Package 'BKTR'

August 18, 2024

Version 0.2.0

Title Bayesian Kernelized Tensor Regression

Description Facilitates scalable spatiotemporally varying coefficient modelling with Bayesian kernelized tensor regression.

The important features of this package are:

- (a) Enabling local temporal and spatial modeling of the relationship between the response variable and covariates.
- (b) Implementing the model described by Lei et al. (2023) <doi:10.48550/arXiv.2109.00046>.
- (c) Using a Bayesian Markov Chain Monte Carlo (MCMC) algorithm to sample from the posterior distribution of the model parameters.
- (d) Employing a tensor decomposition to reduce the number of estimated parameters.
- (e) Accelerating tensor operations and enabling graphics processing unit (GPU) acceleration with the 'torch' package.

```
Depends R (>= 4.0.0)
Encoding UTF-8
Imports torch (>= 0.13.0), R6, R6P, ggplot2, ggmap, data.table
License MIT + file LICENSE
RoxygenNote 7.2.3
Collate 'samplers.R' 'tensor ops.R' 'result logger.R'
      'likelihood_evaluator.R' 'distances.R' 'kernels.R' 'bktr.R'
      'examples.R' 'plots.R' 'utils.R'
Suggests knitr, rmarkdown, R.rsp
LazyData true
VignetteBuilder knitr, rmarkdown, R.rsp
BugReports https://github.com/julien-hec/BKTR/issues
NeedsCompilation no
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```

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*.Kernel

*.Kernel

Operator overloading for kernel multiplication

Description

Operator overloading for kernel multiplication

Usage

```
## S3 method for class 'Kernel'
k1 * k2
```

Arguments

k1 Kernel: The left kernel to use for compositionk2 Kernel: The right kernel to use for composition

Value

A new KernelMulComposed object.

Examples

```
# Create a new locally periodic kernel
k_loc_per <- KernelSE$new() * KernelPeriodic$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_loc_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_loc_per$kernel_gen()</pre>
```

+.Kernel

Operator overloading for kernel addition

Description

Operator overloading for kernel addition

Usage

```
## S3 method for class 'Kernel'
k1 + k2
```

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Arguments

k1 Kernel: The left kernel to use for compositionk2 Kernel: The right kernel to use for composition

Value

A new KernelAddComposed object.

Examples

```
# Create a new additive kernel
k_rq_plus_per <- KernelRQ$new() + KernelPeriodic$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_rq_plus_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_rq_plus_per$kernel_gen()</pre>
```

BixiData

BIXI Data Class

Description

R6 class encapsulating all BIXI dataframes. It is also possible to use a light version of the dataset by using the is_light parameter. In this case, the dataset is reduced to its first 25 stations and first 50 days. The light version is only used for testing and short examples.

Public fields

```
departure_df The departure dataframe
spatial_features_df The spatial features dataframe
temporal_features_df The temporal features dataframe
spatial_positions_df The spatial positions dataframe
temporal_positions_df The temporal positions dataframe
data_df The data dataframe
is_light Whether the light version of the dataset is used
```

Methods

Public methods:

- BixiData\$new()
- BixiData\$clone()

Method new(): Initialize the BIXI data class

bixi_spatial_features 5

```
Usage:
BixiData$new(is_light = FALSE)

Arguments:
is_light Whether the light version of the dataset is used, defaults to FALSE.

Returns: A new BIXI data instance

Method clone(): The objects of this class are cloneable with this method.

Usage:
BixiData$clone(deep = FALSE)

Arguments:
deep Whether to make a deep clone.
```

Examples

```
# Create a light BIXI data collection instance containing multiple dataframes
# This only uses the first 25 stations and 50 days of the full dataset
bixi_data <- BixiData$new(is_light = TRUE)
# Dataframe containing the position (latitude and longitude) of M stations
bixi_data$spatial_positions_df
# Dataframe containing the time position of N days (O to N-1)
bixi_data$temporal_positions_df
# Dataframe with spatial and temporal features for each day and station (M x N rows)
bixi_data$data_df</pre>
```

Description

These data represent 14 spatial features (columns) for 587 bike sharing stations (rows) located at different geographical coordinates (longitude, latitude) in Montreal. The Montreal based bike sharing company is named BIXI. The first column contains the descriptive label affected to each station and the other columns contain information about the infrastructure, points of interests, walkscore and population surrounding each station for 2019.

Usage

```
data("bixi_spatial_features")
```

Format

A data frame with 587 observations on the following 14 variables.

```
location a character vector area_park a numeric vector
```

len_cycle_path a numeric vector
len_major_road a numeric vector
len_minor_road a numeric vector
num_metro_stations a numeric vector
num_other_commercial a numeric vector
num_restaurants a numeric vector
num_university a numeric vector
num_pop a numeric vector
num_bus_stations a numeric vector
num_bus_routes a numeric vector
walkscore a numeric vector

Source

Wang, X., Cheng, Z., Trépanier, M., & Sun, L. (2021). Modeling bike-sharing demand using a regression model with spatially varying coefficients. Journal of Transport Geography, 93, 103059.

References

Reference for the BIXI station informations: BIXI Montréal (2023). "Open Data." Accessed: 2023-07-11, URL https://bixi.com/en/open-data.

Reference for point of interests and infrastructure informations: DMTI Spatial Inc (2019). "Enhanced Point of Interest (DMTI)." URL https://www.dmtispatial.com.

Reference for Walkscore: Walk Score (2023). "Walk Score Methodology." Accessed: 2023-07-11, URL https://www.walkscore.com/methodology.shtml.

The population information comes from the 2016 Canada census data at a dissemination block level.

bixi_spatial_locations

Spatial Locations of Montreal BIXI Stations in 2019

Description

Data points representing the spatial locations of 587 bike sharing stations for the Montreal based bike sharing company named BIXI. The dataframe contains a label to identify each station and its associated longitude and latitude coordinates.

Usage

```
data("bixi_spatial_locations")
```

bixi_station_departures

Format

A data frame with 587 observations on the following 3 variables.

location a character vector latitude a numeric vector longitude a numeric vector

Source

Wang, X., Cheng, Z., Trépanier, M., & Sun, L. (2021). Modeling bike-sharing demand using a regression model with spatially varying coefficients. Journal of Transport Geography, 93, 103059.

References

BIXI Montréal (2023). "Open Data." Accessed: 2023-07-11, URL https://bixi.com/en/open-data.

bixi_station_departures

Daily Departure from BIXI Stations in 2019

Description

These data capture the number of daily departure for 587 bike sharing stations (rows) through 197 days (columns). The data is limited to the 2019 season of a Montreal based bike sharing company named BIXI.

Usage

```
data("bixi_station_departures")
```

Format

A data frame with 587 rows and 197 columns.

Source

Wang, X., Cheng, Z., Trépanier, M., & Sun, L. (2021). Modeling bike-sharing demand using a regression model with spatially varying coefficients. Journal of Transport Geography, 93, 103059.

References

BIXI Montréal (2023). "Open Data." Accessed: 2023-07-11, URL https://bixi.com/en/opendata.

```
bixi_temporal_features
```

Temporal Features in Montreal applicable to BIXI for 2019

Description

These data represent the temporal features in Montreal applicable to a Montreal based bike sharing company named BIXI. The data include six features (columns) for 196 days (rows). The time column represent the label associated to each captured time for the 2019 season of BIXI. The other columns contain information about Montral weather and applicable holidays for each day.

Usage

```
data("bixi_temporal_features")
```

Format

A data frame with 196 observations on the following 6 variables.

