Package ‘GeoModels’

October 25, 2024

Type Package

Title Procedures for Gaussian and Non Gaussian Geostatistical (Large) Data Analysis

Version 2.0.7

Date 2024-10-05

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Description

Functions for Gaussian and Non Gaussian (bivariate) spatial and spatio-temporal data analysis are provided for a) (fast) simulation of random fields, b) inference for random fields using standard likelihood and a likelihood approximation method called weighted composite likelihood based on pairs and b) prediction using (local) best linear unbiased prediction. Weighted composite likelihood can be very efficient for estimating massive datasets. Both regression and spatial (temporal) dependence analysis can be jointly performed. Flexible covariance models for spatial and spatial-temporal data on Euclidean domains and spheres are provided. There are also many useful functions for plotting and performing diagnostic analysis. Different non Gaussian random fields can be considered in the analysis. Among them, random fields with marginal distributions such as Skew-Gaussian, Student-t, Tukey-h, Sin-Arcsin, Two-piece, Weibull, Gamma, Log-Gaussian, Binomial, Negative Binomial and Poisson. See the URL for the papers associated with this package, as for instance, Bevilacqua and Gaetan (2015) <doi:10.1007/s11222-014-9460-6>, Bevilacqua et al. (2016) <doi:10.1007/s13253-016-0256-3>, Vallejos et al. (2020) <doi:10.1007/978-3-030-56681-4>, Bevilacqua et. al (2020) <doi:10.1002/env.2632>, Bevilacqua et. al (2021) <doi:10.1111/sjos.12447>, Bevilacqua et al. (2022) <doi:10.1016/j.jmva.2022.104949>, Morales-Navarrete et al. (2023) <doi:10.1080/01621459.2022.2140053>, and a large class of examples and tutorials.

Depends R (>= 4.1.0), fields, mapproj, shape, codetools

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Encoding UTF-8

Imports methods, spam, scatterplot3d, dotCall64, FastGP, plotrix, pracma, pbivnorm, zipfR, sn,sp, lamW, nabor, hypergeo,VGAM, data.table, foreach,future, doFuture,progressr,minqa,

Suggests actuar, GoFKernel, optimParallel, numDeriv

Repository CRAN

URL https://vmoprojs.github.io/GeoModels-page/

BugReports https://github.com/vmoprojs/GeoModels/issues

NeedsCompilation yes

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Date/Publication 2024-10-25 21:30:02 UTC

Contents

`anomalies` *Annual precipitation anomalies in U.S.*

Description

A $(7252x3)$-matrix containing lon/lat and yearly total precipitation anomalies registered at 7.352 location sites in USA.

Usage

```
data(anomalies)
```

Format

A numerical matrix of dimension $7252x3$.

Source

Kaufman, C.G., Schervish, M.J., Nychka, D.W. (2008) Covariance tapering for likelihood-based estimation in large spatial data sets. *Journal of the American Statistical Association, Theory & Methods*, **103**, 1545–1555.

`austemp` *Maximum australian temperature*

Description

A matrix containing maximum temperature in Australia in July 2011.

Usage

```
data(austemp)
```

Format

A (446×4)-matrix containing longitude, latitude, maximum temperature, and the 'so called' geometric temperature covariate.

Source

Bevilacqua M., Caamaño C., Morales-Oñate V., Arellano-Valle R. B. (2020) Non-Gaussian Geostatistical Modeling using (skew) t Processes, *Scandinavian Journal of Statistics*.

`CheckBiv` *Checking Bivariate covariance models*

Description

The procedure control if the correlation model is bivariate.

Usage

```
CheckBiv(numbermodel)
```

Arguments

`numbermodel` numeric; the number associated to a given correlation model.

Details

The function check if the correlation model is bivariate.

Value

Return TRUE or FALSE depending if the correlation model is bivariate or not.

Author(s)

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Examples

```
library(GeoModels)
CheckBiv(CkCorrModel("Bi_matern_sep"))
```

CheckDistance *Checking Distance*

Description

The procedure controls the type of distance.

Usage

```
CheckDistance(distance)
```

Arguments

distance	String; the type of distance, for the description see GeoCovmatrix. Default is Eucl. Other possible values are Geod and Chor that is euclidean, geodesic and chordal distance.

Details

The function check if the type of distance is valid.

Value

Returns 0,1,2 for euclidean,geodesic, chordal distances respectively. Otherwise returns NULL.

Author(s)

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CheckSph *Checking if a covariance is valid only on the sphere*

Description

Subroutine called by InitParam. The procedure controls if a covariance model is valid only on the sphere.

Usage

```
CheckSph(numbermodel)
```

Arguments

numbermodel	Numeric; the code number for the covariance model.

Details

The function checks if a covariance is valid only on the sphere

Value

Returns TRUE or FALSE

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

CheckST *Checking SpaceTime covariance models*

Description

The procedure control if the correlation model is spacetime.

Usage

```
CheckST(numbermodel)
```

Arguments

numbermodel	numeric; the number associated to a given correlation model.

Details

The function check if the correlation model is spacetime.

Value

Returns TRUE or FALSE depending if the correlation model is spacetime or not.

Author(s)

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Examples

```
library(GeoModels)
CheckST(CkCorrModel("gneiting"))
```

CkCorrModel *Checking Correlation Model*

Description

The procedure controls if the correlation model inserted is correct.

Usage

```
CkCorrModel(corrmodel)
```

Arguments

corrmodel	String; the name of a correlation model, for the description see GeoCovmatrix.

Details

The procedure controls if the correlation model is correct

Value

Return a number associated to a given correlation model if the model is considered in the package. Otherwise return NULL.

Author(s)

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CkInput *Checking Input*

Description

Subroutine called by the fitting procedures. The procedure controls the the validity of the input inserted by the users.

Usage

```
CkInput(coordx, coordy, coordt, coordx_dyn,corrmodel, data, distance,
          fcall, fixed, grid,likelihood, maxdist, maxtime,
           model, n,  optimizer, param, radius,
          start, taper, tapsep,  type, varest, vartype,
          weighted,copula,X)
```

Arguments

coordx	A numeric $(d \times 2)$-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.
corrmodel	String; the name of a correlation model, for the description see GeoFit.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
data	A numeric vector or a $(n \times d)$-matrix or $(d \times d \times n)$-matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
fcall	String; Fitting to call the fitting procedure and simulation to call the simulation.
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
likelihood	String; the configuration of the composite likelihood. Marginal is the default.

maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag separation in the composite-likelihood.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
model	String; the density associated to the likelihood objects. Gaussian is the default.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
optimizer	String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the default.
param	A numeric vector of parameters, needed only in simulation. See GeoSim.
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default.
taper	String; the name of the tapered correlation function.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see **Details**).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see GeoFit.
weighted	Logical; if TRUE the likelihood objects are weighted. If FALSE (the default) the composite likelihood is not weighted.
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
X	Numeric; Matrix of space-time covariates in the linear mean specification.

Details

Subroutine called by the fitting procedures. The procedure controls the the validity of the input inserted by the users.

Value

A list with the type of error associated with the input parameters.

Author(s)

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See Also

GeoFit

CkLikelihood — *Checking Composite-likelihood Type*

Description

Subroutine called by InitParam. The procedure controls the type of the composite-likelihood inserted by the users.

Usage

```
CkLikelihood(likelihood)
```

Arguments

likelihood	String; the configuration of the composite likelihood. `Marginal` is the default.

Details

The function controls the type of the composite-likelihood inserted by the users.

Value

The function returns a numeric positive integer, or NULL if the likelihood is invalid.

Author(s)

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See Also

GeoFit

CkModel — *Checking Random Field type*

Description

Subroutine called by InitParam. The procedure controls the type of random field inserted by the users.

Usage

```
CkModel(model)
```

Arguments

`model`	String; the density associated to the likelihood objects. `Gaussian` is the default.

Details

The function controls the type of random field inserted by the users.

Value

The function returns a numeric positive integer, or NULL if the model is invalid.

Author(s)

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See Also

GeoFit

`CkType` *Checking Likelihood Objects*

Description

Subroutine called by InitParam. The procedure controls the type of likelihood objects inserted by the users.

Usage

```
CkType(type)
```

Arguments

`type`	String; the type of the likelihood objects. If `Pairwise` (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.

Details

The procedure checks the likelihood Object

Value

The function returns a numeric positive integer, or NULL if the type of likelihood is invalid.

Author(s)

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See Also

`GeoFit`

`CkVarType` *Checking Variance Estimates Type*

Description

Subroutine called by InitParam. The procedure controls the method used to compute the estimates' variances.

Usage

```
CkVarType(type)
```

Arguments

`type`	String; the method used to compute the estimates' variances. If `SubSamp` the estimates' variances are computed by the sub-sampling method, see `GeoFit`.

Details

The procedure controls the method used to compute the estimates' variances

Value

The function returns a numeric positive integer, or NULL if the method is invalid.

Author(s)

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See Also

`GeoFit`

CompIndLik2 *Optimizes the Composite indipendence log-likelihood*

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the indipendence composite log-likelihood.

Usage

```
CompIndLik2(bivariate, coordx, coordy ,coordt,
coordx_dyn, data, flagcorr, flagnuis, fixed,grid,
              lower, model, n, namescorr, namesnuis,
              namesparam,
              numparam, optimizer, onlyvar, parallel,
              param, spacetime, type,
              upper, namesupper, varest, ns, X,
              sensitivity,copula,MM)
```

Arguments

bivariate	Logical; if `TRUE` then the data come froma a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric (d×2)-matrix (where `d` is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; `coordy` is interpreted only if `coordx` is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is `NULL` then a spatial random field is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
data	A numeric vector or a $(n \times d)$-matrix or $(d \times d \times n)$-matrix of observations.
flagcorr	A numeric vector of binary values denoting which paramerters of the correlation function will be estimated.
flagnuis	A numeric vector of binary values denoting which nuisance paramerters will be estimated.
fixed	A numeric vector of parameters that will be considered as known values.
grid	Logical; if `FALSE` (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if `TRUE` then $(d \times d \times n)$-matrix is considered.
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is `L-BFGS-B` or `nlminb` or `optimize`. The names of the list must be the same of the names in the `start` list.
model	Numeric; the id value of the density associated to the likelihood objects.

n	Numeric; number of trials in a binomial random fields.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numparam	Numeric; the number of parameters to be maximised.
optimizer	String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
parallel	Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.
param	A numeric vector of parameters values.
spacetime	Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
namesupper	String; the names of the upper limit of the parameters.
varest	Logical; if TRUE the estimate variances and standard errors are returned. FALSE is the default.
ns	Numeric; Number of (dynamical) temporal instants.
X	Numeric; Matrix of space-time covariates in the linear mean specification.
sensitivity	Logical; if TRUE then the sensitivy matrix is computed
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
MM	Numeric;a non constant fixed mean

Value

Return a list from an optim call.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

GeoFit

CompLik *Optimizes the Composite log-likelihood*

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

Usage

```
CompLik(copula,bivariate, coordx, coordy ,coordt,
coordx_dyn, corrmodel, data,distance,  flagcorr,
flagnuis, fixed, GPU, grid,likelihood,local,lower,
                 model, n, namescorr, namesnuis,
                 namesparam,
               numparam, numparamcorr, optimizer,
               onlyvar, parallel, param,
               spacetime, type,upper, varest, vartype,
               weigthed, winconst, winstp,winconst_t,
               winstp_t, ns, X,sensitivity,MM,aniso)
```

Arguments

copula	String; the type of copula. It can be "Clayton" or "Gaussian"
bivariate	Logical; if TRUE then the data come froma a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric ($d\times2$)-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a ($n \times d$)-matrix or ($d \times d \times n$)-matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
flagcorr	A numeric vector of binary values denoting which paramerters of the correlation function will be estimated.
flagnuis	A numeric vector of binary values denoting which nuisance paramerters will be estimated.
fixed	A numeric vector of parameters that will be considered as known values.

GPU	Numeric; if NULL (the default) no GPU computation is performed.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
likelihood	String; the configuration of the compositelikelihood, see GeoFit.
local	Numeric; number of local work-items of the GPU
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
model	Numeric; the id value of the density associated to the likelihood objects.
n	Numeric; number of trials in a binomial random fields.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numparam	Numeric; the number of parameters to be maximised.
numparamcorr	Numeric; the number of correlation parameters.
optimizer	String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
parallel	Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.
param	A numeric vector of parameters' values.
spacetime	Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see GeoFit.
weigthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime)are used.
winconst	Numeric; a positive value for computing the spatial sub-window in the sub-sampling procedure.
winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.

winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
ns	Numeric; Number of (dynamical) temporal instants.
X	Numeric; Matrix of space-time covariates in the linear mean specification.
sensitivity	Logical; if TRUE then the sensitivy matrix is computed
MM	Numeric;a non constant fixed mean
aniso	Logical; should anisotropy be considered?

Details

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood

Value

Return a list from an `optim` call.

Author(s)

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See Also

GeoFit

CompLik2 *Optimizes the Composite log-likelihood*

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximisation of the composite log-likelihood.

Usage

```
CompLik2(copula,bivariate, coordx, coordy ,coordt,
coordx_dyn,corrmodel, data, distance, flagcorr, flagnuis,
         fixed, GPU,grid,likelihood, local,lower,
         model, n, namescorr, namesnuis, namesparam,
         numparam, numparamcorr, optimizer, onlyvar,
         parallel, param, spacetime, type,
         upper, varest, vartype, weigthed, winconst,
         winstp,winconst_t, winstp_t, ns, X,sensitivity,
         colidx,rowidx,neighb,MM,aniso)
```

Arguments

copula	String; the type of copula. It can be "Clayton" or "Gaussian"
bivariate	Logical; if TRUE then the data come froma a bivariate random field. Otherwise from a univariate random field.
coordx	A numeric ($d \times 2$)-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a ($n \times d$)-matrix or ($d \times d \times n$)-matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
flagcorr	A numeric vector of binary values denoting which paramerters of the correlation function will be estimated.
flagnuis	A numeric vector of binary values denoting which nuisance paramerters will be estimated.
fixed	A numeric vector of parameters that will be considered as known values.
GPU	Numeric; if NULL (the default) no GPU computation is performed.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a ($n \times d$)-matrix, instead if TRUE then ($d \times d \times n$)-matrix is considered.
likelihood	String; the configuration of the compositelikelihood, see GeoFit.
local	Numeric; number of local work-items of the GPU
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
model	Numeric; the id value of the density associated to the likelihood objects.
n	Numeric; number of trials in a binomial random fields.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numparam	Numeric; the number of parameters to be maximised.
numparamcorr	Numeric; the number of correlation parameters.
optimizer	String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.

onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
parallel	Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.
param	A numeric vector of parameters' values.
spacetime	Logical; if TRUE the random field is spatial-temporal otherwise is a spatial field.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see GeoFit.
weigthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime)are used.
winconst	Numeric; a positive value for computing the spatial sub-window in the subsampling procedure.
winstp	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the subsampling procedure.
winstp_t	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
ns	Numeric; Number of (dynamical) temporal instants.
X	Numeric; Matrix of space-time covariates in the linear mean specification.
sensitivity	Logical; if TRUE then the sensitivy matrix is computed
colidx	Numeric; Vector of indexes for spatial distances.
rowidx	Numeric; Vector of indexes for spatial distances.
neighb	Numeric; an optional positive integer indicating the order of neighborhood location.
MM	Numeric;a non constant fixed mean
aniso	Logical; should anisotropy be considered?

Value

Return a list from an optim call.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

GeoFit

CorrelationPar *Lists the Parameters of a Correlation Model*

Description

Subroutine called by InitParam and other procedures. The procedure returns a list with the parameters of a given correlation model.

Usage

```
CorrelationPar(corrmodel)
```

Arguments

corrmodel	Integer; an integer associated to a given correlation model.

Details

The function return a list with the Parameters of a Correlation Model

Value

Return a vector string of correlation parameters.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

GeoFit

CorrParam *Lists the Parameters of a Correlation Model*

Description

The procedure returns a list with the names of the parameters of a given correlation model.

Usage

```
CorrParam(corrmodel)
```

Arguments

corrmodel	String: the name associated to a given correlation model.

Details

The function return a list with the Parameters of a Correlation Model

Value

Return a vector string of correlation parameters.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

GeoCovmatrix

Examples

```
require(GeoModels)
################################################################
###
### Example 1. Parameters of the  Matern  model
###
###############################################################

CorrParam("Matern")


################################################################
###
### Example 2. Parameters of the  Generalized Wendland  model
```

```
###
###############################################################

CorrParam("GenWend")


################################################################
###
### Example 3. Parameters of the  Generalized Cauchy  model
###
###############################################################

CorrParam("GenCauchy")


################################################################
###
### Example 4. Parameters of the  space time Gneiting  model
###
###############################################################

CorrParam("Gneiting")


################################################################
###
### Example 5. Parameters of the bi-Matern separable  model.
###            Note that in the bivariate case variance paramters are
###            included
###############################################################

CorrParam("Bi_Matern_sep")
```

GeoAniso *Spatial Anisotropy correction*

Description

Transforms or back-transforms a set of coordinates according to the geometric anisotropy parameters.

Usage

```
GeoAniso(coords, anisopars=c(0,1), inverse = FALSE)
```

Arguments

`coords`	An n x 2 matrix with the coordinates to be transformed.

`anisopars` A bivariate vector with the the anisotropy angle and the anisotropy ratio, respectively. The angle must be given in radians in [0,pi] and the anisotropy ratio must be greater or equal than 1.

`inverse` Logical: Default to FALSE. If TRUE the reverse transformation is performed.

Details

Geometric anisotropy is defined by a linear tranformation from the anisotropic space to the isotropic space that is

$$Y = XRS$$

where X is a matrix with original coordinates (anisotropic space), and Y is a matrix with transformed coordinates (isotropic space). Here R is a rotation matrix with associated anisotropy angle parameter (in $[0, pi]$) and a S is a shrinking matrix with associated anisotropy ratio parameter (greeater or equal than one). The two parameters are specified in the anisopars argument as a bivariate numeric vector. The case $(., 1)$ corresponds to the isotropic case.

Value

Returns a matrix of transformed coordinates

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

---

GeoCorrFct *Spatial and Spatio-temporal correlation or covariance of (non) Gaussian random fields*

Description

The function computes the correlations of a spatial (or spatio-temporal or bivariate spatial) Gaussian or non Gaussian randomm field for a given correlation model and a set of spatial (temporal) distances.

Usage

```
GeoCorrFct(x,t=NULL,corrmodel, model="Gaussian",
distance="Eucl", param, radius=6371,n=1,
covariance=FALSE,variogram=FALSE)
```

Arguments

x	A set of spatial distances.
t	A set of (optional) temporal distances.
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
model	String; the type of RF. See GeoFit.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
param	A list of parameter values required for the covariance model.
radius	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
n	Numeric; the number of trials in a (negative) binomial random fields. Default is 1.
covariance	Logic; if TRUE then the covariance is returned. Default is FALSE
variogram	Logic; if FALSE then the covariance/correlation is returned. Otherwise the associated semivariogram is returned

Value

Returns correlations or covariances values associated to a given parametric spatial and temporal correlation models.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

Examples

```
library(GeoModels)

################################################################
###
### Example 1. Covariance of a Gaussian random field with underlying
### Matern correlation model with nugget
###
###############################################################
# Define the spatial distances
#x = seq(0,1,0.002)
# Correlation Parameters for Matern model
#CorrParam("Matern")
#NuisParam("Gaussian")
# Matern Parameters
#param=list(sill=2,smooth=0.5,scale=0.2/3,nugget=0.2,mean=0)
```

```
#cc= GeoCorrFct(x=x, corrmodel="Matern", covariance=TRUE,
#  param=param,model="Gaussian")
#plot(cc,ylab="Corr",lwd=2,main="Matern correlation",type="l")

################################################################
###
### Example 2. Covariance of a Gaussian random field with underlying
### Generalized Wendland-Matern correlation model
###
###############################################################
#CorrParam("GenWend_Matern")
#NuisParam("Gaussian")
# GenWend Matern Parameters
#param=list(sill=2,smooth=1,scale=0.1,nugget=0,power2=1/4,mean=0)
#cc= GeoCorrFct(x=x, corrmodel="GenWend_Matern", param=param,model="Gaussian",covariance=FALSE)
#plot(cc,ylab="Cov",lwd=2,,main="GenWend covariance",type="l")

################################################################
###
### Example 3. Semivariogram of a Tukeyh random field with underlying
### Generalized Wendland correlation model
###
###############################################################
#CorrParam("GenWend")
#NuisParam("Tukeyh")
#x = seq(0,1,0.005)
#param=list(sill=1,smooth=1,scale=0.5,nugget=0,power2=5,tail=0.1,mean=0)
#cc= GeoCorrFct(x=x, corrmodel="GenWend", param=param,model="Tukeyh",variogram=TRUE)
#plot(cc,ylab="Corr",lwd=2,main="Tukey semivariogram",type="l")

################################################################
###
### Example 4. Semi-Variogram of a LoggGaussian random field with underlying
### Kummer correlation model
###
###############################################################
#CorrParam("Kummer")
#NuisParam("LogGaussian")
# GenWend Matern Parameters
#param=list(smooth=1,sill=0.5,scale=0.1,nugget=0,power2=1,mean=0)
#cc= GeoCorrFct(x=x, corrmodel="Kummer", param=param,model="LogGaussian",
#       ,covariance=TRUE,variogram=TRUE)
#plot(cc,ylab="Semivario",lwd=2,
#  main="LogGaussian semivariogram",type="l")


################################################################
###
### Example 5. Covariance of Poisson random field with underlying
### Matern correlation model
###
###############################################################
#CorrParam("Matern")
```

```
#NuisParam("Poisson")
#x = seq(0,1,0.005)
#param=list(scale=0.6/3,nugget=0,smooth=0.5,mean=2)
#cc= GeoCorrFct(x=x, corrmodel="Matern", param=param,model="Poisson",covariance=TRUE)
#plot(cc,ylab="Cov",lwd=2,
#  main="Poisson covariance",type="l")

################################################################
###
### Example 6.  Space time  semivariogram of a Gaussian random field
### with  separable Matern correlation model
###
###############################################################

## spatial and temporal distances
#h<-seq(0,3,by=0.04)
#times<-seq(0,3,by=0.04)

# Correlation Parameters for the space time separable Matern model
#CorrParam("Matern")
#NuisParam("Gaussian")
# Matern Parameters
#param=list(sill=1,scale_s=0.6/3,scale_t=0.5,nugget=0,mean=0,smooth_s=1.5,smooth_t=0.5)
#cc= GeoCorrFct(x=h,t=times,corrmodel="Matern_Matern", param=param,
#        model="Gaussian",variogram=TRUE)
#plot(cc,lwd=2,type="l")


################################################################
###
### Example 7. Correlation of a bivariate Gaussian random field
### with underlying  separable bivariate  Matern correlation model
###
###############################################################
# Define the spatial distances
#x = seq(0,1,0.005)
# Correlation Parameters for the bivariate sep Matern model
#CorrParam("Bi_Matern")
# Matern Parameters
#param=list(sill_1=1,sill_2=1,smooth_1=0.5,smooth_2=1,smooth_12=0.75,
#           scale_1=0.2/3, scale_2=0.2/3, scale_12=0.2/3,
#           mean_1=0,mean_2=0,nugget_1=0,nugget_2=0,pcol=-0.2)
#cc= GeoCorrFct(x=x, corrmodel="Bi_Matern", param=param,model="Gaussian")
#plot(cc,ylab="corr",lwd=2,type="l")
```

GeoCorrFct_Cop — *Spatial and Spatio-temporal correlation or covariance of (non) Gaussian random fields (copula models)*

Description

The function computes the correlations of a spatial or spatio-temporal or a bivariate spatial Gaussian or non Gaussian copula randomm field with a given covariance model and a set of spatial (temporal) distances.

Usage

```
GeoCorrFct_Cop(x,t=NULL,corrmodel,
model="Gaussian",copula="Gaussian",
distance="Eucl", param, radius=6371,
n=1,covariance=FALSE,variogram=FALSE)
```

Arguments

`x`	A set of spatial distances.
`t`	A set of (optional) temporal distances.
`corrmodel`	String; the name of a correlation model, for the description see the Section **Details**.
`model`	String; the type of RF. See `GeoFit`.
`copula`	String; the type of copula. The two options are Gaussian and Clayton.
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See `GeoFit`.
`param`	A list of parameter values required for the covariance model.
`radius`	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
`n`	Numeric; the number of trials in a (negative) binomial random fields. Default is 1.
`covariance`	Logic; if TRUE then the covariance is returned. Default is FALSE
`variogram`	Logic; if FALSE then the covariance/coorelation is returned. Otherwise the associated semivariogram is returned

Value

Returns a vector of correlations or covariances values associated to a given parametric spatial and temporal correlation models.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

Examples

```
library(GeoModels)

################################################################
###
### Example 1. Correlation of a (mean reparametrized) beta random field with underlying
### Matern correlation model using Gaussian and Clayton copulas
###
###############################################################

# Define the spatial distances
x = seq(0,0.4,0.01)

# Correlation Parameters for Matern model
CorrParam("Matern")
NuisParam("Beta2")
# corr Gaussian copula
param=list(smooth=0.5,sill=1,scale=0.2/3,nugget=0,mean=0,min=0,max=1,shape=0.5)
corr1= GeoCorrFct_Cop(x=x, corrmodel="Matern", param=param,copula="Gaussian",model="Beta2")

plot(corr1,ylab="corr",main="Gauss copula correlation",lwd=2)

# corr Clayton copula
param=list(smooth=0.5,sill=1,scale=0.2/3,nugget=0,mean=0,min=0,max=1,shape=0.5,nu=2)
corr2= GeoCorrFct_Cop(x=x, corrmodel="Matern", param=param,copula="Clayton",model="Beta2")
lines(x,corr2$corr,ylim=c(0,1),lty=2)

plot(corr1,ylab="corr",main="Clayton copula correlation",lwd=2)
```

GeoCovariogram *Computes the fitted variogram model.*

Description

The procedure computes and plots estimated covariance or semivariogram models of a Gaussian or a non Gaussian spatial (temporal or bivariate spatial) random field. It allows to add the empirical estimates in order to compare them with the fitted model.

Usage

```
GeoCovariogram(fitted, distance="Eucl",answer.cov=FALSE,
             answer.vario=FALSE, answer.range=FALSE, fix.lags=NULL,
              fix.lagt=NULL, show.cov=FALSE, show.vario=FALSE,
             show.range=FALSE, add.cov=FALSE, add.vario=FALSE,
             pract.range=95, vario, invisible=FALSE, ...)
```

Arguments

fitted	A fitted object obtained from the GeoFit or GeoWLS procedures.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
answer.cov	Logical; if TRUE a vector with the estimated covariance function is returned; if FALSE (the default) the covariance is not returned.
answer.vario	Logical; if TRUE a vector with the estimated variogram is returned; if FALSE (the default) the variogram is not returned.
answer.range	Logical; if TRUE the estimated pratical range is returned; if FALSE (the default) the pratical range is not returned.
fix.lags	Integer; a positive value denoting the spatial lag to consider for the plot of the temporal profile.
fix.lagt	Integer; a positive value denoting the temporal lag to consider for the plot of the spatial profile.
show.cov	Logical; if TRUE the estimated covariance function is plotted; if FALSE (the default) the covariance function is not plotted.
show.vario	Logical; if TRUE the estimated variogram is plotted; if FALSE (the default) the variogram is not plotted.
show.range	Logical; if TRUE the estimated pratical range is added on the plot; if FALSE (the default) the pratical range is not added.
add.cov	Logical; if TRUE the vector of the estimated covariance function is added on the current plot; if FALSE (the default) the covariance is not added.
add.vario	Logical; if TRUE the vector with the estimated variogram is added on the current plot; if FALSE (the default) the correlation is not added.
pract.range	Numeric; the percent of the sill to be reached.
vario	A Variogram object obtained from the GeoVariogram procedure.
invisible	Logical;If TRUE then a statistic the (sum of the squared diffeence between the empirical semivariogram and the estimated semivariogram) is computed.
...	other optional parameters which are passed to plot functions.

