# Package 'Rtropical'

October 12, 2022

Title Data Analysis Tools over Space of Phylogenetic Trees Using Tropical Geometry
 Version 1.2.1
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Description Process phylogenetic trees with tropical support vector machine and principal component analysis defined with tropical geometry. Details about tropical support vector machine are available in: Tang, X., Wang, H. & Yoshida, R. (2020) <arXiv:2003.00677>. Details about tropical principle component analysis are available in: Page, R., Yoshida, R. & Zhang L. (2020) <doi:10.1093/bioinformatics/btaa564> and Yoshida, R., Zhang, L. & Zhang 018-0493-4>.

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Encoding UTF-8
LazyData true
RoxygenNote 7.1.1

Imports ape, lpSolve, lpSolveAPI, parallel, Rfast, RcppAlgos, caret

**Depends** R (>= 3.5.0)

**Suggests** rmarkdown, knitr, e1071, testthat (>= 3.0.0)

URL https://github.com/HoujieWang/Rtropical

VignetteBuilder knitr

Config/testthat/edition 3

NeedsCompilation no

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Repository CRAN

Date/Publication 2021-11-09 18:50:08 UTC

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all\_trees

Simulated Tree Data with Different Proximity Parameter Value

# Description

Simulated Tree Data with Different Proximity Parameter Value

# Usage

data(all\_trees)

# **Format**

A list of length 12 with each element a sublist containing an ape multiPhylo object with 300 rooted trees on 5 tips and the tree categories.

apicomplexa 3

apicomplexa Apicomplexa gene trees sample data set.
---

# **Description**

Apicomplexa gene trees sample data set.

# Usage

```
apicomplexa
```

### **Format**

An ape multiPhylo object with 268 rooted trees on 8 tips.

### **Source**

Chih-Horng Kuo, John P. Wares, Jessica C. Kissinger. *The Apicomplexan Whole-Genome Phylogeny: An Analysis of Incongruence among Gene Trees Molecular Biology and Evolution, Volume 25, Issue 12, December 2008, Pages 2689–2698.* 

```
as.matrix.multiPhylo Vectorize a Set of Phylognetic Trees
```

# Description

Unifies tip labels of all phylogenetic trees in multiPhylo object the same as the first tree and returns the cophenetic distance of their corresponding chronogram.

# Usage

```
## S3 method for class 'multiPhylo'
as.matrix(x, tipOrder = x[[1]]$tip.label, parallel = FALSE, ncores = 2, ...)
```

# Arguments

x	an object of class multiPhylo
tipOrder	a numeric vector of order of leaf names to which all trees in the multiPhylo object will unified. If not specified on purpose, the tip order of the first tree will be used.
parallel	a logical value indicating if parallel computing should be used. (default: FALSE)
ncores	a numeric value indicating the number of threads utilized for multi-cored CPUs. (default: 2)
	Not used. Other arguments to as vector

as.vector.phylo

# Value

A data matrix with each row a vector representation of a chronogram. Each element of the vector is the distance between two leaves.

# **Examples**

```
data(apicomplexa)
data <- as.matrix(apicomplexa[1: 10]) # matrixize first ten trees</pre>
```

as.vector.phylo

Vectorize a Phylogenetic Tree

# Description

Computes the cophenetic distance and outputs them in a vector of a phylogenetic tree in phylo object

# Usage

```
## S3 method for class 'phylo'
as.vector(x, mode = "any")
```

# **Arguments**

x A object of class phylo

mode The same as base::as.vector. But only numeric output in vector form is

accepted for other functions in Rtropical

### Value

A vector with its elements the distance between two leaves of the tree.

```
library(ape)
tree <- rcoal(5)
tree_vec <- as.vector(tree)</pre>
```

coef.cv.tropsvm 5

coef.cv.tropsvm

Extract Optimal Tropical Hyperplane from a cv.tropsvm object

# Description

Obtain the optimal tropical hyperplane in the form of vectors from a cv.tropsvm object.

### Usage

```
## S3 method for class 'cv.tropsvm'
coef(object, ...)
```

# Arguments

```
object a fitted "cv.tropsvm" object.
... Not used. Other arguments.
```

### Value

An output of the apex of the fitted optimal tropical hyperplane.

coef.tropsvm

Extract Optimal Tropical Hyperplane from a tropsvm object

# **Description**

Obtain the optimal tropical hyperplane in the form of vectors from a tropsvm object.

