Package 'gofedf'

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Description

Compute Maximum likelihood estimates of the parameters in Gamma distribution, Score function evaluated at the sample, and probability inverse transformed (PIT) values of sample.

Usage

```
applyGamma(x, use.rate)
```

Arguments

x a numeric vector.

use.rate logical. If TRUE the rate parameter is returned while estimating MLE. Otherwise the scale is returned.

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Value

a list with three elements.

applyLMNormal

Apply linear model and extract required components for the test

Description

Apply linear model and extract required components for the test

Usage

```
applyLMNormal(x, y)
```

Arguments

x is either a numeric vector or a design matrix. In the design matrix, rows indicate

observations and columns presents explanatory variables.

y is a vector of numeric values with the same number of observations or number

of rows as x.

Value

a list with three elements.

expMLE

Calculate MLE of rate in Exponential dist.

Description

Calculate MLE of rate in Exponential dist.

Usage

```
expMLE(x)
```

Arguments

x a numeric vector of length n

Value

Numeric

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expPIT

Compute probability inverse transform values for Exponential dist.

Description

Compute probability inverse transform values for Exponential dist.

Usage

```
expPIT(x, theta)
```

Arguments

x a numeric vector of length n

theta a numeric vector of length one, containing MLE rate in Exponential dist.

Value

a vector of length n containing the probability inverse transformed (PIT) values

expScore

Compute score function for Exponential distribution.

Description

Compute score function for Exponential distribution.

Usage

```
expScore(x, theta)
```

Arguments

x a numeric vector of length n

theta a numeric vector of length two, containing MLE of parameters in Exponential

dist.

Value

Score matrix with n rows and two columns.

gammaFisherByHessian

gammaFisherByHessian	Compute Fisher information matrix by the negative expected value of
	Hessian matrix in Gamma distribution.

Description

Compute Fisher information matrix by the negative expected value of Hessian matrix in Gamma distribution.

Usage

```
gammaFisherByHessian(theta)
```

Arguments

theta

a numeric vector of length two, containing MLE of parameters in Gamma dist

Value

Fisher information matrix for Gamma distribution

gammaMLE	Compute maximum likelihood estimate of shape and scale parameter
	in Gamma distribution.

Description

Estimate the MLE of shape and scale parameters of the Gamma distribution using the Newton-Raphson method on the profile log-likelihood to estimate the shape parameter.

Usage

```
gammaMLE(x, ur)
```

Arguments

x a numeric vector of length n

ur logical. If TRUE the rate parameter is returned. Otherwise the scale is returned.

Value

a vector of length two with shape and scale/rate.

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gammaPIT

Compute probability inverse transform values for Gamma distribution

Description

Compute probability inverse transform values for Gamma distribution

Usage

```
gammaPIT(x, theta)
```

Arguments

x a numeric vector of length n

theta a numeric vector of length two, containing MLE of parameters in Gamma dist.

Value

a vector of length n containing the probability inverse transformed (PIT) values

gammaScore

Compute score function for Gamma distribution.

Description

Compute score function for Gamma distribution.

Usage

```
gammaScore(x, theta)
```

Arguments

x a numeric vector of length n

theta a numeric vector of length two, containing MLE of parameters in Gamma dist.

Value

Score matrix with n rows and two columns.

glmMLE 7

glmMLE	Compute maximum likelihood estimates for a generalized linear model with Gamma response.

Description

Compute maximum likelihood estimates for a generalized linear model with Gamma response.

Usage

```
glmMLE(fit)
```

Arguments

fit

is an object of class glm and its default value is NULL. If a fit of class glm is provided, the arguments x, y, and 1 will be ignored. We recommend using glm2 function from glm2 package since it provides better convergence while optimizing the likelihood to estimate coefficients of the model by IWLS method. It is required to return design matrix by x = TRUE in glm or glm2 function. For more information on how to do this, refer to the help documentation for the glm or glm2 function.

Value

a numeric vector of estimates.

glmScorePIT	Compute score function for a generalized linear model with Gamma
	response.

Description

Compute score function for a generalized linear model with Gamma response.

Usage

```
glmScorePIT(fit, theta)
```

Arguments

fit TBD

theta a numeric vector of length (p+1), containing MLE of parameters in a linear

model.

Value

Score matrix with n rows and (p+2) columns.

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inversegaussianMLE	Compute the maximum likelihood estimate of parameters in Inverse
	Gaussian distribution with weighted observations.

Description

This function is used in testYourModel function for example purposes.