Details

The function computes the fitted variogram model

Value

Produces a plot. No values are returned.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

References

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Gaetan, C. and Guyon, X. (2010) Spatial Statistics and Modelling. *Spring Verlang, New York*.

See Also

GeoFit.

Examples

```
library(GeoModels)
library(scatterplot3d)

################################################################
###
### Example 1. Plot of fitted covariance and fitted
### and empirical semivariogram from a  Gaussian RF
### with Matern correlation.
###
###############################################################
set.seed(21)
# Set the coordinates of the points:
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel = "Matern"
model = "Gaussian"
mean = 0
sill = 1
nugget = 0
scale = 0.2/3
smooth=0.5

param=list(mean=mean,sill=sill, nugget=nugget, scale=scale, smooth=smooth)
# Simulation of the Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data
I=Inf
start=list(mean=0,scale=scale,sill=sill)
lower=list(mean=-I,scale=0,sill=0)
upper=list(mean= I,scale=I,sill=I)
fixed=list(nugget=nugget,smooth=smooth)
# Maximum composite-likelihood fitting of the Gaussian random field:
fit = GeoFit(data=data,coordx=coords, corrmodel=corrmodel,model=model,
            likelihood="Marginal",type='Pairwise',start=start,
            lower=lower,upper=upper,
            optimizer="nlminb", fixed=fixed,neighb=3)

# Empirical estimation of the variogram:
vario = GeoVariogram(data=data,coordx=coords,maxdist=0.5)
```

```
# Plot of covariance and variogram functions:
GeoCovariogram(fit,show.vario=TRUE, vario=vario,pch=20)

################################################################
###
### Example 2. Plot of fitted covariance and fitted
### and empirical semivariogram from a  Bernoulli
###  RF with Genwend  correlation.
###
###############################################################
set.seed(2111)

model="Binomial";n=1
# Set the coordinates of the points:
x = runif(500, 0, 1)
y = runif(500, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel = "GenWend"
mean = 0
nugget = 0
scale = 0.2
smooth=0
power=4
param=list(mean=mean, nugget=nugget, scale=scale,smooth=0,power2=4)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param,n=n)$data

start=list(mean=0,scale=scale)
fixed=list(nugget=nugget,power2=4,smooth=0)
# Maximum composite-likelihood fitting of the Binomial random field:
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
             likelihood="Marginal",type='Pairwise',start=start,n=n,
             optimizer="BFGS", fixed=fixed,neighb=4)

# Empirical estimation of the variogram:
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.vario=TRUE, vario=vario,pch=20,ylim=c(0,0.3))


################################################################
###
### Example 3. Plot of fitted covariance and fitted
### and empirical semivariogram from a  Weibull  RF
###   with Wend0 correlation.
###
###############################################################
set.seed(111)

model="Weibull";shape=4
```

```
# Set the coordinates of the points:
x = runif(700, 0, 1)
y = runif(700, 0, 1)
coords=cbind(x,y)

# Set the model's parameters:
corrmodel = "Wend0"
mean = 0
nugget = 0
scale = 0.4
power2=4


param=list(mean=mean, nugget=nugget, scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, model=model,param=param)$data

start=list(mean=0,scale=scale,shape=shape)
I=Inf
lower=list(mean=-I,scale=0,shape=0)
upper=list(mean= I,scale=I,shape=I)
fixed=list(nugget=nugget,power2=power2)

fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
             likelihood="Marginal",type='Pairwise',start=start,
             lower=lower,upper=upper,
             optimizer="nlminb", fixed=fixed,neighb=3)

# Empirical estimation of the variogram:
vario = GeoVariogram(data,coordx=coords,maxdist=0.5)

# Plot of covariance and variogram functions:
GeoCovariogram(fit, show.vario=TRUE, vario=vario,pch=20)

################################################################
###
### Example 4. Plot of  fitted  and empirical semivariogram
### from a space time Gaussian random fields
### with double Matern correlation.
###
###############################################################
set.seed(92)
# Define the spatial-coordinates of the points:
x = runif(50, 0, 1)
y = runif(50, 0, 1)
coords=cbind(x,y)
# Define the temporal sequence:
time = seq(0, 10, 1)


param=list(mean=mean,nugget=nugget,
  smooth_s=0.5,smooth_t=0.5,scale_s=0.5/3,scale_t=2/2,sill=sill)
# Simulation of the spatio-temporal Gaussian random field:
```

```
data = GeoSim(coordx=coords, coordt=time, corrmodel="Matern_Matern",param=param)$data

fixed=list(nugget=0, mean=0, smooth_s=0.5,smooth_t=0.5)
start=list(scale_s=0.2, scale_t=0.5, sill=1)
# Maximum composite-likelihood fitting of the space-time Gaussian random field:
fit = GeoFit(data, coordx=coords, coordt=time, corrmodel="Matern_Matern", maxtime=1,
             neighb=3, likelihood="Marginal", type="Pairwise",fixed=fixed, start=start)

# Empirical estimation of spatio-temporal covariance:
vario = GeoVariogram(data,coordx=coords, coordt=time, maxtime=5,maxdist=0.5)

# Plot of the fitted space-time variogram
GeoCovariogram(fit,vario=vario,show.vario=TRUE)

# Plot of covariance, variogram and spatio and temporal profiles:
GeoCovariogram(fit,vario=vario,fix.lagt=1,fix.lags=1,show.vario=TRUE,pch=20)


################################################################
###
### Example 5. Plot of  fitted  and empirical semivariogram
### from a bivariate Gaussian random fields
### with  Matern correlation.
###
###############################################################
set.seed(92)
# Define the spatial-coordinates of the points:
x <- runif(600, 0, 2)
y <- runif(600, 0, 2)
coords <- cbind(x,y)

# Simulation of a bivariate spatial Gaussian RF:
# with a  Bivariate Matern
set.seed(12)
param=list(mean_1=4,mean_2=2,smooth_1=0.5,smooth_2=0.5,smooth_12=0.5,
           scale_1=0.12,scale_2=0.1,scale_12=0.15,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=-0.5)
data <- GeoSim(coordx=coords,corrmodel="Bi_matern",
              param=param)$data



# selecting fixed and estimated parameters
fixed=list(mean_1=4,mean_2=2,nugget_1=0,nugget_2=0,
      smooth_1=0.5,smooth_2=0.5,smooth_12=0.5)
start=list(sill_1=var(data[1,]),sill_2=var(data[2,]),
           scale_1=0.1,scale_2=0.1,scale_12=0.1,
           pcol=cor(data[1,],data[2,]))


# Maximum marginal pairwise likelihood
fitcl<- GeoFit(data=data, coordx=coords, corrmodel="Bi_Matern",
                     likelihood="Marginal",type="Pairwise",
```

```
                    optimizer="BFGS" , start=start,fixed=fixed,
                    neighb=4)
print(fitcl)

# Empirical estimation of spatio-temporal covariance:
vario = GeoVariogram(data,coordx=coords,maxdist=0.4,bivariate=TRUE)
GeoCovariogram(fitcl,vario=vario,show.vario=TRUE,pch=20)
```

GeoCovDisplay *Image plot displaying the pattern of the sparsness of a covariance matrix.*

Description

Image plot displaying the pattern of the sparsness of a covariance matrix.

Usage

```
GeoCovDisplay(covmatrix,limits=FALSE,pch=2)
```

Arguments

covmatrix	An object of class GeoCovmatrix. See the Section **Details**.
limits	Logical; If TRUE and the covariance matrix is spatiotemporal or spatial bivariate then vertical and horizontal lines are added to the image plot.
pch	Type of symbols to use in the image plot.

Details

For a given covariance matrix object (GeoCovmatrix) the function diplays the pattern of the sparsness of a covariance matrix where the white color represents 0 entries and black color represents non zero entries

Value

Produces a plot. No values are returned.

Author(s)

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See Also

GeoCovmatrix

Examples

```
library(GeoModels)


  # Define the spatial-coordinates of the points:
x <- runif(100, 0, 2)
y <- runif(100, 0, 2)
coords=cbind(x,y)
matrix1 <- GeoCovmatrix(coordx=coords, corrmodel="GenWend", param=list(smooth=0,
                      power2=4,sill=1,scale=0.2,nugget=0))

GeoCovDisplay(matrix1)
```

GeoCovmatrix *Spatial and Spatio-temporal Covariance Matrix of (non) Gaussian random fields*

Description

The function computes the covariance matrix associated to a spatial or spatio(-temporal) or a bivariate spatial Gaussian or non Gaussian randomm field with given underlying covariance model and a set of spatial location sites (and temporal instants).

Usage

```
GeoCovmatrix(estobj=NULL,coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel,
          distance="Eucl", grid=FALSE, maxdist=NULL, maxtime=NULL,
        model="Gaussian", n=1, param, anisopars=NULL, radius=6371, sparse=FALSE,
         taper=NULL, tapsep=NULL, type="Standard",copula=NULL,X=NULL,spobj=NULL)
```

Arguments

`estobj` An object of class Geofit that includes information about data, model and estimates.

`coordx` A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.

`coordy` A numeric vector giving 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.

`coordt` A numeric vector giving 1-dimension of temporal coordinates. At the moment implemented only for the Gaussian case. Optional argument, the default is `NULL` then a spatial random field is expected.

coordx_dyn	A list of T numeric ($d_t \times 2$)-matrices containing dynamical (in time) coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid). See GeoFit.
maxdist	Numeric; an optional positive value indicating the marginal spatial compact support in the case of tapered covariance matrix. See GeoFit.
maxtime	Numeric; an optional positive value indicating the marginal temporal compact support in the case of spacetime tapered covariance matrix. See GeoFit.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
model	String; the type of RF. See GeoFit.
param	A list of parameter values required for the covariance model.
anisopars	A list of two elements "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
radius	Numeric; a value indicating the radius of the sphere when using covariance models valid using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE the function return an object of class spam. This option should be used when a parametric compactly supporte covariance is used. Default is FALSE.
taper	String; the name of the taper correlation function if type is Tapering, see the Section **Details**.
tapsep	Numeric; an optional value indicating the separabe parameter in the space-time non separable taper or the colocated correlation parameter in a bivariate spatial taper (see **Details**).
type	String; the type of covariance matrix Standard (the default) or Tapering for tapered covariance matrix.
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
X	Numeric; Matrix of space-time covariates.
spobj	An object of class sp or spacetime

Details

In the spatial case, the covariance matrix of the random vector

$$[Z(s_1),\dots,Z(s_n,)]^T$$

with a specific spatial covariance model is computed. Here n is the number of the spatial location sites.

In the space-time case, the covariance matrix of the random vector

$$[Z(s_1, t_1), Z(s_2, t_1), \ldots, Z(s_n, t_1), \ldots, Z(s_n, t_m)]^T$$

with a specific space time covariance model is computed. Here m is the number of temporal instants.

In the bivariate case, the covariance matrix of the random vector

$$[Z_1(s_1), Z_2(s_1), \ldots, Z_1(s_n), Z_2(s_n)]^T$$

with a specific spatial bivariate covariance model is computed.

The location site s_i can be a point in the d-dimensional euclidean space with $d = 2$ or a point (given in lon/lat degree format) on a sphere of arbitrary radius.

A list with all the implemented space and space-time and bivariate correlation models is given below. The argument `param` is a list including all the parameters of a given correlation model specified by the argument `corrmodel`. For each correlation model one can check the associated parameters' names using `CorrParam`. In what follows $\kappa > 0$, $\beta > 0$, $\alpha, \alpha_s, \alpha_t \in (0, 2]$, and $\gamma \in [0, 1]$. The associated parameters in the argument `param` are `smooth`, `power2`, `power`, `power_s`, `power_t` and `sep` respectively. Moreover let $1(A) = 1$ when A is true and 0 otherwise.

- Spatial correlation models:
  1. $GenCauchy$ (generalised $Cauchy$ in Gneiting and Schlater (2004)) defined as:
  $$R(h) = (1 + h^{\alpha})^{-\beta/\alpha}$$
  If h is the geodesic distance then $\alpha \in (0, 1]$.
  2. $Matern$ defined as:
  $$R(h) = 2^{1-\kappa}\Gamma(\kappa)^{-1}h^{\kappa}K_{\kappa}(h)$$
  If h is the geodesic distance then $\kappa \in (0, 0.5]$
  3. $Kummer$ (Kummer hypergeometric in Ma and Bhadra (2022)) defined as:
  $$R(h) = \Gamma(\kappa + \alpha)U(\alpha, 1 - \kappa, 0.5h^2)/\Gamma(\kappa + \alpha)$$
  $U(., ., .)$ is the Kummer hypergeometric function. If h is the geodesic distance then $\kappa \in (0, 0.5]$
  4. $Kummer\_Matern$ It is a rescaled version of the $Kummer$ model that is h must be divided by $(2 * (1 + \alpha))^{0.5}$. When α goes to infinity it is the Matern model.
  5. $Wave$ defined as:
  $$R(h) = sin(h)/h$$
  This model is valid only for dimensions less than or equal to 3.
  6. $GenWend$ (Generalized Wendland in Bevilacqua et al.(2019)) defined as:
  $$R(h) = A(1 - h^2)^{\beta+\kappa}F(\beta/2, (\beta + 1)/2, 2\beta + \kappa + 1, 1 - h^2)1(h \in [0, 1])$$
  where $\mu \geq 0.5(d + 1) + \kappa$ and $A = (\Gamma(\kappa)\Gamma(2\kappa + \beta + 1))/(\Gamma(2\kappa)\Gamma(\beta + 1 - \kappa)2^{\beta+1})$ and $F(.,.,.)$ is the Gaussian hypergeometric function. The cases $\kappa = 0, 1, 2$ correspond to the $Wend0$, $Wend1$ and $Wend2$ models respectively.

7. $GenWend\_Matern$ (Generalized Wendland Matern in Bevilacqua et al. (2022)). It is defined as a rescaled version of the Generalized Wendland that is h must be divided by $(\Gamma(\beta+2\kappa+1)/\Gamma(\beta))^{1/(1+2\kappa)}$. When β goes to infinity it is the Matern model.
8. $GenWend\_Matern2$ (Generalized Wendland Matern second parametrization. It is defined as a rescaled version of the Generalized Wendland that is h must be multiplied by β and the smoothness parameter is $\kappa - 0.5$. When β goes to infinity it is the Matern model.
9. $Hypergeometric$ (Parsimonious Hypegeometric model in Alegria and Emery (2022)) defined as the model $Hypergeometric2$ with the restriction $\beta = \alpha$.
10. $Multiquadric$ defined as:

$$R(h) = (1-\alpha 0.5)^{2\beta}/(1+(\alpha 0.5)^2 - \alpha cos(h))^{\beta}, \quad h \in [0,\pi]$$

This model is valid on the unit sphere and h is the geodesic distance.
11. $Sinpower$ defined as:

$$R(h) = 1 - (sin(h/2))^{\alpha}, \quad h \in [0,\pi]$$

This model is valid on the unit sphere and h is the geodesic distance.
12. $Smoke$ (F family in Alegria et al. (2021)) defined as:

$$R(h) = K * F(1/\alpha, 1/\alpha + 0.5, 2/\alpha + 0.5 + \kappa), \quad h \in [0,\pi]$$

where $K = (\Gamma(a)\Gamma(i))/\Gamma(i)\Gamma(o))$. This model is valid on the unit sphere and h is the geodesic distance.

- Spatio-temporal correlation models.
  - Non-separable models:
    1. $Gneiting$ defined as:

$$R(h,u) = e^{-h^{\alpha_s}/((1+u^{\alpha_t})^{0.5\gamma\alpha_s})}/(1+u^{\alpha_t})$$

    2. $Gneiting\_GC$

$$R(h,u) = e^{-u^{\alpha_t}/((1+h^{\alpha_s})^{0.5\gamma\alpha_t})}/(1+h^{\alpha_s})$$

    where h can be both the euclidean and the geodesic distance
    3. $Iacocesare$

$$R(h,u) = (1+h^{\alpha_s}+u_t^{\alpha})^{-\beta}$$

    4. $Porcu$

$$R(h,u) = (0.5(1+h^{\alpha_s})^{\gamma} + 0.5(1+u^{\alpha_t})^{\gamma})^{-\gamma^{-1}}$$

    5. $Porcu1$

$$R(h,u) = (e^{-h^{\alpha_s}(1+u^{\alpha_t})^{0.5\gamma\alpha_s}})/((1+u^{\alpha_t})^{1.5})$$

    6. $Stein$

$$R(h,u) = (h^{\psi(u)}K_{\psi(u)}(h))/(2^{\psi(u)}\Gamma(\psi(u)+1))$$

    where $\psi(u) = \nu + u^{0.5\alpha_t}$
    7. $Gneiting\_mat\_S$, defined as:

$$R(h,u) = \phi(u)^{\tau_t} Mat(h\phi(u)^{-\beta}, \nu_s)$$

    where $\phi(u) = (1+u^{0.5\alpha_t})$, $\tau_t \geq 3.5 + \nu_s$, $\beta \in [0,1]$

8. $Gneiting\_mat\_T$, defined interchanging h with u in $Gneiting\_mat\_S$
9. $Gneiting\_wen\_S$, defined as:

$$R(h,u) = \phi(u)^{\tau_t} GenWend(h\phi(u)^{\beta}, \nu_s, \mu_s)$$

where $\phi(u) = (1 + u^{0.5\alpha_t})$, $\tau_t \geq 2.5 + 2\nu_s$, $\beta \in [0,1]$

10. $Gneiting\_wen\_T$, defined interchanging h with u in $Gneiting\_wen\_S$
11. $Multiquadric\_st$ defined as:

$$R(h,u) = ((1 - 0.5\alpha_s)^2/(1 + (0.5\alpha_s)^2 - \alpha_s\psi(u)cos(h)))^{a_s}, \quad h \in [0,\pi]$$

where $\psi(u) = (1 + (u/a_t)^{\alpha_t})^{-1}$. This model is valid on the unit sphere and h is the geodesic distance.

12. $Sinpower\_st$ defined as:

$$R(h,u) = (e^{\alpha_s cos(h)\psi(u)/a_s}(1 + \alpha_s cos(h)\psi(u)/a_s))/k$$

where $\psi(u) = (1+(u/a_t)^{\alpha_t})^{-1}$ and $k = (1+\alpha_s/a_s)exp(\alpha_s/a_s), \quad h \in [0,\pi]$ This model is valid on the unit sphere and h is the geodesic distance.

– Separable models.

Space-time separable correlation models are easly obtained as the product of a spatial and a temporal correlation model, that is

$$R(h,u) = R(h)R(u)$$

Several combinations are possible:

1. Exp_Exp defined as:

$$R(h,u) = Exp(h)Exp(u)$$

2. $Matern\_Matern$ defined as:

$$R(h,u) = Matern(h;\kappa_s)Matern(u;\kappa_t)$$

3. $GenWend\_GenWend$ defined as

$$R(h,u) = GenWend(h;\kappa_s,\mu_s)GenWend(u;\kappa_t,\mu_t)$$

.

4. $Stable\_Stable$ defined as:

$$R(h,u) = Stable(h;\alpha_s)Stable(u;\alpha_t)$$

Note that some models are nested. (The Exp_Exp with $Matern\_Matern$ for instance.)

- Spatial bivariate correlation models (see below):
  1. Bi_Matern (Bivariate full Matern model)
  2. Bi_Matern_contr (Bivariate Matern model with contrainsts)
  3. Bi_Matern_sep (Bivariate separable Matern model)
  4. Bi_LMC (Bivariate linear model of coregionalization)
  5. Bi_LMC_contr (Bivariate linear model of coregionalization with constraints)

6. Bi_Wendx (Bivariate full Wendland model)
7. Bi_Wendx_contr (Bivariate Wendland model with contrainsts)
8. Bi_Wendx_sep (Bivariate separable Wendland model)
9. Bi_Smoke (Bivariate full Smoke model on the unit sphere)

- Spatial taper.
  For spatial covariance tapering the taper functions are:
  1. $Bohman$ defined as:
  $$T(h) = (1-h)(sin(2\pi h)/(2\pi h)) + (1-cos(2\pi h))/(2\pi^2 h)1_{[0,1]}(h)$$
  2. $Wendlandx, \quad x = 0, 1, 2$ defined as:
  $$T(h) = Wendx(h; x + 2), x = 0, 1, 2$$
  .
- Spatio-temporal tapers.
  For spacetime covariance tapering the taper functions are:
  1. $Wendlandx\_Wendlandy$ (Separable tapers) $x, y = 0, 1, 2$ defined as:
  $$T(h, u) = Wendx(h; x + 2)Wendy(h; y + 2), x, y = 0, 1, 2.$$
  2. $Wendlandx\_time$ (Non separable temporal taper) $x = 0, 1, 2$ defined as: $Wenx\_time$, $x = 0, 1, 2$ assuming $\alpha_t = 2$, $\mu_s = 3.5 + x$ and $\gamma \in [0, 1]$ to be fixed using `tapsep`.
  3. $Wendlandx\_space$ (Non separable spatial taper) $x = 0, 1, 2$ defined as: $Wenx\_space$, $x = 0, 1, 2$ assuming $\alpha_s = 2$, $\mu_t = 3.5 + x$ and $\gamma \in [0, 1]$ to be fixed using `tapsep`.
- Spatial bivariate taper (see below).
  1. $Bi\_Wendlandx, \quad x = 0, 1, 2$

Remarks:
In what follows we assume $\sigma^2, \sigma_1^2, \sigma_2^2, \tau^2, \tau_1^2, \tau_2^2, a, a_s, a_t, a_{11}, a_{22}, a_{12}, \kappa_{11}, \kappa_{22}, \kappa_{12}, f_{11}, f_{12}, f_{21}, f_{22}$ positive.

The associated names of the parameters in `param` are `sill`, `sill_1`,`sill_2`, `nugget`, `nugget_1`,`nugget_2`, `scale`,`scale_s`,`scale_t`, `scale_1`,`scale_2`,`scale_12`, `smooth_1`,`smooth_2`,`smooth_12`, `a_1`,`a_12`,`a_21`,`a_2` respectively.

Let $R(h)$ be a spatial correlation model given in standard notation. Then the covariance model applied with arbitrary variance, nugget and scale equals to σ^2 if $h = 0$ and

$$C(h) = \sigma^2(1 - \tau^2)R(h/a, , ...), \quad h > 0$$

with nugget parameter τ^2 between 0 and 1. Similarly if $R(h, u)$ is a spatio-temporal correlation model given in standard notation, then the covariance model is σ^2 if $h = 0$ and $u = 0$ and

$$C(h, u) = \sigma^2(1 - \tau^2)R(h/a_s, u/a_t, ...) \quad h > 0, u > 0$$

Here '...' stands for additional parameters.

The bivariate models implemented are the following :

1. Bi_Matern defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i\sigma_j + \tau_i^2 1(i=j,h=0))Matern(h/a_{ij},\kappa_{ij}) \quad i,j=1,2. \quad h \geq 0$$

   where $\rho = \rho_{12} = \rho_{21}$ is the correlation colocated parameter and $\rho_{ii} = 1$. The model Bi_Matern_sep (separable matern) is a special case when $a = a_{11} = a_{12} = a_{22}$ and $\kappa = \kappa_{11} = \kappa_{12} = \kappa_{22}$. The model Bi_Matern_contr (constrained matern) is a special case when $a_{12} = 0.5(a_{11} + a_{22})$ and $\kappa_{12} = 0.5(\kappa_{11} + \kappa_{22})$

2. $Bi\_GenWend$ defined as:

$$C_{ij}(h) = \rho_{ij}(\sigma_i\sigma_j + \tau_i^2 1(i=j,h=0))GenWend(h/a_{ij},\nu_{ij},\kappa_i j) \quad i,j=1,2. \quad h \geq 0$$

   where $\rho = \rho_{12} = \rho_{21}$ is the correlation colocated parameter and $\rho_{ii} = 1$. The model $Bi\_GenWend\_sep$ (separable Genwendland) is a special case when $a = a_{11} = a_{12} = a_{22}$ and $\mu = \mu_{11} = \mu_{12} = \mu_{22}$. The model $Bi\_GenWend\_contr$ (constrained Genwendland) is a special case when $a_{12} = 0.5(a_{11} + a_{22})$ and $\mu_{12} = 0.5(\mu_{11} + \mu_{22})$

3. Bi_LMC defined as:

$$C_{ij}(h) = \sum_{k=1}^{2}(f_{ik}f_{jk} + \tau_i^2 1(i=j,h=0))R(h/a_k)$$

   where $R(h)$ is a correlation model. The model Bi_LMC_contr is a special case when $f = f_{12} = f_{21}$. Bivariate LMC models, in the current version of the package, is obtained with $R(h)$ equal to the exponential correlation model.