# Usage

```
## S3 method for class 'tropsvm'
coef(object, ...)
```

# **Arguments**

```
object a fitted "tropsvm" object.
... Not used. Other arguments.
```

### Value

An output of the apex of the fitted optimal tropical hyperplane.

6 cv.tropsvm

cv.tropsvm	Cross-Validation for Tropical Support Vector Machines	

### **Description**

Conduct k-fold cross validation for tropsvm and return an object "cv. tropsvm".

### Usage

```
cv.tropsvm(x, y, parallel = FALSE, nfold = 10, nassignment = 10, ncores = 2)
```

# **Arguments**

x a data matrix, of dimension nobs x nvars; each row is an observation vector.

y a response vector with one label for each row/component of x.

parallel a logical value indicating if parallel computing should be used. (default: FALSE)

nfold a numeric value of the number of data folds for cross-validation. (default: 10)

nassignment a numeric value indicating the size of the parameter grid of assignments. (de-

fault: 10)

ncores a numeric value indicating the number of threads utilized for multi-cored CPUs.

(default: 2)

### Value

object with S3 class cv. tropsvm containing the fitted model, including:

apex The negative apex of the fitted optimal tropical hyperplane.

assignment The best assignment tuned by cross-validation.

index The best classification method tuned by cross-validation.

levels The name of each category, consistent with categories in y.

accuracy The validation accuracy for each fold.

nfold The number of folds used in cross-validation.

### See Also

summary, predict, coef and the tropsvm function.

```
# data generation
library(Rfast)
set.seed(101)
e <- 20
n <- 10</pre>
```

lungfish 7

```
N <- 10
s <- 5
x <- rbind(
  rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(n, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
y \leftarrow as.factor(c(rep(1, n), rep(2, n)))
newx <- rbind(</pre>
  rmvnorm(N, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(N, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
newy <- as.factor(rep(c(1, 2), each = N))
# train the tropical svm
cv_tropsvm_fit <- cv.tropsvm(x, y, parallel = FALSE)</pre>
summary(cv_tropsvm_fit)
coef(cv_tropsvm_fit)
# test with new data
pred <- predict(cv_tropsvm_fit, newx)</pre>
# check with accuracy
table(pred, newy)
# compute testing accuracy
sum(pred == newy) / length(newy)
```

lungfish

Coelacanths genome and transcriptome data

### **Description**

Coelacanths genome and transcriptome data

# Usage

lungfish

# Format

An ape multiPhylo object with 1193 rooted trees on 10 tips.

#### **Source**

Tom M. W. Nye, Xiaoxian Tang, Grady Weyenberg and Ruriko Yoshida. *Principal component analysis and the locus of the Fréchet mean in the space of phylogenetic trees, Biometrika, Volume 104, Issue 4, December 2017, Pages 901–922.* 

8 predict.cv.tropsvm

plot.	troppca
-------	---------

Plot the Tropical Principal Components with Data Projections

### **Description**

Visualize the second order tropical principle components in troppca as a tropical triangle with projections on a two-dimensional plot via tropical isometry.

# Usage

```
## S3 method for class 'troppca'
plot(x, plab = NULL, fw = FALSE, ...)
```

### **Arguments**

x a fitted troppca object.

plab a vector of labels of all points in the given data matrix. Not needed for unlabeled

data. (default: NULL)

fw a logical variable to determine if to add Fermat-Weber point of the data projec-

tion. (default: FALSE)

... Not used. Other arguments to plot

# Value

plot. troppca does not return anything other than the plot.

predict.cv.tropsvm

Predict Method for Tropical Support Vector Machines based on Cross-Validation

# **Description**

Predicts values based upon a model trained by cv. tropsvm.

### Usage

```
## S3 method for class 'cv.tropsvm'
predict(object, newx, ...)
```

# Arguments

object a fitted "cv.tropsvm" object.

newx a data matrix, of dimension nobs x nvars used as testing data.

. . . Not used. Other arguments to predict.

predict.tropsvm 9

### Value

A vector of predicted values of a vector of labels.

#### See Also

summary, coef and the cv. tropsvm function.

