Package 'DiscreteDists'

September 13, 2024

Title Discrete Statistical Distributions
Version 1.0.0
Description Implementation of new discrete statistical distributions. Each distribution includes the traditional functions as well as an additional function called the family function, which can be used to estimate parameters within the 'gamlss' framework.
License MIT + file LICENSE
RdMacros Rdpack
Imports gamlss, gamlss.dist, pracma, Rdpack, Rcpp
LinkingTo Rcpp
Encoding UTF-8
RoxygenNote 7.3.1
Suggests knitr, rmarkdown
<pre>URL https://github.com/fhernanb/DiscreteDists</pre>
BugReports https://github.com/fhernanb/DiscreteDists/issues
NeedsCompilation yes
Author Freddy Hernandez-Barajas [aut, cre]
Maintainer Freddy Hernandez-Barajas <fhernanb@unal.edu.co></fhernanb@unal.edu.co>
Repository CRAN
Date/Publication 2024-09-13 18:10:06 UTC
Contents
add

2 add

	dDBH	5
	dDGEII	7
	dDIKUM	10
	dDLD	13
	dDMOLBE	16
	DGEII	18
	dGGEO	20
	dHYPERPO	23
	dHYPERPO2	26
	dHYPERPO_single	28
		29
	DIKUM	30
	DLD	31
	DMOLBE	33
	dPOISXL	35
	f11_cpp	37
		38
	HYPERPO	40
	HYPERPO2	41
	mean_var_hp	43
	plot_discrete_cdf	45
	•	46
Index	4	48

Sum of One-Dimensional Functions

Description

add

Sum of One-Dimensional Functions

Usage

```
add(f, lower, upper, ..., abs.tol = .Machine$double.eps)
```

Arguments

f	an R function taking a numeric first argument and returning a numeric vector of the same length.
lower	the lower limit of sum. Can be infinite.
upper	the upper limit of sum. Can be infinite.
	additional arguments to be passed to f.
abs.tol	absolute accuracy requested.

Value

This function returns the sum value.

DBH 3

Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

Examples

DBH

The Discrete Burr Hatke family

Description

The function DBH() defines the Discrete Burr Hatke distribution, one-parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
DBH(mu.link = "logit")
```

Arguments

 ${\it mu.link}$

defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log)

Details

The Discrete Burr-Hatke distribution with parameters μ has a support 0, 1, 2, ... and density given by

$$f(x|\mu) = (\frac{1}{x+1} - \frac{\mu}{x+2})\mu^x$$

The pmf is log-convex for all values of $0 < \mu < 1$, where $\frac{f(x+1;\mu)}{f(x;\mu)}$ is an increasing function in x for all values of the parameter μ .

Note: in this implementation we changed the original parameters λ for μ , we did it to implement this distribution within gamlss framework.

DBH DBH

Value

Returns a gamlss.family object which can be used to fit a Discrete Burr-Hatke distribution in the gamlss() function.

Author(s)

Valentina Hurtado Sepulveda, <vhurtados@unal.edu.co>

References

El-Morshedy M, Eliwa MS, Altun E (2020). "Discrete Burr-Hatke distribution with properties, estimation methods and regression model." *IEEE access*, **8**, 74359–74370.

See Also

dDBH.

```
# Example 1
# Generating some random values with
# known mu
y < - rDBH(n=1000, mu=0.74)
library(gamlss)
mod1 <- gamlss(y~1, family=DBH,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
# Extracting the fitted values for mu
# using the inverse logit function
inv_logit <- function(x) exp(x) / (1+exp(x))
inv_logit(coef(mod1, parameter="mu"))
# Example 2
# Generating random values under some model
\# A function to simulate a data set with Y \sim DBH
gendat <- function(n) {</pre>
  x1 <- runif(n)</pre>
      <-inv_logit(-3 + 5 * x1)
 y <- rDBH(n=n, mu=mu)
  data.frame(y=y, x1=x1)
}
datos <- gendat(n=150)</pre>
mod2 <- NULL
mod2 <- gamlss(y~x1, family=DBH, data=datos,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
```

dDBH 5

dDBH

The Discrete Burr Hatke distribution

Description

These functions define the density, distribution function, quantile function and random generation for the Discrete Burr Hatke distribution with parameter μ .

Usage

```
dDBH(x, mu, log = FALSE)
pDBH(q, mu, lower.tail = TRUE, log.p = FALSE)
qDBH(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rDBH(n, mu = 1)
```

Arguments

x, q	vector of (non-negative integer) quantiles.
mu	vector of the mu parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \mathrel{<=} x]$, otherwise, $P[X \mathrel{>} x]$.
р	vector of probabilities.
n	number of random values to return

Details

The Discrete Burr-Hatke distribution with parameters μ has a support 0, 1, 2, ... and density given by

$$f(x|\mu) = (\frac{1}{x+1} - \frac{\mu}{x+2})\mu^x$$

6 dDBH

The pmf is log-convex for all values of $0 < \mu < 1$, where $\frac{f(x+1;\mu)}{f(x;\mu)}$ is an increasing function in x for all values of the parameter μ .

Note: in this implementation we changed the original parameters λ for μ , we did it to implement this distribution within gamlss framework.

Value

dDBH gives the density, pDBH gives the distribution function, qDBH gives the quantile function, rDBH generates random deviates.

Author(s)

Valentina Hurtado Sepulveda, <vhurtados@unal.edu.co>

References

El-Morshedy M, Eliwa MS, Altun E (2020). "Discrete Burr-Hatke distribution with properties, estimation methods and regression model." *IEEE access*, **8**, 74359–74370.

See Also

DBH.

