# Package 'KFPCA'

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Title Kendall Functional Principal Component Analysis
Version 2.0
Description Implementation for Kendall functional principal component analysis. Kendall functional principal component analysis is a robust functional principal component analysis technique for non-Gaussian functional/longitudinal data. The crucial function of this package is KF-PCA() and KFPCA_reg(). Moreover, least square estimates of functional principal component scores are also provided. Refer to Rou Zhong, Shishi Liu, Haocheng Li, Jingxiao Zhang. (2021) <a href="mailto:arXiv:2102.01286">arXiv:2102.01286</a> >. Rou Zhong, Shishi Liu, Haocheng Li, Jingxiao Zhang. (2021) <a href="mailto:doi:10.1016/j.jmva.2021.104864">doi:10.1016/j.jmva.2021.104864</a> >.
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CD4

### **Description**

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A dataset containing the logarithm of CD4 cell counts for 190 patients with AIDS from June 1997 to January 2002. The data come from a human immunodeficiency virus (HIV) study by Wohl et al. (2005) and can be obtained from Cao et al. (2015).

### Usage

CD4

#### **Format**

A data frame with 741 rows and 3 variables:

PATIENT Patient ID.

CD4OBS Logarithm of CD4 cell counts.

CD4DATE Day of measurement.

### References

David A. Wohl, Donglin Zeng, Paul Stewart, Nicolas Glomb, Timothy Alcorn, Suzanne Jones, Jean Handy, Susan Fiscus, Adriana Weinberg, Deepthiman Gowda, and Charles van der Horst (2005). "Cytomegalovirus viremia, mortality, and end-organ disease among patients with aids receiving potent antiretroviral therapies." Journal of Acquired Immune Deficiency Syndromes, 38(5):538-544.

Hongyuan Cao, Donglin Zeng, and Jason P. Fine (2015). "Regression analysis of sparse asynchronous longitudinal data." Journal of The Royal Statistical Society Series B-statistical Methodology, 77(4):755-776.

FPCscoreLSE 3

recscorelse Least square estimates of functional principal component scores	FPCscoreLSE	Least square estimates of functional principal component scores
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### **Description**

Least square estimates (LSE) of functional principal component scores.

### Usage

```
FPCscoreLSE(Lt, Ly, kern, bw, FPC_dis, RegGrid, more = FALSE)
```

### **Arguments**

`		
	Lt	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the observation time in ascending order for each subject.
	Ly	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the measurements of each subject at the observation time correspond to Lt.
	kern	A character denoting the kernel type; 'epan' (Epanechnikov), 'unif' (Uniform), 'quar' (Quartic), 'gauss' (Gaussian).
	bw	A scalar denoting the bandwidth for mean function estimate.
	FPC_dis	A nRegGrid by nK matrix containing the eigenfunction estimates at RegGrid, where nRegGrid is the length of RegGrid and nK is the number of FPCs.
	RegGrid	A vector of the equally spaced time points in the support interval.
	more	Logical; If FALSE, only the estimates of FPC scores are returned. If TRUE, the mean function estimates and the eigenfunction estimates at all observation time points are also returned.

### Value

If more = FALSE, a n by nK matrix containing the estimates of the FPC scores is returned, where n is the sample size. If more = TRUE, a list containing the following components is returned:

score a *n* by nK matrix containing the estimates of the FPC scores.

meanest\_fine Mean function estimates at all observation time points.

FPC\_dis\_fine Eigenfunction estimates at all observation time points.

```
# Generate data n <-100 interval <-c(0, 10) lambda_1 <-9 #the first eigenvalue lambda_2 <-1.5 #the second eigenvalue eigfun <- list() eigfun[[1]] <- function(x){cos(pi * x/10)/sqrt(5)} eigfun[[2]] <- function(x){sin(pi * x/10)/sqrt(5)}
```

4 GenDataKL

GenDataKL

Generate functional/longitudinal data via KL expansion

### Description

Generate functional/longitudinal data via Karhunen-Loève expansion.

#### Usage

```
GenDataKL(n, interval, sparse, regular, meanfun, score, eigfun, sd)
```

### **Arguments**

n	number of sample size.
interval	A vector of length two denoting the supporting interval.
sparse	A vector denoting the possible numbers of observation size. The elements are chosen with equal chance. The length of sparse must be one if regular = TRUE.
regular	Logical; If TRUE, the observation grids are equally-spaced.
meanfun	A function for the mean.
score	A $n$ by nK matrix containing the estimates of the FPC scores, where nK is the number of FPCs.
eigfun	A list containing the eigenfunctions.
sd	A scalar denoting the standard deviation of measurement errors.