### **Examples**

```
# data generation
library(Rfast)
e <- 20
n <- 10
N <- 10
s <- 5
x <- rbind(</pre>
  rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(n, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
)
y \leftarrow as.factor(c(rep(1, n), rep(2, n)))
newx <- rbind(</pre>
  rmvnorm(N, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(N, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
)
newy <- as.factor(rep(c(1, 2), each = N))
# train the tropical svm
cv_tropsvm_fit <- cv.tropsvm(x, y, parallel = FALSE)</pre>
# test with new data
pred <- predict(cv_tropsvm_fit, newx)</pre>
# check with accuracy
table(pred, newy)
# compute testing accuracy
sum(pred == newy) / length(newy)
```

predict.tropsvm

Predict Method for Tropical Support Vector Machines

# Description

Predicts values based upon a model trained by tropsvm.

#### Usage

```
## S3 method for class 'tropsvm'
predict(object, newx, ...)
```

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# **Arguments**

```
object a fitted tropsvm object.

newx a data matrix, of dimension nobs x nvars used as testing data.

... Not used. Other arguments to predict.
```

# Value

A vector of predicted values of a vector of labels.

#### See Also

summary, coef and the tropsvm function.

```
# data generation
library(Rfast)
e <- 100
n <- 10
N <- 10
s <- 5
x <- rbind(</pre>
  rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(n, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
)
y \leftarrow as.factor(c(rep(1, n), rep(2, n)))
newx <- rbind(</pre>
  rmvnorm(N, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(N, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
newy <- as.factor(rep(c(1, 2), each = N))
# train the tropical svm
tropsvm_fit <- tropsvm(x, y, auto.assignment = TRUE, ind = 1)</pre>
# test with new data
pred <- predict(tropsvm_fit, newx)</pre>
# check with accuracy
table(pred, newy)
# compute testing accuracy
sum(pred == newy) / length(newy)
```

read.nexus.to.data.matrix

```
read.nexus.to.data.matrix
```

Read NEXUS-formatted trees from two categories into a data matrix

# **Description**

Read NEXUS-formatted trees from two categories into a data matrix

### Usage

```
read.nexus.to.data.matrix(data.file1, data.file2)
```

# Arguments

data.file1	A data set with trees from one category.
data.file2	A data set with trees from the other category.

### Value

A data matrix with the first x rows corresponding the x trees in the first file and the last y rows are the trees from the second file.

```
read.tree.to.data.matrix
```

Read Newick-formatted trees in two categories into a data matrix

# **Description**

Read Newick-formatted trees in two categories into a data matrix

### Usage

```
read.tree.to.data.matrix(data.file1, data.file2)
```

# Arguments

data.file2 A file containing trees in Newick form in an assumed different category.

# Value

```
read.tree.to.data.matrix has the same return as read.nexus.to.data.matrix.
```

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sim\_trees

Simulated Tree Data

# Description

Simulated Tree Data

### Usage

```
data(sim_trees)
```

#### **Format**

An ape multiPhylo object with 300 rooted trees on 5 tips.

summary.cv.tropsvm

Summarize an Analysis of Cross-Validated Tropical Support Vector Machine

# **Description**

Return a summary with a more detailed explanation of the object "cv.tropsvm".

### Usage

```
## S3 method for class 'cv.tropsvm'
summary(object, ...)
```

# **Arguments**

```
object a fitted "cv.tropsvm" object.
... Not used. Other arguments to summary.
```

#### Value

A summary of the crucial information of a tropical support vector machine is printed, including the selected best assignment and classification methods and the validation accuracy of each data fold. The summary section of classification methods specifies the sectors and their intersections used to classify points of two different categories.

#### See Also

```
predict, coef and the cv. tropsvm function.
```

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### **Examples**

```
# data generation
library(Rfast)
e <- 20
n <- 10
N <- 10
s <- 5
x <- rbind(
  rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(n, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
)
y <- as.factor(c(rep(1, n), rep(2, n)))</pre>
newx <- rbind(</pre>
  rmvnorm(N, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(N, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
newy <- as.factor(rep(c(1, 2), each = N))
# train the tropical svm with cross-validation
cv_tropsvm_fit <- cv.tropsvm(x, y, parallel = FALSE)</pre>
summary(cv_tropsvm_fit)
```

tropdet

Tropical Determinant of a Matrix

# Description

Compute the tropical determinant for a given matrix. This is equivalent to solving an assignment problem.