```
time a IDate
humidity a numeric vector
max_temp_f a numeric vector
mean_temp_c a numeric vector
total_precip_mm a numeric vector
holiday a numeric vector
```

Source

Lei, M., Labbe, A., & Sun, L. (2021). Scalable Spatiotemporally Varying Coefficient Modelling with Bayesian Kernelized Tensor Regression. arXiv preprint arXiv:2109.00046.

References

The weather data is sourced from the Environment and Climate Change Canada Historical Climate Data website.

The holiday column is specifying if a date is a holiday or not, according to the Quebec government.

bixi_temporal_locations

Temporal indices for the 2019 BIXI season

Description

These data represent 196 temporal indices (rows) related to each day of the 2019 season of Montreal based bike sharing company named BIXI. The time column represent the label associated to each day and the time_index column represent the location in time space of each day when compared to each other. Since no days are missing and they are all spaced by exactly one day, the time_index is simply a range from 0 to 195.

Usage

```
data("bixi_temporal_locations")
```

Format

A data frame with 196 observations on the following 2 variables.

time a IDate time_index a numeric vector

Source

Lei, M., Labbe, A., & Sun, L. (2021). Scalable Spatiotemporally Varying Coefficient Modelling with Bayesian Kernelized Tensor Regression. arXiv preprint arXiv:2109.00046.

BKTRRegressor

R6 class encapsulating the BKTR regression elements

Description

A BKTRRegressor holds all the key elements to accomplish the MCMC sampling algorithm (**Algorithm 1** of the paper).

Public fields

data_df The dataframe containing all the covariates through time and space (including the response variable)

y The response variable tensor

omega The tensor indicating which response values are not missing

covariates The tensor containing all the covariates

covariates_dim The dimensions of the covariates tensor

logged_params_tensor The tensor containing all the sampled hyperparameters tau The precision hyperparameter spatial_decomp The spatial covariate decomposition temporal_decomp The temporal covariate decomposition covs_decomp The feature covariate decomposition result_logger The result logger instance used to store the results of the MCMC sampling has_completed_sampling Boolean showing wheter the MCMC sampling has been completed spatial_kernel The spatial kernel used temporal_kernel The temporal kernel used spatial_positions_df The dataframe containing the spatial positions temporal_positions_df The dataframe containing the temporal positions spatial_params_sampler The spatial kernel hyperparameter sampler temporal_params_sampler The temporal kernel hyperparameter sampler tau_sampler The tau hyperparameter sampler precision_matrix_sampler The precision matrix sampler spatial_ll_evaluator The spatial likelihood evaluator temporal_ll_evaluator The temporal likelihood evaluator rank_decomp The rank of the CP decomposition burn_in_iter The number of burn in iterations sampling_iter The number of sampling iterations max_iter The total number of iterations a_0 The initial value for the shape in the gamma function generating tau b_0 The initial value for the rate in the gamma function generating tau formula The formula used to specify the relation between the response variable and the covariates spatial_labels The spatial labels temporal_labels The temporal labels feature_labels The feature labels geo_coords_projector The geographic coordinates projector

Active bindings

summary A summary of the BKTRRegressor instance
beta_covariates_summary A dataframe containing the summary of the beta covariates
y_estimates A dataframe containing the y estimates
imputed_y_estimates A dataframe containing the imputed y estimates
beta_estimates A dataframe containing the beta estimates
hyperparameters_per_iter_df A dataframe containing the beta estimates per iteration
decomposition_tensors List of all used decomposition tensors

Methods

Public methods:

```
• BKTRRegressor$new()
```

- BKTRRegressor\$mcmc_sampling()
- BKTRRegressor\$predict()
- BKTRRegressor\$get_iterations_betas()
- BKTRRegressor\$get_beta_summary_df()
- BKTRRegressor\$clone()

Method new(): Create a new BKTRRegressor object.

```
Usage:
BKTRRegressor$new(
  data_df,
  spatial_positions_df,
  temporal_positions_df,
  rank_decomp = 10,
  burn_in_iter = 500,
  sampling_iter = 500,
  formula = NULL,
  spatial_kernel = KernelMatern$new(smoothness_factor = 3),
  temporal_kernel = KernelSE$new(),
  sigma_r = 0.01,
  a_0 = 1e_06,
  b_0 = 1e_06
 has_geo_coords = TRUE,
  geo_coords_scale = 10
)
```

Arguments:

data_df data.table: A dataframe containing all the covariates through time and space. It is important that the dataframe has a two indexes named 'location' and 'time' respectively. The dataframe should also contain every possible combinations of 'location' and 'time' (i.e. even missing rows should be filled present but filled with NaN). So if the dataframe has 10 locations and 5 time points, it should have 50 rows (10 x 5). If formula is None, the dataframe should contain the response variable 'Y' as the first column. Note that the covariate columns cannot contain NaN values, but the response variable can.

spatial_positions_df data.table: Spatial kernel input tensor used to calculate covariates' distance. Vector of length equal to the number of location points.

temporal_positions_df data.table: Temporal kernel input tensor used to calculate covariate distance. Vector of length equal to the number of time points.

 $rank_decomp$ Integer: Rank of the CP decomposition (Paper – R). Defaults to 10.

burn_in_iter Integer: Number of iteration before sampling (Paper $-K_1$). Defaults to 500.

sampling_iter Integer: Number of sampling iterations (Paper – K_2). Defaults to 500.

formula A Wilkinson R formula to specify the relation between the response variable 'Y' and the covariates. If Null, the first column of the data frame will be used as the response variable and all the other columns will be used as the covariates. Defaults to Null.

 $spatial_kernel \ \ Kernel: Spatial \ kernel \ Used. \ Defaults to a \ KernelMatern(smoothness_factor=3). \\ temporal_kernel \ \ Kernel: \ Temporal \ kernel \ used. \ Defaults to \ KernelSE(). \\$

sigma_r Numeric: Variance of the white noise process (τ^{-1}) defaults to 1E-2.

- a_0 Numeric: Initial value for the shape (α) in the gamma function generating tau defaults to 1E-6.
- b_0 Numeric: Initial value for the rate (β) in the gamma function generating tau defaults to 1E-6.

has_geo_coords Boolean: Whether the spatial positions df use geographic coordinates (latitude, longitude). Defaults to TRUE.

geo_coords_scale Numeric: Scale factor to convert geographic coordinates to euclidean 2D space via Mercator projection using x & y domains of [-scale/2, +scale/2]. Only used if has_geo_coords is TRUE. Defaults to 10.

Returns: A new BKTRRegressor object.

Method mcmc_sampling(): Launch the MCMC sampling process.

For a predefined number of iterations:

- 1. Sample spatial kernel hyperparameters
- 2. Sample temporal kernel hyperparameters
- 3. Sample the precision matrix from a wishart distribution
- 4. Sample a new spatial covariate decomposition
- 5. Sample a new feature covariate decomposition
- 6. Sample a new temporal covariate decomposition
- 7. Calculate respective errors for the iterations
- 8. Sample a new tau value
- 9. Collect all the important data for the iteration

Usage:

BKTRRegressor\$mcmc_sampling()

Returns: NULL Results are stored and can be accessed via summary()

Method predict(): Use interpolation to predict betas and response values for new data.

Usage:

```
BKTRRegressor$predict(
  new_data_df,
  new_spatial_positions_df = NULL,
  new_temporal_positions_df = NULL,
  jitter = 1e-05
)
```

Arguments:

new_data_df data.table: New covariates. Must have the same columns as the covariates used to fit the model. The index should contain the combination of all old spatial coordinates with all new temporal coordinates, the combination of all new spatial coordinates with all old temporal coordinates, and the combination of all new spatial coordinates with all new temporal coordinates.

new_spatial_positions_df data.table or NULL: A data frame containing the new spatial positions. Defaults to NULL.

new_temporal_positions_df data.table or NULL: A data frame containing the new temporal positions. Defaults to NULL.

jitter Numeric or NULL: A small value to add to the diagonal of the precision matrix. Defaults to NULL.

Returns: List: A list of two dataframes. The first represents the beta forecasted for all new spatial locations or temporal points. The second represents the forecasted response for all new spatial locations or temporal points.

Method get_iterations_betas(): Return all sampled betas through sampling iterations for a given set of spatial, temporal and feature labels. Useful for plotting the distribution of sampled beta values.

