# Package 'cocons'

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Title Covariate-Based Covariance Functions for Nonstationary Spatial

Type Package

```
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Description
      Estimation, prediction, and simulation of nonstationary Gaussian process with modular covariate-
      based covariance functions.
      Sources of nonstationarity, such as trend, variance, geometric anisotropy, smooth-
      ness, and nugget, can be considered based on spatial characteristics.
      An induced compact-
      supported nonstationary covariance function is provided, enabling fast and memory-
      efficient computations when handling densely sampled domains.
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2 Contents

## **Contents**

Index

cocons-package	3
coco	3
coco-class	6
cocoOptim	6
cocoPredict	8
cocoSim	10
cov_rns	12
cov_rns_classic	12
cov_rns_pred	13
cov_rns_taper	14
cov_rns_taper_pred	14
getAIC	15
getBIC	16
getBoundaries	16
getBoundariesV2	17
getBoundariesV3	18
getCIs	19
getCovMatrix	19
getCRPS	20
getDesignMatrix	21
getEstims	22
getHessian	22
getLoglik	23
getLogScore	23
getModelLists	24
getModHess	25
GetNeg2loglikelihood	25
GetNeg2loglikelihoodProfile	26
GetNeg2loglikelihoodTaper	27
	28
getScale	29
getSpatEffects	29
getTrend	30
	30
holes_bm	31
is.formula	32
	32
plotOptimInfo	33
. <u>.</u>	34
	34
•	35

**36** 

cocons-package 3

cocons-package	Covariate-based Covariance Functions for Nonstationary Gaussian Processes

#### **Description**

Provides routines and methods for estimating and predicting nonstationary Gaussian process models with modular covariate-based covariance functions. Several sources of nonstationarity can be modeled based on spatial information, including a trend, marginal standard deviation, local geometric anisotropy, local nugget, and spatially varying smoothness. Each of these components is modeled separately. An induced compact-supported nonstationary covariance function is provided to speed up computations when handling densly sampled domains. Model parameters are estimated via maximum likelihood (and flavours of it, such as penalized and profile maximum likelihood). A variety of functions are also included to compute prediction metrics and to visualize, simulate, and summarize these types of models. Details of the models can be found in the vignette and in coco.

#### Disclaimer

This package is provided "as is" without warranty of any kind, either express or implied. Backwards compatibility will not be offered until later versions.

#### Author(s)

```
Federico Blasi [aut, cre], <federico.blasi@uzh.ch>
```

#### **Examples**

```
## Not run:
    vignette("cocons", package = "cocons")
    methods(class = "coco")
## End(Not run)
```

coco

Creates a coco S4 object

#### **Description**

Creates an S4 object of class coco, which is the centerpiece of the **cocons** package. The function provides a set of consistency checks for ensuring the suitability of the different objects involved.

## Usage

```
coco(type, data, locs, z, model.list, info, output = list())
```

4 coco

#### **Arguments**

type	(character) One of two available types "dense" or "sparse". See description.
data	$({\tt data.frame})A{\tt data.frame}withcovariatesinformation,where{\tt colnames}({\tt data})matchesmodel.\\listspecification.$
locs	(matrix) A matrix with spatial locations.
Z	(vector or matrix) A matrix of $n \times r$ response realizations, one realization per column. When considering only one realization, a vector can also be provided.
model.list	(list) A list specifying a model for each aspect of the spatial structure.
info	(list or NULL) A list specifying characteristics of the coco object.
output	(list or NULL) Empty or the resulting object from running optimParallel, adding to this a list with boundary information (check getBoundaries to check the expected structure).

#### **Details**

Two types of coco objects are available, each assuming a different type of covariance matrix for the Gaussian process. Type "dense" builds dense covariance matrices (non zero elements), while type "sparse" builds sparse covariance matrices by tapering the dense covariance matrix with a compact isotropic compact-supported correlation matrix [1]. Type "sparse" allows a set of efficient algorithms, thus making it more suitable for large sample sizes.

An important component of the coco S4 class is the model.list specification, involving individual formulas provided as a list, where each of them specifies a covariate-based parametric model for a specific source of nonstationarity. It involves "trend" for the spatial trend, the "std.dev" for the marginal standard deviation, "scale", "aniso" and "tilt", each of them shaping specific aspects of the local spatial geometrically anisotropy structure, "smooth" handling local smoothness, and "nugget" handling the local nugget effect. The models are defined as:

Source	Related to	Description	Model
mean	$\mu$	Mean function	$oldsymbol{X}_1oldsymbol{eta}$
std.dev	$\sigma^X$	Marginal standard deviation	$\exp(0.5\boldsymbol{X}_2\boldsymbol{\alpha})$
scale	$\mathbf{\Sigma}^{X}$	Local scale	$\exp(\boldsymbol{X}_3\boldsymbol{ heta}_1)$
aniso	$oldsymbol{\Sigma}^{X}$	Local geometric anisotropy	$\exp(\boldsymbol{X}_4 \boldsymbol{\theta}_2)$
tilt	$oldsymbol{\Sigma}^X$	(Restricted) local tilt	$\cos(\operatorname{logit}^{-1}(\boldsymbol{X}_{5}\boldsymbol{ heta}_{3}))$
smooth	$ u^X$	Local smoothness	$( u_u -  u_l)/(1 + \exp(-\boldsymbol{X}_6 \boldsymbol{\phi})) +  u_l$
nugget	$\sigma^X_\epsilon$	Local micro-scale variability	$\exp(\boldsymbol{X}_{7}\boldsymbol{\zeta})$

where  $\beta$ ,  $\alpha$ ,  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ ,  $\phi$ , and  $\zeta$  are the parameter vectors of each model,  $\nu_l$ , and  $\nu_u$  are the lower and upper bounds limiting the range of variation of the spatially-varying smoothness, and where  $X_\ell$  relates to a specific design matrix defined by the specific models for each of the source of nonstationarity.

Lastly, arguments for the "info" list argument involve:

• "lambda": (numeric) a positive scalar specifying the regularization parameter. Larger values penalizes highly-smoothed long-tailed covariance functions.

coco 5

• "smooth.limits": (numeric vector) specifying the range of variation for the spatially varying smoothness (e.g. c(0.5, 2.5)).

- "taper": (numeric) specifying the desired taper function from the spam package (only for "sparse" coco objects).
- "delta": (numeric) specifying the taper range/scale (only for "sparse" coco objects).
- "skip.scale": (integer vector) By default, all covariates are scaled. skip.scale allows to specify the index of those variables in data that should not be scaled during the optimization.

#### Value

(S4) An S4 object of class coco.

#### Author(s)

Federico Blasi

#### References

[1] Furrer, Reinhard, Marc G. Genton, and Douglas Nychka. "Covariance tapering for interpolation of large spatial datasets." Journal of Computational and Graphical Statistics 15.3 (2006): 502-523.

#### See Also

