:10.1002 env.752=""> and Ibrahim et.al. (2015) <doi:10.1002 sim.6728="">. Sampling of the power parameter is achieved via either independence Metropolis-Hastings or random walk Metropolis-Hastings based on transformation.</doi:10.1002></doi:10.1002>
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BerMNPP_MCMC1
BerMNPP_MCMC2
BerNPP_MCMC
BerOMNPP_MCMC1
BerOMNPP_MCMC2
LaplacelogC
LMMNPP_MCMC1
LMMNPP_MCMC2
LMNPP_MCMC
LMOMNPP_MCMC1

2 BerMNPP_MCMC1

	_MOMNPP_MCMC2	22
	ogCdelta	24
	ogCknot	
	oglikBerD0	
	oglikNormD0	27
	ModeDeltaBerNPP	28
	ModeDeltaLMNPP	30
	ModeDeltaMultinomialNPP	31
	ModeDeltaNormalNPP	33
	ModeDeltaPoisNPP	35
	MultinomialNPP_MCMC	37
	NormalNPP_MCMC	39
	PHData	41
	PoiMNPP_MCMC1	42
	PoiMNPP_MCMC2	44
	PoiOMNPP_MCMC1	45
	PoiOMNPP_MCMC2	47
	PoissonNPP_MCMC	48
	SPDData	50
	VaccineData	51
Index		5 3
D = =MA	D MCMC Compline for Domestill Domestic Melling Historic	1
BerMN	P_MCMC1 MCMC Sampling for Bernoulli Population with Multiple Historical Data using Normalized Power Prior	aı
	Data using Normanzea Lower Littor	

Description

Incorporate multiple historical data sets for posterior sampling of a Bernoulli population using the normalized power prior. The Metropolis-Hastings algorithm, with either an independence proposal or a random walk proposal on the logit scale, is applied for the power parameter δ . Gibbs sampling is utilized for the model parameter p.

Usage

Arguments

n0	A non-negative integer vector representing the number of trials in historical data.
y0	A non-negative integer vector denoting the number of successes in historical data.
n	A non-negative integer indicating the number of trials in the current data.
У	A non-negative integer for the number of successes in the current data.

BerMNPP_MCMC1 3

prior_p a vector of the hyperparameters in the prior distribution $Beta(\alpha,\beta)$ for p. prior_delta_alpha

a vector of the hyperparameter α in the prior distribution $Beta(\alpha, \beta)$ for each δ .

prior_delta_beta

a vector of the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for each δ .

prop_delta_alpha

a vector of the hyperparameter α in the proposal distribution $Beta(\alpha,\beta)$ for

each δ

prop_delta_beta

a vector of the hyperparameter β in the proposal distribution $Beta(\alpha,\beta)$ for

each δ .

delta_ini the initial value of δ in MCMC sampling. prop_delta the class of proposal distribution for δ .

rw_delta the stepsize(variance of the normal distribution) for the random walk proposal

of logit δ . Only applicable if prop_delta = 'RW'.

nsample specifies the number of posterior samples in the output.

burnin the number of burn-ins. The output will only show MCMC samples after bunrin.

thin the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling δ . The normalized power prior distribution is

$$\frac{\pi_0(\delta)\pi_0(\theta)\prod_{k=1}^{K}L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta)\prod_{k=1}^{K}L(\theta|D_{0k})^{\delta_k}d\theta}.$$

Here $\pi_0(\delta)$ and $\pi_0(\theta)$ are the initial prior distributions of δ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and δ_k is the corresponding power parameter.

Value

A list of class "NPP" comprising:

acceptrate Acceptance rate in MCMC sampling for δ via the Metropolis-Hastings algo-

rithm.

p Posterior distribution of the model parameter p.

delta Posterior distribution of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

BerMNPP_MCMC2

References

4

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. Statistics in Medicine 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. Environmetrics 17:95-106.

See Also

```
BerMNPP_MCMC2; BerOMNPP_MCMC1; BerOMNPP_MCMC2
```

Examples

```
BerMNPP_MCMC1(n0 = c(275, 287), y0 = c(92, 125), n = 39, y = 17,
              prior_p = c(1/2, 1/2), prior_delta_alpha = c(1/2, 1/2),
              prior_delta_beta = c(1/2,1/2),
              prop_delta_alpha = c(1,1)/2, prop_delta_beta = c(1,1)/2,
              delta_ini = NULL, prop_delta = "IND",
              nsample = 2000, burnin = 500, thin = 2)
```

BerMNPP_MCMC2

MCMC Sampling for Bernoulli Population of multiple historical data using Normalized Power Prior

Description

Multiple historical data are combined individually. The NPP of multiple historical data is the product of the NPP of each historical data. Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter p, Gibbs sampling is used.

Usage

```
BerMNPP_MCMC2(n0, y0, n, y, prior_p, prior_delta_alpha, prior_delta_beta,
              prop_delta_alpha, prop_delta_beta, delta_ini, prop_delta,
              rw_delta, nsample, burnin, thin)
```

Arguments

```
a non-negative integer vector: number of trials in historical data.
n0
y0
                   a non-negative integer vector: number of successes in historical data.
                   a non-negative integer: number of trials in the current data.
n
                   a non-negative integer: number of successes in the current data.
٧
                   a vector of the hyperparameters in the prior distribution Beta(\alpha, \beta) for p.
prior_p
prior_delta_alpha
```

a vector of the hyperparameter α in the prior distribution $Beta(\alpha, \beta)$ for each δ .

BerMNPP_MCMC2 5

prior_delta_beta

a vector of the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for each δ .

prop_delta_alpha

a vector of the hyperparameter α in the proposal distribution $Beta(\alpha,\beta)$ for

each δ .

prop_delta_beta

a vector of the hyperparameter β in the proposal distribution $Beta(\alpha, \beta)$ for

each δ .

delta_ini the initial value of δ in MCMC sampling. prop_delta the class of proposal distribution for δ .

rw_delta the stepsize (variance of the normal distribution) for the random walk proposal

of logit δ . Only applicable if prop_delta = 'RW'.

nsample specifies the number of posterior samples in the output.

burnin the number of burn-ins. The output will only show MCMC samples after burnin.

thin the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling δ . The normalized power prior distribution is

$$\pi_0(\delta) \prod_{k=1}^K \frac{\pi_0(\theta) L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta) L(\theta|D_{0k})^{\delta_k} d\theta}.$$

Here $\pi_0(\delta)$ and $\pi_0(\theta)$ are the initial prior distributions of δ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and δ_k is the corresponding power parameter.

Value

A list of class "NPP" with three elements:

acceptrate the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

p posterior of the model parameter p. delta posterior of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

6 BerNPP_MCMC

See Also

```
BerMNPP_MCMC1; BerOMNPP_MCMC1; BerOMNPP_MCMC2
```

Examples

BerNPP_MCMC

MCMC Sampling for Bernoulli Population using Normalized Power Prior

Description

Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter p, Gibbs sampling is used.

Usage

Arguments

Data.Cur a non-negative integer vector of two elements: c(number of trials, number of

successes) in the current data.

Data. Hist a non-negative integer vector of two elements: c(number of trials, number of

successes) in the historical data.

CompStat a list of four elements that represents the "compatibility(sufficient) statistics" for

p. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. Note: in Bernoulli population providing CompStat is equivalent to provide the data

summary as in Data. Cur and Data. Cur.

no is the number of trials in the historical data.

y0 is the number of successes in the historical data.

n1 is the number of trials in the current data.

y1 is the number of successes in the current data.

BerNPP_MCMC 7

prior a list of the hyperparameters in the prior for both p and δ .

p. alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for p. p. beta is the hyperparameter β in the prior distribution $Beta(\alpha,\beta)$ for p. delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for

 δ .

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

MCMCmethod sampling method for δ in MCMC. It can be either 'IND' for independence pro-

posal; or 'RW' for random walk proposal on logit scale.

rw.logit.delta the stepsize(variance of the normal distribution) for the random walk proposal

of logit δ . Only applicable if MCMCmethod = 'RW'.

ind.delta.alpha

specifies the first parameter α when independent proposal $Beta(\alpha, \beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

ind.delta.beta specifies the first parameter β when independent proposal $Beta(\alpha,\beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

nsample specifies the number of posterior samples in the output.

control.mcmc a list of three elements used in posterior sampling.

delta.ini is the initial value of δ in MCMC sampling.

burnin is the number of burn-ins. The output will only show MCMC samples

after bunrin.

thin is the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling δ , and the deviance information criteria.

Value

A list of class "NPP" with four elements:

p posterior of the model parameter p. delta posterior of the power parameter δ .

acceptance the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

DIC the deviance information criteria for model diagnostics.

