# Package 'widals'

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Type Package

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# Description

Fit, forecast, predict massive spacio-temporal data

# **Details**

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Date: 2019-12-07 License: GPL (>=2)

The two essential functions are widals.snow and widals.predict, both contain an Adaptive Least Squares (ALS) prediction stage and complementary 'stochastic adjustment' stage. The function H.als.b solely fits with ALS.

This package offers the user a metaheuristic stochastic search to locate the scalar WIDALS hyperparameters. The function MSS.snow along with helper functions fun.load serve this end. In fairness, providing some useful amount of generality makes this aspect of widals a bit challenging to learn. The user new to this package should expect to spend a couple hours playing with the examples before effectively applying these functions to their own data.

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#### See Also

Package LatticeKrig.

applystnd.Hs

Standardize Spacial Covariates with Existing Object

# Description

Standardize spacial covariates with respect to both the space and time dimensions

## Usage

```
applystnd.Hs(Hs0, x)
```

# **Arguments**

Hs0 Spacial covariates (of interpolation sites). An  $n^* \times p_s$  numeric matrix. x Spacial standardization object, as created by stnd.Hs.

#### Value

```
An n^* x p_s matrix.
```

#### See Also

```
stnd.Hst.ls, applystnd.Hst.ls.
```

```
n.all <- 21
Hs.all <- cbind(1, rnorm(n.all, 1, 0.1), rnorm(n.all, -200, 21))
ndx.interp <- c(1,3,5)
ndx.support <- I(1:n.all)[ -ndx.interp ]

Hs <- Hs.all[ndx.support, , drop=FALSE]

xsns.obj <- stnd.Hs(Hs)
Hs0 <- Hs.all[ndx.interp, , drop=FALSE]</pre>
```

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```
sHs0 <- applystnd.Hs(Hs0, xsns.obj)
sHs0

xsns.obj$sHs

crossprod(xsns.obj$sHs) / nrow(Hs)

crossprod(sHs0) / nrow(sHs0)

## The function is currently defined as function (Hs0, x)
{
    sHs0 <- t((t(Hs0) - x$h.mean)/x$h.sd)
    if (x$intercept) {
        sHs0[, 1] <- 1/sqrt(x$n)
    }
    return(sHs0)
}</pre>
```

applystnd.Hst.ls

Standardize Space-Time Covariates with Existing Object

#### **Description**

Standardize spacio-temporal covariates with respect to both the space and time dimensions

# Usage

```
applystnd.Hst.ls(Hst0.ls, x)
```

# Arguments

Hst0.1s Space-time covariates (of interpolation sites). A list of length  $\tau$ , each element should be a  $n^*$  x  $p_s t$  numeric matrix.

x Space-time standardization object, as created by stnd.Hst.1s.

#### Value

An unnamed list of length  $\tau$ , each element a  $n^*$  x  $p_s t$  numeric matrix.

```
tau <- 20
n.all <- 10

Hst.ls.all <- list()
for(tt in 1:tau) {</pre>
```

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```
Hst.ls.all[[tt]] <- cbind(rnorm(n.all, 1, 0.1), rnorm(n.all, -200, 21))</pre>
ndx.interp <- c(1,3,5)
ndx.support <- I(1:n.all)[ -ndx.interp ]</pre>
Hst.ls <- subsetsites.Hst.ls(Hst.ls.all, ndx.support)</pre>
xsnst.obj <- stnd.Hst.ls(Hst.ls)</pre>
Hst0.ls <- subsetsites.Hst.ls(Hst.ls.all, ndx.interp)</pre>
sHst0.ls <- applystnd.Hst.ls(Hst0.ls, xsnst.obj)</pre>
Hst.sumup(xsnst.obj$sHst.ls)
Hst.sumup(sHst0.ls)
## The function is currently defined as
function (Hst0.ls, x)
    tau <- length(Hst0.ls)</pre>
    sHst0.ls <- list()
    for (i in 1:tau) {
        sHst0.ls[[i]] \leftarrow t((t(Hst0.ls[[i]]) - x$h.mean)/x$h.sd)
    return(sHst0.ls)
  }
```

create.rm.ndx.ls

Cross-Validation Indices

# **Description**

Create a list of vectors of indices to remove for *k*-fold cross-validation

# Usage

```
create.rm.ndx.ls(n, xincmnt = 10)
```

#### **Arguments**

n Number of sites. A scalar integer.

xincmnt How many cv folds, i.e., k.

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#### **Details**

The name of the object produced by this function is commonly rm.ndx in this documentation. See MSS. snow for a reminder that this object is passed out-of-scope when using MSS. snow.

In this package rm.ndx is used by Hals.fastcv.snow and widals.snow; however, creating this object as a list using this function is only necessary when using widals.snow with cv=2 (i.e., 'true' cross-validation).

#### Value

An unnamed list of integer (>0) vectors.

#### **Examples**

```
n <- 100
xincmnt <- 7
rm.ndx <- create.rm.ndx.ls(n=n, xincmnt=xincmnt)</pre>
rm.ndx
####### if we want randomization of indices:
n <- 100
xincmnt <- 7
rm.ndx <- create.rm.ndx.ls(n=n, xincmnt=xincmnt)</pre>
rnd.ndx <- sample(I(1:n))</pre>
for(i in 1:length(rm.ndx)) { rm.ndx[[i]] <- rnd.ndx[rm.ndx[[i]]] }</pre>
rm.ndx
## The function is currently defined as
function (n, xincmnt = 10)
{
    rm.ndx.ls <- list()</pre>
    for (i in 1:xincmnt) {
        xrm.ndxs <- seq(i, n + xincmnt, by = xincmnt)</pre>
        xrm.ndxs <- xrm.ndxs[xrm.ndxs <= n]</pre>
        rm.ndx.ls[[i]] <- xrm.ndxs</pre>
    return(rm.ndx.ls)
}
```

crispify

Observation-Space Stochastic Correction

# Description

Improve observation-space predictions using 'left over' spacial correlation between model residuals

#### Usage

```
crispify(locs1, locs2, Z.delta, z.lags.vec, geodesic, alpha, flatten, self.refs,
lags, stnd.d = FALSE, log10cutoff = -16)
```

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# Arguments

locs1	Locations of supporting sites. An $n \times 3$ matrix, first column is spacial $x$ , second column is spacial $y$ , third column contains relative temporal 'distance'. If the geodesic is TRUE, make sure latitude is in the first column.
locs2	Locations of interpolation sites. An $n^* \times 3$ matrix, where $n^*$ is the number of interpolation sites. See locs1 above.
Z.delta	Observed residuals. A $\tau$ x $x$ matrix.
z.lags.vec	Temporal lags. An integer vector or scalar.
geodesic	Use geodesic distance? Boolean. If true, distance (used internally) is in units kilometers.
alpha	The WIDALS distance rate hyperparameter. A scalar non-negative number.
flatten	The WIDALS 'flattening' hyperparameter. A scalar non-negative number. Typically between 0 and some number slightly greater than 1. When 0, no crispification.
self.refs	Which sites are self-referencing? An integer vector of (zero-based) lag indices, OR a scalar set to $-1$ . This argument only has meaning when locs1 is identical to locs2. If the lags argument is, say, 0, then it would be pointless to smooth predictions with existing values. In this case, we can set self.refs = 0. If locs1 is NOT the same as locs2, then set this argument to $-1$ .
lags	Temporal lags. An integer vector or scalar. E.g., if the data's time increment is daily, then lags = $c(-1,0,1)$ would have crispify smooth today's predictions using yesterdays, today's, and tomorrow's observed residuals.
stnd.d	Spacial compression. Boolean.
log10cutoff	Weight threshold. A scalar number. A value of, e.g., -10, will instruct crispify to ignore weights less than 10^(-10) when smoothing.

# **Details**

This function is called inside widals.predict and widals.snow. It may be useful for the user in building their own WIDALS model extensions.

# Value

A  $\tau$  x x matrix.

