# Package 'BivGeo'

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Type Package

Title Basu-Dhar Bivariate Geometric Distribution

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<b>Description</b> Computes the joint probability mass function (pmf), the joint cumulative function (cdf), the joint survival function (sf), the correlation coefficient, the covariance, the crossfactorial moment and generate random deviates for the Basu-Dhar bivariate geometric distribution as well the joint probability mass, cumulative and survival function assuming the presence of a cure fraction given by the standard bivariate mixture cure fraction model. The package also computes the estimators based on the method of moments.
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## **Description**

This function computes the cross-factorial moment for the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

## Usage

cfbivgeo(theta)

## **Arguments**

theta

vector (of length 3) containing values of the parameters  $\theta_1, \theta_2$  and  $\theta_3$  of the Basu-Dhar bivariate Geometric distribution. For real data applications, use the maximum likelihood estimates or Bayesian estimates to get the cross-factorial moment.

## **Details**

The cross-factorial moment between X and Y, assuming the Basu-Dhar bivariate geometric distribution, is given by,

$$E[XY] = \frac{1 - \theta_1 \theta_2 \theta_3^2}{(1 - \theta_1 \theta_3)(1 - \theta_2 \theta_3)(1 - \theta_1 \theta_2 \theta_3)}$$

Note that the cross-factorial moment is always positive.

### Value

**cfbivgeo** computes the cross-factorial moment for the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

Invalid arguments will return an error message.

#### Author(s)

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## **Source**

cfbivgeo is calculated directly from the definition.

#### References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, 42, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

## **Examples**

```
cfbivgeo(theta = c(0.5, 0.5, 0.7))
# [1] 2.517483
cfbivgeo(theta = c(0.2, 0.5, 0.7))
# [1] 1.829303
cfbivgeo(theta = c(0.8, 0.9, 0.1))
# [1] 1.277864
cfbivgeo(theta = c(0.9, 0.9, 0.9))
# [1] 35.15246
```

corbivgeo

Correlation Coefficient for the Basu-Dhar Bivariate Geometric Distribution

## **Description**

This function computes the correlation coefficient analogous of the Pearson correlation coefficient for the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

## Usage

```
corbivgeo(theta)
```

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#### **Arguments**

theta

vector (of length 3) containing values of the parameters  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  of the Basu-Dhar bivariate Geometric distribution. For real data applications, use the maximum likelihood estimates or Bayesian estimates to get the correlation coefficient.

#### **Details**

The correlation coefficient between X and Y, assuming the Basu-Dhar bivariate geometric distribution, is given by,

$$\rho = \frac{(1 - \theta_3)(\theta_1 \theta_2)^{1/2}}{1 - \theta_1 \theta_2 \theta_3}$$

Note that the correlation coefficient is always positive which implies that the Basu-Dhar bivariate geometric distribution is useful for bivariate lifetimes with positive correlation.

## Value

corbivgeo computes the correlation coefficient analogous to the Pearson correlation coefficient for the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

Invalid arguments will return an error message.

#### Author(s)

#### Source

corbivgeo is calculated directly from the definition.

#### References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, 42, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

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## **Examples**

```
corbivgeo(theta = c(0.5, 0.5, 0.7))
# [1] 0.1818182
corbivgeo(theta = c(0.2, 0.5, 0.7))
# [1] 0.102009
corbivgeo(theta = c(0.8, 0.9, 0.1))
# [1] 0.822926
corbivgeo(theta = c(0.9, 0.9, 0.9))
# [1] 0.3321033
```

covbivgeo

Covariance for the Basu-Dhar Bivariate Geometric Distribution

## **Description**

This function computes the covariance for the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

## Usage

```
covbivgeo(theta)
```

## **Arguments**

theta

vector (of length 3) containing values of the parameters  $\theta_1, \theta_2$  and  $\theta_3$  of the Basu-Dhar bivariate Geometric distribution. For real data applications, use the maximum likelihood estimates or Bayesian estimates to get the covariance.

#### **Details**

The covariance between X and Y, assuming the Basu-Dhar bivariate geometric distribution, is given by,

$$Cov(X,Y) = \frac{\theta_1\theta_2\theta_3(1-\theta_3)}{(1-\theta_1\theta_3)(1-\theta_2\theta_3)(1-\theta_1\theta_2\theta_3)}$$

Note that the covariance is always positive.

#### Value

covbivgeo computes the covariance for the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

Invalid arguments will return an error message.