#### Value

A list containing the following components:

Lt	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the observation time in ascending order for each subject.
Ly	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the measurements of each subject at the observation time correspond to Lt.

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### **Examples**

GetGCVbw1D

Bandwidth selection through GCV for one-dimension cases

### **Description**

Bandwidth selection through generalized cross-validation (GCV) for one-dimension cases.

#### **Usage**

```
GetGCVbw1D(Lt, Ly, kern, dataType = "Sparse")
```

#### **Arguments**

Lt	A list of <i>n</i> vectors, where <i>n</i> is the sample size. Each entry contains the observation time in ascending order for each subject.
Ly	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the measurements of each subject at the observation time correspond to Lt.
kern	A character denoting the kernel type; 'epan'(Epanechnikov), 'unif'(Uniform), 'quar'(Quartic), 'gauss'(Gaussian).
dataType	A character denoting the data type; 'Sparse'-default, 'Dense'.

### Value

A scalar denoting the optimal bandwidth.

```
# Generate data
n <- 100
interval <- c(0, 10)
lambda_1 <- 9 #the first eigenvalue
lambda_2 <- 1.5 #the second eigenvalue
eigfun <- list()
eigfun[[1]] <- function(x){cos(pi * x/10)/sqrt(5)}
eigfun[[2]] <- function(x){sin(pi * x/10)/sqrt(5)}</pre>
```

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GetGCVbw2D

Bandwidth selection through GCV for two-dimension cases

#### **Description**

Bandwidth selection through generalized cross-validation (GCV) for two-dimension cases.

#### Usage

```
GetGCVbw2D(tPairs, yin, Lt, kern, ObsGrid, RegGrid, dataType = "Sparse")
```

### **Arguments**

tPairs	A matrix with two columns containing the pairs of time points.
yin	A vector denoting the corresponding values.
Lt	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the observation time in ascending order for each subject.
kern	A character denoting the kernel type; 'epan' (Epanechnikov), 'unif' (Uniform), 'quar' (Quartic), 'gauss' (Gaussian).
ObsGrid	A vector containing all observation grids in ascending order.
RegGrid	A vector of the equally spaced time points in the support interval.
dataType	A character denoting the data type; 'Sparse'-default, 'Dense'.

#### Value

A scalar denoting the optimal bandwidth.

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kernfun

Kernel Functions

### **Description**

Some common-used kernel functions.

#### Usage

```
kernfun(type)
```

#### Arguments

type

A character denoting the kernel type; 'epan'(Epanechnikov), 'unif'(Uniform), 'quar'(Quartic), 'gauss'(Gaussian).

### Value

The corresponding kernel function.

```
x <- seq(-2, 2, 0.01)
par(mfrow = c(2,2))
plot(x, kernfun("epan")(x), type = "l", main = "Epanechnikov")
plot(x, kernfun("unif")(x), type = "l", main = "Uniform")
plot(x, kernfun("quar")(x), type = "l", main = "Quartic")
plot(x, kernfun("gauss")(x), type = "l", main = "Gaussian")
par(mfrow = c(1,1))</pre>
```

KFPCA

KFPCA Kendall Functional Principal Component Analysis (KFPCA) for sparse design

### Description

KFPCA for non-Gaussian functional data with sparse design or longitudinal data.

### Usage

```
KFPCA(
 Lt,
 Ly,
  interval,
 dataType = "Sparse",
 nΚ,
  kern = "epan",
 bw,
  kernK = "epan",
  bwK = "GCV",
  kernmean = "epan",
 bwmean = "GCV",
  nRegGrid,
  fdParobj,
 more = TRUE
)
```