Value

Returns an object of class `GeoCovmatrix`. An object of class `GeoCovmatrix` is a list containing at most the following components:

`bivariate`	Logical:`TRUE` if the Gaussian random field is bivariaete otherwise `FALSE`;
`coordx`	A d-dimensional vector of spatial coordinates;
`coordy`	A d-dimensional vector of spatial coordinates;
`coordt`	A t-dimensional vector of temporal coordinates;
`coordx_dyn`	A list of t matrices of spatial coordinates;
`covmatrix`	The covariance matrix if `type` is`Standard`. An object of class spam if `type` is `Tapering` or `Standard` and `sparse` is TRUE.
`corrmodel`	String: the correlation model;
`distance`	String: the type of spatial distance;
`grid`	Logical:`TRUE` if the spatial data are in a regular grid, otherwise `FALSE`;
`nozero`	In the case of tapered matrix the percentage of non zero values in the covariance matrix. Otherwise is NULL.
`maxdist`	Numeric: the marginal spatial compact support if `type` is `Tapering`;
`maxtime`	Numeric: the marginal temporal compact support if `type` is `Tapering`;
`n`	The number of trial for Binomial RFs

namescorr	String: The names of the correlation parameters;
numcoord	Numeric: the number of spatial coordinates;
numtime	Numeric: the number the temporal coordinates;
model	The type of RF, see GeoFit.
param	Numeric: The covariance parameters;
tapmod	String: the taper model if type is Tapering. Otherwise is NULL.
spacetime	TRUE if spatio-temporal and FALSE if spatial covariance model;
sparse	Logical: is the returned object of class spam? ;

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See Also

GeoKrig, GeoSim, GeoFit

Examples

```
library(GeoModels)
################################################################
###
### Example 1.Estimated  spatial covariance matrix associated to
### a WCL estimates using the Matern correlation model
###
###############################################################

set.seed(3)
N=300  # number of location sites
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
# Set the covariance model's parameters:
corrmodel <- "Matern"
mean=0.5;
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5

param<-list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth)
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param)$data
fixed<-list(nugget=nugget,smooth=smooth)
start<-list(mean=mean,scale=scale,sill=sill)

fit0 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
                    neighb=3,likelihood="Conditional",optimizer="BFGS",
                    type="Pairwise", start=start,fixed=fixed)
print(fit0)

#estimated covariance matrix using Geofit object
mm=GeoCovmatrix(fit0)$covmatrix
#estimated covariance matrix
mm1 = GeoCovmatrix(coordx=coords,corrmodel=corrmodel,
           param=c(fit0$param,fit0$fixed))$covmatrix
sum(mm-mm1)

################################################################
###
### Example 2. Spatial covariance matrix associated to
### the GeneralizedWendland-Matern correlation model
###
###############################################################

# Correlation Parameters for Gen Wendland model
CorrParam("GenWend_Matern")
```

```
# Gen Wendland Parameters
param=list(sill=1,scale=0.04,nugget=0,smooth=0,power2=1.5)

matrix2 = GeoCovmatrix(coordx=coords, corrmodel="GenWend_Matern", param=param,sparse=TRUE)

# Percentage of no zero values
matrix2$nozero


################################################################
###
### Example 3. Spatial covariance matrix associated to
### the Kummer correlation model
###
###############################################################

# Correlation Parameters for kummer model
CorrParam("Kummer")
param=list(sill=1,scale=0.2,nugget=0,smooth=0.5,power2=1)

matrix3 = GeoCovmatrix(coordx=coords, corrmodel="Kummer", param=param)

matrix3$covmatrix[1:4,1:4]


################################################################
###
### Example 4. Covariance matrix associated to
### the space-time double Matern correlation model
###
###############################################################

# Define the temporal-coordinates:
times = seq(1, 4, 1)

# Correlation Parameters for double Matern model
CorrParam("Matern_Matern")

# Define covariance parameters
param=list(scale_s=0.3,scale_t=0.5,sill=1,smooth_s=0.5,smooth_t=0.5)

# Simulation of a spatial Gaussian random field:
matrix4 = GeoCovmatrix(coordx=coords, coordt=times, corrmodel="Matern_Matern",
                     param=param)

dim(matrix4$covmatrix)

################################################################
###
### Example 5. Spatial Covariance matrix associated to
### a  skew gaussian RF with Matern correlation model
###
###############################################################
```

```
param=list(sill=1,scale=0.3/3,nugget=0,skew=4,smooth=0.5)
# Simulation of a spatial Gaussian random field:
matrix5 = GeoCovmatrix(coordx=coords,  corrmodel="Matern", param=param,
                     model="SkewGaussian")

# covariance matrix
matrix5$covmatrix[1:4,1:4]

################################################################
###
### Example 6. Spatial Covariance matrix associated to
### a  Weibull RF with Genwend correlation model
###
###############################################################

param=list(scale=0.3,nugget=0,shape=4,mean=0,smooth=1,power2=5)
# Simulation of a spatial Gaussian random field:
matrix6 = GeoCovmatrix(coordx=coords,  corrmodel="GenWend", param=param,
                     sparse=TRUE,model="Weibull")

# Percentage of no zero values
matrix6$nozero

################################################################
###
### Example 7. Spatial Covariance matrix associated to
### a  binomial gaussian RF with Generalized Wendland correlation model
###
###############################################################

param=list(mean=0.2,scale=0.2,nugget=0,power2=4,smooth=0)
# Simulation of a spatial Gaussian random field:
matrix7 = GeoCovmatrix(coordx=coords,  corrmodel="GenWend", param=param,n=5,
                     sparse=TRUE,
                     model="Binomial")

as.matrix(matrix7$covmatrix)[1:4,1:4]


################################################################
###
### Example 8.  Covariance matrix associated to
### a bivariate Matern exponential correlation model
###
###############################################################

set.seed(8)
# Define the spatial-coordinates of the points:
x = runif(4, 0, 1)
y = runif(4, 0, 1)
coords = cbind(x,y)
```

```
# Parameters
param=list(mean_1=0,mean_2=0,sill_1=1,sill_2=2,
           scale_1=0.1,  scale_2=0.1,  scale_12=0.1,
           smooth_1=0.5, smooth_2=0.5, smooth_12=0.5,
            nugget_1=0,nugget_2=0,pcol=-0.25)

# Covariance matrix
matrix8 = GeoCovmatrix(coordx=coords, corrmodel="Bi_matern", param=param)$covmatrix

matrix8
```

GeoCV *n-fold kriging Cross-validation*

Description

The procedure use the GeoKrig or GeoKrigloc function to compute n-fold kriging cross-validation using informations from a GeoFit object. The function returns some prediction scores.

Usage

```
GeoCV(fit, K=100, estimation=TRUE, optimizer=NULL,
     lower=NULL, upper=NULL, n.fold=0.05,local=FALSE,
    neighb=NULL, maxdist=NULL,maxtime=NULL,sparse=FALSE,
    type_krig="Simple", which=1, parallel=FALSE, ncores=NULL)
```

Arguments

fit	An object of class GeoFit.
K	The number of iterations in cross-validation.
estimation	Logical; if TRUE then an estimation is performed at each iteration and the estimates are used in the prediction. Otherwise the estimates in the object fit are used.
optimizer	The type of optimization algorithm if estimation is TRUE. See GeoFit for details. If NULL then the optimization algorithm of the object fit is chosen.
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize if estimation is TRUE.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize if estimation is TRUE.
n.fold	Numeric; the percentage of data to be deleted (and predicted) in the cross-validation procedure.
local	Logical; If local is TRUE, then local kriging is performed. The default is FALSE.

neighb	Numeric; an optional positive integer indicating the order of neighborhood if local kriging is performed.
maxdist	Numeric; an optional positive value indicating the distance in the spatial neighborhood if local kriging is performed.
maxtime	Numeric; an optional positive value indicating the distance in the temporal neighborhood if local kriging is performed.
sparse	Logical; if TRUE kriging and simulation are computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
type_krig	String; the type of kriging. If Simple (the default) then simple kriging is performed. If Optim then optimal kriging is performed for some non-Gaussian RFs
which	Numeric; In the case of bivariate cokriging it indicates which variable to predict. It can be 1 or 2
parallel	Logical; if TRUE then the estimation step is parallelized
ncores	Numeric; number of cores involved in parallelization.

Value

Returns an object containing the following informations:

predicted	A list of the predicted values in the CV procedure;
data_to_pred	A list of the data to predict in the CV procedure;
mae	The vector of mean absolute error in the CV procedure;
mad	The vector of median absolute error in the CV procedure;
brie	The vector of brie score in the CV procedure;
rmse	The vector of root mean squared error in the CV procedure;
lscore	The vector of log-score in the CV procedure;
crps	The vector of continuous ranked probability score in the CV procedure;

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See Also

GeoKrig.

Examples

```
 library(GeoModels)

################################################################
########### Examples of spatial kriging ############
################################################################

model="Gaussian"
set.seed(79)
x = runif(400, 0, 1)
y = runif(400, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel = "GenWend"
mean=0; sill=5; nugget=0
scale=0.2;smooth=0;power2=4

param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth,power2=power2)

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,
              param=param)$data

## estimation with pairwise likelihood
fixed=list(nugget=nugget,smooth=0,power2=power2)
start=list(mean=0,scale=scale,sill=1)
I=Inf
lower=list(mean=-I,scale=0,sill=0)
upper=list(mean= I,scale=I,sill=I)
# Maximum pairwise likelihood fitting :
fit = GeoFit(data, coordx=coords, corrmodel=corrmodel,model=model,
                    likelihood='Marginal', type='Pairwise',neighb=3,
                    optimizer="nlminb", lower=lower,upper=upper,
                    start=start,fixed=fixed)

#a=GeoCV(fit,K=100,estimation=TRUE,parallel=TRUE)
#a$rmse
```

GeoDoScores   *Computation of drop-one predictive scores*

Description

The function computes RMSE, MAE, LSCORE, CRPS predictive scores based on drop-one prediction for a spatial, spatiotemporal and bivariate Gaussian RFs

Usage

```
GeoDoScores(data,  method="cholesky", matrix)
```

Arguments

data	A d-dimensional vector (a single spatial realisation) or a a$(t \times d)$-matrix (a single spatial-temporal realisation). or a a$(2 \times d)$-matrix (a single bivariate realisation).
method	String; the type of matrix decomposition used in the computation of the predictive scores. Default is `cholesky`. The other possible choices is `svd`.
matrix	An object of class GeoCovmatrix. See the Section **Details**.

Details

For a given covariance matrix object (`GeoCovmatrix`) and a given spatial, spatiotemporal or bivariare realization from a Gaussian random field, the function computes four predictive scores based on drop-one prediction.

Value

Returns a list containing the following informations:

RMSE	Root-mean-square error predictive score
MAE	Mean absolute error predictive score
LSCORE	Logarithmic predictive score
CRPS	Continuous ranked probability predictive score

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See Also

`GeoCovmatrix`

Examples

```
library(GeoModels)

################################################################
######### Examples of predictive score computation  ############
################################################################
set.seed(8)
  # Define the spatial-coordinates of the points:
x <- runif(500, 0, 2)
```

```
y <- runif(500, 0, 2)
coords=cbind(x,y)
matrix1 <- GeoCovmatrix(coordx=coords, corrmodel="Matern", param=list(smooth=0.5,
                      sill=1,scale=0.2,nugget=0))

data <- GeoSim(coordx=coords, corrmodel="Matern", param=list(mean=0,smooth=0.5,
                      sill=1,scale=0.2,nugget=0))$data

Pr_scores <- GeoDoScores(data,matrix=matrix1)

Pr_scores
```

GeoFit *Max-Likelihood-Based Fitting of Gaussian and non Gaussian random fields.*

Description

Maximum weighted composite-likelihood fitting for Gaussian and some Non-Gaussian univariate spatial, spatio-temporal and bivariate spatial random fieldss The function allows to fix any of the parameters and setting upper/lower bound in the optimization. Different methods of optimization can be used.

Usage

```
GeoFit(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL,copula=NULL,
      corrmodel=NULL,distance="Eucl",fixed=NULL,anisopars=NULL,
      est.aniso=c(FALSE,FALSE),GPU=NULL, grid=FALSE, likelihood='Marginal',
     local=c(1,1),  lower=NULL,maxdist=Inf,neighb=NULL,
      maxtime=Inf, memdist=TRUE,method="cholesky",
      model='Gaussian',n=1, onlyvar=FALSE ,
      optimizer='Nelder-Mead', parallel=FALSE,
      radius=6371,  sensitivity=FALSE,sparse=FALSE,
      start=NULL, taper=NULL, tapsep=NULL,
      type='Pairwise', upper=NULL, varest=FALSE,
      vartype='SubSamp', weighted=FALSE, winconst=NULL, winstp=NULL,
      winconst_t=NULL, winstp_t=NULL,X=NULL,nosym=FALSE,spobj=NULL,spdata=NULL)
```

Arguments

`data` A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid). For the description see the Section **Details**.

`coordx` A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.

coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random fields is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
fixed	An optional named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated.
anisopars	A list of two elements: "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
est.aniso	A bivariate logical vector providing which anisotropic parameters must be estimated.
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
likelihood	String; the configuration of the composite likelihood. Marginal is the default, see the Section **Details**.
local	Numeric; number of local work-items of the OpenCL setup
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or bobyqa or optimize. The names of the list must be the same of the names in the start list.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite or tapered likelihood computation. See the Section **Details** for more information.
neighb	Numeric; an optional positive integer indicating the order of neighborhood in the composite likelihood computation. See the Section **Details** for more information.
maxtime	Numeric; an optional positive integer indicating the order of temporal neighborhood in the composite likelihood computation.
memdist	Logical; if TRUE then all the distances useful in the composite likelihood estimation are computed before the optimization. FALSE is deprecated.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.

model	String; the type of random fields and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section **Details**.
n	Numeric; number of trials in a binomial random fields; number of successes in a negative binomial random fields
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
optimizer	String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are nlm, BFGS, SANN, L-BFGS-B and nlminb and bobyqa. In these last three cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
parallel	Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
sensitivity	Logical; if TRUE then the sensitivy matrix is computed
sparse	Logical; if TRUE then maximum likelihood is computed using sparse matrices algorithms (spam packake).It should be used with compactly supported covariance models.FALSE is the default.
start	An optional named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see **Details**).
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see **Details**).
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods (see **Details**).
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or bobyqa or nlminb or optimize. The names of the list must be the same of the names in the start list.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. For composite likelihood estimation it is deprecated. Use sensitivity TRUE and update the object using the function GeoVarestbootstrap FALSE is the default.
vartype	String; (SubSamp the default) the type of method used for computing the estimates' variances, see the Section **Details**.
weighted	Logical; if TRUE the likelihood objects are weighted, see the Section **Details**. If FALSE (the default) the composite likelihood is not weighted.
winconst	Numeric; a bivariate positive vector for computing the spatial sub-window in the sub-sampling procedure. See **Details** for more information.
winstp	Numeric; a value in $(0,1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure. The case 1 correspond to no overlapping. See **Details** for more information.

`winconst_t`	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure. See **Details** for more information.
`winstp_t`	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure. The case 1 correspond to no overlapping. See **Details** for more information.
`X`	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
`nosym`	Logical; if TRUE simmetric weights are not considered. This allows a faster but less efficient CL estimation.
`spobj`	An object of class sp or spacetime
`spdata`	Character:The name of data in the sp or spacetime object

Details

`GeoFit` provides weighted composite likelihood estimation based on pairs and independence composite likelihood estimation for Gaussian and non Gaussian random fields. Specifically, marginal and conditional pairwise likelihood are considered for each type of random field (Gaussian and not Gaussian). The optimization method is specified using `optimizer`. The default method is `Nelder-mead` and other available methods are `nlm`, `BFGS`, `SANN`, `L-BFGS-B`, `bobyqa` and `nlminb`. In the last three cases upper and lower bounds constraints in the optimization can be specified using `lower` and `upper` parameters.

Depending on the dimension of `data` and on the name of the correlation model, the observations are assumed as a realization of a spatial, spatio-temporal or bivariate random field. Specifically, with `data`, `coordx`, `coordy`, `coordt` parameters:

- If `data` is a numeric d-dimensional vector, `coordx` and `coordy` are two numeric d-dimensional vectors (or `coordx` is $(d \times 2)$-matrix and `coordy=NULL`), then the data are interpreted as a single spatial realisation observed on d spatial sites;
- If `data` is a numeric $(t \times d)$-matrix, `coordx` and `coordy` are two numeric d-dimensional vectors (or `coordx` is $(d \times 2)$-matrix and `coordy=NULL`), `coordt` is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a random fields observed on `d` spatial sites and for `t` times.
- If `data` is a numeric $(2 \times d)$-matrix, `coordx` and `coordy` are two numeric d-dimensional vectors (or `coordx` is $(d \times 2)$-matrix and `coordy=NULL`), then the data are interpreted as a single spatial realisation of a bivariate random fields observed on `d` spatial sites.
- If `data` is a list, `coordxdyn` is a list and `coordt` is a numeric t-dimensional vector, then the data are interpreted as a single spatial-temporal realisation of a random fields observed on dynamical spatial sites (different locations sites for each temporal instants) and for `t` times.

Is is also possible to specify a matrix of covariates using `X`. Specifically:

- In the spatial case `X` must be a $(d \times k)$ covariates matrix associated to `data` a numeric d-dimensional vector;
- In the spatiotemporal case `X` must be a $(N \times k)$ covariates matrix associated to `data` a numeric $(t \times d)$-matrix, where $N = t \times d$;
- In the spatiotemporal case `X` must be a $(N \times k)$ covariates matrix associated to `data` a numeric $(t \times d)$-matrix, where $N = 2 \times d$;

The `corrmodel` parameter allows to select a specific correlation function for the random fields. (See `GeoCovmatrix`).

The `distance` parameter allows to consider differents kinds of spatial distances. The settings alternatives are:

1. `Eucl`, the euclidean distance (default value);
2. `Chor`, the chordal distance;
3. `Geod`, the geodesic distance;

The `likelihood` parameter represents the composite-likelihood configurations. The settings alternatives are:

1. `Conditional`, the composite-likelihood is formed by conditionals likelihoods;
2. `Marginal`, the composite-likelihood is formed by marginals likelihoods (default value);
3. `Full`, the composite-likelihood turns out to be the standard likelihood;

It must be coupled with the `type` parameter that can be fixed to

1. `Pairwise`, the composite-likelihood is based on pairs;
2. `Independence`, the composite-likelihood is based on indepedence;
3. `Standard`, this is the option for the standard likelihood;

The possible combinations are:

1. `likelihood="Marginal"` and `type="Pairwise"` for maximum marginal pairwise likelihood estimation (the default setting)
2. `likelihood="Conditional"` and `type="Pairwise"` for maximum conditional pairwise likelihood estimation
3. `likelihood="Marginal"` and `type="Independence"` for maximum independence composite likelihood estimation
4. `likelihood="Full"` and `type="Standard"` for maximum stardard likelihood estimation

The first three combinations can be used for any model. The standard likelihood can be used only for some specific model.

The `model` parameter indicates the type of random field considered. The available options are:

random fields with marginal symmetric distribution:

- `Gaussian`, for a Gaussian random field.
- `StudentT`, for a StudentT random field (see Bevilacqua M., Caamaño C., Arellano Valle R.B., Morales-Oñate V., 2020).
- `Tukeyh`, for a Tukeyh random field.
- `Tukeyh2`, for a Tukeyhh random field. (see Caamaño et al., 2023)
- `Logistic`, for a Logistic random field.

random fields with positive values and right skewed marginal distribution:

- `Gamma` for a Gamma random fields (see Bevilacqua M., Caamano C., Gaetan, 2020)

- Weibull for a Weibull random fields (see Bevilacqua M., Caamano C., Gaetan, 2020)
- LogGaussian for a LogGaussian random fields (see Bevilacqua M., Caamano C., Gaetan, 2020)
- LogLogistic for a LogLogistic random fields.

random fields with with possibly asymmetric marginal distribution:

- SkewGaussian for a skew Gaussian random field (see Alegrıa et al. (2017)).
- SinhAsinh for a Sinh-arcsinh random field (see Blasi et. al 2022).
- TwopieceGaussian for a Twopiece Gaussian random field (see Bevilacqua et. al 2022).
- TwopieceTukeyh for a Twopiece Tukeyh random field (see Bevilacqua et. al 2022).

random fields with for directional data

- Wrapped for a wrapped Gaussian random field (see Alegria A., Bevilacqua, M., Porcu, E. (2016))

random fields with marginal counts data

- Poisson for a Poisson random field (see Morales-Navarrete et. al 2021).
- PoissonZIP for a zero inflated Poisson random field (see Morales-Navarrete et. al 2021).
- Binomial for a Binomial random field.
- BinomialNeg for a negative Binomial random field.
- BinomialNegZINB for a zero inflated negative Binomial random field.

random fields using Gaussian and Clayton copula (see Bevilacqua, Alvarado and Caamaño, 2023) with the following marginal distribution:

- Gaussian for Gaussian random field.
- Beta2 for Beta random field.

For a given model the associated parameters are given by nuisance and correlation parameters. In order to obtain the nuisance parameters associated with a specific model use NuisParam. In order to obtain the correlation parameters associated with a given correlation model use CorrParam.

All the nuisance and covariance parameters must be specified by the user using the start and the fixed parameter. Specifically:

The option start sets the starting values parameters in the optimization process for the parameters to be estimated. The option fixed parameter allows to fix some of the parameters.

Regression parameters in the linear specification must be specified as mean,mean1,..meank (see NuisParam). In this case a matrix of covariates with suitable dimension must be specified using X. In the case of a single mean then X should not be specified and it is interpreted as a vector of ones. It is also possible to fix a vector of spatial or spatio-temporal means (estimated with other methods for instance).

Two types of binary weights can be used in the weighted composite likelihood estimation based on pairs, one based on neighboords and one based on distances.

In the first case binary weights are set to 1 or 0 depending if the pairs are neighboords of a certain order (1, 2, 3, ..) specified by the parameter (neighb). This weighting scheme is effecient for large-data sets since the computation of the 'useful' pairs in based on fast nearest neighbour search (Caamaño et al., 2023).

In the second case, binary weights are set to 1 or 0 depending if the pairs have distance lower than (`maxdist`). This weighting scheme is less inefficient for large data. The same arguments of `neighb` applies for `maxtime` that sets the order (1, 2, 3, ..) of temporal neighboords in spatial-temporal field.

For estimation of the variance-covariance matrix of the weighted composite likelihood estimates the option `sensitivity=TRUE` must be specified. Then the `GeoFit` object must be updated using the function `GeoVarestbootstrap`. This allows to estimate the Godambe Information matrix (see Bevilacqua et. al. (2012) , Bevilacqua and Gaetan (2013)). Then standard error estimation, confidence intervals, pvalues and composite likelihood information critera can be obtained.

The option `varest=TRUE` is deprecated.

Value

Returns an object of class `GeoFit`. An object of class `GeoFit` is a list containing at most the following components:

`bivariate`	Logical:`TRUE` if the Gaussian random fields is bivariate, otherwise `FALSE`;
`clic`	The composite information criterion, if the full likelihood is considered then it coincides with the Akaike information criterion;
`coordx`	A d-dimensional vector of spatial coordinates;
`coordy`	A d-dimensional vector of spatial coordinates;
`coordt`	A t-dimensional vector of temporal coordinates;
`coordx_dyn`	A list of dynamical (in time) spatial coordinates;
`conf.int`	Confidence intervals for standard maximum likelihood estimation;
`convergence`	A string that denotes if convergence is reached;
`copula`	The type of copula;
`corrmodel`	The correlation model;
`data`	The vector or matrix or array of data;
`distance`	The type of spatial distance;
`fixed`	A list of the fixed parameters;
`iterations`	The number of iteration used by the numerical routine;
`likelihood`	The configuration of the composite likelihood;
`logCompLik`	The value of the log composite-likelihood at the maximum;
`maxdist`	The maximum spatial distance used in the weigthed composite likelihood. If no spatial distance is specified then it is NULL;
`maxtime`	The order of temporal neighborhood in the composite likelihood computation.
`message`	Extra message passed from the numerical routines;
`model`	The density associated to the likelihood objects;
`missp`	True if a misspecified Gaussian model is ued in the composite likelihhod;
`n`	The number of trials in a binominal random fields;the number of successes in a negative Binomial random fieldss;

neighb	The order of spatial neighborhood in the composite likelihood computation.
ns	The number of (different) location sites in the bivariate case;
nozero	In the case of tapered likelihood the percentage of non zero values in the covariance matrix. Otherwise is NULL.
numcoord	The number of spatial coordinates;
numtime	The number of the temporal realisations of the random fields;
param	A list of the parameters' estimates;
radius	The radius of the sphere in the case of great circle distance;
stderr	The vector of standard errors for standard maximum likelihood estimation;
sensmat	The sensitivity matrix;
varcov	The matrix of the variance-covariance of the estimates;
varimat	The variability matrix;
vartype	The method used to compute the variance of the estimates;
type	The type of the likelihood objects.
winconst	The constant used to compute the window size in the spatial sub-sampling;
winstp	The step used for moving the window in the spatial sub-sampling;
winconst_t	The constant used to compute the window size in the spatio-temporal sub-sampling;
winstp_	The step used for moving the window in the spatio-temporal sub-sampling;
X	The matrix of covariates;

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Sub-sampling estimation:

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Examples

```
library(GeoModels)

###############################################################
############ Examples of spatial Gaussian random fieldss ################
###############################################################


# Define the spatial-coordinates of the points:
set.seed(3)
N=300  # number of location sites
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)

# Define spatial matrix covariates and regression parameters
X=cbind(rep(1,N),runif(N))
```

```
mean <- 0.2
mean1 <- -0.5

# Set the covariance model's parameters:
corrmodel <- "Matern"
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5


param<-list(mean=mean,mean1=mean1,sill=sill,nugget=nugget,scale=scale,smooth=smooth)

# Simulation of the spatial Gaussian random fields:
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param,X=X)$data



################################################################
###
### Example 0. Maximum independence composite likelihood fitting of
### a Gaussian random fields (no dependence parameters)
###
###############################################################
# setting starting parameters to be estimated
start<-list(mean=mean,mean1=mean1,sill=sill)

fit1 <- GeoFit(data=data,coordx=coords,likelihood="Marginal",
                    type="Independence", start=start,X=X)
print(fit1)


################################################################
###
### Example 1. Maximum conditional pairwise likelihood fitting of
### a Gaussian random fields using BFGS
###
###############################################################
# setting fixed and starting parameters to be estimated
fixed<-list(nugget=nugget,smooth=smooth)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)

fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
                    neighb=3,likelihood="Conditional",optimizer="BFGS",
                    type="Pairwise", start=start,fixed=fixed,X=X)
print(fit1)

################################################################
###
### Example 2. Standard Maximum likelihood fitting of
### a Gaussian random fields using nlminb
###
###############################################################
```

```
# Define the spatial-coordinates of the points:
set.seed(3)
N=250  # number of location sites
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)

param<-list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth)

data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param)$data

# setting fixed and parameters to be estimated
fixed<-list(nugget=nugget,smooth=smooth)
start<-list(mean=mean,scale=scale,sill=sill)

I=Inf
lower<-list(mean=-I,scale=0,sill=0)
upper<-list(mean=I,scale=I,sill=I)
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
                    optimizer="nlminb",upper=upper,lower=lower,
                    likelihood="Full",type="Standard",
                    start=start,fixed=fixed)
print(fit2)


###############################################################
############ Examples of spatial non-Gaussian random fieldss #############
###############################################################


################################################################
###
### Example 3. Maximum pairwise likelihood fitting of a Weibull  random fields
### with Generalized Wendland correlation with Nelder-Mead
###
###############################################################
set.seed(524)
# Define the spatial-coordinates of the points:
N=300
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
shape=2
scale=0.2
smooth=0

model="Weibull"
corrmodel="GenWend"
param=list(mean=mean,mean1=mean1,scale=scale,
                     shape=shape,nugget=nugget,power2=4,smooth=smooth)
```

```
# Simulation of a  non stationary weibull random fields:
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,
           param=param)$data


fixed<-list(nugget=nugget,power2=4,smooth=smooth)
start<-list(mean=mean,mean1=mean1,scale=scale,shape=shape)

# Maximum independence likelihood:
fit <- GeoFit(data=data, coordx=coords, X=X,
           likelihood="Marginal",type="Independence", corrmodel=corrmodel,
         ,model=model, start=start, fixed=fixed)
print(unlist(fit$param))

## estimating  dependence parameter fixing vector mean   parameter
Xb=as.numeric(X %*% c(mean,mean1))
fixed<-list(nugget=nugget,power2=4,smooth=smooth,mean=Xb)
start<-list(scale=scale,shape=shape)

# Maximum conditional composite-likelihood fitting of the random fields:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    neighb=3,likelihood="Conditional",type="Pairwise",
                    optimizer="Nelder-Mead",
                    start=start,fixed=fixed)
print(unlist(fit1$param))



### joint estimation  of the dependence parameter and  mean   parameters
fixed<-list(nugget=nugget,power2=4,smooth=smooth)
start<-list(mean=mean,mean1=mean1,scale=scale,shape=shape)
fit2 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    neighb=3,likelihood="Conditional",type="Pairwise",X=X,
                    optimizer="Nelder-Mead",
                    start=start,fixed=fixed)
print(unlist(fit2$param))



################################################################
###
### Example 4. Maximum pairwise likelihood fitting of
### a Skew-Gaussian spatial  random fields with Wendland correlation
###
###############################################################
set.seed(261)
model="SkewGaussian"
# Define the spatial-coordinates of the points:
x <- runif(500, 0, 1)
y <- runif(500, 0, 1)
coords <- cbind(x,y)

corrmodel="Wend0"
```

```
mean=0;nugget=0
sill=1
skew=-4.5
power2=4
c_supp=0.2

# model parameters
param=list(power2=power2,skew=skew,
             mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param)$data

plot(density(data))
fixed=list(power2=power2,nugget=nugget)
start=list(scale=c_supp,skew=skew,mean=mean,sill=sill)
lower=list(scale=0,skew=-I,mean=-I,sill=0)
upper=list(scale=I,skew=I,mean=I,sill=I)
# Maximum marginal pairwise likelihood:
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    neighb=3,likelihood="Marginal",type="Pairwise",
                    optimizer="bobyqa",lower=lower,upper=upper,
                    start=start,fixed=fixed)
print(unlist(fit1$param))


################################################################
###
### Example 5. Maximum pairwise likelihood fitting of
### a Binomial random fields with exponential correlation
###
###############################################################

set.seed(422)
N=250
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
mean=0.1; mean1=0.8; mean2=-0.5 # regression parameters
X=cbind(rep(1,N),runif(N),runif(N)) # marix covariates
corrmodel <- "Wend0"
param=list(mean=mean,mean1=mean1,mean2=mean2,nugget=0,scale=0.2,power2=4)
# Simulation of the spatial Binomial-Gaussian random fields:
data <- GeoSim(coordx=coords, corrmodel=corrmodel, model="Binomial", n=10,X=X,
             param=param)$data


## estimating the marginal parameters using independence cl
fixed <- list(power2=4,scale=0.2,nugget=0)
start <- list(mean=mean,mean1=mean1,mean2=mean2)

# Maximum independence likelihood:
fit <- GeoFit(data=data, coordx=coords,n=10, X=X,
           likelihood="Marginal",type="Independence",corrmodel=corrmodel,
         ,model="Binomial", start=start, fixed=fixed)
```

```
print(fit)


## estimating  dependence parameter fixing vector mean   parameter
Xb=as.numeric(X %*% c(mean,mean1,mean2))
fixed <- list(nugget=0,power2=4,mean=Xb)
start <- list(scale=0.2)

# Maximum conditional pairwise likelihood:
fit1 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel,n=10,
          likelihood="Conditional",type="Pairwise",  neighb=3
         ,model="Binomial", start=start, fixed=fixed)

print(fit1)


## estimating jointly marginal   and dependnce parameters
fixed <- list(nugget=0,power2=4)
start <- list(mean=mean,mean1=mean1,mean2=mean2,scale=0.2)

# Maximum conditional pairwise likelihood:
fit2 <- GeoFit(data=data, coordx=coords, corrmodel=corrmodel,n=10, X=X,
          likelihood="Conditional",type="Pairwise",  neighb=3
         ,model="Binomial", start=start, fixed=fixed)

print(fit2)


################################################################
######### Examples of Gaussian spatio-temporal random fieldss ###########
################################################################
set.seed(52)
# Define the temporal sequence:
time <- seq(1, 9, 1)

# Define the spatial-coordinates of the points:
x <- runif(20, 0, 1)
y <- runif(20, 0, 1)
coords=cbind(x,y)

# Set the covariance model's parameters:
scale_s=0.2/3;scale_t=1
smooth_s=0.5;smooth_t=0.5
sill=1
nugget=0
mean=0

param<-list(mean=0,scale_s=scale_s,scale_t=scale_t,
 smooth_t=smooth_t, smooth_s=smooth_s ,sill=sill,nugget=nugget)

# Simulation of the spatial-temporal Gaussian random fields:
data <- GeoSim(coordx=coords,coordt=time,corrmodel="Matern_Matern",
```

```
              param=param)$data

################################################################
###
### Example 6. Maximum pairwise likelihood fitting of a
### space time Gaussian random fields with double-exponential correlation
###
###############################################################
# Fixed parameters
fixed<-list(nugget=nugget,smooth_s=smooth_s,smooth_t=smooth_t)
# Starting value for the estimated parameters
start<-list(mean=mean,scale_s=scale_s,scale_t=scale_t,sill=sill)

# Maximum composite-likelihood fitting of the random fields:
fit <- GeoFit(data=data,coordx=coords,coordt=time,
                    corrmodel="Matern_Matern",maxtime=1,neighb=3,
                    likelihood="Marginal",type="Pairwise",
                     start=start,fixed=fixed)
print(fit)



###############################################################
######### Examples of a bivariate Gaussian  random fields ###########
###############################################################

################################################################
### Example 7. Maximum pairwise  likelihood fitting of a
### bivariate Gaussian random fields with separable Bivariate  matern
### (cross) correlation model
###############################################################

# Define the spatial-coordinates of the points:
set.seed(89)
x <- runif(300, 0, 1)
y <- runif(300, 0, 1)
coords=cbind(x,y)
# parameters
param=list(mean_1=0,mean_2=0,scale=0.1,smooth=0.5,sill_1=1,sill_2=1,
           nugget_1=0,nugget_2=0,pcol=0.2)

# Simulation of a spatial bivariate Gaussian random fields:
data <- GeoSim(coordx=coords, corrmodel="Bi_Matern_sep",
              param=param)$data

# selecting fixed and estimated parameters
fixed=list(mean_1=0,mean_2=0,nugget_1=0,nugget_2=0,smooth=0.5)
start=list(sill_1=var(data[1,]),sill_2=var(data[2,]),
           scale=0.1,pcol=cor(data[1,],data[2,]))


# Maximum marginal pairwise likelihood
fitcl<- GeoFit(data=data, coordx=coords, corrmodel="Bi_Matern_sep",
```

```
                    likelihood="Marginal",type="Pairwise",
                    optimizer="BFGS" , start=start,fixed=fixed,
                    neighb=c(4,4,4))
print(fitcl)
```