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#### Author(s)

#### Source

covbivgeo is calculated directly from the definition.

#### References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, 42, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

## **Examples**

```
covbivgeo(theta = c(0.5, 0.5, 0.7))
# [1] 0.1506186
covbivgeo(theta = c(0.2, 0.5, 0.7))
# [1] 0.04039471
covbivgeo(theta = c(0.8, 0.9, 0.1))
# [1] 0.0834061
covbivgeo(theta = c(0.9, 0.9, 0.9))
# [1] 7.451626
```

dbivgeo

Joint Probability Mass Function for the Basu-Dhar Bivariate Geometric Distribution

## **Description**

This function computes the joint probability mass function of the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

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## Usage

```
dbivgeo1(x, y = NULL, theta, log = FALSE)
dbivgeo2(x, y = NULL, theta, log = FALSE)
```

#### **Arguments**

matrix or vector containing the data. If x is a matrix then it is considered as x Х the first column and y the second column (y argument need be setted to NULL). Additional columns and y are ignored. vector containing the data of y. It is used only if x is also a vector. Vectors x and У y should be of equal length. vector (of length 3) containing values of the parameters  $\theta_1, \theta_2$  and  $\theta_3$  of the theta Basu-Dhar bivariate Geometric distribution. The parameters are restricted to

 $0 < \theta_i < 1, i = 1, 2 \text{ and } 0 < \theta_3 \le 1.$ 

logical argument for calculating the log probability or the probability function. log

The default value is FALSE.

## **Details**

The joint probability mass function for a random vector (X, Y) following a Basu-Dhar bivariate geometric distribution could be written in two forms. The first form is described by:

$$P(X = x, Y = y) = \theta_1^{x-1} \theta_2^{y-1} \theta_3^{z_1} - \theta_1^{x} \theta_2^{y-1} \theta_3^{z_2} - \theta_1^{x-1} \theta_2^{y} \theta_2^{z_3} + \theta_1^{x} \theta_2^{y} \theta_3^{z_4}$$

where x, y > 0 are positive integers and  $z_1 = \max(x - 1, y - 1), z_2 = \max(x, y - 1), z_3 =$  $\max(x-1,y), z_4 = \max(x,y)$ . The second form is given by the conditions:

If X < Y, then

$$P(X = x, Y = y) = \theta_1^{x-1} (\theta_2 \theta_3)^{y-1} (1 - \theta_2 \theta_3) (1 - \theta_1)$$

If X = Y, then

$$P(X = x, Y = y) = (\theta_1 \theta_2 \theta_3)^{x-1} (1 - \theta_1 \theta_3 - \theta_2 \theta_3 + \theta_1 \theta_2 \theta_3)$$

If X > Y, then

$$P(X = x, Y = y) = \theta_2^{y-1} (\theta_1 \theta_3)^{x-1} (1 - \theta_1 \theta_3) (1 - \theta_2)$$

## Value

dbivgeo1 gives the values of the probability mass function using the first form of the joint pmf. dbivgeo2 gives the values of the probability mass function using the second form of the joint pmf. Invalid arguments will return an error message.

#### Author(s)

Ricardo P. Oliveira < rpuziol.oliveira@gmail.com> Jorge Alberto Achcar <achcar@fmrp.usp.br>

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#### Source

dbivgeo1 and dbivgeo2 are calculated directly from the definitions.

#### References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, 42, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

#### See Also

Geometric for the univariate geometric distribution.

## **Examples**

```
# If x and y are integer numbers:
dbivgeo1(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = FALSE)
# [1] 0.16128
dbivgeo2(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = FALSE)
# [1] 0.16128
# If x is a matrix:
matr <- matrix(c(1,2,3,5), ncol = 2)
dbivgeo1(x = matr, y = NULL, theta = c(0.2, 0.4, 0.7), log = FALSE)
# [1] 0.0451584000 0.0007080837
dbivgeo2(x = matr, y = NULL, theta = c(0.2, 0.4, 0.7), log = FALSE)
# [1] 0.0451584000 0.0007080837
# If log = TRUE:
dbivgeo1(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = TRUE)
# [1] -1.824613
dbivgeo2(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = TRUE)
# [1] -1.824613
```

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dbivgeocure	Joint Probability Mass Function for the Basu-Dhar Bivariate Geomet- ric Distribution in Presence of Cure Fraction

## **Description**

This function computes the joint probability mass function of the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values in presence of cure fraction.