Author(s)

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References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
MultinomialNPP_MCMC; NormalNPP_MCMC; PoissonNPP_MCMC
```

Examples

```
\label{eq:bernPP_MCMC} \begin{aligned} \text{BerNPP\_MCMC}(\text{Data.Cur} = \text{c}(493, \ 473), \ \text{Data.Hist} = \text{c}(680, \ 669), \\ \text{prior} &= \text{list}(\text{p.alpha} = 0.5, \ \text{p.beta} = 0.5, \ \text{delta.alpha} = 1, \ \text{delta.beta} = 1), \\ \text{MCMCmethod} &= \text{'RW'}, \ \text{rw.logit.delta} = 1, \ \text{nsample} = 5000, \\ \text{control.mcmc} &= \text{list}(\text{delta.ini} = \text{NULL}, \ \text{burnin} = 2000, \ \text{thin} = 5)) \end{aligned}
```

BerOMNPP_MCMC1

MCMC Sampling for Bernoulli Population of multiple ordered historical data using Normalized Power Prior

Description

Multiple ordered historical data are incorporated together. Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter γ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter p, Gibbs sampling is used.

Usage

Arguments

n0	a non-negative integer vector: number of trials in historical data.
y0	a non-negative integer vector: number of successes in historical data.
n	a non-negative integer: number of trials in the current data.
у	a non-negative integer: number of successes in the current data.
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2,, \alpha_K)$ for γ .
prior_p	a vector of the hyperparameters in the prior distribution $Beta(\alpha,\beta)$ for p .
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1,\alpha_2,,\alpha_K)$ for γ .
gamma_ini	the initial value of γ in MCMC sampling.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burnin.
thin	the thinning parameter in MCMC sampling.
adjust	Logical, indicating whether or not to adjust the parameters of the proposal distribution.

BerOMNPP_MCMC1 9

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling γ . The normalized power prior distribution is given by:

$$\frac{\pi_0(\gamma)\pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)}d\theta}.$$

Here, $\pi_0(\gamma)$ and $\pi_0(\theta)$ are the initial prior distributions of γ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and $\sum_{i=1}^k \gamma_i$ is the corresponding power parameter.

Value

A list of class "NPP" with three elements:

acceptrate the acceptance rate in MCMC sampling for γ using Metropolis-Hastings algo-

rithm.

p posterior of the model parameter p.

delta posterior of the power parameter δ . It is equal to the cumulative sum of γ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

BerMNPP_MCMC1, BerMNPP_MCMC2, BerOMNPP_MCMC2

BerOMNPP_MCMC2

BerOMNPP_MCMC2	MCMC Sampling for Bernoulli Population of multiple ordered historical data using Normalized Power Prior
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Description

Multiple ordered historical data are combined individually. Conduct posterior sampling for Bernoulli population with normalized power prior. For the power parameter γ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter p, Gibbs sampling is used.

Usage

Arguments

n0	a vector of non-negative integers: numbers of trials in historical data.
у0	a vector of non-negative integers: numbers of successes in historical data.
n	a non-negative integer: number of trials in the current data.
у	a non-negative integer: number of successes in the current data.
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1,\alpha_2,,\alpha_K)$ for γ .
prior_p	a vector of the hyperparameters in the prior distribution $Beta(\alpha,\beta)$ for p .
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1,\alpha_2,,\alpha_K)$ for γ .
gamma_ini	the initial value of γ in MCMC sampling.
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after burn-in.
thin	the thinning parameter in MCMC sampling.
adjust	Whether or not to adjust the parameters of the proposal distribution.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling γ . The normalized power prior distribution is

$$\pi_0(\gamma) \prod_{k=1}^K \frac{\pi_0(\theta) L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\theta) L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\theta}.$$

Here $\pi_0(\gamma)$ and $\pi_0(\theta)$ are the initial prior distributions of γ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and $\sum_{i=1}^k \gamma_i$ is the corresponding power parameter.

LaplacelogC 11

Value

A list of class "NPP" with three elements:

acceptrate the acceptance rate in MCMC sampling for γ using Metropolis-Hastings algo-

rithm.

p posterior of the model parameter p.

delta posterior of the power parameter δ . It is equal to the cumulative sum of γ

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
BerMNPP_MCMC1; BerMNPP_MCMC2; BerOMNPP_MCMC1
```

Examples

```
BerOMNPP_MCMC2(n0 = c(275, 287), y0 = c(92, 125), n = 39, y = 17, prior_gamma=c(1,1,1)/3, prior_p=c(1/2,1/2), gamma_ind_prop=rep(1,3)/2, gamma_ini=NULL, nsample = 2000, burnin = 500, thin = 2, adjust = FALSE)
```

LaplacelogC

A Function to Calculate $logC(\delta)$ Based on Laplace Approximation

Description

The function assumes that the prior of the model parameters is very flat that had very minor impact on the shape of the power prior (posterior based on the D0).

Usage

```
LaplacelogC(delta, loglikmle, detHessian, ntheta)
```

Arguments

delta	the power parameter between 0 and 1. The function returns $logC(\delta)$
loglikmle	a scalar; the loglikelihood of the historical data evaluated at the maximum likelihood estimates based on the historical data
detHessian	determinant of the Hessian matrix evaluated at the loglikelihood function with respect to the maximum likelihood estimates based on the historical data
ntheta	an positive integer indicating number of parameters in the model

12 LMMNPP_MCMC1

Value

 $logC(\delta)$ based on the Laplace approximation. Can be used for the posterior sampling in the normalized power prior.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

logCknot

LMMNPP_MCMC1	MCMC Sampling for Linear Regression Model of multiple historical
	data using Normalized Power Prior

Description

Multiple historical data are incorporated together. Conduct posterior sampling for Linear Regression Model with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameters (β, σ^2) , Gibbs sampling is used.

Usage

```
LMMNPP_MCMC1(D0, X, Y, a0, b, mu0, R, delta_ini, prop_delta, prior_delta_alpha, prior_delta_beta, prop_delta_alpha, prop_delta_beta, rw_delta, nsample, burnin, thin)
```

Arguments

DØ	a list of k elements representing k historical data, where the i^{th} element corresponds to the i^{th} historical data named as "D0i".
X	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
Υ	a vector of individual level of the response y in the current data.
a0	a positive shape parameter for inverse-gamma prior on model parameter σ^2 .
b	a positive scale parameter for inverse-gamma prior on model parameter σ^2 .

LMMNPP_MCMC1 13

mu0 a vector of the mean for prior $\beta | \sigma^2$.

R a inverse matrix of the covariance matrix for prior $\beta | \sigma^2$.

delta_ini the initial value of δ in MCMC sampling. prop_delta the class of proposal distribution for δ .

prior_delta_alpha

a vector of the hyperparameter α in the prior distribution $Beta(\alpha, \beta)$ for each δ .

prior_delta_beta

a vector of the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for each δ .

prop_delta_alpha

a vector of the hyperparameter α in the proposal distribution $Beta(\alpha,\beta)$ for

each δ .

prop_delta_beta

a vector of the hyperparameter β in the proposal distribution $Beta(\alpha,\beta)$ for

each δ .

rw_delta the stepsize(variance of the normal distribution) for the random walk proposal

of logit δ . Only applicable if prop_delta = 'RW'.

nsample specifies the number of posterior samples in the output.

burnin the number of burn-ins. The output will only show MCMC samples after bunrin.

thin the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling δ . Let $\theta = (\beta, \sigma^2)$, the normalized power prior distribution is

$$\frac{\pi_0(\delta)\pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta)\prod_{k=1}^K L(\theta|D_{0k})^{\delta_k} d\theta}.$$

Here $\pi_0(\delta)$ and $\pi_0(\theta)$ are the initial prior distributions of δ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and δ_k is the corresponding power parameter.

