# Package 'trawl'

October 14, 2022

Type Package					
Title Estimation and Simulation of Trawl Processes					
Version 0.2.2					
Author Almut E. D. Veraart					
Maintainer Almut E. D. Veraart <a.veraart@imperial.ac.uk></a.veraart@imperial.ac.uk>					
Description Contains R functions for simulating and estimating integer-valued trawl processes as described in the article Veraart (2019), "Modeling, simulation and inference for multivariate time series of counts using trawl processes", Journal of Multivariate Analysis, 169, pages 110-129, <doi:10.1016 j.jmva.2018.08.012=""> and for simulating random vectors from the bivariate negative binomial and the bi- and trivariate logarithmic series distributions.</doi:10.1016>					
License GPL-3					
Encoding UTF-8					
LazyData true					
RoxygenNote 7.1.1					
<b>Depends</b> R (>= 4.0.0)					
<b>Imports</b> DEoptim, ggplot2, ggpubr, graphics, MASS, rootSolve, Runuran, stats, squash, TSA					
Suggests knitr, rmarkdown, testthat					
VignetteBuilder knitr					
NeedsCompilation no					
Repository CRAN					
<b>Date/Publication</b> 2021-02-22 17:30:02 UTC					
R topics documented:					
acf_DExp       2         acf_Exp       3         acf_LM       4					

2 acf\_DExp

im																												6
n																												6
																												7
																												8
or																												8
ov																												9
																												9
																												10
																												11
																												12
sson																												12
																												13
ection																												14
ction_Exp																												15
ction_LM																												16
																												16
																												17
1																												17
																												18
																												19
gg																												19
awl																												20
Trawl																												21
																												23
																												23
																												24
																												25
sim																												25
																												27
	or ov	or	or	or ov	or	or ov ov	or ov ov	im n or ov sson section section_Exp section_LM																				

acf\_DExp

Autocorrelation function of the double exponential trawl function

## Description

This function computes the autocorrelation function associated with the double exponential trawl function.

## Usage

```
acf_DExp(x, w, lambda1, lambda2)
```

acf\_Exp 3

#### **Arguments**

X	The argument (lag) at which the autocorrelation function associated with the double exponential trawl function will be evaluated
W	parameter in the double exponential trawl
lambda1	parameter in the double exponential trawl
lambda2	parameter in the double exponential trawl

#### **Details**

The trawl function is parametrised by parameters  $0 \le w \le 1$  and  $\lambda_1, \lambda_2 > 0$  as follows:

$$g(x) = we^{\lambda_1 x} + (1 - w)e^{\lambda_2 x}$$
, for  $x \le 0$ .

Its autocorrelation function is given by:

$$r(x) = (we^{-\lambda_1 x}/\lambda_1 + (1-w)e^{-\lambda_2 x}/\lambda_2)/c$$
, for  $x \ge 0$ ,

where

$$c = w/\lambda_1 + (1 - w)/\lambda_2.$$

#### Value

The autocorrelation function of the double exponential trawl function evaluated at x

#### **Examples**

```
acf_DExp(1,0.3,0.1,2)
```

acf\_Exp

Autocorrelation function of the exponential trawl function

#### **Description**

This function computes the autocorrelation function associated with the exponential trawl function.

#### Usage

```
acf_Exp(x, lambda)
```

#### **Arguments**

x The argument (lag) at which the autocorrelation function associated with the exponential trawl function will be evaluated

1

lambda parameter in the exponential trawl

4 acf\_LM

#### **Details**

The trawl function is parametrised by the parameter  $\lambda > 0$  as follows:

$$g(x) = e^{\lambda x}$$
, for  $x \le 0$ .

Its autocorrelation function is given by:

$$r(x) = e^{-\lambda x}$$
, for  $x \ge 0$ .

#### Value

The autocorrelation function of the exponential trawl function evaluated at x

#### **Examples**

```
acf_Exp(1,0.1)
```

acf\_LM

Autocorrelation function of the long memory trawl function

#### **Description**

This function computes the autocorrelation function associated with the long memory trawl function.

#### Usage

## Arguments

X	The argument (lag) at which the autocorrelation function associated with the
	long memory trawl function will be evaluated
alpha	parameter in the long memory trawl
Н	parameter in the long memory trawl

#### **Details**

The trawl function is parametrised by the two parameters H>1 and  $\alpha>0$  as follows:

$$g(x)=(1-x/\alpha)^{-H}, \text{ for } x\leq 0.$$

Its autocorrelation function is given by

$$r(x) = (1 + x/\alpha)^{(1-H)}$$
, for  $x \ge 0$ .

