# Package 'MultiRobust'

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Define a Generalized Linear Model

## **Description**

Define a generalized linear model. All the arguments in glm are allowed except for data. Supported types of family include gaussian, binomial, poisson, Gamma and inverse.gaussian.

## Usage

```
glm.work(formula, family = gaussian, weights = NULL, ...)
```

## **Arguments**

formula	The formula of the model to be fitted.
family	The distribution of the response variable and the link function to be used in the model.
weights	The prior weights to be used in the model.
	Addition arguments for the function glm.

## See Also

glm.

## **Examples**

```
# A logistic regression with response R and covariates X1 and X2 mis1 \leftarrow glm.work(formula = R \sim X1 + X2, family = binomial(link = logit))
```

MR.mean

Multiply Robust Estimation of the Marginal Mean

## **Description**

MR.mean() is used to estimate the marginal mean of a variable which is subject to missingness. Multiple missingness probability models and outcome regression models can be accommodated.

## Usage

```
MR.mean(response, reg.model = NULL, mis.model = NULL, moment = NULL,
  order = 1, data, bootstrap = FALSE, bootstrap.size = 300,
  alpha = 0.05)
```

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## **Arguments**

response	The response variable of interest whose marginal mean is to be estimated.
reg.model	A list of outcome regression models defined by glm.work.
mis.model	A list of missingness probability models defined by ${\tt glm.work}$ . The dependent variable is always specified as R.
moment	A vector of auxiliary variables whose moments are to be calibrated.
order	A numeric value. The order of moments up to which to be calibrated.
data	A data frame with missing data encoded as NA.
bootstrap	Logical. If bootstrap = TRUE, the bootstrap will be applied to calculate the standard error and construct a Wald confidence interval.
bootstrap.size	A numeric value. Number of bootstrap resamples generated if bootstrap = $\ensuremath{TRUE}.$
alpha	Significance level used to construct the $100(1\mbox{ - alpha})\%$ Wald confidence interval.

#### Value

mu	The estimated value of the marginal mean.
SE	The bootstrap standard error of mu when bootstrap = TRUE.
CI	A Wald-type confidence interval based on ${\tt mu}$ and SE when ${\tt bootstrap} = {\tt TRUE}.$
weights	The calibration weights if any reg. model, mis. model or moment is specified.

## References

Han, P. and Wang, L. (2013). Estimation with missing data: beyond double robustness. *Biometrika*, **100**(2), 417–430.

Han, P. (2014). A further study of the multiply robust estimator in missing data analysis. *Journal of Statistical Planning and Inference*, **148**, 101–110.

## **Examples**

```
# Simulated data set
set.seed(123)
n <- 400
gamma0 <- c(1, 2, 3)
alpha0 <- c(-0.8, -0.5, 0.3)
X <- runif(n, min = -2.5, max = 2.5)
p.mis <- 1 / (1 + exp(alpha0[1] + alpha0[2] * X + alpha0[3] * X ^ 2))
R <- rbinom(n, size = 1, prob = 1 - p.mis)
a.x <- gamma0[1] + gamma0[2] * X + gamma0[3] * exp(X)
Y <- rnorm(n, a.x, sd = sqrt(4 * X ^ 2 + 2))
dat <- data.frame(X, Y)
dat[R == 0, 2] <- NA</pre>
# Define the outcome regression models and missingness probability models
reg1 <- glm.work(formula = Y ~ X + exp(X), family = gaussian)
```

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```
\label{eq:continuous_section} $\operatorname{reg2} < -\operatorname{glm.work}(\operatorname{formula} = Y \sim X + \operatorname{I}(X \wedge 2), \; \operatorname{family} = \operatorname{gaussian})$$ $\operatorname{mis1} < -\operatorname{glm.work}(\operatorname{formula} = R \sim X + \operatorname{I}(X \wedge 2), \; \operatorname{family} = \operatorname{binomial}(\operatorname{link} = \operatorname{logit}))$$ $\operatorname{mis2} < -\operatorname{glm.work}(\operatorname{formula} = R \sim X + \operatorname{exp}(X), \; \operatorname{family} = \operatorname{binomial}(\operatorname{link} = \operatorname{cloglog}))$$ $\operatorname{MR.mean}(\operatorname{response} = Y, \; \operatorname{reg.model} = \operatorname{list}(\operatorname{reg1}, \; \operatorname{reg2}), \\ & \operatorname{mis.model} = \operatorname{list}(\operatorname{mis1}, \; \operatorname{mis2}), \; \operatorname{data} = \operatorname{dat})$$ $\operatorname{MR.mean}(\operatorname{response} = Y, \; \operatorname{moment} = \operatorname{c}(X), \; \operatorname{order} = 2, \; \operatorname{data} = \operatorname{dat})$$
```

MR.quantile

Multiply Robust Estimation of the Marginal Quantile

## **Description**

MR.quantile() is used to estimate the marginal quantile of a variable which is subject to missingness. Multiple missingness probability models and imputation models are allowed.

## Usage

