# Package 'ui'

# October 12, 2022

Title Uncertainty Intervals and Sensitivity Analysis for Missing Data
Version 0.1.1
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Description Implements functions to derive uncertainty intervals for (i) regression (linear and probit) parameters when outcome is missing not at random (non-ignorable missingness) introduced in Genbaeck, M., Stanghellini, E., de Luna, X. (2015) <doi:10.1007 s00362-014-0610-x=""> and Genbaeck, M., Ng, N., Stanghellini, E., de Luna, X. (2018) <doi:10.1007 s10433-017-0448-x="">; and (ii) double robust and outcome regression estimators of average causal effects (on the treated) with possibly unobserved confounding introduced in Genbaeck, M., de Luna, X. (2018) <doi:10.1111 biom.13001="">.</doi:10.1111></doi:10.1007></doi:10.1007>
<b>Depends</b> R (>= $3.5$ )
Imports Matrix, maxLik, mvtnorm, numDeriv, graphics, stats
Suggests MASS
Encoding UTF-8
LazyData true
License GPL-2
RoxygenNote 6.1.1
NeedsCompilation no
Repository CRAN
<b>Date/Publication</b> 2019-11-11 13:10:02 UTC
R topics documented:
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gridrho.f

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gridrho.f Support function for ui.causal

# Description

Divides the rho interval into a grid

#### Usage

```
gridrho.f(rho, gridn, rho.plotrange, plot)
```

# Arguments

rho interval that should be divided

gridn number of gridpoints

rho.plotrange a larger interval of grids to be used in a plot

plot whether or not the larger interval of grids should be created

grr 3

grr

Gradient for the loglikelihood used by ui.probit

#### Description

This function derives the gradient in order for ui.probit to run faster.

# Usage

```
grr(par, rho, X.z = X.z, X.y = X.y, y = y, z = z)
```

#### Arguments

par	Coefficients.
rho	Rho.
X.z	Covariate matrix for missingness.
X.y	Covariate matrix for outcome.
у	Outcome.
Z	Missing or not.

hess

Hessian for the loglikelihood used by ui.probit

#### Description

This function derives the hessian in order for ui.probit to run faster.

#### Usage

```
hess(par, rho, X.z = X.z, X.y = X.y, y = y, z = z)
```

par	Coefficients.
rho	Rho.
X.z	Covariate matrix for missingness.
X.y	Covariate matrix for outcome.
у	Outcome.
z	Missing or not.

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interv.p

Print interval in parantesis

#### **Description**

This function allows you to print an interval (vector of two elements) in a parantesis single element.

#### Usage

```
interv.p(v, digits = 3)
```

#### Arguments

v Lower and upper bounds. digits Number of decimals.

lambda0

Inverse Mills rato

#### **Description**

This function allows you to calculate the inverse Mills ratio.

# Usage

lambda0(x)

#### Arguments

Х

Vector

lambda1

Inverse Mills rato

# Description

This function allows you to calculate the inverse Mills ratio.

#### Usage

lambda1(x)

#### Arguments

Х

Vector

LogL.probit 5

telihood used by ui.probit
----------------------------

# Description

This function derives the Loglikelihood for ui.probit.

# Usage

```
LogL.probit(par, rho, X.z = X.z, X.y = X.y, y = y, z = z)
```

# Arguments

par	Coeficient values the logliklihood should be drived at.
rho	The value of the sensitivity parameter.
X.z	covariate matrix for missingness mechanism
X.y	covariate matrix for the outcome regression
у	outcome vector
z	indicator of wether y is missing or not

Log1.sandACT	Loglikelohood used in sandwich estimator of average causal effect on
	the treated for DR

# Description

Loglikelohood used in sandwich estimator of average causal effect on the treated for DR, support function for ui.causal

# Usage

```
Logl.sandACT(x, X, z)
```

# Arguments

Χ	coefficents.

X Covariate matrix.

z Missing or not.