```
spam::cov.wend1()
```

#### **Examples**

```
## Not run:
locs <- expand.grid(seq(0,1,length.out = 10),</pre>
seq(0,1,length.out = 10))
toydata <- data.frame('x' = locs[,1])</pre>
set.seed(1)
z <- rnorm(100)
model.list <- list('mean' = 0,</pre>
                     'std.dev' = formula( ~ 1),
                     'scale' = formula( \sim 1 + x),
                     'aniso' = 0,
                     'tilt' = 0,
                     'smooth' = 3/2,
                     'nugget' = -Inf)
coco_object <- coco(type = 'dense',</pre>
                      data = toydata,
                      locs = as.matrix(locs),
                     z = z,
                     model.list = model.list)
```

6 cocoOptim

```
coco_object
## End(Not run)
```

coco-class

An S4 class to store information

#### **Description**

An S4 class to store information

#### **Slots**

type (character) One of two available types "dense" or "sparse". See description.

data (data.frame) A data.frame with covariates information, where colnames(data) matches model.list specification

locs (numeric matrix) a matrix with locs matching data

z (numeric matrix) A matrix of dimension n x p with response values

model.list (list) A list specifying a model for each aspect of the spatial structure.

info (list) a list with information about the coco object

output (list) if building an already fitted coco object (not the standard approach), then requires an output from Optimparallel output, including as well boundaries, etc.

#### Author(s)

Federico Blasi

cocoOptim

Optimizer for Nonstationary Spatial Models

#### **Description**

This function estimates the spatial model parameters using the L-BFGS-B optimizer [1].

## Usage

```
cocoOptim(coco.object, boundaries = list(), ncores = "auto",
optim.type, safe, optim.control)
```

cocoOptim 7

#### **Arguments**

coco.object (S4) A coco object.

boundaries (list) If provided, a list containing lower, initial, and upper values for the pa-

rameters, as defined by getBoundaries. If not provided, these values are auto-

matically computed with global lower and upper bounds set to -2 and 2.

ncores (character or integer) The number of threads to use for the optimization. If

set to "auto", the number of threads is chosen based on system capabilities or a

fraction of the available cores.

optim.type (character) The optimization approach. Options include:

• "mle": Classical Maximum Likelihood estimation.

 "pmle": Profile Maximum Likelihood, factoring out the spatial trend for dense objects or the global marginal variance parameter for sparse objects.

safe (logical) If TRUE, the function avoids Cholesky decomposition errors due to

ill-posed covariance matrices by returning a pre-defined large value. Defaults to

TRUE.

optim.control (list) A list of settings to be passed to the optimParallel function [2].

#### Value

(S4) An optimized S4 object of class coco.

#### Author(s)

Federico Blasi

#### References

- [1] Byrd, Richard H., et al. "A limited memory algorithm for bound constrained optimization." SIAM Journal on scientific computing 16.5 (1995): 1190-1208.
- [2] Gerber, Florian, and Reinhard Furrer. "optimParallel: An R package providing a parallel version of the L-BFGS-B optimization method." R Journal 11.1 (2019): 352-358.

#### See Also

[optimParallel]

#### **Examples**

8 cocoPredict

