

# Package ‘HCPclust’

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**Title** Hierarchical Conformal Prediction for Clustered Data with Missing Responses

**Version** 0.1.1

**Description** Implements hierarchical conformal prediction for clustered data with missing responses. The method uses repeated cluster-level splitting and within-cluster subsampling to accommodate dependence, and inverse-probability weighting to correct distribution shift induced by missingness. Conditional densities are estimated by inverting fitted conditional quantiles (linear quantile regression or quantile regression forests), and p-values are aggregated across resampling and splitting steps using the Cauchy combination test.

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**URL** <https://github.com/judywangstat/HCP>

**BugReports** <https://github.com/judywangstat/HCP/issues>

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### fit\_cond\_density\_quantile

*Estimate conditional density  $\pi(y|x)$  via quantile process + quotient estimator*

### Description

Fits a conditional quantile function  $\hat{Q}_Y(\tau | x)$  using pooled observed data (working-independence), and estimates the conditional density through the quotient estimator along the quantile curve:

$$\hat{\pi}\{\hat{Q}(\tau | x) | x\} = \frac{2h(\tau)}{\hat{Q}(\tau + h(\tau) | x) - \hat{Q}(\tau - h(\tau) | x)}.$$

For numerical stability, the quantile curve can be monotone-adjusted (isotonic regression), and tail decay extrapolation can be used before interpolation to  $\pi(y | x)$ .

### Usage

```
fit_cond_density_quantile(
  dat,
  y_col = "Y",
  delta_col = "delta",
  x_cols,
  taus = seq(0.05, 0.95, by = 0.01),
  h = NULL,
  method = c("rq", "qrf"),
  enforce_monotone = TRUE,
  tail_decay = TRUE,
  num_extra_points = 10L,
  decay_factor = 0.8,
  dens_floor = 1e-10,
  eps = 1e-08,
  gap_min = 0.01,
  seed = NULL,
  ...
)
```

### Arguments

dat	data.frame in long format, containing outcome, missingness indicator, and covariates.
-----	---

y_col	name of outcome column (observed Y, may contain NA).
delta_col	name of missingness indicator (1 observed, 0 missing).
x_cols	character vector of covariate column names (include time if desired).
taus	grid of quantile levels in (0,1) at which the quantile process is evaluated.
h	Bandwidth(s) for quotient. Either a scalar or a numeric vector of length length(taus). If NULL, a tau-specific bandwidth vector $h(\tau)$ is computed via <code>quantreg::bandwidth.rq</code> , and automatically shrunk near the boundaries to ensure $\tau \pm h(\tau) \in (0, 1)$ .
method	quantile engine: "rq" (linear quantile regression) or "qrf" (quantile random forest).
enforce_monotone	logical; if TRUE, apply isotonic regression to the predicted quantile curve in $\tau$ for each $x$ to reduce quantile crossing.
tail_decay	logical; if TRUE, add extra tail points with geometric decay before interpolation.
num_extra_points	number of extra tail points on each side when tail_decay=TRUE.
decay_factor	decay factor in (0,1) for tail densities when tail_decay=TRUE.
dens_floor	lower bound for density to avoid numerical issues.
eps	small stabilizer for denominator <code>pmax(Qplus-Qminus, eps)</code> .
gap_min	minimum spacing for tail extrapolation points.
seed	optional seed.
...	extra arguments passed to the underlying quantile engine: rq passed to <code>quantreg::rq.fit</code> , e.g. <code>rq_method="br"</code> . qrf passed to <code>quantregForest::quantregForest</code> , e.g. <code>ntree=500</code> .

**Value**

A list containing fitted objects and prediction functions:

`predict_Q(x_new, taus_use)` Returns the estimated conditional quantiles

$$\hat{Q}_Y(\tau | x)$$

for  $\tau \in (0, 1)$  specified by `taus_use`, evaluated at new covariate values `x_new`. The output is a numeric matrix with one row per covariate vector  $x$  and one column per quantile level  $\tau$ .

`predict_density(x_new, y_new)` Returns the estimated conditional density

$$\hat{\pi}(y | x),$$

evaluated at specified (x,y) pairs. The inputs `x_new` and `y_new` are paired row-wise, so that the r-th row of `x_new` is evaluated at `y_new[r]`.