Value

A list of class "NPP" with four elements:

acceptrate the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

beta posterior of the model parameter β in vector or matrix form.

sigma posterior of the model parameter σ^2 . delta posterior of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

14 LMMNPP_MCMC1

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
LMMNPP_MCMC2; LMOMNPP_MCMC1; LMOMNPP_MCMC2
```

```
## Not run:
set.seed(1234)
sigsq0 = 1
n01 = 100
theta01 = c(0, 1, 1)
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01%*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigsq0))
D01 = cbind(X01, Y01)
n02 = 70
theta02 = c(0, 2, 3)
X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02%*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigsq0))
D02 = cbind(X02, Y02)
n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03%*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigsq0))
D03 = cbind(X03, Y03)
D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)
n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X%*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigsq0))
LMMNPP_MCMC1(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)),
             delta_ini=NULL, prior_delta_alpha=c(1,1,1), prior_delta_beta=c(1,1,1),
             prop\_delta\_alpha=c(1,1,1), \ prop\_delta\_beta=c(1,1,1),
             prop_delta="RW", rw_delta=0.9, nsample=5000, burnin=1000, thin=3)
## End(Not run)
```

LMMNPP_MCMC2 15

LMMNPP_MCMC2	MCMC Sampling for Linear Regression Model of multiple historical data using Normalized Power Prior
	· ·

Description

Multiple historical data are combined individually. The NPP of multiple historical data is the product of the NPP of each historical data. Conduct posterior sampling for Linear Regression Model with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameters (β, σ^2) , Gibbs sampling is used.

Usage

```
LMMNPP_MCMC2(D0, X, Y, a0, b, mu0, R, delta_ini, prop_delta, prior_delta_alpha, prior_delta_beta, prop_delta_alpha, prop_delta_beta, rw_delta, nsample, burnin, thin)
```

Arguments

D0	a list of k elements representing k historical data, where the i^{th} element corresponds to the i^{th} historical data named as "D0i".	
X	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.	
Υ	a vector of individual level of the response y in the current data.	
a0	a positive shape parameter for inverse-gamma prior on model parameter σ^2 .	
b	a positive scale parameter for inverse-gamma prior on model parameter σ^2 .	
mu0	a vector of the mean for prior $\beta \sigma^2$.	
R	a inverse matrix of the covariance matrix for prior $\beta \sigma^2$.	
delta_ini	the initial value of δ in MCMC sampling.	
prop_delta	the class of proposal distribution for δ .	
prior_delta_alpha		
	a vector of the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for each δ .	
prior_delta_bet	ta	
	a vector of the hyperparameter β in the prior distribution $Beta(\alpha,\beta)$ for each δ .	
prop_delta_alph	na	
	a vector of the hyperparameter α in the proposal distribution $Beta(\alpha,\beta)$ for each δ .	
prop_delta_beta		
	a vector of the hyperparameter β in the proposal distribution $Beta(\alpha,\beta)$ for each $\delta.$	
rw_delta	the stepsize(variance of the normal distribution) for the random walk proposal of logit δ . Only applicable if prop_delta = 'RW'.	

16 LMMNPP_MCMC2

nsample specifies the number of posterior samples in the output.

burnin the number of burn-ins. The output will only show MCMC samples after bunrin.

thin the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling δ . Let $\theta = (\beta, \sigma^2)$, the normalized power prior distribution is

$$\pi_0(\delta) \prod_{k=1}^K \frac{\pi_0(\theta) L(\theta|D_{0k})^{\delta_k}}{\int \pi_0(\theta) L(\theta|D_{0k})^{\delta_k} d\theta}.$$

Here $\pi_0(\delta)$ and $\pi_0(\theta)$ are the initial prior distributions of δ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and δ_k is the corresponding power parameter.

Value

A list of class "NPP" with four elements:

acceptrate the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

beta posterior of the model parameter β in vector or matrix form.

sigma posterior of the model parameter σ^2 . delta posterior of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
LMMNPP_MCMC1; LMOMNPP_MCMC1; LMOMNPP_MCMC2
```

```
## Not run:
set.seed(1234)
sigsq0 = 1

n01 = 100
theta01 = c(0, 1, 1)
```

LMNPP_MCMC 17

```
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01%*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigsq0))
D01 = cbind(X01, Y01)
n02 = 70
theta02 = c(0, 2, 3)
X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02%*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigsq0))
D02 = cbind(X02, Y02)
n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03%*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigsq0))
D03 = cbind(X03, Y03)
D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)
n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X%*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigsq0))
 LMMNPP\_MCMC2(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)), R=diag(c(1/64,1/64,1/64)), R=diag(c(1/64,1/64)), R=diag(c(1/64,1/64))
                                      delta_ini=NULL, prior_delta_alpha=c(1,1,1), prior_delta_beta=c(1,1,1),
                                      prop_delta_alpha=c(1,1,1), prop_delta_beta=c(1,1,1),
                                      prop_delta="RW", rw_delta=0.9, nsample=5000, burnin=1000, thin=5)
## End(Not run)
```

LMNPP_MCMC

MCMC Sampling for Normal Linear Model using Normalized Power Prior

Description

Conduct posterior sampling for normal linear model with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the regression parameter β and σ^2 , Gibbs sampling is used.

Usage

18 LMNPP_MCMC

Arguments

y.Cur a vector of individual level of the response y in current data.

y. Hist a vector of individual level of the response y in historical data.

x.Cur a vector or matrix or data frame of covariate observed in the current data. If

more than 1 covariate available, the number of rows is equal to the number of

observations.

x. Hist a vector or matrix or data frame of covariate observed in the historical data. If

more than 1 covariate available, the number of rows is equal to the number of

observations.

prior a list of the hyperparameters in the prior for model parameters (β, σ^2) and δ .

The form of the prior for model parameter (β, σ^2) is in the section "Details".

a a positive hyperparameter for prior on model parameters. It is the power \boldsymbol{a} in

formula $(1/\sigma^2)^a$; See details.

b equals 0 if a flat prior is used for β . Equals 1 if a normal prior is used for β ;

See details.

mu0 a vector of the mean for prior $\beta | \sigma^2$. Only applicable if b = 1.

Rinv inverse of the matrix R. The covariance matrix of the prior for $\beta | \sigma^2$ is

 $\sigma^2 R^{-1}$.

delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for

 δ .

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

MCMCmethod sampling method for δ in MCMC. It can be either 'IND' for independence pro-

posal; or 'RW' for random walk proposal on logit scale.

 $\verb"rw.logit.delta" the stepsize (variance of the normal distribution) for the random walk proposal$

of logit δ . Only applicable if MCMCmethod = 'RW'.

ind.delta.alpha

specifies the first parameter α when independent proposal $Beta(\alpha, \beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

ind.delta.beta specifies the first parameter β when independent proposal $Beta(\alpha,\beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

nsample specifies the number of posterior samples in the output.

control.mcmc a list of three elements used in posterior sampling.

delta. ini is the initial value of δ in MCMC sampling.

burnin is the number of burn-ins. The output will only show MCMC samples

after bunrin.

thin is the thinning parameter in MCMC sampling.

Details

If b=1, prior for (β,σ) is $(1/\sigma^2)^a*N(mu0,\sigma^2R^{-1})$, which includes the g-prior. If b=0, prior for (β,σ) is $(1/\sigma^2)^a$. The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate when sampling δ , and the deviance information criteria.

LMNPP_MCMC 19

Value

A list of class "NPP" with five elements:

beta posterior of the model parameter β in vector or matrix form.

sigmasq posterior of the model parameter σ^2 . delta posterior of the power parameter δ .

acceptance the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

DIC the deviance information criteria for model diagnostics.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. *Bayesian Statistics* 4: Proceedings of the Fourth Valencia International Meeting, Bernardo, J.M, Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds., 35-60, Clarendon Press:Oxford.

Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proceedings of the Royal Statistical Society of London, Series A 186:453-461*.

See Also

BerNPP_MCMC; MultinomialNPP_MCMC; PoissonNPP_MCMC; NormalNPP_MCMC

LMOMNPP_MCMC1

LMOMNPP_MCMC1	MCMC Sampling for Linear Regression Model of multiple historical data using Ordered Normalized Power Prior
	data using Ordered Normalized Power Prior

Description

Multiple historical data are incorporated together. Conduct posterior sampling for Linear Regression Model with ordered normalized power prior. For the power parameter γ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameters (β, σ^2) , Gibbs sampling is used.

Usage

```
LMOMNPP_MCMC1(D0, X, Y, a0, b, mu0, R, gamma_ini, prior_gamma, gamma_ind_prop, nsample, burnin, thin, adjust)
```

Arguments

D0	a list of k elements representing k historical data, where the i^{th} element corresponds to the i^{th} historical data named as "D0i".
X	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
Υ	a vector of individual level of the response y in the current data.
a0	a positive shape parameter for inverse-gamma prior on model parameter σ^2 .
b	a positive scale parameter for inverse-gamma prior on model parameter σ^2 .
mu0	a vector of the mean for prior $\beta \sigma^2$.
R	a inverse matrix of the covariance matrix for prior $\beta \sigma^2$.
gamma_ini	the initial value of γ in MCMC sampling.
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1,\alpha_2,,\alpha_K)$ for γ .
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1,\alpha_2,,\alpha_K)$ for γ .
nsample	specifies the number of posterior samples in the output.
burnin	the number of burn-ins. The output will only show MCMC samples after bunrin.
thin	the thinning parameter in MCMC sampling.
adjust	Whether or not to adjust the parameters of the proposal distribution.