```
Usage:

BKTRRegressor$get_iterations_betas(
    spatial_label,
    temporal_label,
    feature_label
)

Arguments:

spatial_label String: The spatial label for which we want to get the betas
temporal_label String: The temporal label for which we want to get the betas
feature_label String: The feature label for which we want to get the betas

Returns: A list containing the sampled betas through iteration for the given labels
```

Method get_beta_summary_df(): Get a summary of estimated beta values. If no labels are given, then the summary is for all the betas. If labels are given, then the summary is for the given labels.

```
Usage:
BKTRRegressor$get_beta_summary_df(
   spatial_labels = NULL,
   temporal_labels = NULL,
   feature_labels = NULL
)
```

spatial_labels vector: The spatial labels used in summary. If NULL, then all spatial labels are used. Defaults to NULL.

temporal_labels vector: The temporal labels used in summary. If NULL, then all temporal labels are used. Defaults to NULL.

feature_labels vector: The feature labels used in summary. If NULL, then all feature labels are used. Defaults to NULL.

Returns: A new data.table with the beta summary for the given labels.

Method clone(): The objects of this class are cloneable with this method.

Usage:

Arguments:

```
BKTRRegressor$clone(deep = FALSE)

Arguments:
deep Whether to make a deep clone.
```

Examples

```
# Create a BIXI data collection instance containing multiple dataframes
bixi_data <- BixiData$new(is_light = TRUE) # Use light version for example</pre>
# Create a BKTRRegressor instance
bktr_regressor <- BKTRRegressor$new(</pre>
  formula = nb_departure ~ 1 + mean_temp_c + area_park,
 data_df <- bixi_data$data_df,</pre>
 spatial_positions_df = bixi_data$spatial_positions_df,
 temporal_positions_df = bixi_data$temporal_positions_df,
 burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
# Launch the MCMC sampling
bktr_regressor$mcmc_sampling()
# Get the summary of the bktr regressor
summary(bktr_regressor)
# Get estimated response variables for missing values
bktr_regressor$imputed_y_estimates
# Get the list of sampled betas for given spatial, temporal and feature labels
bktr_regressor$get_iterations_betas(
 spatial_label = bixi_data$spatial_positions_df$location[1],
 temporal_label = bixi_data$temporal_positions_df$time[1],
 feature_label = 'mean_temp_c')
# Get the summary of all betas for the 'mean_temp_c' feature
bktr_regressor$get_beta_summary_df(feature_labels = 'mean_temp_c')
## PREDICTION EXAMPLE ##
# Create a light version of the BIXI data collection instance
bixi_data <- BixiData$new(is_light = TRUE)</pre>
# Simplify variable names
data_df <- bixi_data$data_df</pre>
spa_pos_df <- bixi_data$spatial_positions_df</pre>
temp_pos_df <- bixi_data$temporal_positions_df</pre>
# Keep some data aside for prediction
new_spa_pos_df <- spa_pos_df[1:2, ]</pre>
new_temp_pos_df <- temp_pos_df[1:5, ]</pre>
reg_spa_pos_df <- spa_pos_df[-(1:2), ]</pre>
reg_temp_pos_df <- temp_pos_df[-(1:5), ]</pre>
reg_data_df_mask <- data_df$location %in% reg_spa_pos_df$location &</pre>
 data_df$time %in% reg_temp_pos_df$time
reg_data_df <- data_df[reg_data_df_mask, ]</pre>
```

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```
new_data_df <- data_df[!reg_data_df_mask, ]

# Launch mcmc sampling on regression data
bktr_regressor <- BKTRRegressor$new(
    formula = nb_departure ~ 1 + mean_temp_c + area_park,
    data_df = reg_data_df,
    spatial_positions_df = reg_spa_pos_df,
    temporal_positions_df = reg_temp_pos_df,
    burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Predict response values for new data
bktr_regressor$predict(
    new_data_df = new_data_df,
    new_spatial_positions_df = new_spa_pos_df,
    new_temporal_positions_df = new_temp_pos_df)</pre>
```

CompositionOps

Kernel Composition Operations

Description

Kernel Composition Operations Enum. Possibilities of operation between two kernels to generate a new composed kernel. The values are: MUL and ADD.

Usage

CompositionOps

Format

An object of class list of length 2.

Kernel

Base R6 class for Kernels

Description

Abstract base class for kernels (Should not be instantiated)

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Public fields

```
kernel_variance The variance of the kernel
jitter_value The jitter value to add to the kernel matrix
distance_matrix The distance matrix between points in a tensor format
name The kernel's name
parameters The parameters of the kernel (list of KernelParameter)
covariance_matrix The covariance matrix of the kernel in a tensor format
positions_df The positions of the points in a dataframe format
has_dist_matrix Identify if the kernel has a distance matrix or not
```

Methods

Public methods:

```
• Kernel$new()
```

- Kernel\$core_kernel_fn()
- Kernel\$add_jitter_to_kernel()
- Kernel\$kernel_gen()
- Kernel\$set_positions()
- Kernel\$plot()
- Kernel\$clone()

```
Method new(): Kernel abstract base constructor
```

```
Usage.
```

Kernel\$new(kernel_variance, jitter_value)

Arguments:

kernel_variance Numeric: The variance of the kernel

jitter_value Numeric: The jitter value to add to the kernel matrix

Returns: A new Kernel object.

Method core_kernel_fn(): Abstract method to compute the core kernel's covariance matrix

Usage:

Kernel\$core_kernel_fn()

Method add_jitter_to_kernel(): Method to add jitter to the kernel's covariance matrix

Usage:

Kernel\$add_jitter_to_kernel()

Method kernel_gen(): Method to compute the kernel's covariance matrix

Usage:

Kernel\$kernel_gen()

Method set_positions(): Method to set the kernel's positions and compute the distance matrix

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```
Usage:
Kernel$set_positions(positions_df)
Arguments:
positions_df Dataframe: The positions of the points in a dataframe format

Method plot(): Method to plot the kernel's covariance matrix

Usage:
Kernel$plot(show_figure = TRUE)

Arguments:
show_figure Boolean: If TRUE, the figure is shown, otherwise it is returned

Returns: If show_figure is TRUE, the figure is shown, otherwise it is returned

Method clone(): The objects of this class are cloneable with this method.

Usage:
Kernel$clone(deep = FALSE)

Arguments:
deep Whether to make a deep clone.
```

KernelAddComposed

R6 class for Kernels Composed via Addition

Description

R6 class automatically generated when adding two kernels together.

Super classes

```
BKTR::Kernel -> BKTR::KernelComposed -> KernelAddComposed
```

Methods

Public methods:

- KernelAddComposed\$new()
- KernelAddComposed\$clone()

Method new(): Create a new KernelAddComposed object.

Usage:

KernelAddComposed\$new(left_kernel, right_kernel, new_name)

Arguments:

left_kernel Kernel: The left kernel to use for composition right_kernel Kernel: The right kernel to use for composition

new_name String: The name of the composed kernel

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Returns: A new KernelAddComposed object.

Method clone(): The objects of this class are cloneable with this method.