# See Also

```
widals.predict, widals.snow.
```

```
######### here's an itty-bitty example
######## simulate itty-bitty data
```

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```
tau <- 21 #### number of time points
d.alpha <- 2
R.scale <- 1
sigma2 <- 0.01
F <- 1
Q <- 0
n.all <- 14 ##### number of spacial locations
set.seed(9999)
library(SSsimple)
locs.all <- cbind(runif(n.all, -1, 1), runif(n.all, -1, 1)) #### random location of sensors
D.mx <- distance(locs.all, locs.all, FALSE) #### distance matrix</pre>
#### create measurement variance using distance and covariogram
R.all <- exp(-d.alpha*D.mx) + diag(sigma2, n.all)</pre>
Hs.all <- matrix(1, n.all, 1) #### constant mean function</pre>
##### use SSsimple to simulate system
xsssim <- SS.sim(F=F, H=Hs.all, Q=Q, R=R.all, length.out=tau, beta0=0)
Z.all <- xsssim$Z ###### system observation matrix</pre>
####### suppose use the global mean as a prediction
z.mean <- mean(Z.all)</pre>
Z.delta <- Z.all - z.mean
z.lags.vec <- rep(0, n.all)</pre>
geodesic <- FALSE</pre>
alpha <- 5
flatten <- 1
## emmulate cross-validation, i.e.,
## don't use observed site values to predict themselves (zero-based)
self.refs <- 0
lags <- 0
locs1 <- cbind(locs.all, rep(0, n.all))</pre>
locs2 <- cbind(locs.all, rep(0, n.all))</pre>
Z.adj <- crispify(locs1, locs2, Z.delta, z.lags.vec, geodesic, alpha,</pre>
    flatten, self.refs, lags, stnd.d = FALSE, log10cutoff = -16)
Z.adj
```

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```
Z.hat <- z.mean + Z.adj
sqrt( mean( (Z.all - Z.hat)^2 ) )
######### set flatten to zero -- this means no crispification
Z.adj <- crispify(locs1, locs2, Z.delta, z.lags.vec, geodesic, alpha, flatten=0, self.refs, lags, stnd.d = FALSE, log10cutoff = -16)
Z.adj
Z.hat <- z.mean + Z.adj
sqrt( mean( (Z.all - Z.hat)^2 ) )</pre>
```

distance

Spacial Distance

# **Description**

Calculate spacial distance between two sets of locations (in two-space)

# Usage

```
distance(locs1, locs2, geodesic = FALSE)
```

# Arguments

locs1	First set of locations. E.g., supporting sites: An $n \times 2$ matrix. If geodesic is set to true, make sure to place latitude in first column.
locs2	Second set of locations. E.g., interpolation sites: An $n^*$ x 2 matrix. If geodesic is set to true, make sure to place latitude in first column.
geodesic	Use geodesic distance? Boolean.

## **Details**

If geodesic is set to FALSE, Euclidean distance is returned; if TRUE, Earth's geodesic distance is returned in units kilometers.

#### Value

An  $n \times n^*$  matrix.

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```
locs1 \leftarrow cbind(c(-1, -1, 1, 1), c(-1, 1, -1, 1))
locs2 \leftarrow cbind(c(0), c(0))
distance(locs1, locs2)
locs1 \leftarrow cbind(c(32, 0), c(-114, -114))
locs2 \leftarrow cbind(c(0), c(0))
distance(locs1, locs2, TRUE)
####### separation of one deg long at 88 degs lat (near North-Pole) is (appx)
locs1 \leftarrow cbind(c(88), c(-114))
locs2 \leftarrow cbind(c(88), c(-115))
distance(locs1, locs2, TRUE)
###### separation of one deg long at 0 degs lat (Equator) is (appx)
locs1 <- cbind( c(0), c(-114) )
locs2 <- cbind(c(0), c(-115))
distance(locs1, locs2, TRUE)
## The function is currently defined as
function (locs1, locs2, geodesic = FALSE)
             dyn.load("~/Files/Creations/C/distance.so")
          n1 <- nrow(locs1)</pre>
          n2 <- nrow(locs2)
          d.out <- rep(0, n1 * n2)
           if (geodesic) {
                      D.Mx <- .C("distance_geodesic_AB", as.double(locs1[,</pre>
                                 1] * pi/180), as.double(locs1[, 2] * pi/180), as.double(locs2[,
                                 1] * pi/180), as.double(locs2[, 2] * pi/180), as.double(d.out),
                                 as.integer(n1), as.integer(n2))[[5]]
           }
           else {
                      \label{eq:decomposition} D.Mx <- .C("distance\_AB", as.double(locs1[, 1]), as.double(locs1
                                 2]), as.double(locs2[, 1]), as.double(locs2[, 2]),
                                 as.double(d.out), as.integer(n1), as.integer(n2))[[5]]
          D.out <- matrix(D.Mx, n1, n2)</pre>
           return(D.out)
```

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dlog.norm

Local Search Function

# Description

Local hyperparameter exponentiated-normal search function

# Usage

```
dlog.norm(n, center, sd)
```

# Arguments

n Sample size. A positive scalar integer.

center Exponential of the mean. A numeric scalar (or vector).

sd Standard deviation. A numeric scalar (or vector).

#### **Details**

This function can be used by MSS. snow.

#### Value

A numeric vector of length n.

#### See Also

```
unif.mh, MSS.snow, fun.load.
```

```
x <- dlog.norm(100, 1, 1)
hist(x)
## The function is currently defined as
function (n, center, sd)
{
    return(exp(rnorm(n, log(center), sd)))
}</pre>
```

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fun.load

Stochastic Search Helper Functions

# **Description**

Functions that assign values and functions needed by MSS. snow

#### Usage

```
fun.load.hals.a()
fun.load.hals.fill()
fun.load.widals.a()
fun.load.widals.fill()
```

#### **Details**

Please see MSS. snow and examples.

#### Value

Nothing. The central role of these functions is the creation of four functions required by MSS.snow: FUN.MH, FUN.GP, FUN.I, and FUN.EXIT. These four functions are assigned to the Global Environment. This fun.load suite of functions also passes needed objects (out-of-scope) to snowfall threads if the global user-made variable run.parallel is set to TRUE.

#### See Also

MSS.snow

```
### Here's an itty bitty example:
### we use stochastic search to find the minimum number in a vector
### GP isn't used here, and hence neither are p.ndx.ls nor f.d
### however, we still need to create them since MSS.snow requires their existence
## Not run:

fun.load.simpleExample <- function() {
    if( run.parallel ) {
        sfExport("xx")
    }
    p.ndx.ls <- list( c(1) )</pre>
```

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```
p.ndx.ls <<- p.ndx.ls
    f.d <- list( dlog.norm )</pre>
    f.d <<- f.d
    FUN.MH <- function(jj, GP.mx, X) {</pre>
        our.cost <- sample(xx, 1)</pre>
    }
    FUN.MH <<- FUN.MH
    FUN.GP <- NULL
    FUN.GP <<- FUN.GP
    FUN.I <- function(envmh, X) {</pre>
     cat("Hello, I have found an even smaller number in xx ---> ", envmh$current.best, "\n")
    FUN.I <<- FUN.I
    FUN.EXIT <- function(envmh, X) {</pre>
        cat( "Done", "\n" )
    FUN.EXIT <<- FUN.EXIT
xx <- 1:600
GP \leftarrow c(1)
run.parallel <- TRUE
sfInit(TRUE, 2)
MH.source <- fun.load.simpleExample</pre>
MH.source()
MSS.snow(MH.source, Inf, p.ndx.ls, f.d, matrix(1, nrow=28), 28, 7)
sfStop()
### Here's another itty bitty example:
### we use stochastic search to find the mean of a vector
### i.e., the argmin? of sum ( x - ? )^2
fun.load.simpleExample2 <- function() {</pre>
   if( run.parallel ) {
```

}

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```
sfExport("xx")
    }
    p.ndx.ls \leftarrow list(c(1))
    p.ndx.ls <<- p.ndx.ls</pre>
    f.d <- list( unif.mh )</pre>
    f.d <<- f.d
    FUN.MH <- function(jj, GP.mx, X) {</pre>
        our.cost <- sum( ( xx - GP.mx[jj, 1] )^2 )</pre>
        return(our.cost)
    FUN.MH <<- FUN.MH
    FUN.GP <- NULL
    FUN.GP <<- FUN.GP
    FUN.I <- function(envmh, X) {</pre>
        cat( "Improvement ---> ", envmh$current.best, " ---- " , envmh$GP, "\n" )
    FUN.I <<- FUN.I
    FUN.EXIT <- function(envmh, X) {</pre>
        our.cost <- envmh$current.best</pre>
        GP <- envmh$GP
        cat( "Done", "\n" )
        cat( envmh$GP, our.cost, "\n" )
    FUN.EXIT <<- FUN.EXIT
}
##set.seed(99999)
xx <- rnorm(300, 5, 10)
GP \leftarrow c(1)
run.parallel <- TRUE
sfInit(TRUE, 2)
MH.source <- fun.load.simpleExample2</pre>
MH.source()
MSS.snow(MH.source, Inf, p.ndx.ls, f.d, matrix(1/10, nrow=140, ncol=length(GP)), 140, 14)
sfStop()
##### in fact:
mean(xx)
## End(Not run)
```

fuse.Hst.ls

fuse.Hst.ls

Merge Contemporaneous Space-Time Covariates

## **Description**

Fuse together two lists of spacio-temporal covariates

# Usage

```
fuse.Hst.ls(Hst.ls1, Hst.ls2)
```

# **Arguments**

Hst.1s1 Space-time covariates. A list of length au, each element should be a numeric n x  $p_1$  matrix.

Hst.1s2 Space-time covariates. A list of length au, each element should be a numeric n x  $p_2$  matrix.

#### Value

An unnamed list of length  $\tau$ , each element will be a numeric  $n \times (p_1 + p_2)$  matrix.