Usage

```
inversegaussianMLE(obs, ...)
```

Arguments

obs a numeric vector of sample observations.

. . . a list of additional parameters to define the likelihood.

Value

The function compute the MLE of parameters in Inverse Gaussian distribution and returns a vector of estimates. The first and second elements of the vector are MLE of the mean and shape, respectively.

inversegaussianPIT

Compute the probability transformed values for a sample from Inverse Gaussian distribution.

Description

This function is used in testYourModel function for example purposes.

Usage

```
inversegaussianPIT(obs, ...)
```

Arguments

obs A numeric vector of sample observations.

... A list of additional parameters to define the likelihood.

Value

A numeric vector of probability transformed values of sample observations.

inversegaussianScore 9

inversegaussianScore Compute the score function of the Inverse Gaussian distribution based on a sample.

Description

This function is used in testYourModel function for example purposes.

Usage

```
inversegaussianScore(obs, ...)
```

Arguments

obs a numeric vector of sample observations.

a list of additional parameters to define the likelihood.

Value

The score matrix with n rows (number of sample observations) and 2 columns (mean and shape).

lmFisherByHessian Compute Fisher information matrix in the case of linear model with Normal residuals.

Description

Compute Fisher information matrix in the case of linear model with Normal residuals.

Usage

```
lmFisherByHessian(x, y, theta)
```

Arguments

x a matrix with n rows and p columns containing the explanatory variables.

y a numeric vector of length n containing the response variable.

theta a numeric vector of length (p+1), containing MLE of parameters in a linear

model.

Value

Fisher information matrix for linear models.

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1mMLE

Compute maximum likelihood estimates for linear models

Description

Compute maximum likelihood estimates for linear models

Usage

```
lmMLE(x, y)
```

Arguments

x a matrix with n rows and p columns containing the explanatory variables.

y a numeric vector of length n containing the response variable.

Value

a numeric vector of estimates.

lmPIT

Compute probability inverse transform values for linear models.

Description

Compute probability inverse transform values for linear models.

Usage

```
lmPIT(x, y, theta)
```

Arguments

x a matrix with n rows and p columns containing the explanatory variables.

y a numeric vector of length n containing the response variable.

theta a numeric vector of length (p+1), containing MLE of parameters in a linear

model.

Value

a vector of length n containing the probability inverse transformed (PIT) values

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1mScore

Compute score function for linear models.

Description

Compute score function for linear models.

Usage

```
lmScore(x, y, theta)
```

Arguments

x a matrix with n rows and p columns containing the explanatory variables.

y a numeric vector of length n containing the response variable.

theta a numeric vector of length (p+1), containing MLE of parameters in a linear

model.

Value

Score matrix with n rows and (p+1) columns.

normalFisherByHessian Compute Fisher information matrix by the negative expected value of Hessian matrix in Normal distribution.

Description

Compute Fisher information matrix by the negative expected value of Hessian matrix in Normal distribution.

Usage

```
normalFisherByHessian(theta)
```

Arguments

theta a numeric vector of length two, containing MLE of parameters in Normal dist

Value

Fisher information matrix for Normal distribution

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normalMLE

Compute MLE estimate for Normal

Description

Compute MLE estimate for Normal

Usage

```
normalMLE(x)
```

Arguments

x a numeric vector of length n

Value

a numeric vector of length two, containing MLE of parameters in Normal dist.

normalPIT

Compute probability inverse transform values for Normal distribution

Description

Compute probability inverse transform values for Normal distribution

Usage

```
normalPIT(x, theta)
```

Arguments

x a numeric vector of length n

theta a numeric vector of length two, containing MLE of parameters in Normal dist.

Value

a vector of length n containing the probability inverse transformed (PIT) values

normalScore 13

normalScore

Compute score function for Normal dist

Description

Compute score function for Normal dist

Usage

```
normalScore(x, theta)
```

Arguments

x a numeric vector of length n

theta a numeric vector of length two, containing MLE of parameters in Normal dist.

Value

Score matrix with n rows and two columns.

testExponential

Apply Goodness of Fit Test for Exponential Distribution

Description

Performs the goodness-of-fit test based on empirical distribution function to check if an i.i.d sample follows an Exponential distribution.

Usage

```
testExponential(
   x,
   ngrid = length(x),
   gridpit = FALSE,
   hessian = FALSE,
   method = "cvm"
)
```

Arguments

x a non-empty numeric vector of sample data.

ngrid the number of equally spaced points to discretize the (0,1) interval for computing

the covariance function.