```
Usage:
KernelAddComposed$clone(deep = FALSE)
Arguments:
deep Whether to make a deep clone.
```

Examples

```
# Create a new additive kernel
k_rq_plus_per <- KernelAddComposed$new(
  left_kernel = KernelRQ$new(),
  right_kernel = KernelPeriodic$new(),
  new_name = 'SE + Periodic Kernel'
)
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_rq_plus_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_rq_plus_per$kernel_gen()</pre>
```

KernelComposed

R6 class for Composed Kernels

Description

R6 class for Composed Kernels

Super class

```
BKTR::Kernel -> KernelComposed
```

Public fields

```
name The kernel's name
parameters The parameters of the kernel (list of KernelParameter)
left_kernel The left kernel to use for composition
right_kernel The right kernel to use for composition
composition_operation The operation to use for composition
has_dist_matrix Identify if the kernel has a distance matrix or not
```

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Methods

```
Public methods:
```

```
• KernelComposed$new()
  • KernelComposed$core_kernel_fn()
  • KernelComposed$set_positions()
  • KernelComposed$clone()
Method new(): Create a new KernelComposed object.
 Usage:
 KernelComposed$new(left_kernel, right_kernel, new_name, composition_operation)
 Arguments:
 left_kernel Kernel: The left kernel to use for composition
 right_kernel Kernel: The right kernel to use for composition
 new_name String: The name of the composed kernel
 composition_operation CompositionOps: The operation to use for composition
Method core_kernel_fn(): Method to compute the core kernel's covariance matrix
 Usage:
 KernelComposed$core_kernel_fn()
 Returns: The core kernel's covariance matrix
Method set_positions(): Method to set the kernel's positions and compute the distance ma-
trix
 Usage:
 KernelComposed$set_positions(positions_df)
 Arguments:
 positions_df Dataframe: The positions of the points in a dataframe format
 Returns: NULL, set the kernel's positions and compute the distance matrix
Method clone(): The objects of this class are cloneable with this method.
 Usage:
 KernelComposed$clone(deep = FALSE)
 Arguments:
 deep Whether to make a deep clone.
```

Examples

```
# Create a new locally periodic kernel
k_loc_per <- KernelComposed$new(
  left_kernel = KernelSE$new(),
  right_kernel = KernelPeriodic$new(),
  new_name = 'Locally Periodic Kernel',
  composition_operation = CompositionOps$MUL
)</pre>
```

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```
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, \emptyset, 3), y=c(-2, \emptyset, 2))
k_loc_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_loc_per$kernel_gen()
```

KernelMatern

R6 class for Matern Kernels

Description

R6 class for Matern Kernels

Super class

```
BKTR::Kernel -> KernelMatern
```

Public fields

lengthscale The lengthscale parameter instance of the kernel smoothness_factor The smoothness factor of the kernel has_dist_matrix Identify if the kernel has a distance matrix or not

Methods

Public methods:

```
• KernelMatern$new()
```

- KernelMatern\$get_smoothness_kernel_fn()
- KernelMatern\$core_kernel_fn()
- KernelMatern\$clone()

Method new(): Create a new KernelMatern object.

```
Usage:
KernelMatern$new(
  smoothness_factor = 5,
  lengthscale = KernelParameter$new(2),
  kernel_variance = 1,
  jitter_value = NULL
)
Arguments:
```

smoothness_factor Numeric: The smoothness factor of the kernel (1, 3 or 5) lengthscale KernelParameter: The lengthscale parameter instance of the kernel kernel_variance Numeric: The variance of the kernel jitter_value Numeric: The jitter value to add to the kernel matrix

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```
Method get_smoothness_kernel_fn(): Method to the get the smoothness kernel function for a given integer smoothness factor
```

Usage:

KernelMatern\$get_smoothness_kernel_fn()

Returns: The smoothness kernel function

Method core_kernel_fn(): Method to compute the core kernel's covariance matrix

Usage:

KernelMatern\$core_kernel_fn()

Returns: The core kernel's covariance matrix

Method clone(): The objects of this class are cloneable with this method.

Usage:

KernelMatern\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

Examples

```
# Create a new Matern 3/2 kernel
k_matern <- KernelMatern$new(smoothness_factor = 3)
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_matern$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_matern$kernel_gen()</pre>
```

 ${\tt Kernel MulComposed}$

R6 class for Kernels Composed via Multiplication

Description

R6 class automatically generated when multiplying two kernels together.

Super classes

```
BKTR::Kernel -> BKTR::KernelComposed -> KernelMulComposed
```

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Methods

Public methods:

```
• KernelMulComposed$new()
```

• KernelMulComposed\$clone()

```
Method new(): Create a new KernelMulComposed object.
    Usage:
    KernelMulComposed$new(left_kernel, right_kernel, new_name)
    Arguments:
```

left_kernel Kernel: The left kernel to use for composition
right_kernel Kernel: The right kernel to use for composition

Returns: A new KernelMulComposed object.

new_name String: The name of the composed kernel

Method clone(): The objects of this class are cloneable with this method.

```
Usage:
KernelMulComposed$clone(deep = FALSE)
Arguments:
deep Whether to make a deep clone.
```

Examples

```
# Create a new locally periodic kernel
k_loc_per <- KernelMulComposed$new(
  left_kernel = KernelSE$new(),
  right_kernel = KernelPeriodic$new(),
  new_name = 'Locally Periodic Kernel'
)
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_loc_per$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_loc_per$kernel_gen()</pre>
```

KernelParameter

R6 class for kernel's hyperparameter

Description

KernelParameter contains all information and behaviour related to a kernel parameters.

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Public fields

```
value The hyperparameter mean's prior value or its constant value is_fixed Says if the kernel parameter is fixed or not (if fixed, there is no sampling) lower_bound The hyperparameter's minimal value during sampling upper_bound The hyperparameter's maximal value during sampling slice_sampling_scale The sampling range's amplitude hparam_precision Precision of the hyperparameter kernel The kernel associated with the parameter (it is set at kernel instanciation) name The kernel parameter's name
```

Active bindings

full_name The kernel parameter's full name

Methods

Public methods:

Usage:

```
• KernelParameter$new()
```

- KernelParameter\$set_kernel()
- KernelParameter\$clone()

Method new(): Create a new KernelParameter object.

```
Usage:
KernelParameter$new(
  value,
  is_fixed = FALSE,
  lower_bound = DEFAULT_LBOUND,
  upper_bound = DEFAULT_UBOUND,
  slice_sampling_scale = log(10),
  hparam_precision = 1
)
Arguments:
value Numeric: The hyperparameter mean's prior value (Paper - \phi) or its constant value
is_fixed Boolean: Says if the kernel parameter is fixed or not (if fixed, there is no sampling)
lower_bound Numeric: Hyperparameter's minimal value during sampling (Paper - \phi_{min})
upper_bound Numeric: Hyperparameter's maximal value during sampling (Paper - \phi_{max})
slice_sampling_scale Numeric: The sampling range's amplitude (Paper - \rho)
hparam_precision Numeric: The hyperparameter's precision
Returns: A new KernelParameter object.
```

Method set_kernel(): Set Kernel for a given KernelParameter object.

KernelParameter\$set_kernel(kernel, param_name)

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```
Arguments:
```

kernel Kernel: The kernel to associate with the parameter

param_name String: The parameter's name

Returns: NULL, set a new kernel for the parameter

Method clone(): The objects of this class are cloneable with this method.