6 plot.uicausal

ML.probit	Fit maximum likelihood for fixed values of rho

#### Description

This is a support function for ui.probit

#### Usage

```
ML.probit(out.formula, mis.formula = NULL, data, rho = c(-0.5, 0.5), progress = TRUE, method = "NR")
```

#### **Arguments**

out.formula	Formula for outcome regression.
mis.formula	Formula for regression model for the missingness mechanism.
data	Data frame containing the variables in the formulas
rho	Vector containing the values of rho for which we want to fit the likelihood.
progress	If TRUE prints out process time for each maximazation of the likelihood.
method	Maximazation method to be passed through maxLik

|--|

#### Description

Plot function for objects returned from ui.causal. Plots confidence intervals for different values of rho and the uncertainty interval.

# Usage

```
## S3 method for class 'uicausal'
plot(x, DR = TRUE, main = "", xlab = NULL,
  ylab = "", ...)
```

x	An object of class uicausal
DR	If TRUE the doubly robust estimator is plotted, otherwise the outcome regression estimator is plotted.
main	Main title, default is no title.
xlab	Title for xaxis, default is expression(rho).
ylab	Title for y axis, default is no title.
	Additional arguments, use is discouraged.

plot.uiols 7

#### **Description**

Plot function for objects returned from ui.ols. Plots confidence intervals, coefficients and significans assuming ignorability and the uncertainty interval under non-ignorability.

#### Usage

```
## S3 method for class 'uiols'
plot(x, plot.all = TRUE, which = NA,
  intercept = FALSE, ylab = NULL, col = c("black", "red"), ...)
```

#### **Arguments**

X	An object of class uiols
plot.all	If TRUE, plots all covariates.
which	Specify which variables should be plotted by either sending in their names in a vector or a vector with their numbers (1 intercept, 2 for the first covariate etc.).
intercept	If TRUE, also plots the intercept.
ylab	Vector of names for the y-axis, default is the variable names.
col	Vector containing the color of confidence intervals (default black) and uncertainty intervals (default red).
	Additional arguments, use is discouraged.

|--|

#### Description

Plot function for objects returned from ui.probit. Plots confidence intervals, coefficients and significans assuming ignorability and the uncertainty interval under non-ignorability.

#### Usage

```
## S3 method for class 'uiprobit'
plot(x, plot.all = TRUE, which = NA,
  intercept = FALSE, ylab = NULL, col = c("black", "red"), ...)
```

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#### Arguments

x	An object of class uiprobit
plot.all	If TRUE, plots all covariates.
which	Specify which variables should be plotted by either sending in their names in a vector or a vector with their numbers (1 for the first covariate, 2 for the second etc.).To plot the intercept, set intercept as TRUE.
intercept	If TRUE, also plots the intercept.
ylab	Vector of names for the y-axis, default is the variable names.
col	Vector containing the color of confidence intervals (default black) and uncertainty intervals (default red).
	Additional arguments, use is discouraged.

print.uicausal

Print function for object of class uicausal

# Description

Print function for object of class uicausal

# Usage

```
## S3 method for class 'uicausal'
print(x, digits = 3, digitsci = digits,
    digitsui = digits, ...)
```

x	An object of returned from ui.causal
digits	number of digits to be printed.
digitsci	number of digits to be printed in the confidence interval.
digitsui	number of digits to be printed in the uncertainty interval.
	Additional arguments, use is discouraged.

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#### Description

Prints objects of class uiols

#### Usage

```
## S3 method for class 'uiols'
print(x, digits = 3, digitsci = digits,
   digitsui = digits, ...)
```

#### **Arguments**

X	an objects returned from ui.ols
digits	number of digits to be printed.
digitsci	number of digits to be printed in the confidence interval.
digitsui	number of digits to be printed in the uncertainty interval.
	Additional arguments, use is discouraged.

print.uiprobit Prints objects of class uiprobit

#### Description

Prints objects of class uiprobit

#### Usage

```
## S3 method for class 'uiprobit'
print(x, digits = 3, digitsci = digits,
    digitsui = digits, ...)
```

```
x an objects returned from ui.probit
digits number of digits to be printed.
digitsci number of digits to be printed in the confidence interval.
digitsui number of digits to be printed in the uncertainty interval.
... Additional arguments, use is discouraged.
```

profile.uiols

profile.uicausal	Plot of UI and CI
	· · · · · · · · · · · · · · · · · · ·

# Description

Plot function for objects returned from ui.causal. Plots confidence intervals for different values of rho0=rho1=rho.