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logical. If TRUE (the default value), the parameter ngrid is ignored and (0,1) interval is divided based on probability inverse transformed values obtained from the sample. If FALSE, the interval is divided into ngrid equally spaced points for computing the covariance function.

hessian logical. If TRUE the Fisher information matrix is estimated by the observed Hessian Matrix based on the sample. If FALSE (the default value) the Fisher information matrix is estimated by the variance of the observed score matrix.

method a character string indicating which goodness-of-fit statistic is to be computed. The default value is 'cvm' for the Cramer-von-Mises statistic. Other options include 'ad' for the Anderson-Darling statistic, and 'both' to compute both cvm and ad.

Value

A list of two containing the following components:

- Statistic: the value of goodness-of-fit statistic.
- p-value: the approximate p-value for the goodness-of-fit test based on empirical distribution function. if method = 'cvm' or method = 'ad', it returns a numeric value for the statistic and p-value. If method = 'both', it returns a numeric vector with two elements and one for each statistic.

Examples

```
set.seed(123)
n <- 50
sim_data <- rexp(n, rate = 2)
testExponential(x = sim_data)</pre>
```

testGamma

Apply Goodness of Fit Test for Gamma Distribution

Description

Performs the goodness-of-fit test based on empirical distribution function to check if an i.i.d sample follows a Gamma distribution.

Usage

```
testGamma(
   x,
   ngrid = length(x),
   gridpit = FALSE,
   hessian = FALSE,
   rate = TRUE,
   method = "cvm"
)
```

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Arguments

X	a non-empty numeric vector of sample data.
ngrid	the number of equally spaced points to discretize the $(0,1)$ interval for computing the covariance function.
gridpit	logical. If TRUE (the default value), the parameter ngrid is ignored and (0,1) interval is divided based on probability inverse transformed values obtained from the sample. If FALSE, the interval is divided into ngrid equally spaced points for computing the covariance function.
hessian	logical. If TRUE the Fisher information matrix is estimated by the observed Hessian Matrix based on the sample. If FALSE (the default value) the Fisher information matrix is estimated by the variance of the observed score matrix.
rate	logical. If TRUE (the default value), the rate is estimated in Gamma distribution. If FALSE, scale is estimated. See GammaDist for more details.
method	a character string indicating which goodness-of-fit statistic is to be computed. The default value is 'cvm' for the Cramer-von-Mises statistic. Other options include 'ad' for the Anderson-Darling statistic, and 'both' to compute both cvm and ad.

Value

A list of two containing the following components:

- Statistic: the value of goodness-of-fit statistic.
- p-value: the approximate p-value for the goodness-of-fit test based on empirical distribution function. if method = 'cvm' or method = 'ad', it returns a numeric value for the statistic and p-value. If method = 'both', it returns a numeric vector with two elements and one for each statistic.

Examples

```
set.seed(123)
sim_data <- rgamma(n = 50, shape = 3)
testGamma(x = sim_data)
sim_data <- runif(n = 50)
testGamma(x = sim_data)</pre>
```

testGLMGamma

Apply Goodness of Fit Test to the Residuals of a Generalized Linear Model with Gamma Link Function

Description

testGLMGamma is used to check the validity of Gamma assumption for the response variable when fitting generalized linear model. Common link functions in glm can be used here.

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Usage

```
testGLMGamma(
    x,
    y,
    fit = NULL,
    1 = "log",
    hessian = FALSE,
    start.value = NULL,
    control = NULL,
    method = "cvm"
)
```

Arguments

fit

1

Х	is either a numeric vector or a design matrix. In the design matrix, rows indicate
	observations and columns presents covariats.

y is a vector of numeric values with the same number of observations or number of rows as x.

is an object of class glm and its default value is NULL. If a fit of class glm is provided, the arguments x, y, and 1 will be ignored. We recommend using glm2 function from glm2 package since it provides better convergence while optimizing the likelihood to estimate coefficients of the model by IWLS method. It is required to return design matrix by x = TRUE in glm or glm2 function. For more information on how to do this, refer to the help documentation for the glm

or glm2 function.

a character vector indicating the link function that should be used for Gamma family. Some common link functions for Gamma family are 'log' and 'inverse'.

For more details see make.link from stats package in R.

hessian logical. If TRUE the Fisher information matrix is estimated by the observed Hessian Matrix based on the sample. If FALSE (the default value) the Fisher infor-

mation matrix is estimated by the variance of the observed score matrix.

start.value a numeric value or vector. This is the same as start argument in glm or glm2.