### Arguments

Lt	A list of <i>n</i> vectors, where <i>n</i> is the sample size. Each entry contains the observation time in ascending order for each subject.
Ly	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the measurements of each subject at the observation time correspond to Lt.
interval	A vector of length two denoting the supporting interval.
dataType	A character denoting the data type; 'Sparse'-default, 'Dense'.
nK	An integer denoting the number of FPCs.
kern	A character denoting the kernel type for the Nadaraya-Watson estimators; 'epan'(Epanechnikov)-default, 'unif'(Uniform), 'quar'(Quartic), 'gauss'(Gaussian).
bw	A scalar denoting the bandwidth for the Nadaraya-Watson estimators.
kernK	A character denoting the kernel type for the estimation of the Kendall's tau function; 'epan'(Epanechnikov)-default, 'unif'(Uniform), 'quar'(Quartic), 'gauss'(Gaussian).
bwK	The bandwidth for the estimation of the Kendall's tau function. If is.numeric(bwK) == T, bwK is exactly the bandwidth. If bwK == "GCV", the bandwidth is chosen by GCV. (default: "GCV")

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kernmean A character denoting the kernel type for the estimation of the mean function;

'epan' (Epanechnikov)-default, 'unif' (Uniform), 'quar' (Quartic), 'gauss' (Gaussian).

The bandwidth for the estimation of the mean function. If is.numeric(bwmean)

== T, bwmean is exactly the bandwidth. If bwmean == "GCV", the bandwidth is

chosen by GCV. (default: "GCV")

nRegGrid An integer denoting the number of equally spaced time points in the supporting

interval. The eigenfunctions and mean function are estimated at these equally

spaced time points.

fdParobj A functional parameter object for the smoothing of the eigenfunctions. For more

detail, see smooth.basis.

more Logical; If FALSE, estimates of FPC scores and predictions of trajectories are

not returned.

#### Value

A list containing the following components:

ObsGrid A vector containing all observation time points in ascending order.

RegGrid A vector of the equally spaced time points in the support interval.

bwmean A scalar denoting the bandwidth for the mean function estimate.

kernmean A character denoting the kernel type for the estimation of the mean function

bwK A scalar denoting the bandwidth for the Kendall's tau function estimate.

kernK A character denoting the kernel type for the estimation of the Kendall's tau

function

mean A vector of length nRegGrid denoting the mean function estimate.

KendFun A nRegGrid by nRegGrid matrix denoting the Kendall's tau function estimate.

FPC\_dis A nRegGrid by nK matrix containing the eigenfunction estimates at RegGrid.

FPC\_smooth A functional data object for the eigenfunction estimates.

score A n by nK matrix containing the estimates of the FPC scores, where n is the

sample size. The results are returned when more = TRUE.

X\_fd A functional data object for the prediction of trajectories. The results are re-

turned when more = TRUE.

Xest\_ind A list containing the prediction of each trajectory at their own observation time

points. The results are returned when more = TRUE.

Lt The input 'Lt'.

Ly The input 'Ly'.

CompTime A scalar denoting the computation time.

#### References

Rou Zhong, Shishi Liu, Haocheng Li, Jingxiao Zhang (2021). "Robust Functional Principal Component Analysis for Non-Gaussian Longitudinal Data." Journal of Multivariate Analysis, https://doi.org/10.1016/j.jmva.2021.

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#### **Examples**

```
# Generate data
n <- 100
interval <- c(0, 10)
lambda_1 <- 9 #the first eigenvalue</pre>
lambda_2 <- 1.5 #the second eigenvalue</pre>
eigfun <- list()
eigfun[[1]] \leftarrow function(x)(cos(pi * x/10)/sqrt(5))
eigfun[[2]] \leftarrow function(x)\{sin(pi * x/10)/sqrt(5)\}
score <- cbind(rnorm(n, 0, sqrt(lambda_1)), rnorm(n, 0, sqrt(lambda_2)))</pre>
DataNew <- GenDataKL(n, interval = interval, sparse = 6:8, regular = FALSE,
                      meanfun = function(x)\{0\}, score = score,
                      eigfun = eigfun, sd = sqrt(0.1)
basis <- fda::create.bspline.basis(interval, nbasis = 13, norder = 4,</pre>
                                breaks = seq(0, 10, length.out = 11))
# KFPCA
Klist <- KFPCA(DataNew$Lt, DataNew$Ly, interval, nK = 2, bw = 1,</pre>
                nRegGrid = 51, fdParobj = basis)
plot(Klist$FPC_smooth)
```

KFPCA\_reg

Kendall Functional Principal Component Analysis (KFPCA) for dense and regular design

### Description

KFPCA for non-Gaussian functional data with dense and regular design.