### Value

The autocorrelation function of the long memory trawl function evaluated at x

acf\_supIG 5

## **Examples**

acf\_supIG

Autocorrelation function of the supIG trawl function

#### **Description**

This function computes the autocorrelation function associated with the supIG trawl function.

#### Usage

```
acf_supIG(x, delta, gamma)
```

## **Arguments**

X	The argument (lag) at which the autocorrelation function associated with the
	supIG trawl function will be evaluated
delta	parameter in the supIG trawl

gamma parameter in the supIG trawl

#### **Details**

The trawl function is parametrised by the two parameters  $\delta \geq 0$  and  $\gamma \geq 0$  as follows:

$$g(x) = (1 - 2x\gamma^{-2})^{-1/2} \exp(\delta \gamma (1 - (1 - 2x\gamma^{-2})^{1/2})), \text{ for } x \le 0.$$

It is assumed that  $\delta$  and  $\gamma$  are not simultaneously equal to zero. Its autocorrelation function is given by:

$$r(x)=\exp(\delta\gamma(1-\sqrt{1+2x/\gamma^2})), \text{ for } x\geq 0.$$

#### Value

The autocorrelation function of the supIG trawl function evaluated at x

#### **Examples**

```
acf_supIG(1,0.3,0.1)
```

6 Bivariate\_NBsim

Bivariate\_LSDsim

Simulates from the bivariate logarithmic series distribution

#### **Description**

Simulates from the bivariate logarithmic series distribution

#### Usage

Bivariate\_LSDsim(N, p1, p2)

#### **Arguments**

N	number of data points to be simulated
p1	parameter $p_1$ of the bivariate logarithmic series distribution
p2	parameter $p_2$ of the bivariate logarithmic series distribution

#### **Details**

The probability mass function of a random vector  $X = (X_1, X_2)'$  following the bivariate logarithmic series distribution with parameters  $0 < p_1, p_2 < 1$  with  $p := p_1 + p_2 < 1$  is given by

$$P(X_1 = x_1, X_2 = x_2) = \frac{\Gamma(x_1 + x_2)}{x_1! x_2!} \frac{p_1^{x_1} p_2^{x_2}}{(-\log(1 - p))},$$

for  $x_1, x_2 = 0, 1, 2, \ldots$  such that  $x_1 + x_2 > 0$ . The simulation proceeds in two steps: First,  $X_1$  is simulated from the modified logarithmic distribution with parameters  $\tilde{p}_1 = p_1/(1-p_2)$  and  $\delta_1 = \log(1-p_2)/\log(1-p)$ . Then we simulate  $X_2$  conditional on  $X_1$ . We note that  $X_2|X_1 = x_1$  follows the logarithmic series distribution with parameter  $p_2$  when  $p_1 = 0$ , and the negative binomial distribution with parameters  $p_1 = 0$ , when  $p_2 = 0$ , and the negative binomial distribution with parameters  $p_2 = 0$ .

#### Value

An  $N \times 2$  matrix with N simulated values from the bivariate logarithmic series distribution

Bivariate\_NBsim

Simulates from the bivariate negative binomial distribution

#### **Description**

Simulates from the bivariate negative binomial distribution

#### Usage

```
Bivariate_NBsim(N, kappa, p1, p2)
```

BivLSD\_Cor 7

#### **Arguments**

N	number of data points to be simulated
kappa	parameter $\boldsymbol{\kappa}$ of the bivariate negative binomial distribution
p1	parameter $p_1$ of the bivariate negative binomial distribution
p2	parameter $p_2$ of the bivariate negative binomial distribution

#### **Details**

A random vector  $\mathbf{X} = (X_1, X_2)'$  is said to follow the bivariate negative binomial distribution with parameters  $\kappa, p_1, p_2$  if its probability mass function is given by

$$P(\mathbf{X} = \mathbf{x}) = \frac{\Gamma(x_1 + x_2 + \kappa)}{x_1! x_2! \Gamma(\kappa)} p_1^{x_1} p_2^{x_2} (1 - p_1 - p_2)^{\kappa},$$

where, for  $i = 1, 2, x_i \in \{0, 1, ...\}$ ,  $0 < p_i < 1$  such that  $p_1 + p_2 < 1$  and  $\kappa > 0$ .