The value is a starting point in iteratively reweighted least squares (IRLS) algo-

rithm for estimating the MLE of coefficients in the model.

control a list of parameters to control the fitting process in glm or glm2 function. For

more details, see glm. control.

method a character string indicating which goodness-of-fit statistic is to be computed.

The default value is 'cvm' for the Cramer-von-Mises statistic. Other options include 'ad' for the Anderson-Darling statistic, and 'both' to compute both cvm

and ad.

Value

A list of three containing the following components:

• Statistic: the value of goodness-of-fit statistic.

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• p-value: the approximate p-value for the goodness-of-fit test based on empirical distribution function. if method = 'cvm' or method = 'ad', it returns a numeric value for the statistic and p-value. If method = 'both', it returns a numeric vector with two elements and one for each statistic.

• converged: logical to indicate if the IWLS algorithm have converged or not.

Examples

```
set.seed(123)
n <- 50
p <- 5
x <- matrix( rnorm(n*p, mean = 10, sd = 0.1), nrow = n, ncol = p)
b <- runif(p)
e <- rgamma(n, shape = 3)
y <- exp(x %*% b) * e
testGLMGamma(x, y, l = 'log')
myfit <- glm(y ~ x, family = Gamma('log'), x = TRUE, y = TRUE)
testGLMGamma(fit = myfit)</pre>
```

testLMNormal

Apply Goodness of Fit Test to Residuals of a Linear Model

Description

testLMNormal is used to check the normality assumption of residuals in a linear model. This function can take the response variable and design matrix, fit a linear model, and apply the goodness-of-fit test. Conveniently, it can take an object of class "lm" and directly applies the goodness-of-fit test. The function returns a goodness-of-fit statistic along with an approximate pvalue.

Usage

```
testLMNormal(
    x,
    y,
    fit = NULL,
    ngrid = length(y),
    gridpit = FALSE,
    hessian = FALSE,
    method = "cvm"
)
```

Arguments

x is either a numeric vector or a design matrix. In the design matrix, rows indicate observations and columns presents covariats.

y is a vector of numeric values with the same number of observations or number of rows as x.

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fit	an object of class "lm" returned by lm function in stats package. The default value of fit is NULL. If any object is provided, x and y will be ignored and the class of object is checked. If you pass an object to fit make sure to return the design matrix by setting $x = TRUE$ and the response variable by setting in $y = TRUE$ in lm function. To read more about this see the help documentation for lm function or see the example below.
ngrid	the number of equally spaced points to discretize the $(0,1)$ interval for computing the covariance function.
gridpit	logical. If TRUE (the default value), the parameter ngrid is ignored and $(0,1)$ interval is divided based on probability inverse transformed values obtained from the sample. If FALSE, the interval is divided into ngrid equally spaced points for computing the covariance function.
hessian	logical. If TRUE the Fisher information matrix is estimated by the observed Hessian Matrix based on the sample. If FALSE (the default value) the Fisher information matrix is estimated by the variance of the observed score matrix.
method	a character string indicating which goodness-of-fit statistic is to be computed. The default value is 'cvm' for the Cramer-von-Mises statistic. Other options include 'ad' for the Anderson-Darling statistic, and 'both' to compute both cvm and ad.

Value

A list of two containing the following components:

- Statistic: the value of goodness-of-fit statistic.
- p-value: the approximate p-value for the goodness-of-fit test based on empirical distribution function. if method = 'cvm' or method = 'ad', it returns a numeric value for the statistic and p-value. If method = 'both', it returns a numeric vector with two elements and one for each statistic.

Examples

```
set.seed(123)
n <- 50
p <- 5
x <- matrix( runif(n*p), nrow = n, ncol = p)
e <- rnorm(n)
b <- runif(p)
y <- x %*% b + e
testLMNormal(x, y)
# Or pass lm.fit object directly:
lm.fit <- lm(y ~ x, x = TRUE, y = TRUE)
testLMNormal(fit = lm.fit)</pre>
```

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Apply Goodness of Fit Test for Normal Distribution

Description

Performs the goodness-of-fit test based on empirical distribution function to check if an i.i.d sample follows a Normal distribution.