```
coco_object <- coco(type = 'dense',</pre>
                    data = holes[[1]][1:100,],
                    locs = as.matrix(holes[[1]][1:100,1:2]),
                    z = holes[[1]][1:100,]$z,
                    model.list = model.list)
optim_coco <- cocoOptim(coco_object,</pre>
boundaries = getBoundaries(coco_object,
lower.value = -3, 3))
plotOptimInfo(optim_coco)
plot(optim_coco)
plot(optim_coco, type = 'ellipse')
plot(optim\_coco, type = 'correlations', index = c(2,3,5))
summary(optim_coco)
getEstims(optim_coco)
## End(Not run)
```

cocoPredict

Prediction Routines for Nonstationary Spatial Models

## **Description**

Computes point predictions and standard errors based on conditional Gaussian distributions for nonstationary spatial models.

#### Usage

```
cocoPredict(coco.object, newdataset, newlocs, type = 'mean', ...)
```

(S4) A fitted coco object.

#### **Arguments**

coco.object

newdataset (data.frame) A data.frame containing the covariates present in model.list at the prediction locations. (matrix) A matrix specifying the prediction locations, matching newdataset newlocs index. (character) Specifies whether to return only the point prediction ('mean') or type

both the point prediction and prediction standard errors ('pred').

Additional arguments. If coco. object contains multiple realizations, the argument index.pred can be used to specify which realization of coco.object@z

should be used for the predictions.

cocoPredict 9

#### Value

A list containing:

- trend: The systematic large-scale variability.
- mean: The stochastic mean.
- sd.pred: The standard errors, when type = 'pred' is specified.

#### Author(s)

Federico Blasi

#### **Examples**

```
## Not run:
# Stationary model
model.list_stat <- list('mean' = 0,</pre>
'std.dev' = formula( ~ 1),
'scale' = formula( ~ 1),
'aniso' = 0,
'tilt' = 0,
'smooth' = 3/2,
'nugget' = -Inf)
model.list_ns <- list('mean' = 0,</pre>
'std.dev' = formula( \sim 1 + cov_x + cov_y),
'scale' = formula( \sim 1 + cov_x + cov_y),
'aniso' = 0,
'tilt' = 0,
'smooth' = 3/2,
'nugget' = -Inf)
coco_object <- coco(type = 'dense',</pre>
data = holes[[1]][1:100, ],
locs = as.matrix(holes[[1]][1:100, 1:2]),
z = holes[[1]][1:100, ]$z,
model.list = model.list_stat)
optim_coco_stat <- cocoOptim(coco_object,</pre>
boundaries = getBoundaries(coco_object,
lower.value = -3, 3))
coco_preds_stat <- cocoPredict(optim_coco_stat, newdataset = holes[[2]],</pre>
newlocs = as.matrix(holes[[2]][, 1:2]),
type = "pred")
# Update model
coco_object@model.list <- model.list_ns</pre>
```

10 cocoSim

```
optim_coco_ns <- cocoOptim(coco_object,
boundaries = getBoundaries(coco_object,
lower.value = -3, 3))

coco_preds_ns <- cocoPredict(optim_coco_ns, newdataset = holes[[2]],
newlocs = as.matrix(holes[[2]][, 1:2]),
type = "pred")

par(mfrow = c(1, 3))

fields::quilt.plot(main = "full data", holes[[1]][, 1:2],
holes[[1]]$z, xlim = c(-1, 1), ylim = c(-1, 1))

fields::quilt.plot(main = "stationary se", holes[[2]][, 1:2],
coco_preds_stat$sd.pred, xlim = c(-1, 1), ylim = c(-1, 1))

fields::quilt.plot(main = "nonstationary se", holes[[2]][, 1:2],
coco_preds_ns$sd.pred, xlim = c(-1, 1), ylim = c(-1, 1))</pre>
```

cocoSim

Marginal and conditional simulation of nonstationary Gaussian processes

#### **Description**

draw realizations of stationary and nonstationary Gaussian processes with covariate-based covariance functions.

#### Usage

```
cocoSim(coco.object, pars, n, seed, standardize,
type = 'classic', sim.type = NULL, cond.info = NULL)
```

#### **Arguments**

coco.object	(S4) A coco object.
pars	(numeric vector or NULL) A vector of parameter values associated with model.list. If coco.object is a fitted object, and pars is NULL, it get pars from coco.object\@output\$pars (and also sets 'type' to 'diff').
n	(integer) Number of realizations to simulate.
seed	(integer or NULL) Seed for random number generation. Defaults to NULL.
standardize	(logical) Indicates whether the provided covariates should be standardized (TRUE) or not (FALSE). Defaults to TRUE.

cocoSim 11

type	(character) Specifies whether the parameters follow a classical parameter-
	ization ('classic') or a difference parameterization ('diff'). Defaults to
	'classic'. For sparse coco objects, only 'diff' is allowed.
sim.type	(character) If set to 'cond', a conditional simulation is performed.
cond.info	(list) A list containing additional information required for conditional simulation.

#### **Details**

#' The argument sim.type = 'cond' specifies a conditional simulation, requiring cond.info to be provided. cond.info is a list including newdataset, a data.frame containing covariates present in model.list at the simulation locations, and newlocs, a matrix specifying the locations corresponding to the simulation, with indexing that matches newdataset.

The argument type = 'classic' assumes a simplified parameterization for the covariance function, with log-parameterizations applied to the parameters std.dev, scale, and smooth.

#### Value