GeoFit2 *Max-Likelihood-Based Fitting of Gaussian and non Gaussian RFs.*

Description

Maximum weighted composite-likelihood fitting for Gaussian and some Non-Gaussian univariate spatial, spatio-temporal and bivariate spatial RFs. A first preliminary estimation is performed using independence composite-likelihood for the marginal parameters of the model. The estimates are then used as starting values in the second final estimation step. The function allows to fix any of the parameters and setting upper/lower bound in the optimization.

Usage

```
GeoFit2(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL,
  copula=NULL,corrmodel,distance="Eucl",fixed=NULL,
     anisopars=NULL,est.aniso=c(FALSE,FALSE),GPU=NULL,
     grid=FALSE, likelihood='Marginal',
     local=c(1,1),  lower=NULL,maxdist=Inf,neighb=NULL,
      maxtime=Inf, memdist=TRUE,method="cholesky",
      model='Gaussian',n=1, onlyvar=FALSE ,
      optimizer='Nelder-Mead', parallel=FALSE,
      radius=6371,  sensitivity=FALSE,sparse=FALSE,
       start=NULL, taper=NULL, tapsep=NULL,
      type='Pairwise', upper=NULL, varest=FALSE,
      vartype='SubSamp', weighted=FALSE, winconst=NULL, winstp=NULL,
      winconst_t=NULL, winstp_t=NULL,X=NULL,nosym=FALSE,spobj=NULL,spdata=NULL)
```

Arguments

`data` A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid). For the description see the Section **Details**.

`coordx` A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.

`coordy` A numeric vector assigning 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.

coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
fixed	An optional named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated.
anisopars	A list of two elements: "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
est.aniso	A bivariate logical vector providing which anisotropic parameters must be estimated.
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
likelihood	String; the configuration of the composite likelihood. Marginal is the default, see the Section **Details**.
local	Numeric; number of local work-items of the OpenCL setup
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or bobyqa or optimize. The names of the list must be the same of the names in the start list.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite or tapered likelihood computation. See the Section **Details** for more information.
neighb	Numeric; an optional positive integer indicating the order of neighborhood in the composite likelihood computation. See the Section **Details** for more information.
maxtime	Numeric; an optional positive integer indicating the order of temporal neighborhood in the composite likelihood computation.
memdist	Logical; if TRUE then all the distances useful in the composite likelihood estimation are computed before the optimization. FALSE is deprecated.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section **Details**.
n	Numeric; number of trials in a binomial RF; number of successes in a negative binomial RF

onlyvar
: Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.

optimizer
: String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are nlm, BFGS, SANN, L-BFGS-B and nlminb and bobyqa. In these last three cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.

parallel
: Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.

radius
: Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.

sensitivity
: Logical; if TRUE then the sensitivy matrix is computed

sparse
: Logical; if TRUE then maximum likelihood is computed using sparse matrices algorithms (spam packake).It should be used with compactly supported covariance models.FALSE is the default.

start
: An optional named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see **Details**).

taper
: String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.

tapsep
: Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see **Details**).

type
: String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods (see **Details**).

upper
: An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or bobyqa or optimize. The names of the list must be the same of the names in the start list.

varest
: Logical; if TRUE the estimates' variances and standard errors are returned. For composite likelihood estimation it is deprecated. Use sensitivity TRUE and update the object using the function GeoVarestbootstrap FALSE is the default.

vartype
: String; (SubSamp the default) the type of method used for computing the estimates' variances, see the Section **Details**.

weighted
: Logical; if TRUE the likelihood objects are weighted, see the Section **Details**. If FALSE (the default) the composite likelihood is not weighted.

winconst
: Numeric; a bivariate positive vector for computing the spatial sub-window in the sub-sampling procedure. See **Details** for more information.

winstp
: Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure. The case 1 correspond to no overlapping. See **Details** for more information.

winconst_t
: Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure. See **Details** for more information.

winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure. The case 1 correspond to no overlapping. See **Details** for more information.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
nosym	Logical; if TRUE simmetric weights are not considered. This allows a faster but less efficient CL estimation.
spobj	An object of class sp or spacetime
spdata	Character:The name of data in the sp or spacetime object

Details

The function `GeoFit2` is similar to the function `GeoFit`. However `GeoFit2` performs a preliminary estimation using maximum indenpendence composite likelihood of the marginal parameters of the model and then use the obtained estimates as starting value in the global weighted composite likelihood estimation (that includes marginal and dependence parameters). This allows to obtain "good" starting values in the optimization algorithm for the marginal parameters.

Value

Returns an object of class `GeoFit`. An object of class `GeoFit` is a list containing at most the following components:

bivariate	Logical:`TRUE` if the Gaussian RF is bivariate, otherwise `FALSE`;
clic	The composite information criterion, if the full likelihood is considered then it coincides with the Akaike information criterion;
coordx	A d-dimensional vector of spatial coordinates;
coordy	A d-dimensional vector of spatial coordinates;
coordt	A t-dimensional vector of temporal coordinates;
coordx_dyn	A list of dynamical (in time) spatial coordinates;
conf.int	Confidence intervals for standard maximum likelihood estimation;
convergence	A string that denotes if convergence is reached;
copula	The type of copula;
corrmodel	The correlation model;
data	The vector or matrix or array of data;
distance	The type of spatial distance;
fixed	A list of the fixed parameters;
iterations	The number of iteration used by the numerical routine;
likelihood	The configuration of the composite likelihood;
logCompLik	The value of the log composite-likelihood at the maximum;
maxdist	The maximum spatial distance used in the weigthed composite likelihood. If no spatial distance is specified then it is NULL;
maxtime	The maximum temporal distance used in the weigthed composite likelihood. If no spatial distance is specified then it is NULL;

`message` Extra message passed from the numerical routines;

`model` The density associated to the likelihood objects;

`missp` True if a misspecified Gaussian model is ued in the composite likelihhod;

`n` The number of trials in a binominal RF;the number of successes in a negative Binomial RFs;

`neighb` The order of spatial neighborhood in the composite likelihood computation.

`ns` The number of (different) location sites in the bivariate case;

`nozero` In the case of tapered likelihood the percentage of non zero values in the covariance matrix. Otherwise is NULL.

`numcoord` The number of spatial coordinates;

`numtime` The number of the temporal realisations of the RF;

`param` A list of the parameters' estimates;

`radius` The radius of the sphere in the case of great circle distance;

`stderr` The vector of standard errors for standard maximum likelihood estimation;

`sensmat` The sensitivity matrix;

`varcov` The matrix of the variance-covariance of the estimates;

`varimat` The variability matrix;

`vartype` The method used to compute the variance of the estimates;

`type` The type of the likelihood objects.

`winconst` The constant used to compute the window size in the spatial sub-sampling;

`winstp` The step used for moving the window in the spatial sub-sampling;

`winconst_t` The constant used to compute the window size in the spatio-temporal sub-sampling;

`winstp_` The step used for moving the window in the spatio-temporal sub-sampling;

`X` The matrix of covariates;

Author(s)

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Examples

```
library(GeoModels)


################################################################
############ Examples of spatial Gaussian RFs ################
################################################################

#################################################################
###
```

```
### Example 1 : Maximum pairwise conditional likelihood fitting
###  of a Gaussian RF with Matern correlation
###
###############################################################
model="Gaussian"
# Define the spatial-coordinates of the points:
set.seed(3)
N=400  # number of location sites
x <- runif(N, 0, 1)
set.seed(6)
y <- runif(N, 0, 1)
coords <- cbind(x,y)

# Define spatial matrix covariates
X=cbind(rep(1,N),runif(N))

# Set the covariance model's parameters:
corrmodel <- "Matern"
mean <- 0.2
mean1 <- -0.5
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5
param<-list(mean=mean,mean1=mean1,sill=sill,nugget=nugget,scale=scale,smooth=smooth)

# Simulation of the spatial Gaussian RF:
data <- GeoSim(coordx=coords,model=model,corrmodel=corrmodel, param=param,X=X)$data

fixed<-list(nugget=nugget,smooth=smooth)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)

################################################################
###
### Maximum pairwise likelihood fitting of
### Gaussian RFs with exponential correlation.
###
###############################################################
fit1 <- GeoFit2(data=data,coordx=coords,corrmodel=corrmodel,
                    optimizer="BFGS",neighb=3,likelihood="Conditional",
                    type="Pairwise", start=start,fixed=fixed,X=X)
print(fit1)



###############################################################
############ Examples of spatial non-Gaussian RFs #############
###############################################################


################################################################
###
```

```
### Example 2. Maximum pairwise likelihood fitting of
### a LogGaussian  RF with Generalized Wendland correlation
###
###############################################################
set.seed(524)
# Define the spatial-coordinates of the points:
N=500
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)
X=cbind(rep(1,N),runif(N))
mean=1; mean1=2 # regression parameters
nugget=0
sill=0.5
scale=0.2
smooth=0

model="LogGaussian"
corrmodel="GenWend"
param=list(mean=mean,mean1=mean1,sill=sill,scale=scale,
                    nugget=nugget,power2=4,smooth=smooth)
# Simulation of a  non stationary LogGaussian RF:
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model,X=X,
           param=param)$data

fixed<-list(nugget=nugget,power2=4,smooth=smooth)
start<-list(mean=mean,mean1=mean1,scale=scale,sill=sill)
I=Inf
lower<-list(mean=-I,mean1=-I,scale=0,sill=0)
upper<-list(mean= I,mean1= I,scale=I,sill=I)

# Maximum pairwise composite-likelihood fitting of the RF:
fit <- GeoFit2(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    neighb=3,likelihood="Conditional",type="Pairwise",X=X,
                    optimizer="nlminb",lower=lower,upper=upper,
                    start=start,fixed=fixed)
print(unlist(fit$param))


################################################################
###
### Example 3. Maximum pairwise likelihood fitting of
### SinhAsinh   RFs with Wendland0 correlation
###
###############################################################
set.seed(261)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
x <- runif(500, 0, 1)
y <- runif(500, 0, 1)
coords <- cbind(x,y)

corrmodel="Wend0"
```

```
mean=0;nugget=0
sill=1
skew=-0.5
tail=1.5
power2=4
c_supp=0.2

# model parameters
param=list(power2=power2,skew=skew,tail=tail,
             mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data <- GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param)$data

plot(density(data))
fixed=list(power2=power2,nugget=nugget)
start=list(scale=c_supp,skew=skew,tail=tail,mean=mean,sill=sill)
# Maximum pairwise likelihood:
fit1 <- GeoFit2(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    neighb=3,likelihood="Marginal",type="Pairwise",
                    start=start,fixed=fixed)
print(unlist(fit1$param))
```

GeoKrig *Spatial (bivariate) and spatio temporal optimal linear prediction for Gaussian and non Gaussian random fields.*

Description

For a given set of spatial location sites (and temporal instants), the function computes optimal linear prediction and associated mean square error for the Gaussian and non Gaussian case.

Usage

```
GeoKrig(estobj=NULL,data, coordx, coordy=NULL, coordt=NULL,
coordx_dyn=NULL, corrmodel,distance="Eucl",
    grid=FALSE, loc, maxdist=NULL, maxtime=NULL,
    method="cholesky", model="Gaussian", n=1,nloc=NULL,mse=FALSE,
    lin_opt=TRUE,  param, anisopars=NULL,radius=6371, sparse=FALSE,
    taper=NULL,tapsep=NULL, time=NULL, type="Standard",type_mse=NULL,
     type_krig="Simple",weigthed=TRUE,which=1,
     copula=NULL, X=NULL,Xloc=NULL,Mloc=NULL,spobj=NULL,spdata=NULL)
```

Arguments

`estobj` An object of class Geofit that includes information about data, model and estimates.

data
: A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.

coordx
: A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates used for prediction. d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.

coordy
: A numeric vector giving 1-dimension of spatial coordinates used for prediction; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.

coordt
: A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is NULL then a spatial random field is expected.

coordx_dyn
: A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL

corrmodel
: String; the name of a correlation model, for the description see the Section **Details**.

distance
: String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.

grid
: Logical; if FALSE (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).

lin_opt
: Logical;If TRUE (default) then optimal (pairwise) linear kriging is computed. Otherwise optimal (pairwise) kriging is computed in the mean square sense.

loc
: A numeric $(n \times 2)$-matrix (where n is the number of spatial sites) giving 2-dimensions of spatial coordinates to be predicted.

maxdist
: Numeric; an optional positive value indicating the maximum spatial compact support in the case of covariance tapering kriging.

maxtime
: Numeric; an optional positive value indicating the maximum temporal compact support in the case of covasriance tapering kriging.

method
: String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.

n
: Numeric; the number of trials in a binomial random fields. Default is 1.

nloc
: Numeric; the number of trials of the locations sites to be predicted in a binomial random fields type II. Default is 1.

mse
: Logical; if TRUE (the default) MSE of the kriging predictor is computed

model
: String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section **Details**.

param
: A list of parameter values required for the correlation model.See the Section **Details**.

anisopars
: A list of two elements: "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.

radius	Numeric: the radius of the sphere if coordinates are passed in lon/lat format;
sparse	Logical; if TRUE kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
taper	String; the name of the taper correlation function, see the Section **Details**.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see **Details**).
time	A numeric $(m \times 1)$ vector (where m is the number of temporal instants) giving the temporal instants to be predicted; the default is NULL then only spatial prediction is performed.
type	String; if Standard then standard kriging is performed;if Tapering then kriging with covariance tapering is performed;if Pairwise then pairwise kriging is performed
type_mse	String; if Theoretical then theoretical MSE pairwise kriging is computed. If SubSamp then an estimation based on subsampling is computed.
type_krig	String; the type of kriging. If Simple (the default) then simple kriging is performed. If Optim then optimal kriging is performed for some non-Gaussian RFs
weigthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime)are used in the pairwise kriging.
which	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
Xloc	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.
Mloc	Numeric; Vector of spatio(temporal) estimated means associated to predicted locations.
spobj	An object of class sp or spacetime
spdata	Character:The name of data in the sp or spacetime object

Details

Best linear unbiased predictor and associated mean square error is computed for Gaussian and some non Gaussian cases. Specifically, for a spatial or spatio-temporal or spatial bivariate dataset, given a set of spatial locations and temporal istants and a correlation model corrmodel with some fixed parameters and given the type of RF (model) the function computes simple kriging, for the specified spatial locations loc and temporal instants time, providing also the respective mean square error. For the choice of the spatial or spatio temporal correlation model see details in GeoCovmatrix function. The list param specifies mean and covariance parameters, see CorrParam and GeoCovmatrix for details. The type_krig parameter indicates the type of kriging. In the case of simple kriging, the known mean can be specified by the parameter mean in the list param (See examples).

Value

Returns an object of class Kg. An object of class Kg is a list containing at most the following components:

- bivariate — TRUE if spatial bivariate cokriging is performed, otherwise FALSE;
- coordx — A d-dimensional vector of spatial coordinates used for prediction;
- coordy — A d-dimensional vector of spatial coordinates used for prediction;
- coordt — A t-dimensional vector of temporal coordinates used for prediction;
- corrmodel — String: the correlation model;
- covmatrix — The covariance matrix if type is Standard. An object of class spam if type is Tapering
- data — The vector or matrix or array of data used for prediction
- distance — String: the type of spatial distance;
- grid — TRUE if the spatial data used for prediction are observed in a regular grid, otherwise FALSE;
- loc — A $(n \times 2)$-matrix of spatial locations to be predicted.
- n — The number of trial for Binomial RFs
- nozero — In the case of tapered simple kriging the percentage of non zero values in the covariance matrix. Otherwise is NULL.
- numcoord — Numeric:he number d of spatial coordinates used for prediction;
- numloc — Numeric: the number n of spatial coordinates to be predicted;
- numtime — Numeric: the number d of the temporal instants used for prediction;
- numt — Numeric: the number m of the temporal instants to be predicted;
- model — The type of RF, see GeoFit.
- param — Numeric: The covariance parameters;
- pred — A $(m \times n)$-matrix of spatio or spatio temporal kriging prediction;
- radius — Numeric: the radius of the sphere if coordinates are pssed in lon/lat format;
- spacetime — TRUE if spatio-temporal kriging and FALSE if spatial kriging;
- tapmod — String: the taper model if type is Tapering. Otherwise is NULL.
- time — A m-dimensional vector of temporal coordinates to be predicted;
- type — String: the type of kriging (Standard or Tapering).
- type_krig — String: the type of kriging.
- mse — A $(m \times n)$-matrix of spatio or spatio temporal mean square error kriging prediction;

Author(s)

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References

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York.

See Also

GeoCovmatrix

Examples

```
library(GeoModels)
################################################################
########### Examples of spatial kriging ############
################################################################

################################################################
###
### Example 1. Spatial  kriging of a
### Gaussian random fields with Gen wendland correlation.
###
################################################################

model="Gaussian"
set.seed(79)
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel = "GenWend"
mean=0; sill=5; nugget=0
scale=0.2;smooth=0;power2=4

param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth,power2=power2)

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,
              param=param)$data

## estimation with pairwise likelihood
fixed=list(nugget=nugget,smooth=0,power2=power2)
start=list(mean=0,scale=scale,sill=1)
I=Inf
lower=list(mean=-I,scale=0,sill=0)
upper=list(mean= I,scale=I,sill=I)
# Maximum pairwise likelihood fitting :
fit = GeoFit(data, coordx=coords, corrmodel=corrmodel,model=model,
             likelihood='Marginal', type='Pairwise',neighb=3,
             optimizer="nlminb", lower=lower,upper=upper,
             start=start,fixed=fixed)

# locations to predict
xx=seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))
```

```
## first option
#param=append(fit$param,fit$fixed)
#pr=GeoKrig(loc=loc_to_pred,coordx=coords,corrmodel=corrmodel,
#       model=model,param=param,data=data,mse=TRUE)

## second option using object GeoFit
pr=GeoKrig(fit,loc=loc_to_pred,mse=TRUE)


colour = rainbow(100)

opar=par(no.readonly = TRUE)
par(mfrow=c(1,3))
quilt.plot(coords,data,col=colour)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
           xlab="",ylab="",
           main=" Kriging ")

# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")
par(opar)

#################################################################
###
### Example 2. Spatial  kriging of a Skew
### Gaussian random fields with Matern correlation.
###
#################################################################
model="SkewGaussian"
set.seed(79)
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel = "Matern"
mean=0
sill=2
nugget=0
scale=0.1
smooth=0.5
skew=3
param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth,skew=skew)

# Simulation of the spatial skew Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,model=model,
              param=param)$data

fixed=list(nugget=nugget,smooth=smooth)
start=list(mean=0,scale=scale,sill=1,skew=skew)
I=Inf
```

```
lower=list(mean=-I,scale=0,sill=0,skew=-I)
upper=list(mean= I,scale=I,sill=I,skew=I)
# Maximum pairwise likelihood fitting :
fit = GeoFit2(data, coordx=coords, corrmodel=corrmodel,model=model,
              likelihood='Marginal', type='Pairwise',neighb=3,
              optimizer="nlminb", lower=lower,upper=upper,
              start=start,fixed=fixed)

# locations to predict
xx=seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))
## optimal linear kriging
pr=GeoKrig(fit,loc=loc_to_pred,mse=TRUE)

colour = rainbow(100)

opar=par(no.readonly = TRUE)
par(mfrow=c(1,3))
quilt.plot(coords,data,col=colour)
# simple kriging map prediction
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
           xlab="",ylab="",
           main=" Kriging ")

# simple kriging MSE map prediction variance
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
           xlab="",ylab="",main="Std error")
par(opar)

################################################################
###
### Example 3. Spatial  kriging  of a
###   Gamma random field with mean spatial regression
###
###############################################################
set.seed(312)
model="Gamma"
corrmodel = "GenWend"
# Define the spatial-coordinates of the points:
NN=300
coords=cbind(runif(NN),runif(NN))
## matrix covariates
a0=rep(1,NN)
a1=runif(NN,0,1)
X=cbind(a0,a1)
##Set model parameters
shape=2
## regression parameters
mean = 1;mean1= -0.2
# correlation parameters
nugget = 0;power2=4
scale = 0.3;smooth=0
```

```
## simulation
param=list(shape=shape,nugget=nugget,mean=mean,mean1=mean1,
           scale=scale,power2=power2,smooth=smooth)
data = GeoSim(coordx=coords,corrmodel=corrmodel, param=param,
              model=model,X=X)$data

#####starting and fixed parameters
fixed=list(nugget=nugget,power2=power2,smooth=smooth)
start=list(mean=mean,mean1=mean1, scale=scale,shape=shape)

## estimation with pairwise likelihood
fit2 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel,X=X,
              neighb=3,likelihood="Conditional",type="Pairwise",
              start=start,fixed=fixed, model = model)

# locations to predict with associated covariates
xx=seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))
NP=nrow(loc_to_pred)
a0=rep(1,NP)
a1=runif(NP,0,1)
Xloc=cbind(a0,a1)

#optimal linear  kriging
pr=GeoKrig(fit2,loc=loc_to_pred,Xloc=Xloc,sparse=TRUE,mse=TRUE)

## map
opar=par(no.readonly = TRUE)
par(mfrow=c(1,3))
quilt.plot(coords,data,main="Data")
map=matrix(pr$pred,ncol=length(xx))
mapmse=matrix(pr$mse,ncol=length(xx))
image.plot(xx, xx, map,
           xlab="",ylab="",main="Kriging ")

image.plot(xx, xx, mapmse,
           xlab="",ylab="",main="MSE")
par(opar)


################################################################
########### Examples of spatio temporal kriging ############
################################################################

################################################################
###
### Example 4. Spatio temporal simple kriging of n locations
### sites and m temporal instants for a Gaussian random fields
### with estimated double Wendland correlation.
###
###############################################################
model="Gaussian"
# Define the spatial-coordinates of the points:
```

```
x = runif(300, 0, 1)
y = runif(300, 0, 1)
coords=cbind(x,y)
times=1:4

# Define model correlation modek and associated parameters
corrmodel="Wend0_Wend0"
param=list(nugget=0,mean=0,power2_s=4,power2_t=4,
           scale_s=0.2,scale_t=2,sill=1)

# Simulation of the space time Gaussian random field:
set.seed(31)
data=GeoSim(coordx=coords,coordt=times,corrmodel=corrmodel,sparse=TRUE,
            param=param)$data

# Maximum pairwise likelihood fitting of the space time random field:
start = list(scale_s=0.15,scale_t=2,sill=1,mean=0)
fixed = list(nugget=0,power2_s=4,power2_t=4)

fit = GeoFit(data, coordx=coords, coordt=times, model=model, corrmodel=corrmodel,
             likelihood='Conditional', type='Pairwise',start=start,fixed=fixed,
             neighb=3,maxtime=1)

# locations to predict
xx=seq(0,1,0.04)
loc_to_pred=as.matrix(expand.grid(xx,xx))
#  Define the times to predict
times_to_pred=2

pr=GeoKrig(fit,loc=loc_to_pred,time=times_to_pred,sparse=TRUE,mse=TRUE)

opar=par(no.readonly = TRUE)
par(mfrow=c(1,3))
zlim=c(-2.5,2.5)
colour = rainbow(100)


quilt.plot(coords,data[2,] ,col=colour,main = paste(" data  at Time 2"))
image.plot(xx, xx, matrix(pr$pred,ncol=length(xx)),col=colour,
           main = paste(" Kriging at Time 2"),ylab="")
image.plot(xx, xx, matrix(pr$mse,ncol=length(xx)),col=colour,
           main = paste("Std err Time at time 2"),ylab="")


par(opar)


################################################################
###
### Example r. Spatial bivariate  simple cokriging of n locations
### sites  for a bivariate Gaussian random fields
### with estimated Matern correlation.
###
```

```
################################################################
#set.seed(6)
#NN=1500 # number of spatial locations
#x = runif(NN, 0, 1);
#y = runif(NN, 0, 1)
#coords=cbind(x,y)

## setting parameters
#mean_1 = 2; mean_2= -1
#nugget_1 =0;nugget_2=0
#sill_1 =0.5; sill_2 =1;

### correlation parameters
#CorrParam("Bi_Matern")
#scale_1=0.2/3; scale_2=0.15/3; scale_12=0.5*(scale_2+scale_1)
#smooth_1=smooth_2=smooth_12=0.5
#pcol = -0.4
#param= list(nugget_1=nugget_1,nugget_2=nugget_2,
#          sill_1=sill_1,sill_2=sill_2,
#          mean_1=mean_1,mean_2=mean_2,
#          smooth_1=smooth_1, smooth_2=smooth_2,smooth_12=smooth_12,
#          scale_1=scale_1, scale_2=scale_2,scale_12=scale_12,
#          pcol=pcol)

## simulation
#data = GeoSim(coordx=coords, corrmodel="Bi_Matern",model=model,param=param)$data

#fixed=list(mean_1=mean_1,mean_2=mean_2, nugget_1=nugget_1,nugget_2=nugget_2,
#       smooth_1=smooth_1, smooth_2=smooth_2,smooth_12=smooth_12)

#start=list( sill_1=sill_1,sill_2=sill_2,
#        scale_1=scale_1,scale_2=scale_2,scale_12=scale_12, pcol=pcol)

## estimation with maximum likelihood
#fit = GeoFit(data=data,coordx=coords, corrmodel="Bi_Matern",
 #likelihood="Full",type="Standard",optimizer="BFGS",
  #likelihood="Marginal",type="Pairwise",optimizer="BFGS",neighb=5,
  #start=start,fixed=fixed)

###### co-kriging for the fist component ##############
#xx=seq(0,1,0.022)
#loc_to_pred=as.matrix(expand.grid(xx,xx))
#pr1 = GeoKrig(fit,which=1,mse=TRUE,loc=loc_to_pred)

#opar=par(no.readonly = TRUE)
#par(mfrow=c(1,2))
#zlim=c(-2.5,2.5)
#colour = rainbow(100)
#quilt.plot(coords,data[1,] ,col=colour,main = paste(" Fist component"))
#quilt.plot(loc_to_pred,pr1$pred,col=colour,
#           main = paste(" Kriging first component"),ylab="")
#par(opar)
```