```
set.seed(9999)
tau <- 5
n <- 7
p1 <- 2
Hst.ls1 <- list()</pre>
for(i in 1:tau) { Hst.ls1[[i]] <- matrix(rnorm(n*p1), nrow=n) }</pre>
p2 <- 3
Hst.ls2 <- list()</pre>
for(i in 1:tau) { Hst.ls2[[i]] <- matrix(rnorm(n*p2), nrow=n) }</pre>
fuse.Hst.ls(Hst.ls1, Hst.ls2)
## The function is currently defined as
function (Hst.ls1, Hst.ls2)
    tau <- length(Hst.ls1)</pre>
    for (i in 1:tau) {
        Hst.ls1[[i]] <- cbind(Hst.ls1[[i]], Hst.ls2[[i]])</pre>
    return(Hst.ls1)
  }
```

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H.als.b Adaptive Least Squares
--------------------------------

# Description

Adaptive Least Squares expecially for large spacio-temporal data

# Usage

```
H.als.b(Z, Hs, Ht, Hst.ls, rho, reg, b.lag = -1, Hs0 = NULL, Ht0 = NULL, Hst0.ls = NULL)
```

# Arguments

Z	Space-time data. A $\tau$ x $n$ numeric matrix.			
Hs	Spacial covariates (of supporting sites). An $n \ge p_s$ numeric matrix.			
Ht	Temporal covariates (of supporting sites). A $\tau$ x $p_t$ numeric matrix.			
Hst.ls	Space-time covariates (of supporting sites). A list of length $\tau$ , each element should be a $n \times p_s t$ numeric matrix.			
rho	ALS signal-to-noise ratio (SNR). A non-negative scalar.			
reg	ALS regularizer. A non-negative scalar.			
b.lag	ALS lag. A scalar integer, typically -1 (a-prior), or 0 (a-posteriori).			
Hs0	Spacial covariates (of interpolation sites). An $n^* \ge p_s$ matrix, or NULL.			
Ht0	Temporal covariates (of interpolation sites). A $\tau$ x $p_t$ matrix, or NULL. If not NULL, I cannot imagine a scenario where this shouldn't be Ht.			
Hst0.ls	Space-time covariates (of interpolation sites). A list of length $\tau$ , each element should be a numeric $n$ x $p_s t$ matrix.			

# Value

Α	named	list.
---	-------	-------

Z.hat	A $\tau$ x <i>n</i> matrix, the <i>i</i> th row of which is the ALS prediction of the supporting sites at time <i>i</i> .
В	A $ au$ x $(p_s+p_t+p_st)$ matrix, the <i>i</i> th row of which is the ALS state (partial slopes) prediction at time <i>i</i> .
Z0.hat	A $\tau$ x $n^*$ matrix, the <i>i</i> th row of which is the ALS prediction of the interpolation sites at time <i>i</i> .
inv.LHH	A $(p_s+p_t+p_st)$ x $(p_s+p_t+p_st)$ matrix. This is the (ALS predicted) covariate precision matrix at time $\tau$ .
ALS.g	The ALS gain at time $ au$ .

H.als.b

## **Examples**

```
set.seed(99999)
library(SSsimple)
tau <- 70
n.all <- 14
Hs.all <- matrix(rnorm(n.all), nrow=n.all)</pre>
Ht <- matrix(rnorm(tau*2), nrow=tau)</pre>
Hst.ls.all <- list()</pre>
for(i in 1:tau) { Hst.ls.all[[i]] <- matrix(rnorm(n.all*2), nrow=n.all) }</pre>
Hst.combined <- list()</pre>
for(i in 1:tau) {
    Hst.combined[[i]] <- cbind( Hs.all, matrix(Ht[i, ], nrow=n.all, ncol=ncol(Ht),</pre>
    byrow=TRUE), Hst.ls.all[[i]] )
}
####### use SSsimple to simulate
sssim.obj <- SS.sim.tv( 0.999, Hst.combined, 0.01, diag(1, n.all), tau )</pre>
ndx.support <- 1:10
ndx.interp <- 11:14
Z.all <- sssim.obj$Z</pre>
Z <- Z.all[ , ndx.support]</pre>
Z0 <- Z.all[ , ndx.interp]</pre>
Hst.ls <- subsetsites.Hst.ls(Hst.ls.all, ndx.support)</pre>
Hst0.ls <- subsetsites.Hst.ls(Hst.ls.all, ndx.interp)</pre>
Hs <- Hs.all[ ndx.support, , drop=FALSE]</pre>
Hs0 <- Hs.all[ ndx.interp, , drop=FALSE]
xrho <- 1/10
xreg <- 1/10
xALS <- H.als.b(Z=Z, Hs=Hs, Ht=Ht, Hst.ls=Hst.ls, rho=xrho, reg=xreg, b.lag=-1,
Hs0=Hs0, Ht0=Ht, Hst0.ls=Hst0.ls)
test.rng <- 20:tau
errs.sq <- (Z0 - xALS$Z0.hat)^2
sqrt( mean(errs.sq[test.rng, ]) )
```

########## calculate the 'effective standard errors' (actually 'effective prediction

18 H.Earth.solar

H.Earth.solar

Solar Radiation

# **Description**

Calculate Incident Solar Area (ISA)

# Usage

```
H.Earth.solar(x, y, dateDate)
```

# **Arguments**

x Longitude. Numeric vector of length n. y Latitude. Numeric vector of length n. dateDate Posix date. Numeric vector of length  $\tau$ .

#### **Details**

This function returns a spacio-temporal covariate list (Earth's ISA is space-time *non-seperable*). A negative value indicates that at that time (list index), and at that location (matrix row), the sun is below the horizon all day.

#### Value

An unnamed list of length  $\tau$ , each element of which is an  $n \ge 1$  matrix.

```
lat <- c(0, -88)
lon <- c(0, 0)
dateDate <- strptime( c('20120621', '20120320'), '%Y%m%d')
H.Earth.solar(lon, lat, dateDate)
## The function is currently defined as function (x, y, dateDate)
{
    Hst.ls <- list()</pre>
```

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```
n <- length(y)</pre>
  tau <- length(dateDate)</pre>
  equinox <- strptime("20110320", "%Y%m%d")</pre>
  for (i in 1:tau) {
      this.date <- dateDate[i]</pre>
      dfe <- as.integer(difftime(this.date, equinox, units = "day"))</pre>
      psi <- 23.5 * sin(2 * pi * dfe/365.25)
      psi
      eta <- 90 - (360/(2 * pi)) * acos(cos(2 * pi * y/360) *
           cos(2 * pi * psi/360) + sin(2 * pi * y/360) * sin(2 *
           pi * psi/360))
      surface.area <- sin(2 * pi * eta/360)</pre>
      surface.area
      Hst.ls[[i]] <- cbind(surface.area)</pre>
  return(Hst.ls)
}
```

Hals.fastcv.snow

ALS Spacial Cross-Validation

# **Description**

Fit Adaptive Least Squares with k-fold cross-validation

# Usage

```
Hals.fastcv.snow(j, rm.ndx, Z, Hs, Ht, Hst.ls, GP.mx)
```

# **Arguments**

j	Index used by snowfall. A scalar integer. Which row of GP.mx to use for the ALS hyperparameters, GP.
rm.ndx	A list of vectors of indices to remove for <i>k</i> -fold cross-validation.
Z	Data. A $\tau$ x $n$ numeric matrix.
Hs	Spacial covariates. An $n \times p_s$ numeric matrix.
Ht	Temporal covariates. An $\tau$ x $p_t$ numeric matrix.
Hst.ls	Space-time covariates. A list of length $\tau$ , each element containing a $n \ge p_s t$ numeric matrix.
GP.mx	Hyperparameters. A $k.glob \times 2$ non-negative matrix. See MSS. snow.

#### Value

A  $\tau$  x n numeric matrix. The ALS cross-validated predictions of Z.