## Examples

```

## -----
## Case A: Conditional density evaluated at a single point (x, y)
## -----
## This illustrates the most basic usage: estimating pi(y | x)
## at one covariate value x and one response value y.

dat <- generate_clustered_mar(
  n = 200, m = 4, d = 2,
  target_missing = 0.3, seed = 1
)
fit <- fit_cond_density_quantile(
  dat,
  y_col = "Y", delta_col = "delta",
  x_cols = c("X1", "X2"),
  taus = seq(0.05, 0.95, by = 0.02),
  method = "rq",
  seed = 1
)
## a single covariate value x
x1 <- matrix(c(0.2, -1.0), nrow = 1)
colnames(x1) <- c("X1", "X2")
## estimate pi(y | x) at y = 0.5
fit$predict_density(x1, y_new = 0.5)

## -----
## Case B: Conditional density as a function of y (density curve)
## -----
## Here we fix x and evaluate pi(y | x) over a grid of y values,
## which produces an estimated conditional density curve.

y_grid <- seq(-3, 3, length.out = 201)
## reuse the same x by repeating it to match the y-grid
x_rep <- x1[rep(1, length(y_grid)), , drop = FALSE]
f_grid <- fit$predict_density(x_rep, y_grid)

## -----
## True conditional density under the data generator
## -----
## Data are generated as:
##   Y = X^T beta + b + eps,
##   b ~ N(0, sigma_b^2),   eps ~ N(0, sigma_eps^2)
## Hence the marginal conditional density is:
##   Y | X = x ~ N(x^T beta, sigma_b^2 + sigma_eps^2)

beta_true <- c(0.5, 0.6)
sigma_b_true <- 0.7
sigma_eps_true <- 1.0
mu_true <- drop(x1 %*% beta_true)
sd_true <- sqrt(sigma_b_true^2 + sigma_eps_true^2)
f_true <- stats::dnorm(y_grid, mean = mu_true, sd = sd_true)

```

```

## -----
## Visualization: estimated vs true conditional density
## (use smooth.spline on log-density for a smoother display)
## -----


## smooth the estimated curve for visualization
ok <- is.finite(f_grid) & (f_grid > 0)
sp <- stats::smooth.spline(y_grid[ok], log(f_grid[ok]), spar = 0.85)
f_smooth <- exp(stats::predict(sp, y_grid)$y)

ymax <- max(c(f_smooth, f_true), na.rm = TRUE)
plot(
  y_grid, f_smooth,
  type = "l", lwd = 2,
  xlab = "y",
  ylab = expression(hat(pi)(y ~ " | " ~ x)),
  ylim = c(0, 1.2 * ymax),
  main = "Conditional density at a fixed x: estimated vs true"
)
grid(col = "gray85", lty = 1)
lines(y_grid, f_true, lwd = 2, lty = 2)
legend(
  "topright",
  legend = c("Estimated (smoothed)", "True (generator)"),
  lty = c(1, 2), lwd = c(2, 2), bty = "n"
)

```

**fit\_missingness\_propensity***Fit missingness propensity model  $P(\delta=1 | X)$  from pooled data***Description**

Fits the missingness propensity  $\pi(x) = \mathbb{P}(\delta = 1 | x)$  under a marginal missingness model using pooled observations. Estimation can be carried out using logistic regression, Generalized Random Forests (GRF), or gradient boosting (xgboost). Both continuous and discrete covariates are supported; categorical variables are automatically expanded into dummy variables via `model.matrix()`.

**Usage**

```
fit_missingness_propensity(
  dat,
  delta_col = "delta",
  x_cols,
  method = c("logistic", "grf", "boosting"),
```

```
  eps = 1e-06,
  ...
)
```

## Arguments

dat	A <code>data.frame</code> containing <code>delta_col</code> and <code>x_cols</code> . Can be any user-supplied dataset; <code>generate_clustered_mar()</code> is used only in examples.
delta_col	Name of missingness indicator column (1 observed, 0 missing).
x_cols	Character vector of covariate column names used to predict missingness.
method	One of "logistic", "grf", "boosting".
eps	Clipping level applied to the estimated missingness propensity $\hat{\pi}(x)$ , truncating predictions to $[\epsilon, 1 - \epsilon]$ .
...	Extra arguments passed to the learner: <code>logistic</code> passed to <code>stats::glm</code> . <code>grf</code> passed to <code>grf::probability_forest</code> . <code>boosting</code> passed to <code>xgboost::xgb.train</code> via <code>params=</code> and <code>nrounds=</code> .