#### Value

An  $N \times 2$  matrix with N simulated values from the bivariate negative binomial distribution

BivLSD_Cor	Computes the correlation of the components of a bivariate vector fol-
	lowing the bivariate logarithmic series distribution

#### **Description**

Computes the correlation of the components of a bivariate vector following the bivariate logarithmic series distribution

## Usage

```
BivLSD_Cor(p1, p2)
```

## Arguments

p1	parameter $p_1$ of the bivariate logarithmic series distribution
p2	parameter $p_2$ of the bivariate logarithmic series distribution

#### Value

Correlation of the components of a bivariate vector following the bivariate logarithmic series distribution

8 BivModLSD\_Cor

BivLSD_Cov	Computes the covariance of the components of a bivariate vector fol-
	lowing the bivariate logarithmic series distribution

#### **Description**

Computes the covariance of the components of a bivariate vector following the bivariate logarithmic series distribution

## Usage

```
BivLSD_Cov(p1, p2)
```

#### **Arguments**

p1	parameter $p_1$ of the bivariate logarithmic series distribution
p2	parameter $p_2$ of the bivariate logarithmic series distribution

#### Value

Covariance of the components of a bivariate vector following the bivariate logarithmic series distribution

BivModLSD_Cor	Computes the correlation of the components of a bivariate vector fol-
	lowing the bivariate modified logarithmic series distribution

## Description

Computes the correlation of the components of a bivariate vector following the bivariate modified logarithmic series distribution

#### Usage

```
BivModLSD_Cor(delta, p1, p2)
```

#### **Arguments**

delta	parameter $\delta$ of the bivariate modified logarithmic series distribution
p1	parameter $p_1$ of the bivariate modified logarithmic series distribution
p2	parameter $p_2$ of the bivariate modified logarithmic series distribution

#### Value

Covariance of the components of a bivariate vector following the bivariate modified logarithmic series distribution

BivModLSD\_Cov 9

BivModLSD_Cov	Computes the covariance of the components of a bivariate vector fol-
_	lowing the bivariate modified logarithmic series distribution

## Description

Computes the covariance of the components of a bivariate vector following the bivariate modified logarithmic series distribution

## Usage

```
BivModLSD_Cov(delta, p1, p2)
```

## Arguments

delta	parameter $\delta$ of the bivariate modified logarithmic series distribution
p1	parameter $p_1$ of the bivariate modified logarithmic series distribution
p2	parameter $p_2$ of the bivariate modified logarithmic series distribution

#### Value

Covariance of the components of a bivariate vector following the bivariate modified logarithmic series distribution

fit_DExptrawl	Fits the trawl function consisting of the weighted sum of two exponen-
	tial functions

## Description

Fits the trawl function consisting of the weighted sum of two exponential functions

## Usage

```
fit_DExptrawl(x, Delta = 1, GMMlag = 5, plotacf = FALSE, lags = 100)
```

## Arguments

Х	vector of equidistant time series data
Delta	interval length of the time grid used in the time series, the default is 1
GMMlag	lag length used in the GMM estimation, the default is 5
plotacf	binary variable specifying whether or not the empirical and fitted autocorrelation function should be plotted
lags	number of lags to be used in the plot of the autocorrelation function

10 fit\_Exptrawl

#### **Details**

The trawl function is parametrised by the three parameters  $0 \le w \le 1$  and  $\lambda_1, \lambda_2 > 0$  as follows:

$$g(x) = we^{\lambda_1 x} + (1 - w)e^{\lambda_2 x}$$
, for  $x \le 0$ .

The Lebesgue measure of the corresponding trawl set is given by  $w/\lambda_1 + (1-w)/\lambda_2$ .

#### Value

w: the weight parameter (restricted to be in [0,0.5] for identifiability reasons)

lambda1: the first memory parameter (denoted by  $\lambda_1$  above)

lambda2: the second memory parameter (denoted by  $\lambda_2$  above)

LM: The Lebesgue measure of the trawl set associated with the double exponential trawl

fit\_Exptrawl

Fits an exponential trawl function to equidistant time series data

#### **Description**

Fits an exponential trawl function to equidistant time series data

#### Usage

## **Arguments**

x vector of equidistant time series data

Delta interval length of the time grid used in the time series, the default is 1

plotacf binary variable specifying whether or not the empirical and fitted autocorrelation

function should be plotted

lags number of lags to be used in the plot of the autocorrelation function

#### **Details**

The trawl function is parametrised by the parameter  $\lambda > 0$  as follows:

$$g(x) = e^{\lambda x}$$
, for  $x \le 0$ .