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#### See Also

```
Hals.snow, MSS.snow.
```

```
set.seed(99999)
library(SSsimple)
tau <- 70
n.all <- 14
Hs.all <- matrix(rnorm(n.all), nrow=n.all)</pre>
Ht <- matrix(rnorm(tau*2), nrow=tau)</pre>
Hst.ls.all <- list()</pre>
for(i in 1:tau) { Hst.ls.all[[i]] <- matrix(rnorm(n.all*2), nrow=n.all) }</pre>
Hst.combined <- list()</pre>
for(i in 1:tau) {
    Hst.combined[[i]] <- cbind( Hs.all, matrix(Ht[i, ], nrow=n.all,</pre>
    ncol=ncol(Ht), byrow=TRUE), Hst.ls.all[[i]] )
}
####### use SSsimple to simulate
sssim.obj <- SS.sim.tv( 0.999, Hst.combined, 0.01, diag(1, n.all), tau )</pre>
Z.all <- sssim.obj$Z</pre>
Z \leftarrow Z.all
n <- n.all
Hst.ls <- Hst.ls.all
Hs <- Hs.all
xrho <- 1/10
xreg <- 1/10
GP.mx <- matrix(c(xrho, xreg), nrow=1)</pre>
rm.ndx <- create.rm.ndx.ls(n, 10)</pre>
Zcv <- Hals.fastcv.snow(j=1, rm.ndx, Z, Hs, Ht, Hst.ls, GP.mx)</pre>
test.rng <- 20:tau
```

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```
errs.sq <- (Z - Zcv)^2
sqrt( mean(errs.sq[test.rng, ]) )</pre>
```

Effective Standard Errors

# Description

Calculate the ALS so-called 'effective standard errors'

# Usage

```
Hals.ses(Z, Hs, Ht, Hst.ls, rho, reg, b.lag, test.rng)
```

# Arguments

Z	Space-time data. A $\tau$ x $n$ numeric matrix.
Hs	Spacial covariates (of supporting sites). An $n \ge p_s$ numeric matrix.
Ht	Temporal covariates (of supporting sites). A $\tau$ x $p_t$ numeric matrix.
Hst.ls	Space-time covariates (of supporting sites). A list of length $\tau$ , each element should be a numeric $n$ x $p_s t$ matrix.
rho	ALS signal-to-noise ratio (SNR). A non-negative scalar.
reg	ALS regularizer. A non-negative scalar.
b.lag	ALS lag. A scalar integer, typically -1 (a-prior), or 0 (a-posteriori).
test.rng	Temporal test range. A vector of temporal indices of the model test range.

# Value

A	named	list.
---	-------	-------

estimates	A $p_s+p_t+p_st$ x 2 matrix, each row giving the ALS partial slope estimate/prediction at time $\tau$ , and the 'effective standard error (prediction error)' for the partial slope.
inv.LHH	A $(p_s+p_t+p_st)$ x $(p_s+p_t+p_st)$ matrix. This is the (ALS predicted) covariate precision matrix at time $\tau$ .
ALS.g	The ALS gain at time $ au$ .

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#### **Examples**

```
## Please see the example in H.als.b

## The function is currently defined as
function (Z, Hs, Ht, Hst.ls, rho, reg, b.lag, test.rng)
{
   tau <- nrow(Z)
        xALS <- H.als.b(Z = Z, Hs = Hs, Ht = Ht, Hst.ls = Hst.ls,
            rho = rho, reg = reg, b.lag = b.lag, Hs0 = NULL, Ht0 = NULL,
            Hst0.ls = NULL)
   rmse <- sqrt(mean((Z[test.rng, ] - xALS$Z.hat[test.rng, ])^2))
   rmse
   als.se <- rmse * sqrt(xALS$ALS.g) * sqrt(diag(xALS$inv.LHH))
   return(list(estimates = cbind(xALS$B[tau, ], als.se), inv.LHH = xALS$inv.LHH,
            ALS.g = xALS$ALS.g))
}</pre>
```

Hals.snow

Fit ALS

# **Description**

Fit Adaptive Least Squares

# Usage

```
Hals.snow(j, Z, Hs, Ht, Hst.ls, b.lag, GP.mx)
```

# **Arguments**

j	Index used by snowfall. A scalar integer. Which row of GP.mx to use for the ALS hyperparameters, GP.
Z	Data. A $\tau$ x $n$ numeric matrix.
Hs	Spacial covariates. An $n \times p_s$ numeric matrix.
Ht	Temporal covariates. An $\tau$ x $p_t$ numeric matrix.
Hst.ls	Space-time covariates. A list of length $\tau$ , each element containing a $n \ge p_s t$ numeric matrix.
b.lag	ALS lag. A scalar integer, typically -1 (a-prior), or 0 (a-posteriori).
GP.mx	Hyperparameters. A $k.glob \times 2$ non-negative matrix. See MSS. snow.

#### Value

A  $\tau$  x n numeric matrix. The ALS predictions of Z.

#### See Also

```
Hals.fastcv.snow, MSS.snow.
```

Hst.sumup 23

#### **Examples**

```
set.seed(9999)
library(SSsimple)
tau <- 280
n.all <- 35
Hs.all <- matrix(rnorm(n.all), nrow=n.all)</pre>
Ht <- matrix(rnorm(tau*2), nrow=tau)</pre>
Hst.ls.all <- list()</pre>
for(i in 1:tau) { Hst.ls.all[[i]] <- matrix(rnorm(n.all*3), nrow=n.all) }</pre>
Hst.combined <- list()</pre>
for(i in 1:tau) {
    Hst.combined[[i]] <- cbind( Hs.all, matrix(Ht[i, ], nrow=n.all,</pre>
    ncol=ncol(Ht), byrow=TRUE), Hst.ls.all[[i]] )
}
####### use SSsimple to simulate
sssim.obj <- SS.sim.tv( 0.999, Hst.combined, 0.1, diag(1, n.all), tau )</pre>
Z.all <- sssim.obj$Z</pre>
Z \leftarrow Z.all
n <- n.all
Hst.ls <- Hst.ls.all
Hs <- Hs.all
xrho <- 1/10
xreg <- 1/10
b.lag <- -1
GP.mx <- matrix(c(xrho, xreg), nrow=1)</pre>
Zcv <- Hals.snow(j=1, Z, Hs, Ht, Hst.ls, b.lag, GP.mx)</pre>
test.rng <- 20:tau
errs.sq \leftarrow (Z - Zcv)^2
sqrt( mean(errs.sq[test.rng, ]) )
```

Hst.sumup

Create Covariance Matrix

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## **Description**

Calculate the covariance matrix of all model covariates

# Usage

```
Hst.sumup(Hst.ls, Hs = NULL, Ht = NULL)
```

# **Arguments**

Hst.ls	Space-time covariates. A list of length $\tau$ , each element containing a $n$ x $p_s t$ numeric matrix.
Hs	Spacial covariates. An $n \times p_s$ numeric matrix.
Ht	Temporal covariates. An $\tau$ x $p_t$ numeric matrix.

## **Details**

Important: The order of the arguments in this function is NOT the same as in the returned covariance matrix. The order in the covariance matrix is the same as in other functions in this package: Hs, Ht, Hst.ls.

#### Value

```
A (p_s + p_t + p_s t) x (p_s + p_t + p_s t) numeric, symmetrix, non-negative definite matrix.
```

```
tau <- 20
n <- 10
Ht <- cbind(sin(1:tau), cos(1:tau))

Hs <- cbind(rnorm(10), rnorm(n, 5, 49))

Hst.ls <- list()
for(tt in 1:tau) {
   Hst.ls[[tt]] <- cbind(rnorm(n, 1, 0.1), rnorm(n, -200, 21))
}

Hst.sumup(Hst.ls, Hs, Ht)

############ standardize all covariates

x1 <- stnd.Hst.ls(Hst.ls, NULL)$sHst.ls
x2 <- stnd.Hs(Hs, NULL, FALSE)$sHs
x3 <- stnd.Ht(Ht, n)</pre>
```

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```
Hst.sumup(x1, x2, x3)
## The function is currently defined as
function (Hst.ls, Hs = NULL, Ht = NULL)
    tau <- length(Hst.ls)</pre>
    if(tau < 1) { tau <- nrow(Ht) }
    if(is.null(tau)) { tau <- 10 ; cat("tau assumed to be 10.", "\n") }
    n <- nrow(Hst.ls[[1]])</pre>
    if(is.null(n)) { n <- nrow(Hs) }</pre>
    big.sum <- 0
    for (i in 1:tau) {
        if (!is.null(Ht)) {
            Ht.mx <- matrix(Ht[i, ], n, ncol(Ht), byrow = TRUE)</pre>
        }
        else {
            Ht.mx <- NULL
        big.sum <- big.sum + crossprod(cbind(Hs, Ht.mx, Hst.ls[[i]]))</pre>
    }
    return(big.sum)
  }
```

load.Hst.ls.2Zs

Load Observations into Space-Time Covariates

# **Description**

Insert an observation matrix into space-time covariates, but segregate based on missing values

#### Usage

```
load.Hst.ls.2Zs(Z, Z.na, Hst.ls.Z, xwhich, rgr.lags = c(0))
```

# Arguments

Z	Observation data. A $\tau$ x $n$ numeric matrix.
Z.na	Missing data indicator. A $\tau$ x $n$ boolean matrix.
Hst.ls.Z	Space-time covariates. A list of length $\tau$ , each element should be a numeric $n$ x $p_s t$ matrix.
xwhich	Which column-pair of Hst.1s.Z[[i]] to insert into the $i$ th row of Z. A scalar positive integer. By 'column-pair', we mean, e.g., a value of 1 will fill columns 1 and 2, a value of 2 will fill columns 3 and 4, a value of 3 will fill columns 5 and 6, etc.
rgr.lags	Temporal lagging of Z. A scalar integer.