GeoKrigloc *Spatial (bivariate) and spatio temporal optimal linear local prediction for Gaussian and non Gaussian RFs.*

Description

For a given set of spatial location sites (and temporal instants), the function computes optmal local linear prediction and the associated mean squared error for the Gaussian and non Gaussian case using a spatial (temporal) neighborhood computed using the function GeoNeighborhood

Usage

```
GeoKrigloc(estobj=NULL,data, coordx, coordy=NULL, coordt=NULL,
    coordx_dyn=NULL, corrmodel, distance="Eucl", grid=FALSE,
    loc, neighb=NULL, maxdist=NULL,
    maxtime=NULL, method="cholesky",
    model="Gaussian", n=1,nloc=NULL, mse=FALSE,
    param, anisopars=NULL,radius=6371,
    sparse=FALSE, time=NULL, type="Standard",type_mse=NULL,
    type_krig="Simple",weigthed=TRUE,
    which=1, copula=NULL,X=NULL,Xloc=NULL,
    Mloc=NULL,spobj=NULL,spdata=NULL,parallel=FALSE,ncores=NULL)
```

Arguments

estobj	An object of class Geofit that includes information about data, model and estimates.
data	A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid) giving the data used for prediction.
coordx	A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates used for prediction. d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates used for prediction; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates used for prediction; the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL

corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
grid	Logical; if FALSE (the default) the data used for prediction are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
loc	A numeric $(n \times 2)$-matrix (where n is the number of spatial sites) giving 2-dimensions of spatial coordinates to be predicted.
neighb	Numeric; an optional positive integer indicating the order of the neighborhood.
maxdist	Numeric; an optional positive value indicating the distance in the spatial neighborhood.
maxtime	Numeric; an optional positive integer value indicating the order of the temporal neighborhood.
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
n	Numeric; the number of trials in a binomial random fields. Default is 1.
nloc	Numeric; the number of trials of the locations sites to be predicted in the binomial random field. If missing then a rounded mean of n is considered.
mse	Logical; if TRUE (the default) MSE of the kriging predictor is computed
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section **Details**.
param	A list of parameter values required for the correlation model.See the Section **Details**.
anisopars	A list of two elements: "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
radius	Numeric: the radius of the sphere if coordinates are passed in lon/lat format;
sparse	Logical; if TRUE kriging is computed with sparse matrices algorithms using spam package. Default is FALSE. It should be used with compactly supported covariances.
time	A numeric $(m \times 1)$ vector (where m is the number of temporal instants) giving the temporal instants to be predicted; the default is NULL then only spatial prediction is performed.
type	String; if Standard then standard kriging is performed;if Tapering then kriging with covariance tapering is performed;if Pairwise then pairwise kriging is performed
type_mse	String; if Theoretical then theoretical MSE pairwise kriging is computed. If SubSamp then an estimation based on subsampling is computed.
type_krig	String; the type of kriging. If Simple (the default) then simple kriging is performed. If Optim then optimal kriging is performed for some non-Gaussian RFs
weigthed	Logical; if TRUE then decreasing weigths coming from a compactly supported correlation function with compact support maxdist (maxtime)are used in the pairwise kriging.

which	Numeric; In the case of bivariate (tapered) cokriging it indicates which variable to predict. It can be 1 or 2
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
Xloc	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification associated to predicted locations.
Mloc	Numeric; Vector of spatio(temporal) estimated means associated to predicted locations.
spobj	An object of class sp or spacetime
spdata	Character:The name of data in the sp or spacetime object
parallel	Logical; if TRUE then the prediction computation is parallelized
ncores	Numeric; number of cores involved in parallelization.

Details

This function use the GeoKrig with a spatial or spatio-temporal neighborhood computed using the function GeoNeighborhood. The neighborhood is specified with the option maxdist and maxtime.

Value

Returns an object of class Kg. An object of class Kg is a list containing at most the following components:

bivariate	TRUE if spatial bivariate cokriging is performed, otherwise FALSE;
coordx	A d-dimensional vector of spatial coordinates used for prediction;
coordy	A d-dimensional vector of spatial coordinates used for prediction;
coordt	A t-dimensional vector of temporal coordinates used for prediction;
corrmodel	String: the correlation model;
covmatrix	The covariance matrix if type is Standard. An object of class spam if type is Tapering
data	The vector or matrix or array of data used for prediction
distance	String: the type of spatial distance;
grid	TRUE if the spatial data used for prediction are observed in a regular grid, otherwise FALSE;
loc	A $(n \times 2)$-matrix of spatial locations to be predicted.
n	The number of trial for Binomial RFs
nozero	In the case of tapered simple kriging the percentage of non zero values in the covariance matrix. Otherwise is NULL.
numcoord	Numeric:he number d of spatial coordinates used for prediction;
numloc	Numeric: the number n of spatial coordinates to be predicted;
numtime	Numeric: the number d of the temporal instants used for prediction;
numt	Numeric: the number m of the temporal instants to be predicted;

model	The type of RF, see GeoFit.
param	Numeric: The covariance parameters;
pred	A $(m \times n)$-matrix of spatio or spatio temporal kriging prediction;
radius	Numeric: the radius of the sphere if coordinates are pssed in lon/lat format;
spacetime	TRUE if spatio-temporal kriging and FALSE if spatial kriging;
tapmod	String: the taper model if type is Tapering. Otherwise is NULL.
time	A m-dimensional vector of temporal coordinates to be predicted;
type	String: the type of kriging (Standard or Tapering).
type_krig	String: the type of kriging.
mse	A $(m \times n)$-matrix of spatio or spatio temporal mean square error kriging prediction;

Author(s)

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References

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See Also

GeoCovmatrix

Examples

```
################################################################
############### Examples of Spatial local kriging  #############
################################################################
require(GeoModels)
####
model="Gaussian"

# Define the spatial-coordinates of the points:
set.seed(759)
x = runif(1000, 0, 1)
y = runif(1000, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
corrmodel = "GenWend"
mean=0; sill=1
nugget=0; scale=0.2
param=list(mean=mean,sill=sill,nugget=nugget,smooth=0,
```

```
scale=scale,power2=4)

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,
              param=param)$data

# Maximum pairwise likelihood fitting of the space time random field:

start=list(scale=scale,sill=sill,mean=mean)
fixed=list(power2=4,smooth=0,nugget=0)
fit = GeoFit(data, coordx=coords, corrmodel=corrmodel,
              start=start,fixed=fixed,
             likelihood='Conditional', type='Pairwise',
              neighb=3)

# locations to predict
loc_to_pred=matrix(runif(8),4,2)
################################################################
###
### Example 1. Comparing spatial kriging with local kriging for
### a Gaussian random field with GenWend correlation.
###
###############################################################
param=append(fit$param,fit$fixed)
pr=GeoKrig(fit,loc=loc_to_pred,mse=TRUE)

pr_loc=GeoKrigloc(fit,loc=loc_to_pred,neighb=100,mse=TRUE)

pr$pred;
pr_loc$pred


############################################################
#### Example: spatio temporal  Gaussian local kriging ######
############################################################


require(GeoModels)
require(fields)
set.seed(78)
coords=cbind(runif(100),runif(100))
coordt=seq(0,5,0.25)
corrmodel="Matern_Matern"
param=list(nugget=0,mean=0,scale_s=0.2/3,scale_t=0.25/3,sill=2,
  smooth_s=0.5,smooth_t=0.5)

data = GeoSim(coordx=coords, coordt=coordt,
     corrmodel=corrmodel, param=param)$data


# Maximum pairwise likelihood fitting of the space time random field:
start = list(scale_s=0.2/3,scale_t=0.25,sill=2,mean=0)
fixed = list(smooth_s=0.5,smooth_t=0.5,nugget=0)
```

```
I=Inf
lower=list(scale_s=0,scale_t=0,sill=0,mean=-I)
upper=list(scale_s=I,scale_t=I,sill=I,mean=I)
fit = GeoFit(data, coordx=coords, coordt=coordt, model=model, corrmodel=corrmodel,
             likelihood='Conditional', type='Pairwise',start=start,fixed=fixed,
             optimizer="nlminb",lower=lower,upper=upper,
              neighb=3,maxtime=1)

##  four location to predict
loc_to_pred=matrix(runif(8),4,2)
## three temporal instants to predict
time=c(0.5,1.5,3.5)


pr=GeoKrig(fit,loc=loc_to_pred,time=time,mse=TRUE)
pr_loc=GeoKrigloc(fit,loc=loc_to_pred,time=time,
  neigh=25,maxtime=1, mse=TRUE)

##  full and local prediction
pr$pred
pr_loc$pred



############################################################
#### Example: spatio bivariate  Gaussian local cokriging ######
############################################################
#set.seed(6)
#NN=1500 # number of spatial locations
#x = runif(NN, 0, 1);
#y = runif(NN, 0, 1)
#coords=cbind(x,y)

## setting parameters
#mean_1 = 2; mean_2= -1
#nugget_1 =0;nugget_2=0
#sill_1 =0.5; sill_2 =1;

### correlation parameters
#CorrParam("Bi_Matern")
#scale_1=0.2/3; scale_2=0.15/3; scale_12=0.5*(scale_2+scale_1)
#smooth_1=smooth_2=smooth_12=0.5
#pcol = -0.4
#param= list(nugget_1=nugget_1,nugget_2=nugget_2,
#          sill_1=sill_1,sill_2=sill_2,
#          mean_1=mean_1,mean_2=mean_2,
#          smooth_1=smooth_1, smooth_2=smooth_2,smooth_12=smooth_12,
#          scale_1=scale_1, scale_2=scale_2,scale_12=scale_12,
#          pcol=pcol)

## simulation
#data = GeoSim(coordx=coords, corrmodel="Bi_Matern",model=model,param=param)$data
```

```
#fixed=list(mean_1=mean_1,mean_2=mean_2, nugget_1=nugget_1,nugget_2=nugget_2,
#       smooth_1=smooth_1, smooth_2=smooth_2,smooth_12=smooth_12)

#start=list( sill_1=sill_1,sill_2=sill_2,
#        scale_1=scale_1,scale_2=scale_2,scale_12=scale_12, pcol=pcol)

## estimation with maximum likelihood
#fit = GeoFit(data=data,coordx=coords, corrmodel="Bi_Matern",
#likelihood="Full",type="Standard",optimizer="BFGS",
 # likelihood="Marginal",type="Pairwise",optimizer="BFGS",neighb=5,
  #start=start,fixed=fixed)

###### co-kriging for the fist component ##############
#xx=seq(0,1,0.022)
#loc_to_pred=as.matrix(expand.grid(xx,xx))
#pr1 = GeoKrigloc(fit,which=1,mse=TRUE,loc=loc_to_pred,neighb=100)

#opar=par(no.readonly = TRUE)
#par(mfrow=c(1,2))
#zlim=c(-2.5,2.5)
#colour = rainbow(100)
#quilt.plot(coords,data[1,] ,col=colour,main = paste(" Fist component"))
#quilt.plot(loc_to_pred,pr1$pred,col=colour,
#          main = paste(" Kriging first component"),ylab="")
#par(opar)
```

GeoNA *Deleting NA values (missing values) from a spatial or spatio-temporal dataset.*

Description

The function deletes NA values from a spatial or spatio-temporal dataset

Usage

```
GeoNA(data, coordx, coordy=NULL, coordt=NULL,
coordx_dyn=NULL, grid=FALSE, X=NULL, setting="spatial")
```

Arguments

data A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid) giving the data.

coordx A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. d-dimensional vector giving 1-dimension of

	spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.
`coordy`	A numeric vector giving 1-dimension of spatial coordinates ; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.
`coordt`	A numeric vector giving 1-dimension of temporal coordinates; the default is `NULL` then a spatial random field is expected.
`coordx_dyn`	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
`grid`	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
`X`	Numeric; Matrix of spatio(temporal) covariates in the linear mean specification.
`setting`	String; are data spatial, spatio-temporal or spatial bivariate (respectively `spatial`, `spacetime`, `bivariate`)

Value

Returns a list containing the following components:

`coordx`	A d-dimensional vector of spatial coordinates;
`coordy`	A d-dimensional vector of spatial coordinates;
`coordt`	A t-dimensional vector of temporal coordinates;
`data`	The data without NAvalues
`grid`	`TRUE` if the spatial data are observed in a regular grid, otherwise `FALSE`;
`perc`	The percentage of NA values .
`setting`	Are data of spatial or spatio-temporal or spatial bivariate type
`X`	Covariates matrix

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Examples

```
library(GeoModels)

# Define the spatial-coordinates of the points:
set.seed(79)
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords=cbind(x,y)
# Set the exponential cov parameters:
```

```
corrmodel = "Matern"
mean=0
sill=1
nugget=0
scale=0.3/3
smooth=0.5
param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth)

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,
              param=param)$data


data[1:100]=NA
# removing NA
a=GeoNA(data,coordx=coords)
a$perc # percentage of NA values
#a$coordx# spatial coordinates without missing values
#a$data # data without missinng values
```

`GeoNeighborhood` *Spatio (temporal) neighborhood selection for local kriging.*

Description

Given a set of spatio (temporal) locations and data, the procedure selects a spatio (temporal) neighborhood associated to some given spatio (temporal) locations. The neighborhood is computed using a fixed spatio (temporal) threshold or considering a fixed number of spatio (temporal) neighbors.

Usage

```
GeoNeighborhood(data=NULL, coordx, coordy=NULL,
coordt=NULL, coordx_dyn=NULL, bivariate=FALSE,
                distance="Eucl", grid=FALSE,
                loc, neighb=NULL,maxdist=NULL,
                maxtime=NULL, radius=6371, time=NULL,
                X=NULL,M=NULL,spobj=NULL,spdata=NULL,
                parallel=FALSE,ncores=NULL)
```

Arguments

`data` An optional d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid).

coordx	A numeric ($d \times 2$)-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a ($d \times 2$)-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
bivariate	If TRUE then data is considered as spatial bivariate data.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
loc	A (1×2)-matrix giving the spatial coordinate of the location for which a neighborhood is computed .
neighb	Numeric; an optional positive integer indicating the order of spatial neighborhood.
maxdist	Numeric; a positive value indicating the maximum spatial distance considered in the spatial neighborhood selection.
maxtime	Numeric; an optional positive integer indicating the order of temporal neighborhood.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
time	Numeric; a value giving the temporal instant for which a neighborhood is computed.
X	Numeric; an optional Matrix of spatio (temporal) covariates.
M	Numeric; an estimated spatio (temporal) mean vector.
spobj	An object of class sp or spacetime
spdata	Character:The name of data in the sp or spacetime object
parallel	Logical; if TRUE then parallelizzation is used
ncores	Numeric; number of cores involved in parallelization.

Value

Returns a list containing the following informations:

coordx	A list of the matrix coordinates of the computed spatial neighborhood ;
coordt	A vector of the computed temporal neighborhood;
data	A list of the vector of data associated with the spatio (temporal) neighborhood;

distance	The type of spatial distance;
numcoord	The vector of numbers of location sites involved the spatial neighborhood;
numtime	The vector of numbers of temporal insttants involved the temporal neighborhood;
radius	The radius of the sphere if coordinates are passed in lon/lat format;
spacetime	TRUE if spatio-temporal and FALSE if spatial RF;
X	The matrix of spatio (temporal) covariates associated with the computed spatio (temporal) neighborhood;

Author(s)

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Examples

```
library(GeoModels)
##########################################
#### Example: spatial neighborhood  ######
##########################################
set.seed(75)
coords=cbind(runif(500),runif(500))

param=list(nugget=0,mean=0,scale=0.2,sill=1,
            power2=4,smooth=1)

data_all = GeoSim(coordx=coords, corrmodel="GenWend",
                         param=param)$data

plot(coords)
##two locations
loc_to_pred=matrix(c(0.3,0.5,0.7,0.2),2,2)

points(loc_to_pred,pch=20)
neigh=GeoNeighborhood(data_all, coordx=coords,
                  loc=loc_to_pred,neighb=8)

# two Neighborhoods
neigh$coordx
points(neigh$coordx[[1]],pch=20,col="red")
points(neigh$coordx[[2]],pch=20,col="blue")
# associated data
neigh$data


###################################################
#### Example: spatio temporal spatial neighborhood#
###################################################
```

```
set.seed(78)
coords=matrix(runif(80),40,2)
coordt=seq(0,6,0.25)

param=list(nugget=0,mean=0,scale_s=0.2/3,scale_t=0.25/3,sill=2)

data_all = GeoSim(coordx=coords, coordt=coordt,corrmodel="Exp_Exp",
                         param=param)$data
##  two location to predict
loc_to_pred=matrix(runif(4),2,2)
## three temporal instants to predict
time=c(1,2)

plot(coords,xlim=c(0,1),ylim=c(0,1))
points(loc_to_pred,pch=20)

neigh=GeoNeighborhood(data_all, coordx=coords,  coordt=coordt,
                  loc=loc_to_pred,time=time,neighb=3,maxtime=0.5)

# first spatio-temporal neighborhoods
# with  associated data
neigh$coordx[[1]]
neigh$coordt[[1]]
neigh$data[[1]]

###################################################
#### Example: bivariate  spatial neighborhood #####
###################################################

set.seed(79)
coords=matrix(runif(100),50,2)

param=list(mean_1=0,mean_2=0,scale=0.12,smooth=0.5,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=0.5)

data_all = GeoSim(coordx=coords,corrmodel="Bi_matern_sep",
                 param=param)$data
##  two location to predict
loc_to_pred=matrix(runif(4),2,2)

neigh=GeoNeighborhood(data_all, coordx=coords,bivariate=TRUE,
                  loc=loc_to_pred,maxdist=0.25)

plot(coords)
points(loc_to_pred,pch=20)
points(neigh$coordx[[1]],col="red",pch=20)
points(neigh$coordx[[2]],col="red",pch=20)
```

GeoNeighbSelect *An heuristic algorithm for spatial or spatiotemoral optimal neighboord selection for pairwise composite likelihood estimation.*

Description

The procedure performs different pairwise composite likelihood estimation using user's specified spatial or spatiotemporal neighboords in the weight function. The neighbor minimizing the sum of the squared differences between the estimated semivariogram and the empirical semivariogram is selected.

Usage

```
GeoNeighbSelect(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL,
    copula=NULL,corrmodel=NULL, distance="Eucl",fixed=NULL,anisopars=NULL,
   est.aniso=c(FALSE,FALSE), grid=FALSE, likelihood='Marginal',lower=NULL,maxdist_v=Inf,
   neighb=c(1,2,3,4,5),maxtime=Inf, maxtime_v=Inf,memdist=TRUE,model='Gaussian',
    n=1,numbins=20, ncores=NULL,optimizer='Nelder-Mead', parallel=FALSE,
    bivariate=FALSE,radius=6371, start=NULL,type='Pairwise', upper=NULL,
    weighted=FALSE,X=NULL,nosym=FALSE,spobj=NULL,spdata=NULL)
```

Arguments

data	A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid). For the description see the Section **Details**.
coordx	A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random fields is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
fixed	An optional named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated.

anisopars	A list of two elements: "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
est.aniso	A bivariate logical vector providing which anisotropic parameters must be estimated.
grid	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
likelihood	String; the configuration of the composite likelihood. `Marginal` is the default (see Section **Details** in GeoFit).
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is `L-BFGS-B` or `nlminb` or `bobyqa` or `optimize`. The names of the list must be the same of the names in the `start` list.
maxdist_v	Numeric; an optional positive value indicating the maximum spatial distance considered in the semivariogram estimation
neighb	Numeric; a vector of positive integers indicating the order of neighborhood in the weight function of composite likelihood (see Section **Details** in GeoFit).
maxtime	Numeric; an optional positive integer indicating the order of temporal neighborhood in the composite likelihood computation.
maxtime_v	Numeric; an optional positive integer indicating the order of temporal neighborhood in the semivariogram estimation.
memdist	Logical; if `TRUE` then all the distances useful in the composite likelihood estimation are computed before the optimization. `FALSE` is deprecated.
model	String; the type of random fields and therefore the densities associated to the likelihood objects. `Gaussian` is the default, see the Section **Details** in GeoFit.
n	Numeric; number of trials in a binomial random fields; number of successes in a negative binomial random fields
numbins	Numeric; an integer indicating the number of bins in the empirical semivariogram
ncores	Numeric; the number of cores involved in the parallelization
optimizer	String; the optimization algorithm (see `optim` for details). `Nelder-Mead` is the default. Other possible choices are `nlm`, `BFGS`, `SANN`, `L-BFGS-B` and `nlminb` and `bobyqa`. In these last three cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function `optimize` is used.
parallel	Logical; if `TRUE` the procedure is parallelized using dofuture.
bivariate	Logical; if `TRUE` the bivariate case is considered.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
start	An optional named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. `NULL` is the default (see Section **Details** in GeoFit).
type	String; the type of the likelihood objects. If `Pairwise` (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods (see Section **Details** in GeoFit).

upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or bobyqa or nlminb or optimize. The names of the list must be the same of the names in the start list.
weighted	Logical; if TRUE the likelihood objects are weighted (see Section **Details** in GeoFit). If FALSE (the default) the composite likelihood is not weighted.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
nosym	Logical; if TRUE simmetric weights are not considered. This allows a faster but less efficient CL estimation.
spobj	An object of class sp or spacetime
spdata	Character:The name of data in the sp or spacetime object

Details

The procedure performs different pairwise composite likelihood estimation using user's specified spatial or spatiotemporal neighboords in the weight function. The neighbor minimizing the sum of the squared differences between the estimated semivariogram and the empirical semivariogram is selected.

Value

Returns a list with information on the best selected neighbor.

Author(s)

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Examples

```
library(GeoModels)
# Define the spatial-coordinates of the points:
set.seed(32)
N=500  # number of location sites
x <- runif(N, 0, 1)
y <- runif(N, 0, 1)
coords <- cbind(x,y)


mean <- 0.2
# Set the covariance model's parameters:
corrmodel <- "Matern"
sill <- 1
nugget <- 0
scale <- 0.2/3
smooth=0.5

model="Gaussian"
```

```
param<-list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth)

# Simulation
data <- GeoSim(coordx=coords,corrmodel=corrmodel, param=param,model=model)$data

I=Inf
fixed<-list(nugget=nugget)
start<-list(mean=mean,scale=scale,smooth=smooth,sill=sill)
lower<-list(mean=-I,scale=0,sill=0,smooth=0)
upper<-list(mean=I,scale=I,sill=I,smooth=I)


neighb=c(2,3,4,5,6,7,8) ## trying  different neighbs
selK <- GeoNeighbSelect(data=data,coordx=coords,corrmodel=corrmodel,
                        maxdist_v=0.25,model=model,numbins=20,neighb=neighb,
                        likelihood="Conditional",type="Pairwise",parallel=FALSE,
                        optimizer="nlminb",lower=lower,upper=upper,
                        start=start,fixed=fixed)
print(selK$best_neighb) ## selected neighbor
```

GeoNeighIndex *Spatial or spatiotemporal near neighbour indices.*

Description

The function returns the indices associated with a given spatial (temporal) neighbour and/or distance

Usage

```
GeoNeighIndex(coordx,coordy=NULL,coordx_dyn=NULL,
coordt=NULL,distance="Eucl",neighb=4,maxdist=NULL,
maxtime=1,radius=6371,bivariate=FALSE)
```

Arguments

coordx	A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector assigning 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.

coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
neighb	Numeric; an optional (vector of) positive integer indicating the order of neighborhood. See the Section **Details** for more information.
maxdist	A numeric value denoting the spatial distance **Details**.
maxtime	A numeric value denoting the temporal distance **Details**.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
bivariate	Logical; if FALSE (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are intrepreted as a a realization from a bivariate field.

Details

The function returns the spatial or spatiotemporal indices of the pairs tha are neighboords of a certain order and/or with a certain fixed distance

Value

Returns a list containing the following components:

colidx	First vector of indices
rowidx	Second vector of indices
lags	Vector of spatial distances
lagt	Vector of temporal distances

Author(s)

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Examples

```
require(GeoModels)
NN = 400
coords = cbind(runif(NN),runif(NN))
scale=0.5/3
param = list(mean=0,sill=1,nugget=0,scale=0.5/3,smooth=0.5)
corrmodel = "Matern";

param = list(mean=0,sill=1,nugget=0,scale=scale,smooth=0.5)
set.seed(951)
data = GeoSim(coordx = coords,corrmodel = corrmodel,
```

```
                    model = "Gaussian",param = param)$data

sel=GeoNeighIndex(coordx=coords,maxdist=0.05)

data1=data[sel$colidx]; data2=data[sel$rowidx]
## plotting pairs  that are neighboord of order 5
plot(data1,data2,xlab="",ylab="",main="h-scatterplot, dist=0.05")
```

GeoNosymindices *GeoNosymindices.*

Description

Given a matrix of indices and associated distances the function return a matrix of indices and associated distances, deleting the symmetric indices.

Usage

```
GeoNosymindices(X,Y)
```

Arguments

X	A matrix of indices
Y	Associated distances

Details

The function return the matrix of indices and associated distances, deleting the symmetric indices.