The Lebesgue measure of the corresponding trawl set is given by  $1/\lambda$ .

#### Value

lambda: the memory parameter  $\lambda$  in the exponential trawl

LM: the Lebesgue measure of the trawl set associated with the exponential trawl, i.e.  $1/\lambda$ .

fit\_LMtrawl 11

	Fits a long memory trawl function to equidistant univariate time series lata
--	---

## Description

Fits a long memory trawl function to equidistant univariate time series data

## Usage

```
fit_LMtrawl(x, Delta = 1, GMMlag = 5, plotacf = FALSE, lags = 100)
```

## Arguments

x	vector of equidistant time series data
Delta	interval length of the time grid used in the time series, the default is 1
GMMlag	lag length used in the GMM estimation, the default is 5
plotacf	binary variable specifying whether or not the empirical and fitted autocorrelation function should be plotted
lags	number of lags to be used in the plot of the autocorrelation function

#### **Details**

The trawl function is parametrised by the two parameters H>1 and  $\alpha>0$  as follows:

$$g(x) = (1 - x/\alpha)^{-H}$$
, for  $x \le 0$ .

The Lebesgue measure of the corresponding trawl set is given by  $\alpha/(1-H)$ .

#### Value

alpha: parameter in the long memory trawl

H: parameter in the long memory trawl

LM: The Lebesgue measure of the trawl set associated with the long memory trawl

fit\_marginalPoisson

fit\_marginalNB

Fist a negative binomial distribution as marginal law

#### **Description**

Fist a negative binomial distribution as marginal law

#### Usage

```
fit_marginalNB(x, LM, plotdiag = FALSE)
```

#### Arguments

x vector of equidistant time series data

LM Lebesgue measure of the estimated trawl

plotdiag binary variable specifying whether or not diagnostic plots should be provided

#### **Details**

The moment estimator for the parameters of the negative binomial distribution are given by

$$\hat{\theta} = 1 - E(X) / Var(X),$$

and

$$\hat{m} = \mathrm{E}(X)(1-\hat{\theta})/(\widehat{\mathrm{LM}}\hat{\theta}).$$

#### Value

m: parameter in the negative binomial marginal distribution

theta: parameter in the negative binomial marginal distribution

a: Here  $a = \theta/(1-\theta)$ . This is given for an alternative parametrisation of the negative binomial marginal distribution

 $\verb|fit_marginalPoisson||$ 

Fits a Poisson distribution as marginal law

## Description

Fits a Poisson distribution as marginal law

## Usage

```
fit_marginalPoisson(x, LM, plotdiag = FALSE)
```

fit\_supIGtrawl 13

## **Arguments**

x vector of equidistant time series dataLM Lebesgue measure of the estimated trawl

plotdiag binary variable specifying whether or not diagnostic plots should be provided

#### **Details**

The moment estimator for the Poisson rate parameter is given by

$$\hat{v} = E(X)/\widehat{LM}$$
.

#### Value

v: the rate parameter in the Poisson marginal distribution

fit\_supIGtrawl

Fits a supIG trawl function to equidistant univariate time series data

## Description

Fits a supIG trawl function to equidistant univariate time series data

#### Usage

#### **Arguments**

x	vector of equidistant time series data
Delta	interval length of the time grid used in the time series, the default is 1
GMMlag	lag length used in the GMM estimation, the default is 5
plotacf	binary variable specifying whether or not the empirical and fitted autocorrelation function should be plotted

number of lags to be used in the plot of the autocorrelation function

#### **Details**

lags

The trawl function is parametrised by the two parameters  $\delta \geq 0$  and  $\gamma \geq 0$  as follows:

$$g(x) = (1 - 2x\gamma^{-2})^{-1/2} \exp(\delta \gamma (1 - (1 - 2x\gamma^{-2})^{1/2})), \text{ for } x \le 0.$$

It is assumed that  $\delta$  and  $\gamma$  are not simultaneously equal to zero. The Lebesgue measure of the corresponding trawl set is given by  $\gamma/\delta$ .