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#### **Details**

This function, along with load.Hst.ls.Z, allows the user to convert a set of observations into covariates for another set of observations. Unlike load.Hst.ls.Z, this function *splits* Z based on the argument Z.na. Values associated with FALSE elements of Z.na are placed into the first column of the specified column-pair of Hst.ls.Z, Values associated with TRUE elements of Z.na are placed into the second column of the specified column-pair of Hst.ls.Z (all other values in in the specified column-pair of Hst.ls.Z are zeroed).

#### Value

An unnamed list of length  $\tau$ , each element will be a numeric  $n \times p_s t$  matrix.

#### See Also

```
load.Hst.ls.Z.
```

```
###### here's an itty-bitty example

tau <- 7
n <- 5

Z <- matrix(1, tau, n)

Z.na <- matrix(FALSE, tau, n)
Z.na[2:3, 4] <- TRUE

Z[Z.na] <- 2

Hst.ls <- list()
for(i in 1:tau) { Hst.ls[[i]] <- matrix(rnorm(n*4), nrow=n) }

load.Hst.ls.2Zs(Z, Z.na, Hst.ls.Z=Hst.ls, 1, 0)

########### insert into cols 3 and 4

load.Hst.ls.2Zs(Z, Z.na, Hst.ls.Z=Hst.ls, 2, 0)</pre>
```

load.Hst.ls.Z

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Load Observations into Space-Time Covariates

# Description

Insert an observation matrix into space-time covariates

# Usage

```
load.Hst.ls.Z(Z, Hst.ls.Z, xwhich, rgr.lags = c(0))
```

# **Arguments**

Z	Observation data. A $\tau$ x $n$ numeric matrix.
Hst.ls.Z	Space-time covariates. A list of length $ au$ , each element should be a numeric $n$ x $p_s t$ matrix.
xwhich	Which column of $Hst.ls.Z[[i]]$ to insert into the $i$ th row of $Z$ . A scalar positive integer.
rgr.lags	Temporal lagging of Z. A scalar integer.

#### **Details**

This function, along with load.Hst.ls.2Zs, allows the user to convert a set of observations into covariates for another set of observations.

# Value

An unnamed list of length  $\tau$ , each element will be a numeric  $n \times p_s t$  matrix.

#### See Also

```
load.Hst.ls.2Zs.
```

```
###### here's an itty-bitty example

tau <- 7
n <- 5

Z <- matrix(1, tau, n)

Hst.ls <- list()
for(i in 1:tau) { Hst.ls[[i]] <- matrix(rnorm(n*4), nrow=n) }</pre>
```

```
load.Hst.ls.Z(Z, Hst.ls.Z=Hst.ls, 1, 0)

########## insert into col 3

load.Hst.ls.Z(Z, Hst.ls.Z=Hst.ls, 3, 0)

########### lag Z examples

Z <- matrix(1:tau, tau, n)

######### lag -1 Z

load.Hst.ls.Z(Z, Hst.ls.Z=Hst.ls, 1, -1)

######### lag 0 Z -- default

load.Hst.ls.Z(Z, Hst.ls.Z=Hst.ls, 1, 0)

######### lag +1 Z

load.Hst.ls.Z(Z, Hst.ls.Z=Hst.ls, 1, +1)</pre>
```

MSS.snow

Metaheuristic Stochastic Search

# **Description**

Locate WIDALS hyperparameters

# Usage

```
MSS.snow(FUN.source, current.best, p.ndx.ls, f.d, sds.mx, k.glob, k.loc.coef, X = NULL)
```

# **Arguments**

FUN.source	Search function definitions (see Details). A path to source code, or function, e.g., fun.load.widals.a.
current.best	An initial cost. A scalar. Setting to NA will cause MSS. snow to make an initial pass over the data to create an initial cost to beat.
p.ndx.ls	Hyperparameter indices (of GP) to search. A list of vectors. For example, list( $c(1,2)$ , $c(3,4,5)$ ) will instruct MSS. snow, for each local search, to search over the first two hyperparameters as a pair, then to search the last three as a group.

f.d	Local search functions. A list of functions (one for each element of GP). Typically, for WIDALS, all five will be dlog.norm.
sds.mx	The standard deviations for f.d. An $k.glob \times q$ matrix, where $q$ is the number of hyperparameters, i.e., the length of GP.
k.glob	The number of global searches. A scalar integer.
k.loc.coef	The coeficient for the number of local searches to make. A scalar integer.
Х	A placeholder for values to be passed between functions inside MSS. snow (see Details).

#### **Details**

This function requires the presence of a number of values and functions out-of-scope. It is assumed that these are available in the Global Environment. They are: run.parallel (boolean), FUN.MH (a function that creates, for a given GP, a cost), FUN.GP (a function that applies constraints to GP), FUN.I (a function that does something when local searches have reduced the cost), FUN.EXIT (a function that does something when MSS.snow is done).

Examine the code for fun.load.widals.a for an example of the four functions described above. Note that these four functions may themselves require objects out-of-scope.

In general, for a given R session, special care should be taken concerning the naming and assigning of the following objects: Z (the space-time data), Z.na (a boolean matrix indicating missing values in Z), locs (site locations), Hs (spacial covariates), Ht (temporal covariates), Hst.ls (space-time covariates), lags (temporal lag vector), b.lag (the ALS lag), cv (cross-validation switch), xgeodesic (boolean), ltco (weight cut-off), GP (hyperparameter vector), run.parralel (boolean), stnd.d (boolean), train.rng (time index vector), test.rng (time index vector).

#### Value

Nothing. After completion, the best hyperparameters, GP, are assigned to the Global Environment.

#### See Also

```
Hals.fastcv.snow, Hals.snow, widals.snow.
```

```
##### simulate a state-space system (using pkg SSsimple)
## Not run:
### using dontrun because of excessive run time for CRAN submission
set.seed(9999)
library(SSsimple)
tau <- 77 #### number of time points</pre>
```

```
d.alpha <- 2
R.scale <- 1
sigma2 <- 0.01
F <- 0.999
Q <- 0.1
udom <- (0:300)/100
plot( udom, R.scale * exp(-d.alpha*udom) , type="1", col="red" ) #### see the covariogram
n.all <- 70 ##### number of spacial locations</pre>
set.seed(9999)
locs.all <- cbind(runif(n.all, -1, 1), runif(n.all, -1, 1)) #### random location of sensors</pre>
D.mx <- distance(locs.all, locs.all, FALSE) #### distance matrix</pre>
#### create measurement variance using distance and covariogram
R.all <- exp(-d.alpha*D.mx) + diag(sigma2, n.all)</pre>
Hs.all <- matrix(1, n.all, 1) #### constant mean function</pre>
##### use SSsimple to simulate system
xsssim <- SS.sim(F=F, H=Hs.all, Q=Q, R=R.all, length.out=tau, beta0=0)</pre>
Z.all <- xsssim$Z ###### system observation matrix</pre>
######## now make assignments required by MSS.snow
##### randomly remove five sites to serve as interpolation points
ndx.interp <- sample(1:n.all, size=5)</pre>
ndx.support <- I(1:n.all)[ -ndx.interp ] ##### support sites</pre>
######### what follows are important assignments,
######## since MSS.snow and the four helper functions
######## will look for these in the Global Environment
######## to commence fitting the model (as noted in Details above)
train.rng <- 30:(tau) ; test.rng <- train.rng</pre>
Z <- Z.all[ , ndx.support ]</pre>
Hs <- Hs.all[ ndx.support, , drop=FALSE]</pre>
locs <- locs.all[ndx.support, , drop=FALSE]</pre>
Ht <- NULL
Hst.ls <- NULL
lags <- c(0)
b.lag <- c(-1)
```

```
cv <- -2
xgeodesic <- FALSE
stnd.d <- FALSE
ltco <- -10
GP \leftarrow c(1/10, 1, 20, 20, 1) ### -- initial hyperparameter values
run.parallel <- TRUE</pre>
if( cv=2 ) { rm.ndx <- create.rm.ndx.ls( <math>nrow(Hs), 14 ) } else { rm.ndx <- 1:nrow(Hs) }
rgr.lower.limit <- 10^{-7}; d.alpha.lower.limit <- 10^{-3}; rho.upper.limit <- 10^{4}
########## tell snowfall to use two threads for local searches
sfInit(TRUE, cpus=2)
fun.load.widals.a()
####### now, finally, search for best fit over support
####### Note that p.ndx.ls and f.d are produced inside fun.load.widals.a()
MSS.snow(fun.load.widals.a, NA, p.ndx.ls, f.d, matrix(1/10, 10, length(GP)), 10, 7)
sfStop()
####### we can use these hyperparameters to interpolate to the
\#\#\#\#\#\# deliberately removed sites, and measure MSE, RMSE
Z0.hat <- widals.predict(Z, Hs, Ht, Hst.ls, locs, lags, b.lag,</pre>
Hs0=Hs.all[ ndx.interp, , drop=FALSE ],
Hst0.ls=NULL, locs0=locs.all[ ndx.interp, , drop=FALSE],
geodesic = xgeodesic, wrap.around = NULL, GP, stnd.d = stnd.d, ltco = ltco)
resids.wid <- ( Z.all[ , ndx.interp ] - Z0.hat )</pre>
mse.wid <- mean( resids.wid[ test.rng, ]^2 )</pre>
mse.wid
sqrt(mse.wid)
Z.all <- xsssim$Z</pre>
Z.missing <- Z.all</pre>
Z.na.all <- matrix( sample(c(TRUE, FALSE), size=n.all*tau, prob=c(0.01, 0.99), replace=TRUE),</pre>
tau, n.all)
Z.missing[ Z.na.all ] <- NA</pre>
Z <- Z.missing
Z[ is.na(Z) ] <- mean(Z, na.rm=TRUE)</pre>
X <- list("Z.fill"=Z)</pre>
Z.na <- Z.na.all</pre>
Hs <- Hs.all
```