## Value

A list containing:

- `method` The estimation method used.
- `fit` The fitted missingness propensity model.
- `predict` A function `predict(x_new)` that returns the estimated missingness propensity  $\hat{\pi}(x) = \mathbb{P}(\delta = 1 | x)$  evaluated at new covariate values `x_new`, with predictions clipped to  $[\epsilon, 1 - \epsilon]$ .

## Examples

```
dat <- generate_clustered_mar(
  n = 80, m = 4, d = 2,
  alpha0 = -0.4, alpha = c(-1.0, 0.8),
  target_missing = 0.30,
  seed = 1
)
x_cols <- c("X1", "X2")

## Logistic regression
fit_log <- fit_missingness_propensity(dat, "delta", x_cols, method = "logistic")
p_log <- fit_log$predict(dat[, x_cols, drop = FALSE])
head(p_log)

## Compare with other methods
## True propensity under the generator
s <- attr(dat, "alpha_shift")
eta <- (-0.4) + (-1.0) * dat$X1 + 0.8 * dat$X2
pi_true <- 1 / (1 + exp(-pmin(pmax(eta, -30), 30)))
```

```

fit_grf <- fit_missingness_propensity(
  dat, "delta", x_cols,
  method = "grf", num.trees = 800, num.threads = 1
)
fit_xgb <- fit_missingness_propensity(
  dat, "delta", x_cols,
  method = "boosting",
  nrounds = 300,
  params = list(max_depth = 3, eta = 0.05, subsample = 0.8, colsample_bytree = 0.8),
  nthread = 1
)

p_grf <- fit_grf$predict(dat[, x_cols, drop = FALSE])
p_xgb <- fit_xgb$predict(dat[, x_cols, drop = FALSE])

op <- par(mfrow = c(1, 3))
plot(pi_true, p_log, pch = 16, cex = 0.5,
     xlab = "True pi(x)", ylab = "Estimated pi-hat(x)", main = "Logistic"); abline(0, 1, lwd = 2)
plot(pi_true, p_grf, pch = 16, cex = 0.5,
     xlab = "True pi(x)", ylab = "Estimated pi-hat(x)", main = "GRF"); abline(0, 1, lwd = 2)
plot(pi_true, p_xgb, pch = 16, cex = 0.5,
     xlab = "True pi(x)", ylab = "Estimated pi-hat(x)", main = "Boosting"); abline(0, 1, lwd = 2)
par(op)

```

## generate\_clustered\_mar

*Simulate clustered continuous outcomes with covariate-dependent MAR missingness*

**Description**

Simulates clustered data  $\{(X_{i,j}, Y_{i,j}, \delta_{i,j})\}$  under a hierarchical subject-level model with covariate-dependent Missing at Random (MAR) missingness:  $\delta \perp Y | X$ . Covariates  $X_{i,j}$  are fully observed, while outcomes  $Y_{i,j}$  may be missing.

Data are generated according to the following mechanisms:

- **Between-subject level:** subject random intercepts  $b_i \sim N(0, \sigma_b^2)$  induce within-cluster dependence, corresponding to latent subject-specific laws  $P_i$ .
- **Outcomes:** for each measurement  $j = 1, \dots, m_i$ ,

$$Y_{i,j} = X_{i,j}^\top \beta + b_i + \varepsilon_{i,j},$$

where, for each subject  $i$ , the within-cluster errors  $\{\varepsilon_{i,j}\}_{j=1}^{m_i}$  are mutually independent with  $\varepsilon_{i,j} \sim N(0, \sigma_\varepsilon^2)$  when  $\text{rho} = 0$ . When  $\text{rho} \neq 0$ , they follow a stationary first-order autoregressive process (AR(1)) within the cluster:

$$\varepsilon_{i,j} = \rho \varepsilon_{i,j-1} + \eta_{i,j}, \quad \eta_{i,j} \sim N(0, \sigma_\varepsilon^2(1 - \rho^2)),$$

which implies  $\text{Var}(\varepsilon_{i,j}) = \sigma_\varepsilon^2$  and  $\text{Cov}(\varepsilon_{i,j}, \varepsilon_{i,j+k}) = \sigma_\varepsilon^2 \rho^{|k|}$  for all  $k$ .