### Usage

```
KFPCA_reg(Lt, Ly, nGrid, nK, fdParobj)
```

### **Arguments**

Lt	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the observation time in ascending order for each subject. The observation times are the same for each subject.
Ly	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the measurements of each subject at the observation time correspond to Lt.
nGrid	An integer denoting the number of observation time for each subject.
nK	An integer denoting the number of FPCs.
fdParobj	A functional parameter object for the smoothing of mean function and eigenfunctions. For more detail, see smooth.basis.

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#### Value

A list containing the following components:

meanfd A functional data object for the mean function estimates.

FPC\_list A list containing nK functional data objects, which are the eigenfunction estimates.

score A n by nK matrix containing the estimates of the FPC scores, where n is the

sample size.

CompTime A scalar denoting the computation time.

#### References

Rou Zhong, Shishi Liu, Haocheng Li, Jingxiao Zhang (2021). "Functional principal component analysis estimator for non-Gaussian data." <arXiv: https://arxiv.org/abs/2102.01286>.

### **Examples**

```
# Generate data
n <- 100
interval <-c(0, 10)
lambda_1 <- 16 #the first eigenvalue</pre>
lambda_2 <- 9 #the second eigenvalue</pre>
eigfun <- list()
eigfun[[1]] <- function(x)\{\cos(pi * x/10)/sqrt(5)\}
eigfun[[2]] <- function(x)\{\sin(pi * x/10)/sqrt(5)\}
score <- cbind(rnorm(n, 0, sqrt(lambda_1)), rnorm(n, 0, sqrt(lambda_2)))</pre>
DataNew <- GenDataKL(n, interval = interval, sparse = 51, regular = TRUE,
                      meanfun = function(x)\{0\}, score = score,
                      eigfun = eigfun, sd = sqrt(0.25))
basis <- fda::create.bspline.basis(interval, nbasis = 13, norder = 4,
                               breaks = seq(0, 10, length.out = 11))
#KFPCA
Klist <- KFPCA_reg(DataNew$Lt, DataNew$Ly, nGrid = 51, nK = 2, fdParobj = basis)</pre>
plot(Klist$FPC_list[[1]])
plot(Klist$FPC_list[[2]])
```

MeanEst

Local linear estimates of mean function

### **Description**

Local linear estimates of mean function.

### Usage

```
MeanEst(Lt, Ly, kern, bw, gridout)
```

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### Arguments

Lt	A list of <i>n</i> vectors, where <i>n</i> is the sample size. Each entry contains the observation time in ascending order for each subject.
Ly	A list of $n$ vectors, where $n$ is the sample size. Each entry contains the measurements of each subject at the observation time correspond to Lt.
kern	A character denoting the kernel type; 'epan'(Epanechnikov), 'unif'(Uniform), 'quar'(Quartic), 'gauss'(Gaussian).
bw	A scalar denoting the bandwidth.
gridout	A vector denoting the time points that the mean function need to be estimated.

#### Value

A list containing the following components:

Grid A vector denoting the time points that the mean function need to be estimated.

Mean A vector containing the mean function estimates.

### **Examples**

```
# Generate data
n <- 100
interval \leftarrow c(0, 10)
lambda_1 <- 9 #the first eigenvalue</pre>
lambda_2 <- 1.5 #the second eigenvalue</pre>
eigfun <- list()
eigfun[[1]] \leftarrow function(x)\{cos(pi * x/10)/sqrt(5)\}
eigfun[[2]] \leftarrow function(x){sin(pi * x/10)/sqrt(5)}
score <- cbind(rnorm(n, 0, sqrt(lambda_1)), rnorm(n, 0, sqrt(lambda_2)))</pre>
DataNew <- GenDataKL(n, interval = interval, sparse = 6:8, regular = FALSE,
                      meanfun = function(x)\{x\}, score = score,
                      eigfun = eigfun, sd = sqrt(0.1)
# Mean function estimate at all observation time points
bwOpt <- GetGCVbw1D(DataNew$Lt, DataNew$Ly, kern = "epan")</pre>
meanest <- MeanEst(DataNew$Lt, DataNew$Ly, kern = "epan", bw = bwOpt,</pre>
                    gridout = sort(unique(unlist(DataNew$Lt))))
plot(meanest$Grid, meanest$mean)
```

predict.KFPCA

Predict FPC scores

### Description

Predict FPC scores using least square estimate (LSE) for a new sample.