## Value

delta: parameter in the supIG trawl gamma: parameter in the supIG trawl

LM: The Lebesgue measure of the trawl set associated with the supIG trawl

14 fit\_trawl\_intersection

```
fit_trawl_intersection
```

Finds the intersection of two trawl sets

#### **Description**

Finds the intersection of two trawl sets

#### Usage

```
fit_trawl_intersection(
  fct1 = base::c("Exp", "DExp", "supIG", "LM"),
  fct2 = base::c("Exp", "DExp", "supIG", "LM"),
  lambda11 = 0,
  lambda12 = 0,
 w1 = 0,
  delta1 = 0,
  gamma1 = 0,
  alpha1 = 0,
 H1 = 0,
  lambda21 = 0,
  lambda22 = 0,
 w2 = 0,
 delta2 = 0,
  gamma2 = 0,
 alpha2 = 0,
 H2 = 0,
 LM1,
 LM2,
  plotdiag = FALSE
)
```

## Arguments

```
fct1
                  specifies the type of the first trawl function
fct2
                  specifies the type of the second trawl function
lambda11, lambda12, w1
                  parameters of the (double) exponential trawl functions of the first process
delta1, gamma1
                  parameters of the supIG trawl functions of the first process
                  parameters of the long memory trawl function of the first process
alpha1, H1
lambda21, lambda22, w2
                  parameters of the (double) exponential trawl functions of the second process
                  parameters of the supIG trawl functions of the second process
delta2, gamma2
                  parameters of the long memory trawl function of the second process
alpha2, H2
                  Lebesgue measure of the first trawl
LM1
```

LM2 Lebesgue measure of the second trawl

plotdiag binary variable specifying whether or not diagnostic plots should be provided

#### **Details**

Computes  $R_{12}(0) = \text{Leb}(A_1 \cap A_2)$  based on two trawl functions  $g_1$  and  $g_2$ .

#### Value

The Lebesgue measure of the intersection of the two trawl sets

fit\_trawl\_intersection\_Exp

Finds the intersection of two exponential trawl sets

## **Description**

Finds the intersection of two exponential trawl sets

#### Usage

```
fit_trawl_intersection_Exp(lambda1, lambda2, LM1, LM2, plotdiag = FALSE)
```

## Arguments

lambda1, lambda2

parameters of the two exponential trawls

LM1 Lebesgue measure of the first trawl

LM2 Lebesgue measure of the second trawl

plotdiag binary variable specifying whether or not diagnostic plots should be provided

#### **Details**

Computes  $R_{12}(0) = \text{Leb}(A_1 \cap A_2)$  based on two trawl functions  $g_1$  and  $g_2$ .

#### Value

The Lebesgue measure of the intersection of the two trawl sets

LSD\_Mean

```
fit_trawl_intersection_LM
```

Finds the intersection of two long memory (LM) trawl sets

#### **Description**

Finds the intersection of two long memory (LM) trawl sets

#### Usage

```
fit_trawl_intersection_LM(alpha1, H1, alpha2, H2, LM1, LM2, plotdiag = FALSE)
```

## Arguments

```
alpha1, H1, alpha2, H2
```

parameters of the two long memory trawls

LM1 Lebesgue measure of the first trawl
LM2 Lebesgue measure of the second trawl

plotdiag binary variable specifying whether or not diagnostic plots should be provided

#### **Details**

```
Computes R_{12}(0) = \text{Leb}(A_1 \cap A_2) based on two trawl functions g_1 and g_2.
```

#### Value

the Lebesgue measure of the intersection of the two trawl sets

LSD\_Mean

Computes the mean of the logarithmic series distribution

## Description

Computes the mean of the logarithmic series distribution

#### Usage

```
LSD_Mean(p)
```

### **Arguments**

p parameter of the logarithmic series distribution

LSD\_Var

#### **Details**

A random variable X has logarithmic series distribution with parameter 0 if

$$P(X = x) = (-1)/(\log(1-p))p^x/x$$
, for  $x = 1, 2, ...$ 

#### Value

Mean of the logarithmic series distribution

LSD\_Var

Computes the variance of the logarithmic series distribution

#### **Description**

Computes the variance of the logarithmic series distribution

#### Usage

LSD\_Var(p)