Usage

```
testNormal(
   x,
   ngrid = length(x),
   gridpit = TRUE,
   hessian = FALSE,
   method = "cvm"
)
```

Arguments

Х	a non-empty numeric vector of sample data.
ngrid	the number of equally spaced points to discretize the $(0,1)$ interval for computing the covariance function.
gridpit	logical. If TRUE (the default value), the parameter ngrid is ignored and (0,1) interval is divided based on probability inverse transformed values obtained from the sample. If FALSE, the interval is divided into ngrid equally spaced points for computing the covariance function.
hessian	logical. If TRUE the Fisher information matrix is estimated by the observed Hessian Matrix based on the sample. If FALSE (the default value) the Fisher information matrix is estimated by the variance of the observed score matrix.
method	a character string indicating which goodness-of-fit statistic is to be computed. The default value is 'cvm' for the Cramer-von-Mises statistic. Other options include 'ad' for the Anderson-Darling statistic, and 'both' to compute both cvm and ad.

Value

A list of two containing the following components:

- Statistic: the value of goodness-of-fit statistic.
- p-value: the approximate p-value for the goodness-of-fit test based on empirical distribution function. if method = 'cvm' or method = 'ad', it returns a numeric value for the statistic and p-value. If method = 'both', it returns a numeric vector with two elements and one for each statistic.

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Examples

```
set.seed(123)
sim_data <- rnorm(n = 50)
testNormal(x = sim_data)
sim_data <- rgamma(50, shape = 3)
testNormal(x = sim_data)</pre>
```

testYourModel

Apply the Goodness of Fit Test Based on Empirical Distribution Function to Any Likelihood Model.

Description

This function applies the goodness-of-fit test based on empirical distribution function. It requires certain inputs depending on whether the model involves parameter estimation or not. If the model is known and there is no parameter estimation, the function requires the sample as a vector and the probability transformed (or pit) values of the sample. This ought to be a vector as well. If there is parameter estimation in the model, the function additionally requires the score as a matrix with n rows and p columns, where n is the sample size and p is the number of estimated parameters. The function checks if the score is zero at the estimated parameter (which is assumed to be the maximum likelihood estimate).

Usage

```
testYourModel(
    x,
    pit,
    score = NULL,
    ngrid = length(x),
    gridpit = TRUE,
    precision = 1e-09,
    method = "cvm"
)
```

Arguments

X	a non-empty numeric vector of sample data.
pit	The probability transformed (or pit) values of the sample which ought to be a numeric vector with the same size as x.
score	The default value is null and refers to no parameter estimation case. If there is parameter estimation, the score matrix must be a matrix with n rows and p columns, where n is the sample size and p is the number of estimated parameters.
ngrid	the number of equally spaced points to discretize the $(0,1)$ interval for computing the covariance function.

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gridpit logical. If TRUE (the default value), the parameter ngrid is ignored and (0,1) interval is divided based on probability inverse transformed values obtained from the sample. If FALSE, the interval is divided into ngrid equally spaced points for computing the covariance function.

precision The theory behind goodness-of-fit test based on empirical distribution function

(edf) works well if the MLE is indeed the root of derivative of log likelihood function. A precision of 1e-9 (default value) is used to check this. A warning message is generated if the score evaluated at MLE is not close enough to zero.

method a character string indicating which goodness-of-fit statistic is to be computed.

The default value is 'cvm' for the Cramer-von-Mises statistic. Other options include 'ad' for the Anderson-Darling statistic, and 'both' to compute both cvm

and ad.

Value

A list of two containing the following components:

- Statistic: the value of goodness-of-fit statistic.
- p-value: the approximate p-value for the goodness-of-fit test based on empirical distribution function. if method = 'cvm' or method = 'ad', it returns a numeric value for the statistic and p-value. If method = 'both', it returns a numeric vector with two elements and one for each statistic.

Examples

```
# Example: Inverse Gaussian (IG) distribution with weights
# Set the seed to reproduce example.
set.seed(123)
# Set the sample size
n <- 50
# Assign weights
weights <- rep(1.5,n)
# Set mean and shape parameters for IG distribution.
           <- 2
mio
           <- 2
1ambda
# Generate a random sample from IG distribution with weighted shape.
sim_data <- statmod::rinvgauss(n, mean = mio, shape = lambda * weights)</pre>
# Compute MLE of parameters, score matrix, and pit values.
             <- inversegaussianMLE(obs = sim_data, w = weights)</pre>
ScoreMatrix <- inversegaussianScore(obs = sim_data, w = weights, mle = theta_hat)</pre>
pitvalues
             <- inversegaussianPIT(obs = sim_data , w = weights, mle = theta_hat)</pre>
# Apply the goodness-of-fit test.
testYourModel(x = sim_data, pit = pitvalues, score = ScoreMatrix)
```

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