```
# Example 1
# Plotting the mass function for different parameter values
plot(x=0:5, y=dDBH(x=0:5, mu=0.1),
     type="h", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", ylim=c(0, 1),
     main="Probability mu=0.1")
plot(x=0:10, y=dDBH(x=0:10, mu=0.5),
     type="h", lwd=2, col="tomato", las=1,
     ylab="P(X=x)", xlab="X", ylim=c(0, 1),
     main="Probability mu=0.5")
plot(x=0:15, y=dDBH(x=0:15, mu=0.9),
     type="h", lwd=2, col="green4", las=1,
     ylab="P(X=x)", xlab="X", ylim=c(0, 1),
     main="Probability mu=0.9")
# Example 2
# Checking if the cumulative curves converge to 1
x_max <- 15
cumulative_probs1 <- pDBH(q=0:x_max, mu=0.1)</pre>
cumulative_probs2 <- pDBH(q=0:x_max, mu=0.5)</pre>
cumulative_probs3 <- pDBH(q=0:x_max, mu=0.9)</pre>
```

dDGEII 7

```
plot(x=0:x_max, y=cumulative_probs1, col="dodgerblue",
     type="o", las=1, ylim=c(0, 1),
     main="Cumulative probability for Burr-Hatke",
     xlab="X", ylab="Probability")
points(x=0:x_max, y=cumulative_probs2, type="o", col="tomato")
points(x=0:x_max, y=cumulative_probs3, type="o", col="green4")
legend("bottomright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=0.1",
                 "mu=0.5",
                 "mu=0.9"))
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
mu <- 0.4
x_max <- 10
probs1 <- dDBH(x=0:x_max, mu=mu)</pre>
names(probs1) <- 0:x_max</pre>
x \leftarrow rDBH(n=1000, mu=mu)
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside = TRUE, names.arg = nombres,</pre>
              col=c("dodgerblue3","firebrick3"), las=1,
              xlab="X", ylab="Proportion")
legend("topright",
       legend=c("Theoretical", "Simulated"),
       bty="n", lwd=3,
       col=c("dodgerblue3", "firebrick3"), lty=1)
# Example 4
# Checking the quantile function
mu <- 0.97
p <- seq(from=0, to=1, by = 0.01)
qxx <- qDBH(p, mu, lower.tail = TRUE, log.p = FALSE)</pre>
plot(p, qxx, type="s", lwd=2, col="green3", ylab="quantiles",
     main="Quantiles of BH(mu=0.97)")
```

dDGEII

Discrete generalized exponential distribution - a second type

Description

These functions define the density, distribution function, quantile function and random generation for the Discrete generalized exponential distribution a second type with parameters μ and σ .

8 dDGEII

Usage

```
dDGEII(x, mu = 0.5, sigma = 1.5, log = FALSE)
pDGEII(q, mu = 0.5, sigma = 1.5, lower.tail = TRUE, log.p = FALSE)
rDGEII(n, mu = 0.5, sigma = 1.5)
qDGEII(p, mu = 0.5, sigma = 1.5, lower.tail = TRUE, log.p = FALSE)
```

Arguments

x, q vector of (non-negative integer) quantiles.

mu vector of the mu parameter.
sigma vector of the sigma parameter.

log, log.p logical; if TRUE, probabilities p are given as log(p).

lower.tail logical; if TRUE (default), probabilities are $P[X \le x]$, otherwise, P[X > x].

n number of random values to return.

p vector of probabilities.

Details

The DGEII distribution with parameters μ and σ has a support 0, 1, 2, ... and mass function given by

$$f(x|\mu,\sigma) = (1-\mu^{x+1})^{\sigma} - (1-\mu^x)^{\sigma}$$

with $0 < \mu < 1$ and $\sigma > 0$. If $\sigma = 1$, the DGEII distribution reduces to the geometric distribution with success probability $1 - \mu$.

Note: in this implementation we changed the original parameters p to μ and α to σ , we did it to implement this distribution within gamlss framework.

Value

dDGEII gives the density, pDGEII gives the distribution function, qDGEII gives the quantile function, rDGEII generates random deviates.

Author(s)

Valentina Hurtado Sepulveda, <vhurtados@unal.edu.co>

References

Nekoukhou V, Alamatsaz MH, Bidram H (2013). "Discrete generalized exponential distribution of a second type." *Statistics*, **47**(4), 876-887.

See Also

DGEII.

dDGEII 9

```
# Plotting the mass function for different parameter values
x_max <- 40
probs1 <- dDGEII(x=0:x_max, mu=0.1, sigma=5)</pre>
probs2 <- dDGEII(x=0:x_max, mu=0.5, sigma=5)</pre>
probs3 <- dDGEII(x=0:x_max, mu=0.9, sigma=5)</pre>
# To plot the first k values
plot(x=0:x_max, y=probs1, type="o", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", main="Probability for DGEII",
     ylim=c(0, 0.60))
points(x=0:x_max, y=probs2, type="o", lwd=2, col="tomato")
points(x=0:x_max, y=probs3, type="o", lwd=2, col="green4")
legend("topright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=0.1, sigma=5",
                "mu=0.5, sigma=5",
                "mu=0.9, sigma=5"))
# Example 2
# Checking if the cumulative curves converge to 1
#plot1
x_max <- 10
plot_discrete_cdf(x=0:x_max,
                  fx=dDGEII(x=0:x_max, mu=0.3, sigma=15),
                  col="dodgerblue",
                  main="CDF for DGEII",
                  1wd=3)
legend("bottomright", legend="mu=0.3, sigma=15",
       col="dodgerblue", lty=1, lwd=2, cex=0.8)
#plot2
plot_discrete_cdf(x=0:x_max,
                  fx=dDGEII(x=0:x_max, mu=0.5, sigma=30),
                  col="tomato",
                  main="CDF for DGEII",
                  1wd=3)
legend("bottomright", legend="mu=0.5, sigma=30",
       col="tomato", lty=1, lwd=2, cex=0.8)
#plot3
plot_discrete_cdf(x=0:x_max,
                  fx=dDGEII(x=0:x_max, mu=0.5, sigma=50),
                  col="green4",
                  main="CDF for DGEII",
                  1wd=3)
legend("bottomright", legend="mu=0.5, sigma=50",
       col="green4", lty=1, lwd=2, cex=0.8)
```

10 dDIKUM