#### **Arguments**

р

parameter of the logarithmic series distribution

#### **Details**

A random variable X has logarithmic series distribution with parameter 0 if

$$P(X = x) = (-1)/(\log(1-p))p^x/x$$
, for  $x = 1, 2, ...$ 

## Value

Variance of the logarithmic series distribution

ModLSD\_Mean

Computes the mean of the modified logarithmic series distribution

## Description

Computes the mean of the modified logarithmic series distribution

## Usage

ModLSD\_Mean(delta, p)

18 ModLSD\_Var

#### **Arguments**

delta parameter  $\delta$  of the modified logarithmic series distribution parameter of the modified logarithmic series distribution

#### **Details**

A random variable X has modified logarithmic series distribution with parameters  $0 \le \delta < 1$  and  $0 if <math>P(X = 0) = (1 - \delta)$  and

$$P(X = x) = (1 - \delta)(-1)/(\log(1 - p))p^x/x$$
, for  $x = 1, 2, ...$ 

## Value

Mean of the modified logarithmic series distribution

ModLSD\_Var

Computes the variance of the modified logarithmic series distribution

## Description

Computes the variance of the modified logarithmic series distribution

#### Usage

```
ModLSD_Var(delta, p)
```

## **Arguments**

delta parameter  $\delta$  of the modified logarithmic series distribution parameter of the modified logarithmic series distribution

#### **Details**

A random variable X has modified logarithmic series distribution with parameters  $0 \le \delta < 1$  and  $0 if <math>P(X = 0) = (1 - \delta)$  and

$$P(X = x) = (1 - \delta)(-1)/(\log(1 - p))p^x/x$$
, for  $x = 1, 2, ...$ 

#### Value

Mean of the modified logarithmic series distribution

plot\_2and1hist

plot_2and1hist	Plots the bivariate histogram of two time series together with the univariate histograms

## Description

Plots the bivariate histogram of two time series together with the univariate histograms

#### Usage

```
plot_2and1hist(x, y)
```

#### **Arguments**

x vector of equidistant time series data

y vector of equidistant time series data (of the same length as x)

#### **Details**

This function plots the bivariate histogram of two time series together with the univariate histograms

#### Value

no return value

plot_2and1hist_gg	Plots the bivariate histogram of two time series together with the uni-
	variate histograms using ggplot2

## Description

Plots the bivariate histogram of two time series together with the univariate histograms using gg-plot2

#### Usage

```
plot_2and1hist_gg(x, y, bivbins = 50, xbins = 30, ybins = 30)
```

## Arguments

X	vector of equidistant time series data
У	vector of equidistant time series data (of the same length as x)
bivbins	number of bins in the bivariate histogram
xbins	number of bins in the histogram of x
vhins	number of bins in the histogram of v

20 sim\_BivariateTrawl

#### **Details**

This function plots the bivariate histogram of two time series together with the univariate histograms

#### Value

no return value

sim\_BivariateTrawl

Simulates a bivariate trawl process

## **Description**

Simulates a bivariate trawl process

## Usage

```
sim_BivariateTrawl(
  t,
 Delta = 1,
  burnin = 10,
 marginal = base::c("Poi", "NegBin"),
  dependencetype = base::c("fullydep", "dep"),
  trawl1 = base::c("Exp", "DExp", "supIG", "LM"),
trawl2 = base::c("Exp", "DExp", "supIG", "LM"),
  v1 = 0,
  v2 = 0,
  v12 = 0,
  kappa1 = 0,
  kappa2 = 0,
  kappa12 = 0,
  a1 = 0,
  a2 = 0,
  lambda11 = 0,
  lambda12 = 0,
 w1 = 0,
  delta1 = 0,
  gamma1 = 0,
  alpha1 = 0,
 H1 = 0,
  lambda21 = 0,
  lambda22 = 0,
 w2 = 0,
  delta2 = 0,
  gamma2 = 0,
 alpha2 = 0,
 H2 = 0
)
```

sim\_UnivariateTrawl 21

#### **Arguments**

t parameter which specifying the length of the time interval [0,t] for which a

simulation of the trawl process is required.