Value

Returns a list containing the following components:

xy	Matrix of indices
d	Associated distance

Author(s)

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`GeoOutlier` *Spatio (temporal) outliers detection*

Description

Given a set of spatio (temporal) locations and data, the procedure select the spatial or spatiotemporal ouliers using a specific algorithm.

Usage

```
GeoOutlier(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL,
            distance="Eucl", grid=FALSE,  neighb=10,alpha=0.001,
            method="Z-Median", radius=6371, bivariate=FALSE,X=NULL)
```

Arguments

`data`	An optional d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid).
`coordx`	A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.
`coordy`	A numeric vector giving 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.
`coordt`	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is `NULL` then a spatial RF is expected.
`coordx_dyn`	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details** of `GeoFit`.
`grid`	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
`neighb`	Numeric; an optional positive integer indicating the order of neighborhoodused for Z-Median algorithm.
`alpha`	Numeric; a numeric value between 0 and 1 used for Z-Median algorithm.
`method`	String; The name of the algorithm for detecting spatial ouliers. Default is Z-median proposed in Chen et al. (2008)
`radius`	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
`bivariate`	If TRUE then data is considered as spatial bivariate data.
`X`	Numeric; an optional Matrix of spatio (temporal) covariates.

Value

Return a matrix or a list containing the dected spatial or spatio-temporal outliers

Author(s)

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References

Chen D, Lu C, Kou Y, Chen F (2008) On detecting spatial outliers. Geoinformatica 12:455–475

Bevilacqua M., Caamaño C., Arellano-Valle R. B., Camilo Gomez C. (2022) A class of random fields with two-piece marginal distributions for modeling point-referenced data with spatial outliers. *Test* 10.1007/s11749-021-00797-5

Examples

```
library(GeoModels)
set.seed(1428)
NN = 1500
coords = cbind(runif(NN),runif(NN))
###
scale=0.5/3
corrmodel = "Matern";

param = list(mean=0,sill=1,nugget=0,scale=scale,smooth=0.5,skew=0)
data = GeoSim(coordx = coords,corrmodel = corrmodel,
                  model = "TwoPieceGaussian",param = param)$data

K=15         #parameter for outliers detection alghoritm
alpha=0.005  #parameter for outliers detection alghoritm
outlier=GeoOutlier(data=data, coordx = coords,neighb=K,alpha=alpha)
quilt.plot(coords,data)
for (i in 1:nrow(outlier)) plotrix::draw.circle(outlier[i,1], outlier[i,2],radius=0.02,lwd=2)
nrow(outlier) # number of outliers

param = list(mean=0,sill=1,nugget=0.4,scale=scale,smooth=0.5)
data = GeoSim(coordx = coords,corrmodel = corrmodel,
                  model = "Gaussian",param = param)$data

K=15         #parameter for outliers detection alghoritm
alpha=0.005  #parameter for outliers detection alghoritm
outlier=GeoOutlier(data=data, coordx = coords,neighb=K,alpha=alpha)
quilt.plot(coords,data)
for (i in 1:nrow(outlier)) plotrix::draw.circle(outlier[i,1], outlier[i,2],radius=0.02,lwd=2)
nrow(outlier) # number of outliers
```

GeoPit *Probability integral or normal score tranformation*

Description

The procedure for a given GeoFit object applies the probability integral tranformation or the normal score transformation to the data

Usage

```
GeoPit(object,type="Uniform")
```

Arguments

object	A GeoFit object
.	
type	The type of transformation. If "Uniform" then the probability integral tranformation is performed. If "Gaussian" then the normal score transformation is performed.

Value

Returns an (updated) object of class GeoFit

Author(s)

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Examples

```
library(GeoModels)

model="Beta2"
copula="Clayton"

set.seed(221)
NN=800
x <- runif(NN);y <- runif(NN)
coords=cbind(x,y)


shape=1.5
scale=0.2;power2=4
smooth=0
nugget=0
```

```
nu=8

corrmodel="GenWend"

min=-2;max=1
mean=0


param=list(smooth=smooth,power2=power2, min=min,max=max,
             mean=mean, nu=nu,
             scale=scale,nugget=nugget,shape=shape)

optimizer="nlminb"

data <- GeoSimCopula(coordx=coords, corrmodel=corrmodel,
model=model,param=param,copula=copula)$data

I=50
fixed<-list(nugget=nugget,sill=1,scale=scale,smooth=smooth,power2=power2,min=min,max=max,nu=nu)
start<-list(shape=shape,mean=mean)
lower<-list(shape=0,mean=-I)
upper<-list(shape=10,mean=I)

#### maximum independence likelihood
fit1 <- GeoFit(data=data,coordx=coords,corrmodel=corrmodel,
model=model,likelihood="Marginal",type="Independence",
                      optimizer=optimizer,lower=lower,
                      upper=upper,copula=copula,
                    start=start,fixed=fixed)

## PIT transformation
aa=GeoPit(fit1,type="Uniform")
hist(aa$data,freq=FALSE)
GeoScatterplot(aa$data,coords,neighb=c(1,2))
## Normal score transformation
bb=GeoPit(fit1,type="Gaussian")
hist(bb$data,freq=FALSE)
GeoScatterplot(bb$data,coords,neighb=c(1,2))
```

GeoQQ *Quantile-quantile plot*

Description

Based on a GeoFit object, the procedure plots a quantile-quantile plot or compares the fitted density with the histogram of the data. It is useful as diagnostic tool.

Usage

```
GeoQQ(fit,type="Q",add=FALSE,ylim=c(0,1),breaks=10,...)
```

Arguments

`fit`	A GeoFit object possibly obtained from `GeoResiduals`.
`type`	The type of plot. If Q then a qq-plot (default) is performed. If D then a comparison between histrogram and the estimated marginal density is performed
`add`	Logical; if TRUE the the estimated density ia added over an existing one
`ylim`	Numeric; a vector of length 2 used for the ylab parameter of the histogram plot.
`breaks`	Numeric; an integer number specifyng the number of cells ofthe histogram plot if the option type=D is chosen.
`...`	Optional parameters passed to the plot function.

Value

Produces a plot. No values are returned.

Author(s)

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Examples

```
library(GeoModels)


##################
### Example 1
##################
set.seed(21)
model="Tukeyh";tail=0.1
N=400 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)

# regression parameters
mean = 5
mean1=0.8

X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
```

```
nugget = 0
scale = 0.3
power2=4


param=list(mean=mean,mean1=mean1, sill=sill, nugget=nugget,
           scale=scale,tail=tail,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data

start=list(mean=mean,mean1=mean1, scale=scale,tail=tail)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
                    likelihood="Conditional",type='Pairwise',start=start,
                    fixed=fixed,neighb=4)

res=GeoResiduals(fit)
GeoQQ(res,type="Q")
GeoQQ(res,type="D",lwd=2,ylim=c(0,0.5),breaks=20)


##################
### Example 2
##################
set.seed(21)
model="Weibull";shape=1.5
N=600 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)


# regression parameters
mean = 0

# correlation parameters:
corrmodel = "Matern"
smooth=0.5
nugget = 0
scale = 0.2/3


param=list(mean=mean, sill=1, nugget=nugget,
             scale=scale,smooth=smooth, shape=shape)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel,model=model,param=param)$data

start=list(mean=mean, scale=scale,shape=shape)
I=Inf
lower=list(mean=-I, scale=0,shape=0)
upper=list(mean= I, scale=I,shape=I)
```

```
I=Inf
fixed=list(nugget=nugget,sill=1,smooth=smooth)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,
                    likelihood="Conditional",type='Pairwise',start=start,
                    optimizer="nlminb",lower=lower,upper=upper,
                    fixed=fixed,neighb=3)
GeoQQ(fit,type="Q")
GeoQQ(fit,type="D",lwd=2,ylim=c(0,1),breaks=20)
```

GeoResiduals *Computes fitted covariance and/or variogram*

Description

The procedure return a GeoFit object associated to the estimated residuals. For a random field Y defined on the real line (Gaussian, Skew Gaussian, Tukeyh etcc) they are computed as (Y-m)/sqrt(v) where m and v are the estimated mean and variance respectively. For a random field Y defined on the positive real line (Gamma, Weibull, Log-Gaussian) they are computed as Y/m where m is estimated mean.

Usage

```
GeoResiduals(fit)
```

Arguments

fit	A fitted object obtained from the GeoFit.

Value

Returns an (updated) object of class `GeoFit`

Author(s)

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See Also

GeoFit.

Examples

```
library(GeoModels)



##############
###Example 1
##############
set.seed(211)
model="Gaussian";
N=700 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)

# regression parameters
mean = 5
mean1=0.8

X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
nugget = 0
scale = 0.3
power2=4

param=list(mean=mean,mean1=mean1, sill=sill, nugget=nugget,
             scale=scale,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data

start=list(mean=mean,mean1=mean1, scale=scale,sill=sill)
fixed=list(nugget=nugget,power2=power2)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
                    likelihood="Conditional",type='Pairwise',start=start,
                    fixed=fixed,neighb=3)

res=GeoResiduals(fit)
mean(res$data) # should be approx 0
var(res$data) # should be approx 1
# checking goodness of fit marginal model
GeoQQ(res);GeoQQ(res,type="D",col="red",ylim=c(0,0.5),breaks=20);
# Empirical estimation of the variogram for the residuals:
vario = GeoVariogram(res$data,coordx=coords,maxdist=0.5)
# Comparison between empirical amd estimated semivariogram for the residuals
GeoCovariogram(res, show.vario=TRUE, vario=vario,pch=20)
```

```
##############
###Example 2
##############
model="Weibull";shape=4
N=700 # number of location sites
# Set the coordinates of the points:
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)


# regression parameters
mean = 5
mean1=0.8

X=cbind(rep(1,N),runif(N))
# correlation parameters:
corrmodel = "Wend0"
sill = 1
nugget = 0
scale = 0.3
power2=4

param=list(mean=mean,mean1=mean1, sill=sill, nugget=nugget,
           scale=scale,shape=shape,power2=power2)
# Simulation of the Gaussian RF:
data = GeoSim(coordx=coords, corrmodel=corrmodel, X=X,model=model,param=param)$data

I=Inf
start=list(mean=mean,mean1=mean1, scale=scale,shape=shape)
lower=list(mean=-I,mean1=-I, scale=0,shape=0)
upper=list(mean= I,mean1= I, scale=I,shape=I)
fixed=list(nugget=nugget,sill=sill,power2=power2)
# Maximum composite-likelihood fitting
fit = GeoFit(data,coordx=coords, corrmodel=corrmodel,model=model,X=X,
                    likelihood="Conditional",type='Pairwise',start=start,
                   optimizer="nlminb", lower=lower,upper=upper,
                    fixed=fixed,neighb=3)


res=GeoResiduals(fit)
mean(res$data) # should be approx 1
# checking goodness of fit marginal model
GeoQQ(res);GeoQQ(res,type="D",col="red",ylim=c(0,1.7),breaks=20);
# Empirical estimation of the variogram for the residuals:
vario = GeoVariogram(res$data,coordx=coords,maxdist=0.5)
# Comparison between empirical amd estimated semivariogram for the residuals
GeoCovariogram(res, show.vario=TRUE, vario=vario,pch=20)
```

GeoScatterplot *h-scatterplot for space and space-time data.*

Description

The function produces h-scatterplots for the spatial, spatio-temporal and bivariate setting.

Usage

```
GeoScatterplot(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL,
          distance='Eucl', grid=FALSE, maxdist=NULL,neighb=NULL,
          times=NULL, numbins=4, radius=6371, bivariate=FALSE,...)
```

Arguments

`data`	A d-dimensional vector (a single spatial realisation) or a $(n \times d)$-matrix (n iid spatial realisations) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$-array (n iid spatial realisations on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$-array (n iid spatial-temporal realisations) or or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid) or an $(d \times d \times t \times n)$-array ($n$ iid spatial-temporal realisations on regular grid). See `GeoFit` for details.
`coordx`	A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.
`coordy`	A numeric vector assigning 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.
`coordt`	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is `NULL` then a spatial random field is expected.
`coordx_dyn`	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details** of `GeoFit`.
`grid`	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.
`maxdist`	A numeric value denoting the spatial maximum distance, see the Section **Details**.
`neighb`	Numeric; an optional positive integer indicating the order of neighborhood. See the Section **Details** for more information.
`times`	A numeric vector denoting the temporal instants involved **Details**.
`numbins`	A numeric value denoting the numbers of bins, see the Section **Details**.

radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
bivariate	Logical; if FALSE (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are intrepreted as a a realization from a bivariate field.
...	Optional parameters passed to the plot function.

Details

h-scatterplot is the plot of the pair values that are neighborhood of a certain order or with distances belonging to a certain interval. In the first case a (vector of) neighborhood must be specified. In the second case a maximum distance (maxdist) and a number of lag-bins (numbins) must be specified. The method based on neighborhoods is recommended in particular for large datasets.

Value

Produces a plot. No values are returned.

Author(s)

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Examples

```
library(GeoModels)
set.seed(514)

NN = 600
coords = cbind(runif(NN),runif(NN))

param = list(mean=0,sill=1,nugget=0,power2=4,scale=0.4,smooth=0)

corrmodel = "GenWend"; model = "Gaussian"

data = GeoSim(coordx = coords,corrmodel = corrmodel,
                  model = model,param = param)$data

# h-scatterplots for given a vector of neighborhoods
GeoScatterplot(data,coords,neighb=c(2,4))
```

GeoScores *Computation of predictive scores*

Description

The function computes some predictive scores for a spatial, spatiotemporal and bivariate Gaussian RFs

Usage

```
GeoScores(data_to_pred, probject=NULL,pred=NULL,mse=NULL,
   score=c("brie","crps","lscore","pit","pe"))
```

Arguments

`data_to_pred`	A numeric vector of data to predict about a response
`probject`	A Geokrig object obtained using the function Geokrig
`pred`	A numeric vector with predictions for the response.
`mse`	a numeric vector with prediction variances.
`score`	A character defining what statistic of the prediction errors should be computed. Possible values are lscore, crps, brie and pe. In the latter case scores based on prediction errors such as rmse, mae, mad are computed. Finally, the character pit allows to compute the probability integral transform for each value

Details

GeoScores computes the items required to evaluate the diagnostic criteria proposed by Gneiting et al. (2007) for assessing the calibration and the sharpness of probabilistic predictions of (cross-) validation data. To this aim, GeoScores uses the assumption that the prediction errors are Gaussian with zero mean and standard deviations equal to the Kriging standard errors. This assumption is an approximation if the errors are not Gaussian.

Value

Returns a list containing the following informations:

`LSCORE`	Logarithmic predictive score
`CRPS`	Continuous ranked probability predictive score
`RMSE`	Root mean squared error
`MAE`	Mean absolute error
`MAD`	Median absolute error
`PIT`	A vector of probability integral transformation

Author(s)

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References

Gneiting T. and Raftery A. *Strictly Proper Scoring Rules, Prediction, and Estimation*. Journal of the American Statistical Association, **102**

Examples

```
library(GeoModels)

################################################################
######### Examples of predictive score computation  ############
################################################################

library(GeoModels)
model="Gaussian"
set.seed(79)
N=1000
x = runif(N, 0, 1)
y = runif(N, 0, 1)
coords=cbind(x,y)

# Set the exponential cov parameters:
corrmodel = "GenWend"
mean=0; sill=5; nugget=0
scale=0.2;smooth=0;power2=4

param=list(mean=mean,sill=sill,nugget=nugget,scale=scale,smooth=smooth,power2=power2)

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel,
              param=param)$data


sel=sample(1:N,N*0.8)
coords_est=coords[sel,]; coords_to_pred=coords[-sel,]
data_est=data[sel]; data_to_pred=data[-sel]

## estimation with pairwise likelihood
fixed=list(nugget=nugget,smooth=0,power2=power2)
start=list(mean=0,scale=scale,sill=1)
I=Inf
lower=list(mean=-I,scale=0,sill=0)
upper=list(mean= I,scale=I,sill=I)
# Maximum pairwise likelihood fitting :
fit = GeoFit(data_est, coordx=coords_est, corrmodel=corrmodel,model=model,
                    likelihood='Marginal', type='Pairwise',neighb=3,
```

```
                    optimizer="nlminb", lower=lower,upper=upper,
                    start=start,fixed=fixed)
# locations to predict
xx=seq(0,1,0.03)
loc_to_pred=as.matrix(expand.grid(xx,xx))

pr=GeoKrig(loc=coords_to_pred,coordx=coords_est,corrmodel=corrmodel,
       model=model,param= param, data=data_est,mse=TRUE)

Pr_scores =GeoScores(data_to_pred,pred=pr$pred,mse=pr$mse)
Pr_scores$rmse;Pr_scores$brie
hist(Pr_scores$pit,freq=FALSE)
```

GeoSim *Simulation of Gaussian and non Gaussian Random Fields.*

Description

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields. The function return a realization of a random Field for a given covariance model and covariance parameters.Simulation is based on Cholesky decomposition.

Usage

```
GeoSim(coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel, distance="Eucl",
    GPU=NULL, grid=FALSE, local=c(1,1),method="cholesky", model='Gaussian', n=1, param,
    anisopars=NULL,radius=6371, sparse=FALSE,X=NULL,spobj=NULL,nrep=1,progress=TRUE)
```

Arguments

coordx	A numeric ($d \times 2$)-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a ($d \times 2$)-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.

`GPU`	Numeric; if `NULL` (the default) no GPU computation is performed.
`grid`	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
`local`	Numeric; number of local work-items of the GPU
`method`	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is `svd`.
`model`	String; the type of RF and therefore the densities associated to the likelihood objects. `Gaussian` is the default, see the Section **Details**.
`n`	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1.
`param`	A list of parameter values required in the simulation procedure of RFs, see **Examples**.
`anisopars`	A list of two elements "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
`radius`	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
`sparse`	Logical; if `TRUE` then cholesky decomposition is performed using sparse matrices algorithms (spam packake). It should be used with compactly supported covariance models.`FALSE` is the default.
`X`	Numeric; Matrix of space-time covariates.
`spobj`	An object of class sp or spacetime
`nrep`	Numeric; Numbers of indipendent replicates.
`progress`	Logic; If TRUE then a progress bar is shown.

Value

Returns an object of class `GeoSim`. An object of class `GeoSim` is a list containing at most the following components:

`bivariate`	Logical:`TRUE` if the Gaussian RF is bivariate, otherwise `FALSE`;
`coordx`	A d-dimensional vector of spatial coordinates;
`coordy`	A d-dimensional vector of spatial coordinates;
`coordt`	A t-dimensional vector of temporal coordinates;
`coordx_dyn`	A list of dynamical (in time) spatial coordinates;
`corrmodel`	The correlation model; see `GeoCovmatrix`.
`data`	The vector or matrix or array of data, see `GeoFit`;
`distance`	The type of spatial distance;
`method`	The method of simulation
`model`	The type of RF, see `GeoFit`.
`n`	The number of trial for Binomial RFs;the number of successes in a negative Binomial RFs;

numcoord	The number of spatial coordinates;
numtime	The number the temporal realisations of the RF;
param	The vector of parameters' estimates;
radius	The radius of the sphere if coordinates are passed in lon/lat format;
spacetime	TRUE if spatio-temporal and FALSE if spatial RF;
nrep	The number of indipendent replicates;

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Examples

```
library(GeoModels)
library(mapproj)


################################################################
###
### Example 1. Simulation of a spatial Gaussian RF
### with Matern and Generalized Wendland correlations
###############################################################

# Define the spatial-coordinates of the points:
x <- runif(500);y <- runif(500)
coords=cbind(x,y)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(coordx=coords, corrmodel="Matern", param=list(smooth=0.5,
             mean=0,sill=1,scale=0.4/3,nugget=0))$data

set.seed(261)
data2 <- GeoSim(coordx=coords,  corrmodel="GenWend", param=list(smooth=0,
              power2=4,mean=0,sill=1,scale=0.4,nugget=0))$data
opar=par(no.readonly = TRUE)
par(mfrow=c(1,2))
quilt.plot(coords,data1,main="Matern",xlab="",ylab="")
quilt.plot(coords,data2,main="Wendland",xlab="",ylab="")
par(opar)


################################################################
###
### Example 2. Simulation of a spatial geometric RF
### with  underlying Wend0 correlation
###
################################################################
```

```
# Define the spatial-coordinates of the points:
x <- runif(800);y <- runif(800)
coords <- cbind(x,y)
set.seed(251)
# Simulation of a spatial Binomial RF:
sim <- GeoSim(coordx=coords, corrmodel="Wend0",
             model="BinomialNeg",n=1,sparse=TRUE,
             param=list(nugget=0,mean=0,scale=.2,power2=4))

quilt.plot(coords,sim$data,nlevel=max(sim$data),col=terrain.colors(max(sim$data+1)))


################################################################
###
### Example 3. Simulation of a spatial Weibull  RF
### with  underlying Matern correlation on a regular grid
###
###############################################################
# Define the spatial-coordinates of the points:
x <- seq(0,1,0.032)
y <- seq(0,1,0.032)
set.seed(261)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="Matern",model="Weibull",
         param=list(shape=1.2,mean=0,scale=0.3/3,nugget=0,smooth=0.5))$data
image.plot(x,y,data1,main="Weibull RF",xlab="",ylab="")



################################################################
###
### Example 4. Simulation of a spatial t  RF
### with  with  underlying Generalized Wendland correlation
###
###############################################################
# Define the spatial-coordinates of the points:
x <- seq(0,1,0.03)
y <- seq(0,1,0.03)
set.seed(268)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSim(x,y,grid=TRUE, corrmodel="GenWend",model="StudentT", sparse=TRUE,
         param=list(df=1/4,mean=0,sill=1,scale=0.3,nugget=0,smooth=1,power2=5))$data
image.plot(x,y,data1,col=terrain.colors(100),main="Student-t RF",xlab="",ylab="")




################################################################
###
### Example 5. Simulation of a sinhasinh RF
###   with  underlying Wend0 correlation.
###
```

```
################################################################

# Define the spatial-coordinates of the points:
x <- runif(500, 0, 2)
y <- runif(500, 0, 2)
coords <- cbind(x,y)
set.seed(261)
corrmodel="Wend0"
# Simulation of a spatial Gaussian RF:
param=list(power2=4,skew=0,tail=1,
             mean=0,sill=1,scale=0.2,nugget=0)  ## gaussian case
data0 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
plot(density(data0),xlim=c(-7,7))

param=list(power2=4,skew=0,tail=0.7,
             mean=0,sill=1,scale=0.2,nugget=0) ## heavy tails
data1 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data1),lty=2)

param=list(power2=4,skew=0.5,tail=1,
             mean=0,sill=1,scale=0.2,nugget=0)  ## asymmetry
data2 <- GeoSim(coordx=coords, corrmodel=corrmodel,
               model="SinhAsinh", param=param,sparse=TRUE)$data
lines(density(data2),lty=3)


#################################################################
###
### Example 6. Simulation of a bivariate Gaussian RF
### with  bivariate Matern correlation model
###
################################################################

# Define the spatial-coordinates of the points:
x <- runif(500, 0, 2)
y <- runif(500, 0, 2)
coords <- cbind(x,y)

# Simulation of a bivariate spatial Gaussian RF:
# with a separable Bivariate Matern
set.seed(12)
param=list(mean_1=4,mean_2=2,smooth_1=0.5,smooth_2=0.5,smooth_12=0.5,
           scale_1=0.12,scale_2=0.1,scale_12=0.15,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=0.5)
data <- GeoSim(coordx=coords,corrmodel="Bi_matern",
              param=param)$data
opar=par(no.readonly = TRUE)
par(mfrow=c(1,2))
quilt.plot(coords,data[1,],col=terrain.colors(100),main="1",xlab="",ylab="")
quilt.plot(coords,data[2,],col=terrain.colors(100),main="2",xlab="",ylab="")
par(opar)
```

```
##################################################################
###
### Example 7. Simulation of a  spatio temporal Gaussian RF.
### observed on  dynamical location sites with double exponential correlation
###
#################################################################

# Define the dynamical spatial-coordinates of the points:

coordt=1:5
coordx_dyn=list()
maxN=30
set.seed(8)
for(k in 1:length(coordt))
{
NN=sample(1:maxN,size=1)
x <- runif(NN, 0, 1)
y <- runif(NN, 0, 1)
coordx_dyn[[k]]=cbind(x,y)
}
coordx_dyn

param<-list(nugget=0,mean=0,scale_s=0.2/3,scale_t=2/3,sill=1)
data <- GeoSim(coordx_dyn=coordx_dyn, coordt=coordt, corrmodel="Exp_Exp",
                     param=param)$data
## spatial realization at first temporal instants
data[[1]]
## spatial realization at third temporal instants
data[[3]]




##################################################################
###
### Example 8. Simulation of a Gaussian RF
###  with a Wend0 correlation in the north emisphere of the planet earth
### using geodesic distance
#################################################################
distance="Geod";radius=6371

NN=3000 ## total point on the sphere on lon/lat format
set.seed(80)
coords=cbind(runif(NN,-180,180),runif(NN,0,90))
## Set the wendland parameters
corrmodel <- "Wend0"
param<-list(mean=0,sill=1,nugget=0,scale=1000,power2=3)
# Simulation of a spatial Gaussian RF on the sphere
#set.seed(2)
data <- GeoSim(coordx=coords,corrmodel=corrmodel,sparse=TRUE,
               distance=distance,radius=radius,param=param)$data
```

```
#require(globe)
#globe::globeearth(eye=place("newyorkcity"))
#globe::globepoints(loc=coords,pch=20,col = cm.colors(length(data),alpha=0.1)[rank(data)])
```

GeoSimapprox *Fast simulation of Gaussian and non Gaussian Random Fields.*

Description

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields using approximate methods of simulation (circulant embeeding and turning bands) (see Examples).

Usage

```
GeoSimapprox(coordx, coordy=NULL, coordt=NULL,
coordx_dyn=NULL,corrmodel, distance="Eucl",GPU=NULL,
grid=FALSE,local=c(1,1),max.ext=1,
method="TB", L=1000,model='Gaussian',
n=1,param,anisopars=NULL, radius=6371,X=NULL,spobj=NULL,
nrep=1,progress=TRUE)
```

Arguments

coordx	A numeric $(d \times 2)$-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a $(d \times 2)$-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
GPU	Numeric; if NULL (the default) no GPU computation is performed.

`grid`	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
`local`	Numeric; number of local work-items of the GPU
`max.ext`	Numeric; The maximum extension of the simulation window (for the spatial CE method).
`method`	String; the type of approximation method. Default is `TB` that is the turning band method. The other possible choice is and `CE` (circular embeeding).
`L`	Numeric; the number of lines in the turning band method.
`model`	String; the type of RF and therefore the densities associated to the likelihood objects. `Gaussian` is the default, see the Section **Details**.
`n`	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1.
`param`	A list of parameter values required in the simulation procedure of RFs, see **Examples**.
`anisopars`	A list of two elements "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
`radius`	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
`X`	Numeric; Matrix of space-time covariates.
`spobj`	An object of class sp or spacetime
`nrep`	Numeric; Numbers of indipendent replicates.
`progress`	Logic; If TRUE then a progress bar is shown.