#### Usage

```
## S3 method for class 'uicausal'
profile(fitted, DR = TRUE, main = "", xlab = NULL,
  ylab = "", ...)
```

#### **Arguments**

fitted	An object of class uicausal
DR	If TRUE, plots both DR if FALSE OR.
main	Main title, default is no title.
xlab	Title for x-axis, default is expression(rho).
ylab	Title for y-axis, default is the variable names.
	Additional arguments, use is discouraged.

profile.uiols Plot of UI and CI

#### **Description**

Plot function for objects returned from ui.ols. Plots confidence intervals for different values of rho and the uncertainty interval.

# Usage

```
## S3 method for class 'uiols'
profile(fitted, plot.all = TRUE, which = NA,
  intercept = FALSE, xlab = NULL, ylab = NULL, ...)
```

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#### Arguments

fitted	An object of class uiols
plot.all	If TRUE, plots all covariates.
which	Specify which variables should be plotted by either sending in their names in a vector or a vector with their numbers (1 intercept, 2 for the first covariate etc.).
intercept	If TRUE, also plots the intercept.
xlab	Title for x-axis, default is expression(rho).
ylab	Title for y-axis, default is the variable names.
	Additional arguments, for instance margins.

profile.uiprobit Plot of UI and CI

# Description

Plot function for objects returned from ui.probit. Plots confidence intervals for different values of rho and the uncertainty interval.

#### Usage

```
## S3 method for class 'uiprobit'
profile(fitted, plot.all = TRUE, which = NA,
  intercept = FALSE, xlab = NULL, ylab = NULL, cex.lab = 2,
  mar = c(6, 6, 2, 2), ...)
```

fitted	An object of class uiprobit
plot.all	If TRUE, plots all covariates.
which	Specify which variables should be plotted by either sending in their names in a vector or a vector with their numbers (1 intercept, 2 for the first covariate etc.).
intercept	If TRUE, also plots the intercept.
xlab	Title for x-axis, default is expression(rho).
ylab	Title for y-axis, default is the variable names.
cex.lab	Size of lables.
mar	Margin around panels in plot.
	Additional arguments, use is discouraged.

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sandACT	Calculates standard error of Average causal effect on the treated

#### Description

This is a support function for ui.causal and calculates standard error of Average causal effect on the treated for the doubly robust estimator.

#### Usage

```
sandACT(deltasigma1, X, Xz, y, z, u, BetaOLSy0, phat, NaivEst, n1, n0, N,
    p, pz)
```

#### Arguments

deltasigma1	Coefficients.
Χ	Covariate matrix outcome.
Xz	Covariate matrix treatment.
у	Outcome vector.
Z	Missingness indicator.
u	Fitted values from propensity score regression.
BetaOLSy0	Coefficients from non-treated regression
phat	Fitted propensity scores.
NaivEst	Naiv estimates.
n1	Number of treated.
n0	Number of non-treated.
N	Total number.
р	Number of covariates outcome regression.
pz	Number of covariates treatment regression.
sandImpACE	Calculates standard error of Average causal effect

#### **Description**

This is a support function for ui.causal and calculates standard error of Average causal effect for the regression imputation estimator.

#### Usage

```
sandImpACE(X, y, z, BetaOLSy0, BetaOLSy1, NaivEst, N, p)
```

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# Arguments

Χ	Covariate matrix.
у	Outcome vector.
z	missingness indicator.
BetaOLSy0	Coefficients from non-treated regression.
BetaOLSy1	Coefficients from treated regression.
NaivEst	Naiv estimates.
N	Total number.
р	Number of covariates outcome regression.

sandImpACT Calculates standard error of Average causal effect on the treated

# Description

This is a support function for ui.causal and calculates standard error of Average causal effect on the treated for the regression imputation estimator.