```
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
x_max < -15
probs1 <- dDGEII(x=0:x_max, mu=0.5, sigma=5)</pre>
names(probs1) <- 0:x_max</pre>
x <- rDGEII(n=1000, mu=0.5, sigma=5)</pre>
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside=TRUE, names.arg=nombres,</pre>
               col=c('dodgerblue3','firebrick3'), las=1,
               xlab='X', ylab='Proportion')
legend('topright',
       legend=c('Theoretical', 'Simulated'),
       bty='n', lwd=3,
       col=c('dodgerblue3','firebrick3'), lty=1)
# Example 4
# Checking the quantile function
mu <- 0.5
sigma <- 5
p <- seq(from=0, to=1, by=0.01)
qxx <- qDGEII(p=p, mu=mu, sigma=sigma, lower.tail=TRUE, log.p=FALSE)</pre>
plot(p, qxx, type="s", lwd=2, col="green3", ylab="quantiles",
     main="Quantiles of DDGEII(mu=0.5, sigma=5)")
```

dDIKUM

The discrete Inverted Kumaraswamy distribution

Description

These functions define the density, distribution function, quantile function and random generation for the discrete Inverted Kumaraswamy, DIKUM(), distribution with parameters μ and σ .

Usage

```
dDIKUM(x, mu = 1, sigma = 5, log = FALSE)
pDIKUM(q, mu = 1, sigma = 5, lower.tail = TRUE, log.p = FALSE)
rDIKUM(n, mu = 1, sigma = 5)
```

dDIKUM 11

Arguments

x, q vector of (non-negative integer) quantiles.

mu vector of the mu parameter.

sigma vector of the sigma parameter.

log, log.p logical; if TRUE, probabilities p are given as log(p).

lower.tail logical; if TRUE (default), probabilities are $P[X \le x]$, otherwise, P[X > x].

n number of random values to return.

p vector of probabilities.

Details

The discrete Inverted Kumaraswamy distribution with parameters μ and σ has a support 0, 1, 2, ... and density given by

$$f(x|\mu,\sigma) = (1 - (2+x)^{-\mu})^{\sigma} - (1 - (1+x)^{-\mu})^{\sigma}$$

with $\mu > 0$ and $\sigma > 0$.

Note: in this implementation we changed the original parameters α and β for μ and σ respectively, we did it to implement this distribution within gamlss framework.

Value

dDIKUM gives the density, pDIKUM gives the distribution function, qDIKUM gives the quantile function, rDIKUM generates random deviates.

Author(s)

Daniel Felipe Villa Rengifo, <dvilla@unal.edu.co>

References

EL-Helbawy AA, Hegazy MA, AL-Dayian GR, Abd EL-Kader RE (2022). "A Discrete Analog of the Inverted Kumaraswamy Distribution: Properties and Estimation with Application to COVID-19 Data." *Pakistan Journal of Statistics & Operation Research*, **18**(1).

See Also

DIKUM.

12 dDIKUM

```
# Plotting the mass function for different parameter values
x_max <- 30
probs1 <- dDIKUM(x=0:x_max, mu=1, sigma=5)</pre>
probs2 <- dDIKUM(x=0:x_max, mu=1, sigma=20)</pre>
probs3 <- dDIKUM(x=0:x_max, mu=1, sigma=50)</pre>
# To plot the first k values
plot(x=0:x_max, y=probs1, type="o", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", main="Probability for Inverted Kumaraswamy Distribution",
     ylim=c(0, 0.12))
points(x=0:x_max, y=probs2, type="o", lwd=2, col="tomato")
points(x=0:x_max, y=probs3, type="o", lwd=2, col="green4")
legend("topright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=1, sigma=5",
                 "mu=1, sigma=20"
                 "mu=1, sigma=50"))
# Example 2
# Checking if the cumulative curves converge to 1
x_max < -500
cumulative_probs1 <- pDIKUM(q=0:x_max, mu=1, sigma=5)</pre>
cumulative_probs2 <- pDIKUM(q=0:x_max, mu=1, sigma=20)</pre>
cumulative_probs3 <- pDIKUM(q=0:x_max, mu=1, sigma=50)</pre>
plot(x=0:x_max, y=cumulative_probs1, col="dodgerblue",
     type="o", las=1, ylim=c(0, 1),
     main="Cumulative probability for Inverted Kumaraswamy Distribution",
     xlab="X", ylab="Probability")
points(x=0:x_max, y=cumulative_probs2, type="o", col="tomato")
points(x=0:x_max, y=cumulative_probs3, type="o", col="green4")
legend("bottomright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=1, sigma=5",
                "mu=1, sigma=20",
                "mu=1, sigma=50"))
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
x_max <- 20
probs1 <- dDIKUM(x=0:x_max, mu=3, sigma=20)</pre>
names(probs1) <- 0:x_max</pre>
x <- rDIKUM(n=1000, mu=3, sigma=20)
probs2 <- prop.table(table(x))</pre>
```

dDLD 13

```
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside = TRUE, names.arg = nombres,</pre>
              col=c('dodgerblue3','firebrick3'), las=1,
              xlab='X', ylab='Proportion')
legend('topright',
       legend=c('Theoretical', 'Simulated'),
       bty='n', lwd=3,
       col=c('dodgerblue3','firebrick3'), lty=1)
# Example 4
# Checking the quantile function
mu <- 1
sigma <- 5
p \le seq(from=0.01, to=0.99, by=0.1)
qxx <- qDIKUM(p=p, mu=mu, sigma=sigma, lower.tail=TRUE, log.p=FALSE)</pre>
plot(p, qxx, type="s", lwd=2, col="green3", ylab="quantiles",
     main="Quantiles of HP(mu = sigma = 3)")
```

dDLD

The Discrete Lindley distribution

Description

These functions define the density, distribution function, quantile function and random generation for the Discrete Lindley distribution with parameter μ .

Usage

```
dDLD(x, mu, log = FALSE)
pDLD(q, mu, lower.tail = TRUE, log.p = FALSE)
qDLD(p, mu, lower.tail = TRUE, log.p = FALSE)
rDLD(n, mu = 0.5)
```

Arguments

