Package 'gamCopula'

October 13, 2022

Type Package
Title Generalized Additive Models for Bivariate Conditional Dependence Structures and Vine Copulas
Version 0.0-7
Date 2020-02-05
Maintainer Thibault Vatter < thibault.vatter@gmail.com>
Depends R (>= $3.1.0$)
Imports graphics, stats, utils, VineCopula (>= 2.0.0), mgcv, MASS, gsl, numDeriv, methods, copula, igraph (>= 1.0.0), parallel, foreach, doParallel
Description Implementation of various inference and simulation tools to apply generalized additive models to bivariate dependence structures and non-simplified vine copulas.
License GPL-3
LazyLoad yes
<pre>BugReports https://github.com/tvatter/gamCopula/issues</pre>
<pre>URL https://github.com/tvatter/gamCopula</pre>
RoxygenNote 7.0.2
NeedsCompilation no
Author Thomas Nagler [aut], Thibault Vatter [aut, cre]
Repository CRAN
Date/Publication 2020-02-05 16:10:02 UTC
R topics documented:
gamCopula-package AIC.gamBiCop BIC.gamBiCop BiCopEta2Par 1

gamCo	opula-package	General Structur				s for	· Biv	aria	ite C	ond	ition	ıal 1	Эер	enc	denc	re
Index																56
	summary.gamVine		• •	 • •	• •	 • •	• •		• •	• •	• •	• •			• •	55
	summary.gamBiCop															
	RVM2GVC															
	plot.gamVine															
	plot.gamBiCop															
	nobs.gamBiCop															
	logLik.gamBiCop .															
	gamVineStructureSe															
	gamVineSimulate .															
	gamVineSeqFit															
	gamVinePDF															
	gamVineNormalize															
	gamVineFamily															
	gamVineCopSelect															
	gamVine-class															
	gamVine															
	gamBiCopSimulate															
	gamBiCopSelect .															
	gamBiCopPredict .															
	gamBiCopPDF															
	gamBiCopFit															19
	gamBiCopCDF			 		 										17
	gamBiCop-class			 		 										16
	gamBiCop			 		 										16
	formula.gamBiCop															
	EDF															
	dim.gamVine															
	condBiCopSim															
	BiCopPar2Eta			 		 										-11

Description

This package implements inference and simulation tools to apply generalized additive models to bivariate dependence structures and vine copulas.

More references in Vatter and Chavez-Demoulin (2015), and Vatter and Nagler (2016).

Details

Package: gamCopula Type: Package Version: 0.0-7 Date: 2020-02-05 License: GPL-3

Author(s)

Thibault Vatter and Thomas Nagler

Maintainer: Thibault Vatter <thibault.vatter@gmail.com>

References

Aas, K., C. Czado, A. Frigessi, and H. Bakken (2009) Pair-copula constructions of multiple dependence. Insurance: Mathematics and Economics 44 (2), 182-198.

Brechmann, E. C., C. Czado, and K. Aas (2012) Truncated regular vines in high dimensions with applications to financial data. Canadian Journal of Statistics 40 (1), 68-85.

Dissmann, J. F., E. C. Brechmann, C. Czado, and D. Kurowicka (2013) Selecting and estimating regular vine copulae and application to financial returns. Computational Statistics & Data Analysis, 59 (1), 52-69.

Vatter, T. and V. Chavez-Demoulin (2015) Generalized Additive Models for Conditional Dependence Structures. Journal of Multivariate Analysis, 141, 147-167.

Vatter, T. and T. Nagler (2016) Generalized additive models for non-simplified pair-copula constructions. https://arxiv.org/abs/1608.01593

Wood, S.N. (2004) Stable and efficient multiple smoothing parameter estimation for generalized additive models. Journal of the American Statistical Association, 99, 673-686.

Wood, S.N. (2006) Generalized Additive Models: an introduction with R. Chapman and Hall/CRC.

See Also

The present package is heavily relying on the mgcv and VineCopula packages, as it basically extends and mix both of them.

```
##### A gamBiCop example
require(copula)
require(mgcv)
set.seed(0)
## Simulation parameters (sample size, correlation between covariates,
## Gaussian copula family)
```

```
n <- 5e2
rho <- 0.5
fam <- 1
## A calibration surface depending on three variables
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm \leftarrow (Tf - Ti)/2
    a \leftarrow -(b/3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
  return(a + b * (t - Tm)^2),
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a \leftarrow b * (1 - 2 * Tf * pi/(f * Tf * pi +
                                   cos(2 * f * pi * (Tf - Ti))
                                 - cos(2 * f * pi * Ti)))
    return((a + b)/2 + (b - a) * sin(2 * f * pi * (t - Ti))/2)},
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf/8) {
    Tm \leftarrow (Tf - Ti)/2
    a \leftarrow (b * s * sqrt(2 * pi)/Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * \exp(-(t - Tm)^2/(2 * s^2))))
## Display the calibration surface
par(mfrow = c(1, 3), pty = "s", mar = c(1, 1, 4, 1))
u <- seq(0, 1, length.out = 100)
sel \leftarrow matrix(c(1, 1, 2, 2, 3, 3), ncol = 2)
jet.colors <- colorRamp(c("#00007F", "blue", "#007FFF", "cyan", "#7FFF7F",</pre>
                           "yellow", "#FF7F00", "red", "#7F0000"))
jet <- function(x) rgb(jet.colors(exp(x/3)/(1 + exp(x/3))),
                        maxColorValue = 255)
for (k in 1:3) {
    tmp <- outer(u, u, function(x, y)</pre>
      eta0 + calib.surf[[sel[k,1]]](x) + calib.surf[[sel[k, 2]]](y))
    persp(u, u, tmp, border = NA, theta = 60, phi = 30, zlab = "",
          col = matrix(jet(tmp), nrow = 100),
          xlab = paste("X", sel[k, 1], sep = ""),
          ylab = paste("X", sel[k,2], sep = ""),
          main = paste("eta0+f", sel[k, 1],
                         "(X", sel[k, 1], ") +f",sel[k, 2],
                        "(X", sel[k, 2], ")", sep = ""))
}
## 3-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 3),</pre>
                                   c("unif"), list(list(min = 0, max = 1)),
                                   marginsIdentical = TRUE)
X <- rMvdc(n, covariates.distr)</pre>
## U in [0,1]x[0,1] with copula parameter depending on X
U \leftarrow condBiCopSim(fam, function(x1,x2,x3)  {eta0+sum(mapply(function(f,x))
  f(x), calib.surf, c(x1,x2,x3)), X[,1:3], par2 = 6, return.par = TRUE)
## Merge U and X
data <- data.frame(U$data,X)</pre>
```

```
names(data) <- c(paste("u",1:2,sep=""),paste("x",1:3,sep=""))</pre>
## Display the data
dev.off()
plot(data[, "u1"], data[, "u2"], xlab = "U1", ylab = "U2")
## Model fit with a basis size (arguably) too small
## and unpenalized cubic spines
pen <- FALSE
basis0 <- c(3, 4, 4)
formula <-\infty(x1, k = basis0[1], bs = "cr", fx = !pen) +
  s(x2, k = basis0[2], bs = "cr", fx = !pen) +
  s(x3, k = basis0[3], bs = "cr", fx = !pen)
system.time(fit0 <- gamBiCopFit(data, formula, fam))</pre>
## Model fit with a better basis size and penalized cubic splines (via min GCV)
pen <- TRUE
basis1 <- c(3, 10, 10)
formula \leftarrow \sim s(x1, k = basis1[1], bs = "cr", fx = !pen) +
  s(x2, k = basis1[2], bs = "cr", fx = !pen) +
  s(x3, k = basis1[3], bs = "cr", fx = !pen)
system.time(fit1 <- gamBiCopFit(data, formula, fam))</pre>
## Extract the gamBiCop objects and show various methods
(res <- sapply(list(fit0,fit1), function(fit){fit$res}))</pre>
metds <- list('logLik'=logLik,'AIC'=AIC,'BIC'=BIC,'EDF'=EDF)</pre>
lapply(res, function(x) sapply(metds, function(f) f(x)))
## Comparison between fitted, true smooth and spline approximation for each
## true smooth function for the two basis sizes
fitted <- lapply(res, function(x) gamBiCopPredict(x, data.frame(x1=u,x2=u,x3=u),</pre>
                                                  type = "terms")$calib)
true <- vector("list", 3)</pre>
for (i in 1:3) {
    y <- eta0+calib.surf[[i]](u)</pre>
    true[[i]]$true <- y - eta0</pre>
    temp <- gam(y \sim s(u, k = basis0[i], bs = "cr", fx = TRUE))
    true[[i]]$approx <- predict.gam(temp, type = "terms")</pre>
    temp <- gam(y \sim s(u, k = basis1[i], bs = "cr", fx = FALSE))
    true[[i]]$approx2 <- predict.gam(temp, type = "terms")</pre>
}
## Display results
par(mfrow = c(1, 3), pty = "s")
yy <- range(true, fitted)</pre>
yy[1] <- yy[1] * 1.5
for(k in 1:3){
  plot(u, true[[k]]$true, type = "1", ylim = yy,
       xlab = paste("Covariate",k), ylab = paste("Smooth",k))
  lines(u, true[[k]]$approx, col = "red", lty = 2)
  lines(u, fitted[[1]][, k], col = "red")
  lines(u, fitted[[2]][, k], col = "green")
```

```
lines(u, true[[k]]$approx2, col = "green", lty = 2)
  legend("bottomleft", cex = 0.6, lty = c(1, 1, 2, 1, 2),
         c("True", "Fitted", "Appox 1", "Fitted 2", "Approx 2"),
         col = c("black", "red", "red", "green", "green"))
}
##### A gamVine example
set.seed(0)
## Simulation parameters
# Sample size
n <- 1e3
# Copula families
familyset <-c(1:2,301:304,401:404)
# Define a 4-dimensional R-vine tree structure matrix
d <- 4
Matrix < c(2,3,4,1,0,3,4,1,0,0,4,1,0,0,0,1)
Matrix <- matrix(Matrix,d,d)</pre>
nnames <- paste("X", 1:d, sep = "")</pre>
## A function factory
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm \leftarrow (Tf - Ti)/2
    a \leftarrow -(b/3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b * (t - Tm)^2)},
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a \leftarrow b * (1 - 2 * Tf * pi/(f * Tf * pi +
                                  cos(2 * f * pi * (Tf - Ti))
                                 - cos(2 * f * pi * Ti)))
    return((a + b)/2 + (b - a) * sin(2 * f * pi * (t - Ti))/2)},
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf/8) {
    Tm \leftarrow (Tf - Ti)/2
    a \leftarrow (b * s * sqrt(2 * pi)/Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * \exp(-(t - Tm)^2/(2 * s^2))))
## Create the model
# Define gam-vine model list
count <- 1
model <- vector(mode = "list", length = d*(d-1)/2)</pre>
sel \leftarrow seq(d, d^2-d, by = d)
# First tree
for (i in 1:(d-1)) {
  # Select a copula family
  family <- sample(familyset, 1)</pre>
  model[[count]]$family <- family</pre>
  # Use the canonical link and a randomly generated parameter
  if (is.element(family,c(1,2))) {
    model[[count]]$par <- tanh(rnorm(1)/2)</pre>
    if (family == 2) {
```

```
model[[count]]$par2 <- 2+exp(rnorm(1))</pre>
    }
  } else {
    if (is.element(family,c(401:404))) {
      rr <- rnorm(1)
      model[[count]]$par <- sign(rr)*(1+abs(rr))</pre>
      model[[count]]$par <- rnorm(1)</pre>
    }
    model[[count]]$par2 <- 0</pre>
  count <- count + 1
}
# A dummy dataset
data <- data.frame(u1 = runif(1e2), u2 = runif(1e2), matrix(runif(1e2*d),1e2,d))</pre>
# Trees 2 to (d-1)
for(j in 2:(d-1)){
  for(i in 1:(d-j)){
    # Select a copula family
    family <- sample(familyset, 1)</pre>
    # Select the conditiong set and create a model formula
    cond <- nnames[sort(Matrix[(d-j+2):d,i])]</pre>
    tmpform \leftarrow paste("\sim", paste(paste("s(", cond, ", k=10, bs='cr')",
                                        sep = ""), collapse=" + "))
    1 <- length(cond)</pre>
    temp <- sample(3, 1, replace = TRUE)</pre>
    # Spline approximation of the true function
    m <- 1e2
    x \leftarrow matrix(seq(0,1,length.out=m), nrow = m, ncol = 1)
    if(1 != 1){
      tmp.fct <- paste("function(x){eta0+",</pre>
                         paste(sapply(1:1, function(x)
                           paste("calib.surf[[",temp[x],"]](x[",x,"])",
                                  sep="")), collapse="+"),"}",sep="")
      tmp.fct <- eval(parse(text = tmp.fct))</pre>
      x <- eval(parse(text = paste0("expand.grid(",</pre>
                                      paste0(rep("x",1), collapse = ","),")",
                                      collapse = "")))
      y \leftarrow apply(x,1,tmp.fct)
    }else{
      tmp.fct <- function(x) eta0+calib.surf[[temp]](x)</pre>
      colnames(x) <- cond
      y <- tmp.fct(x)
    }
    # Estimate the gam model
    form <- as.formula(paste0("y", tmpform))</pre>
    dd <- data.frame(y, x)</pre>
    names(dd) \leftarrow c("y", cond)
```