- **MAR missingness:** outcomes are observed with probability

$$\Pr(\delta_{i,j} = 1 \mid X_{i,j}) = \text{logit}^{-1}(\alpha_0 + \alpha^\top X_{i,j}),$$

which depends only on covariates, ensuring  $\delta \perp Y \mid X$ . If `target_missing` is provided, the intercept  $\alpha_0$  is automatically calibrated (via a deterministic root-finding procedure on the expected missing proportion) so that the *marginal missing proportion* is close to `target_missing`.

## Usage

```
generate_clustered_mar(
  n,
  m = 4L,
  d = 2L,
  beta = NULL,
  sigma_b = 0.7,
  sigma_eps = 1,
  rho = 0,
  hetero_gamma = 0,
  x_dist = c("normal", "bernoulli", "uniform"),
  x_params = NULL,
  alpha0 = -0.2,
  alpha = NULL,
  target_missing = NULL,
  seed = NULL
)
```

## Arguments

<code>n</code>	Number of clusters (subjects).
<code>m</code>	Cluster size. Either a single positive integer (common $m_i = m$ ) or an integer vector of length <code>n</code> specifying $m_i$ for each subject.
<code>d</code>	Covariate dimension.
<code>beta</code>	Population regression coefficients for $Y \mid X$ (length <code>d</code> ). If <code>NULL</code> , defaults to <code>seq(0.5, 0.5 + 0.1*(d-1), by=0.1)</code> .
<code>sigma_b</code>	SD of subject random intercept $b_i$ .
<code>sigma_eps</code>	Marginal SD of within-subject errors $\varepsilon_{i,j}$ .
<code>rho</code>	AR(1) correlation parameter within cluster for $\varepsilon_{i,j}$ .
<code>hetero_gamma</code>	Optional heteroskedasticity parameter; a value of 0 yields the standard homoskedastic model, while nonzero values induce covariate-dependent error variance through the first covariate $X_1$ .
<code>x_dist</code>	Distribution for covariates: "normal", "bernoulli", or "uniform".
<code>x_params</code>	Optional list of distribution parameters for <code>x_dist</code> .
<code>alpha0</code>	Missingness intercept $\alpha_0$ . If <code>target_missing</code> is not <code>NULL</code> , the effective intercept becomes $\alpha_0 + s$ , where $s$ is a calibrated shift.
<code>alpha</code>	Missingness slopes (length <code>d</code> ). If <code>NULL</code> , defaults to zeros.

<b>target_missing</b>	Target <i>marginal missing proportion</i> defined as the empirical average of the fitted missing probabilities $1 - \pi(X_{i,j})$ over all observations, where $\pi(x) = \Pr(\delta = 1   X = x)$ . If NULL, no calibration.
<b>seed</b>	Optional RNG seed.

**Value**

A `data.frame` in long format with one row per measurement:

- id** Cluster index.
- j** Within-cluster index.
- Y** Observed outcome; NA if missing.
- Y\_full** Latent complete outcome.
- delta** Observation indicator (1 observed, 0 missing).
- X1..Xd** Covariates.

Attributes:

- m\_i** Integer vector of cluster sizes  $(m_1, \dots, m_n)$ .
- target\_missing** Target marginal missing proportion used for calibration, defined as the empirical average of missing probabilities over all observations.
- alpha\_shift** Calibrated global intercept shift  $s$  added to the missingness linear predictor  $\alpha_0 + s + \alpha^\top X_{i,j}$  (present only when `target_missing` is provided).
- missing\_rate** Sample missing rate  $N^{-1} \sum I(\delta_{i,j} = 0)$ . This may deviate from `target_missing` due to Bernoulli sampling variability.