### Usage

```
tropdet(x)
```

# Arguments

Х

a square matrix

# Value

The determinant of the given matrix,

```
R <- matrix(sample(1:9, 9), nrow = 3)
tropdet(R)</pre>
```

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tropFW

Tropical Fermat-Weber Point

# **Description**

Compute the tropical Fermat-Weber (FW) point for a given data matrix. The FW point minimizes the summed tropical distance to the trees described in the data matrix.

# Usage

```
tropFW(x)
```

### **Arguments**

Χ

a data matrix, of dimension nobs x nvars; each row is an observation vector.

### Value

A list containing:

fw The fermat-weber point.

distsum The sum of distance from each observation to the fermat-weber point.

# **Examples**

```
x <- matrix(rnorm(100), ncol = 10)
tropFW(x)</pre>
```

troppca.linsp

Tropical Principal Component Analysis by Tropical Linear Space

# **Description**

Approximate the principal component as a tropical linear space for a given data matrix and returns the results as an object of class troppca.

# Usage

```
troppca.linsp(x, pcs = 2, iteration = list(), ncores = 2)
```

troppca.linsp2poly 15

#### **Arguments**

x a data matrix, of size n x e, with each row an observation vector. e is the dimen-

sion of the tropical space

pcs a numeric value indicating the order of principal component. (default: 2)

iteration a list with arguments controlling the iteration of the algorithm.

**exhaust** a logical variable indicating if to iterate over all possible combinations of the linear space based on the given data matrix x. If FALSE, please input a number of iteration for niter. If TRUE, please enter 0 for niter and this function will iterate over all possible combinations of linear space. This

could be time consuming when x is large. (default: FALSE)

niter a numeric variable indicating the number of iterations. (default: 100)

a numeric value indicating the number of threads utilized for multi-cored CPUs.

(default: 2)

#### Value

ncores

A list of S3 class "troppca", including:

pc The principal component as a tropical linear space

obj The tropical PCA objective, the sum of tropical distance from each point to the

projection.

projection The projections of all data points.

type The geometry of principal component.

#### **Examples**

```
library(Rfast)
n <- 100
e <- 10
sig2 <- 1
x <- rbind(rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(sig2, e)))
troppca_fit <- troppca.linsp(x)</pre>
```

troppca.linsp2poly

Tropical Principal Component Analysis by Polytope Converted from Linear Space

# Description

Approximate the principal component as a tropical polytope converted from tropical linear space for a given data matrix via MCMC and return the results as an object of class troppca.

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### Usage

```
troppca.linsp2poly(x, pcs = 2, nsample = 1000, ncores = 2)
```

# **Arguments**

a data matrix, of size n x e, with each row an observation vector. e is the dimen-Χ

sion of the tropical space#'

pcs a numeric value indicating the order of principal component. (default: 2)

a numeric value indicating the number of samples of MCMC. (default: 1000) nsample ncores

a numeric value indicating the number of threads utilized for multi-cored CPUs.

(default: 2)

### Value

A list of S3 class "troppca", including:

The principal component as a tropical linear space рс

obj The tropical PCA objective, the sum of tropical distance from each point to the

projection.

The projections of all data points. projection

The geometry of principal component. type

# Examples

```
library(Rfast)
n <- 50
e <- 50
s <- 5
x <- rbind(
  rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(n, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
)
troppca_fit <- troppca.linsp2poly(x)</pre>
```

troppca.poly

Tropical Principal Component Analysis by Tropical Polytope

# **Description**

Approximates the principal component as a tropical polytope for a given data matrix via MCMC and return the results as an object of class troppca.

tropproj.linsp 17

### Usage

```
troppca.poly(x, pcs = 2, nsample = 1000, ncores = 2)
```

# **Arguments**

x a data matrix, of size n x e, with each row an observation vector. e is the dimension of the tropical space#'

pcs a numeric value indicating the order of principal component. (default: 2)

scarple a numeric value indicating the number of samples of MCMC (default: 1000)

nsample a numeric value indicating the number of samples of MCMC. (default: 1000) a numeric value indicating the number of threads utilized for multi-cored CPUs.

(default: 2)

### Value

A list of S3 class "troppca", including:

pc The principal component as a tropical linear space

obj The tropical PCA objective, the sum of tropical distance from each point to the

projection.

projection The projections of all data points.

type The geometry of principal component.