```
(matrix) a matrix dim(data)[1] x n.
```

#### Author(s)

Federico Blasi

#### See Also

coco

#### **Examples**

```
## Not run:
model.list <- list('mean' = 0,</pre>
                     'std.dev' = formula( \sim 1 + cov_x + cov_y),
                     'scale' = formula( \sim 1 + cov_x + cov_y),
                    'aniso' = 0,
                    'tilt' = 0,
                     'smooth' = 0.5,
                     'nugget' = -Inf)
coco_object <- coco(type = 'dense',</pre>
                     data = holes[[1]][1:1000,],
                     locs = as.matrix(holes[[1]][1:1000,1:2]),
                     z = holes[[1]][1:1000,]$z,
                     model.list = model.list)
coco_sim <- cocoSim(coco.object = coco_object,</pre>
             pars = c(0,0.25,0.25, \text{ # pars related to std.dev})
             log(0.25), 1, -1),
                                   # pars related to scale
             n = 1,
```

12 cov\_rns\_classic

```
standardize = TRUE)
fields::quilt.plot(coco_object@locs,coco_sim)
## End(Not run)
```

cov\_rns

Dense covariance function (difference parameterization)

## Description

Dense covariance function (difference parameterization)

## Usage

```
cov_rns(theta, locs, x_covariates, smooth_limits)
```

## Arguments

theta vector of parameters
locs a matrix with locations
x\_covariates design data.frame
smooth\_limits smooth limits

#### Value

dense covariance matrix

cov\_rns\_classic

Dense covariance function (classic parameterization)

## Description

Dense covariance function (classic parameterization)

#### Usage

```
cov_rns_classic(theta, locs, x_covariates)
```

## Arguments

theta vector of parameters
locs a matrix with locations
x\_covariates design data.frame

cov\_rns\_pred 13

#### Value

dense covariance matrix with classic parameterization

cov\_rns\_pred

Dense covariance function

## Description

Dense covariance function

## Usage

```
cov_rns_pred(
   theta,
   locs,
   locs_pred,
   x_covariates,
   x_covariates_pred,
   smooth_limits
)
```

#### **Arguments**

theta vector of parameters

locs a matrix with locations

locs\_pred a matrix with prediction locations

x\_covariates design data.frame

 $x\_covariates\_pred$ 

design data.frame at prediction locations

smooth\_limits smooth limits

#### Value

dense covariance matrix

14 cov\_rns\_taper\_pred

cov\_rns\_taper

Sparse covariance function

## Description

Sparse covariance function

## Usage

```
cov_rns_taper(
   theta,
   locs,
   x_covariates,
   colindices,
   rowpointers,
   smooth_limits
)
```

## Arguments

theta vector of parameters
locs a matrix with locations
x\_covariates design data.frame
colindices from spam object
rowpointers from spam object
smooth\_limits smooth limits

#### Value

sparse covariance matrix between locs and pred\_locs

cov\_rns\_taper\_pred

Sparse covariance function

## Description

Sparse covariance function

getAIC 15

#### Usage

```
cov_rns_taper_pred(
  theta,
  locs,
  locs_pred,
  x_covariates,
  x_covariates_pred,
  colindices,
  rowpointers,
  smooth_limits
)
```

## Arguments

theta vector of parameters locs a matrix with locations

locs\_pred a matrix with prediction locations

x\_covariates design data.frame

x\_covariates\_pred

design data.frame at prediction locations

colindices from spam object rowpointers from spam object smooth\_limits smooth limits

#### Value

sparse covariance matrix at locs

getAIC Retrieve AIC

#### **Description**

Retrieve the Akaike information criterion from a fitted coco object.

## Usage

```
getAIC(coco.object)
```

## Arguments

```
coco.object (S4) a fitted coco S4 object.
```

## Value

```
(numeric) the associated AIC value
```

16 getBoundaries

#### Author(s)

Federico Blasi

getBIC

Retrieve BIC

#### **Description**

Retrieve BIC from a fitted coco object.

#### Usage

```
getBIC(coco.object)
```

#### **Arguments**

coco.object

(\$4) a fitted coco \$4 object.

#### Value

(numeric) the associated BIC value

#### Author(s)

Federico Blasi

getBoundaries

Simple build of boundaries

## Description

provides a generic set of upper and lower bounds for the L-BFGS-B routine

## Usage