```
MR.quantile(response, tau = 0.5, imp.model = NULL, mis.model = NULL,
moment = NULL, order = 1, L = 30, data, bootstrap = FALSE,
bootstrap.size = 300, alpha = 0.05)
```

## **Arguments**

response The response variable of interest whose marginal quantile	is to be estimated.
tau A numeric value in $(0,1)$ . The quantile to be estimated.	
imp.model A list of imputation models defined by glm.work.	
mis.model A list of missingness probability models defined by glm. variable is always specified as R.	work. The dependent
moment A vector of auxiliary variables whose moments are to be or	alibrated.
order A numeric value. The order of moments up to which to be	calibrated.
L Number of imputations.	
data A data frame with missing data encoded as NA.	
bootstrap Logical. If bootstrap = TRUE, the bootstrap will be ap standard error and construct a Wald confidence interval.	blied to calculate the
bootstrap.size A numeric value. Number of bootstrap resamples gene $TRUE.$	rated if bootstrap =
alpha Significance level used to construct the 100(1 - alpha)% Val.	Vald confidence inter-

## Value

q	The estimated value of the marginal quantile.
SE	The bootstrap standard error of q when bootstrap = TRUE.
CI	A Wald-type confidence interval based on q and SE when bootstrap = TRUE.
weights	The calibration weights if any imp.model. mis.model or moment is specified.

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#### References

Han, P., Kong, L., Zhao, J. and Zhou, X. (2019). A general framework for quantile estimation with incomplete data. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*. **81**(2), 305–333.

## **Examples**

```
# Simulated data set
set.seed(123)
n <- 400
gamma0 <- c(1, 2, 3)
alpha0 <- c(-0.8, -0.5, 0.3)
X \leftarrow runif(n, min = -2.5, max = 2.5)
p.mis <-1 / (1 + exp(alpha0[1] + alpha0[2] * X + alpha0[3] * X ^ 2))
R \leftarrow rbinom(n, size = 1, prob = 1 - p.mis)
a.x \leftarrow gamma0[1] + gamma0[2] * X + gamma0[3] * exp(X)
Y \leftarrow rnorm(n, a.x, sd = sqrt(4 * X ^ 2 + 2))
dat <- data.frame(X, Y)</pre>
dat[R == 0, 2] \leftarrow NA
# Define the outcome regression models and missingness probability models
imp1 \leftarrow glm.work(formula = Y \sim X + exp(X), family = gaussian)
imp2 \leftarrow glm.work(formula = Y \sim X + I(X ^ 2), family = gaussian)
mis1 <- glm.work(formula = R \sim X + I(X \wedge 2), family = binomial(link = logit))
mis2 \leftarrow glm.work(formula = R \sim X + exp(X), family = binomial(link = cloglog))
MR.quantile(response = Y, tau = 0.25, imp.model = list(imp1, imp2),
             mis.model = list(mis1, mis2), L = 10, data = dat)
MR.quantile(response = Y, tau = 0.25, moment = c(X), order = 2, data = dat)
```

MR.quantreg

Multiply Robust Estimation for Quantile Regression

## Description

MR.quantreg() is used for quantile regression with missing responses and/or missing covariates. Multiple missingness probability models and imputation models are allowed.

#### Usage