**Examples**

```
dat <- generate_clustered_mar(
  n = 200, m = 5, d = 2,
  alpha0 = -0.2, alpha = c(-1.0, 0.0),
  target_missing = 0.30,
  seed = 1
)
mean(dat$delta == 0)      # ~0.30
attr(dat, "alpha_shift") # calibrated shift
```

## Description

Constructs a marginal conformal prediction region for a new covariate value  $x_{n+1}$  under clustered data with missing outcomes, following the HCP framework:

- **(1) Model fitting.** Fit a pooled conditional density model  $\hat{\pi}(y | x)$  using `fit_cond_density_quantile`, together with a marginal missingness propensity model  $\hat{p}(x) = \mathbb{P}(\delta = 1 | x)$  using `fit_missingness_propensity`, both estimated on a subject-level training split.
- **(2) Subsampled calibration.** Repeatedly construct calibration sets by randomly drawing one observation per subject from the calibration split.
- **(3) Weighted conformal scoring.** Compute weighted conformal  $p$ -values over a candidate grid using the nonconformity score  $R(x, y) = -\hat{\pi}(y | x)$  and inverse-propensity weights  $w(x) = 1/\hat{p}(x)$  under a MAR assumption.
- **(4) Aggregation.** Aggregate dependent  $p$ -values across subsamples (B) and data splits (S) using either the Cauchy combination test (CCT/ACAT) or the arithmetic mean.

The prediction region is returned as a subset of the supplied grid:

$$\hat{C}(x_{n+1}; \alpha) = \{y \in \mathcal{Y} : p_{\text{final}}(y) > \alpha\}.$$

## Usage

```
hcp_conformal_region(
  dat,
  id_col,
  y_col = "Y",
  delta_col = "delta",
  x_cols,
  x_test,
  y_grid,
  alpha = 0.1,
  train_frac = 0.5,
  S = 5,
  B = 5,
  combine_B = c("cct", "mean"),
  combine_S = c("cct", "mean"),
  seed = NULL,
  return_details = FALSE,
  dens_method = c("rq", "qrf"),
  dens_taus = seq(0.05, 0.95, by = 0.02),
  dens_h = NULL,
  enforce_monotone = TRUE,
  tail_decay = TRUE,
  prop_method = c("logistic", "grf", "boosting"),
  prop_eps = 1e-06,
  ...
)
```

## Arguments

dat	A data.frame containing clustered observations. Must include id_col, y_col, delta_col, and all columns in x_cols.
id_col	Subject/cluster identifier column name.
y_col	Outcome column name.
delta_col	Missingness indicator column name (1 observed, 0 missing).
x_cols	Covariate column names used for both density estimation and missingness propensity.
x_test	New covariate value(s). A numeric vector (treated as one row), or a numeric matrix/data.frame with nrow(x_test)=K test points and ncol(x_test)=length(x_cols) covariates.
y_grid	Numeric vector of candidate $y$ values at which to evaluate conformal $p$ -values.
alpha	Miscoverage level in (0,1). Region keeps $y$ with $p(y) > \alpha$ .
train_frac	Fraction of subjects assigned to training in each split.
S	Number of independent subject-level splits.
B	Number of subsamples per split (one observation per subject per subsample).
combine_B	Combine $p$ -values across B subsamples: "cct" (default) or "mean".
combine_S	Combine $p$ -values across S splits: "cct" (default) or "mean".
seed	Optional seed for reproducibility.
return_details	Logical; if TRUE, also return split-level p-values and split metadata.
dens_method	Density/quantile engine for <code>fit_cond_density_quantile</code> : "rq" or "qrf".
dens_taus	Quantile grid passed to <code>fit_cond_density_quantile</code> .
dens_h	Bandwidth(s) passed to <code>fit_cond_density_quantile</code> .
enforce_monotone	Passed to <code>fit_cond_density_quantile</code> .
tail_decay	Passed to <code>fit_cond_density_quantile</code> .
prop_method	Missingness propensity method for <code>fit_missingness_propensity</code> : "logistic", "grf", or "boosting".
prop_eps	Clipping level for propensity predictions used by <code>fit_missingness_propensity</code> .
...	Extra arguments passed to <code>fit_missingness_propensity</code> .