```
getBoundaries(x, lower.value, upper.value)
```

#### Arguments

x (S4) or (list) a coco.object or a par.pos list (as output from getDesignMatrix) lower.value (numeric vector) if provided, provides a vector filled with values lower.value. (numeric vector) if provided, provides a vector filled with values upper.value.

## Value

(list) a list with boundaries and simple init values for the optim L-BFGS-B routine

getBoundariesV2

#### Author(s)

Federico Blasi

getBoundariesV2	Simple build of boundaries (v2)	

#### **Description**

provides a generic set of upper and lower bounds for the L-BFGS-B routine

#### Usage

```
getBoundariesV2(coco.object, mean.limits, std.dev.limits,
scale.limits, aniso.limits, tilt.limits, smooth.limits, nugget.limits)
```

#### **Arguments**

```
coco.object
                   (S4) a coco object.
mean.limits
                   (numeric vector) a vector of c(lower,init,upper) values for the associated param.
std.dev.limits
                  (numeric vector) a vector of c(lower,init,upper) values for the associated param.
scale.limits
                   (numeric vector) a vector of c(lower,init,upper) values for the associated param.
aniso.limits
                   (numeric vector) a vector of c(lower,init,upper) values for the associated param.
tilt.limits
                   (numeric vector) a vector of c(lower,init,upper) values for the associated param.
smooth.limits
                   (numeric vector) a vector of c(lower,init,upper) values for the associated param.
                   (numeric vector) a vector of c(lower,init,upper) values for the associated param.
nugget.limits
```

#### Value

(list) a list with boundaries for the optim L-BFGS-B routine

#### Author(s)

Federico Blasi

18 getBoundariesV3

getBoundariesV3 Simple build of boundaries (v3)

#### **Description**

provides a generic set of upper and lower bounds for the L-BFGS-B routine

#### Usage

```
getBoundariesV3(coco.object, mean.limits, global.lower,
std.dev.max.effects,
scale.max.effects, aniso.max.effects, tilt.max.effects,
smooth.max.effects, nugget.max.effects)
```

## **Arguments**

coco.object (S4) a coco object. mean.limits (numeric vector) a vector of c(lower,init,upper) values for the associated param. global.lower (numeric vector) a vector of c(lower, init, upper) values for the associated param. std.dev.max.effects (numeric vector) a vector of c(lower,init,upper) values for the associated param. scale.max.effects (numeric vector) a vector of c(lower,init,upper) values for the associated param. aniso.max.effects (numeric vector) a vector of c(lower,init,upper) values for the associated param. tilt.max.effects (numeric vector) a vector of c(lower,init,upper) values for the associated param. smooth.max.effects (numeric vector) a vector of c(lower,init,upper) values for the associated param. nugget.max.effects (numeric vector) a vector of c(lower,init,upper) values for the associated param.

#### Value

(list) a list with boundaries for the optim L-BFGS-B routine

#### Author(s)

Federico Blasi

getCIs 19

getCIs	Compute approximate confidence intervals for a coco object	ţ
_		

## Description

Compute approximate confidence intervals for a (fitted) coco object.

#### Usage

```
getCIs(coco.object, inv.hess, alpha = 0.05)
```

#### Arguments

coco.object (S4) a fitted coco S4 object.

inv.hess (matrix) Inverse of the Hessian. getHessian.

alpha (numeric) confidence level.

#### Value

(numeric matrix) a matrix with approximate confidence intervals for each parameter in the model.

#### Author(s)

Federico Blasi

getCovMatrix	Covariance matrix for "coco" class

#### **Description**

Compute the covariance matrix of coco.object.

#### Usage

```
getCovMatrix(coco.object, type = 'global', index = NULL)
```

## Arguments

coco.object (S4) a fitted coco() object.

type (character) whether 'global' to retrieve the regular covariance matrix, or

'local' to retrieve global covariance. based on the local aspects of a specific

location (not implemented yet).

index (integer) index to perform local covariance matrix (not implemented yet).

20 getCRPS

#### Value

(matrix or S4) a n x n covariance matrix (for 'dense' coco objects) or a S4 spam object (for 'sparse' coco objects).

#### Author(s)

Federico Blasi

## **Examples**

```
## Not run:
model.list <- list('mean' = 0,</pre>
                     'std.dev' = formula( \sim 1 + cov_x + cov_y),
                    'scale' = formula( \sim 1 + cov_x + cov_y),
                    'aniso' = 0,
                    'tilt' = 0,
                    'smooth' = 3/2,
                    'nugget' = -Inf)
coco_object <- coco(type = 'dense',</pre>
                     data = holes[[1]][1:100,],
                     locs = as.matrix(holes[[1]][1:100,1:2]),
                     z = holes[[1]][1:100,]$z,
                     model.list = model.list)
optim_coco <- cocoOptim(coco_object,</pre>
boundaries = getBoundaries(coco_object,
lower.value = -3, 3))
getCovMatrix(optim_coco)
## End(Not run)
```

getCRPS

Based on a set of predictions computes the Continuous Ranked Probability Score

#### **Description**

Retrieves the Continuous Ranked Probability Score (CRPS) [1].

## Usage