## Usage

```
dbivgeocure(x, y, theta, phi11, log = FALSE)
```

## **Arguments**

х	matrix or vector containing the data. If x is a matrix then it is considered as x the first column and y the second column (y argument need be setted to NULL). Additional columns and y are ignored.
У	vector containing the data of y. It is used only if x is also a vector. Vectors x and y should be of equal length.
theta	vector (of length 3) containing values of the parameters $\theta_1, \theta_2$ and $\theta_3$ of the Basu-Dhar bivariate Geometric distribution. The parameters are restricted to $0 < \theta_i < 1, i = 1, 2$ and $0 < \theta_3 \leq 1$ .
phi11	real number containing the value of the cure fraction incidence parameter $\phi_{11}$ restricted to $0<\phi_{11}<1$ and $\phi_{11}+\phi_{10}+\phi_{01}+\phi_{00}=1$ where $\phi_{10},\phi_{01}$ and $\phi_{00}$ are the complementary cure fraction incidence parameters for the joint cdf and sf functions.
log	logical argument for calculating the log probability or the probability function. The default value is FALSE.

## **Details**

The joint probability mass function for a random vector (X, Y) following a Basu-Dhar bivariate geometric distribution in presence of cure fraction could be written as:

$$P(X=x,Y=y) = \phi_{11}(\theta_1^{x-1}\theta_2^{y-1}\theta_3^{z_1} - \theta_1^x\theta_2^{y-1}\theta_3^{z_2} - \theta_1^{x-1}\theta_2^y\theta_2^{z_3} + \theta_1^x\theta_2^y\theta_3^{z_4})$$

where x, y > 0 are positive integers and  $z_1 = \max(x - 1, y - 1), z_2 = \max(x, y - 1), z_3 = \max(x - 1, y), z_4 = \max(x, y).$ 

## Value

dbivgeocure gives the values of the probability mass function in presence of cure fraction.

Invalid arguments will return an error message.

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#### Author(s)

#### Source

dbivgeocure is calculated directly from the definition.

## References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

## See Also

Geometric for the univariate geometric distribution.

## **Examples**

```
# If log = FALSE:
dbivgeocure(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), phi11 = 0.4, log = FALSE)
# [1] 0.064512
# If log = TRUE:
dbivgeocure(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), phi11 = 0.4, log = TRUE)
# [1] -2.740904
```

mombivgeo

Moments Estimator for the Basu-Dhar Bivariate Geometric Distribution

## **Description**

This function computes the estimators based on the method of the moments for each parameter of the Basu-Dhar bivariate geometric distribution.

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## Usage

mombivgeo(x, y)

## Arguments

x matrix or vector containing the data. If x is a matrix then it is considered as x the first column and y the second column (y argument need be setted to NULL).

Additional columns and y are ignored.

y vector containing the data of y. It is used only if x is also a vector. Vectors x and

y should be of equal length.

#### **Details**

The moments estimators of  $\theta_1, \theta_2, \theta_3$  of the Basu-Dhar bivariate geometric distribution are given by:

$$\hat{\theta}_1 = \frac{\bar{Y}(1 - \bar{W})}{\bar{W}(1 - \bar{Y})}$$

$$\hat{\theta}_2 = \frac{\bar{X}(\bar{W} - 1)}{\bar{W}(\bar{X} - 1)}$$

$$\hat{\theta}_3 = \frac{\bar{X}(\bar{X}-1)(\bar{Y}-1)}{(\bar{W}-1)\bar{X}\bar{Y}}$$

## Value

mombivgeo gives the values of the moments estimator.

Invalid arguments will return an error message.

## Author(s)

Jorge Alberto Achcar <achcar@fmrp.usp.br>

#### Source

mombivgeo is calculated directly from the definition.

## References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, 42, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

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de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

## See Also

Geometric for the univariate geometric distribution.

## **Examples**

```
# Generate the data set:
set.seed(123)
x1 < - rbivgeo1(n = 1000, theta = c(0.5, 0.5, 0.7))
set.seed(123)
x2 < - rbivgeo2(n = 1000, theta = c(0.5, 0.5, 0.7))
# Compute de moment estimator by:
mombivgeo(x = x1, y = NULL) # For data set x1
              [,1]
# theta1 0.5053127
# theta2 0.5151873
# theta3 0.6640406
mombivgeo(x = x2, y = NULL) # For data set x2
              [,1]
# theta1 0.4922327
# theta2 0.5001577
# theta3 0.6993893
```

pbivgeo

Joint Cumulative Function for the Basu-Dhar Bivariate Geometric Distribution

## Description

This function computes the joint cumulative function of the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