Usage:

KernelParameter\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

Examples

```
# A kernel parameter can be a constant value
const_param <- KernelParameter$new(7, is_fixed = TRUE)
# It can otherwise be sampled and have its value updated through sampling
samp_param <- KernelParameter$new(1, lower_bound = 0.1,
    upper_bound = 10, slice_sampling_scale = 4)
# A kernel parameter can be associated with any type of kernel
KernelPeriodic$new(period_length = const_param, lengthscale = samp_param)</pre>
```

KernelPeriodic

R6 class for Periodic Kernels

Description

R6 class for Periodic Kernels

Super class

```
BKTR::Kernel -> KernelPeriodic
```

Public fields

lengthscale The lengthscale parameter instance of the kernel period_length The period length parameter instance of the kernel has_dist_matrix Identify if the kernel has a distance matrix or not name The kernel's name

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Methods

```
Public methods:
```

```
• KernelPeriodic$new()
  • KernelPeriodic$core_kernel_fn()
  • KernelPeriodic$clone()
Method new(): Create a new KernelPeriodic object.
 Usage:
 KernelPeriodic$new(
    lengthscale = KernelParameter$new(2),
   period_length = KernelParameter$new(2),
   kernel_variance = 1,
    jitter_value = NULL
 )
 Arguments:
 lengthscale KernelParameter: The lengthscale parameter instance of the kernel
 period_length KernelParameter: The period length parameter instance of the kernel
 kernel_variance Numeric: The variance of the kernel
 jitter_value Numeric: The jitter value to add to the kernel matrix
 Returns: A new KernelPeriodic object.
Method core_kernel_fn(): Method to compute the core kernel's covariance matrix
 KernelPeriodic$core_kernel_fn()
 Returns: The core kernel's covariance matrix
Method clone(): The objects of this class are cloneable with this method.
 Usage:
 KernelPeriodic$clone(deep = FALSE)
 Arguments:
 deep Whether to make a deep clone.
```

Examples

```
# Create a new Periodic kernel
k_periodic <- KernelPeriodic$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_periodic$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_periodic$kernel_gen()</pre>
```

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KernelRQ

R6 class for Rational Quadratic Kernels

Description

R6 class for Rational Quadratic Kernels

Super class

```
BKTR::Kernel -> KernelRQ
```

Public fields

```
lengthscale The lengthscale parameter instance of the kernel alpha The alpha parameter instance of the kernel has_dist_matrix The distance matrix between points in a tensor format name The kernel's name
```

Methods

Public methods:

```
• KernelRQ$new()
```

- KernelRQ\$core_kernel_fn()
- KernelRQ\$clone()

Method new(): Create a new KernelRQ object.

```
Usage:
KernelRQ$new(
  lengthscale = KernelParameter$new(2),
  alpha = KernelParameter$new(2),
  kernel_variance = 1,
  jitter_value = NULL
)
```

Arguments:

lengthscale KernelParameter: The lengthscale parameter instance of the kernel alpha KernelParameter: The alpha parameter instance of the kernel kernel_variance Numeric: The variance of the kernel jitter_value Numeric: The jitter value to add to the kernel matrix *Returns:* A new KernelRQ object.

Method core_kernel_fn(): Method to compute the core kernel's covariance matrix *Usage*:

```
KernelRQ$core_kernel_fn()
```

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Returns: The core kernel's covariance matrix

Method clone(): The objects of this class are cloneable with this method.

```
Usage:
KernelRQ$clone(deep = FALSE)
Arguments:
deep Whether to make a deep clone.
```

Examples

```
# Create a new RQ kernel
k_rq <- KernelRQ$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_rq$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_rq$kernel_gen()</pre>
```

KernelSE

R6 class for Square Exponential Kernels

Description

R6 class for Square Exponential Kernels

Super class

```
BKTR::Kernel -> KernelSE
```

Public fields

lengthscale The lengthscale parameter instance of the kernel has_dist_matrix Identify if the kernel has a distance matrix or not name The kernel's name

Methods

Public methods:

- KernelSE\$new()
- KernelSE\$core_kernel_fn()
- KernelSE\$clone()

Method new(): Create a new KernelSE object.

Usage:

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```
KernelSE$new(
    lengthscale = KernelParameter$new(2),
   kernel_variance = 1,
    jitter_value = NULL
 )
 Arguments:
 lengthscale KernelParameter: The lengthscale parameter instance of the kernel
 kernel_variance Numeric: The variance of the kernel
 jitter_value Numeric: The jitter value to add to the kernel matrix
 Returns: A new KernelSE object.
Method core_kernel_fn(): Method to compute the core kernel's covariance matrix
 KernelSE$core_kernel_fn()
 Returns: The core kernel's covariance matrix
Method clone(): The objects of this class are cloneable with this method.
 Usage:
 KernelSE$clone(deep = FALSE)
 Arguments:
 deep Whether to make a deep clone.
```

Examples

```
# Create a new SE kernel
k_se <- KernelSE$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_se$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_se$kernel_gen()</pre>
```

KernelWhiteNoise

R6 class for White Noise Kernels

Description

R6 class for White Noise Kernels

Super class

```
BKTR::Kernel -> KernelWhiteNoise
```

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Public fields

```
has_dist_matrix Identify if the kernel has a distance matrix or not name The kernel's name
```

Methods

Public methods:

- KernelWhiteNoise\$new()
- KernelWhiteNoise\$core_kernel_fn()
- KernelWhiteNoise\$clone()

Method new():

```
Usage:
KernelWhiteNoise$new(kernel_variance = 1, jitter_value = NULL)
Arguments:
kernel_variance Numeric: The variance of the kernel
jitter_value Numeric: The jitter value to add to the kernel matrix
Returns: A new KernelWhiteNoise object.
```

Method core_kernel_fn(): Method to compute the core kernel's covariance matrix

Usage:

KernelWhiteNoise\$core_kernel_fn()

Returns: The core kernel's covariance matrix

Method clone(): The objects of this class are cloneable with this method.

Usage:

KernelWhiteNoise\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

Examples

```
# Create a new white noise kernel
k_white_noise <- KernelWhiteNoise$new()
# Set the kernel's positions
positions_df <- data.frame(x=c(-4, 0, 3), y=c(-2, 0, 2))
k_white_noise$set_positions(positions_df)
# Generate the kernel's covariance matrix
k_white_noise$kernel_gen()</pre>
```

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plot_beta_dists

Plot Beta Coefficients Distribution

Description

Plot the distribution of beta values for a given list of labels.

Usage

```
plot_beta_dists(
   bktr_reg,
   labels_list,
   show_figure = TRUE,
   fig_width = 9,
   fig_height = 6,
   fig_resolution = 200
)
```

Arguments

bktr_reg BKTRRegressor: BKTRRegressor object.

labels_list List: List of labels tuple (spatial, temporal, feature) for which to plot the beta

distribution through iterations

show_figure Boolean: Whether to show the figure. Defaults to True.

fig_width Integer: Figure width. Defaults to 9. fig_height Integer: Figure height. Defaults to 6.

fig_resolution Numeric: Figure resolution PPI. Defaults to 200.

Value

ggplot or NULL: ggplot object or NULL if show_figure is set to FALSE.

Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
    data_df <- bixi_data$data_df,
    spatial_positions_df = bixi_data$spatial_positions_df,
    temporal_positions_df = bixi_data$temporal_positions_df,
    burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot temporal beta coefficients for the first station and the first feature
spa_lab <- bixi_data$spatial_positions_df$location[3]
plot_beta_dists(
    bktr_regressor,</pre>
```

```
labels_list = list(
    c(spa_lab, '2019-04-15', 'area_park'),
    c(spa_lab, '2019-04-16', 'area_park'),
    c(spa_lab, '2019-04-16', 'mean_temp_c')
),
)
```

plot_covariates_beta_dists

Plot Beta Coefficients Distribution Regrouped by Covariates

Description

Plot the distribution of beta estimates regrouped by covariates.