## Value

If `return_details`=FALSE (default), a list with:

```
region Length-K list; region[[k]] is the subset of y_grid with p_final[k, ] > alpha.
lo_hi K x 2 matrix with columns c("lo", "hi") giving min/max of region[[k]] (NA if empty).
p_final K x length(y_grid) matrix of final p-values on y_grid.
y_grid The candidate grid used.
```

If `return_details`=TRUE, also includes:

```
p_split An array with dimensions c(S, K, length(y_grid)) of split-level p-values.
split_meta Train subject IDs for each split.
```

## Examples

```
dat <- generate_clustered_mar(n = 200, m = 4, d = 2, target_missing = 0.30, seed = 1)
y_grid <- seq(-4, 4, length.out = 200)
x_test <- matrix(c(0.2, -1.0), nrow = 1); colnames(x_test) <- c("X1", "X2")

res <- hcp_conformal_region(
  dat, id_col = "id",
  y_col = "Y", delta_col = "delta",
  x_cols = c("X1", "X2"),
  x_test = x_test,
  y_grid = y_grid,
  alpha = 0.1,
  S = 2, B = 2,
  seed = 1
)

## interval endpoints on the y-grid (outer envelope)
c(lo = min(res$region[[1]]), hi = max(res$region[[1]]))
```

**hcp\_predict\_targets**    *HCP prediction wrapper for multiple measurements with optional per-patient Bonferroni*

## Description

Wraps [hcp\\_conformal\\_region](#) to produce conformal prediction regions for a collection of measurements, possibly including multiple measurements per individual.

Based on the structure of the test dataset, the prediction mode is determined automatically as follows, where  $P$  denotes the number of patients (clusters) and  $M$  denotes the number of measurements per patient:

- $P = 1, M = 1$ : Predict a single patient with a single measurement.
- $P = 1, M > 1$ : Predict a single patient with multiple measurements (e.g., repeated or longitudinal measurements for the same patient). If per-patient simultaneous prediction is desired, optional per-patient Bonferroni calibration can be applied.
- $P > 1, M = 1$ : Predict multiple patients, each with a single measurement. Predictions are performed independently at the nominal level  $\alpha$ , without Bonferroni calibration.
- $P > 1, M > 1$ : Predict multiple patients, each with multiple measurements. When per-patient simultaneous coverage is desired, a Bonferroni correction can be applied by using an effective level  $\alpha/M_p$  for each measurement, yielding Bonferroni-adjusted marginal prediction regions for patient  $p$ .

**Usage**

```
hcp_predict_targets(
  dat,
  test,
  pid_col = "pid",
  x_cols,
  y_grid,
  alpha = 0.1,
  bonferroni = FALSE,
  return_region = FALSE,
  id_col = "id",
  y_col = "Y",
  delta_col = "delta",
  ...
)
```

**Arguments**

<code>dat</code>	Training/calibration data passed to <a href="#">hcp_conformal_region</a> .
<code>test</code>	A data.frame of test measurements, where each row corresponds to a single measurement. The test data must follow one of the four clustered settings $P = 1, M = 1, P = 1, M > 1, P > 1, M = 1$ , or $P > 1, M > 1$ , where $P$ is the number of patients (clusters) and $M$ is the number of measurements per patient. The data.frame must include a patient identifier specified by <code>pid_col</code> and all covariate columns listed in <code>x_cols</code> . Repeated values of <code>pid_col</code> indicate multiple measurements (e.g., repeated or longitudinal measurements) for the same patient.
<code>pid_col</code>	Column in <code>test</code> giving the patient (cluster/subject) identifier. Default "pid".
<code>x_cols</code>	Covariate column names (e.g., <code>c("X1")</code> ).
<code>y_grid</code>	Candidate y-grid passed to <a href="#">hcp_conformal_region</a> .
<code>alpha</code>	Nominal miscoverage level in (0,1) passed to <a href="#">hcp_conformal_region</a> .
<code>bonferroni</code>	Logical; if TRUE, apply per-patient Bonferroni only when a patient has multiple test measurements (i.e., $M_p > 1$ ). If FALSE, always use level $\alpha$ .
<code>return_region</code>	Logical; if TRUE, return the full region (subset of <code>y_grid</code> ) for each row.
<code>id_col, y_col, delta_col</code>	Column names in <code>dat</code> for patient ID, outcome, and missingness indicator.
<code>...</code>	Additional arguments forwarded to <a href="#">hcp_conformal_region</a> (e.g., <code>S, B, combine_B, combine_S, dens_method, prop_method, seed</code> ).