8 AIC.gamBiCop

```
b <- gam(form, data = dd)
    #plot(x[,1],(y-fitted(b))/y)
    # Create a dummy gamBiCop object
    tmp <- gamBiCopFit(data = data, formula = form, family = 1, n.iters = 1)$res</pre>
    # Update the copula family and the model coefficients
    attr(tmp, "model")$coefficients <- coefficients(b)</pre>
    attr(tmp, "model")$smooth <- b$smooth</pre>
    attr(tmp, "family") <- family</pre>
    if (family == 2) {
      attr(tmp, "par2") <- 2+exp(rnorm(1))</pre>
    model[[count]] <- tmp</pre>
    count <- count+1
  }
}
# Create the gamVineCopula object
GVC <- gamVine(Matrix=Matrix,model = model,names=nnames)</pre>
print(GVC)
## Not run:
## Simulate and fit the model
sim <- gamVineSimulate(n, GVC)</pre>
fitGVC <- gamVineSeqFit(sim, GVC, verbose = TRUE)</pre>
fitGVC2 <- gamVineCopSelect(sim, Matrix, verbose = TRUE)</pre>
## Plot the results
par(mfrow=c(3,4))
plot(GVC, ylim = c(-2.5, 2.5))
plot(fitGVC, ylim = c(-2.5, 2.5))
plot(fitGVC2, ylim = c(-2.5, 2.5))
## End(Not run)
```

AIC.gamBiCop

Akaike's An Information Criterion for a gamBiCop Object

Description

Function calculating Akaike's 'An Information Criterion' (AIC) for an object of the class gamBiCop (note that the models are usually fitted by penalized likelihood maximization).

Usage

```
## S4 method for signature 'gamBiCop'
AIC(object, ..., k = 2)
```

BIC.gamBiCop 9

Arguments

object An object of the class gamBiCop.

... un-used in this class

k numeric, the penalty per parameter to be used; the default k = 2 is the classical

AIC.

Value

A numeric value with the corresponding AIC.

See Also

AIC and BIC.

BIC.gamBiCop

Schwarz's Bayesian Information Criterion for a gamBiCop Object

Description

Function calculating the Schwarz's Bayesian Information Criterion (BIC) for an object of the class gamBiCop (note that the models are usually fitted by penalized likelihood maximization).

Usage

```
## S4 method for signature 'gamBiCop'
BIC(object, ...)
```

Arguments

object An object of the class gamBiCop.

... un-used in this class

Value

A numeric value with the corresponding BIC.

See Also

AIC and BIC.

10 BiCopEta2Par

BiCopEta2Par	Copula Parameter of a Bivariate Copula for a Given Value of the Calibration Function

Description

Computes the (first) copula parameter of a bivariate copula for a given value of the calibration function (eta).

Usage

```
BiCopEta2Par(family, eta)
```

Arguments

family A copula family: 1 Gaussian, 2 Student t, 301 Double Clayton type I (standard

and rotated 90 degrees), 302 Double Clayton type II (standard and rotated 270 degrees), 303 Double Clayton type III (survival and rotated 90 degrees), 304 Double Clayton type IV (survival and rotated 270 degrees), 401 Double Gumbel type I (standard and rotated 90 degrees), 402 Double Gumbel type II (standard and rotated 270 degrees), 403 Double Gumbel type III (survival and rotated 90

degrees), 404 Double Gumbel type IV (survival and rotated 270 degrees).

eta The calibration function.

Value

The value of the first copula parameter, depending on the copula parameter and family as:

- 1 Gaussian, $f(x) = \tanh(x/2)$
- 2 Student t, $f(x) = \tanh(x/2)$
- 301 Double Clayton type I (standard and rotated 90 degrees), f(x) = x
- 302 Double Clayton type II (standard and rotated 270 degrees), f(x) = x
- 303 Double Clayton type III (survival and rotated 90 degrees), f(x) = x
- 304 Double Clayton type IV (survival and rotated 270 degrees), f(x) = x
- 401 Double Gumbel type I (standard and rotated 90 degrees), f(x) = x*(1+abs(x))/abs(x)
- 402 Double Gumbel type II (standard and rotated 270 degrees), f(x) = x*(1+abs(x))/abs(x)
- 403 Double Gumbel type III (survival and rotated 90 degrees), f(x) = x*(1+abs(x))/abs(x)
- 404 Double Gumbel type IV (survival and rotated 270 degrees) f(x) = x*(1+abs(x))/abs(x).

See Also

BiCopEta2Par or BiCopPar2Tau and BiCopTau2Par from VineCopula.

BiCopPar2Eta 11

	BiCopPar2Eta	Calibration Function of a Bivariate Copula for a Given Parameter's Value
--	--------------	--

Description

Computes the calibration function (eta) of a bivariate copula for a given value of the (first) copula parameter.

Usage

```
BiCopPar2Eta(family, par)
```

Arguments

family A copula family: 1 Gaussian, 2 Student t, 301 Double Clayton type I (standard

> and rotated 90 degrees), 302 Double Clayton type II (standard and rotated 270 degrees), 303 Double Clayton type III (survival and rotated 90 degrees), 304 Double Clayton type IV (survival and rotated 270 degrees), 401 Double Gumbel type I (standard and rotated 90 degrees), 402 Double Gumbel type II (standard and rotated 270 degrees), 403 Double Gumbel type III (survival and rotated 90

degrees), 404 Double Gumbel type IV (survival and rotated 270 degrees).

The (first) copula parameter par

Value

The value of the calibration function, depending on the copula parameter and family as:

- 1 Gaussian, f(x) = 2*atanh(x)
- 2 Student t, f(x) = 2*atanh(x)
- 301 Double Clayton type I (standard and rotated 90 degrees), f(x) = x
- 302 Double Clayton type II (standard and rotated 270 degrees), f(x) = x
- 303 Double Clayton type III (survival and rotated 90 degrees), f(x) = x
- 304 Double Clayton type IV (survival and rotated 270 degrees), f(x) = x
- 401 Double Gumbel type I (standard and rotated 90 degrees), f(x) = x*(1-1/abs(x))
- 402 Double Gumbel type II (standard and rotated 270 degrees), f(x) = x*(1-1/abs(x))
- 403 Double Gumbel type III (survival and rotated 90 degrees), f(x) = x*(1-1/abs(x))
- 404 Double Gumbel type IV (survival and rotated 270 degrees) f(x) = x*(1-1/abs(x)).

See Also

BiCopEta2Par or BiCopPar2Tau and BiCopTau2Par from VineCopula.

12 condBiCopSim

condBiCopSim Simulation from a Conditional Bivariate Copula	condBiCopSim	Simulation from a Conditional Bivariate Copula	
---	--------------	--	--

Description

Simulates from a conditional bivariate copula, where each copula parameter takes a different value, depending on the calibration function and covariates.

Usage

```
condBiCopSim(family, calib.fnc, X, par2 = 0, return.par = TRUE, tau = TRUE)
```

Arguments

family	family A copula family: 1 Gaussian, 2 Student t, 3 Clayton, 4 Gumbel, 5 Frank, 13 Survival Clayton, 14 Survival Gumbel, 23 Rotated (90 degrees) Clayton, 24 Rotated (90 degrees) Gumbel, 33 Rotated (270 degrees) Clayton and 34 Rotated (270 degrees) Gumbel.
calib.fnc	A calibration function.
X	A vector (if calib.fnc takes a single argument) or matrix (if calib.fnc takes multiple arguments) of covariates values.
par2	The second copula parameter (for the Student t), default par $2 = 0$.
return.par	Should the parameter (and calibration function) be returned as well (default return.par = TRUE)?
tau	Should the calibration function (and the model) be specified for the copula parameter or Kendall's tau (default tau = TRUE)?

Value

If return.par = TRUE, then the function returns a list with:

- data, a matrix with two columns containing the simulated data,
- par, a vector containing the values of the copula parameter,
- and eta, a vector containing the values of the calibration function.

If return.par = FALSE, then the function simply returns data, a matrix with two columns containing the simulated data.