Delta parameter  $\Delta$  specifying the length of the time step, the default is 1

burnin parameter specifying the length of the burn-in period at the beginning of the

simulation

marginal parameter specifying the marginal distribution of the trawl

dependence type Parameter specifying the type of dependence

trawl1 parameter specifying the type of the first trawl function

trawl2 parameter specifying the type of the second trawl function

v1, v2, v12 parameters of the Poisson distribution

kappa1, kappa2, kappa12, a1, a2

parameters of the (possibly bivariate) negative binomial distribution

lambda11, lambda12, w1

parameters of the exponential (or double-exponential) trawl function of the first

process

delta1, gamma1 parameters of the supIG trawl function of the first process

alpha1, H1 parameter of the long memory trawl of the first process

lambda21, lambda22, w2

parameters of the exponential (or double-exponential) trawl function of the sec-

ond process

delta2, gamma2 parameters of the supIG trawl function of the second process

alpha2, H2 parameter of the long memory trawl of the second process

#### **Details**

This function simulates a bivariate trawl process with either Poisson or negative binomial marginal law. For the trawl function there are currently four choices: exponential, double-exponential, supIG or long memory. More details on the precise simulation algorithm is available in the vignette.

sim\_UnivariateTrawl

Simulates a univariate trawl process

#### **Description**

Simulates a univariate trawl process

22 sim\_UnivariateTrawl

#### Usage

```
sim_UnivariateTrawl(
  t,
 Delta = 1,
 burnin = 10,
 marginal = base::c("Poi", "NegBin"),
 trawl = base::c("Exp", "DExp", "supIG", "LM"),
 v = 0,
 m = 0,
  theta = 0,
 lambda1 = 0,
 lambda2 = 0,
 w = 0,
 delta = 0,
 gamma = 0,
 alpha = 0,
 H = 0
)
```

## Arguments

t	parameter which specifying the length of the time interval $\left[0,t\right]$ for which a simulation of the trawl process is required.
Delta	parameter $\Delta$ specifying the length of the time step, the default is $\boldsymbol{1}$
burnin	parameter specifying the length of the burn-in period at the beginning of the simulation
marginal	parameter specifying the marginal distribution of the trawl
trawl	parameter specifying the type of trawl function
V	parameter of the Poisson distribution
m	parameter of the negative binomial distribution
theta	parameter $\theta$ of the negative binomial distribution
lambda1	parameter $\lambda_1$ of the exponential (or double-exponential) trawl function
lambda2	parameter $\lambda_2$ of the double-exponential trawl function
W	parameter of the double-exponential trawl function
delta	parameter $\delta$ of the supIG trawl function
gamma	parameter $\gamma$ of the supIG trawl function
alpha	parameter $\alpha$ of the long memory trawl function
Н	parameter of the long memory trawl function

## **Details**

This function simulates a univariate trawl process with either Poisson or negative binomial marginal law. For the trawl function there are currently four choices: exponential, double-exponential, supIG or long memory. More details on the precise simulation algorithm is available in the vignette.

trawl\_DExp 23

trawl	DEvn
ti awı	DEXD

Evaluates the double exponential trawl function

#### **Description**

Evaluates the double exponential trawl function

#### Usage

```
trawl_DExp(x, w, lambda1, lambda2)
```

#### **Arguments**

x the argument at which the double exponential trawl function will be evaluated

w parameter in the double exponential trawl

lambda1 the parameter  $\lambda_1$  in the double exponential trawl lambda2 the parameter  $\lambda_2$  in the double exponential trawl

#### **Details**

The trawl function is parametrised by parameters  $0 \le w \le 1$  and  $\lambda_1, \lambda_2 > 0$  as follows:

$$g(x) = we^{\lambda_1 x} + (1 - w)e^{\lambda_2 xz}$$
, for  $x \le 0$ .

#### Value

The double exponential trawl function evaluated at x

+		1	E v	n
ι	raw	⊥_	ᄄΧ	μ

Evaluates the exponential trawl function

#### **Description**

Evaluates the exponential trawl function

## Usage

```
trawl_Exp(x, lambda)
```

#### **Arguments**

x the argument at which the exponential trawl function will be evaluated

lambda the parameter  $\lambda$  in the exponential trawl

24 trawl\_LM

#### **Details**

The trawl function is parametrised by parameter  $\lambda > 0$  as follows:

$$g(x) = e^{\lambda x}$$
, for  $x \le 0$ .