```
x, q vector of (non-negative integer) quantiles.  
mu vector of positive values of this parameter.  
log, log.p logical; if TRUE, probabilities p are given as log(p).  
lower.tail logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].  
p vector of probabilities.  
n number of random values to return.
```

14 dDLD

Details

The Discrete Lindley distribution with parameters μ has a support 0, 1, 2, ... and density given by

$$f(x|\mu) = \frac{e^{-\mu x}}{1+\mu} \left[\mu (1 - 2e^{-\mu}) + (1 - e^{-\mu})(1 + \mu x) \right]$$

Note: in this implementation we changed the original parameters θ for μ , we did it to implement this distribution within gamlss framework.

Value

dDLD gives the density, pDLD gives the distribution function, qDLD gives the quantile function, rDLD generates random deviates.

Author(s)

Yojan Andrés Alcaraz Pérez, <yalcaraz@unal.edu.co>

References

Bakouch HS, Jazi MA, Nadarajah S (2014). "A new discrete distribution." *Statistics*, **48**(1), 200–240.

See Also

DLD.

```
# Example 1
# Plotting the mass function for different parameter values
plot(x=0:25, y=dDLD(x=0:25, mu=0.2),
     type="h", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", ylim=c(0, 0.1),
    main="Probability mu=0.2")
plot(x=0:15, y=dDLD(x=0:15, mu=0.5),
     type="h", lwd=2, col="tomato", las=1,
     ylab="P(X=x)", xlab="X", ylim=c(0, 0.25),
     main="Probability mu=0.5")
plot(x=0:8, y=dDLD(x=0:8, mu=1),
     type="h", lwd=2, col="green4", las=1,
     ylab="P(X=x)", xlab="X", ylim=c(0, 0.5),
     main="Probability mu=1")
plot(x=0:5, y=dDLD(x=0:5, mu=2),
     type="h", lwd=2, col="red", las=1,
     ylab="P(X=x)", xlab="X", ylim=c(0, 1),
    main="Probability mu=2")
# Example 2
```

dDLD 15

```
# Checking if the cumulative curves converge to 1
x_max <- 10
cumulative\_probs1 <- pDLD(q=0:x\_max, mu=0.2)
cumulative_probs2 <- pDLD(q=0:x_max, mu=0.5)</pre>
cumulative_probs3 <- pDLD(q=0:x_max, mu=1)</pre>
cumulative_probs4 <- pDLD(q=0:x_max, mu=2)</pre>
plot(x=0:x_max, y=cumulative_probs1, col="dodgerblue",
     type="o", las=1, ylim=c(0, 1),
     main="Cumulative probability for Lindley",
     xlab="X", ylab="Probability")
points(x=0:x_max, y=cumulative_probs2, type="o", col="tomato")
points(x=0:x_max, y=cumulative_probs3, type="o", col="green4")
points(x=0:x_max, y=cumulative_probs4, type="o", col="magenta")
legend("bottomright",
       col=c("dodgerblue", "tomato", "green4", "magenta"), lwd=3,
       legend=c("mu=0.2",
                 "mu=0.5",
                 "mu=1",
                 "mu=2"))
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
mu <- 0.6
x \leftarrow rDLD(n = 1000, mu = mu)
x_{max} <- max(x)
probs1 <- dDLD(x = 0:x_max, mu = mu)
names(probs1) <- 0:x_max</pre>
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside = TRUE, names.arg = nombres,</pre>
              col=c('dodgerblue3','firebrick3'), las=1,
              xlab='X', ylab='Proportion')
legend('topright',
       legend=c('Theoretical', 'Simulated'),
       bty='n', lwd=3,
       col=c('dodgerblue3','firebrick3'), lty=1)
# Example 4
# Checking the quantile function
mu <- 0.9
p <- seq(from=0, to=1, by=0.01)
qxx <- qDLD(p, mu, lower.tail = TRUE, log.p = FALSE)</pre>
plot(p, qxx, type="S", lwd=2, col="green3", ylab="quantiles",
```

16 dDMOLBE

```
main="Quantiles of DL(mu=0.9)")
```

dDMOLBE

The DMOLBE distribution

Description

These functions define the density, distribution function, quantile function and random generation for the Discrete Marshall–Olkin Length Biased Exponential DMOLBE distribution with parameters μ and σ .

Usage

```
dDMOLBE(x, mu = 1, sigma = 1, log = FALSE)
pDMOLBE(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rDMOLBE(n, mu = 1, sigma = 1)
qDMOLBE(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
```

Arguments

x, q	vector of (non-negative integer) quantiles.
mu	vector of the mu parameter.
sigma	vector of the sigma parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X <= x]$, otherwise, $P[X > x]$.
n	number of random values to return.
р	vector of probabilities.

Details

The DMOLBE distribution with parameters μ and σ has a support 0, 1, 2, ... and mass function given by

```
f(x|\mu,\sigma) = \frac{\sigma((1+x/\mu)\exp(-x/\mu) - (1+(x+1)/\mu)\exp(-(x+1)/\mu))}{(1-(1-\sigma)(1+x/\mu)\exp(-x/\mu))((1-(1-\sigma)(1+(x+1)/\mu)\exp(-(x+1)/\mu))} with \mu > 0 and \sigma > 0
```

Value

dDMOLBE gives the density, pDMOLBE gives the distribution function, qDMOLBE gives the quantile function, rDMOLBE generates random deviates.

dDMOLBE 17

Author(s)

Olga Usuga, <olga.usuga@udea.edu.co>

References

Aljohani HM, Ahsan-ul-Haq M, Zafar J, Almetwally EM, Alghamdi AS, Hussam E, Muse AH (2023). "Analysis of Covid-19 data using discrete Marshall-Olkinin Length Biased Exponential: Bayesian and frequentist approach." *Scientific Reports*, **13**(1), 12243.

See Also

DMOLBE.