Usage

```
plot_covariates_beta_dists(
  bktr_reg,
  feature_labels = NULL,
  show_figure = TRUE,
  fig_width = 9,
  fig_height = 6,
  fig_resolution = 200
)
```

Arguments

bktr_reg BKTRRegressor: BKTRRegressor object.

feature_labels Array or NULL: Array of feature labels for which to plot the beta estimates

distribution. If NULL plot for all features.

show_figure Boolean: Whether to show the figure. Defaults to True.

fig_width Integer: Figure width. Defaults to 9.
fig_height Integer: Figure height. Defaults to 6.

fig_resolution Numeric: Figure resolution PPI. Defaults to 200.

Value

ggplot or NULL: ggplot object or NULL if show_figure is set to FALSE.

Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
   formula = 'nb_departure ~ 1 + area_park + mean_temp_c + total_precip_mm',
   data_df <- bixi_data$data_df,
   spatial_positions_df = bixi_data$spatial_positions_df,
   temporal_positions_df = bixi_data$temporal_positions_df,
   burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot beta estimates distribution for all features
plot_covariates_beta_dists(bktr_regressor)
# Or plot for a subset of features
plot_covariates_beta_dists(bktr_regressor, c('area_park', 'mean_temp_c'))</pre>
```

plot_hyperparams_dists

Plot Hyperparameters Distributions

Description

Plot the distribution of hyperparameters through iterations

Usage

```
plot_hyperparams_dists(
   bktr_reg,
   hyperparameters = NULL,
   show_figure = TRUE,
   fig_width = 9,
   fig_height = 6,
   fig_resolution = 200
)
```

Arguments

bktr_reg BKTRRegressor: BKTRRegressor object.

hyperparameters

Array or NULL: Array of hyperparameters to plot. If NULL, plot all hyperpa-

rameters. Defaults to NULL.

show_figure Boolean: Whether to show the figure. Defaults to True.

fig_width Integer: Figure width. Defaults to 9. fig_height Integer: Figure height. Defaults to 6.

fig_resolution Numeric: Figure resolution PPI. Defaults to 200.

Value

ggplot or NULL: ggplot object or NULL if show_figure is set to FALSE.

Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)</pre>
k_matern <- KernelMatern$new()</pre>
k_periodic <- KernelPeriodic$new()</pre>
bktr_regressor <- BKTRRegressor$new(</pre>
  data_df <- bixi_data$data_df,</pre>
  spatial_kernel = k_matern,
  temporal_kernel = k_periodic,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()
# Plot the distribution of all hyperparameters
plot_hyperparams_dists(bktr_regressor)
# Plot the distribution of the spatial kernel hyperparameters
spa_par_name <- paste0('Spatial - ', k_matern$parameters[[1]]$full_name)</pre>
plot_hyperparams_dists(bktr_regressor, spa_par_name)
# Plot the distribution of the temporal kernel hyperparameters
temp_par_names <- sapply(k_periodic$parameters, function(x) x$full_name)</pre>
temp_par_names <- paste0('Temporal - ', temp_par_names)</pre>
plot_hyperparams_dists(bktr_regressor, temp_par_names)
```

```
plot_hyperparams_traceplot
```

Plot Hyperparameters Traceplot

Description

Plot the evolution of hyperparameters through iterations. (Traceplot)

Usage

```
plot_hyperparams_traceplot(
   bktr_reg,
   hyperparameters = NULL,
   show_figure = TRUE,
   fig_width = 9,
   fig_height = 5.5,
   fig_resolution = 200
)
```

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Arguments

bktr_reg BKTRRegressor object.

hyperparameters

Array or NULL: Array of hyperparameters to plot. If NULL, plot all hyperpa-

rameters. Defaults to NULL.

show_figure Boolean: Whether to show the figure. Defaults to True.

fig_width Integer: Figure width. Defaults to 9.
fig_height Integer: Figure height. Defaults to 5.5.

fig_resolution Numeric: Figure resolution PPI. Defaults to 200.

Value

ggplot or NULL: ggplot object or NULL if show_figure is set to FALSE.

Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)</pre>
k_matern <- KernelMatern$new()</pre>
k_periodic <- KernelPeriodic$new()</pre>
bktr_regressor <- BKTRRegressor$new(</pre>
  data_df <- bixi_data$data_df,</pre>
  spatial_kernel = k_matern,
  temporal_kernel = k_periodic,
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()
# Plot the traceplot of all hyperparameters
plot_hyperparams_traceplot(bktr_regressor)
# Plot the traceplot of the spatial kernel hyperparameters
spa_par_name <- paste0('Spatial - ', k_matern$parameters[[1]]$full_name)</pre>
plot_hyperparams_traceplot(bktr_regressor, spa_par_name)
# Plot the traceplot of the temporal kernel hyperparameters
temp_par_names <- sapply(k_periodic*parameters, function(x) x*full_name)</pre>
temp_par_names <- paste0('Temporal - ', temp_par_names)</pre>
plot_hyperparams_traceplot(bktr_regressor, temp_par_names)
```

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Description

Create a plot of beta values through space for a given temporal point and a set of feature labels. We use ggmap under the hood, so you need to provide a Google or Stadia API token to plot on a map. See: https://cran.r-project.org/web/packages/ggmap/readme/README.html for more details on how to get an API token.

Usage

```
plot_spatial_betas(
   bktr_reg,
   plot_feature_labels,
   temporal_point_label,
   nb_cols = 1,
   use_dark_mode = TRUE,
   show_figure = TRUE,
   zoom = 11,
   google_token = NULL,
   stadia_token = NULL,
   fig_width = 8.5,
   fig_height = 5.5,
   fig_resolution = 200
)
```

Arguments

```
bktr_reg
                  BKTRRegressor: BKTRRegressor object.
plot_feature_labels
                  Array: Array of feature labels to plot.
temporal_point_label
                  String: Temporal point label to plot.
                  Integer: The number of columns to use in the facet grid.
nb_cols
use_dark_mode
                  Boolean: Whether to use a dark mode for the geographic map or not. Defaults
                  to TRUE.
show_figure
                  Boolean: Whether to show the figure. Defaults to True.
zoom
                  Integer: Zoom level for the geographic map. Defaults to 11.
google_token
                  String or NULL: Google API token to use for the geographic map. Defaults to
                  NULL.
stadia_token
                  String or NULL: Stadia API token to use for the geographic map. Defaults to
                  NULL.
fig_width
                  Numeric: Figure width when figure is shown. Defaults to 8.5.
fig_height
                  Numeric: Figure height when figure is shown. Defaults to 5.5.
fig_resolution Numeric: Figure resolution PPI. Defaults to 200.
```

Value

ggplot or NULL: ggplot object or NULL if show_figure is set to FALSE.

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Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)</pre>
bktr_regressor <- BKTRRegressor$new(</pre>
  data_df <- bixi_data$data_df,</pre>
  spatial_positions_df = bixi_data$spatial_positions_df,
  temporal_positions_df = bixi_data$temporal_positions_df,
  burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()
# Plot spatial beta coefficients for the first time point and the two features
# Use your own Google API token instead of 'GOOGLE_API_TOKEN'
plot_spatial_betas(
  bktr_regressor,
  plot_feature_labels = c('mean_temp_c', 'area_park'),
  temporal_point_label = bixi_data$temporal_positions_df$time[1],
  google_token = 'GOOGLE_API_TOKEN')
# We can also use light mode and plot the maps side by side
# Use your own Stadia API token instead of 'STADIA_API_TOKEN'
plot_spatial_betas(
  bktr_regressor,
  plot_feature_labels = c('mean_temp_c', 'area_park', 'total_precip_mm'),
  temporal_point_label = bixi_data$temporal_positions_df$time[10],
  use_dark_mode = FALSE, nb_cols = 3, stadia_token = 'STADIA_API_TOKEN')
```

plot_temporal_betas

Plot Temporal Beta Coefficients

Description

Create a plot of the beta values through time for a given spatial point and a set of feature labels.