LMOMNPP_MCMC1 21

Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling γ . Let $\theta = (\beta, \sigma^2)$, the normalized power prior distribution is

$$\frac{\pi_0(\gamma)\pi_0(\theta)\prod_{k=1}^{K}L(\theta|D_{0k})^{(\sum_{i=1}^{k}\gamma_i)}}{\int \pi_0(\theta)\prod_{k=1}^{K}L(\theta|D_{0k})^{(\sum_{i=1}^{k}\gamma_i)}d\theta}.$$

Here $\pi_0(\gamma)$ and $\pi_0(\theta)$ are the initial prior distributions of γ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and $\sum_{i=1}^k \gamma_i$ is the corresponding power parameter.

Value

A list of class "NPP" with four elements:

acceptrate the acceptance rate in MCMC sampling for γ using Metropolis-Hastings algo-

rithm.

beta posterior of the model parameter β in vector or matrix form.

sigma posterior of the model parameter σ^2 . delta posterior of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
LMMNPP_MCMC1; LMMNPP_MCMC2; LMOMNPP_MCMC2
```

```
## Not run:
set.seed(1234)
sigsq0 = 1

n01 = 100
theta01 = c(0, 1, 1)
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01%*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigsq0))
D01 = cbind(X01, Y01)

n02 = 70
theta02 = c(0, 2, 3)
```

LMOMNPP_MCMC2

```
X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02%*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigsq0))
D02 = cbind(X02, Y02)
n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03%*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigsq0))
D03 = cbind(X03, Y03)
D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)
n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X%*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigsq0))
 LMOMNPP\_MCMC1(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)), \\
              gamma_ini=NULL, prior_gamma=rep(1/4,4), gamma_ind_prop=rep(1/4,4),
              nsample=5000, burnin=1000, thin=5, adjust=FALSE)
## End(Not run)
```

LMOMNPP_MCMC2

MCMC Sampling for Linear Regression Model of multiple historical data using Ordered Normalized Power Prior

Description

Multiple historical data are combined individually. The NPP of multiple historical data is the product of the NPP of each historical data. Conduct posterior sampling for Linear Regression Model with ordered normalized power prior. For the power parameter γ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameters (β, σ^2) , Gibbs sampling is used.

Usage

```
LMOMNPP_MCMC2(D0, X, Y, a0, b, mu0, R, gamma_ini, prior_gamma, gamma_ind_prop, nsample, burnin, thin, adjust)
```

Arguments

D0	a list of k elements representing k historical data, where the i^{th} element corresponds to the i^{th} historical data named as "D0i".
X	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
Υ	a vector of individual level of the response y in the current data.

LMOMNPP_MCMC2 23

a positive shape parameter for inverse-gamma prior on model parameter σ^2 . b a positive scale parameter for inverse-gamma prior on model parameter σ^2 .

mu0 a vector of the mean for prior $\beta | \sigma^2$.

R a inverse matrix of the covariance matrix for prior $\beta | \sigma^2$.

gamma_ini the initial value of γ in MCMC sampling.

prior_gamma a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, ..., \alpha_K)$

for γ .

gamma_ind_prop a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, ..., \alpha_K)$

for γ .

nsample specifies the number of posterior samples in the output.

burnin the number of burn-ins. The output will only show MCMC samples after bunrin.

thin the thinning parameter in MCMC sampling.

adjust Whether or not to adjust the parameters of the proposal distribution.

Details

The outputs include posteriors of the model parameters and power parameter, acceptance rate in sampling γ . Let $\theta = (\beta, \sigma^2)$, the normalized power prior distribution is

$$\pi_0(\gamma) \prod_{k=1}^K \frac{\pi_0(\theta) L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\theta) L(\theta|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\theta}.$$

Here $\pi_0(\gamma)$ and $\pi_0(\theta)$ are the initial prior distributions of γ and θ , respectively. $L(\theta|D_{0k})$ is the likelihood function of historical data D_{0k} , and $\sum_{i=1}^k \gamma_i$ is the corresponding power parameter.

Value

A list of class "NPP" with four elements:

acceptrate the acceptance rate in MCMC sampling for γ using Metropolis-Hastings algo-

rithm.

beta posterior of the model parameter β in vector or matrix form.

sigma posterior of the model parameter σ^2 . delta posterior of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

24 logCdelta

See Also

```
LMMNPP_MCMC1; LMMNPP_MCMC2; LMOMNPP_MCMC1
```

Examples

```
## Not run:
set.seed(1234)
sigsq0 = 1
n01 = 100
theta01 = c(0, 1, 1)
X01 = cbind(1, rnorm(n01, mean=0, sd=1), runif(n01, min=-1, max=1))
Y01 = X01%*%as.vector(theta01) + rnorm(n01, mean=0, sd=sqrt(sigsq0))
D01 = cbind(X01, Y01)
n02 = 70
theta02 = c(0, 2, 3)
X02 = cbind(1, rnorm(n02, mean=0, sd=1), runif(n02, min=-1, max=1))
Y02 = X02%*%as.vector(theta02) + rnorm(n02, mean=0, sd=sqrt(sigsq0))
D02 = cbind(X02, Y02)
n03 = 50
theta03 = c(0, 3, 5)
X03 = cbind(1, rnorm(n03, mean=0, sd=1), runif(n03, min=-1, max=1))
Y03 = X03%*%as.vector(theta03) + rnorm(n03, mean=0, sd=sqrt(sigsq0))
D03 = cbind(X03, Y03)
D0 = list(D01, D02, D03)
n0 = c(n01, n02, n03)
n = 100
theta = c(0, 3, 5)
X = cbind(1, rnorm(n, mean=0, sd=1), runif(n, min=-1, max=1))
Y = X%*%as.vector(theta) + rnorm(n, mean=0, sd=sqrt(sigsq0))
LMOMNPP_MCMC1(D0=D0, X=X, Y=Y, a0=2, b=2, mu0=c(0,0,0), R=diag(c(1/64,1/64,1/64)),
              gamma_ini=NULL, prior_gamma=rep(1/4,4), gamma_ind_prop=rep(1/4,4),
              nsample=5000, burnin=1000, thin=5, adjust=FALSE)
## End(Not run)
```

logCdelta

A Function to Interpolate $logC(\delta)$ Based on Its Values on Selected Knots

Description

The function returns the interpolated value (a scalar) of $logC(\delta)$ based on its results on selected knots, given input vector of δ .

logCknot 25

Usage

```
logCdelta(delta, deltaknot, lCknot)
```

Arguments

delta a scalar of the input value of δ .

deltaknot a vector of the knots for δ . It should be selected before conduct the sampling.

1Cknot a vector of the values $logC(\delta)$ on selected knots, coming from the function

logCknot.

Value

A sequence of the values, $logC(\delta)$ on selected knots.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

loglikNormD0; loglikBerD0; logCknot

logCknot

A Function to Calculate $logC(\delta)$ on Selected Knots

Description

The function returns a sequence of the values, $logC(\delta)$ on selected knots, given input vector of δ .

Usage

```
logCknot(deltaknot, llikf0)
```

Arguments

deltaknot a vector of the knots for δ . It should be selected before conduct the sampling.

11ikf0 a matrix of the log-likelihoods of class "npp".

26 loglikBerD0

Value

A sequence of the values, $logC(\delta)$ on selected knots.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

loglikNormD0; loglikBerD0; logCdelta

loglikBerD0

A Function to Calculate Log-likelihood of the Historical Data, Given Matrix-valued Parameters, for Bernoulli Population

Description

The function returns a matrix of class "npp", each element is a log-likelihood of the historical data. It is an intermediate step to calculate the "normalizing constant" $C(\delta)$ in the normalized power prior, for the purpose of providing a flexible implementation. Users can specify their own likelihood function of the same class following this structure.

Usage

```
loglikBerD0(D0, thetalist, ntheta = 1)
```

Arguments

D0 a vector of each observation(binary) in historical data.

thetalist a list of parameter values. The number of elements is equal to ntheta. Each

element is a matrix. The sample should come from the posterior of the powered likelihood for historical data, with each column corresponds to a distinct value of the power parameter δ (the corresponding power parameter increases from left to right). The number of rows is the number of Monte Carlo samples for each δ fixed. The number of columns is the number of selected knots (number

of distinct δ).

ntheta a positive integer indicating number of parameters to be estimated in the model.

Default is 1 for Bernoulli.

loglikNormD0 27

Value

A numeric matrix of log-likelihood, for the historical data given the matrix(or array)-valued parameters.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

loglikNormD0; logCknot; logCdelta

loglikNormD0	A Function to Calculate Log-likelihood of the Historical Data, Given Array-valued Parameters, for Normal Population
loglikNormD0	·

Description

The function returns a matrix of class "npp", each element is a log-likelihood of the historical data. It is an intermediate step to calculate the "normalizing constant" $C(\delta)$ in the normalized power prior, for the purpose of providing a flexible implementation. Users can specify their own likelihood function of the same class following this structure.

Usage

```
loglikNormD0(D0, thetalist, ntheta = 2)
```

Arguments

DØ	a vector	of each	observation	in historical data.
שע	a vector	or cacii	oosci vation	III IIIStoricai uata.

thetalist a list of parameter values. The number of elements is equal to ntheta. Each

element is a matrix. The sample should come from the posterior of the powered likelihood for historical data, with each column corresponds to a distinct value of the power parameter δ (the corresponding power parameter increases from left to right). The number of rows is the number of Monte Carlo samples for each δ fixed. The number of columns is the number of selected knots (number

of distinct δ).

ntheta a positive integer indicating number of parameters to be estimated in the model.

28 ModeDeltaBerNPP

Value

A numeric matrix of log-likelihood, for the historical data given the matrix(or array)-valued parameters.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
loglikBerD0; logCknot; logCdelta
```

ModeDeltaBerNPP

Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Bernoulli Population

Description

The function returns the posterior mode of the power parameter δ in Bernoulli population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

Usage

Arguments

Data.Cur a non-negative integer vector of two elements: c(number of success, number of

failure) in the current data.