```
getCRPS(z.pred, mean.pred, sd.pred)
```

getDesignMatrix 21

#### **Arguments**

```
z.pred (numeric vector).
mean.pred (numeric vector).
sd.pred (numeric vector).
```

#### Value

(numeric vector) retrieves CRPS.

## Author(s)

Federico Blasi

#### References

[1] Gneiting, Tilmann, and Adrian E. Raftery. "Strictly proper scoring rules, prediction, and estimation." Journal of the American statistical Association 102.477 (2007): 359-378.

getDesignMatrix

Create an efficient design matrix based on a list of aspect models

#### **Description**

Creates a unique design matrix based on model specification for each of the different potentially spatially varying aspects.

#### Usage

```
getDesignMatrix(model.list, data)
```

#### **Arguments**

model.list (list) a list of formulas, one for each source of nonstationarity, specifying the

models.

data (data.frame) a data.frame.

#### Value

(list) a list with two elements: a design matrix of dimension (n x p), and a par.pos object, indexing columns of the design matrix to each of the spatially-varying functions.

#### Author(s)

Federico Blasi

22 getHessian

getEstims

Retrieve estimates from a fitted coco object

## **Description**

Retrieve estimates from a fitted coco object.

#### Usage

```
getEstims(coco.object)
```

## Arguments

```
coco.object (S4) a fitted coco S4 object.
```

#### Value

(list) a list with the estimates parameters for the different aspects

#### Author(s)

Federico Blasi

getHessian

getHessian

#### **Description**

numerically approximate the Hessian. Hessians of parameters based on "pmle" are based on full likelihoods.

#### Usage

```
getHessian(coco.object, ncores = parallel::detectCores() - 1,
eps = .Machine$double.eps^(1/4))
```

## Arguments

```
coco.object (S4) a fitted coco object.
```

ncores (integer) number of cores used for the computation.

eps (numeric)...

## Value

(numeric matrix) a symmetric matrix pxp of the approximated (observed) Hessian

getLoglik 23

#### Author(s)

Federico Blasi

getLoglik

Retrieve the loglikelihood value

## Description

Retrieve the loglikelihood value from a fitted coco object.

#### Usage

```
getLoglik(coco.object)
```

#### **Arguments**

```
coco.object
```

(S4) a fitted coco S4 object.

#### Value

(numeric) wrap for value from a OptimParallel object

#### Author(s)

Federico Blasi

getLogScore

Based on a set of predictions computes the Log-Score

#### **Description**

```
Computes the Log-Score [1].
```

## Usage

```
getLogScore(z.pred, mean.pred, sd.pred)
```

## Arguments

```
z.pred (numeric vector).
mean.pred (numeric vector).
sd.pred (numeric vector).
```

## Value

```
(numeric vector) retrieves Log-Score.
```

24 getModelLists

#### Author(s)

Federico Blasi

#### References

[1] Gneiting, Tilmann, and Adrian E. Raftery. "Strictly proper scoring rules, prediction, and estimation." Journal of the American statistical Association 102.477 (2007): 359-378.

getModelLists

Builds the necessary input for building covariance matrices

## Description

Returns a list of parameter vectors for each of the aspects.

#### Usage

```
getModelLists(theta, par.pos, type = 'diff')
```

## Arguments

theta	(numeric vector)	a vector of	f length p,	where p i	s the num	ber of parameters
-------	------------------	-------------	-------------	-----------	-----------	-------------------

for each of the models.

par.pos (list) a list detailing in which position of each aspect the elements of theta

should be placed. Expected to be par.pos output of getDesignMatrix.

type (character) whether parameters are related to a classical parameterization

('classic') or a difference parameterization 'diff'. Default set to 'diff'.

#### Value

(list) a list of different spatial aspects and mean required for the cov.rns functions

#### Author(s)

Federico Blasi

getModHess 25

## **Description**

Based on the inverse of the Hessian (based on the difference parameterization for the std.dev and scale parameters), retrieves the modified inverse of the hessian (i.e. std.dev and scale).

## Usage

```
getModHess(coco.object, inv.hess)
```

## Arguments

```
coco.object (S4) a fitted coco S4 object.
inv.hess (matrix) Inverse of the Hessian.
```

#### Value

(numeric matrix) the modified inverse of the hessian matrix

#### Author(s)

Federico Blasi

```
{\tt GetNeg2loglikelihood} \quad \textit{GetNeg2loglikelihood}
```

## **Description**

compute the negative 2 log likelihood based on theta

## Usage

```
GetNeg2loglikelihood(theta, par.pos, locs, x_covariates,
smooth.limits, z, n, lambda, safe = TRUE)
```

#### **Arguments**