Usage

```
plot_temporal_betas(
  bktr_reg,
  plot_feature_labels,
  spatial_point_label,
  date_format = "%Y-%m-%d",
  show_figure = TRUE,
  fig_width = 8.5,
  fig_height = 5.5,
  fig_resolution = 200
)
```

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Arguments

```
bktr_reg
                  BKTRRegressor: BKTRRegressor object.
plot_feature_labels
                  Array: Array of feature labels to plot.
spatial_point_label
                  String: Spatial point label to plot.
date_format
                  String: Format of the date to use in bktr dataframes for the time. Defaults to
                  '%Y-%m-%d'.
show_figure
                  Boolean: Whether to show the figure. Defaults to True.
fig_width
                 Numeric: Figure width when figure is shown. Defaults to 8.5.
fig_height
                 Numeric: Figure height when figure is shown. Defaults to 5.5.
fig_resolution Numeric: Figure resolution PPI. Defaults to 200.
```

Value

ggplot or NULL: ggplot object or NULL if show_figure is set to FALSE.

Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
    data_df <- bixi_data$data_df,
    spatial_positions_df = bixi_data$spatial_positions_df,
    temporal_positions_df = bixi_data$temporal_positions_df,
    burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot temporal beta coefficients for the first station and the two features
plot_temporal_betas(
    bktr_regressor,
    plot_feature_labels = c('mean_temp_c', 'area_park'),
    spatial_point_label = bixi_data$spatial_positions_df$location[1])</pre>
```

plot_y_estimates

Plot Y Estimates

Description

Plot y estimates vs observed y values.

Usage

```
plot_y_estimates(
   bktr_reg,
   show_figure = TRUE,
   fig_width = 5,
   fig_height = 5,
   fig_resolution = 200,
   fig_title = "y estimates vs observed y values"
)
```

Arguments

bktr_reg BKTRRegressor: BKTRRegressor object.

show_figure Boolean: Whether to show the figure. Defaults to True.

fig_width Numeric: Figure width when figure is shown. Defaults to 5.

fig_height Numeric: Figure height when figure is shown. Defaults to 5.

fig_resolution Numeric: Figure resolution PPI when figure is shown. Defaults to 200.

fig_title String or NULL: Figure title if provided. Defaults to 'y estimates vs observed y values'

Value

ggplot or NULL: ggplot object or NULL if show_figure is set to FALSE.

Examples

```
# Launch MCMC sampling on a light version of the BIXI dataset
bixi_data <- BixiData$new(is_light = TRUE)
bktr_regressor <- BKTRRegressor$new(
   data_df <- bixi_data$data_df,
   spatial_positions_df = bixi_data$spatial_positions_df,
   temporal_positions_df = bixi_data$temporal_positions_df,
   burn_in_iter = 5, sampling_iter = 10) # For example only (too few iterations)
bktr_regressor$mcmc_sampling()

# Plot Y estimates vs observed y values
plot_y_estimates(bktr_regressor)</pre>
```

print.BKTRRegressor

Print the summary of a BKTRRegressor instance

Description

Print the summary of a BKTRRegressor instance

39 reshape_covariate_dfs

Usage

```
## S3 method for class 'BKTRRegressor'
print(x, ...)
```

Arguments

A BKTRRegressor instance Х

Additional arguments to comply with generic function

reshape_covariate_dfs Function used to transform covariates coming from two dataframes one for spatial and one for temporal into a single dataframe with the right shape for the BKTR Regressor. This is useful when the temporal covariates do not vary trough space and the spatial covariates do not vary trough time (Like in the BIXI example). The function also adds a column for the target variable at the beginning of the dataframe.

Description

Function used to transform covariates coming from two dataframes one for spatial and one for temporal into a single dataframe with the right shape for the BKTR Regressor. This is useful when the temporal covariates do not vary trough space and the spatial covariates do not vary trough time (Like in the BIXI example). The function also adds a column for the target variable at the beginning of the dataframe.

Usage

```
reshape_covariate_dfs(spatial_df, temporal_df, y_df, y_column_name = "y")
```

Arguments

data.table: Spatial covariates dataframe with an index named location and a spatial_df

shape of (n_locations, n_spatial_covariates)

temporal_df data.table: Temporal covariates dataframe with an index named time and a shape

of (n_times, n_temporal_covariates)

data.table: The dataframe containing the target variable. It should have a shape y_df

of (n locations, n times). The columns and index names of this dataframe

should be correspond to the one of the spatial df and temporal df.

y_column_name string: The name of the target variable column in y_df. Default to 'y'.

Value

data.table: The reshaped covariates dataframe with a shape of (n_locations * n_times, 1 + n_spatial_covariates + n_temporal_covariates). The first two columns are the indexes (location, time), the following column is the target variable and the other columns are the covariates.

Examples

```
# Let's reshape the BIXI dataframes without normalization
new_data_df <- reshape_covariate_dfs(
    spatial_df = BKTR::bixi_spatial_features,
    temporal_df = BKTR::bixi_temporal_features,
    y_df = BKTR::bixi_station_departures,
    y_column_name = 'whole_nb_departure')
# The resulting dataframe has the right shape to be a BKTRRegressor data_df
head(new_data_df)</pre>
```

```
simulate_spatiotemporal_data
```

Simulate Spatiotemporal Data Using Kernel Covariances.

Description

Simulate Spatiotemporal Data Using Kernel Covariances.

Usage

```
simulate_spatiotemporal_data(
  nb_locations,
  nb_time_points,
  nb_spatial_dimensions,
  spatial_scale,
  time_scale,
  spatial_covariates_means,
  temporal_covariates_means,
  spatial_kernel,
  temporal_kernel,
  noise_variance_scale
)
```

Arguments

Value

A list containing 4 dataframes: - 'data_df' contains the response variable and the covariates - 'spatial_positions_df' contains the spatial locations and their coordinates - 'temporal_positions_df' contains the time points and their coordinates - 'beta_df' contains the true beta coefficients

Examples

```
# Simulate data with 20 locations, 30 time points, in 2D spatial dimensions
# with 3 spatial covariates and 2 temporal covariates
simu_data <- simulate_spatiotemporal_data(</pre>
   nb_locations=20,
   nb_time_points=30,
   nb_spatial_dimensions=2,
   spatial_scale=10,
   time_scale=10,
   spatial_covariates_means=c(0, 2, 4),
   temporal_covariates_means=c(1, 3),
   spatial_kernel=KernelMatern$new(),
   temporal_kernel=KernelSE$new(),
   noise_variance_scale=1)
# The dataframes are similar to bixi_data, we have:
# - data_df
head(simu_data$data_df)
# - spatial_positions_df
head(simu_data$spatial_positions_df)
# - temporal_positions_df
head(simu_data$temporal_positions_df)
# We also obtain the true beta coefficients used to simulate the data
head(simu_data$beta_df)
```

summary.BKTRRegressor Summarize a BKTRRegressor instance

Description

Summarize a BKTRRegressor instance

Usage

```
## S3 method for class 'BKTRRegressor'
summary(object, ...)
```

Arguments

object A BKTRRegressor instance
... Additional arguments to comply with generic function

TensorOperator

R6 singleton that contains the configuration for the tensor backend

Description

Tensor backend configuration and methods for all the tensor operations in BKTR

Super class