```
# Example 1
# Plotting the mass function for different parameter values
x_max <- 20
probs1 <- dDMOLBE(x=0:x_max, mu=0.5, sigma=0.5)
probs2 <- dDMOLBE(x=0:x_max, mu=5, sigma=0.5)</pre>
probs3 <- dDMOLBE(x=0:x_max, mu=1, sigma=2)</pre>
# To plot the first k values
plot(x=0:x_max, y=probs1, type="o", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", main="Probability for DMOLBE",
     ylim=c(0, 0.80))
points(x=0:x_max, y=probs2, type="o", lwd=2, col="tomato")
points(x=0:x_max, y=probs3, type="o", lwd=2, col="green4")
legend("topright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=0.5, sigma=0.5",
                "mu=5, sigma=0.5",
                "mu=1, sigma=2"))
# Example 2
# Checking if the cumulative curves converge to 1
x_max <- 20
cumulative_probs1 <- pDMOLBE(q=0:x_max, mu=0.5, sigma=0.5)</pre>
cumulative_probs2 <- pDMOLBE(q=0:x_max, mu=5, sigma=0.5)</pre>
cumulative_probs3 <- pDMOLBE(q=0:x_max, mu=1, sigma=2)</pre>
plot(x=0:x_max, y=cumulative_probs1, col="dodgerblue",
     type="o", las=1, ylim=c(0, 1),
     main="Cumulative probability for DMOLBE",
     xlab="X", ylab="Probability")
points(x=0:x_max, y=cumulative_probs2, type="o", col="tomato")
points(x=0:x_max, y=cumulative_probs3, type="o", col="green4")
legend("bottomright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=0.5, sigma=0.5",
                "mu=5, sigma=0.5",
                "mu=1, sigma=2"))
```

DGEII

```
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
x_max <- 15
probs1 <- dDMOLBE(x=0:x_max, mu=5, sigma=0.5)</pre>
names(probs1) <- 0:x_max</pre>
x <- rDMOLBE(n=1000, mu=5, sigma=0.5)</pre>
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside = TRUE, names.arg = nombres,</pre>
              col=c('dodgerblue3','firebrick3'), las=1,
              xlab='X', ylab='Proportion')
legend('topright',
       legend=c('Theoretical', 'Simulated'),
       bty='n', 1wd=3,
       col=c('dodgerblue3','firebrick3'), lty=1)
# Example 4
# Checking the quantile function
mu <- 3
sigma <-3
p <- seq(from=0, to=1, by=0.01)
qxx <- qDMOLBE(p=p, mu=mu, sigma=sigma, lower.tail=TRUE, log.p=FALSE)</pre>
plot(p, qxx, type="s", lwd=2, col="green3", ylab="quantiles",
     main="Quantiles of DMOLBE(mu = 3, sigma = 3)")
```

DGEII

The DGEII distribution

Description

The function DGEII() defines the Discrete generalized exponential distribution, Second type, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
DGEII(mu.link = "logit", sigma.link = "log")
```

DGEII 19

Arguments

mu.link defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log).

sigma.link defines the sigma.link, with "log" link as the default for the sigma.

Details

The DGEII distribution with parameters μ and σ has a support 0, 1, 2, ... and mass function given by

$$f(x|\mu,\sigma) = (1 - \mu^{x+1})^{\sigma} - (1 - \mu^x)^{\sigma}$$

with $0 < \mu < 1$ and $\sigma > 0$. If $\sigma = 1$, the DGEII distribution reduces to the geometric distribution with success probability $1 - \mu$.

Note: in this implementation we changed the original parameters p to μ and α to σ , we did it to implement this distribution within gamlss framework.

Value

Returns a gamlss.family object which can be used to fit a DGEII distribution in the gamlss() function.

Author(s)

Valentina Hurtado Sepúlveda, <vhurtados@unal.edu.co>

References

Nekoukhou V, Alamatsaz MH, Bidram H (2013). "Discrete generalized exponential distribution of a second type." *Statistics*, **47**(4), 876-887.

See Also

dDGEII.

20 dGGEO

```
inv_logit(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))
# Example 2
# Generating random values under some model
# A function to simulate a data set with Y ~ GGEO
gendat <- function(n) {</pre>
  x1 <- runif(n)</pre>
  x2 <- runif(n)</pre>
        <- inv_logit(1.7 - 2.8*x1)
  sigma <- exp(0.73 + 1*x2)
  y <- rDGEII(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
set.seed(1234)
datos <- gendat(n=100)</pre>
mod2 <- gamlss(y~x1, sigma.fo=~x2, family=DGEII, data=datos,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
# Example 3
# Number of accidents to 647 women working on H. E. Shells
# for 5 weeks. Taken from
# Nekoukhou V, Alamatsaz MH, Bidram H (2013) page 886.
y \leftarrow rep(x=0:5, times=c(447, 132, 42, 21, 3, 2))
mod3 <- gamlss(y~1, family=DGEII,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
# Extracting the fitted values for mu and sigma
# using the inverse link function
inv_logit <- function(x) 1/(1 + exp(-x))
inv_logit(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))
```

dGGE0

The GGEO distribution

Description

These functions define the density, distribution function, quantile function and random generation for the Generalized Geometric distribution with parameters μ and σ .

dGGEO 21

Usage

```
dGGEO(x, mu = 0.5, sigma = 1, log = FALSE)
pGGEO(q, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rGGEO(n, mu = 0.5, sigma = 1)
qGGEO(p, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
```

Arguments

x, q vector of (non-negative integer) quantiles.

mu vector of the mu parameter. sigma vector of the sigma parameter.

log, log.p logical; if TRUE, probabilities p are given as log(p).

lower.tail logical; if TRUE (default), probabilities are $P[X \le x]$, otherwise, P[X > x].

n number of random values to return.

p vector of probabilities.