#### Usage

```
sandImpACT(X, y, z, BetaOLSy0, NaivEst, n1, N, p)
```

Χ	Covariate matrix.
у	Outcome vector.
z	missingness indicator
BetaOLSy0	Coefficients from non-treated regression
NaivEst	Naiv estimates.
n1	Number of treated.
N	Total number.
р	Number of covariates outcome regression.

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		-	
Se	0	П	5

#### Calculation of se for OLS

#### Description

This function calculates the se for UI based on OLS when we have MNAR data, for ui.ols.

#### Usage

```
se.ols(X, sigmaOLScor, u, gridrho)
```

#### **Arguments**

X Covariate matrix.

sigmaOLScor Output from sigmaOLScor1
u Fitted values from mis.model.

gridrho Values of rho.

sigmaOLScor0

Correction of OLS sigma for causal effects

#### **Description**

This function is a bias correction of the residual standard deviation under MNAR, for ui.causal.

#### Usage

```
sigmaOLScor0(X, sigmaOLS, n, p, u, gridrho)
```

#### **Arguments**

sigmaOLS Residual sd from outcome regression.

n Number of complete cases.

p Number of covariates outcome regression.

u Fitted values from propensity score regression.

gridrho Values of rho.

sigmaOLScor1 15

sigmaOLScor1	Correction of OLS sigma	

#### Description

This function is a bias correction of the residual standard deviation under MNAR, used by ui.causal and ui.ols.

#### Usage

```
sigmaOLScor1(X, sigmaOLS, n, p, u, gridrho)
```

#### Arguments

Χ	Covariate matrix outcome.
sigmaOLS	Residual sd from outcome regression.
n	Number of complete cases.
p	Number of covariates outcome regression.
u	Fitted values from propensity score regression.
gridrho	Values of rho.

ui.causal	Uncertainty intervals for Average Causal Effects

#### **Description**

This function allows you to derive uncertainty intervals for the average causal effect (ACE) or the average causal effect on the treated (ACT). The function uses a regression imputation estimator and a doubly robust estimator. The uncertainty intervals can be used as a sensitivity analysis to unconfoundedness. Note that rho=0 render the same results as assuming no unobserved confounding.

# Usage

```
ui.causal(out.formula, treat.formula, data, rho = c(-0.3, 0.3), rho0 = NULL, rho1 = NULL, ACT = FALSE, sand = TRUE, gridn = 21, plot = TRUE, rho.plotrange = c(-0.5, 0.5), alpha = 0.05)
```

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#### **Arguments**

out.formula Formula for the outcome regression models treat.formula Formula for the propensity score model (regression model for treatment assignment). data.frame containing the variables in the formula. data rho Pre-specified interval for rho0 and rho1. Pre-specified value of rho0, if an interval it has to be the same as rho1. rho0 rho1 Pre-specified value of rho1, if an interval it has to be the same as rho0. ACT If TRUE Average Causal effect of the Treated is calculated, if FALSE Average Causal effect is calculated. Default is FALSE. sand Specifies which estimator of the standard errors should be used for OR, see details. gridn Number of fixed points within the rho interval for which sigma0 and sigma1 should be estimated. If TRUE the function runs slightly slower but you will be able to plot your results plot using plot.uicausal. rho.plotrange an interval larger than rho for the plot using plot.uicausal. alpha Default 0.05 corresponding to a confidence level of 95 for CI and UI.