See Also

gamBiCopFit and gamBiCopSimulate.

condBiCopSim 13

```
require(copula)
set.seed(0)
## Simulation parameters (sample size, correlation between covariates,
## Gaussian copula family)
n <- 2e2
rho <- 0.5
fam <- 1
## A calibration surface depending on three variables
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b \star (t - Tm)<sup>2</sup>)
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a \leftarrow b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  },
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * \exp(-(t - Tm)^2 / (2 * s^2)))
)
## Display the calibration surface
par(mfrow = c(1, 3), pty = "s", mar = c(1, 1, 4, 1))
u \leftarrow seq(0, 1, length.out = 100)
sel \leftarrow matrix(c(1, 1, 2, 2, 3, 3), ncol = 2)
jet.colors <- colorRamp(c(</pre>
  "#00007F", "blue", "#007FFF", "cyan", "#7FFF7F",
  "yellow", "#FF7F00", "red", "#7F0000"
jet \leftarrow function(x) rgb(jet.colors(exp(x / 3) / (1 + exp(x / 3))),
    maxColorValue = 255
  )
for (k in 1:3) {
  tmp <- outer(u, u, function(x, y)</pre>
    eta0 + calib.surf[[sel[k, 1]]](x) + calib.surf[[sel[k, 2]]](y))
  persp(u, u, tmp,
    border = NA, theta = 60, phi = 30, zlab = "",
    col = matrix(jet(tmp), nrow = 100),
    xlab = paste("X", sel[k, 1], sep = ""),
    ylab = paste("X", sel[k, 2], sep = ""),
    main = paste("eta0+f", sel[k, 1],
```

14 dim.gamVine

```
"(X", sel[k, 1], ") +f", sel[k, 2],
      "(X", sel[k, 2], ")",
      sep = ""
 )
}
## 3-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 3),</pre>
  c("unif"), list(list(min = 0, max = 1)),
  marginsIdentical = TRUE
X <- rMvdc(n, covariates.distr)</pre>
## U in [0,1]x[0,1] with copula parameter depending on X
U <- condBiCopSim(fam, function(x1, x2, x3) {
  eta0 + sum(mapply(function(f, x)
    f(x), calib.surf, c(x1, x2, x3)))
, X[, 1:3], par2 = 6, return.par = TRUE)
## Merge U and X
data <- data.frame(U$data, X)</pre>
names(data) <- c(paste("u", 1:2, sep = ""), paste("x", 1:3, sep = ""))</pre>
## Display the data
dev.off()
plot(data[, "u1"], data[, "u2"], xlab = "U1", ylab = "U2")
```

dim.gamVine

Dimension of an Object of the Class gamVine

Description

Retrieve the dimension of an object of the class gamVine.

Usage

```
## S4 method for signature 'gamVine' \dim(x)
```

Arguments

Χ

An object of the class gamVine.

Value

Dimension of the gamVine object.

See Also

gamVine.

EDF

EDF

Equivalent Degrees of Freedom for an Object of the Class gamBiCop

Description

Function calculating the Equivalent Degrees of Freedom (EDF) for a gamBiCop object. It basically sums the edf of the gamObject for each smooth component.

Usage

```
EDF(object)
```

Arguments

object

An object of the class gamBiCop.

Value

Estimated degrees of freedom for each smooth component.

formula.gamBiCop

Model Formula of the gamBiCop Object

Description

Extracts the gam formula from an object of the class gamBiCop. This function is a wrapper to formula.gam from the mgcv package.

Usage

```
## S4 method for signature 'gamBiCop'
formula(x, ...)
```

Arguments

x An object of the class gamBiCop.

... un-used in this class

See Also

formula.gam function from the mgcv package.

16 gamBiCop-class

		_	
~ nm	Dί	\sim	r
gam	υт	Cυ	L

Construction of a gamBiCop Class Object

Description

Constructs an object of the class gamBiCop.

Usage

```
gamBiCop(family, model, par2 = 0, tau = TRUE)
```

Arguments

family A copula family: 1 Gaussian, 2 Student t, 5 Frank, 301 Double Clayton
--

I (standard and rotated 90 degrees), 302 Double Clayton type II (standard and rotated 270 degrees), 303 Double Clayton type III (survival and rotated 90 degrees), 304 Double Clayton type IV (survival and rotated 270 degrees), 401 Double Gumbel type I (standard and rotated 90 degrees), 402 Double Gumbel type II (standard and rotated 270 degrees), 403 Double Gumbel type III (survival and rotated 90 degrees), 404 Double Gumbel type IV (survival and rotated 270

degrees).

model A gamObject as return by the gam function from the mgcv package.

par2 Second parameter for the Student t-copula.

tau FALSE for a calibration function specified for the Copula parameter or TRUE (de-

fault) for a calibration function specified for Kendall's tau.

Value

An object of the class gamBiCop.

See Also

gamBiCop, gamBiCopFit, gamBiCopPredict and gamBiCopSimulate.

gamBiCop-class	
Fallintron crass	

The gamBiCop Class

Description

gamBiCop is an S4 class to store a Generalized Additive Model for bivariate copula a parameter or Kendall's tau. Objects can be created by calls of the form new("gamBiCop", ...), or by function gamBiCop.

gamBiCopCDF 17

Slots

family A copula family: 1 Gaussian, 2 Student t, 5 Frank, 301 Double Clayton type I (standard and rotated 90 degrees), 302 Double Clayton type II (standard and rotated 270 degrees), 303 Double Clayton type III (survival and rotated 90 degrees), 304 Double Clayton type IV (survival and rotated 270 degrees), 401 Double Gumbel type I (standard and rotated 90 degrees), 402 Double Gumbel type II (standard and rotated 270 degrees), 403 Double Gumbel type III (survival and rotated 90 degrees), 404 Double Gumbel type IV (survival and rotated 270 degrees).

model A gamObject as return by the gam function from the mgcv package.

par2 Second parameter for the Studen t-copula.

tau FALSE (default) for a calibration function specified for the Copula parameter or TRUE for a calibration function specified for Kendall's tau.

See Also

gamBiCopFit, gamBiCopPredict and gamBiCopSimulate.

gamBiCopCDF	Conditional distribution function of a Generalized Additive model for the copula parameter or Kendall's tau

Description

This function returns the distribution function of a bivariate conditional copula, where either the copula parameter or the Kendall's tau is modeled as a function of the covariates.

Usage

```
gamBiCopCDF(object, newdata = NULL)
```

Arguments

object gamBiCop-class object.

newdata (Same as in predict.gam from the mgcv package) A matrix or data frame con-

taining the values of the model covariates at which predictions are required. If this is not provided then the distribution corresponding to the original data are returned. If newdata is provided then it should contain all the variables needed

for prediction: a warning is generated if not.

Value

The conditional density.

See Also

gamBiCop and gamBiCopPredict.

18 gamBiCopCDF

```
require(copula)
set.seed(0)
## Simulation parameters (sample size, correlation between covariates,
## Gaussian copula family)
n <- 2e2
rho <- 0.5
fam <- 1
## A calibration surface depending on three variables
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b \star (t - Tm)<sup>2</sup>)
  },
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a <- b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      -\cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  },
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## 3-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 3),</pre>
  c("unif"), list(list(min = 0, max = 1)),
  marginsIdentical = TRUE
X <- rMvdc(n, covariates.distr)</pre>
colnames(X) \leftarrow paste("x", 1:3, sep = "")
## U in [0,1]x[0,1] with copula parameter depending on X
U <- condBiCopSim(fam, function(x1, x2, x3) {
  eta0 + sum(mapply(function(f, x)
    f(x), calib.surf, c(x1, x2, x3)))
, X[, 1:3], par2 = 6, return.par = TRUE)
## Merge U and X
data <- data.frame(U$data, X)</pre>
names(data) \leftarrow c(paste("u", 1:2, sep = ""), paste("x", 1:3, sep = ""))
## Model fit with penalized cubic splines (via min GCV)
basis <- c(3, 10, 10)
formula <- \sim s(x1, k = basis[1], bs = "cr") +
```

```
s(x2, k = basis[2], bs = "cr") +
s(x3, k = basis[3], bs = "cr")
system.time(fit <- gamBiCopFit(data, formula, fam))
## Evaluate the conditional density
gamBiCopCDF(fit$res)</pre>
```

gamBiCopFit

Maximum penalized likelihood estimation of a Generalized Additive model for the copula parameter or Kendall's tau.

Description

This function estimates the parameter(s) of a Generalized Additive model (gam) for the copula parameter or Kendall's tau. It solves the maximum penalized likelihood estimation for the copula families supported in this package by reformulating each Newton-Raphson iteration as a generalized ridge regression, which is solved using the mgcv package.

Usage

```
gamBiCopFit(
  data,
  formula = ~1,
  family = 1,
  tau = TRUE,
  method = "FS",
  tol.rel = 0.001,
  n.iters = 10,
  verbose = FALSE,
  ...
)
```

Arguments

data

A list, data frame or matrix containing the model responses, (u1,u2) in [0,1]x[0,1], and covariates required by the formula.

formula

A gam formula (see gam, formula.gam and gam.models from mgcv).

family

A copula family: 1 Gaussian, 2 Student t, 5 Frank, 301 Double Clayton type I (standard and rotated 90 degrees), 302 Double Clayton type II (standard and rotated 270 degrees), 303 Double Clayton type III (survival and rotated 90 degrees), 304 Double Clayton type IV (survival and rotated 270 degrees), 401 Double Gumbel type I (standard and rotated 90 degrees), 402 Double Gumbel type II (standard and rotated 270 degrees), 403 Double Gumbel type III (survival and rotated 90 degrees), 404 Double Gumbel type IV (survival and rotated 270 degrees).

tau

FALSE (default) for a calibration function specified for the Copula parameter or TRUE for a calibration function specified for Kendall's tau.

method	'NR' for Newton-Raphson and 'FS' for Fisher-scoring (default).
tol.rel	Relative tolerance for 'FS'/'NR' algorithm.
n.iters	Maximal number of iterations for 'FS'/'NR' algorithm.
verbose	TRUE if informations should be printed during the estimation and FALSE (default) for a silent version.
	Additional parameters to be passed to gam from mgcv.

Value

gamBiCopFit returns a list consisting of

```
res S4 gamBiCop-class object.

method 'FS' for Fisher-scoring (default) and 'NR' for Newton-Raphson.

tol.rel relative tolerance for 'FS'/'NR' algorithm.

n.iters maximal number of iterations for 'FS'/'NR' algorithm.

trace the estimation procedure's trace.

conv 0 if the algorithm converged and 1 otherwise.
```