Data. Hist a non-negative integer vector of two elements: c(number of success, number of

failure) in the historical data.

ModeDeltaBerNPP 29

CompStat

a list of four elements that represents the "compatibility(sufficient) statistics" for p. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. Note: in Bernoulli population providing CompStat is equivalent to provide the data summary as in Data.Cur and Data.Cur.

n0 is the number of trials in the historical data.
y0 is the number of successes in the historical data.
n1 is the number of trials in the current data.
y1 is the number of successes in the current data.

npoints

is a non-negative integer scalar indicating number of points on a regular spaced grid between [0, 1], where we calculate the log of the posterior and search for the mode.

tne m

prior a list of the hyperparameters in the prior for both p and δ .

p. alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for p. p. beta is the hyperparameter β in the prior distribution $Beta(\alpha,\beta)$ for p. delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for δ

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

Details

See example.

Value

A numeric value between 0 and 1.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

ModeDeltaNormalNPP; ModeDeltaPoisNPP; ModeDeltaMultinomialNPP

```
ModeDeltaBerNPP(Data.Cur = c(100, 40), Data.Hist = c(100, 40), npoints = 1000, prior = list(p.alpha = 1, p.beta = 1, delta.alpha = 1, delta.beta = 1))

ModeDeltaBerNPP(Data.Cur = c(100, 40), Data.Hist = c(100, 35), npoints = 1000,
```

30 ModeDeltaLMNPP

```
prior = list(p.alpha = 1, p.beta = 1, delta.alpha = 1, delta.beta = 1)) ModeDeltaBerNPP(Data.Cur = c(100, 40), Data.Hist = c(100, 50), npoints = 1000, \\ prior = list(p.alpha = 1, p.beta = 1, delta.alpha = 1, delta.beta = 1))
```

ModeDeltaLMNPP

Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Normal Linear Model

Description

The function returns the posterior mode of the power parameter δ in normal linear model. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

Usage

```
ModeDeltaLMNPP(y.Cur, y.Hist, x.Cur = NULL, x.Hist = NULL, npoints = 1000, prior = list(a = 1.5, b = 0, mu0 = 0, Rinv = matrix(1, nrow = 1), delta.alpha = 1, delta.beta = 1))
```

Arguments

O	
y.Cur	a vector of individual level of the response y in current data.
y.Hist	a vector of individual level of the response y in historical data.
x.Cur	a vector or matrix or data frame of covariate observed in the current data. If more than 1 covariate available, the number of rows is equal to the number of observations.
x.Hist	a vector or matrix or data frame of covariate observed in the historical data. If more than 1 covariate available, the number of rows is equal to the number of observations.
npoints	is a non-negative integer scalar indicating number of points on a regular spaced grid between [0, 1], where we calculate the log of the posterior and search for the mode.
prior	a list of the hyperparameters in the prior for model parameters (β, σ^2) and δ . The form of the prior for model parameter (β, σ^2) is in the section "Details". a a positive hyperparameter for prior on model parameters. It is the power a in formula $(1/\sigma^2)^a$; See details.
	b equals 0 if a flat prior is used for β . Equals 1 if a normal prior is used for β ; See details.
	mu0 a vector of the mean for prior $\beta \sigma^2$. Only applicable if b = 1.
	Rinv inverse of the matrix R . The covariance matrix of the prior for $\beta \sigma^2$ is σ^2R^{-1} .
	delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for δ .

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

Details

If b=1, prior for (β,σ) is $(1/\sigma^2)^a*N(mu0,\sigma^2R^{-1})$, which includes the g-prior. If b=0, prior for (β,σ) is $(1/\sigma^2)^a$. The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate when sampling δ , and the deviance information criteria.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. *Bayesian Statistics* 4: Proceedings of the Fourth Valencia International Meeting, Bernardo, J.M, Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds., 35-60, Clarendon Press:Oxford.

Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proceedings of the Royal Statistical Society of London, Series A* 186:453-461.

See Also

 ${\tt ModeDeltaBerNPP; ModeDeltaNormalNPP; ModeDeltaMultinomialNPP; ModeDeltaNormalNPP; ModeDeltaNormalNPP;$

ModeDeltaMultinomialNPP

Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Multinomial Population

Description

The function returns the posterior mode of the power parameter δ in multinomial population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

Usage

```
ModeDeltaMultinomialNPP(Data.Cur, Data.Hist, CompStat = list(n0 = NULL, n1 = NULL), npoints = 1000, prior = list(theta.dir.alpha = c(0.5, 0.5, 0.5), delta.alpha = 1, delta.beta = 1))
```

Arguments

Data. Cur a non-negative integer vector of K elements: c(number of success in group 1,

number of success in group 2, ..., number of success in group K) in the current

data.

Data. Hist a non-negative integer vector of K elements: c(number of success in group 1,

number of success in group 2, ..., number of success in group K) in the historical

data.

CompStat a list of two elements that represents the "compatibility(sufficient) statistics" for

 θ . Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data. Cur and Data. Hist will be ignored. Note: in multinomial case providing CompStat is equivalent to provide the data summary

as in Data. Cur and Data. Cur.

n0 is a non-negative integer vector of K elements for compatible statistics in historical data: c(number of success in group 1, number of success in group 2,

..., number of success in group K).

n1 is a non-negative integer vector of K elements for compatible statistics in current data: c(number of success in group 1, number of success in group 2, ...,

number of success in group K).

npoints is a non-negative integer scalar indicating number of points on a regular spaced

grid between [0, 1], where we calculate the log of the posterior and search for

the mode.

prior a list of the hyperparameters in the prior for both p and δ .

theta.dir is a vector of K elements of the hyperparameter α in the prior dis-

tribution $Dir(\alpha[1], \alpha[2], ..., \alpha[K])$ for θ .

delta. alpha a scalar, the hyperparameter α in the prior distribution $Beta(\alpha, \beta)$

for δ .

delta. beta a scalar, the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$

for δ .

Details

See example.

Value

A numeric value between 0 and 1.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

ModeDeltaNormalNPP 33

See Also

ModeDeltaBerNPP: ModeDeltaNormalNPP: ModeDeltaPoisNPP

Examples

ModeDeltaNormalNPP

Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Normal Population

Description

The function returns the posterior mode of the power parameter δ in multinomial population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

Usage

Arguments

Data.Cur a vector of individual level current data.

Data.Hist a vector of individual level historical data.

CompStat a list of six elements(scalar) that represents the "compatibility(sufficient) statis-

tics" for model parameters. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist $\frac{1}{2}$

will be ignored.

n0 is the sample size of historical data.

mean0 is the sample mean of the historical data. var0 is the sample variance of the historical data. 34 ModeDeltaNormalNPP

n1 is the sample size of current data.

mean1 is the sample mean of the current data. var1 is the sample variance of the current data.

npoints is a non-negative integer scalar indicating number of points on a regular spaced

grid between [0, 1], where we calculate the log of the posterior and search for

the mode.

a list of the hyperparameters in the prior for both (μ, σ^2) and δ . The form of the prior

prior for model parameter (μ, σ^2) is $(1/\sigma^2)^a$. When a=1 it corresponds to the reference prior, and when a=1.5 it corresponds to the Jeffrey's prior.

a is the power a in formula $(1/\sigma^2)^a$, the prior for (μ, σ^2) jointly.

delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha, \beta)$ for

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

Details

See example.

Value

A numeric value between 0 and 1.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. Statistics in Medicine 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. Environmetrics 17:95-106.

Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. Bayesian Statistics 4: Proceedings of the Fourth Valencia International Meeting, Bernardo, J.M, Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds., 35-60, Clarendon Press:Oxford.

Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. Proceedings of the Royal Statistical Society of London, Series A 186:453-461.

See Also

ModeDeltaBerNPP; ModeDeltaMultinomialNPP; ModeDeltaPoisNPP

```
ModeDeltaNormalNPP(CompStat = list(n0 = 50, mean0 = 0, var0 = 1,
                                  n1 = 50, mean1 = 0, var1 = 1), npoints = 1000,
                   prior = list(a = 1.5, delta.alpha = 1, delta.beta = 1))
```

ModeDeltaPoisNPP 35

ModeDeltaPoisNPP

Calculate Posterior Mode of the Power Parameter in Normalized Power Prior with Grid Search, Poisson Population

Description

The function returns the posterior mode of the power parameter δ in multinomial population. It calculates the log of the posterior density (up to a normalizing constant), and conduct a grid search to find the approximate mode.

Usage

Arguments

CompStat

Data.Cur a non-negative integer vector of each observed current data.

Data.Hist a non-negative integer vector of each observed historical data.

a list of four elements that represents the "compatibility(sufficient) statistics" for λ . Default is NULL so the fitting will be based on the data. If the CompStat is

provided then the inputs in Data. Cur and Data. Hist will be ignored.

 $n\emptyset$ is the number of observations in the historical data.

mean0 is the sample mean of the historical data. n1 is the number of observations in the current data.

mean1 is the sample mean of the current data.

npoints is a non-negative integer scalar indicating number of points on a regular spaced

grid between [0, 1], where we calculate the log of the posterior and search for

the mode.

prior a list of the hyperparameters in the prior for both λ and δ . A Gamma distribution

is used as the prior of λ , and a Beta distribution is used as the prior of δ .

lambda. shape is the shape (hyper)parameter in the prior distribution Gamma(shape, scale)

for λ .

36 ModeDeltaPoisNPP

lambda. scale is the scale (hyper)parameter in the prior distribution Gamma(shape, scale) for λ .

delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for δ

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

Details

See example.

Value

A numeric value between 0 and 1.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

ModeDeltaBerNPP; ModeDeltaNormalNPP; ModeDeltaMultinomialNPP

MultinomialNPP_MCMC MCMC Sampling for Multinomial Population using Normalized Power Prior

Description

Conduct posterior sampling for multinomial population with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter vector θ , Gibbs sampling is used. Assume the prior for model parameter θ comes from a Dirichlet distribution.

Usage

Arguments

Data.Cur

a non-negative integer vector of K elements: c(number of success in group 1, number of success in group 2, ..., number of success in group K) in the current data.

Data.Hist

a non-negative integer vector of K elements: c(number of success in group 1, number of success in group 2, ..., number of success in group K) in the historical data.

CompStat

a list of two elements that represents the "compatibility(sufficient) statistics" for θ . Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored. Note: in multinomial case providing CompStat is equivalent to provide the data summary as in Data.Cur and Data.Cur.

n0 is a non-negative integer vector of K elements for compatible statistics in historical data: c(number of success in group 1, number of success in group 2, ..., number of success in group K).

n1 is a non-negative integer vector of K elements for compatible statistics in current data: c(number of success in group 1, number of success in group 2, ..., number of success in group K).

prior

a list of the hyperparameters in the prior for both p and δ .

theta.dir is a vector of K elements of the hyperparameter α in the prior distribution $Dir(\alpha[1], \alpha[2], ..., \alpha[K])$ for θ .

delta. alpha a scalar, the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for δ .

delta. beta a scalar, the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$

for δ .

MCMCmethod sampling method for δ in MCMC. It can be either 'IND' for independence pro-

posal; or 'RW' for random walk proposal on logit scale.

rw.logit.delta the stepsize(variance of the normal distribution) for the random walk proposal

of logit δ . Only applicable if MCMCmethod = 'RW'.

ind.delta.alpha

specifies the first parameter α when independent proposal $Beta(\alpha, \beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

ind.delta.beta specifies the first parameter β when independent proposal $Beta(\alpha,\beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

nsample specifies the number of posterior samples in the output.

control.mcmc a list of three elements used in posterior sampling.

delta. ini is the initial value of δ in MCMC sampling.

burnin is the number of burn-ins. The output will only show MCMC samples

after bunrin

thin is the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling δ , and the deviance information criteria.

Value

A list of class "NPP" with four elements:

p posterior of the model parameter θ . delta posterior of the power parameter δ .

acceptance the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

DIC the deviance information criteria for model diagnostics.

Author(s)

Tianyu Bai <tianyu.bai24@gmail.com> Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

BerNPP_MCMC; NormalNPP_MCMC; PoissonNPP_MCMC

NormalNPP_MCMC 39

Examples

NormalNPP MCMC

MCMC Sampling for Normal Population using Normalized Power Prior

Description

Conduct posterior sampling for normal population with normalized power prior. The initial prior $\pi(\mu|\sigma^2)$ is a flat prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter μ and σ^2 , Gibbs sampling is used.

Usage

Arguments

Data.Cur a vector of individual level current data.

Data.Hist a vector of individual level historical data.

CompStat

a list of six elements(scalar) that represents the "compatibility(sufficient) statistics" for model parameters. Default is NULL so the fitting will be based on the data. If the CompStat is provided then the inputs in Data.Cur and Data.Hist will be ignored.

n0 is the sample size of historical data.

mean0 is the sample mean of the historical data.

var0 is the sample variance of the historical data.

n1 is the sample size of current data.

mean1 is the sample mean of the current data.

var1 is the sample variance of the current data.

40 NormalNPP_MCMC

prior a list of the hyperparameters in the prior for both (μ, σ^2) and δ . The form of the

prior for model parameter (μ, σ^2) is $(1/\sigma^2)^a$. When a=1 it corresponds to the

reference prior, and when a=1.5 it corresponds to the Jeffrey's prior. a is the power a in formula $(1/\sigma^2)^a$, the prior for (μ, σ^2) jointly.

delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for

 δ .

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

MCMCmethod sampling method for δ in MCMC. It can be either 'IND' for independence pro-

posal; or 'RW' for random walk proposal on logit scale.

rw.logit.delta the stepsize(variance of the normal distribution) for the random walk proposal

of logit δ . Only applicable if MCMCmethod = 'RW'.

ind.delta.alpha

specifies the first parameter α when independent proposal $Beta(\alpha, \beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

ind.delta.beta specifies the first parameter β when independent proposal $Beta(\alpha,\beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

nsample specifies the number of posterior samples in the output.

control.mcmc a list of three elements used in posterior sampling.

delta. ini is the initial value of δ in MCMC sampling.

burnin is the number of burn-ins. The output will only show MCMC samples

after bunrin.

thin is the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling δ , and the deviance information criteria.

Value

A list of class "NPP" with five elements:

mu posterior of the model parameter μ . sigmasq posterior of the model parameter σ^2 . delta posterior of the power parameter δ .

acceptance the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

DIC the deviance information criteria for model diagnostics.

Author(s)

Zifei Han <hanzifei1@gmail.com>

PHData 41

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

Berger, J.O. and Bernardo, J.M. (1992). On the development of reference priors. *Bayesian Statistics* 4: *Proceedings of the Fourth Valencia International Meeting, Bernardo, J.M, Berger, J.O., Dawid, A.P. and Smith, A.F.M. eds.*, 35-60, Clarendon Press:Oxford.

Jeffreys, H. (1946). An Invariant Form for the Prior Probability in Estimation Problems. *Proceedings of the Royal Statistical Society of London, Series A 186:453-461*.

See Also

```
BerNPP_MCMC; MultinomialNPP_MCMC; PoissonNPP_MCMC;
```

Examples

PHData

PH Data on four sites in Virginia

Description

The dataset is used to assess if there is site impairment. The site impairment is defined as whether the pH values at a site indicate that the site violates a (lower) standard of 6.0 more than 10% of the time.

```
data("PHData")
```

42 PoiMNPP_MCMC1

Format

A data frame with 325 observations on the following 3 variables.

Station the site number, labeled as 1 to 4

Data. Time indicator of historical data (coded as 0) or current data (coded as 1)

PH value of PH on the site

Examples

data(PHData)

PoiMNPP_MCMC1 MCMC Sampling for Poisson Population using Normalized Power

Prior with Multiple Historical Data

Description

This function incorporates multiple sets of historical data for posterior sampling in a Poisson population using a normalized power prior. The power parameter δ uses a Metropolis-Hastings algorithm, which can be either an independence proposal or a random walk proposal on its logit scale. For the model parameter λ , Gibbs sampling is employed.