#### **Details**

In order to visualize the results, you can use plot.uicausal. Details about estimators can be found in Genbäck and de Luna (2018)

The standard errors are calculated with the following estimators:

DR ACE - simplified sandwich estimator

DR ACT - sandwich estimator

OR ACE - if sand=TRUE sandwich estimator (default and recommended), if sand=FALSE large sample variance

OR ACT - if sand=TRUE sandwich estimator (default and recommended), if sand=FALSE large sample variance

#### Value

#### A list containing:

call	The matched call
rho0	The rage of rho0 from which the ui is calculated
rho1	If ACT==FALSE,range of rho1 from which the ui is calculated
out.model0	Outcome regression model for non-treated.
out.model1	Outcome regression model for treated.
treat.model	Regression model for treatment mechanism (propensity score).
sigma0	Consistent estimate of sigma0 for different values of rho0

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sigma1	Consistent estimate of sigma1 for different values of rho1
DR	DR inference, confidence intervals for different pre-specified values of rho for the OR estimator, uncertainty interval, coefficient estimates, confounding bias, indentification interval, standard error etc.
OR	OR inference, confidence intervals for different pre-specified values of rho for the OR estimator, uncertainty interval, coefficient estimates, confounding bias, indentification interval, standard error etc.

#### Author(s)

Minna Genbäck

#### References

Genbäck, M., de Luna, X. (2018). Causal Inference Accounting for Unobserved Confounding after Outcome Regression and Doubly Robust Estimation. *Biometrics*. DOI: 10.1111/biom.13001

#### **Examples**

```
library(MASS)
n<-500
delta < -c(-0.3, 0.65)
rho<-0.3
X<-cbind(rep(1,n),rnorm(n))</pre>
x < -X[,-1]
s0<-2
error<-mvrnorm(n, c(0,0,0), matrix(c(1,0.6,0.9,0.6,4,0.54,0.9,0.54,9), ncol=3))
zstar<-X%*%delta+error[,1]</pre>
z<- zstar>0
y1<-ifelse(x<(-1),0.2*x-0.1*x^2, ifelse(x<1,0.3*x, ifelse(x<3,0.4-0.1*x^2,-0.2-0.1*x)))+error[,3]
y0<-ifelse(x<1.5, x-0.4*x^2, ifelse(x<2, -0.15-0.25*x+0.5*x^2, 1.85-0.25*x))+error[,2]
y<-y0
y[z==1]<-y1[z==1]
data<-data.frame(y,z,x)</pre>
ui<-ui.causal(y~x, z~x, data=data, rho=c(0,0.3), ACT=FALSE)
ui
plot(ui)
profile(ui)
mean(y1-y0)
ui<-ui.causal(y~x, z~x, data=data, rho=c(0,0.3), ACT=TRUE)
plot(ui)
mean(y1[z==1]-y0[z==1])
```

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ui.ols Uncertainty intervals for OLS regression	
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#### Description

This function allows you to derive uncertainty intervals for OLS regression when there is missing data in the continuous outcome. The uncertainty intervals can be used as a sensitivity analysis to ignorability (missing at random). Note that rho=0 render the same results as a complete case analysis.

#### Usage

```
ui.ols(out.formula, mis.formula = NULL, data, rho = c(-0.3, 0.3), alpha = 0.05, gridn = 101)
```

#### Arguments

out.fo	rmula	Formula for outcome regression.
mis.fo	rmula	Formula for missingness mechanism. If NULL the same covariates as in the outcome regression will be used.
data		data.frame containing the variables in the formula.
rho		The limits of rho for which the uncertainty interval should be constructed.
alpha		Default 0.05 corresponding to a confidence level of 95 for CI and UI.
gridn		The number of distinct points within the interval rho at which confidence intervals should be constructed. Default is 101.

#### **Details**

In order to visualize the results, you can use plot.uiols, or profile.uiols.

#### Value

#### A list containing:

call	The matched call
ci	Confidence intervals for different values of rho
ui	Uncertainty intervals
coef	Estimated coefficients (outcome regression) for different values of rho
out.model	Outcome regression model when rho=0.
mis.model	Regression model for missingness mechanism (selection).
rho	The range of rho for which we want to construct an uncertainty interval
gridrho	The values of rho for which bias and standard errors are derived
sigma	Consistant estimate of sigma
se	Standard error for different values of rho
ciols	Confidence intervals from a complete case analysis
ident.bound	Bounds for the coefficient estimates.

ui.probit

#### Author(s)

Minna Genbäck

#### References

Genbäck, M., Stanghellini, E., de Luna, X. (2015). Uncertainty Intervals for Regression Parameters with Non-ignorable Missingness in the Outcome. *Statistical Papers*, 56(3), 829-847.