See Also

gamBiCop and gamBiCopSimulate.

```
require(copula)
require(mgcv)
set.seed(0)
## Simulation parameters (sample size, correlation between covariates,
## Gaussian copula family)
n <- 5e2
rho <- 0.5
fam <- 1
## A calibration surface depending on three variables
eta0 <- 1
calib.surf <- list(</pre>
 calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
   Tm <- (Tf - Ti) / 2
   a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
   return(a + b \star (t - Tm)^2)
 calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
   a <- b * (1 - 2 * Tf * pi / (f * Tf * pi +
     cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
   return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
 },
```

```
calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## Display the calibration surface
par(mfrow = c(1, 3), pty = "s", mar = c(1, 1, 4, 1))
u <- seq(0, 1, length.out = 100)
sel \leftarrow matrix(c(1, 1, 2, 2, 3, 3), ncol = 2)
jet.colors <- colorRamp(c(</pre>
  "#00007F", "blue", "#007FFF", "cyan", "#7FFF7F",
  "yellow", "#FF7F00", "red", "#7F0000"
))
jet \leftarrow function(x) rgb(jet.colors(exp(x / 3) / (1 + exp(x / 3))),
    maxColorValue = 255
  )
for (k in 1:3) {
  tmp <- outer(u, u, function(x, y)</pre>
    eta0 + calib.surf[[sel[k, 1]]](x) + calib.surf[[sel[k, 2]]](y))
  persp(u, u, tmp,
    border = NA, theta = 60, phi = 30, zlab = "",
    col = matrix(jet(tmp), nrow = 100),
    xlab = paste("X", sel[k, 1], sep = ""),
    ylab = paste("X", sel[k, 2], sep = ""),
    main = paste("eta0+f", sel[k, 1],
      "(X", sel[k, 1], ") +f", sel[k, 2],
      "(X", sel[k, 2], ")",
      sep = ""
    )
 )
}
## 3-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 3),</pre>
  c("unif"), list(list(min = 0, max = 1)),
  marginsIdentical = TRUE
X <- rMvdc(n, covariates.distr)</pre>
## U in [0,1]x[0,1] with copula parameter depending on X
U <- condBiCopSim(fam, function(x1, x2, x3) {
  eta0 + sum(mapply(function(f, x)
    f(x), calib.surf, c(x1, x2, x3)))
}, X[, 1:3], par2 = 6, return.par = TRUE)
## Merge U and X
data <- data.frame(U$data, X)</pre>
names(data) <- c(paste("u", 1:2, sep = ""), paste("x", 1:3, sep = ""))</pre>
## Display the data
dev.off()
```

```
plot(data[, "u1"], data[, "u2"], xlab = "U1", ylab = "U2")
## Model fit with a basis size (arguably) too small
## and unpenalized cubic spines
pen <- FALSE
basis0 <- c(3, 4, 4)
formula \leftarrow \sim s(x1, k = basis0[1], bs = "cr", fx = !pen) +
  s(x2, k = basis0[2], bs = "cr", fx = !pen) +
  s(x3, k = basis0[3], bs = "cr", fx = !pen)
system.time(fit0 <- gamBiCopFit(data, formula, fam))</pre>
## Model fit with a better basis size and penalized cubic splines (via min GCV)
pen <- TRUE
basis1 <- c(3, 10, 10)
formula \leftarrow \sim s(x1, k = basis1[1], bs = "cr", fx = !pen) +
  s(x2, k = basis1[2], bs = "cr", fx = !pen) +
  s(x3, k = basis1[3], bs = "cr", fx = !pen)
system.time(fit1 <- gamBiCopFit(data, formula, fam))</pre>
## Extract the gamBiCop objects and show various methods
(res <- sapply(list(fit0, fit1), function(fit) {</pre>
  fit$res
}))
metds <- list("logLik" = logLik, "AIC" = AIC, "BIC" = BIC, "EDF" = EDF)</pre>
lapply(res, function(x) sapply(metds, function(f) f(x)))
## Comparison between fitted, true smooth and spline approximation for each
## true smooth function for the two basis sizes
fitted <- lapply(res, function(x) gamBiCopPredict(x, data.frame(x1 = u, x2 = u, x3 = u),
    type = "terms"
  )$calib)
true <- vector("list", 3)</pre>
for (i in 1:3) {
  y <- eta0 + calib.surf[[i]](u)
  true[[i]]$true <- y - eta0</pre>
  temp <- gam(y \sim s(u, k = basis0[i], bs = "cr", fx = TRUE))
  true[[i]]$approx <- predict.gam(temp, type = "terms")</pre>
  temp <- gam(y \sim s(u, k = basis1[i], bs = "cr", fx = FALSE))
  true[[i]]$approx2 <- predict.gam(temp, type = "terms")</pre>
## Display results
par(mfrow = c(1, 3), pty = "s")
yy <- range(true, fitted)</pre>
yy[1] \leftarrow yy[1] * 1.5
for (k in 1:3) {
  plot(u, true[[k]]$true,
    type = "1", ylim = yy,
    xlab = paste("Covariate", k), ylab = paste("Smooth", k)
  lines(u, true[[k]]$approx, col = "red", lty = 2)
  lines(u, fitted[[1]][, k], col = "red")
```

gamBiCopPDF 23

```
lines(u, fitted[[2]][, k], col = "green")
lines(u, true[[k]]$approx2, col = "green", lty = 2)
legend("bottomleft",
   cex = 0.6, lty = c(1, 1, 2, 1, 2),
   c("True", "Fitted", "Appox 1", "Fitted 2", "Approx 2"),
   col = c("black", "red", "red", "green", "green")
)
}
```

gamBiCopPDF

Conditional density function of a Generalized Additive model for the copula parameter or Kendall's tau

Description

This function returns the density of a bivariate conditional copula, where either the copula parameter or the Kendall's tau is modeled as a function of the covariates.

Usage

```
gamBiCopPDF(object, newdata = NULL)
```

Arguments

object

gamBiCop-class object.

newdata

(Same as in predict.gam from the mgcv package) A matrix or data frame containing the values of the model covariates at which predictions are required, along with two columns named "u1" and "u2". If this is not provided then the density corresponding to the original data are returned. If newdata is provided then it should contain all the variables needed for prediction: a warning is generated if not.

Value

The conditional density.

See Also

```
gamBiCop and gamBiCopPredict.
```

```
require(copula)
set.seed(0)

## Simulation parameters (sample size, correlation between covariates,
## Gaussian copula family)
n <- 2e2
rho <- 0.5</pre>
```

24 gamBiCopPDF

```
fam <- 1
## A calibration surface depending on three variables
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b \star (t - Tm)^2)
  },
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a <- b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  },
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * \exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## 3-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 3),</pre>
  c("unif"), list(list(min = 0, max = 1)),
  marginsIdentical = TRUE
X <- rMvdc(n, covariates.distr)</pre>
colnames(X) \leftarrow paste("x", 1:3, sep = "")
## U in [0,1]x[0,1] with copula parameter depending on X
U <- condBiCopSim(fam, function(x1, x2, x3) {
  eta0 + sum(mapply(function(f, x)
    f(x), calib.surf, c(x1, x2, x3)))
, X[, 1:3], par2 = 6, return.par = TRUE)
## Merge U and X
data <- data.frame(U$data, X)</pre>
names(data) \leftarrow c(paste("u", 1:2, sep = ""), paste("x", 1:3, sep = ""))
## Model fit with penalized cubic splines (via min GCV)
basis <- c(3, 10, 10)
formula \leftarrow \sim s(x1, k = basis[1], bs = "cr") +
  s(x2, k = basis[2], bs = "cr") +
  s(x3, k = basis[3], bs = "cr")
system.time(fit <- gamBiCopFit(data, formula, fam))</pre>
## Evaluate the conditional density
gamBiCopPDF(fit$res)
```

gamBiCopPredict 25

gamBiCopPredict	Predict method of a Generalized Additive model for the copula parameter or Kendall's tau

Description

Predict method of a Generalized Additive model for the copula parameter or Kendall's tau

Usage

```
gamBiCopPredict(
  object,
  newdata = NULL,
  target = "calib",
  alpha = 0,
  type = "link"
)
```

Arguments

object gamBiCop-class object.

newdata (Same as in predict.gam from the mgcv package) A matrix or data frame con-

taining the values of the model covariates at which predictions are required. If this is not provided then predictions corresponding to the original data are returned. If newdata is provided then it should contain all the variables needed

for prediction: a warning is generated if not.

target Either 'calib', 'par' or 'tau' or a combination of those. 'calib' (de-

fault) corresponds to the calibration function, 'par' to the copula parameter

and 'tau' to Kendall's tau.

alpha In (0,1) to return the corresponding confidence interval.

type (Similar as in predict.gam from the mgcv package, only active for type =

'calib'). When this has the value 'link' (default), the calibration function is returned. When type = 'terms' each component of the linear predictor is returned separately (possibly with standard errors): this includes parametric model components, followed by each smooth component, but excludes any offset and any intercept. When type = 'lpmatrix' then a matrix is returned which yields the values of the linear predictor (minus any offset) when post-multiplied by the

parameter vector (in this case alpha is ignored).

Value

If target = 'calib', then a list with 1 item calib. If target = 'par', target = 'tau' or target = c('par', 'tau'), then a list with 2, 2 or 3 items, namely calib and par, tau and par, or calib, tau and par.

If alpha is in (0,1), then a additional items of the list are calib.CI as well as e.g. par.CI and/or tau.CI depending on the value of target.

26 gamBiCopPredict

Otherwise, if type = 'lpmatrix' (only active for type = 'calib'), then a matrix is returned which will give a vector of linear predictor values (minus any offset) at the supplied covariate values, when applied to the model coefficient vector (similar as predict.gam from the mgcv).

See Also

gamBiCop and gamBiCopFit.

```
require(copula)
set.seed(0)
## Simulation parameters (sample size, correlation between covariates,
## Clayton copula family)
n <- 5e2
rho <- 0.5
fam <- 1
## A calibration surface depending on three variables
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b * (t - Tm)^2)
  },
  calib.sin \leftarrow function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a \leftarrow b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## 3-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 3),</pre>
  c("unif"), list(list(min = 0, max = 1)),
  marginsIdentical = TRUE
X <- rMvdc(n, covariates.distr)</pre>
colnames(X) \leftarrow paste("x", 1:3, sep = "")
## U in [0,1]x[0,1] with copula parameter depending on X
U <- condBiCopSim(fam, function(x1, x2, x3) {
  eta0 + sum(mapply(function(f, x)
    f(x), calib.surf, c(x1, x2, x3)))
```

gamBiCopSelect 27

```
}, X[, 1:3], par2 = 6, return.par = TRUE)

## Merge U and X
data <- data.frame(U$data, X)
names(data) <- c(paste("u", 1:2, sep = ""), paste("x", 1:3, sep = ""))

## Model fit with penalized cubic splines (via min GCV)
basis <- c(3, 10, 10)
formula <- ~ s(x1, k = basis[1], bs = "cr") +
    s(x2, k = basis[2], bs = "cr") +
    s(x3, k = basis[3], bs = "cr")
system.time(fit <- gamBiCopFit(data, formula, fam))

## Extract the gamBiCop objects and show various methods
(res <- fit$res)
EDF(res)
pred <- gamBiCopPredict(fit$res, X, target = c("calib", "par", "tau"))</pre>
```

gamBiCopSelect

Selection and Maximum penalized likelihood estimation of a Generalized Additive model (gam) for the copula parameter or Kendall's tau.

Description

This function selects an appropriate bivariate copula family for given bivariate copula data using one of a range of methods. The corresponding parameter estimates are obtained by maximum penalized likelihood estimation, where each Newton-Raphson iteration is reformulated as a generalized ridge regression solved using the mgcv package.