Value

Returns an object of class `GeoSim`. An object of class `GeoSim` is a list containing at most the following components:

`bivariate`	Logical:`TRUE` if the Gaussian RF is bivariate, otherwise `FALSE`;
`coordx`	A d-dimensional vector of spatial coordinates;
`coordy`	A d-dimensional vector of spatial coordinates;
`coordt`	A t-dimensional vector of temporal coordinates;
`coordx_dyn`	A list of dynamical (in time) spatial coordinates;
`corrmodel`	The correlation model; see `GeoCovmatrix`.
`data`	The vector or matrix or array of data, see `GeoFit`;
`distance`	The type of spatial distance;
`method`	The method of simulation
`model`	The type of RF, see `GeoFit`.
`n`	The number of trial for Binomial RFs;the number of successes in a negative Binomial RFs;
`numcoord`	The number of spatial coordinates;

numtime	The number the temporal realisations of the RF;
param	The vector of parameters' estimates;
radius	The radius of the sphere if coordinates are passed in lon/lat format;
spacetime	TRUE if spatio-temporal and FALSE if spatial RF;
nrep	The number of indipendent replicates;

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Examples

```
library(GeoModels)


################################################################
###
### Example 1. Simulation of a large spatial Gaussian RF
###            with  Matern  covariance model
###            using circulant embeeding method
###            It works only for regular grid
###            for any typpe of correlation model
###############################################################
set.seed(68)
x = seq(0,1,0.006)
y = seq(0,1,0.006)
param=list(smooth=1.5,mean=0,sill=1,scale=0.2/3,nugget=0)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSimapprox(coordx=x,coordy=y, grid=TRUE,corrmodel="Matern", model="Gaussian",
                      method="CE",param=param)$data
fields::image.plot( matrix(data1, length(x), length(y), byrow = TRUE) )

################################################################
###
### Example 2. Simulation of a large spatial Gaussian RF
```

```
###            with  Matern  covariance model
###            using Turning band method
###            It works for (ir)regular grid
###            for the Matern model
###############################################################
set.seed(68)
x = runif(8000)
y = runif(8000)
coords=cbind(x,y)
param=list(smooth=0.5,mean=0,sill=1,scale=0.1,nugget=0)
# Simulation of a spatial Gaussian RF with Matern correlation function
data1 <- GeoSimapprox(coords, corrmodel="Matern", model="Gaussian",
                      method="TB",param=param)$data
quilt.plot(coords,data1)



################################################################
###
### Example 3. Simulation of a large spacetime Gaussian RF
###            with separable matern  covariance model
###            using  Circular embeeding method
###            It works  for (large) regular time grid
###############################################################
set.seed(68)
coordt <- (0:100)
coords <- cbind( runif(100, 0 ,1), runif(100, 0 ,1))
param <- list(mean  = 0, sill = 1, nugget = 0.25,
              scale_s = 0.05, scale_t = 2,
              smooth_s = 0.5, smooth_t = 0.5)
# Simulation of a spatial Gaussian RF with Matern correlation function
param<-list(nugget=0,mean=0,scale_s=0.2/3,scale_t=2/3,sill=1,smooth_s=0.5,smooth_t=0.5)

data <- GeoSimapprox(coordx=coords, coordt=coordt, corrmodel="Matern_Matern",
                     model="Gaussian",method="CE",param=param)$data
dim(data)

################################################################
###
### Example 4. Simulation of a large spacetime Gaussian RF
###            with separable GenWend covariance model
###            using  Circular embeeding method in time
###############################################################
set.seed(68)
# Simulation of a spatial Gaussian RF with Matern correlation function
param<-list(nugget=0,mean=0,scale_s=0.2,scale_t=3,sill=1,
             smooth_s=0,smooth_t=0, power2_s=4,power2_t=4)

data <- GeoSimapprox(coordx=coords, coordt=coordt, corrmodel="GenWend_GenWend",
                     model="Gaussian",method="CE",param=param)$data
dim(data)
```

```
################################################################
###
### Example 6. Simulation of a large bivariate Gaussian RF
### with  bivariate Matern correlation model
###
###############################################################

# Define the spatial-coordinates of the points:
#x <- runif(20000, 0, 2)
#y <- runif(20000, 0, 2)
#coords <- cbind(x,y)

# Simulation of a bivariate spatial Gaussian RF:
# with a  Bivariate Matern
#set.seed(12)
#param=list(mean_1=4,mean_2=2,smooth_1=0.5,smooth_2=0.5,smooth_12=0.5,
#           scale_1=0.12,scale_2=0.1,scale_12=0.15,
#           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,pcol=0.5)
#data <- GeoSimapprox(coordx=coords,corrmodel="Bi_matern",
#              param=param,method="TB",L=1000)$data
#opar=par(no.readonly = TRUE)
#par(mfrow=c(1,2))
#quilt.plot(coords,data[1,],col=terrain.colors(100),main="1",xlab="",ylab="")
#quilt.plot(coords,data[2,],col=terrain.colors(100),main="2",xlab="",ylab="")
```

GeoSimCopula — *Simulation of Gaussian and non Gaussian Random Fields using copula.*

Description

Simulation of Gaussian and some non Gaussian spatial, spatio-temporal and spatial bivariate random fields using Gaussian or Clayton copula. The function return a realization of a Random Field for a given covariance model and covariance parameters. Simulation is based on Cholesky decomposition.

Usage

```
GeoSimCopula(coordx, coordy=NULL, coordt=NULL,
coordx_dyn=NULL, corrmodel, distance="Eucl",
      GPU=NULL, grid=FALSE, local=c(1,1),
      method="cholesky", model='Gaussian', n=1, param,
      anisopars=NULL,radius=6371, sparse=FALSE,
      copula="Gaussian",seed=NULL, X=NULL,spobj=NULL,nrep=1)
```

Arguments

coordx	A numeric ($d \times 2$)-matrix (where d is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius radius are passed in lon/lat format expressed in decimal degrees.
coordy	A numeric vector giving 1-dimension of spatial coordinates; coordy is interpreted only if coordx is a numeric vector or grid=TRUE otherwise it will be ignored. Optional argument, the default is NULL then coordx is expected to be numeric a ($d \times 2$)-matrix.
coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial RF is expected.
coordx_dyn	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description see the Section **Details**.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
GPU	Numeric; if NULL (the default) no GPU computation is performed.
grid	Logical; if FALSE (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites (irregular grid).
local	Numeric; number of local work-items of the GPU
method	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd.
model	String; the type of RF and therefore the densities associated to the likelihood objects. Gaussian is the default, see the Section **Details**.
n	Numeric; the number of trials for binomial RFs. The number of successes in the negative Binomial RFs. Default is 1.
param	A list of parameter values required in the simulation procedure of RFs, see **Examples**.
anisopars	A list of two elements "angle" and "ratio" i.e. the anisotropy angle and the anisotropy ratio, respectively.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake). It should be used with compactly supported covariance models.FALSE is the default.
copula	String; the type of copula. It can be "Clayton" or "Gaussian"
seed	Numeric; an integer used in set.seed function to reproduce the simulation.
X	Numeric; Matrix of space-time covariates.
spobj	An object of class sp or spacetime
nrep	Numeric; Numbers of indipendent replicates.

Value

Returns an object of class GeoSimCopula. An object of class GeoSimCopula is a list containing at most the following components:

bivariate	Logical:TRUE if the Gaussian RF is bivariate, otherwise FALSE;
coordx	A d-dimensional vector of spatial coordinates;
coordy	A d-dimensional vector of spatial coordinates;
coordt	A t-dimensional vector of temporal coordinates;
coordx_dyn	A list of dynamical (in time) spatial coordinates;
corrmodel	The correlation model; see GeoCovmatrix.
data	The vector or matrix or array of data, see GeoFit;
distance	The type of spatial distance;
method	The method of simulation
model	The type of RF, see GeoFit.
n	The number of trial for Binomial RFs;the number of successes in a negative Binomial RFs;
numcoord	The number of spatial coordinates;
numtime	The number the temporal realisations of the RF;
param	A list of the parameters
radius	The radius of the sphere if coordinates are passed in lon/lat format;
randseed	The seed used for the random simulation;
spacetime	TRUE if spatio-temporal and FALSE if spatial RF;
copula	The type of copula

Author(s)

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References

Bevilacqua M., Alvarado E., Caamano C. (2024) A flexible Clayton-like spatial copula with application to bounded support data. *Journal of Multivariate Analysis* **201**

Examples

```
library(GeoModels)

################################################################
###
### Example q. Simulation of a reparametrized Beta RF
### for beta regression
```

```
### with Gaussian and Clayton Copula
### with  underlying Wendland correlation.
###
###############################################################
set.seed(261)
NN=1400
x <- runif(NN);y <- runif(NN)
coords=cbind(x,y)

shape1=3
shape2=3
smooth=0


corrmodel="GenWend"
min=0;max=1


X=cbind(rep(1,NN),runif(NN))

NuisParam("Beta2",num_betas=2,copula="Gaussian")
CorrParam("GenWend")


#### Gaussian copula
param=list(smooth=smooth,power2=4, min=min,max=max,
             mean=0.1,mean1=0.1,scale=0.3,nugget=0,shape=5)

data <- GeoSimCopula(coordx=coords, corrmodel=corrmodel, model="Beta2",param=param,
  copula="Gaussian",sparse=TRUE,X=X)$data

quilt.plot(coords,data)


#### Clayton copula
NuisParam("Beta2",num_betas=2,copula="Clayton")
CorrParam("GenWend")
param=list(smooth=smooth,power2=4, min=min,max=max,
             mean=0.2,mean1=0.1,scale=0.3,nugget=0,shape=6,nu=4)
data1 <- GeoSimCopula(coordx=coords, corrmodel=corrmodel, model="Beta2",param=param,
  copula="Clayton",sparse=TRUE,X=X)$data

hist(data1,freq=FALSE)
quilt.plot(coords,data1)
```

GeoTests *Statistical Hypothesis Tests for Nested Models*

Description

The function performs statistical hypothesis tests for nested models based on composite or standard likelihood versions of Wald-type and Wilks-type (likelihood ratio) statistics.

Usage

```
GeoTests(object1, object2, ..., statistic)
```

Arguments

`object1`	An object of class `GeoFit`.
`object2`	An object of class `GeoFit` that is a nested model within `object1`.
`...`	Further successively nested objects.
`statistic`	String; the name of the statistic used within the hypothesis test (see **Details**).

Details

The implemented hypothesis tests for nested models are based on the following statistics:

1. Wald-type (`Wald`);
2. Likelihood ratio or Wilks-type (`Wilks` under standard likelihood); For composite likelihood available variants of the basic version are:
   - Rotnitzky and Jewell adjustment (`WilksRJ`);
   - Satterhwaite adjustment (`WilksS`);
   - Chandler and Bate adjustment (`WilksCB`);
   - Pace, Salvan and Sartori adjustment (`WilksPSS`);

More specifically, consider an p-dimensional random vector $\mathbf{Y}$ with probability density function $f(\mathbf{y};\theta)$, where $\theta \in \Theta$ is a q-dimensional vector of parameters. Suppose that $\theta = (\psi, \tau)$ can be partitioned in a q'-dimensional subvector ψ and q''-dimensional subvector τ. Assume also to be interested in testing the specific values of the vector ψ. Then, one can use some statistical hypothesis tests for testing the null hypothesis $H_0 : \psi = \psi_0$ against the alternative $H_1 : \psi \neq \psi_0$. Composite likelihood versions of 'Wald' statistis have the usual asymptotic chi-square distribution with q' degree of freedom. The Wald-type statistic is

$$W = (\hat{\psi} - \psi_0)^T (G^{\psi\psi})^{-1}(\hat{\theta})(\hat{\psi} - \psi_0),$$

where $G_{\psi\psi}$ is the $q' \times q'$ submatrix of the Godambe or Fisher information pertaining to ψ and $\hat{\theta}$ is the maximum likelihood estimator from the full model. This statistic can be called from the routine `GeoTests` assigning at the argument `statistic` the value: `Wald`.

Alternatively to the Wald-type statistic one can use the composite version of the Wilks-type or likelihood ratio statistic, given by

$$W = 2[C\ell(\hat{\theta};\mathbf{y}) - C\ell\{\psi_0, \hat{\tau}(\psi_0);\mathbf{y}\}].$$

In the composite likelihood case, the asymptotic distribution of the composite likelihood ratio statistic is given by

$$W \dot{\sim} \sum_i \lambda_i \chi^2,$$

for $i = 1, \dots, q'$, where χ_i^2 are q' iid copies of a chi-square one random variable and $\lambda_1, \dots, \lambda_{q'}$ are the eigenvalues of the matrix $(H^{\psi\psi})^{-1}G^{\psi\psi}$. There exist several adjustments to the composite likelihood ratio statistic in order to get an approximated $\chi_{q'}^2$. For example, Rotnitzky and Jewell (1990) proposed the adjustment $W' = W/\bar{\lambda}$ where $\bar{\lambda}$ is the average of the eigenvalues λ_i. This statistic can be called within the routine by the value: `WilksRJ`. A better solution is proposed by Satterhwaite (1946) defining $W'' = \nu W/(q'\bar{\lambda})$, where $\nu = (\sum_i \lambda)^2/\sum_i \lambda_i^2$ for $i = 1 \dots, q'$, is the effective number of the degree of freedom. Note that in this case the distribution of the likelihood ratio statistic is a chi-square random variable with ν degree of freedom. This statistic can be called from the routine assigning the value: `WilksS`. For the adjustments suggested by Chandler and Bate (2007) we refere to the article (see **References**). This versions can be called from the routine assigning respectively the values: `WilksCB`.

Value

An object of class `c("data.frame")`. The object contain a table with the results of the tested models. The rows represent the responses for each model and the columns the following results:

`Num.Par`	The number of the model's parameters.
`Diff.Par`	The difference between the number of parameters of the model in the previous row and those in the actual row.
`Df`	The effective number of degree of freedom of the chi-square distribution.
`Chisq`	The observed value of the statistic.
`Pr(>chisq)`	The p-value of the quantile `Chisq` computed using a chi-squared distribution with `Df` degrees of freedom.

Author(s)

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Rotnitzky, A. and Jewell, N. P. (1990). Hypothesis Testing of Regression Parameters in Semiparametric Generalized Linear Models for Cluster Correlated Data. *Biometrika*, **77**, 485–497.

Satterthwaite, F. E. (1946). An Approximate Distribution of Estimates of Variance Components. *Biometrics Bulletin*, **2**, 110–114.

Varin, C., Reid, N. and Firth, D. (2011). An Overview of Composite Likelihood Methods. *Statistica Sinica*, **21**, 5–42.

See Also

`GeoFit`.

Examples

```
library(GeoModels)

################################################################
###
### Example 1. Test on the parameter
### of a regression model using conditional composite likelihood
###
###############################################################
set.seed(342)
model="Gaussian"
# Define the spatial-coordinates of the points:
NN=1500
x = runif(NN, 0, 1)
y = runif(NN, 0, 1)
coords = cbind(x,y)
# Parameters
mean=1; mean1=-1.25;  # regression parameters
nugget=0; sill=1

# matrix covariates
X=cbind(rep(1,nrow(coords)),runif(nrow(coords)))

# model correlation
corrmodel="Wend0"
power2=4;c_supp=0.15

# simulation
param=list(power2=power2,mean=mean,mean1=mean1,
              sill=sill,scale=c_supp,nugget=nugget)
data = GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param,X=X)$data

I=Inf
##### H1: (regressian mean)
fixed=list(nugget=nugget,power2=power2)
start=list(mean=mean,mean1=mean1,scale=c_supp,sill=sill)

lower=list(mean=-I,mean1=-I,scale=0,sill=0)
upper=list(mean=I,mean1=I,scale=I,sill=I)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH1 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
              likelihood="Conditional",type="Pairwise",sensitivity=TRUE,
                   lower=lower,upper=upper,neighb=3,
                   optimizer="nlminb",X=X,
                    start=start,fixed=fixed)

unlist(fitH1$param)

##### H0: (constant mean i.e beta1=0)
fixed=list(power2=power2,nugget=nugget,mean1=0)
start=list(mean=mean,scale=c_supp,sill=sill)
```

```
lower0=list(mean=-I,scale=0,sill=0)
upper0=list(mean=I,scale=I,sill=I)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH0 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
            likelihood="Conditional",type="Pairwise",sensitivity=TRUE,
                      lower=lower0,upper=upper0,neighb=3,
                   optimizer="nlminb",X=X,
                    start=start,fixed=fixed)
unlist(fitH0$param)

# not run
##fitH1=GeoVarestbootstrap(fitH1,K=100,optimizer="nlminb",
##                       lower=lower, upper=upper)
##fitH0=GeoVarestbootstrap(fitH0,K=100,optimizer="nlminb",
##                       lower=lower0, upper=upper0)

#  Composite likelihood Wald and ratio statistic statistic tests
#  rejecting null  hypothesis beta1=0
##GeoTests(fitH1, fitH0 ,statistic='Wald')
##GeoTests(fitH1, fitH0 , statistic='WilksS')
##GeoTests(fitH1, fitH0 , statistic='WilksCB')



################################################################
###
### Example 2. Parametric test of Gaussianity
### using SinhAsinh random fields using standard likelihood
###
###############################################################
set.seed(99)
model="SinhAsinh"
# Define the spatial-coordinates of the points:
NN=200
x = runif(NN, 0, 1)
y = runif(NN, 0, 1)
coords = cbind(x,y)
# Parameters
mean=0; nugget=0; sill=1
### skew and tail parameters
skew=0;tail=1   ## H0 is Gaussianity
# underlying model correlation
corrmodel="Wend0"
power2=4;c_supp=0.2

# simulation from Gaussian  model (H0)
param=list(power2=power2,skew=skew,tail=tail,
             mean=mean,sill=sill,scale=c_supp,nugget=nugget)
data = GeoSim(coordx=coords, corrmodel=corrmodel,model=model, param=param)$data


##### H1: SinhAsinh model
```

```
fixed=list(power2=power2,nugget=nugget,mean=mean)
start=list(scale=c_supp,skew=skew,tail=tail,sill=sill)

lower=list(scale=0,skew=-I, tail=0,sill=0)
upper=list(scale=I,skew= I,tail=I,sill=I)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH1 = GeoFit2(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                   likelihood="Full",type="Standard",varest=TRUE,
                   lower=lower,upper=upper,
                   optimizer="nlminb",
                    start=start,fixed=fixed)

unlist(fitH1$param)

##### H0: Gaussianity (i.e tail1=1, skew=0 fixed)
fixed=list(power2=power2,nugget=nugget,mean=mean,tail=1,skew=0)
start=list(scale=c_supp,sill=sill)
lower=list(scale=0,sill=0)
upper=list(scale=2,sill=5)
# Maximum pairwise composite-likelihood fitting of the RF:
fitH0 = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
                    likelihood="Full",type="Standard",varest=TRUE,
                      lower=lower,upper=upper,
                   optimizer="nlminb",
                    start=start,fixed=fixed)

unlist(fitH0$param)

#  Standard likelihood Wald and ratio statistic statistic tests
#  not rejecting null  hypothesis tail=1,skew=0 (Gaussianity)
GeoTests(fitH1, fitH0,statistic='Wald')
GeoTests(fitH1, fitH0,statistic='Wilks')
```

GeoVarestbootstrap — *Update a* GeoFit *object using parametric bootstrap for std error estimation*

Description

The procedure update a GeoFit object computing stderr estimation, confidence intervals and p-values using parametric bootstrap.

Usage

```
GeoVarestbootstrap(fit,K=100,sparse=FALSE, GPU=NULL,local=c(1,1),
  optimizer=NULL, lower=NULL, upper=NULL,
  method="cholesky",alpha=0.95, L=3000,parallel=FALSE,ncores=NULL)
```

Arguments

fit	A fitted object obtained from the GeoFit.
K	The number of simulations in the parametric bootstrap.
sparse	Logical; if TRUE then cholesky decomposition is performed using sparse matrices algorithms (spam packake).
GPU	Numeric; if NULL (the default) no OpenCL computation is performed. The user can choose the device to be used. Use DeviceInfo() function to see available devices, only double precision devices are allowed
local	Numeric; number of local work-items of the OpenCL setup
optimizer	The type of optimization algorithm (see GeoFit for details). If NULL then the optimization algorithm of the object fit is chosen.
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize.
method	String; The method of simulation. Default is cholesky. For large data set three options are TB or CE (see the GeoSimapprox) function.
alpha	Numeric; The level of the confidence interval.
L	Numeric; the number of lines in the turning band method.
parallel	Logical; if TRUE then the estimation step is parallelized
ncores	Numeric; number of cores involved in parallelization.

Details

The function update a GeoFit object estimating stderr estimation and confidence interval using parametric bootstrap.

Value

Returns an (updated) object of class GeoFit.

Author(s)

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See Also

GeoFit.

Examples

```
library(GeoModels)

################################################################
###
### Example 1. Test on the parameter
### of a regression model using conditional composite likelihood
###
###############################################################
set.seed(342)
model="Gaussian"
# Define the spatial-coordinates of the points:
NN=3500
x = runif(NN, 0, 1)
y = runif(NN, 0, 1)
coords = cbind(x,y)
# Parameters
mean=1; mean1=-1.25;  # regression parameters
 sill=1 # variance

# matrix covariates
X=cbind(rep(1,nrow(coords)),runif(nrow(coords)))

# model correlation
corrmodel="Matern"
smooth=0.5;scale=0.1; nugget=0;

# simulation
param=list(smooth=smooth,mean=mean,mean1=mean1,
              sill=sill,scale=scale,nugget=nugget)
data = GeoSim(coordx=coords, corrmodel=corrmodel,
                model=model, param=param,X=X)$data

I=Inf

fixed=list(nugget=nugget,smooth=smooth)
start=list(mean=mean,mean1=mean1,scale=scale,sill=sill)

lower=list(mean=-I,mean1=-I,scale=0,sill=0)
upper=list(mean=I,mean1=I,scale=I,sill=I)
# Maximum pairwise composite-likelihood fitting of the RF:
fit = GeoFit(data=data,coordx=coords,corrmodel=corrmodel, model=model,
              likelihood="Conditional",type="Pairwise",sensitivity=TRUE,
                   lower=lower,upper=upper,neighb=3,
                   optimizer="nlminb",X=X,
                    start=start,fixed=fixed)

unlist(fit$param)


#fit_update=GeoVarestbootstrap(fit,K=100,parallel=TRUE)
```

```
#fit_update$stderr
#fit_update$conf.int
#fit_update$pvalues
```

GeoVariogram *Empirical semi-variogram estimation*

Description

The function returns an empirical estimate of the semi-variogram for spatio (temporal) and bivariate random fields.

Usage

```
GeoVariogram(data, coordx, coordy=NULL, coordt=NULL,
coordx_dyn=NULL,cloud=FALSE, distance="Eucl",
              grid=FALSE, maxdist=NULL,neighb=NULL,
              maxtime=NULL, numbins=NULL,
              radius=6371, type='variogram',bivariate=FALSE)
```

Arguments

`data`	A d-dimensional vector (a single spatial realisation) or a $(n \times d)$-matrix (n iid spatial realisations) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$-array (n iid spatial realisations on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(t \times d \times n)$-array (n iid spatial-temporal realisations) or or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid) or an $(d \times d \times t \times n)$-array ($n$ iid spatial-temporal realisations on regular grid). See `GeoFit` for details.
`coordx`	A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates. Coordinates on a sphere for a fixed radius `radius` are passed in lon/lat format expressed in decimal degrees.
`coordy`	A numeric vector assigning 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.
`coordt`	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is `NULL` then a spatial random field is expected.
`coordx_dyn`	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
`cloud`	Logical; if `TRUE` the semivariogram cloud is computed, otherwise if `FALSE` (the default) the empirical (binned) semivariogram is returned.
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details** of `GeoFit`.

`grid`	Logical; if `FALSE` (the default) the data are interpreted as spatial or spatial-temporal realisations on a set of non-equispaced spatial sites.
`maxdist`	A numeric value denoting the spatial maximum distance, see the Section **Details**.
`neighb`	Numeric; an optional positive integer indicating the order of neighborhood. See the Section **Details** for more information.
`maxtime`	A numeric value denoting the temporal maximum distance, see the Section **Details**.
`numbins`	A numeric value denoting the numbers of bins, see the Section **Details**.
`radius`	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
`type`	A String denoting the type of semivariogram. The option available is : `variogram`.
`bivariate`	Logical; if `FALSE` (the default) the data are interpreted as univariate spatial or spatial-temporal realisations. Otherwise they are intrepreted as a a realization from a bivariate field.

Details

We briefly report the definitions of semi-variogram used for the spatial case. It can be easily extended to the space-time or spatial bivariate case. In the case of a spatial Gaussian random field the sample semivariogram estimator is defined by

$$\hat{\gamma}(h) = 0.5 \sum_{x_i, x_j \in N(h)} (Z(x_i) - Z(x_j))^2 / |N(h)|$$

where $N(h)$ is the set of all the sample pairs whose distances fall into a tolerance region with size h (equispaced intervalls are considered).

The `numbins` parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The `maxdist` parameter indicates the maximum spatial distance below which the shorter distances will be considered in the calculation of the semivariogram.

The `maxdist` parameter can be coupled with the `neighb` parameter. This is useful when handling large dataset.

The `maxtime` parameter indicates the maximum temporal distance below which the shorter distances will be considered in the calculation of the spatio-temoral semivariogram.

Value

Returns an object of class `Variogram`. An object of class `Variogram` is a list containing at most the following components:

`bins`	Adjacent intervals of grouped spatial distances if `cloud=FALSE`. Otherwise if `cloud=TRUE` all the spatial pairwise distances;
`bint`	Adjacent intervals of grouped temporal distances if `cloud=FALSE`. Otherwise if `cloud=TRUE` all the temporal pairwise distances;

`cloud`	If the variogram cloud is returned (`TRUE`) or the empirical variogram (`FALSE`);
`centers`	The centers of the spatial bins;
`distance`	The type of spatial distance;
`lenbins`	The number of pairs in each spatial bin;
`lenbinst`	The number of pairs in each spatial-temporal bin;
`lenbint`	The number of pairs in each temporal bin;
`maxdist`	The maximum spatial distance used for the calculation of the variogram. If no spatial distance is specified then it is NULL;
`maxtime`	The maximum temporal distance used for the calculation of the variogram. If no temporal distance is specified then it is NULL;
`spacetime_dyn`	If the space-time variogram is obtained using dynamical coordinates then it is(`TRUE`).
`variograms`	The empirical spatial variogram;
`variogramst`	The empirical spatial-temporal variogram;
`variogramt`	The empirical temporal variogram;
`type`	The type of estimated variogram

Author(s)

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Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York.

See Also

GeoFit

Examples

```
library(GeoModels)

################################################################
###
### Example 1. Empirical estimation of the semi-variogram from a
### spatial Gaussian random field with exponential correlation.
###
###############################################################
set.seed(514)
# Set the coordinates of the sites:
x = runif(200, 0, 1)
```

```
y = runif(200, 0, 1)
coords = cbind(x,y)
# Set the model's parameters:
corrmodel = "Exponential"
mean = 0
sill = 1
nugget = 0
scale = 0.3/3

# Simulation of the spatial Gaussian random field:
data = GeoSim(coordx=coords, corrmodel=corrmodel, param=list(mean=mean,
              sill=sill, nugget=nugget, scale=scale))$data

# Empirical spatial semi-variogram estimation:
vario = GeoVariogram(coordx=coords,data=data,maxdist=0.6)

plot(vario,pch=20,ylim=c(0,1),ylab="Semivariogram",xlab="Distance")


################################################################
###
### Example 2. Empirical estimation of the variogram from a
### spatio-temporal Gaussian random fields with Gneiting
### correlation function.
###
###############################################################

set.seed(331)
# Define the temporal sequence:
# Set the coordinates of the sites:
x = runif(200, 0, 1)
y = runif(200, 0, 1)
coords = cbind(x,y)
times = seq(1,10,1)

# Simulation of a spatio-temporal Gaussian random field:
data = GeoSim(coordx=coords, coordt=times, corrmodel="gneiting",
              param=list(mean=0,scale_s=0.08,scale_t=0.4,sill=1,
              nugget=0,power_s=1,power_t=1,sep=0.5))$data

# Empirical spatio-temporal semi-variogram estimation:
vario_st = GeoVariogram(data=data, coordx=coords, coordt=times, maxtime=7,maxdist=0.5)

plot(vario_st)

################################################################
###
### Example 3. Empirical estimation of the (cross) semivariograms
### from a bivariate Gaussian random fields with Matern
### correlation function.
###
###############################################################
# Simulation of a bivariate spatial Gaussian random field:
```

```
set.seed(293)
# Define the spatial-coordinates of the points:
x = runif(400, 0, 1)
y = runif(400, 0, 1)
coords=cbind(x,y)

# Simulation of a bivariate Gaussian Random field
# with matern (cross)  covariance function
param=list(mean_1=0,mean_2=0,scale_1=0.1/3,scale_2=0.15/3,scale_12=0.15/3,
           sill_1=1,sill_2=1,nugget_1=0,nugget_2=0,
           smooth_1=0.5,smooth_12=0.5,smooth_2=0.5,pcol=0.3)
data = GeoSim(coordx=coords, corrmodel="Bi_matern", param=param)$data

# Empirical  semi-(cross)variogram estimation:
biv_vario=GeoVariogram(data,coordx=coords, bivariate=TRUE,maxdist=0.5)

plot(biv_vario,pch=20)
```

GeoWLS *WLS of Random Fields*

Description

the function returns the parameters' estimates of a random field obtained by the weigthed least squares estimator.