locs <- locs.all
Ht <- NULL</pre>

```
Hst.ls <- NULL
lags \leftarrow c(0)
b.lag <- c(-1)
cv <- -2
xgeodesic <- FALSE
ltco <- -10
if( cv==2 ) { rm.ndx <- create.rm.ndx.ls( nrow(Hs), 14 ) } else { <math>rm.ndx <- 1:nrow(Hs) }
GP <- c(1/10, 1, 20, 20, 1)
rgr.lower.limit \leftarrow 10^{-7}; d.alpha.lower.limit \leftarrow 10^{-3}; rho.upper.limit \leftarrow 10^{4})
run.parallel <- TRUE
sfInit(TRUE, cpus=2)
fun.load.widals.fill()
MSS.snow(fun.load.widals.fill, NA, p.ndx.ls, f.d,
seq(2, 0.01, length=10)*matrix(1/10, 10, length(GP)), 10, 7, X=X)
sfStop()
sqrt(mean(( (Z.all[train.rng, ] - Z.fill[train.rng, ])^2 )[ Z.na[ train.rng, ] ]))
######### Now Try with ALS alone
Z.all <- xsssim$Z</pre>
GP \leftarrow c(1/10, 1) ### -- initial hyperparameter values
########## tell snowfall to use two threads for local searches
sfInit(TRUE, cpus=2)
fun.load.hals.a()
####### now, finally, search for best fit over support
####### Note that p.ndx.ls and f.d are produced inside fun.load.widals.a()
MSS.snow(fun.load.hals.a, NA, p.ndx.ls, f.d, matrix(1/10, 10, length(GP)), 10, 7)
sfStop()
####### we can use these hyperparameters to interpolate to the deliberately removed sites,
####### and measure MSE, RMSE
hals.obj <- H.als.b(Z, Hs, Ht, Hst.ls, rho=GP[1], reg=GP[2], b.lag = b.lag,
Hs0 = Hs.all[ ndx.interp, , drop=FALSE ], Ht0 = NULL, Hst0.ls = NULL)
Z0.hat <- hals.obj$Z0.hat</pre>
```

O3

```
resids.als <- ( Z.all[ , ndx.interp ] - Z0.hat )</pre>
mse.als <- mean( resids.als[ test.rng, ]^2 )</pre>
mse.als
sqrt(mse.als)
Z.all <- xsssim$Z</pre>
Z.missing <- Z.all</pre>
set.seed(99)
Z.na.all <- matrix( sample(c(TRUE, FALSE), size=n.all*tau, prob=c(0.03, 0.97), replace=TRUE),</pre>
tau, n.all)
Z.missing[ Z.na.all ] <- NA</pre>
Z <- Z.missing
Z[ is.na(Z) ] <- 0 #mean(Z, na.rm=TRUE)</pre>
X <- list("Z.fill"=Z)</pre>
Z.na <- Z.na.all</pre>
Hs <- Hs.all
GP \leftarrow c(1/10, 1) ### -- initial hyperparameter values
sfInit(TRUE, cpus=2)
fun.load.hals.fill()
MSS.snow(fun.load.hals.fill, NA, p.ndx.ls, f.d,
seq(3, 0.01, length=10)*matrix(1, 10, length(GP)), 10, 7, X=X)
sfStop()
sqrt(mean(( (Z.all[train.rng, ] - Z.fill[train.rng, ])^2 )[ Z.na[ train.rng, ] ]))
## End(Not run)
```

03

California Ozone

#### **Description**

Daily airborne Ozone concentrations (ppb) over California, 68 fixed sensors, 2005-2006

#### Usage

```
data(03)
```

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#### **Format**

The format is:

List of 5

\$Z:'data.frame': 730 obs. of 68 variables: Ozone ppb for 68 sensors.

\$locs:'data.frame': 68 obs. of 2 variables: longitude, latitude for 68 sites.

\$helevs: elevations (meters) for 68 sites.

\$locs0: 2358 obs. of 2 variables: longitude, latitude for interpolation grid.

\$helevs0: elevations (meters) for 2358 interpolation grid sites.

#### **Source**

The Ozone data originates from the California Air Resources Board (CARB). The interpolation grid elevations originate from the Google Elevation API.

#### References

The O3 data comes from the California Air Resources Board (CARB).

#### **Examples**

data(03)

rm.cols.Hst.ls

Remove Space-Time Covariates from Model

#### **Description**

Remove spacial covariates from space-time covariate list

## Usage

```
rm.cols.Hst.ls(Hst.ls, rm.col.ndx)
```

#### **Arguments**

Hst.1s Space-time covariates (of supporting sites). A list of length  $\tau$ , each element

should be a numeric  $n \times p_s t$  matrix.

rm.col.ndx Which columns of Hst.ls to remove. A positive scalar integer.

# Value

An unnamed list of length  $\tau$ , each element will be a numeric n x  $p_s t - p_r m$  matrix, where  $p_r m$  is the length of rm.col.ndx.

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#### **Examples**

```
tau <- 21
n <- 7

pst <- 5
Hst.ls <- list()
for(i in 1:tau) { Hst.ls[[i]] <- matrix(1:pst, n, pst, byrow=TRUE) }

rm.cols.Hst.ls(Hst.ls, c(1,3))

## The function is currently defined as function (Hst.ls, rm.col.ndx)
{
    tau <- length(Hst.ls)
    for (i in 1:tau) {
        Hst.ls[[i]] <- Hst.ls[[i]][, -rm.col.ndx, drop = FALSE]
    }
    return(Hst.ls)
}</pre>
```

stnd.Hs

Standardize Spacial Covariates

#### **Description**

Standardize spacial covariates with respect to both the space and time dimensions

# Usage

```
stnd.Hs(Hs, Hs0 = NULL, intercept = TRUE)
```

# **Arguments**

Hs Spacial covariates (of supporting sites). An  $n \times p_s$  numeric matrix. Hs0 Spacial covariates (of interpolation sites). An  $n^* \times p_s$  numeric matrix.

intercept Include intercept term? Boolean.

#### Value

A named list.

sHs An  $n \ge p_s$  numeric matrix. sHs0 An  $n^* \ge p_s$  numeric matrix. h.mean The covariates' mean over space.

h.sd The covariates' standard deviation over space.

n Number of support sites.

intercept The supplied intercept argument.

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#### See Also

```
stnd.Ht, stnd.Hst.ls, applystnd.Hs.
```

# **Examples**

```
##### Please see the examples in Hst.sumup
## The function is currently defined as
function (Hs, Hs0 = NULL, intercept = TRUE)
    n <- nrow(Hs)
    h.mean <- apply(Hs, 2, mean)</pre>
    h.sd \leftarrow apply(t(t(Hs) - h.mean), 2, function(x) {
        sqrt(sum(x^2))
    })
    h.sd[h.sd == 0] <- 1
    sHs \leftarrow t((t(Hs) - h.mean)/h.sd)
    if (intercept) {
        sHs[, 1] <- 1/sqrt(n)
    sHs0 <- NULL
    if (!is.null(Hs0)) {
        sHs0 \leftarrow t((t(Hs0) - h.mean)/h.sd)
        if (intercept) {
             sHs0[, 1] <- 1/sqrt(n)
        }
    ls.out <- list(sHs = sHs, sHs0 = sHs0, h.mean = h.mean, h.sd = h.sd,</pre>
        n = n, intercept = intercept)
    return(ls.out)
  }
```

stnd.Hst.ls

Standardize Space-Time Covariates

# **Description**

Standardize spacio-temporal covariates with respect to both the spacial and time dimensions

#### Usage

```
stnd.Hst.ls(Hst.ls, Hst0.ls = NULL)
```

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# **Arguments**

Hst.ls	Space-time covariates (of supporting sites). A list of length $\tau$ , each element should be a numeric $n$ x $p_s t$ matrix.
Hst0.ls	Space-time covariates (of interpolation sites). A list of length $\tau$ , each element should be a numeric $n^*$ x $p_s t$ matrix.