Usage

```
PoiMNPP_MCMC1(n0, n, prior_lambda, prop_delta, prior_delta_alpha, prior_delta_beta, rw_delta, delta_ini, nsample, burnin, thin)
```

Arguments

n0 A vector of natural numbers: number of successes in historical data.

n A natural number: number of successes in the current data.

prior_lambda A vector of hyperparameters for the prior distribution $Gamma(\alpha, \beta)$ of λ .

prop_delta The class of proposal distribution for δ .

prior_delta_alpha

A vector of hyperparameter α for the prior distribution $Beta(\alpha, \beta)$ for each δ .

prior_delta_beta

A vector of hyperparameter β for the prior distribution $Beta(\alpha, \beta)$ for each δ .

rw_delta The stepsize (variance of the normal distribution) for the random walk proposal

of logit δ . This is only applicable if prop_delta = 'RW'.

delta_ini The initial value for δ in MCMC sampling.

nsample Specifies the number of posterior samples in the output.

burnin The number of burn-ins. Only the MCMC samples after this burn-in will be

shown in the output.

thin The thinning parameter used in MCMC sampling.

PoiMNPP_MCMC1 43

Details

The function returns posteriors for both the model and power parameters, as well as the acceptance rate for sampling δ . The normalized power prior distribution is given by:

$$\frac{\pi_0(\delta)\pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{\delta_k}}{\int \pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{\delta_k} d\lambda}.$$

Here, $\pi_0(\delta)$ and $\pi_0(\lambda)$ are the initial prior distributions for δ and λ , respectively. $L(\lambda|D_{0k})$ is the likelihood function based on historical data D_{0k} , with δ_k being its corresponding power parameter.

Value

A list of class "NPP" comprising:

acceptrate The acceptance rate in MCMC sampling for δ using the Metropolis-Hastings

algorithm.

lambda Posterior samples of the model parameter λ .

delta Posterior samples of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y., and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K., and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

PoiMNPP_MCMC2, PoiOMNPP_MCMC1, PoiOMNPP_MCMC2

Examples

```
\label{eq:poiMNPP_MCMC1} PoiMNPP_MCMC1 (n0 = c(0, 3, 5), n = 3, prior_lambda = c(1, 1/10), prop_delta = "IND", prior_delta_alpha = c(1, 1, 1), prior_delta_beta = c(1, 1, 1), rw_delta = 0.1, delta_ini = NULL, nsample = 2000, burnin = 500, thin = 2)
```

44 PoiMNPP_MCMC2

PoiMNPP_MCMC2	MCMC Sampling for Poisson Population of multiple historical data using Normalized Power Prior
	using Normanized Tower Triol

Description

Multiple historical data are combined individually. Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter λ , Gibbs sampling is used.

Usage

```
PoiMNPP_MCMC2(n0,n,prior_lambda,prop_delta,prior_delta_alpha,
prior_delta_beta,rw_delta, delta_ini,nsample,burnin,thin)
```

Arguments

n0 a natural number vector: number of successes in historical data. a natural number: number of successes in the current data. prior_lambda a vector of the hyperparameters in the prior distribution $Gamma(\alpha, \beta)$ for λ . prop_delta the class of proposal distribution for δ . prior_delta_alpha a vector of the hyperparameter α in the prior distribution $Beta(\alpha, \beta)$ for each δ . prior_delta_beta a vector of the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for each δ . the stepsize(variance of the normal distribution) for the random walk proposal rw_delta of logit δ . Only applicable if prop_delta = 'RW'. delta_ini the initial value of δ in MCMC sampling. nsample specifies the number of posterior samples in the output. burnin the number of burn-ins. The output will only show MCMC samples after bunrin. thin the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling δ . The normalized power prior distribution is

$$\pi_0(\delta) \prod_{k=1}^K \frac{\pi_0(\lambda) L(\lambda|D_{0k})^{\delta_k}}{\int \pi_0(\lambda) L(\lambda|D_{0k})^{\delta_k} d\lambda}.$$

Here $\pi_0(\delta)$ and $\pi_0(\lambda)$ are the initial prior distributions of δ and λ , respectively. $L(\lambda|D_{0k})$ is the likelihood function of historical data D_{0k} , and δ_k is the corresponding power parameter.

PoiOMNPP_MCMC1 45

Value

A list of class "NPP" with three elements:

acceptrate the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

lambda posterior of the model parameter λ . delta posterior of the power parameter δ .

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
PoiMNPP_MCMC1; PoiOMNPP_MCMC1; PoiOMNPP_MCMC2
```

Examples

PoiOMNPP_MCMC1

MCMC Sampling for Poisson Population of multiple ordered historical data using Normalized Power Prior

Description

Multiple ordered historical data are incorporated together. Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter γ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter λ , Gibbs sampling is used.

```
PoiOMNPP_MCMC1(n0,n,prior_gamma,prior_lambda, gamma_ind_prop, gamma_ini,nsample,burnin,thin)
```

Arguments

n0 a natural number vector: number of successes in historical data.

n a natural number: number of successes in the current data.

prior_gamma a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2, ..., \alpha_K)$

for γ .

prior_lambda a vector of the hyperparameters in the prior distribution $Gamma(\alpha, \beta)$ for λ .

gamma_ind_prop a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1, \alpha_2, ..., \alpha_K)$

for γ .

gamma_ini the initial value of γ in MCMC sampling.

nsample specifies the number of posterior samples in the output.

burnin the number of burn-ins. The output will only show MCMC samples after bunrin.

thin the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling γ . The normalized power prior distribution is

$$\frac{\pi_0(\gamma)\pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\lambda)\prod_{k=1}^K L(\lambda|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\lambda}.$$

Here $\pi_0(\gamma)$ and $\pi_0(\lambda)$ are the initial prior distributions of γ and λ , respectively. $L(\lambda|D_{0k})$ is the likelihood function of historical data D_{0k} , and $\sum_{i=1}^k \gamma_i$ is the corresponding power parameter.

Value

A list of class "NPP" with three elements:

acceptrate — the acceptance rate in MCMC sampling for γ using Metropolis-Hastings algo-

rithm.

lambda posterior of the model parameter λ .

delta posterior of the power parameter δ . It is equal to the cumulative sum of γ

Author(s)

Qiang Zhang <zqzjf0408@163.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

PoiMNPP_MCMC1; PoiMNPP_MCMC2; PoiOMNPP_MCMC2

PoiOMNPP_MCMC2 47

Examples

```
PoiOMNPP_MCMC1(n0=c(0,3,5),n=3,prior_gamma=c(1/2,1/2,1/2), prior_lambda=c(1,1/10), gamma_ind_prop=rep(1,4),gamma_ini=NULL, nsample = 2000, burnin = 500, thin = 2)
```

PoiOMNPP_MCMC2 MCMC Sampling for Poisson Population of multiple ordered historical data using Normalized Power Prior

Description

Multiple ordered historical data are combined individually. Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter γ , a Metropolis-Hastings algorithm with independence proposal is used. For the model parameter λ , Gibbs sampling is used.

Usage

```
PoiOMNPP_MCMC2(n0,n,prior_gamma,prior_lambda, gamma_ind_prop,gamma_ini, nsample, burnin, thin)
```

Arguments

n0	a natural number vector: number of successes in historical data.	
n	a natural number: number of successes in the current data.	
prior_gamma	a vector of the hyperparameters in the prior distribution $Dirichlet(\alpha_1, \alpha_2,, \alpha_K)$ for γ .	
prior_lambda	a vector of the hyperparameters in the prior distribution $Gamma(\alpha,\beta)$ for λ .	
gamma_ind_prop	a vector of the hyperparameters in the proposal distribution $Dirichlet(\alpha_1,\alpha_2,,\alpha_K)$ for γ .	
gamma_ini	the initial value of γ in MCMC sampling.	
nsample	specifies the number of posterior samples in the output.	
burnin	the number of burn-ins. The output will only show MCMC samples after bunrin.	
thin	the thinning parameter in MCMC sampling.	

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling γ . The normalized power prior distribution is

$$\pi_0(\gamma) \prod_{k=1}^K \frac{\pi_0(\lambda) L(\lambda|D_{0k})^{(\sum_{i=1}^k \gamma_i)}}{\int \pi_0(\lambda) L(\lambda|D_{0k})^{(\sum_{i=1}^k \gamma_i)} d\lambda}.$$

Here $\pi_0(\gamma)$ and $\pi_0(\lambda)$ are the initial prior distributions of γ and λ , respectively. $L(\lambda|D_{0k})$ is the likelihood function of historical data D_{0k} , and $\sum_{i=1}^k \gamma_i$ is the corresponding power parameter.

48 PoissonNPP_MCMC

Value

A list of class "NPP" with three elements:

acceptrate the acceptance rate in MCMC sampling for γ using Metropolis-Hastings algo-

rithm.

lambda posterior of the model parameter λ .

delta posterior of the power parameter δ . It is equal to the cumulative sum of γ

Author(s)

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References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