Usage

```
GeoWLS(data, coordx, coordy=NULL, coordt=NULL, coordx_dyn=NULL, corrmodel,
              distance="Eucl", fixed=NULL, grid=FALSE, maxdist=NULL,neighb=NULL,
              maxtime=NULL,  model='Gaussian', optimizer='Nelder-Mead',
         numbins=NULL, radius=6371, start=NULL, weighted=FALSE,optimization=TRUE)
```

Arguments

`data` A d-dimensional vector (a single spatial realisation) or a $(d \times d)$-matrix (a single spatial realisation on regular grid) or an $(d \times d \times n)$-array (n iid spatial realisations on regular grid) or a $(t \times d)$-matrix (a single spatial-temporal realisation) or an $(d \times d \times t \times n)$-array (a single spatial-temporal realisation on regular grid). See `GeoFit` for details.

`coordx` A numeric $(d \times 2)$-matrix (where `d` is the number of spatial sites) giving 2-dimensions of spatial coordinates or a numeric d-dimensional vector giving 1-dimension of spatial coordinates.

`coordy` A numeric vector giving 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a $(d \times 2)$-matrix.

coordt	A numeric vector giving 1-dimension of temporal coordinates. Optional argument, the default is NULL then a spatial random field is expected.
coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description (see GeoFit).
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details** of GeoFit.
fixed	A named list giving the values of the parameters that will be considered as known values. The listed parameters for a given correlation function will be not estimated, i.e. if list(nugget=0) the nugget effect is ignored.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
maxdist	A numeric value denoting the maximum distance, see **Details** in GeoFit.
neighb	Numeric; an optional positive integer indicating the order of neighborhood. See **Details** and GeoFit
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered.See **Details** and GeoFit.
model	String; the type of random field. Gaussian is the default, see GeoFit for the different types.
optimizer	String; the optimization algorithm (see optim for details). 'Nelder-Mead' is the default.
numbins	A numeric value denoting the numbers of bins, see the Section **Details**
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure. NULL is the default (see GeoFit).
weighted	Logical; if TRUE then the weighted least square estimator is considered. If FALSE (the default) then the classic least square is used.
optimization	Logical; if TRUE then the weighted least square minimization is performed. Otherwise the weighted least square function is evaluated at the starting value.

Details

The numbins parameter indicates the number of adjacent intervals to consider in order to grouped distances with which to compute the (weighted) lest squares.

The maxdist parameter indicates the maximum distance below which the shorter distances will be considered in the calculation of the (weigthed) least squares.

Value

Returns an object of class WLS. An object of class WLS is a list containing at most the following components:

bins	Adjacent intervals of grouped distances;

`bint`	Adjacent intervals of grouped temporal separations
`centers`	The centers of the bins;
`coordx`	The vector or matrix of spatial coordinates;
`coordy`	The vector of spatial coordinates;
`coordt`	The vector of temporal coordinates;
`convergence`	A string that denotes if convergence is reached;
`corrmodel`	The correlation model;
`data`	The vector or matrix of data;
`distance`	The type of spatial distance;
`fixed`	The vector of fixed parameters;
`iterations`	The number of iteration used by the numerical routine;
`maxdist`	The maximum spatial distance used for the calculation of the variogram used in least square estimation. If no spatial distance is specified then it is NULL;
`maxtime`	The maximum temporal distance used for the calculation of the variogram used in least square estimation. If no temporal distance is specified then it is NULL;
`message`	Extra message passed from the numerical routines;
`model`	The type of random fields;
`numcoord`	The number of spatial coordinates;
`numtime`	The number the temporal realisations of the random field;
`param`	The vector of parameters' estimates;
`variograms`	The empirical spatial variogram;
`variogramt`	The empirical temporal variogram;
`variogramst`	The empirical spatial-temporal variogram;
`weighted`	A logical value indicating if its the weighted method;
`wls`	The value of the least squares at the minimum.

Author(s)

Moreno Bevilacqua, `<moreno.bevilacqua89@gmail.com>`,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, `<victor.morales@uv.cl>`, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, `<chcaaman@ubiobio.cl>`,https://www.researchgate.net/profile/Christian-Caamano

References

Cressie, N. A. C. (1993) *Statistics for Spatial Data*. New York: Wiley.

Gaetan, C. and Guyon, X. (2010) *Spatial Statistics and Modelling*. Spring Verlang, New York.

See Also

`GeoFit`, `optim`

Examples

```
library(GeoModels)


# Set the coordinates of the sites:

set.seed(211)
x <- runif(200, 0, 1)
set.seed(98)
y <- runif(200, 0, 1)
coords <- cbind(x,y)

################################################################
###
### Example 1. Least square fitting of a Gaussian random field
### with exponential correlation.
###
###############################################################

# Set the model's parameters:
corrmodel <- "Exponential"
mean <- 0
sill <- 1
nugget <- 0
scale <- 0.15/3
param <- list(mean=0,sill=sill, nugget=nugget, scale=scale)
# Simulation of the Gaussian random field:
set.seed(2)
data <- GeoSim(coordx=coords, corrmodel=corrmodel, param=param)$data

fixed=list(nugget=0,mean=mean)
start=list(scale=scale,sill=sill)
# Least square fitting of the random field:
fit <- GeoWLS(data=data,coordx=coords, corrmodel=corrmodel,
         fixed=fixed,start=start,maxdist=0.5)

# Results:
print(fit)
```

`Jamaicatemp` *December monthly average temperature in Jamaica between 1970-2000*

Description

A $(13530x3)$-matrix containing spatial december average temperature (°C) observed between 1970-2000 in Jamaica with associated UTM coordinates. The data has been retrieved using the package **geodata** that allows to download climate data from WorldClim version 2.1. UTM coordinates has been obtained using zone 18 and datum WGS84 and rescaled by 1000 to have distances in KM.

Usage

```
data(Jamaicatemp)
```

Format

A numerical matrix of dimension $13530x3$.

Source

Fick, S.E., Hijmans, R.J. (2017) WorldClim 2: new 1km spatial resolution climate surfaces for global land areas. *International Journal of Climatology*, **37**, 4302–4315.

`Lik` *Optimizes the Log Likelihood*

Description

Subroutine called by GeoFit. The procedure estimates the model parameters by maximization of the log-likelihood.

Usage

```
Lik(copula,bivariate,coordx,coordy,coordt,
coordx_dyn,corrmodel,data,fixed,flagcor,flagnuis,
           grid,lower,mdecomp,model,namescorr,
           namesnuis,namesparam,numcoord,
           numpairs,numparamcor,numtime,optimizer,
           onlyvar,parallel,param,radius,setup,
           spacetime,sparse,varest,taper,type,
           upper,ns,X,neighb,MM,aniso)
```

Arguments

`copula`	String; the type of copula. It can be "Beta" or "Gaussian"
`bivariate`	Logical; if `TRUE` then the data come froma a bivariate random field. Otherwise from a univariate random field.
`coordx`	A numeric ($d \times 2$)-matrix (where `d` is the number of spatial sites) assigning 2-dimensions of spatial coordinates or a numeric d-dimensional vector assigning 1-dimension of spatial coordinates.
`coordy`	A numeric vector assigning 1-dimension of spatial coordinates; `coordy` is interpreted only if `coordx` is a numeric vector or `grid=TRUE` otherwise it will be ignored. Optional argument, the default is `NULL` then `coordx` is expected to be numeric a ($d \times 2$)-matrix.
`coordt`	A numeric vector assigning 1-dimension of temporal coordinates. Optional argument, the default is `NULL` then a spatial random field is expected.

coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	Numeric; the id of the correlation model.
data	A numeric vector or a $(n \times d)$-matrix or $(d \times d \times n)$-matrix of observations.
flagcor	A numeric vector of flags denoting which correlation parameters have to be estimated.
flagnuis	A numeric verctor of flags denoting which nuisance parameters have to estimated.
fixed	A numeric vector of parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
lower	An optional named list giving the values for the lower bound of the space parameter when the optimizer is L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
model	Numeric; the id value of the density associated to the likelihood objects.
namescorr	String; the names of the correlation parameters.
namesnuis	String; the names of the nuisance parameters.
namesparam	String; the names of the parameters to be maximised.
numcoord	Numeric; the number of coordinates.
numpairs	Numeric; the number of pairs.
numparamcor	Numeric; the number of the correlation parameters.
numtime	Numeric; the number of temporal observations.
mdecomp	String; the type of matrix decomposition used in the simulation. Default is cholesky. The other possible choices is svd (Singular values decomposition).
optimizer	String; the optimization algorithm (see optim for details). Nelder-Mead is the default. Other possible choices are nlm, BFGS L-BFGS-B and nlminb. In these last two cases upper and lower bounds can be passed by the user. In the case of one-dimensional optimization, the function optimize is used.
onlyvar	Logical; if TRUE (and varest is TRUE) only the variance covariance matrix is computed without optimizing. FALSE is the default.
parallel	Logical; if TRUE optmization is performed using optimParallel using the maximum number of cores, when optimizer is L-BFGS-B.FALSE is the default.
param	A numeric vector of parameters.
sparse	Logical; if TRUE then maximum likelihood is computed using sparse matrices algorithms.FALSE is the default.
radius	Numeric; the radius of the sphere when considering data on a sphere.
ns	Numeric: vector of number of location sites for each temporal instants
setup	A List of useful components for the estimation based on the maximum tapered likelihood.
spacetime	Logical; if the random field is spatial (FALSE) or spatio-temporal (TRUE).

varest	Logical; if TRUE the estimate' variances and standard errors are returned. FALSE is the default.
taper	String; the name of the taper correlation function.
type	String; the type of the likelihood objects. If Pairwise (the default) then the marginal composite likelihood is formed by pairwise marginal likelihoods.
upper	An optional named list giving the values for the upper bound of the space parameter when the optimizer is or L-BFGS-B or nlminb or optimize. The names of the list must be the same of the names in the start list.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
neighb	Numeric;parameter for vecchia approximation using GPvecchia package
MM	Numeric;a non constant fixed mean
aniso	Logical; should anisotropy be considered?

Value

Return a list from an optim call.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

GeoFit

---

madagascarph *Soil ph of Madagascar*

Description

A $(300649x3)$-matrix containing lon/lat coordinates and soil ph in H20 at 300649 location sites in Madagascar at one meter depth over the period 2008–2014. Data obtained using the Geodata package with the function soil_af. For more info, see https://www.isric.org/projects/soil-property-maps-africa-250-m-resolution

Usage

```
data(madagascarph)
```

Format

A numerical matrix of dimension $300649x3$.

Source

Hengl T, Heuvelink GBM, Kempen B, Leenaars JGB, Walsh MG, Shepherd KD, et al. (2015) Mapping Soil Properties of Africa at 250 m Resolution: Random Forests Significantly Improve Current Predictions. PLoS ONE 10(6): e0125814. doi:10.1371/journal.pone.0125814

MatDecomp *Matrix decomposition*

Description

Matrix decomposition.

Usage

```
MatDecomp(mtx, method)
```

Arguments

mtx	numeric; a square positive or semipositive definite matrix.
method	string; the type of matrix decomposition. Two possible choices: cholesky and svd.

Details

Decomposition of a square positive or positive semidefinite matrix.

Value

Return a matrix decomposition

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

MatSqrt, MatInv, MatLogDet

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

Description

Square root, inverse and log determinant of a (semi)positive definite matrix, given a matrix decomposition.

Usage

```
MatSqrt(mat.decomp,method)
MatInv(mtx)
MatLogDet(mat.decomp,method)
```

Arguments

`mtx`	numeric; a squared symmetric positive definite matrix.
`mat.decomp`	numeric; a matrix decomposition.
`method`	string; the type of matrix decomposition. Two possible choices: `cholesky` and `svd`.

Value

The function returna a square root or inverse or log determinant of a (semi)positive definite matrix using the function in the FastGP package.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

MatDecomp

Examples

```
library(GeoModels)
################################################################
###
### Example 1. Inverse of Covariance matrix associated to
### a Matern correlation model
###
###############################################################
```

```
# Define the spatial-coordinates of the points:
x <- runif(15, 0, 1)
y <- runif(15, 0, 1)
coords <- cbind(x,y)
# Matern Parameters
param=list(smooth=0.5,sill=1,scale=0.2,nugget=0)
a=matrix <- GeoCovmatrix(coordx=coords, corrmodel="Matern", param=param)

## decomposition with cholesky method
b=MatDecomp(a$covmat,method="cholesky")
## inverse of covariance matrix
inverse=MatInv(a$covmat)
```

NuisParam *Lists the Nuisance Parameters of a Random Field*

Description

The procedure returns a list with the nuisance parameters of a given random field model.

Usage

```
NuisParam(model, bivariate=FALSE,num_betas=c(1,1),copula=NULL)
```

Arguments

model	String; the name of a random field.
bivariate	Logical; if FALSE (the default) the correlation model is univariate spatial or spatial-temporal. Otherwise is bivariate.
num_betas	Numerical; the nunber of mean parameters in the linear specification (default is 1)
copula	The type of copula.

Details

The function returns a list with the nuisance parameters of a given random field model.

Value

Return a vector string of nuisance parameters.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

GeoFit

Examples

```
library(GeoModels)

NuisParam("Gaussian")

NuisParam("Binomial")

NuisParam("Weibull",num_betas=2)

NuisParam("SkewGaussian", num_betas=3)

NuisParam("SinhAsinh")

NuisParam("Beta2",copula="Clayton")

NuisParam("StudentT")
## note that in the bivariate case sill and nugget are considered as correlation parameteres
NuisParam("Gaussian", bivariate=TRUE)
```

NuisParam2 *Internal function handling Nuisance Parameters of a Random Field*

Description

Internal function handling Nuisance Parameters of a Random Field.

Usage

```
NuisParam2(model, bivariate=FALSE,num_betas=c(1,1),copula=NULL)
```

Arguments

model	String; the name of a random field.
bivariate	Logical; if FALSE (the default) the correlation model is univariate spatial or spatial-temporal. Otherwise is bivariate.
num_betas	Numerical; the nunber of mean parameters in the linear specification (default is 1)
copula	The type of copula.

Details

The function returns a list with the nuisance parameters of a given random field model.

Value

Return a vector string of nuisance parameters.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

See Also

GeoFit

---

plot.GeoCorrFct *Plot Spatial and Spatio-temporal correlation or covariance of (non) Gaussian random fields*

Description

Plot Spatial and Spatio-temporal correlation or covariance of (non) Gaussian random fields for a given set of spatial or spatiotemporal distances GeoCorrFct.

Usage

```
## S3 method for class 'GeoCorrFct'
plot(x,type="p", ...)
```

Arguments

x	an object of the class "GeoCorrFct"
type	The type of graphic. The possible options are "p" and "l". If "p" then a point type graphic is displayed. Otherwise a lines type graphic displayed.
...	Other graphical options arguments. plot

Details

Plot Spatial and Spatio-temporal correlation or covariance of (non) Gaussian random fields

Value

Produces a plot. No values are returned.

See Also

GeoCorrFct for examples.

plot.GeoVariogram — *Plot empirical spatial, spatio-temporal and spatial bivariate semi-Variogram*

Description

Plot empirical spatial, spatio-temporal and spatial bivariate semi-Variogram using on object `GeoVariogram`.

Usage

```
## S3 method for class 'GeoVariogram'
plot(x, ...)
```

Arguments

`x`	an object of the class `"GeoVariogram"`
`...`	other arguments to be passed to the function `plot`

Details

This function plots empirical semi variogram in the spatial, spatio-temporal and spatial bivariate case

Value

Produces a plot. No values are returned.

See Also

`GeoVariogram` for variogram computation and examples.

SimCE — *Circulant embeeding simulation*

Description

Subroutine called by GeoSimapprox. The procedure return a simulation on a regular grid from a standard spatial Gaussian random field with a specified correlation model

Usage

```
SimCE(M,N,x,y,corrmodel,param,mean.val, max.ext)
```

Arguments

`M`	Numeric; the dimension of x
`N`	Numeric; the dimension of y
`x`	A numeric M-dimensional vector giving 1-dimension of spatial coordinates.
`y`	A numeric N-dimensional vector giving 1-dimension of spatial coordinates.
`corrmodel`	String; the name of a correlation model.
`param`	A list of parameter values required in the simulation procedure.
`mean.val`	The mean of the random field.
`max.ext`	The maximum extension of the simulation window.

Value

Return a list from an `optim` call.

Author(s)

Moreno Bevilacqua, `<moreno.bevilacqua89@gmail.com>`,`https://sites.google.com/view/moreno-bevilacqua/home`, Víctor Morales Oñate, `<victor.morales@uv.cl>`, `https://sites.google.com/site/moralesonatevictor/`, Christian", Caamaño-Carrillo, `<chcaaman@ubiobio.cl>`,`https://www.researchgate.net/profile/Christian-Caamano`

See Also

`GeoSimapprox`

`sp2Geo` *Extracting information from an sp or spacetime object*

Description

Extracting information from an sp or spacetime object

Usage

```
sp2Geo(spobj,spdata = NULL)
```

Arguments

`spobj`	An object of class sp or spacetime
`spdata`	Character: The name of data in the sp or spacetime object

Details

The function accepts as input a sp or spacetime object and the name of the data of interest in the object and it returns a list with some useful informations for Geomodels functions.

Value

A list with spatio-temporal informations

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

Examples

```
# Define the spatial-coordinates of the points:
set.seed(3)
N <- 30  # number of location sites
x <- runif(N, 0, 1)
set.seed(6)
y <- runif(N, 0, 1)
coords <- cbind(x,y)

# Define spatial matrix covariates and regression parameters
X <- cbind(rep(1,N),runif(N))
# Define spatial matrix dependent variable
Y <- rnorm(nrow(X))

obj1 <- sp::SpatialPoints(coords)
obj2 <- sp::SpatialPointsDataFrame(coords,data = data.frame(X,Y))

# sp2Geo info extraction
b <- sp2Geo(obj2,spdata  = "Y")
class(b)
b
```

---

spanish_wind *August monthly average wind speed in Spain between 1970-2000*

Description

A $(6000x3)$-matrix containing lon/lat and august monthly average wind speed (2 m above the ground, meter/second) registered at 6000 location sites in the Iberian peninsula. Data obtained from WorldClim version 2.1

Usage

```
data(spanish_wind)
```

Format

A numerical matrix of dimension $6000x3$.

Source

Fick, S.E., Hijmans, R.J. (2017) WorldClim 2: new 1km spatial resolution climate surfaces for global land areas. *International Journal of Climatology*, **37**, 4302–4315.

StartParam *Initializes the Parameters for Estimation Procedures*

Description

Subroutine called by the fitting procedures. The procedure initializes the parameters for the fitting procedure.

Usage

```
StartParam(coordx, coordy, coordt,coordx_dyn, corrmodel, data, distance, fcall,
          fixed, grid,likelihood,  maxdist, neighb, maxtime, model, n,  param,
          parscale, paramrange, radius,  start, taper, tapsep,
          type,typereal,varest, vartype, weighted, winconst,
          winstp, winconst_t, winstp_t,copula,X,memdist,nosym)
```

Arguments

`coordx`	A numeric ($d \times 2$)-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
`coordy`	A numeric vector assigning 1-dimension of coordinates; `coordy` is interpreted only if `coordx` is a numeric vector otherwise it will be ignored.
`coordt`	A numeric vector assigning 1-dimension of temporal coordinates.
`coordx_dyn`	A list of m numeric ($d_t \times 2$)-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is `NULL`
`corrmodel`	String; the name of a correlation model.
`data`	A numeric vector or a ($n \times d$)-matrix or ($d \times d \times n$)-matrix of observations.
`distance`	String; the name of the spatial distance. The default is `Eucl`, the euclidean distance. See the Section **Details**.
`fcall`	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
`fixed`	A named list giving the values of the parameters that will be considered as known values.
`grid`	Logical; if `FALSE` (the default) the data are interpreted as a vector or a ($n \times d$)-matrix, instead if `TRUE` then ($d \times d \times n$)-matrix is considered.
`likelihood`	String; the configuration of the composite likelihood.
`maxdist`	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.

neighb	Numeric; an optional positive integer indicating the order of neighborhood in the composite likelihood computation. See the Section **Details** for more information.
maxtime	Numeric; an optional positive value indicating the maximum temporal lag considered in the composite-likelihood computation.
radius	Numeric; the radius of the sphere in the case of lon-lat coordinates. The default is 6371, the radius of the earth.
model	String; the density associated to the likelihood objects. `Gaussian` is the default.
n	Numeric; number of trials for binomial random fields.
param	A numeric vector of parameter values required in the simulation procedure of random fields.
parscale	A numeric vector of scaling factor to improve the maximizing procedure, see `optim`.
paramrange	A numeric vector of parameters ranges, see `optim`.
start	A named list with the initial values of the parameters that are used by the numerical routines in maximization procedure.
taper	String; the name of the type of covariance matrix. It can be `Standard` (the default value) or `Tapering` for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time adaptive taper (see **Details**).
type	String; the type of likelihood objects. Temporary value set to be "WLeastSquare" (weigthed least-square) in order to compute the starting values.
typereal	String; the real type of likelihood objects. See `GeoFit`.
varest	Logical; if `TRUE` the estimates' variances and standard errors are returned. `FALSE` is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section **Details**.
weighted	Logical; if `TRUE` the likelihood objects are weighted, see `GeoFit`.
winconst	Numeric; a positive value for computing the spatial sub-window in the subsampling procedure.
winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the subsampling procedure.
winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
copula	The type of copula.
X	Numeric; Matrix of space-time covariates.
memdist	Logical; if `TRUE` then the distances in the composite likelihood are computed before the optmization.
nosym	Logical; if TRUE two simmetric weights are not considered

Details

Internal function called by WlsStart.

Value

A list with a set of useful informations in the estimation procedure.

Author(s)

Moreno Bevilacqua, <moreno.bevilacqua89@gmail.com>,https://sites.google.com/view/moreno-bevilacqua/home, Víctor Morales Oñate, <victor.morales@uv.cl>, https://sites.google.com/site/moralesonatevictor/, Christian", Caamaño-Carrillo, <chcaaman@ubiobio.cl>,https://www.researchgate.net/profile/Christian-Caamano

---

winds *Irish Daily Wind Speeds*

Description

A matrix containing daily wind speeds, in kilometers per hour, from 1961 to 1978 at 12 sites in Ireland

Usage

```
data(winds)
```

Format

A (6574×11)-matrix containing wind speed observations.

Source

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

winds.coords *Weather Stations of the Irish Daily Wind Speeds*

Description

A data frame containing information about the weather stations where the data are recorded in Ireland.

Usage

```
data(winds.coords)
```

Format

A data frame containing site - the name of the city (character), abbr - the abbrevation (character), elev - the elevation (numeric), lat - latitude (numeric) and lon - longitude.

Source

Haslett, J. and Raftery, A. E. (1989), Space-time modelling with long-memory dependence: assessing Ireland's wind-power resource (with discussion), *Applied Statistics*, 38, 1–50.

WlsStart *Computes Starting Values based on Weighted Least Squares*

Description

Subroutine called by GeoFit. The function returns opportune starting values for the composite-likelihood fitting procedure based on weigthed least squares.

Usage

```
WlsStart(coordx, coordy, coordt, coordx_dyn, corrmodel, data, distance, fcall,
        fixed, grid,likelihood, maxdist, neighb,maxtime, model, n, param,
        parscale, paramrange, radius,start, taper, tapsep, type, varest,
     vartype, weighted, winconst,winconst_t, winstp_t, winstp,copula,X,memdist,nosym)
```

Arguments

coordx	A numeric $(d \times 2)$-matrix (where d is the number of points) assigning 2-dimensions of coordinates or a numeric vector assigning 1-dimension of coordinates.
coordy	A numeric vector assigning 1-dimension of coordinates; coordy is interpreted only if coordx is a numeric vector otherwise it will be ignored.
coordt	A numeric vector assigning 1-dimension of temporal coordinates.

coordx_dyn	A list of m numeric $(d_t \times 2)$-matrices containing dynamical (in time) spatial coordinates. Optional argument, the default is NULL
corrmodel	String; the name of a correlation model, for the description.
data	A numeric vector or a $(n \times d)$-matrix or $(d \times d \times n)$-matrix of observations.
distance	String; the name of the spatial distance. The default is Eucl, the euclidean distance. See the Section **Details**.
fcall	String; "fitting" to call the fitting procedure and "simulation" to call the simulation procedure.
fixed	A named list giving the values of the parameters that will be considered as known values.
grid	Logical; if FALSE (the default) the data are interpreted as a vector or a $(n \times d)$-matrix, instead if TRUE then $(d \times d \times n)$-matrix is considered.
likelihood	String; the configuration of the composite likelihood.
maxdist	Numeric; an optional positive value indicating the maximum spatial distance considered in the composite-likelihood computation.
neighb	Numeric; an optional positive integer indicating the order of neighborhood in the composite likelihood computation. See the Section **Details** for more information.
maxtime	Numeric; an optional positive value indicating the maximum temporal separation considered in the composite-likelihood computation.
model	String; the name of the model. Here the default is NULL.
n	Numeric; number of trials in a binomial random field.
param	A numeric vector of parameter values required in the simulation procedure of random fields.
parscale	A numeric vector with scaling values for improving the maximisation routine.
paramrange	A numeric vector with the range of the parameter space.
radius	Numeric; a value indicating the radius of the sphere when using the great circle distance. Default value is the radius of the earth in Km (i.e. 6371)
start	A numeric vector with starting values.
taper	String; the name of the type of covariance matrix. It can be Standard (the default value) or Tapering for taperd covariance matrix.
tapsep	Numeric; an optional value indicating the separabe parameter in the space time quasi taper (see **Details**).
type	String; the type of estimation method.
varest	Logical; if TRUE the estimates' variances and standard errors are returned. FALSE is the default.
vartype	String; the type of estimation method for computing the estimate variances, see the Section **Details**.
weighted	Logical; if TRUE the likelihood objects are weighted, see GeoFit.
winconst	Numeric; a positive value for computing the spatial sub-window in the subsampling procedure.

winstp	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the spatial sub-sampling procedure.
winconst_t	Numeric; a positive value for computing the temporal sub-window in the sub-sampling procedure.
winstp_t	Numeric; a value in $(0, 1]$ for defining the the proportion of overlapping in the temporal sub-sampling procedure.
copula	The type of copula.
X	Numeric; Matrix of spatio(temporal)covariates in the linear mean specification.
memdist	Logical; if TRUE then the distances in the composite likelihood are computed before the optmization.
nosym	Logical; if TRUE two simmetric weights are not considered

Details

Internal function called by GeoFit.

Value

A list with a set of useful informations in the estimation procedure.

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See Also

GeoFit.

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