Usage

```
gamBiCopSelect(
  udata.
  lin.covs = NULL,
  smooth.covs = NULL,
  familyset = NA,
  rotations = TRUE,
  familycrit = "AIC",
  level = 0.05,
  edf = 1.5,
  tau = TRUE,
 method = "FS",
  tol.rel = 0.001,
  n.iters = 10,
 parallel = FALSE,
  verbose = FALSE,
  select.once = TRUE,
)
```

28 gamBiCopSelect

Arguments

udata A matrix or data frame containing the model responses, (u1,u2) in [0,1]x[0,1]A matrix or data frame containing the parametric (i.e., linear) covariates. lin.covs smooth.covs A matrix or data frame containing the non-parametric (i.e., smooth) covariates. (Similar to BiCopSelect from the VineCopula package) Vector of bivariate familyset copula families to select from. If familyset = NA (default), selection among all possible families is performed. Coding of bivariate copula families: 1 Gaussian, 2 Student t, 5 Frank, 301 Double Clayton type I (standard and rotated 90 degrees), 302 Double Clayton type II (standard and rotated 270 degrees), 303 Double Clayton type III (survival and rotated 90 degrees), 304 Double Clayton type IV (survival and rotated 270 degrees), 401 Double Gumbel type I (standard and rotated 90 degrees), 402 Double Gumbel type II (standard and rotated 270 degrees), 403 Double Gumbel type III (survival and rotated 90 degrees), 404 Double Gumbel type IV (survival and rotated 270 degrees). If TRUE, all rotations of the families in familyset are included. rotations familycrit Character indicating the criterion for bivariate copula selection. Possible choices: familycrit = 'AIC' (default) or 'BIC', as in BiCopSelect from the VineCopula package. level Numerical; significance level of the test for removing individual predictors (default: level = 0.05). edf Numerical; if the estimated EDF for individual predictors is smaller than edf but the predictor is still significant, then it is set as linear (default: edf = 1.5). FALSE for a calibration function specified for the Copula parameter or TRUE (detau fault) for a calibration function specified for Kendall's tau. method 'FS' for Fisher-scoring (default) and 'NR' for Newton-Raphson. tol.rel Relative tolerance for 'FS'/'NR' algorithm. Maximal number of iterations for 'FS'/'NR' algorithm. n.iters parallel TRUE for a parallel estimation across copula families. verbose TRUE prints informations during the estimation. if TRUE the GAM structure is only selected once, for the family that appears first select.once in familyset.

Value

gamBiCopFit returns a list consisting of

res S4 gamBiCop-class object.

method 'FS' for Fisher-scoring and 'NR' for Newton-Raphson.

tol.rel relative tolerance for 'FS'/'NR' algorithm.

n.iters maximal number of iterations for 'FS'/'NR' algorithm.

Additional parameters to be passed to gam

trace the estimation procedure's trace.

conv 0 if the algorithm converged and 1 otherwise.

gamBiCopSelect 29

See Also

gamBiCop and gamBiCopFit.

```
require(copula)
set.seed(0)
## Simulation parameters (sample size, correlation between covariates,
## Student copula with 4 degrees of freedom)
n <- 5e2
rho <- 0.9
fam <- 2
par2 <- 4
## A calibration surface depending on four variables
eta0 <- 1
calib.surf <- list(</pre>
  calib.lin <- function(t, Ti = 0, Tf = 1, b = 2) {
    return(-2 + 4 * t)
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b \star (t - Tm)<sup>2</sup>)
  calib.sin \leftarrow function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a \leftarrow b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## 6-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 6),</pre>
  c("unif"), list(list(min = 0, max = 1)),
  marginsIdentical = TRUE
)
X <- rMvdc(n, covariates.distr)</pre>
colnames(X) \leftarrow paste("x", 1:6, sep = "")
## U in [0,1]x[0,1] depending on the four first columns of X
U <- condBiCopSim(fam, function(x1, x2, x3, x4) {
  eta0 + sum(mapply(function(f, x)
    f(x), calib.surf, c(x1, x2, x3, x4)))
}, X[, 1:4], par2 = 4, return.par = TRUE)
```

30 gamBiCopSimulate

```
## Not run:
## Selection using AIC (about 30sec on single core)
## Use parallel = TRUE to speed-up....
system.time(best <- gamBiCopSelect(U$data, smooth.covs = X))
print(best$res)
EDF(best$res) ## The first function is linear
## Plot only the smooth component
par(mfrow = c(2, 2))
plot(best$res)
## End(Not run)</pre>
```

gamBiCopSimulate

Simulate from gamBiCop-class object

Description

Simulate from gamBiCop-class object

Usage

```
gamBiCopSimulate(
  object,
  newdata = NULL,
  N = NULL,
  return.calib = FALSE,
  return.par = FALSE,
  return.tau = FALSE
)
```

Arguments

object gamBiCop-class object.

newdata (same as in predict.gam from the mgcv package) A matrix or data frame con-

taining the values of the model covariates at which simulations are required. If this is not provided then simulations corresponding to the original data are

returned.

N sample size.

return.calib should the calibration function (TRUE) be returned or not (FALSE)?

return.par should the copula parameter (TRUE) be returned or not (FALSE)?

return. tau should the Kendall's tau (TRUE) be returned or not (FALSE)?

gamBiCopSimulate 31

Value

A list with 1 item data. When N is smaller or larger than the newdata's number of rows (or the number of rows in the original data if newdata is not provided), then N observations are sampled uniformly (with replacement) among the row of newdata (or the rows of the original data if newdata is not provided).

If return.calib = TRUE, return.par = TRUE and/or return.tau = TRUE, then the list also contains respectively items calib, par and/or tau.

```
require(copula)
set.seed(1)
## Simulation parameters (sample size, correlation between covariates,
## Gaussian copula family)
n <- 5e2
rho <- 0.5
fam <- 1
## A calibration surface depending on three variables
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b \star (t - Tm)^2)
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a <- b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      -\cos(2 * f * pi * Ti))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## 3-dimensional matrix X of covariates
covariates.distr <- mvdc(normalCopula(rho, dim = 3),</pre>
  c("unif"), list(list(min = 0, max = 1)),
  marginsIdentical = TRUE
X <- rMvdc(n, covariates.distr)</pre>
colnames(X) \leftarrow paste("x", 1:3, sep = "")
## U in [0,1]x[0,1] with copula parameter depending on X
U <- condBiCopSim(fam, function(x1, x2, x3) {
  eta0 + sum(mapply(function(f, x)
```

32 gamVine

```
f(x), calib.surf, c(x1, x2, x3)))
}, X[, 1:3], par2 = 6, return.par = TRUE)

## Merge U and X
data <- data.frame(U$data, X)
names(data) <- c(paste("u", 1:2, sep = ""), paste("x", 1:3, sep = ""))

## Model fit with penalized cubic splines (via min GCV)
basis <- c(3, 10, 10)
formula <- ~ s(x1, k = basis[1], bs = "cr") +
    s(x2, k = basis[2], bs = "cr") +
    s(x3, k = basis[3], bs = "cr")
system.time(fit <- gamBiCopFit(data, formula, fam))

## Extract the gamBiCop objects and show various methods
(res <- fit$res)
EDF(res)
sim <- gamBiCopSimulate(fit$res, X)</pre>
```

gamVine

Construction of a gamVine Class Object

Description

Constructs an object of the class gamVine.

Usage

```
gamVine(Matrix, model, names = NA, covariates = NA)
```

Arguments

Matrix lower triangular d x d matrix that defines the tree structure.

model list containing d x (d-1)/2 lists with three numeric items (family, par and par2)

and/or objects of the class gamBiCop.

names vector of d names.

covariates vector of names for the covariates.

Value

An object of the class gamVine.

See Also

 ${\tt gamVine}, {\tt RVineMatrix}, {\tt gamBiCop\ gamVineSeqFit}, {\tt gamVineCopSelect}, {\tt gamVineStructureSelect} \\ {\tt and\ gamVineSimulate}.$

gamVine-class 33

Description

gamVine is an S4 class to store a conditional and potentially non-simplified pair-copula construction. Objects can be created by calls of the form new("gamVine", ...), or by function gamVine.

Slots

Matrix Lower triangular d x d matrix that defines the tree structure.

model list containing d x (d-1)/2 lists with three numeric items (family, par and par2) and/or gamBiCop objects.

names vector of d names.

covariates vector of names for the exogenous covariates.

See Also

gamVine, RVineMatrix, gamBiCop gamVineSeqFit, gamVineCopSelect, gamVineStructureSelect and gamVineSimulate.

gamVineCopSelect Sequential pair-copula selection and maximum penalized likelihood estimation of a GAM-Vine model.

Description

This function select the copula family and estimates the parameter(s) of a Generalized Additive model (GAM) Vine model, where GAMs for individual edges are specified either for the copula parameter or Kendall's tau. It solves the maximum penalized likelihood estimation for the copula families supported in this package by reformulating each Newton-Raphson iteration as a generalized ridge regression, which is solved using the mgcv package.

Usage

```
gamVineCopSelect(
  data,
  Matrix,
  lin.covs = NULL,
  smooth.covs = NULL,
  simplified = FALSE,
  familyset = NA,
  rotations = TRUE,
  familycrit = "AIC",
```

34 gamVineCopSelect

```
level = 0.05,
trunclevel = NA,
tau = TRUE,
method = "FS",
tol.rel = 0.001,
n.iters = 10,
parallel = FALSE,
verbose = FALSE,
select.once = TRUE,
...
)
```

Arguments

data A matrix or data frame containing the data in [0,1]^d.

Matrix Lower triangular d x d matrix that defines the R-vine tree structure.

lin.covs A matrix or data frame containing the parametric (i.e., linear) covariates (default:

lin.covs = NULL).

smooth.covs A matrix or data frame containing the non-parametric (i.e., smooth) covariates

(default: smooth.covs = NULL).

simplified If TRUE, then a simplified vine is fitted (which is possible only if there are ex-

ogenous covariates). If FALSE (default), then a non-simplified vine is fitted.

familyset An integer vector of pair-copula families to select from (the independence cop-

ula MUST NOT be specified in this vector unless one wants to fit an independence vine!). The vector has to include at least one pair-copula family that allows for positive and one that allows for negative dependence. Not listed copula families might be included to better handle limit cases. If familyset = NA (default), selection among all possible families is performed. Coding of pair-copula families: 1 Gaussian, 2 Student t, 3 Clayton, 4 Gumbel, 13 Survival Clayton, 14 Survival Gumbel, 23 Rotated (90 degrees) Clayton, 24 Rotated (90 degrees) Gumbel, 33 Rotated (270 degrees) Clayton and 34 Rotated (270 degrees)

grees) Gumbel.

rotations If TRUE, all rotations of the families in familyset are included.

familycrit Character indicating the criterion for bivariate copula selection. Possible choices:

familycrit = 'AIC' (default) or 'BIC', as in BiCopSelect from the VineCopula

package.

level Numerical; Passed to gamBiCopSelect, it is the significance level of the test

for removing individual predictors (default: level = 0.05) for each conditional

pair-copula.

trunclevel Integer; level of truncation.

tau TRUE (default) for a calibration function specified for Kendall's tau or FALSE for

a calibration function specified for the Copula parameter.

method 'NR' for Newton-Raphson and 'FS' for Fisher-scoring (default).

tol.rel Relative tolerance for 'FS'/'NR' algorithm.

n. iters Maximal number of iterations for 'FS'/'NR' algorithm.

gamVineCopSelect 35

parallel	TRUE (default) for parallel selection of copula family at each edge or FALSE for the sequential version. for the Copula parameter.
verbose	TRUE if informations should be printed during the estimation and FALSE (default) for a silent version. from $mgcv$.
select.once	if TRUE the GAM structure is only selected once, for the family that appears first in family set .
	Additional parameters to be passed to gam from mgcv.