Details

The GGEO distribution with parameters μ and σ has a support 0, 1, 2, ... and mass function given by

$$f(x|\mu,\sigma) = \frac{\sigma \mu^x (1-\mu)}{(1-(1-\sigma)\mu^{x+1})(1-(1-\sigma)\mu^x)}$$

with $0 < \mu < 1$ and $\sigma > 0$. If $\sigma = 1$, the GGEO distribution reduces to the geometric distribution with success probability $1 - \mu$.

Note: in this implementation we changed the original parameters θ for μ and α for σ , we did it to implement this distribution within gamlss framework.

Value

dGGEO gives the density, pGGEO gives the distribution function, qGGEO gives the quantile function, rGGEO generates random deviates.

Author(s)

Valentina Hurtado Sepulveda, <vhurtados@unal.edu.co>

References

Gómez-Déniz E (2010). "Another generalization of the geometric distribution." Test, 19, 399-415.

See Also

GGEO.

22 dGGEO

```
# Plotting the mass function for different parameter values
x_max <- 80
probs1 <- dGGEO(x=0:x_max, mu=0.5, sigma=10)
probs2 <- dGGEO(x=0:x_max, mu=0.7, sigma=30)
probs3 <- dGGEO(x=0:x_max, mu=0.9, sigma=50)
# To plot the first k values
plot(x=0:x_max, y=probs1, type="o", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", main="Probability for GGEO",
     ylim=c(0, 0.20))
points(x=0:x_max, y=probs2, type="o", lwd=2, col="tomato")
points(x=0:x_max, y=probs3, type="o", lwd=2, col="green4")
legend("topright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=0.5, sigma=10",
                "mu=0.7, sigma=30"
                "mu=0.9, sigma=50"))
# Checking if the cumulative curves converge to 1
x max <- 10
plot_discrete_cdf(x=0:x_max,
                  fx=dGGEO(x=0:x_max, mu=0.3, sigma=15),
                  col="dodgerblue",
                  main="CDF for GGEO",
                  1wd= 3)
legend("bottomright", legend="mu=0.3, sigma=15", col="dodgerblue",
       lty=1, lwd=2, cex=0.8)
plot_discrete_cdf(x=0:x_max,
                  fx=dGGEO(x=0:x_max, mu=0.5, sigma=30),
                  col="tomato",
                  main="CDF for GGEO",
                  1wd=3)
legend("bottomright", legend="mu=0.5, sigma=30",
       col="tomato", lty=1, lwd=2, cex=0.8)
plot_discrete_cdf(x=0:x_max,
                  fx=dGGEO(x=0:x_max, mu=0.5, sigma=50),
                  col="green4",
                  main="CDF for GGEO",
                  1wd=3)
legend("bottomright", legend="mu=0.5, sigma=50",
       col="green4", lty=1, lwd=2, cex=0.8)
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
```

```
x_max <- 15
probs1 <- dGGEO(x=0:x_max, mu=0.5, sigma=5)
names(probs1) <- 0:x_max</pre>
x <- rGGEO(n=1000, mu=0.5, sigma=5)
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside=TRUE, names.arg=nombres,</pre>
               col=c("dodgerblue3", "firebrick3"), las=1,
               xlab="X", ylab="Proportion")
legend("topright",
       legend=c("Theoretical", "Simulated"),
       bty="n", lwd=3,
       col=c("dodgerblue3","firebrick3"), lty=1)
# Example 4
# Checking the quantile function
mu <- 0.5
sigma <- 5
p <- seq(from=0, to=1, by=0.01)
qxx <- qGGEO(p=p, mu=mu, sigma=sigma, lower.tail=TRUE, log.p=FALSE)</pre>
plot(p, qxx, type="s", lwd=2, col="green3", ylab="quantiles",
     main="Quantiles of GGEO(mu=0.5, sigma=0.5)")
```

dHYPERP0

The hyper-Poisson distribution

Description

These functions define the density, distribution function, quantile function and random generation for the hyper-Poisson, HYPERPO(), distribution with parameters μ and σ .

Usage

```
dHYPERPO(x, mu = 1, sigma = 1, log = FALSE)
pHYPERPO(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rHYPERPO(n, mu = 1, sigma = 1)
qHYPERPO(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
```

Arguments

x, q	vector of (non-negative integer) quantiles.
mu	vector of the mu parameter.
sigma	vector of the sigma parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X <= x]$, otherwise, $P[X > x]$.
n	number of random values to return.
р	vector of probabilities.

Details

The hyper-Poisson distribution with parameters μ and σ has a support 0, 1, 2, ... and density given by

$$f(x|\mu,\sigma) = \frac{\mu^x}{{}_1F_1(1;\mu;\sigma)} \frac{\Gamma(\sigma)}{\Gamma(x+\sigma)}$$
 where the function ${}_1F_1(a;c;z)$ is defined as

$$_{1}F_{1}(a;c;z) = \sum_{r=0}^{\infty} \frac{(a)_{r}}{(c)_{r}} \frac{z^{r}}{r!}$$

and
$$(a)_r = \frac{\gamma(a+r)}{\gamma(a)}$$
 for $a>0$ and r positive integer.

Note: in this implementation we changed the original parameters λ and γ for μ and σ respectively, we did it to implement this distribution within gamlss framework.

Value

dHYPERPO gives the density, pHYPERPO gives the distribution function, qHYPERPO gives the quantile function, rHYPERPO generates random deviates.

Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

References

Sáez-Castillo AJ, Conde-Sánchez A (2013). "A hyper-Poisson regression model for overdispersed and underdispersed count data." *Computational Statistics & Data Analysis*, **61**, 148–157.