```
MR.quantreg(formula, tau = 0.5, imp.model = NULL, mis.model = NULL,
moment = NULL, order = 1, L = 30, data, bootstrap = FALSE,
bootstrap.size = 300, alpha = 0.05, ...)
```

#### **Arguments**

formula The formula of the linear quantile regression model of interest.

tau A numeric value in (0,1). The quantile to be estimated.

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imp.model A list of possibly multiple lists of the form list(list.1, list.2, ..., list.K),

where K is the total number of different imputation models. For the *k*-th imputation model, list.k is a list of possibly multiple models, each of which is defined by glm.work and imputes one single missing variable marginally. See details.

mis.model A list of missingness probability models defined by glm.work. The dependent

variable is always specified as R.

moment A vector of auxiliary variables whose moments are to be calibrated.

Order A numeric value. The order of moments up to which to be calibrated.

L Number of imputations.

data A data frame with missing data encoded as NA.

bootstrap Logical. If bootstrap = TRUE, the bootstrap will be applied to calculate the

standard error and construct a Wald confidence interval.

bootstrap.size A numeric value. Number of bootstrap resamples generated if bootstrap =

TRUE.

alpha Significance level used to construct the 100(1 - alpha)% Wald confidence inter-

val.

... Addition arguments for the function rq.

#### **Details**

The function MR.quantreg() currently deals with data with one missingness pattern. When multiple variables are subject to missingness, their values are missing simultaneously. The method in Han et al. (2019) specifies an imputation model by modeling the joint distribution of the missing variables conditional on the fully observed variables. In contrast, the function MR.quantreg() specifies an imputation model by separately modeling the marginal distribution of each missing variable conditional on the fully observed variables. These marginal distribution models for different missing variables constitute one joint imputation model. Different imputation models do not need to model the marginal distribution of each missing variable differently.

## Value

coefficients The estimated quantile regression coefficients.

SE The bootstrap standard error of coefficients when bootstrap = TRUE.

CI A Wald-type confidence interval based on coefficients and SE when bootstrap

= TRUE.

## References

Han, P., Kong, L., Zhao, J. and Zhou, X. (2019). A general framework for quantile estimation with incomplete data. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*. **81**(2), 305–333.

## See Also

rq.

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## **Examples**

```
# Simulated data set
set.seed(123)
n <- 400
gamma0 <- c(1, 2, 3)
alpha0 <- c(-0.8, -0.5, 0.3)
S \leftarrow runif(n, min = -2.5, max = 2.5) \# auxiliary variables
X1 \leftarrow rbinom(n, size = 1, prob = 0.5) \# covariate X1
X2 <- rexp(n) # covariate X2
p.obs <-1/(1 + \exp(alpha0[1] + alpha0[2] * S + alpha0[3] * S ^ 2)) # non-missingness probability
R <- rbinom(n, size = 1, prob = p.obs)</pre>
a.x <- gamma0[1] + gamma0[2] * X1 + gamma0[3] * X2
Y \leftarrow rnorm(n, a.x)
dat <- data.frame(S, X1, X2, Y)</pre>
dat[R == 0, c(2, 4)] \leftarrow NA \# X1 and Y may be missing
# marginal imputation models for X1
impX1.1 <- glm.work(formula = X1 ~ S, family = binomial(link = logit))</pre>
impX1.2 <- glm.work(formula = X1 ~ S + X2, family = binomial(link = cloglog))</pre>
# marginal imputation models for Y
impY.1 <- glm.work(formula = Y ~ S, family = gaussian)</pre>
impY.2 <- glm.work(formula = Y ~ S + X2, family = gaussian)</pre>
# missingness probability models
mis1 <- glm.work(formula = R ~ S + I(S ^ 2), family = binomial(link = logit))
mis2 <- glm.work(formula = R ~ I(S ^ 2), family = binomial(link = cloglog))
\# this example considers the following K = 3 imputation models for imputing the missing (X1, Y)
imp1 <- list(impX1.1, impY.1)</pre>
imp2 <- list(impX1.1, impY.2)</pre>
imp3 <- list(impX1.2, impY.1)</pre>
results <- MR.quantreg(formula = Y ~ X1 + X2, tau = 0.75, imp.model = list(imp1, imp2, imp3),
                         mis.model = list(mis1, mis2), L = 10, data = dat)
results$coefficients
MR.quantreg(formula = Y \sim X1 + X2, tau = 0.75,
             moment = c(S, X2), order = 2, data = dat)$coefficients
```

MR.reg

Multiply Robust Estimation for (Mean) Regression

## **Description**

MR.reg() is used for (mean) regression under generalized linear models with missing responses and/or missing covariates. Multiple missingness probability models and imputation models are allowed.

## Usage