```
theta (numeric vector) a vector with parameters values.
par.pos (list) par.pos list.
```

 z (numeric vector) a vector of observed values.

n (integer)  $\dim(z)[1]$ .

lambda (numeric) regularization parameter.

safe (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.

Default TRUE.

#### Value

value

## Author(s)

Federico Blasi

 ${\tt GetNeg2loglikelihoodProfile}$ 

GetNeg2loglikelihoodProfile

## Description

compute the negative 2 log likelihood based on theta

#### Usage

```
GetNeg2loglikelihoodProfile(theta, par.pos, locs, x_covariates,
smooth.limits, z, n, x_betas,lambda, safe = TRUE)
```

#### **Arguments**

theta (numeric vector) a vector with parameters values.

par.pos (list) par.pos list.

locs (matrix) spatial location matrix.

x\_covariates (data.frame) design matrix.

smooth.limits (numeric vector) smooth.limits.

z (numeric vector) a vector of observed values.

n (integer)  $\dim(z)[1]$ .

x\_betas (matrix) or (data.frame) design matrix for the trend.

lambda (numeric) regularization parameter.

safe (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.

Default TRUE.

## Value

value

#### Author(s)

Federico Blasi

GetNeg2loglikelihoodTaper

GetNeg2loglikelihoodTaper

## Description

compute the negative 2 log likelihood based on theta

## Usage

```
GetNeg2loglikelihoodTaper(theta, par.pos, ref_taper, locs,
x_covariates, smooth.limits, cholS, z, n, lambda, safe = TRUE)
```

#### **Arguments**

theta (numeric vector) a vector with parameters values.

(list) par.pos list from getDesignMatrix. par.pos

(S4) spam object based on a compact-supported covariance function. ref\_taper

(matrix) spatial location matrix. locs

x\_covariates (data.frame) design matrix.

smooth.limits (numeric vector) smooth.limits. cholS

(\$4) Cholesky object from spam.

(numeric vector) a vector of observed values. Z

(numeric)  $\dim(z)[1]$ .

lambda (numeric) regularization parameter.

safe (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.

Default TRUE.

#### Value

value

#### Author(s)

Federico Blasi

GetNeg2loglikelihoodTaperProfile

Get Neg 2 log like lihood Taper Profile

## **Description**

compute the negative 2 log likelihood based on theta

#### Usage

```
GetNeg2loglikelihoodTaperProfile(theta, par.pos, ref_taper,
locs, x_covariates, smooth.limits, cholS, z, n, lambda, safe = TRUE)
```

#### **Arguments**

theta (numeric vector) a vector with parameters values.

par.pos (list) par.pos list.

ref\_taper (S4) spam object based on a taper based covariance function.

locs (matrix) spatial location matrix.

x\_covariates (data.frame) design matrix.

smooth.limits (numeric vector) smooth.limits.

cholS (S4) Cholesky object from spam.

z (numeric vector) a vector of observed values.

n (integer)  $\dim(z)[1]$ .

lambda (numeric) regularization parameter.

safe (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.

Default TRUE.

#### Value

(numeric)

#### Author(s)

Federico Blasi

getScale 29

getScale Fast and simple standardization for the design matrix.
---

## Description

Centers and scale the design matrix.

## Usage

```
getScale(x, mean.vector = NULL, sd.vector = NULL)
```

## Arguments

х	(S4) or (matrix) a coco object, or a n x p matrix with covariate information to introduce, where the first column is a column of ones.
mean.vector	(numeric vector) if provided, it centers covariates based on this information.
sd.vector	(numeric vector) if provided, it scales covariates based on this information.

#### Value

(list) a list with a scaled design matrix of dimension  $n \times (p+1)$ , and a set of mean and sd vectors employed to scale the matrix

#### Author(s)

Federico Blasi

getSpatEffects	Evaluates the spatially-varying functions from a coco object at locs	

## Description

Evaluates the spatially-varying functions of the nonstationary spatial structure.

#### Usage

```
getSpatEffects(coco.object)
```

## Arguments

```
coco.object (S4) a fitted coco S4 object.
```

## Value

(list) a list with the different estimated surfaces.

30 holes

#### Author(s)

Federico Blasi

getTrend

Computes the spatial trend of a (fitted) coco object

## Description

Compute the trend of the (fitted) coco object.

#### Usage

```
getTrend(coco.object)
```

#### Arguments

coco.object

(S4) a fitted coco S4 object.

#### Value

(numeric vector) a vector with the adjusted trend.

#### Author(s)

Federico Blasi

holes

Holes Data Set

## Description

The synthetic "holes" provides a set of training and test data.frame of a Gaussian process realization with a (inherently dense) nonstationary covariance function. Four holes are present in the training dataset, and the task is to predict them.

## Usage

holes

#### **Format**

A list with training and test data.frame with rows and variables:

- x first spatial coordinate
- y second spatial coordinate
- cox\_x first spatial characteristic
- cov\_y second spatial characteristic
- z response variable

holes\_bm 31

#### **Source**

Source of the data

#### **Examples**

data(holes)

holes\_bm

Holes with trend + multiple realizations Data Set

## Description

The synthetic "holes\_bm" provides a set of training and test data.frame of a Gaussian process realization with a (inherently dense) nonstationary covariance function. Four holes are present in the training dataset, and the task is to predict them. This version provides ten independent realizations of the process, as well as considers a spatial mean effect.

## Usage

holes\_bm

#### **Format**

A list with training, training.z, test, and test.z data.frames with rows and variables:

- x first spatial coordinate
- y second spatial coordinate
- cox\_x first spatial characteristic
- cov\_y second spatial characteristic
- cov\_z third spatial characteristic
- z.i i-th response variable

#### **Source**

Source of the data

## **Examples**

data(holes\_bm)

is.formula

check whether an R object is a formula

## Description

check whether an R object is a formula

#### Usage