See Also

HYPERPO.

```
# Example 1
# Plotting the mass function for different parameter values
x_max <- 30
probs1 <- dHYPERPO(x=0:x_max, mu=5, sigma=0.1)</pre>
```

```
probs2 <- dHYPERPO(x=0:x_max, mu=5, sigma=1.0)</pre>
probs3 <- dHYPERPO(x=0:x_max, mu=5, sigma=1.8)</pre>
# To plot the first k values
plot(x=0:x_max, y=probs1, type="o", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", main="Probability for hyper-Poisson",
     ylim=c(0, 0.20))
points(x=0:x_max, y=probs2, type="o", lwd=2, col="tomato")
points(x=0:x_max, y=probs3, type="o", lwd=2, col="green4")
legend("topright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=5, sigma=0.1",
                 "mu=5, sigma=1.0",
                 "mu=5, sigma=1.8"))
# Example 2
# Checking if the cumulative curves converge to 1
x_max <- 15
cumulative_probs1 <- pHYPERPO(q=0:x_max, mu=5, sigma=0.1)</pre>
cumulative_probs2 <- pHYPERPO(q=0:x_max, mu=5, sigma=1.0)</pre>
cumulative_probs3 <- pHYPERPO(q=0:x_max, mu=5, sigma=1.8)</pre>
plot(x=0:x_max, y=cumulative_probs1, col="dodgerblue",
     type="o", las=1, ylim=c(0, 1),
     main="Cumulative probability for hyper-Poisson",
     xlab="X", ylab="Probability")
points(x=0:x_max, y=cumulative_probs2, type="o", col="tomato")
points(x=0:x_max, y=cumulative_probs3, type="o", col="green4")
legend("bottomright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=5, sigma=0.1",
                "mu=5, sigma=1.0",
                 "mu=5, sigma=1.8"))
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
x_max < -15
probs1 <- dHYPERPO(x=0:x_max, mu=3, sigma=1.1)</pre>
names(probs1) <- 0:x_max</pre>
x <- rHYPERPO(n=1000, mu=3, sigma=1.1)
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside = TRUE, names.arg = nombres,</pre>
              col=c("dodgerblue3", "firebrick3"), las=1,
              xlab="X", ylab="Proportion")
legend("topright",
       legend=c("Theoretical", "Simulated"),
       bty="n", lwd=3,
```

dHYPERP02

The hyper-Poisson distribution (with mu as mean)

Description

These functions define the density, distribution function, quantile function and random generation for the hyper-Poisson in the second parameterization with parameters μ (as mean) and σ as the dispersion parameter.

Usage

```
dHYPERPO2(x, mu = 1, sigma = 1, log = FALSE)
pHYPERPO2(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rHYPERPO2(n, mu = 1, sigma = 1)
qHYPERPO2(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
```

Arguments

```
x, q vector of (non-negative integer) quantiles.

mu vector of positive values of this parameter.

sigma vector of positive values of this parameter.

log, log.p logical; if TRUE, probabilities p are given as log(p).

lower.tail logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].

n number of random values to return

p vector of probabilities.
```

Details

The hyper-Poisson distribution with parameters μ and σ has a support 0, 1, 2, ...

Note: in this implementation the parameter μ is the mean of the distribution and σ corresponds to the dispersion parameter. If you fit a model with this parameterization, the time will increase because an internal procedure to convert μ to λ parameter.

Value

dHYPERP02 gives the density, pHYPERP02 gives the distribution function, qHYPERP02 gives the quantile function, rHYPERP02 generates random deviates.

Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

References

Sáez-Castillo AJ, Conde-Sánchez A (2013). "A hyper-Poisson regression model for overdispersed and underdispersed count data." *Computational Statistics & Data Analysis*, **61**, 148–157.

See Also

HYPERPO2, HYPERPO.

```
# Example 1
# Plotting the mass function for different parameter values
x_max <- 30
probs1 <- dHYPERPO2(x=0:x_max, sigma=0.01, mu=3)</pre>
probs2 <- dHYPERPO2(x=0:x_max, sigma=0.50, mu=5)</pre>
probs3 <- dHYPERPO2(x=0:x_max, sigma=1.00, mu=7)</pre>
# To plot the first k values
plot(x=0:x_max, y=probs1, type="o", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", main="Probability for hyper-Poisson",
     ylim=c(0, 0.30))
points(x=0:x_max, y=probs2, type="o", lwd=2, col="tomato")
points(x=0:x_max, y=probs3, type="o", lwd=2, col="green4")
legend("topright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("sigma=0.01, mu=3",
                 "sigma=0.50, mu=5",
                 "sigma=1.00, mu=7"))
# Example 2
# Checking if the cumulative curves converge to 1
x_max <- 15
cumulative_probs1 <- pHYPERPO2(q=0:x_max, mu=1, sigma=1.5)</pre>
cumulative_probs2 <- pHYPERPO2(q=0:x_max, mu=3, sigma=1.5)</pre>
cumulative_probs3 <- pHYPERPO2(q=0:x_max, mu=5, sigma=1.5)</pre>
```

28 dHYPERPO_single

```
plot(x=0:x_max, y=cumulative_probs1, col="dodgerblue",
     type="o", las=1, ylim=c(0, 1),
     main="Cumulative probability for hyper-Poisson",
     xlab="X", ylab="Probability")
points(x=0:x_max, y=cumulative_probs2, type="o", col="tomato")
points(x=0:x_max, y=cumulative_probs3, type="o", col="green4")
legend("bottomright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("sigma=1.5, mu=1",
                 "sigma=1.5, mu=3",
                 "sigma=1.5, mu=5"))
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
x_max < -15
probs1 <- dHYPERPO2(x=0:x_max, mu=3, sigma=1.1)</pre>
names(probs1) <- 0:x_max</pre>
x <- rHYPERPO2(n=1000, mu=3, sigma=1.1)
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside = TRUE, names.arg = nombres,</pre>
               col=c('dodgerblue3','firebrick3'), las=1,
              xlab='X', ylab='Proportion')
legend('topright',
       legend=c('Theoretical', 'Simulated'),
       bty='n', lwd=3,
       col=c('dodgerblue3','firebrick3'), lty=1)
# Example 4
# Checking the quantile function
mu <- 3
sigma <-3
p <- seq(from=0, to=1, by=0.01)
\mbox{qxx} <- \mbox{qHYPERPO2}(\mbox{p=p, mu=mu, sigma=sigma, lower.tail=TRUE, log.p=FALSE)} \label{eq:partial}
plot(p, qxx, type="s", lwd=2, col="green3", ylab="quantiles",
     main="Quantiles of HP2(mu = sigma = 3)")
```

dHYPERPO_single

Function to obtain the dHYPERPO for a single value x

Description

Function to obtain the dHYPERPO for a single value x

dHYPERPO_vec 29

Usage

```
dHYPERPO_single(x, mu = 1, sigma = 1, log = FALSE)
```

Arguments

x numeric value for x.
 mu numeric value for nu.
 sigma numeric value for sigma.
 log logical value for log.