Value

gamVineCopSelect returns a gamVine-class object.

See Also

gamVineSeqFit,gamVineStructureSelect,gamVine-class,gamVineSimulateandgamBiCopFit.

```
require(mgcv)
set.seed(0)
## Simulation parameters
# Sample size
n <- 1e3
# Copula families
familyset <- c(1:2, 301:304, 401:404)
# Define a 4-dimensional R-vine tree structure matrix
d < - 4
Matrix <- c(2, 3, 4, 1, 0, 3, 4, 1, 0, 0, 4, 1, 0, 0, 0, 1)
Matrix <- matrix(Matrix, d, d)</pre>
nnames <- paste("X", 1:d, sep = "")</pre>
## A function factory
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b \star (t - Tm)^2)
  },
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a <- b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a <- (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
```

36 gamVineCopSelect

```
)
## Create the model
# Define gam-vine model list
count <- 1
model \leftarrow vector(mode = "list", length = d * (d - 1) / 2)
sel \leftarrow seq(d, d^2 - d, by = d)
# First tree
for (i in 1:(d - 1)) {
  # Select a copula family
  family <- sample(familyset, 1)</pre>
  model[[count]]$family <- family</pre>
  # Use the canonical link and a randomly generated parameter
  if (is.element(family, c(1, 2))) {
    model[[count]]$par <- tanh(rnorm(1) / 2)</pre>
    if (family == 2) {
      model[[count]]$par2 <- 2 + exp(rnorm(1))</pre>
    }
  } else {
    if (is.element(family, c(401:404))) {
      rr <- rnorm(1)</pre>
      model[[count]]par <- sign(rr) * (1 + abs(rr))
    } else {
      model[[count]]$par <- rnorm(1)</pre>
    model[[count]]$par2 <- 0</pre>
  count <- count + 1
}
# A dummy dataset
data <- data.frame(u1 = runif(1e2), u2 = runif(1e2), matrix(runif(1e2 * d), 1e2, d))</pre>
# Trees 2 to (d-1)
for (j in 2:(d - 1)) {
  for (i in 1:(d - j)) {
    # Select a copula family
    family <- sample(familyset, 1)</pre>
    # Select the conditiong set and create a model formula
    cond <- nnames[sort(Matrix[(d - j + 2):d, i])]
    tmpform \leftarrow paste("~", paste(paste("s(", cond, ", k=10, bs='cr')",
      sep = ""
    ), collapse = " + "))
    1 <- length(cond)</pre>
    temp <- sample(3, 1, replace = TRUE)</pre>
    # Spline approximation of the true function
    m <- 1e2
    x \leftarrow matrix(seq(0, 1, length.out = m), nrow = m, ncol = 1)
    if (1 != 1) {
```

gamVineCopSelect 37

```
tmp.fct <- paste("function(x){eta0+",</pre>
        paste(sapply(1:1, function(x)
          paste("calib.surf[[", temp[x], "]](x[", x, "])",
          )), collapse = "+"), "}",
        sep = ""
      tmp.fct <- eval(parse(text = tmp.fct))</pre>
      x <- eval(parse(text = paste0("expand.grid(",</pre>
        paste0(rep("x", 1), collapse = ","), ")",
        collapse = ""
      y \leftarrow apply(x, 1, tmp.fct)
    } else {
      tmp.fct <- function(x) eta0 + calib.surf[[temp]](x)</pre>
      colnames(x) <- cond
      y <- tmp.fct(x)
    }
    # Estimate the gam model
    form <- as.formula(paste0("y", tmpform))</pre>
    dd <- data.frame(y, x)</pre>
    names(dd) <- c("y", cond)</pre>
    b <- gam(form, data = dd)
    # plot(x[,1],(y-fitted(b))/y)
    # Create a dummy gamBiCop object
    tmp <- gamBiCopFit(data = data, formula = form, family = 1, n.iters = 1)$res</pre>
    # Update the copula family and the model coefficients
    attr(tmp, "model")$coefficients <- coefficients(b)</pre>
    attr(tmp, "model")$smooth <- b$smooth</pre>
    attr(tmp, "family") <- family</pre>
    if (family == 2) {
      attr(tmp, "par2") <- 2 + exp(rnorm(1))</pre>
    }
    model[[count]] <- tmp</pre>
    count <- count + 1
}
# Create the gamVineCopula object
GVC <- gamVine(Matrix = Matrix, model = model, names = nnames)
print(GVC)
## Not run:
## Simulate and fit the model
sim <- gamVineSimulate(n, GVC)</pre>
fitGVC <- gamVineSeqFit(sim, GVC, verbose = TRUE)</pre>
fitGVC2 <- gamVineCopSelect(sim, Matrix, verbose = TRUE)</pre>
## Plot the results
par(mfrow = c(3, 4))
plot(GVC, ylim = c(-2.5, 2.5))
```

38 gamVineNormalize

```
plot(fitGVC, ylim = c(-2.5, 2.5))
plot(fitGVC2, ylim = c(-2.5, 2.5))
## End(Not run)
```

gamVineFamily

Family Matrix of an Object of the Class gamVine

Description

Return the matrix of copula family (see gamBiCop) corresponding to the model list in the gamVine object.

Usage

```
gamVineFamily(GVC)
```

Arguments

GVC

An object of the class gamVine.

Value

Matrix of copula families corresponding to the model list in the gamVine object.

See Also

gamVine.

gamVineNormalize

Normalize an Object of the Class gamVine

Description

Change the R-vine matrix in the natural order, i.e. with d:1 on the diagonal

Usage

```
gamVineNormalize(GVC)
```

Arguments

GVC

An object of the class gamVine.

gamVinePDF 39

Value

The normalized gamVine object.

See Also

gamVine.

gamVinePDF

Conditional density function of a gamVine

Description

This function returns the density of a conditional pair-copula constructions, where either the copula parameters or the Kendall's taus are modeled as a function of the covariates.

Usage

```
gamVinePDF(object, data)
```

Arguments

object gamVine-class object.

data (Same as in predict.gam from the mgcv package) A matrix or data frame con-

taining the values of the model covariates at which predictions are required, along with a number of additional columns corresponding to the variables in the

pair copula decomposition.

Value

The conditional density.

See Also

 ${\tt gamVine}. {\tt gamVineCopSelect}, {\tt gamVineStructureSelect}, {\tt gamVine-class}, {\tt gamVineSimulate} \ and {\tt gamBiCopFit}.$

Examples

```
require(mgcv)
set.seed(0)

## Simulation parameters
# Sample size
n <- 1e3
# Copula families
familyset <- c(1:2, 301:304, 401:404)
# Define a 4-dimensional R-vine tree structure matrix
d <- 4</pre>
```

40 gamVinePDF

```
Matrix \leftarrow c(2, 3, 4, 1, 0, 3, 4, 1, 0, 0, 4, 1, 0, 0, 0, 1)
Matrix <- matrix(Matrix, d, d)</pre>
nnames <- paste("X", 1:d, sep = "")</pre>
## A function factory
eta0 <- 1
calib.surf <- list(</pre>
  calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b * (t - Tm)^2)
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a <- b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  },
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## Create the model
# Define gam-vine model list
count <- 1
model \leftarrow vector(mode = "list", length = d * (d - 1) / 2)
sel \leftarrow seq(d, d^2 - d, by = d)
# First tree
for (i in 1:(d - 1)) {
  # Select a copula family
  family <- sample(familyset, 1)</pre>
  model[[count]]$family <- family</pre>
  # Use the canonical link and a randomly generated parameter
  if (is.element(family, c(1, 2))) {
    model[[count]]$par <- tanh(rnorm(1) / 2)</pre>
    if (family == 2) {
      model[[count]]$par2 <- 2 + exp(rnorm(1))</pre>
    }
  } else {
    if (is.element(family, c(401:404))) {
      rr <- rnorm(1)</pre>
      model[[count]]$par <- sign(rr) * (1 + abs(rr))</pre>
    } else {
      model[[count]]$par <- rnorm(1)</pre>
    model[[count]]$par2 <- 0</pre>
  count <- count + 1
```

gamVinePDF 41

```
}
# A dummy dataset
data <- data.frame(u1 = runif(1e2), u2 = runif(1e2), matrix(runif(1e2 * d), 1e2, d))</pre>
# Trees 2 to (d-1)
for (j in 2:(d - 1)) {
  for (i in 1:(d - j)) {
    # Select a copula family
    family <- sample(familyset, 1)</pre>
    # Select the conditiong set and create a model formula
    cond <- nnames[sort(Matrix[(d - j + 2):d, i])]</pre>
    tmpform <- paste("~", paste(paste("s(", cond, ", k=10, bs='cr')",</pre>
      sep = ""
    ), collapse = " + "))
    1 <- length(cond)</pre>
    temp <- sample(3, 1, replace = TRUE)</pre>
    # Spline approximation of the true function
    m <- 1e2
    x \leftarrow matrix(seq(0, 1, length.out = m), nrow = m, ncol = 1)
    if (1 != 1) {
      tmp.fct <- paste("function(x){eta0+",</pre>
        paste(sapply(1:1, function(x)
          paste("calib.surf[[", temp[x], "]](x[", x, "])",
          )), collapse = "+"), "}",
        sep = ""
      tmp.fct <- eval(parse(text = tmp.fct))</pre>
      x <- eval(parse(text = paste0("expand.grid(",</pre>
        paste0(rep("x", 1), collapse = ","), ")",
        collapse = ""
      )))
      y \leftarrow apply(x, 1, tmp.fct)
    } else {
      tmp.fct <- function(x) eta0 + calib.surf[[temp]](x)</pre>
      colnames(x) <- cond
      y <- tmp.fct(x)
    # Estimate the gam model
    form <- as.formula(paste0("y", tmpform))</pre>
    dd <- data.frame(y, x)</pre>
    names(dd) \leftarrow c("y", cond)
    b <- gam(form, data = dd)
    # plot(x[,1],(y-fitted(b))/y)
    # Create a dummy gamBiCop object
    tmp <- gamBiCopFit(data = data, formula = form, family = 1, n.iters = 1)$res</pre>
    # Update the copula family and the model coefficients
```

```
attr(tmp, "model")$coefficients <- coefficients(b)</pre>
    \verb|attr(tmp, "model")$smooth <- b$smooth|\\
    attr(tmp, "family") <- family</pre>
    if (family == 2) {
      attr(tmp, "par2") <- 2 + exp(rnorm(1))</pre>
    model[[count]] <- tmp</pre>
    count <- count + 1
  }
}
# Create the gamVineCopula object
GVC <- gamVine(Matrix = Matrix, model = model, names = nnames)
print(GVC)
## Not run:
## Simulate and fit the model
sim <- gamVineSimulate(n, GVC)</pre>
fitGVC <- gamVineSeqFit(sim, GVC, verbose = TRUE)</pre>
fitGVC2 <- gamVineCopSelect(sim, Matrix, verbose = TRUE)</pre>
(gamVinePDF(GVC, sim[1:10, ]))
## Plot the results
dev.off()
par(mfrow = c(3, 4))
plot(GVC, ylim = c(-2.5, 2.5))
plot(fitGVC, ylim = c(-2.5, 2.5))
plot(fitGVC2, ylim = c(-2.5, 2.5))
## End(Not run)
```

gamVineSeqFit

Sequential maximum penalized likelihood estimation of a GAM-Vine model.