#### Value

The exponential trawl function evaluated at x

trawl\_LM

Evaluates the long memory trawl function

## Description

Evaluates the long memory trawl function

## Usage

```
trawl_LM(x, alpha, H)
```

## **Arguments**

x the argument at which the long memory trawl function will be evaluated alpha the parameter  $\alpha$  in the long memory trawl the parameter H in the long memory trawl

## Details

The trawl function is parametrised by the two parameters H>1 and  $\alpha>0$  as follows:

$$g(x) = (1 - x/\alpha)^{-H}$$
, for  $x \le 0$ .

#### Value

the long memory trawl function evaluated at x

trawl\_supIG 25

trawl\_supIG

Evaluates the supIG trawl function

## Description

Evaluates the supIG trawl function

## Usage

```
trawl_supIG(x, delta, gamma)
```

#### **Arguments**

X	the argument at which the supIG trawl function will be evaluated
delta	the parameter $\delta$ in the supIG trawl
gamma	the parameter $\gamma$ in the supIG trawl

#### **Details**

The trawl function is parametrised by the two parameters  $\delta \geq 0$  and  $\gamma \geq 0$  as follows:

$$gd(x) = (1 - 2x\gamma^{-2})^{-1/2} \exp(\delta\gamma(1 - (1 - 2x\gamma^{-2})^{1/2})), \text{ for } x \le 0.$$

It is assumed that  $\delta$  and  $\gamma$  are not simultaneously equal to zero.

#### Value

The supIG trawl function evaluated at x

Trivariate\_LSDsim

Simulates from the trivariate logarithmic series distribution

#### **Description**

Simulates from the trivariate logarithmic series distribution

#### Usage

```
Trivariate_LSDsim(N, p1, p2, p3)
```

#### **Arguments**

N	number of data points to be simulated
p1	parameter $p1$ of the trivariate logarithmic series distribution
p2	parameter $p2$ of the trivariate logarithmic series distribution
р3	parameter $p3$ of the trivariate logarithmic series distribution

26 Trivariate\_LSDsim

#### **Details**

The probability mass function of a random vector  $X = (X_1, X_2, X_3)'$  following the trivariate logarithmic series distribution with parameters  $0 < p_1, p_2, p_3 < 1$  with  $p := p_1 + p_2 + p_3 < 1$  is given by

$$P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = \frac{\Gamma(x_1 + x_2 + x_3)}{x_1! x_2! x_3!} \frac{p_1^{x_1} p_2^{x_2} p_3^{x_3}}{(-\log(1-p))},$$

for  $x_1, x_2, x_3 = 0, 1, 2, \dots$  such that  $x_1 + x_2 + x_3 > 0$ .

The simulation proceeds in two steps: First,  $X_1$  is simulated from the modified logarithmic distribution with parameters  $\tilde{p}_1 = p_1/(1-p_2-p_3)$  and  $\delta_1 = \log(1-p_2-p_3)/\log(1-p)$ . Then we simulate  $(X_2,X_3)'$  conditional on  $X_1$ . We note that  $(X_2,X_3)'|X_1=x_1$  follows the bivariate logarithmic series distribution with parameters  $(p_2,p_3)$  when  $x_1=0$ , and the bivariate negative binomial distribution with parameters  $(x_1,p_2,p_3)$  when  $x_1>0$ .

#### Value

An  $N \times 3$  matrix with N simulated values from the trivariate logarithmic series distribution

## **Index**

```
acf_DExp, 2
acf_Exp, 3
acf_LM, 4
acf_supIG, 5
Bivariate_LSDsim, 6
Bivariate_NBsim, 6
BivLSD_Cor, 7
BivLSD_Cov, 8
BivModLSD_Cor, 8
BivModLSD_Cov, 9
fit_DExptrawl, 9
fit_Exptrawl, 10
fit_LMtrawl, 11
fit_marginalNB, 12
fit_marginalPoisson, 12
fit_supIGtrawl, 13
fit_trawl_intersection, 14
fit_trawl_intersection_Exp, 15
fit_trawl_intersection_LM, 16
LSD_Mean, 16
LSD_Var, 17
ModLSD_Mean, 17
ModLSD_Var, 18
plot_2and1hist, 19
plot_2and1hist_gg, 19
sim_BivariateTrawl, 20
sim_UnivariateTrawl, 21
trawl_DExp, 23
trawl_Exp, 23
trawl_LM, 24
trawl_supIG, 25
Trivariate_LSDsim, 25
```