```
PoiMNPP_MCMC1; PoiMNPP_MCMC2; PoiOMNPP_MCMC1
```

Examples

PoissonNPP_MCMC

MCMC Sampling for Bernoulli Population using Normalized Power Prior

Description

Conduct posterior sampling for Poisson population with normalized power prior. For the power parameter δ , a Metropolis-Hastings algorithm with either independence proposal, or a random walk proposal on its logit scale is used. For the model parameter λ , Gibbs sampling is used.

PoissonNPP_MCMC 49

Arguments

Data.Cur a non-negative integer vector of each observed current data.

Data.Hist a non-negative integer vector of each observed historical data.

CompStat a list of four elements that represents the "compatibility(sufficient) statistics" for

 λ . Default is NULL so the fitting will be based on the data. If the CompStat is

provided then the inputs in Data. Cur and Data. Hist will be ignored.

n0 is the number of observations in the historical data.
mean0 is the sample mean of the historical data.
n1 is the number of observations in the current data.
mean1 is the sample mean of the current data.

prior a list of the hyperparameters in the prior for both λ and δ . A Gamma distribution

is used as the prior of λ , and a Beta distribution is used as the prior of δ .

lambda. shape is the shape (hyper)parameter in the prior distribution Gamma(shape, scale)

for λ .

lambda.scale is the scale (hyper) parameter in the prior distribution Gamma(shape, scale)

for λ .

delta.alpha is the hyperparameter α in the prior distribution $Beta(\alpha,\beta)$ for

 δ .

delta. beta is the hyperparameter β in the prior distribution $Beta(\alpha, \beta)$ for δ .

MCMCmethod sampling method for δ in MCMC. It can be either 'IND' for independence pro-

posal; or 'RW' for random walk proposal on logit scale.

rw.logit.delta the stepsize(variance of the normal distribution) for the random walk proposal

of logit δ . Only applicable if MCMCmethod = 'RW'.

ind.delta.alpha

specifies the first parameter α when independent proposal $Beta(\alpha, \beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

ind.delta.beta specifies the first parameter β when independent proposal $Beta(\alpha, \beta)$ for δ is

used. Only applicable if MCMCmethod = 'IND'

nsample specifies the number of posterior samples in the output.

control.mcmc a list of three elements used in posterior sampling.

delta. ini is the initial value of δ in MCMC sampling.

burnin is the number of burn-ins. The output will only show MCMC samples

after bunrin.

thin is the thinning parameter in MCMC sampling.

Details

The outputs include posteriors of the model parameter(s) and power parameter, acceptance rate in sampling δ , and the deviance information criteria.

Value

A list of class "NPP" with four elements:

lambda posterior of the model parameter λ .

50 SPDData

delta posterior of the power parameter δ .

acceptance the acceptance rate in MCMC sampling for δ using Metropolis-Hastings algo-

rithm.

DIC the deviance information criteria for model diagnostics.

Author(s)

Zifei Han <hanzifei1@gmail.com>

References

Ibrahim, J.G., Chen, M.-H., Gwon, Y. and Chen, F. (2015). The Power Prior: Theory and Applications. *Statistics in Medicine* 34:3724-3749.

Duan, Y., Ye, K. and Smith, E.P. (2006). Evaluating Water Quality: Using Power Priors to Incorporate Historical Information. *Environmetrics* 17:95-106.

See Also

MultinomialNPP_MCMC; NormalNPP_MCMC; BerNPP_MCMC;

Examples

SPDData

Dataset for Diagnostic Test (PartoSure Test, Medical Device) Evaluation for Spontaneous Preterm Delivery

Description

The diagnostic test was developed to aid in rapidly assess the risk of spontaneouspreterm delivery within 7 days from the time of diagnosis in pre-pregnant women with signs and symptoms. The same diagnostic test was used for two populations in US and EU respectively. The number of counts in the four cells (True positive, false positive, false negative, true negative) was recorded.

```
data("SPDData")
```

VaccineData 51

Format

A data frame with 2 observations on the following 5 variables.

Data. Region region where the diagnostic test was conducted

TPDP number of subjects with tested positive and the disease status positive (true positive)

TPDN number of subjects with tested positive but the disease status negative (false positive)

TNDP number of subjects with tested negative and the disease status positive (false negative)

TNDN number of subjects with tested negative and the disease status negative (true negative)

Source

https://www.accessdata.fda.gov/cdrh_docs/pdf16/P160052C.pdf

Examples

data(SPDData)

VaccineData

Dataset of a Vaccine Trial for RotaTeq and Multiple Historical Trials for Control Group

Description

The study was designed to investigate the concomitant use of RotaTeq(Test Vaccine) and some routine pediatric vaccines between 2001-2005. The dataset includes four historical control trials. The purpose of the study is to borrow the historical controls for the non-inferiority trial. The interest is in the response rate to the routine vaccines.

Usage

```
data("VaccineData")
```

Format

A data frame with 6 observations on the following 7 variables.

Data. Time indicator of historical data (coded as 0) or current data (coded as 1).

StudyID character to distinguish different studies.

Group indicator of control group (coded as 0) or treatment group (coded as 1).

Start. Year start year of the trial

End. Year end year of the trial

N total number of patients enrolled and dosed in the group

y total number of patients respond to the vaccine

52 VaccineData

References

Liu, G.F. (2018). A Dynamic Power Prior for Borrowing Historical Data in Noninferiority Trials with Binary Endpoint. *Pharmaceutical Statistics* 17:61-73.

Examples

data(VaccineData)

Index

* Bernoulli	BerMNPP_MCMC2, 4, 4, 9, 11		
VaccineData, 51	BerNPP_MCMC, 6, 19, 38, 41, 50		
* multinomial	BerOMNPP_MCMC1, 4, 6, 8, 11		
SPDData, 50	BerOMNPP_MCMC2, $4, 6, 9, 10$		
* multiple historical data normalized power			
prior	LaplacelogC, 11		
BerMNPP_MCMC1, 2	LMMNPP_MCMC1, 12, 16, 21, 24		
BerMNPP_MCMC2, 4	LMMNPP_MCMC2, <i>14</i> , 15, <i>21</i> , <i>24</i>		
BerOMNPP_MCMC1,8	LMNPP_MCMC, 17		
BerOMNPP_MCMC2, 10	LMOMNPP_MCMC1, 14, 16, 20, 24		
LMMNPP_MCMC1, 12	LMOMNPP_MCMC2, 14, 16, 21, 22		
LMMNPP_MCMC2, 15	logCdelta, 24, 26–28		
LMOMNPP_MCMC1, 20	logCknot, 12, 25, 25, 27, 28		
LMOMNPP_MCMC2, 22	loglikBerD0, 25, 26, 26, 28		
PoiMNPP_MCMC2, 44	loglikNormD0, <i>25–27</i> , 27		
PoiOMNPP_MCMC1,45	W D U D NDD 20 21 22 24 26		
PoiOMNPP_MCMC2, 47	ModeDeltaBerNPP, 28, 31, 33, 34, 36		
* multiple historical data	ModeDeltaLMNPP, 30		
PoiMNPP_MCMC1, 42	ModeDeltaMultinomialNPP, 29, 31, 31, 34, 36		
* normalized power prior	ModeDeltaNormalNPP, 29, 31, 33, 33, 36		
BerNPP_MCMC, 6	ModeDeltaPoisNPP, 29, 33, 34, 35		
LaplacelogC, 11	MultinomialNPP_MCMC, 8, 19, 37, 41, 50		
LMNPP_MCMC, 17	NormalNPP_MCMC, 8, 19, 38, 39, 50		
logCdelta, 24	NOT IIIaINI 1 _PICPIC, 0, 19, 30, 39, 30		
logCknot, 25	PHData, 41		
loglikBerD0,26	PoiMNPP_MCMC1, 42, 45, 46, 48		
loglikNormD0,27	PoiMNPP_MCMC2, 43, 44, 46, 48		
ModeDeltaBerNPP, 28	PoiOMNPP_MCMC1, 43, 45, 45, 48		
ModeDeltaLMNPP, 30	PoiOMNPP_MCMC2, 43, 45, 46, 47		
ModeDeltaMultinomialNPP, 31	PoissonNPP_MCMC, 8, 19, 38, 41, 48		
ModeDeltaNormalNPP, 33			
ModeDeltaPoisNPP, 35	SPDData, 50		
MultinomialNPP_MCMC, 37			
NormalNPP_MCMC, 39	VaccineData, 51		
PoiMNPP_MCMC1, 42			
PoissonNPP_MCMC, 48			
* normal			
PHData, 41			
BerMNPP_MCMC1, $2, 6, 9, 11$			