#### Value

A named list.

```
sHst.ls A list of length \tau, each element a numeric n \times p_s t matrix. sHst0.ls A list of length \tau, each element a n^* \times p_s t matrix h.mean The covariates' mean over space-time. h.sd The covariates' standard deviation over space-time.
```

#### See Also

```
stnd.Ht, stnd.Hs, applystnd.Hst.ls.
```

```
##### Please see the examples in Hst.sumup
## The function is currently defined as
function (Hst.ls, Hst0.ls = NULL)
    tau <- length(Hst.ls)</pre>
    big.sum <- 0
    for (i in 1:tau) {
        big.sum <- big.sum + apply(Hst.ls[[i]], 2, mean)</pre>
    h.mean <- big.sum/tau
    sHst.ls <- list()
    big.sum.mx <- 0
    for (i in 1:tau) {
        sHst.ls[[i]] \leftarrow t(t(Hst.ls[[i]]) - h.mean)
        big.sum.mx <- big.sum.mx + crossprod(sHst.ls[[i]])</pre>
    }
    cov.mx <- big.sum.mx/tau</pre>
    sqrtXX <- 1/sqrt(diag(cov.mx))</pre>
    for (i in 1:tau) {
        sHst.ls[[i]] <- t(t(sHst.ls[[i]]) * sqrtXX)</pre>
    sHst0.1s \leftarrow NULL
    if (!is.null(Hst0.ls)) {
        sHst0.ls <- list()
        for (i in 1:tau) {
             sHst0.ls[[i]] \leftarrow t((t(Hst0.ls[[i]]) - h.mean) * sqrtXX)
        }
```

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stnd.Ht

Standardize Temporal Covariates

# **Description**

Standardize temporal covariates with respect to both the spacial and time dimensions

## Usage

```
stnd.Ht(Ht, n)
```

# Arguments

Ht Temporal covariates (of supporting sites). A  $\tau$  x  $p_t$  numeric matrix.

n Number of sites. A positive scalar integer.

#### Value

A  $\tau$  x  $p_t$  numeric matrix.

# See Also

```
stnd.Hs, stnd.Hst.ls.
```

```
##### Please see the examples in Hst.sumup

## The function is currently defined as
function (Ht, n)
{
    h.mean <- apply(Ht, 2, mean)
    sHt <- t(t(Ht) - h.mean)
    sHt <- t(t(sHt)/apply(sHt, 2, function(x) {
        sqrt(sum(x^2))
    }))
    sHt <- sHt * sqrt(nrow(Ht)/n)
    return(sHt)
}</pre>
```

subsetsites.Hst.ls 39

```
subsetsites.Hst.ls Site-Wise Extract Space-Time Covariates
```

## Description

Extract space-time covariates by site

# Usage

```
subsetsites.Hst.ls(Hst.ls, xmask)
```

#### **Arguments**

Hst.ls Space-time covariates. A list of length  $\tau$ , each element should be a  $n \times p_s t$ 

numeric matrix.

xmask Which sites to remove from Hst.1s. A boolean vector of length n, or a vector

of spacial indices.

#### Value

Space-time covariates. A list of length  $\tau$ , each element a  $c \ge p_s t$  numeric matrix, where c is the number of TRUE's in boolean xmask, or length of index xmask.

```
tau <- 70
n <- 28

Hst.ls <- list()
for(i in 1:tau) { Hst.ls[[i]] <- matrix(rnorm(n*4), nrow=n) }

subsetsites.Hst.ls(Hst.ls, c(1,3,10))

subsetsites.Hst.ls(Hst.ls, c(TRUE, TRUE, rep(FALSE, n-2)))

## The function is currently defined as function (Hst.ls, xmask)
{
    tau <- length(Hst.ls)
    Hst.ls.out <- list()
    for (i in 1:tau) {
        Hst.ls.out[[i]] <- Hst.ls[[i]][xmask, , drop = FALSE]
    }
    return(Hst.ls.out)
}</pre>
```

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unif.mh

Local Search Function

# Description

Search function

# Usage

```
unif.mh(n, center, sd)
```

# Arguments

n Sample size. A positive scalar integer.
center Mean. A numeric scalar (or vector).
sd Standard deviation. A numeric scalar (or vector).

# Value

A numeric vector of length n.

# See Also

```
dlog.norm, MSS.snow.
```

```
x <- unif.mh(100, 1, 1)
hist(x)

## The function is currently defined as
function (n, center, sd)
{
    w <- sd * sqrt(3)
    a <- center - w
    b <- center + w
    x <- runif(n, a, b)
    return(x)
}</pre>
```

unload.Hst.ls 41

unload.Hst.ls

Convert a Space-Time Covariate into Data

# **Description**

Convert a spacio-temporal covariate into contemporaneous data

# Usage

```
unload.Hst.ls(Hst.ls, which.col, rgr.lags)
```

# **Arguments**

Hst.ls	Space-time covariates. A list of length $\tau$ , each element should be a $n \ge p_s t$ numeric matrix.
which.col	Which column of ${\tt Hst.ls[[i]]}$ to insert into the $i{\tt th}$ row of Z. A scalar positive integer.
rgr.lags	Temporal lagging of Z. A scalar integer.

#### Value

A numeric  $\tau$  x n matrix.

#### See Also

```
load.Hst.ls.Z, load.Hst.ls.2Zs.
```

```
###### here's an itty-bitty example

tau <- 7
n <- 5

Hst.ls <- list()
for(i in 1:tau) { Hst.ls[[i]] <- matrix(rnorm(n*4), nrow=n) }

Zh <- unload.Hst.ls(Hst.ls, 1, 0)

## The function is currently defined as
function (Hst.ls, which.col, rgr.lags)
{
    n <- nrow(Hst.ls[[i]])
    tau <- length(Hst.ls)
    Z.out <- matrix(NA, tau, n)
    min.ndx <- max(1, -min(rgr.lags))</pre>
```

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```
for (i in min.ndx:max.ndx) {
     Z.out[i - rgr.lags, ] <- Hst.ls[[i]][, which.col]
}
return(Z.out)
}</pre>
```

widals.predict

WIDALS Interpolation

# **Description**

Interpolate to unmonitored sites using WIDALS

# Usage

```
widals.predict(Z, Hs, Ht, Hst.ls, locs, lags, b.lag, Hs0, Hst0.ls, locs0, geodesic = FALSE, wrap.around = NULL, GP, stnd.d = FALSE, ltco = -16)
```

# **Arguments**

Z	Space-time data. A $\tau$ x $n$ numeric matrix.
Hs	Spacial covariates (of supporting sites). An $n \times p_s$ numeric matrix.
Ht	Temporal covariates (of supporting sites). A $\tau$ x $p_t$ numeric matrix.
Hst.1s	Space-time covariates (of supporting sites). A list of length $\tau$ , each element should be a $n \times p_s t$ numeric matrix.
locs	Locations of supporting sites. An $n \times 2$ numeric matrix, first column is spacial $x$ , second column is spacial $y$ . If the geodesic is TRUE, make sure latitude is in the first column.
lags	Temporal lags for stochastic smoothing. An integer vector or scalar. E.g., if the data's time increment is daily, then lags = $c(-1,0,1)$ would tell the enclosed function crispify smooth today's predictions using yesterdays, today's, and tomorrow's observed residuals.
b.lag	ALS lag. A scalar integer, typically -1 (a-prior), or 0 (a-posteriori).
Hs0	Spacial covariates (of interpolation sites). An $n^*$ x $p_s$ matrix, or NULL.
Hst0.ls	Space-time covariates (of interpolation sites). A list of length $\tau$ , each element should be a numeric $n$ x $p_s t$ matrix.
locs0	Locations of interpolation sites. An $n^*$ x 2 numeric matrix. See locs argument above.
geodesic	Use geodesic distance? Boolean. If true, distance (used internally) is in units kilometers.
wrap.around	**Unused.
GP	Widals hyperparameters. A non-negative vector.
stnd.d	Spacial compression. Boolean.
ltco	Weight threshold. A scalar number. A value of, e.g., -10, will instruct crispify to ignore weights less than 10^(-10) when smoothing.

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#### Value

A  $\tau$  x  $n^*$  matrix. The WIDALS predictions at locs0.

#### See Also