Description

This function estimates the parameter(s) of a Generalized Additive model (GAM) Vine model, where GAMs for individual edges are specified either for the copula parameter or Kendall's tau. It solves the maximum penalized likelihood estimation for the copula families supported in this package by reformulating each Newton-Raphson iteration as a generalized ridge regression, which is solved using the mgcv package.

Usage

```
gamVineSeqFit(
  data,
  GVC,
```

```
covariates = NA,
 method = "FS",
 tol.rel = 0.001,
 n.iters = 10,
  verbose = FALSE,
)
```

Arguments

A matrix or data frame containing the data in [0,1]^d. data GVC A gamVine object. Vector of names for the covariates. covariates 'NR' for Newton-Raphson and 'FS' for Fisher-scoring (default). method Relative tolerance for 'FS'/'NR' algorithm. tol.rel n.iters Maximal number of iterations for 'FS'/'NR' algorithm. verbose TRUE if informations should be printed during the estimation and FALSE (default) for a silent version. Additional parameters to be passed to gam from mgcv.

Value

. . .

gamVineSeqFit returns a gamVine object.

See Also

```
gamVineCopSelect and gamVineStructureSelect
gamVineCopSelect,gamVineStructureSelect,gamVine-class,gamVineSimulateandgamBiCopFit.
```

Examples

```
require(mgcv)
set.seed(0)
## Simulation parameters
# Sample size
n <- 1e3
# Copula families
familyset <- c(1:2, 301:304, 401:404)
# Define a 4-dimensional R-vine tree structure matrix
d < - 4
Matrix \leftarrow c(2, 3, 4, 1, 0, 3, 4, 1, 0, 0, 4, 1, 0, 0, 0, 1)
Matrix <- matrix(Matrix, d, d)</pre>
nnames <- paste("X", 1:d, sep = "")</pre>
## A function factory
eta0 <- 1
calib.surf <- list(</pre>
```

```
calib.quad <- function(t, Ti = 0, Tf = 1, b = 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow -(b / 3) * (Tf^2 - 3 * Tf * Tm + 3 * Tm^2)
    return(a + b \star (t - Tm)^2)
  },
  calib.sin <- function(t, Ti = 0, Tf = 1, b = 1, f = 1) {
    a <- b * (1 - 2 * Tf * pi / (f * Tf * pi +
      cos(2 * f * pi * (Tf - Ti))
      - cos(2 * f * pi * Ti)))
    return((a + b) / 2 + (b - a) * sin(2 * f * pi * (t - Ti)) / 2)
  calib.exp <- function(t, Ti = 0, Tf = 1, b = 2, s = Tf / 8) {
    Tm <- (Tf - Ti) / 2
    a \leftarrow (b * s * sqrt(2 * pi) / Tf) * (pnorm(0, Tm, s) - pnorm(Tf, Tm, s))
    return(a + b * exp(-(t - Tm)^2 / (2 * s^2)))
  }
)
## Create the model
# Define gam-vine model list
count <- 1
model \leftarrow vector(mode = "list", length = d * (d - 1) / 2)
sel \leftarrow seq(d, d^2 - d, by = d)
# First tree
for (i in 1:(d - 1)) {
  # Select a copula family
  family <- sample(familyset, 1)</pre>
  model[[count]]$family <- family</pre>
  # Use the canonical link and a randomly generated parameter
  if (is.element(family, c(1, 2))) {
    model[[count]]$par <- tanh(rnorm(1) / 2)</pre>
    if (family == 2) {
      model[[count]]$par2 <- 2 + exp(rnorm(1))</pre>
    }
  } else {
    if (is.element(family, c(401:404))) {
      rr <- rnorm(1)</pre>
      model[[count]]par <- sign(rr) * (1 + abs(rr))
    } else {
      model[[count]]$par <- rnorm(1)</pre>
    model[[count]]$par2 <- 0</pre>
  }
  count <- count + 1
}
# A dummy dataset
data <- data.frame(u1 = runif(1e2), u2 = runif(1e2), matrix(runif(1e2 * d), 1e2, d))</pre>
# Trees 2 to (d-1)
for (j in 2:(d - 1)) {
```

```
for (i in 1:(d - j)) {
  # Select a copula family
  family <- sample(familyset, 1)</pre>
  # Select the conditiong set and create a model formula
  cond <- nnames[sort(Matrix[(d - j + 2):d, i])]</pre>
  tmpform <- paste("~", paste(paste("s(", cond, ", k=10, bs='cr')",</pre>
    sep = ""
  ), collapse = " + "))
  1 <- length(cond)</pre>
  temp <- sample(3, 1, replace = TRUE)</pre>
  # Spline approximation of the true function
  m <- 1e2
  x \leftarrow matrix(seq(0, 1, length.out = m), nrow = m, ncol = 1)
  if (1 != 1) {
    tmp.fct <- paste("function(x){eta0+",</pre>
      paste(sapply(1:1, function(x)
         paste("calib.surf[[", temp[x], "]](x[", x, "])",
          sep = ""
        )), collapse = "+"), "}",
      sep = ""
    )
    tmp.fct <- eval(parse(text = tmp.fct))</pre>
    x <- eval(parse(text = paste0("expand.grid(",</pre>
      paste0(rep("x", 1), collapse = ","), ")",
      collapse = ""
    )))
    y \leftarrow apply(x, 1, tmp.fct)
  } else {
    tmp.fct <- function(x) eta0 + calib.surf[[temp]](x)</pre>
    colnames(x) <- cond
    y <- tmp.fct(x)
  }
  # Estimate the gam model
  form <- as.formula(paste0("y", tmpform))</pre>
  dd <- data.frame(y, x)</pre>
  names(dd) \leftarrow c("y", cond)
  b <- gam(form, data = dd)
  # plot(x[,1],(y-fitted(b))/y)
  # Create a dummy gamBiCop object
  tmp <- gamBiCopFit(data = data, formula = form, family = 1, n.iters = 1)$res</pre>
  # Update the copula family and the model coefficients
  attr(tmp, "model")$coefficients <- coefficients(b)</pre>
  attr(tmp, "model")$smooth <- b$smooth</pre>
  attr(tmp, "family") <- family</pre>
  if (family == 2) {
    attr(tmp, "par2") <- 2 + exp(rnorm(1))</pre>
  model[[count]] <- tmp</pre>
```

46 gamVineSimulate

```
count <- count + 1
  }
}
# Create the gamVineCopula object
GVC <- gamVine(Matrix = Matrix, model = model, names = nnames)</pre>
print(GVC)
## Not run:
## Simulate and fit the model
sim <- gamVineSimulate(n, GVC)</pre>
fitGVC <- gamVineSeqFit(sim, GVC, verbose = TRUE)</pre>
fitGVC2 <- gamVineCopSelect(sim, Matrix, verbose = TRUE)</pre>
(gamVinePDF(GVC, sim[1:10, ]))
## Plot the results
dev.off()
par(mfrow = c(3, 4))
plot(GVC, ylim = c(-2.5, 2.5))
plot(fitGVC, ylim = c(-2.5, 2.5))
plot(fitGVC2, ylim = c(-2.5, 2.5))
## End(Not run)
```

gamVineSimulate

Simulation from a gamVine-class object

Description

Simulation from a gamVine-class object

Usage

```
gamVineSimulate(n, GVC, U = NULL, newdata = NULL)
```

Arguments

n number of d-dimensional observations to simulate.

GVC A gamVine object.

U If not NULL, U is an (N,d)-matrix of U[0,1] random variates to be transformed to

the copula sample.

newdata If not NULL, which is mandatory when the attribute covariates from GVC is not

NA, newdata is a data frame containing the values of the model covariates at

which simulations are required.

gamVineSimulate 47

Value

A matrix of data simulated from the given gamVine object.

Examples

```
require(VineCopula)
## Example adapted from RVineSim
## Define 5-dimensional R-vine tree structure matrix
Matrix <- c(
  5, 2, 3, 1, 4,
  0, 2, 3, 4, 1,
  0, 0, 3, 4, 1,
  0, 0, 0, 4, 1,
  0, 0, 0, 0, 1
Matrix <- matrix(Matrix, 5, 5)</pre>
## Define R-vine pair-copula family matrix
family <- c(
  0, 1, 3, 4, 4,
  0, 0, 3, 4, 1,
  0, 0, 0, 4, 1,
  0, 0, 0, 0, 3,
  0, 0, 0, 0, 0
family <- matrix(family, 5, 5)</pre>
## Define R-vine pair-copula parameter matrix
par <- c(
  0, 0.2, 0.9, 1.5, 3.9,
  0, 0, 1.1, 1.6, 0.9,
  0, 0, 0, 1.9, 0.5,
  0, 0, 0, 0, 4.8,
  0, 0, 0, 0, 0
par <- matrix(par, 5, 5)</pre>
## Define second R-vine pair-copula parameter matrix
par2 <- matrix(0, 5, 5)</pre>
## Define RVineMatrix object
RVM <- RVineMatrix(</pre>
  Matrix = Matrix, family = family,
  par = par, par2 = par2,
  names = c("V1", "V2", "V3", "V4", "V5")
)
## Convert to gamVine object
GVC <- RVM2GVC(RVM)
```

```
## U[0,1] random variates to be transformed to the copula sample
n <- 1e2
d <- 5
U <- matrix(runif(n * d), nrow = n)

## The output of gamVineSimulate correspond to that of RVineSim
sampleRVM <- RVineSim(n, RVM, U)
sampleGVC <- gamVineSimulate(n, GVC, U)
all.equal(sampleRVM, sampleGVC)

## Fit the two models and compare the estimated parameter
fitRVM <- RVM2GVC(RVineSeqEst(sampleRVM, RVM))
fitGVC <- gamVineSeqFit(sampleGVC, GVC)
all.equal(
    simplify2array(attr(fitRVM, "model")),
    simplify2array(attr(fitGVC, "model"))
)</pre>
```

gamVineStructureSelect

Structure selection and estimation of a GAM-Vine model.

Description

This function select the structure and estimates the parameter(s) of a Generalized Additive model (GAM) Vine model, where GAMs for individual edges are specified either for the copula parameter or Kendall's tau. It solves the maximum penalized likelihood estimation for the copula families supported in this package by reformulating each Newton-Raphson iteration as a generalized ridge regression, which is solved using the mgcv package.