```
MR.reg(formula, family = gaussian, imp.model = NULL,
mis.model = NULL, moment = NULL, order = 1, L = 30, data,
bootstrap = FALSE, bootstrap.size = 300, alpha = 0.05, ...)
```

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#### **Arguments**

formula The formula of the regression model of interest.

family A description of the error distribution and link function to be used for the GLM

of interest.

imp.model A list of possibly multiple lists of the form list(list.1, list.2, ..., list.K),

where K is the total number of different imputation models. For the *k*-th imputation model, list.k is a list of possibly multiple models, each of which is defined by glm.work and imputes one single missing variable marginally. See details.

mis.model A list of missingness probability models defined by glm.work. The dependent

variable is always specified as R.

moment A vector of auxiliary variables whose moments are to be calibrated.

Order A numeric value. The order of moments up to which to be calibrated.

L Number of imputations.

data A data frame with missing data encoded as NA.

bootstrap Logical. If bootstrap = TRUE, the bootstrap will be applied to calculate the

standard error and construct a Wald confidence interval.

bootstrap.size A numeric value. Number of bootstrap resamples generated if bootstrap =

TRUE.

alpha Significance level used to construct the 100(1 - alpha)% Wald confidence inter-

val.

... Addition arguments for the function glm.

#### **Details**

The function MR.reg() currently deals with data with one missingness pattern. When multiple variables are subject to missingness, their values are missing simultaneously. The methods in Han (2016) and Zhang and Han (2019) specify an imputation model by modeling the joint distribution of the missing variables conditional on the fully observed variables. In contrast, the function MR.reg() specifies an imputation model by separately modeling the marginal distribution of each missing variable conditional on the fully observed variables. These marginal distribution models for different missing variables constitute one joint imputation model. Different imputation models do not need to model the marginal distribution of each missing variable differently.

#### Value

coefficients The estimated regression coefficients.

SE The bootstrap standard error of coefficients when bootstrap = TRUE.

CI A Wald-type confidence interval based on coefficients and SE when bootstrap

= TRUE.

## References

Han, P. (2014). Multiply robust estimation in regression analysis with missing data. *Journal of the American Statistical Association*, **109**(507), 1159–1173.

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Han, P. (2016). Combining inverse probability weighting and multiple imputation to improve robustness of estimation. *Scandinavian Journal of Statistics*, **43**, 246–260.

Zhang, S. and Han, P. (2019). A simple implementation of multiply robust estimation for GLMs with missing data. Unpublished manuscript.

#### See Also

glm.

#### **Examples**

```
# Simulated data set
set.seed(123)
n <- 400
gamma0 <- c(1, 2, 3)
alpha0 <- c(-0.8, -0.5, 0.3)
S \leftarrow runif(n, min = -2.5, max = 2.5) \# auxiliary variables
X1 \leftarrow rbinom(n, size = 1, prob = 0.5) \# covariate X1
X2 <- rexp(n) # covariate X2
p.obs <-1 / (1 + exp(alpha0[1] + alpha0[2] * S + alpha0[3] * S ^ 2)) # non-missingness probability
R <- rbinom(n, size = 1, prob = p.obs)</pre>
a.x <- gamma0[1] + gamma0[2] * X1 + gamma0[3] * X2
Y \leftarrow rnorm(n, a.x)
dat <- data.frame(S, X1, X2, Y)</pre>
dat[R == 0, c(2, 4)] \leftarrow NA \# X1 and Y may be missing
# marginal imputation models for X1
impX1.1 <- glm.work(formula = X1 ~ S, family = binomial(link = logit))</pre>
impX1.2 <- glm.work(formula = X1 ~ S + X2, family = binomial(link = cloglog))</pre>
# marginal imputation models for Y
impY.1 <- glm.work(formula = Y ~ S, family = gaussian)</pre>
impY.2 <- glm.work(formula = Y ~ S + X2, family = gaussian)</pre>
# missingness probability models
mis1 \leftarrow glm.work(formula = R \sim S + I(S \wedge 2), family = binomial(link = logit))
mis2 <- glm.work(formula = R ~ I(S ^ 2), family = binomial(link = cloglog))</pre>
# this example considers the following K = 3 imputation models for imputing the missing (X1, Y)
imp1 <- list(impX1.1, impY.1)</pre>
imp2 \leftarrow list(impX1.1, impY.2)
imp3 <- list(impX1.2, impY.1)</pre>
results <- MR.reg(formula = Y ~ X1 + X2, family = gaussian, imp.model = list(imp1, imp2, imp3),
                   mis.model = list(mis1, mis2), L = 10, data = dat)
results$coefficients
MR.reg(formula = Y ~ X1 + X2, family = gaussian,
       moment = c(S, X2), order = 2, data = dat)$coefficients
```

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