#### **Examples**

```
library(MASS)
n<-500
delta < -c(0.5, 0.3, 0.1)
beta<-c(0.8,-0.2,0.3)
X < -cbind(rep(1,n), rnorm(n), rbinom(n,1,0.5))
x < -X[,-1]
rho=0.4
error<-mvrnorm(n, c(0, 0), matrix(c(1, rho*2, rho*2, 4), 2))
zstar<-X%*%delta+error[,1]
z<-as.numeric(zstar>0)
y<-X%*%beta+error[,2]
y[z==0]<-NA
data < -data.frame(y,x,z)
ui < -ui.ols(y \sim X1 + X2, data = data, rho = c(-0.5, 0.5))
ui
plot(ui)
```

ui.probit

Uncertainty intervals for probit regression

#### **Description**

This function allows you to derive uncertainty intervals for probit regression when there is missing data in the binary outcome. The uncertainty intervals can be used as a sensitivity analysis to ignorability (missing at random), and are derived by maximum likelihood. Note that rho=0 render the same results as a complete case analysis.

#### Usage

```
ui.probit(out.formula, mis.formula = NULL, data, rho = c(-0.3, 0.3), progress = TRUE, max.grid = 0.1, alpha = 0.05, method = "NR")
```

#### **Arguments**

out.formula Formula for outcome regression.

mis.formula Formula for missingness mechanism. If NULL the same covariates as in the outcome regression will be used.

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data data.frame containing the variables in the formula.

rho Vector containing the values of rho for which we want to fit the likelihood.

progress If TRUE prints out process time for each maximization of the likelihood.

max.grid Maximum distance between two elements in rho, if two wide there can difficul-

ties with convergence of the maximum likelihood.

alpha Default 0.05 corresponding to a confidence level of 95 for CI and UI.

method Maximization method to be passed through maxLik

#### **Details**

In order to visualize the results, you can use plot.uiprobit or profile.uiprobit.

#### Value

#### A list containing:

coef Estimated coefficients (outcome regression) for different values of rho.

rho The values of rho for which the likelihood is maximized.

vcov Covariance matrix.

ci Confidence intervals for different values of rho.

ui Uncertainty intervals.

out.model Outcome regression model when rho=0.

mis.model Regression model for missingness mechanism (selection).

se Standard errors from outcome regression.

value Value of maximum likelihood for different values of rho.

y Outcome vector.

z Indicator variable of observed outcome.X.y Covariate matrix for outcome regression.

X.z Covariate matrix for missingness mechanism (selection regression model).

max.info Information about the maximization procedure. Includes whether it converged,

message, method and number of iterations.

#### Author(s)

Minna Genbäck

#### References

Genbäck, M., Ng, N., Stanghellini, E., de Luna, X. (2018). Predictors of Decline in Self-reported Health: Addressing Non-ignorable Dropout in Longitudinal Studies of Aging. *European journal of ageing*, 15(2), 211-220.

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#### **Examples**

```
library(MASS)
n<-500
delta<-c(0.5,0.6,0.1,-1,1)
beta<-c(-0.3,-0.5,0,-0.4,-0.3)
X \leftarrow cbind(rep(1,n), rnorm(n), runif(n), rbinom(n,2,0.5), rbinom(n,1,0.5))
x<-X[,-1]
rho=0.4
error<-mvrnorm(n,c(0,0),matrix(c(1,rho,rho,1),2))
zstar<-X%*%delta+error[,1]
z<-as.numeric(zstar>0)
ystar<-X%*%beta+error[,2]
y<-as.integer(ystar>0)
y[z==0]<-NA
\label{eq:data-data} data=data.frame(y=y,x1=x[,1],x2=x[,2],x3=x[,3],x4=x[,4])
m<-ui.probit(y\sim x1+x2+x3+x4,data=data,rho=c(0,0.5))
plot(m)
profile(m)
```

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