#### Usage

```
## S3 method for class 'KFPCA'
predict(object, newLt, newLy, nK, more = FALSE, ...)
```

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### Arguments

object A KFPCA object obtained from KFPCA.

newLt A list of *n* vectors, where *n* is the new sample size. Each entry contains the observation time in ascending order for each new subject.

newLy A list of *n* vectors, where *n* is the new sample size. Each entry contains the measurements of each new subject at the observation time correspond to newLt.

nK An integer denoting the number of FPCs.

more Logical; If FALSE, only the predictions of FPC scores are returned. If TRUE, the mean function estimates and the eigenfunction estimates at the new observation time points are also returned.

Not used.

#### Value

If more = FALSE, a n by nK matrix containing the predictions of the FPC scores is returned, where n is the new sample size. If more = TRUE, a list containing the following components is returned:

score\_new a *n* by nK matrix containing the predictions of the FPC scores.

meanest\_new Mean function estimates at the new observation time points.

FPC\_dis\_new Eigenfunction estimates at the new observation time points.

```
# Generate training data
n <- 100
interval <-c(0, 10)
lambda_1 <- 9 #the first eigenvalue</pre>
lambda_2 <- 1.5 #the second eigenvalue</pre>
eigfun <- list()
eigfun[[1]] \leftarrow function(x)\{cos(pi * x/10)/sqrt(5)\}
eigfun[[2]] \leftarrow function(x){sin(pi * x/10)/sqrt(5)}
score <- cbind(rnorm(n, 0, sqrt(lambda_1)), rnorm(n, 0, sqrt(lambda_2)))</pre>
DataNew <- GenDataKL(n, interval = interval, sparse = 6:8, regular = FALSE,
                      meanfun = function(x)\{0\}, score = score,
                      eigfun = eigfun, sd = sqrt(0.1))
basis <- fda::create.bspline.basis(interval, nbasis = 13, norder = 4,</pre>
                                breaks = seq(0, 10, length.out = 11))
Klist <- KFPCA(DataNew$Lt, DataNew$Ly, interval, nK = 2, bw = 1,</pre>
                nRegGrid = 51, fdParobj = basis)
# Generate test data
n_{\text{test}} < -20
score_test <- cbind(rnorm(n_test, 0, sqrt(lambda_1)),</pre>
                     rnorm(n_test, 0, sqrt(lambda_2)))
Data_test <- GenDataKL(n_test, interval = interval, sparse = 6:8, regular = FALSE,</pre>
                        meanfun = function(x)\{0\}, score = score_test,
                        eigfun = eigfun, sd = sqrt(0.1)
# Prediction
score_pre <- predict(Klist, Data_test$Lt, Data_test$Ly, nK = 2)</pre>
plot(score_test[,1], score_pre[,1])
```

14 SparsePlot

SparsePlot	Sparse plot
Spar Ser IU t	sparse pioi

### **Description**

Create sparse plot to see the sparsity of the data.

#### Usage

```
SparsePlot(Lt, interval, ...)
```

#### **Arguments**

A list of n vectors, where n is the sample size. Each entry contains the observation time in ascending order for each subject.
A vector of length two denoting the supporting interval.
Other arguments passed into plot.

#### **Details**

For the sparse plot, x-axis is the observation time while y-axis represents various subjects.

#### Value

Create the corresponding sparse plot.

```
# Generate data
n <- 100
interval <- c(0, 10)
lambda_1 <- 9 #the first eigenvalue</pre>
lambda_2 <- 1.5 #the second eigenvalue</pre>
eigfun <- list()
eigfun[[1]] \leftarrow function(x)\{cos(pi * x/10)/sqrt(5)\}
eigfun[[2]] \leftarrow function(x)\{sin(pi * x/10)/sqrt(5)\}
score <- cbind(rnorm(n, 0, sqrt(lambda_1)), rnorm(n, 0, sqrt(lambda_2)))</pre>
# DataNew1 and DataNew2 have different sparsity
DataNew1 <- GenDataKL(n, interval = interval, sparse = 6:8, regular = FALSE,
                       meanfun = function(x)\{0\}, score = score,
                       eigfun = eigfun, sd = sqrt(0.1)
DataNew2 <- GenDataKL(n, interval = interval, sparse = 2:4, regular = FALSE,
                       meanfun = function(x)\{0\}, score = score,
                       eigfun = eigfun, sd = sqrt(0.1)
# Create sparse plots
par(mfrow = c(1, 2))
SparsePlot(DataNew1$Lt, interval = interval)
SparsePlot(DataNew2$Lt, interval = interval)
par(mfrow = c(1, 1))
```

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