### **Examples**

```
library(Rfast)
n <- 50
e <- 50
s <- 5
x <- rbind(
    rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
    rmvnorm(n, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
)
troppca_fit <- troppca.poly(x)
plot(troppca_fit)</pre>
```

tropproj.linsp

Projection on Tropical Linear Space

# **Description**

Compute projection of data points on a given tropical linear space.

18 tropproj.poly

### Usage

```
tropproj.linsp(x, V)
```

# **Arguments**

x a data matrix, of size n x e, with each row an observation.

V a data matrix, of dimension s x e, with each row a basis of tropical linear space. e is the dimension of the tropical space and s is the dimension of the linear space.

#### Value

A matrix of projections of all data points.

# **Examples**

```
library(Rfast)
n <- 100
e <- 10
sig2 <- 1
s <- 3
x <- rbind(rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(sig2, e)))
V <- matrix(runif(s * e, -10, 10), nrow = s, ncol = e)
x_proj <- tropproj.linsp(x, V)
head(x_proj)</pre>
```

tropproj.poly

Projection on Tropical Polytope

# **Description**

Project a point onto a given tropical polytope.

# Usage

```
tropproj.poly(x, tconv)
```

### **Arguments**

x a data vector, of length e.

tconv a data matrix, of size e x s, with each column a vertex of the tropical polytope.

e is the dimension of the tropical space and s is the number of vertices of the

polytope

### Value

A projected vector on the given tropical polytope.

tropsvm 19

#### **Examples**

```
# Generate a tropical polytope consisting of three trees each with 5 leaves
library(ape)
pltp <- sapply(1:3, function(i) {
    as.vector(rcoal(5))
})
# Generate an observation and vectorize it
tree <- rcoal(5)
tree_vec <- as.vector(tree)
tropproj.poly(tree_vec, pltp)</pre>
```

tropsvm

Tropical Support Vector Machines

# **Description**

Fit a discriminative two-class classifier via linear programming defined by the tropical hyperplane which maximizes the minimum tropical distance from data points to itself in order to separate the data points into sectors (half-spaces) in the tropical projective torus.

### Usage

```
tropsvm(x, y, auto.assignment = FALSE, assignment = NULL, ind = 1)
```

### Arguments

x a data matrix, of dimension nobs x nvars; each row is an observation vector.

y a response vector with one label for each row/component of x.

auto.assignment

a logical value indicating if to provide an assignment manually. If FALSE, an input is required, otherwise the function automatically finds a good assignment.(default: FALSE)

assignment

a numeric vector of length 4 indicating the sectors of tropical hyperplane that the data will be assigned to. The first and third elements in the assignment are the coordinates of an observed point in data matrix x believed from the first category where the maximum and second maximum of the vector addition between the fitted optimal tropical hyperplane and the point itself are achieved. The meanings for the second and the fourth element in the assignment are the same but for the points in the second category. Namely, the first and second values in the assignment are the indices of sectors where the two point clouds are assigned. Not needed when auto.assignment = TRUE. (default: NULL)

ind

a numeric value or a numeric vector ranging from 1 to 70 indicating which classification method to be used. There are 70 different classification methods. Details of a given method can be retrieved by summary. The different classification methods are proposed to resolve the issue when points fall on the intersections of sectors. Users can have personal choices if better knowledge is assumed. (default: 1)

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### Value

An object with S3 class tropsvm containing the fitted model, including:

apex The negative apex of the fitted optimal tropical hyperplane.

assignment The user-input or auto-found assignment.

index The user-input classification method.

levels The name of each category, consistent with categories in y.

### See Also

predict, coef and the cv. tropsvm function.

```
# data generation
library(Rfast)
e <- 100
n <- 10
N <- 100
s <- 10
x <- rbind(
  rmvnorm(n, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(n, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
y <- as.factor(c(rep(1, n), rep(2, n)))</pre>
newx <- rbind(</pre>
  rmvnorm(N, mu = c(5, -5, rep(0, e - 2)), sigma = diag(s, e)),
  rmvnorm(N, mu = c(-5, 5, rep(0, e - 2)), sigma = diag(s, e))
newy <- as.factor(rep(c(1, 2), each = N))
# train the tropical svm
tropsvm_fit <- tropsvm(x, y, auto.assignment = TRUE, ind = 1)</pre>
coef(tropsvm_fit)
# test with new data
pred <- predict(tropsvm_fit, newx)</pre>
# check with accuracy
table(pred, newy)
# compute testing accuracy
sum(pred == newy) / length(newy)
```

# **Index**

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