**Value**

A list with:

**pred** A data.frame in the same row order as `test`. It contains all columns of `test` plus the effective level `alpha_eff` and the prediction-band endpoints `lo` and `hi` for each measurement.

**region** If `return_region=TRUE`, a list of length `nrow(test)` where each element is the subset of `y_grid` retained in the prediction region for the corresponding test row; otherwise `NULL`.

**meta** A list with summary information, including the number of patients `P`, the per-patient measurement counts `M_by_pid`, and the settings `alpha` and `bonferroni`.

### Note

When per-patient Bonferroni calibration is enabled and a patient has a large number of measurements (e.g.,  $M_p > 10$ ), the effective level  $\alpha/M_p$  may be very small, which can lead to extremely wide prediction regions (potentially spanning the entire `y_grid`). This behavior is an inherent consequence of Bonferroni adjustment and not a numerical issue.

In longitudinal or panel studies, a cluster corresponds to a single individual (subject), and within-cluster points correspond to multiple time points or repeated measurements on the same individual. In this setting, the time variable `time` can be treated as a generic covariate. In the examples below, time is represented by `X1`.

### Examples

```
## -----
## Examples illustrating the four test-data settings:
## (P=1, M=1), (P=1, M>1), (P>1, M=1), and (P>1, M>1)
## -----
set.seed(1)

## training data (fixed across all cases)
dat_train <- generate_clustered_mar(
  n = 200, m = 4, d = 1,
  x_dist = "uniform", x_params = list(min = 0, max = 10),
  target_missing = 0.30,
  seed = 1
)

y_grid <- seq(-6, 6, length.out = 201)

## Case 1: P=1, M=1 (one patient, one measurement)
test_11 <- data.frame(
  pid = 1,
  X1 = 2.5
)
out_11 <- hcp_predict_targets(
  dat = dat_train,
  test = test_11,
  x_cols = "X1",
  y_grid = y_grid,
  alpha = 0.1,
  S = 2, B = 2,
  seed = 1
)
out_11$pred

## Case 2: P=1, M>1 (one patient, multiple measurements)
```

```
test_1M <- data.frame(
  pid = 1,
  X1  = c(1, 3, 7, 9)
)
out_1M <- hcp_predict_targets(
  dat = dat_train,
  test = test_1M,
  x_cols = "X1",
  y_grid = y_grid,
  alpha = 0.1,
  S = 2, B = 2,
  seed = 1
)
out_1M$pred

## Case 3: P>1, M=1 (multiple patients, one measurement each)
test_P1 <- data.frame(
  pid = 1:4,
  X1  = c(2, 4, 6, 8)
)
out_P1 <- hcp_predict_targets(
  dat = dat_train,
  test = test_P1,
  x_cols = "X1",
  y_grid = y_grid,
  alpha = 0.1,
  S = 2, B = 2,
  seed = 1
)
out_P1$pred

## Case 4: P>1, M>1 (multiple patients, multiple measurements per patient)
test_PM <- data.frame(
  pid = c(1,1, 2,2,2, 3,3),
  X1  = c(1,6, 2,5,9, 3,8)
)
out_PM <- hcp_predict_targets(
  dat = dat_train,
  test = test_PM,
  x_cols = "X1",
  y_grid = y_grid,
  alpha = 0.1,
  S = 2, B = 2,
  seed = 1
)
out_PM$pred
```

## Description

Unified plotting function for two common visualizations of HCP prediction intervals:

- mode="band": plot an interval band (lo/hi) versus a 1D covariate (e.g., time X1).
- mode="pid": plot one interval per patient on the x-axis (patients optionally sorted by a covariate).

## Usage

```
plot_hcp_intervals(
  df,
  mode = c("band", "pid"),
  lo_col = "lo",
  hi_col = "hi",
  y_true_col = NULL,
  y_true = NULL,
  show_center = TRUE,
  show_true = TRUE,
  x_col = NULL,
  pid_col = "pid",
  x_sort_col = NULL,
  max_patients = NULL,
  ...
)
```