## Usage

```
pbivgeo(x, y, theta, lower.tail = TRUE)
```

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## **Arguments**

х	matrix or vector containing the data. If x is a matrix then it is considered as x the first column and y the second column (y argument need be setted to NULL). Additional columns and y are ignored.
У	vector containing the data of y. It is used only if x is also a vector. Vectors x and y should be of equal length.
theta	vector (of length 3) containing values of the parameters $\theta_1,\theta_2$ and $\theta_3$ of the Basu-Dhar bivariate Geometric distribution. The parameters are restricted to $0<\theta_i<1,i=1,2$ and $0<\theta_3\leq 1$ .
lower.tail	logical; If TRUE (default), probabilities are $P(X \leq x, Y \leq y)$ otherwise $P(X > x, Y > y)$ .

#### **Details**

The joint cumulative function for a random vector (X, Y) following a Basu-Dhar bivariate geometric distribution could be written as:

$$P(X \le x, Y \le y) = 1 - (\theta_1 \theta_3)^x - (\theta_2 \theta_3)^y + \theta_1^x \theta_2^y \theta_3^{\max(x,y)}$$

and the joint survival function is given by:

$$P(X > x, Y > y) = \theta_1^x \theta_2^y \theta_3^{\max(x,y)}$$

#### Value

pbivgeo gives the values of the cumulative function.

Invalid arguments will return an error message.

#### Author(s)

#### Source

pbivgeo is calculated directly from the definition.

#### References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, 42, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

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## See Also

Geometric for the univariate geometric distribution.

#### **Examples**

```
# If x and y are integer numbers:

pbivgeo(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), lower.tail = TRUE)
# [1] 0.79728

# If x is a matrix:

matr <- matrix(c(1,2,3,5), ncol = 2)
pbivgeo(x = matr, y = NULL, theta = c(0.2,0.4,0.7), lower.tail = TRUE)
# [1] 0.8424384 0.9787478

# If lower.tail = FALSE:

pbivgeo(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), lower.tail = FALSE)
# [1] 0.01568

matr <- matrix(c(1,2,3,5), ncol = 2)
pbivgeo(x = matr, y = NULL, theta = c(0.9,0.4,0.7), lower.tail = FALSE)
# [1] 0.01975680 0.00139404</pre>
```

pbivgeocure

Joint Cumulative Function for the Basu-Dhar Bivariate Geometric Distribution in Presence of Cure Fraction

## **Description**

This function computes the joint cumulative function of the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values in presence of cure fraction.

## Usage

```
pbivgeocure(x, y, theta, phi, lower.tail = TRUE)
```

## **Arguments**

x matrix or vector containing the data. If x is a matrix then it is considered as x the first column and y the second column (y argument need be setted to NULL). Additional columns and y are ignored.

y vector containing the data of y. It is used only if x is also a vector. Vectors x and y should be of equal length.

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theta vector (of length 3) containing values of the parameters  $\theta_1,\theta_2$  and  $\theta_3$  of the Basu-Dhar bivariate Geometric distribution. The parameters are restricted to  $0<\theta_i<1,i=1,2$  and  $0<\theta_3\leq 1$ . phi vector (of length 4) containing values of the cure fraction incidence parameters  $\phi_{11},\phi_{10},\phi_{01}$  and  $\phi_{00}$ . The parameters are restricted to  $\phi_{11}+\phi_{10}+\phi_{01}+\phi_{00}=1$ .

lower.tail logical; If TRUE (default), probabilities are  $P(X \le x, Y \le y)$  otherwise P(X > x, Y > y).

#### **Details**

The joint cumulative function for a random vector (X, Y) following a Basu-Dhar bivariate geometric distribution in presence of cure fraction could be written as:

$$P(X \le x, Y \le y) = 1 - (\phi_{11} + \phi_{10})(\theta_1 \theta_3)^x - (\phi_{01} + \phi_{00}) - (\phi_{11} + \phi_{01})(\theta_2 \theta_3)^y - (\phi_{10} + \phi_{00})$$
$$+\phi_{11}(\theta_1^x \theta_2^y \theta_3^{\max(x,y)}) + \phi_{10}(\theta_1 \theta_3)^x + \phi_{01}(\theta_2 \theta_3)^y + \phi_{00}$$

and the joint survival function is given by:

$$P(X > x, Y > y) = \phi_{11}(\theta_1^x \theta_2^y \theta_3^{\max(x,y)}) + \phi_{10}(\theta_1 \theta_3)^x + \phi_{01}(\theta_2 \theta_3)^y + \phi_{00}$$

#### Value

pbivgeocure gives the values of the cumulative function in presence of cure fraction. Invalid arguments will return an error message.