```
R6P::Singleton -> TensorOperator
```

Public fields

fp_type The floating point type to use for the tensor operations
fp_device The device to use for the tensor operations

Methods

Public methods:

- TensorOperator\$new()
- TensorOperator\$set_params()
- TensorOperator\$get_default_jitter()
- TensorOperator\$tensor()
- TensorOperator\$is_tensor()
- TensorOperator\$eye()
- TensorOperator\$ones()
- TensorOperator\$zeros()
- TensorOperator\$rand()
- TensorOperator\$randn()
- TensorOperator\$randn_like()
- TensorOperator\$arange()
- TensorOperator\$rand_choice()
- TensorOperator\$kronecker_prod()
- TensorOperator\$khatri_rao_prod()
- TensorOperator\$clone()

```
Method new(): Initialize the tensor operator with the given floating point type and device
 TensorOperator$new(fp_type = "float64", fp_device = "cpu")
 Arguments:
 fp_type The floating point type to use for the tensor operations (either "float64" or "float32")
 fp_device The device to use for the tensor operations (either "cpu" or "cuda")
 Returns: A new tensor operator instance
Method set_params(): Set the tensor operator parameters
 TensorOperator$set_params(fp_type = NULL, fp_device = NULL, seed = NULL)
 Arguments:
 fp_type The floating point type to use for the tensor operations (either "float64" or "float32")
 fp_device The device to use for the tensor operations (either "cpu" or "cuda")
 seed The seed to use for the random number generator
Method get_default_jitter(): Get the default jitter value for the floating point type used by
the tensor operator
 Usage:
 TensorOperator$get_default_jitter()
 Returns: The default jitter value for the floating point type used by the tensor operator
Method tensor(): Create a tensor from a vector or matrix of data with the tensor operator dtype
and device
 Usage:
 TensorOperator$tensor(tensor_data)
 Arguments:
 tensor_data The vector or matrix of data to create the tensor from
 Returns: A new tensor with the tensor operator dtype and device
Method is_tensor(): Check if a provided object is a tensor
 Usage:
 TensorOperator$is_tensor(tensor)
 Arguments:
 tensor The object to check
 Returns: A boolean indicating if the object is a tensor
Method eye(): Create a tensor with a diagonal of ones and zeros with the tensor operator dtype
and device for a given dimension
 TensorOperator$eye(eye_dim)
 Arguments:
```

eye_dim The dimension of the tensor to create

Returns: A new tensor with a diagonal of ones and zeros with the tensor operator dtype and device

Method ones(): Create a tensor of ones with the tensor operator dtype and device for a given dimension

Usage:

TensorOperator\$ones(tsr_dim)

Arguments:

tsr_dim The dimension of the tensor to create

Returns: A new tensor of ones with the tensor operator dtype and device

Method zeros(): Create a tensor of zeros with the tensor operator dtype and device for a given dimension

Usage:

TensorOperator\$zeros(tsr_dim)

Arguments:

tsr_dim The dimension of the tensor to create

Returns: A new tensor of zeros with the tensor operator dtype and device

Method rand(): Create a tensor of random uniform values with the tensor operator dtype and device for a given dimension

Usage:

TensorOperator\$rand(tsr_dim)

Arguments:

tsr_dim The dimension of the tensor to create

Returns: A new tensor of random values with the tensor operator dtype and device

Method randn(): Create a tensor of random normal values with the tensor operator dtype and device for a given dimension

Usage:

TensorOperator\$randn(tsr_dim)

Arguments:

tsr_dim The dimension of the tensor to create

Returns: A new tensor of random normal values with the tensor operator dtype and device

Method randn_like(): Create a tensor of random uniform values with the same shape as a given tensor with the tensor operator dtype and device

Usage:

TensorOperator\$randn_like(input_tensor)

Arguments:

input_tensor The tensor to use as a shape reference

Returns: A new tensor of random uniform values with the same shape as a given tensor

Method arange(): Create a tensor of a range of values with the tensor operator dtype and device for a given start and end

```
Usage:
TensorOperator$arange(start, end)
Arguments:
start The start of the range
end The end of the range
```

Returns: A new tensor of a range of values with the tensor operator dtype and device

Method rand_choice(): Choose random values from a tensor for a given number of samples

```
Usage:
TensorOperator$rand_choice(
  choices_tsr,
  nb_sample,
  use_replace = FALSE,
  weights_tsr = NULL
```

Arguments:

choices_tsr The tensor to choose values from

nb_sample The number of samples to choose

use_replace A boolean indicating if the sampling should be done with replacement. Defaults to FALSE

weights_tsr The weights to use for the sampling. If NULL, the sampling is uniform. Defaults to NULL

Returns: A new tensor of randomly chosen values from a tensor

Method kronecker_prod(): Efficiently compute the kronecker product of two matrices in tensor format

Usage:

TensorOperator\$kronecker_prod(a, b)

Arguments:

- a The first tensor
- b The second tensor

Returns: The kronecker product of the two matrices

Method khatri_rao_prod(): Efficiently compute the khatri rao product of two matrices in tensor format having the same number of columns

```
Usage:
```

```
TensorOperator$khatri_rao_prod(a, b)
```

Arguments:

a The first tensor

b The second tensor

Returns: The khatri rao product of the two matrices

Method clone(): The objects of this class are cloneable with this method.

Usage:

TensorOperator\$clone(deep = FALSE)

Arguments.

deep Whether to make a deep clone.

Examples

```
# Set the seed, setup the tensor floating point type and device
TSR$set_params(fp_type='float64', fp_device='cpu', seed=42)
# Create a tensor from a vector
TSR$tensor(c(1, 2, 3))
# Create a tensor from a matrix
TSR$tensor(matrix(c(1, 2, 3, 4), nrow=2))
# Create a 3x3 tensor with a diagonal of ones and zeros elsewhere
TSR$eye(3)
# Create a tensor of ones (with 6 elements, 2 rows and 3 columns)
TSR$ones(c(2, 3))
# Create a tensor of zeros (with 12 elements, 3 rows and 4 columns)
TSR$zeros(c(3, 4))
# Create a tensor of random uniform values (with 6 elements)
TSR$rand(c(2, 3))
# Create a tensor of random normal values (with 6 elements)
TSR$randn(c(2, 3))
# Create a tensor of random normal values with the same shape as a given tensor
tsr_a \leftarrow TSR randn(c(2, 3))
TSR$randn_like(tsr_a)
# Create a tensor of a range of values (1, 2, 3, 4)
TSR$arange(1, 4)
# Choose two random values from a given tensor without replacement
tsr_b <- TSR$rand(6)</pre>
TSR$rand_choice(tsr_b, 2)
# Use the tensor operator to compute the kronecker product of two 2x2 matrices
tsr_c \leftarrow TSR\$tensor(matrix(c(1, 2, 3, 4), nrow=2))
tsr_d \leftarrow TSR\$tensor(matrix(c(5, 6, 7, 8), nrow=2))
TSR$kronecker\_prod(tsr\_c, tsr\_d) # Returns a 4x4 tensor
# Use the tensor operator to compute the khatri rao product of two 2x2 matrices
TSR$khatri_rao_prod(tsr_c, tsr_d) # Returns a 4x2 tensor
# Check if a given object is a tensor
TSR$is_tensor(tsr_d) # Returns TRUE
TSR$is_tensor(TSR$eye(2)) # Returns TRUE
TSR$is_tensor(1) # Returns FALSE
```

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TSR Tensor Operator Singleton

Description

Singleton instance of the TensorOperator class that contains all informations related the tensor API; tensor methods, used data type and used device.

Usage

TSR

Format

An object of class TensorOperator (inherits from Singleton, R6) of length 19.

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