## Arguments

<code>df</code>	A data.frame containing prediction results. It must include the interval endpoints specified by <code>lo_col</code> and <code>hi_col</code> , and the covariate columns required by the chosen plotting mode.
<code>mode</code>	Plotting mode. Use "band" to visualize an interval band as a function of a continuous covariate, or "pid" to visualize one prediction interval per patient on the x-axis.
<code>lo_col</code>	Name of the column containing the lower endpoint of the prediction interval. Default is "lo".
<code>hi_col</code>	Name of the column containing the upper endpoint of the prediction interval. Default is "hi".
<code>y_true_col</code>	Optional name of a column in <code>df</code> containing the true outcome values. Used for overlaying truth points when <code>show_true = TRUE</code> .
<code>y_true</code>	Optional numeric vector of true outcome values with length equal to <code>nrow(df)</code> . If provided, this overrides <code>y_true_col</code> .
<code>show_center</code>	Logical; if <code>TRUE</code> , draw the midpoint of each interval (as a dashed line in <code>mode = "band"</code> or as points in <code>mode = "pid"</code> ).
<code>show_true</code>	Logical; if <code>TRUE</code> , overlay true outcome values when available.
<code>x_col</code>	( <code>mode = "band"</code> ) Name of the covariate column used as the x-axis in the interval band plot (e.g., time or another continuous predictor).

pid_col	(mode = "pid") Name of the column identifying patients (or clusters). Each patient must appear exactly once in df. Default is "pid".
x_sort_col	(mode = "pid") Optional covariate column used to order patients along the x-axis (e.g., "X1"). If NULL, patients are ordered by their IDs.
max_patients	(mode = "pid") Optional maximum number of patients to display. If specified, only the first max_patients patients after sorting are plotted.
...	Additional graphical parameters passed to <code>plot</code> , such as <code>main</code> , <code>xlab</code> , <code>ylab</code> , <code>xlim</code> , or <code>ylim</code> .

## Value

Invisibly returns the data.frame used for plotting:

- For mode = "band", the input df sorted by x\_col.
- For mode = "pid", the input df sorted by pid\_col or x\_sort\_col, if provided.

## Examples

```
## -----
## Two common plots:
## (A) one patient, multiple measurements -> interval band vs X1
## (B) multiple patients, one measurement -> intervals by patient (sorted by X1)
## -----
dat_train <- generate_clustered_mar(
  n = 200, m = 20, d = 1,
  x_dist = "uniform", x_params = list(min = 0, max = 10),
  hetero_gamma = 2.5,
  target_missing = 0.30,
  seed = 1
)
y_grid <- seq(-6, 10, length.out = 201)

## test data with latent truth
dat_test <- generate_clustered_mar(
  n = 100, m = 20, d = 1,
  x_dist = "uniform", x_params = list(min = 0, max = 10),
  hetero_gamma = 2.5,
  seed = 999
)

## ----- Case A: P=1, M>1 (one patient, multiple measurements) -----
pid <- dat_test$id[1]
idx <- which(dat_test$id == pid)
idx <- idx[order(dat_test$X1[idx])][1:10]
test_1M <- data.frame(pid = pid, X1 = dat_test$X1[idx], y_true = dat_test$Y_full[idx])

out_1M <- hcp_predict_targets(
  dat = dat_train, test = test_1M,
  x_cols = "X1", y_grid = y_grid,
  alpha = 0.1,
  S = 2, B = 2,
```

```
seed = 1
)
plot_hcp_intervals(
  out_1M$pred, mode = "band", x_col = "X1",
  y_true_col = "y_true", show_true = TRUE,
  main = "Case A: one patient, multiple time points (band vs time)"
)

## ----- Case B: P>1, M=1 (multiple patients, one measurement each) -----
## take one measurement per patient: j==1 for the first 20 patients
pids <- unique(dat_test$id)[1:20]
test_P1 <- subset(dat_test, id %in% pids & j == 1,
                   select = c(id, X1, Y_full))
names(test_P1) <- c("pid", "X1", "y_true")

out_P1 <- hcp_predict_targets(
  dat = dat_train, test = test_P1,
  x_cols = "X1", y_grid = y_grid,
  alpha = 0.1,
  S = 2, B = 2,
  seed = 1
)
plot_hcp_intervals(
  out_P1$pred, mode = "pid", pid_col = "pid", x_sort_col = "X1",
  y_true_col = "y_true", show_true = TRUE,
  main = "Case B: multiple patients, one time point (by patient)"
)
```

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