```
is.formula(x)
```

## **Arguments**

Х

(ANY) an R object.

#### Value

TRUE/FALSE

#### Author(s)

Federico Blasi

```
plot,coco,missing-method
```

Plot Method for coco objects

#### **Description**

This method plots objects of class coco.

## Usage

```
## S4 method for signature 'coco,missing'
plot(x, y, type = NULL, index = NULL, factr = 0.1, ...)
```

## **Arguments**

x (S4) A fitted object of class coco.

y Not used.

type (character or NULL) The type of plot. NULL or "ellipse" for drawing ellipse of

the convolution kernels.

index (integer vector) For plotting local correlation plots.

factr (numeric) Factor rate for size of ellipses.
... Additional arguments passed to quilt.plot.

plotOptimInfo 33

## Value

Several plots are created.

## Author(s)

Federico Blasi

plotOptimInfo

Plot log info detailed

## Description

plot output of optim

## Usage

```
plotOptimInfo(coco.object, ...)
```

## Arguments

```
coco.object an optimized coco.object ... arguments for par()
```

## Value

Outputs a sequence of plots detailing parameters during the optimization routine

## Author(s)

Federico Blasi

## See Also

```
cocoOptim()
```

34 stripes

show

Show Method for Coco Class

#### **Description**

This method show objects of class 'coco'.

#### Usage

```
## S4 method for signature 'coco'
show(object)
```

#### **Arguments**

object

(S4) An object of class 'coco'.

#### Value

A plot is created.

## Author(s)

Federico Blasi

stripes

Stripes Data Set

#### **Description**

The synthetic "stripes" provides a set of training and test data.frame of a Gaussian process realization with a (inherently sparse) nonstationary covariance function. Several stripes are present in the training dataset, and the task is to predict them.

## Usage

stripes

#### **Format**

A list with training and test data.frame with rows and variables:

x first spatial coordinate

y second spatial coordinate

cox\_x first spatial characteristic

cov\_y second spatial characteristic

cov\_xy third spatial characteristic

z response variable

summary 35

#### Source

Source of the data

## Examples

```
data(stripes)
```

summary

Summary Method for Coco Class

## Description

method summary for objects of class 'coco'.

## Usage

```
## S4 method for signature 'coco'
summary(object, inv.hess = NULL)
```

## Arguments

object (S4) An object of class 'coco'.

inv.hess (numeric matrix or NULL) inverse of the approximated hessian matrix (getHes-

sian)

## Value

summary the coco object

#### Author(s)

Federico Blasi

# **Index**

* datasets holes, 30	<pre>getSpatEffects, 29 getTrend, 30</pre>		
holes_bm, 31	geen end, 50		
stripes, 34	holes, 30		
20. 2005, 0	holes_bm, 31		
coco, <i>3</i> , <i>3</i> , <i>7</i> , <i>8</i> , <i>10</i> , <i>11</i>			
coco(), <i>19</i>	is.formula,32		
coco-class, 6	entimDarallel 4.7		
cocons (cocons-package), 3	optimParallel, 4, 7		
cocons-package, 3	plot,coco,missing-method,32		
cocoOptim, 6	plot,coco-method		
cocoOptim(), 33	(plot,coco,missing-method), 32		
cocoPredict, 8	plotOptimInfo, 33		
cocoSim, 10			
cov_rns, 12	quilt.plot, 32		
cov_rns_classic, 12			
cov_rns_pred, 13	show, 34		
cov_rns_taper, 14	show, coco-method (show), 34		
cov_rns_taper_pred, 14	spam::cov.wend1(), 5		
	stripes, 34		
getAIC, 15	summary, 35		
getBIC, 16	summary, coco-method (summary), 35		
getBoundaries, 4, 7, 16			
getBoundariesV2, 17 getBoundariesV3, 18			
getCIs, 19			
getCovMatrix, 19			
getCRPS, 20			
getDesignMatrix, 16, 21, 24, 27			
getEstims, 22			
getHessian, 19, 22			
getLoglik, 23			
getLogScore, 23			
getModelLists, 24			
getModHess, 25			
GetNeg2loglikelihood, 25			
GetNeg2loglikelihoodProfile, 26			
GetNeg2loglikelihoodTaper, 27			
GetNeg2loglikelihoodTaperProfile, 28			
getScale. 29			