#### Author(s)

#### **Source**

pbivgeocure is calculated directly from the definition.

## References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

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## See Also

Geometric for the univariate geometric distribution.

## **Examples**

```
# If lower.tail = TRUE:

pbivgeocure(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), phi = c(0.2, 0.3, 0.3, 0.2), lower.tail = TRUE)
# [1] 0.159456

matr <- matrix(c(1,2,3,5), ncol = 2)
pbivgeocure(x=matr,y=NULL,theta=c(0.2, 0.4, 0.7),phi=c(0.2, 0.3, 0.3, 0.2),lower.tail = TRUE)
# [1] 0.1684877 0.1957496

# If lower.tail = FALSE:

pbivgeocure(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), phi = c(0.2, 0.3, 0.3, 0.2), lower.tail = FALSE)
# [1] 0.268656

matr <- matrix(c(1,2,3,5), ncol = 2)
pbivgeocure(x=matr,y=NULL,theta=c(0.2, 0.4, 0.7),phi=c(0.2, 0.3, 0.3, 0.2),lower.tail = FALSE)
# [1] 0.2494637 0.2064101</pre>
```

rbivgeo

Generates Random Deviates from the Basu-Dhar Bivariate Geometric Distribution

## **Description**

This function generates random values from the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

## Usage

```
rbivgeo1(n, theta)
rbivgeo2(n, theta)
```

## **Arguments**

n number of observations. If length(n) > 1, the length is taken to be the number

required.

theta vector (of length 3) containing values of the parameters  $\theta_1, \theta_2$  and  $\theta_3$  of the

Basu-Dhar bivariate Geometric distribution. The parameters are restricted to

 $0 < \theta_i < 1, i = 1, 2 \text{ and } 0 < \theta_3 \le 1.$ 

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#### **Details**

The conditional distribution of X given Y is given by:

If X < Y, then

$$P(X = x | Y = y) = \theta_1^{x-1} (1 - \theta_1)$$

If X = Y, then

$$P(X = x | Y = y) = \frac{\theta_1^{x-1} (1 - \theta_1 \theta_3 - \theta_2 \theta_3 + \theta_1 \theta_2 \theta_3)}{1 - \theta_2 \theta_3}$$

If X > Y, then

$$P(X = x | Y = y) = \frac{\theta_1^{x-1} \theta_3^{x-y} (1 - \theta_1 \theta_3) (1 - \theta_2)}{1 - \theta_2 \theta_3}$$

#### Value

rbivgeo1 and rbivgeo2 generate random deviates from the Bash-Dhar bivariate geometric distribution. The length of the result is determined by n, and is the maximum of the lengths of the numerical arguments for the other functions.

Invalid arguments will return an error message.

#### Author(s)

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#### Source

rbivgeo1 generates random deviates using the inverse transformation method. Returns a matrix that the first column corresponds to X generated random values and the second column corresponds to Y generated random values.

rbivgeo2 generates random deviates using the shock model. Returns a matrix that the first column corresponds to X generated random values and the second column corresponds to Y generated random values. See Marshall and Olkin (1967) for more details.

## References

Marshall, A. W., & Olkin, I. (1967). A multivariate exponential distribution. *Journal of the American Statistical Association*, **62**, 317, 30-44.

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, 2, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

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## See Also

Geometric for the univariate geometric distribution.

## **Examples**

```
rbivgeo1(n = 10, theta = c(0.5, 0.5, 0.7))
      [,1] [,2]
# [1,] 2
# [2,]
       3
          1
# [3,]
       1 1
# [4,]
       1 1
# [5,]
       2 2
# [6,]
       1 3
# [7,]
       2 2
# [8,]
       1
          1
# [9,]
       1 1
# [10,]
        2
            2
rbivgeo2(n = 10, theta = c(0.5, 0.5, 0.7))
      [,1] [,2]
 [1,]
       1
# [2,]
        2
            1
# [3,]
           1
        2
# [4,]
        4
          1
# [5,]
       1 1
# [6,]
       2 2
# [7,]
       3 2
# [8,]
       3 1
       3 2
# [9,]
# [10,]
       1 1
```

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