Usage

```
gamVineStructureSelect(
  udata.
 lin.covs = NULL,
  smooth.covs = NULL,
  simplified = TRUE,
  type = 0,
  familyset = NA,
  rotations = TRUE,
  familycrit = "AIC",
  treecrit = "tau",
  level = 0.05,
  trunclevel = NA,
  tau = TRUE,
 method = "FS",
  tol.rel = 0.001,
  n.iters = 10,
```

```
parallel = FALSE,
  verbose = FALSE,
  select.once = TRUE
)
```

Arguments

udata A matrix or data frame containing the data in [0,1]^d.

lin.covs A matrix or data frame containing the parametric (i.e., linear) covariates (default:

lin.covs = NULL).

smooth.covs A matrix or data frame containing the non-parametric (i.e., smooth) covariates

(default: smooth.covs = NULL).

simplified If TRUE (default), then a simplified vine is fitted (which is possible only if there

are exogenous covariates). If FALSE, then a non-simplified vine is fitted.

type = 0 (default) for a R-Vine and type = 1 for a C-Vine.

familyset An integer vector of pair-copula families to select from (the independence cop-

ula MUST NOT be specified in this vector unless one wants to fit an independence vine!). Not listed copula families might be included to better handle limit cases. If familyset = NA (default), selection among all possible families is performed. Coding of pair-copula families: 1 Gaussian, 2 Student t, 5 Frank, 301 Double Clayton type I (standard and rotated 90 degrees), 302 Double Clayton type II (standard and rotated 270 degrees), 303 Double Clayton type III (survival and rotated 90 degrees), 304 Double Clayton type IV (survival and rotated 270 degrees), 401 Double Gumbel type I (standard and rotated 90 degrees), 402 Double Gumbel type II (standard and rotated 270 degrees), 403 Double Gumbel type III (survival and rotated 90 degrees), 404 Double Gumbel type IV (survival

and rotated 270 degrees).

rotations If TRUE, all rotations of the families in familyset are included.

familycrit Character indicating the criterion for bivariate copula selection. Possible choices:

familycrit = 'AIC' (default) or 'BIC', as in BiCopSelect from the VineCopula

package.

treecrit Character indicating how pairs are selected in each tree. treecrit = "tau" uses

the maximum spanning tree of the Kendall's tau (i.e., the tree of maximal overall

dependence), treecrit = "rho" uses the Spearman's rho.

level Numerical; Passed to gamBiCopSelect, it is the significance level of the test

for removing individual predictors (default: level = 0.05) for each conditional

pair-copula.

trunclevel Integer; level of truncation.

tau TRUE (default) for a calibration function specified for Kendall's tau or FALSE for

a calibration function specified for the Copula parameter.

method 'NR' for Newton-Raphson and 'FS' for Fisher-scoring (default).

tol.rel Relative tolerance for 'FS'/'NR' algorithm.

n.iters Maximal number of iterations for 'FS'/'NR' algorithm.

parallel TRUE (default) for parallel selection of copula family at each edge or FALSE for

the sequential version. for the Copula parameter.

verbose TRUE if informations should be printed during the estimation and FALSE (default)

for a silent version. from mgcv.

select.once if TRUE the GAM structure is only selected once, for the family that appears first

in familyset.

Value

gamVineSeqFit returns a gamVine-class object.

See Also

gamVineSeqFit,gamVineCopSelect, gamVine-class, gamVineSimulate and gamBiCopSelect.

Examples

```
require(VineCopula)
set.seed(0)
## An example with a 3-dimensional GAM-Vine
# Sample size
n <- 1e3
# Define a R-vine tree structure matrix
d <- 3
Matrix \leftarrow c(2, 3, 1, 0, 3, 1, 0, 0, 1)
Matrix <- matrix(Matrix, d, d)</pre>
nnames <- paste("x", 1:d, sep = "")</pre>
# Copula families for each edge
fam <- c(301, 401, 1)
# Parameters for the first tree (two unconditional copulas)
par <- c(1, 2)
# Pre-allocate the GAM-Vine model list
model \leftarrow vector(mode = "list", length = d * (d - 1) / 2)
# The first tree contains only the two unconditional copulas
for (i in 1:(d - 1)) {
  model[[count]] \leftarrow list(family = fam[count], par = par[count], par2 = 0)
  count <- count + 1
}
# The second tree contains a unique conditional copula
# In this first example, we take a linear calibration function (10*x-5)
# Set-up a dummy dataset
tmp \leftarrow data.frame(u1 = runif(1e2), u2 = runif(1e2), x1 = runif(1e2))
```

logLik.gamBiCop 51

```
# Set-up an arbitrary linear model for the calibration function
model[[count]] <- gamBiCopFit(tmp, ~x1, fam[count])$res</pre>
# Update the coefficients of the model
attr(model[[count]], "model")$coefficients <- c(-5, 10)</pre>
# Define gamVine object
GVC <- gamVine(Matrix = Matrix, model = model, names = nnames)</pre>
GVC
## Not run:
# Simulate new data
simData <- data.frame(gamVineSimulate(n, GVC))</pre>
colnames(simData) <- nnames</pre>
# Fit data using sequential estimation assuming true model known
summary(fitGVC <- gamVineSeqFit(simData, GVC))</pre>
# Fit data using structure selection and sequential estimation
summary(fitGVC2 <- gamVineStructureSelect(simData, simplified = FALSE))</pre>
## End(Not run)
```

logLik.gamBiCop

Extract the Log-likelihood from a gamBiCop Object

Description

Function to extract the log-likelihood from an object of the class gamBiCop (note that the models are usually fitted by penalized likelihood maximization). This function is used by AIC and BIC.

Usage

```
## S4 method for signature 'gamBiCop'
logLik(object, ...)
```

Arguments

```
object An object of the class gamBiCop.
... un-used in this class
```

Value

```
Standard logLik object: see logLik.
```

See Also

```
AIC and BIC.
```

52 plot.gamBiCop

nobs.gamBiCop

Extract the Number of Observations from gamBiCop Object

Description

Extract the number of 'observations' from a model fit. This is principally intended to be used in computing the BIC (see AIC).

Usage

```
## S4 method for signature 'gamBiCop'
nobs(object, ...)
```

Arguments

object An object of the class gamBiCop.
... un-used in this class

Value

A single number, normally an integer.

See Also

AIC and BIC.

plot.gamBiCop

Plot a gamBiCop Object

Description

Plot from an object of the class gamBiCop. The function is based on (see plot.gam from mgcv).

Usage

```
## S4 method for signature 'gamBiCop,ANY'
plot(x, y, ...)
```

Arguments

x An object of the class gamBiCop.

y Not used with this class.

... additional arguments to be passed to plot.gam.

plot.gamVine 53

Value

This function simply generates plots.

See Also

```
plot.gam from mgcv).
```

plot.gamVine

Plot an Object of the Class gamVine

Description

Plot an object of the class gamVine. The function is based on (see plot.gam from mgcv).

Usage

```
## S4 method for signature 'gamVine,ANY'
plot(x, y, ...)
```

Arguments

x An object of the class gamVine.

y Not used with this class.

... additional arguments to be passed to plot.gam.

Value

This function simply generates plots.

See Also

```
plot.gam from mgcv).
```

RVM2GVC

Transform an Object of the Class R-Vine into an Object of the Class gamVine

Description

Transform an object of the class RVineMatrix into an object of the class gamVine.

Usage

RVM2GVC(RVM)

54 summary.gamBiCop

Arguments

RVM

An object of the class RVineMatrix.

Value

An object of the class gamVine.

See Also

RVineMatrix and gamVine.

 $\verb"summary.gamBiCop"$

Summary for a gamBiCop Object

Description

Takes a gamBiCop object and produces various useful summaries from it.

Usage

```
## S4 method for signature 'gamBiCop'
summary(object, ...)
```

Arguments

object An object of the class gamBiCop.

... unused in this class

Value

A useful summary (see summary.gam from mgcv for more details).

See Also

```
summary.gam from mgcv
```

summary.gamVine 55

summary.gamVine

Summary for an Object of the Class gamVine

Description

Takes an object of the class gamVine and produces various useful summaries from it.

Usage

```
## S4 method for signature 'gamVine'
summary(object, ...)
```

Arguments

object An object of the class gamVine.
... unused in this class

Value

A useful summary (see summary.gam from mgcv for more details).

See Also

```
summary.gam from mgcv
```

Index

```
gamCopula.package (gamCopula-package), 2
* package
    gamCopula-package, 2
                                                   gamObject, 15-17
                                                   gamVine, 14, 32, 32, 33, 38, 39, 43, 46, 47,
AIC, 9, 51, 52
                                                            53-55
AIC, gamBiCop-method (AIC. gamBiCop), 8
                                                   gamVine-class, 33, 46
AIC.gamBiCop, 8
                                                   gamVineCopSelect, 32, 33, 33, 39, 43, 50
                                                   gamVineFamily, 38
BIC, 9, 51, 52
                                                   gamVineNormalize, 38
BIC, gamBiCop-method (BIC. gamBiCop), 9
                                                   gamVinePDF, 39
BIC.gamBiCop, 9
                                                   gamVineSeqFit, 32, 33, 35, 42, 50
BiCopEta2Par, 10, 10, 11
                                                   gamVineSimulate, 32, 33, 35, 39, 43, 46, 50
BiCopPar2Eta, 11
                                                   gamVineStructureSelect, 32, 33, 35, 39, 43,
BiCopPar2Tau, 10, 11
                                                            48
BiCopSelect, 28, 34, 49
BiCopTau2Par, 10, 11
                                                   logLik, 51
                                                   logLik,gamBiCop-method
condBiCopSim, 12
                                                            (logLik.gamBiCop), 51
                                                   logLik.gamBiCop, 51
dim, gamVine-method (dim.gamVine), 14
dim.gamVine, 14
                                                   mgcv, 3, 15–17, 19, 20, 23, 25–27, 30, 33, 35,
                                                            39, 42, 43, 48, 50, 52–55
EDF. 15
                                                   nobs, gamBiCop-method (nobs.gamBiCop), 52
formula,gamBiCop-method
                                                   nobs.gamBiCop, 52
         (formula.gamBiCop), 15
formula.gam, 15, 19
                                                   plot,gamBiCop,ANY-method
formula.gamBiCop, 15
                                                            (plot.gamBiCop), 52
                                                   plot,gamVine,ANY-method(plot.gamVine),
gam, 15–17, 19, 20, 28, 35, 43
gam.models, 19
                                                   plot.gam, 52, 53
gamBiCop, 8, 9, 15, 16, 16, 17, 20, 23, 26, 29,
                                                   plot.gamBiCop, 52
         32, 33, 38, 51, 52, 54
                                                   plot.gamVine, 53
gamBiCop-class, 16, 30
                                                   predict.gam, 17, 23, 25, 26, 30, 39
gamBiCopCDF, 17
gamBiCopFit, 12, 16, 17, 19, 26, 29, 35, 39, 43
                                                   RVineMatrix, 32, 33, 53, 54
gamBiCopPDF, 23
                                                   RVM2GVC, 53
gamBiCopPredict, 16, 17, 23, 25
gamBiCopSelect, 27, 34, 49, 50
                                                   summary, gamBiCop-method
gamBiCopSimulate, 12, 16, 17, 20, 30
                                                            (summary.gamBiCop), 54
gamCopula (gamCopula-package), 2
                                                   summary,gamVine-method
gamCopula-package, 2
                                                            (summary.gamVine), 55
```

INDEX 57

```
summary.gam, 54, 55
summary.gamBiCop, 54
summary.gamVine, 55
VineCopula, 3, 10, 11, 28, 34, 49
```