```
crispify, H.als.b, widals.snow.
```

```
#### similar to example provided in H.als.b.
set.seed(99999)
library(SSsimple)
tau <- 70
n.all <- 14
Hs.all <- matrix(rnorm(n.all), nrow=n.all)</pre>
Ht <- matrix(rnorm(tau*2), nrow=tau)</pre>
Hst.ls.all <- list()</pre>
for(i in 1:tau) { Hst.ls.all[[i]] <- matrix(rnorm(n.all*2), nrow=n.all) }</pre>
Hst.combined <- list()</pre>
for(i in 1:tau) {
    Hst.combined[[i]] <- cbind( Hs.all, matrix(Ht[i, ], nrow=n.all, ncol=ncol(Ht),</pre>
    byrow=TRUE), Hst.ls.all[[i]] )
}
locs.all <- cbind(runif(n.all, -1, 1), runif(n.all, -1, 1))</pre>
D.mx.all <- distance(locs.all, locs.all, FALSE)</pre>
R.all <- exp(-2*D.mx.all) + diag(0.01, n.all)
####### use SSsimple to simulate
sssim.obj <- SS.sim.tv( 0.999, Hst.combined, 0.01, R.all, tau )</pre>
ndx.support <- 1:10
ndx.interp <- 11:14
locs <- locs.all[ndx.support, ]</pre>
locs0 <- locs.all[ndx.interp, ]</pre>
Z.all <- sssim.obj$Z</pre>
Z <- Z.all[ , ndx.support]</pre>
Z0 <- Z.all[ , ndx.interp]</pre>
Hst.ls <- subsetsites.Hst.ls(Hst.ls.all, ndx.support)</pre>
Hst0.ls <- subsetsites.Hst.ls(Hst.ls.all, ndx.interp)</pre>
```

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```
Hs <- Hs.all[ ndx.support, , drop=FALSE]
Hs0 <- Hs.all[ ndx.interp, , drop=FALSE]</pre>
test.rng <- 20:tau
########## use ALS
xrho <- 1/10
xreg <- 1/10
xALS <- H.als.b(Z=Z, Hs=Hs, Ht=Ht, Hst.ls=Hst.ls, rho=xrho, reg=xreg,
b.lag=-1, Hs0=Hs0, Ht0=Ht, Hst0.ls=Hst0.ls)
errs.sq <- (Z0 - xALS$Z0.hat)^2
sqrt( mean(errs.sq[test.rng, ]) )
########### now use WIDALS
GP \leftarrow c(1/10, 1/10, 2, 0, 1)
Zwid <- widals.predict(Z=Z, Hs=Hs, Ht=Ht, Hst.ls=Hst.ls, locs=locs, lags=c(0),</pre>
b.lag=-1, Hs0=Hs0, Hst0.ls=Hst0.ls, locs0=locs0, FALSE, NULL, GP)
errs.sq <- (Z0 - Zwid)^2
sqrt( mean(errs.sq[test.rng, ]) )
```

widals.snow

Fit WIDALS

# **Description**

Locate the WIDALS hyperparameters

#### Usage

```
widals.snow(j, rm.ndx, Z, Hs, Ht, Hst.ls, locs, lags, b.lag, cv = 0, geodesic = FALSE, wrap.around = NULL, GP.mx, stnd.d = FALSE, ltco = -16)
```

# Arguments

j	Index used by snowfall. A scalar integer. Which row of GP.mx to use for the ALS hyperparameters, GP.
rm.ndx	A list of vectors of indices to remove for k-fold cross-validation.
Z	Data. A $\tau$ x $n$ numeric matrix.
Hs	Spacial covariates. An $n \times p_s$ numeric matrix.
Ht	Temporal covariates. A $\tau$ x $p_t$ numeric matrix.
Hst.ls	Space-time covariates. A list of length $\tau$ , each element containing a $n \times p_s t$ numeric matrix.

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locs	Locations of supporting sites. An $n \times 2$ numeric matrix, first column is spacial $x$ , second column is spacial $y$ . If the geodesic is TRUE, make sure latitude is in the first column.
lags	Temporal lags for stochastic smoothing. An integer vector or scalar. E.g., if the data's time increment is daily, then $lags = c(-1,0,1)$ would tell the enclosed function crispify smooth today's predictions using yesterdays, today's, and tomorrow's observed residuals.
b.lag	ALS lag. A scalar integer, typically -1 (a-prior), or 0 (a-posteriori).
CV	Cross-validation switch. Currently takes on a value of -2 or 2. See Details below.
geodesic	Use geodesic distance? Boolean. If true, distance (used internally) is in units kilometers.
wrap.around	**Unused.
GP.mx	Hyperparameters. A $k.glob \times 2$ non-negative matrix. See MSS. snow.
stnd.d	Spacial compression. Boolean.
ltco	Weight threshold. A scalar number. A value of, e.g., -10, will instruct crispify to ignore weights less than 10^(-10) when smoothing.

#### **Details**

When the cv is set to 2, then this function uses spacial k-fold validation, according to the site indices present in rm.ndx. When cv is set to -2, self-referencing sites are given zero-weight, i.e., a site's value is not allowed to contribute to its predicted value.

#### Value

A  $\tau$  x n matrix. The WIDALS predictions at locs.

# See Also

```
crispify, H.als.b, widals.predict.
```

```
set.seed(99999)
library(SSsimple)

tau <- 100
n.all <- 35

Hs.all <- matrix(rnorm(n.all), nrow=n.all)
Ht <- matrix(rnorm(tau*2), nrow=tau)
Hst.ls.all <- list()
for(i in 1:tau) { Hst.ls.all[[i]] <- matrix(rnorm(n.all*2), nrow=n.all) }

Hst.combined <- list()</pre>
```

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```
for(i in 1:tau) {
    Hst.combined[[i]] <- cbind( Hs.all, matrix(Ht[i, ], nrow=n.all, ncol=ncol(Ht),</pre>
    byrow=TRUE), Hst.ls.all[[i]] )
}
locs.all <- cbind(runif(n.all, -1, 1), runif(n.all, -1, 1))</pre>
D.mx.all <- distance(locs.all, locs.all, FALSE)</pre>
R.all \leftarrow exp(-2*D.mx.all) + diag(0.01, n.all)
####### use SSsimple to simulate
sssim.obj <- SS.sim.tv( 0.999, Hst.combined, 0.01, R.all, tau )</pre>
n <- n.all
locs <- locs.all</pre>
Z.all <- sssim.obj$Z</pre>
Z <- Z.all
Hst.ls <- Hst.ls.all
Hs <- Hs.all
test.rng <- 20:tau
########### WIDALS, true cross-validation
rm.ndx <- create.rm.ndx.ls(n, 10)</pre>
cv <- 2
lags <- c(0)
b.lag <- 0
GP \leftarrow c(1/8, 1/12, 5, 0, 1)
GP.mx <- matrix(GP, ncol=length(GP))</pre>
Zwid <- widals.snow(j=1, rm.ndx, Z, Hs, Ht, Hst.ls, locs, lags, b.lag, cv = cv,
geodesic = FALSE, wrap.around = NULL, GP.mx, stnd.d = FALSE, 1tco = -16)
errs.sq <- (Z - Zwid)^2
sqrt( mean(errs.sq[test.rng, ]) )
########### WIDALS, pseudo cross-validation
rm.ndx \leftarrow I(1:n)
cv <- -2
lags <- c(0)
b.lag <- -1
GP \leftarrow c(1/8, 1/12, 5, 0, 1)
GP.mx <- matrix(GP, ncol=length(GP))</pre>
Zwid <- widals.snow(j=1, rm.ndx, Z, Hs, Ht, Hst.ls, locs, lags, b.lag, cv = cv,
```

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```
geodesic = FALSE, wrap.around = NULL, GP.mx, stnd.d = FALSE, ltco = -16)
errs.sq <- (Z - Zwid)^2
sqrt( mean(errs.sq[test.rng, ]) )</pre>
```

Z.clean.up

Clean Data

# Description

A crude, brute-force way to destroy bad values in data.

# Usage

```
Z.clean.up(Z)
```

# **Arguments**

Ζ

Data. A  $\tau$  x n matrix.

#### **Details**

This function replaces intractable values, e.g., NA, or -Inf, in data, with the global mean.

#### Value

A  $\tau$  x n numeric matrix.

# **Examples**

```
tau <- 10
n <- 7

Z <- matrix(1, tau, n)
Z[2,4] <- -Inf
Z[3,4] <- Inf
Z[4,4] <- NA
Z[5,4] <- log(-1)
Z

Z.clean.up(Z)</pre>
```

## The function is currently defined as

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```
function (Z)
{
    Z[Z == Inf | Z == -Inf] <- NA
    Z[is.na(Z) | is.nan(Z)] <- mean(Z, na.rm = TRUE)
    return(Z)
}</pre>
```

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