Value

returns the pmf for a single value x.

dHYPERPO_vec

Function to obtain the dHYPERPO for a vector x

Description

Function to obtain the dHYPERPO for a vector x

Usage

```
dHYPERPO_vec(x, mu, sigma, log)
```

Arguments

x numeric value for x.

mu numeric value for nu.

sigma numeric value for sigma.

log logical value for log.

Value

returns the pmf for a vector.

30 DIKUM

DIKUM

The discrete Inverted Kumaraswamy family

Description

The function DIKUM() defines the discrete Inverted Kumaraswamy distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
DIKUM(mu.link = "log", sigma.link = "log")
```

Arguments

mu.link defines the mu.link, with "log" link as the default for the mu parameter.
sigma.link defines the sigma.link, with "log" link as the default for the sigma.

Details

The discrete Inverted Kumaraswamy distribution with parameters μ and σ has a support 0, 1, 2, ... and density given by

$$f(x|\mu,\sigma) = (1 - (2+x)^{-\mu})^{\sigma} - (1 - (1+x)^{-\mu})^{\sigma}$$

with $\mu > 0$ and $\sigma > 0$.

Note: in this implementation we changed the original parameters α and β for μ and σ respectively, we did it to implement this distribution within gamlss framework.

Value

Returns a gamlss.family object which can be used to fit a discrete Inverted Kumaraswamy distribution in the gamlss() function.

Author(s)

Daniel Felipe Villa Rengifo, <dvilla@unal.edu.co>

References

EL-Helbawy AA, Hegazy MA, AL-Dayian GR, Abd EL-Kader RE (2022). "A Discrete Analog of the Inverted Kumaraswamy Distribution: Properties and Estimation with Application to COVID-19 Data." *Pakistan Journal of Statistics & Operation Research*, **18**(1).

See Also

dDIKUM.

DLD 31

Examples

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(150)
y <- rDIKUM(1000, mu=1, sigma=5)
# Fitting the model
library(gamlss)
mod1 <- gamlss(y ~ 1, sigma.fo = ~1, family=DIKUM,</pre>
               control = gamlss.control(n.cyc=500, trace=FALSE))
# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what='mu'))
exp(coef(mod1, what='sigma'))
# Example 2
# Generating random values under some model
library(gamlss)
# A function to simulate a data set with Y ~ DIKUM
gendat <- function(n) {</pre>
  x1 <- runif(n, min=0.4, max=0.6)
  x2 <- runif(n, min=0.4, max=0.6)</pre>
        \leftarrow exp(1.21 - 3 * x1) # 0.75 approximately
  sigma <- exp(1.26 - 2 * x2) # 1.30 approximately
  y <- rDIKUM(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}
dat <- gendat(n=150)</pre>
# Fitting the model
mod2 <- gamlss(y ~ x1, sigma.fo = ~x2, family = "DIKUM", data=dat,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
```

DLD

The Discrete Lindley family

Description

The function DLD() defines the Discrete Lindley distribution, one-parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
DLD(mu.link = "log")
```

DLD

Arguments

mu.link

defines the mu.link, with "log" link as the default for the mu parameter.

Details

The Discrete Lindley distribution with parameters $\mu>0$ has a support 0,1,2,... and density given by

$$f(x|\mu) = \frac{e^{-\mu x}}{1+\mu} (\mu(1-2e^{-\mu}) + (1-e^{-\mu})(1+\mu x))$$

The parameter μ can be interpreted as a strict upper bound on the failure rate function

The conventional discrete distributions (such as geometric, Poisson, etc.) are not suitable for various scenarios like reliability, failure times, and counts. Consequently, alternative discrete distributions have been created by adapting well-known continuous models for reliability and failure times. Among these, the discrete Weibull distribution stands out as the most widely used. But models like these require two parameters and not many of the known discrete distributions can provide accurate models for both times and counts, which the Discrete Lindley distribution does.

Note: in this implementation we changed the original parameters θ for μ , we did it to implement this distribution within gamlss framework.

Value

Returns a gamlss.family object which can be used to fit a Discrete Lindley distribution in the gamlss() function.

Author(s)

Yojan Andrés Alcaraz Pérez, <yalcaraz@unal.edu.co>

References

Bakouch HS, Jazi MA, Nadarajah S (2014). "A new discrete distribution." *Statistics*, **48**(1), 200–240.

See Also

dDLD.

DMOLBE 33

```
# using the inverse link function
exp(coef(mod1, what='mu'))
# Example 2
# Generating random values under some model
# A function to simulate a data set with Y \sim DLD
gendat <- function(n) {</pre>
  x1 <- runif(n)</pre>
       \leftarrow exp(2 - 4 * x1)
  y <- rDLD(n=n, mu=mu)
  data.frame(y=y, x1=x1)
}
set.seed(1235)
datos <- gendat(n=150)</pre>
mod2 <- NULL
mod2 <- gamlss(y~x1, sigma.fo=~x2, family=DLD, data=datos,</pre>
                  control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
```

DMOLBE

The DMOLBE family

Description

The function DMOLBE() defines the Discrete Marshall-Olkin Length Biased Exponential distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
DMOLBE(mu.link = "log", sigma.link = "log")
```

Arguments

mu.link defines the mu.link, with "log" link as the default for the mu parameter. sigma.link defines the sigma.link, with "log" link as the default for the sigma.

Details

The DMOLBE distribution with parameters μ and σ has a support 0, 1, 2, ... and mass function given by

```
\begin{split} f(x|\mu,\sigma) &= \tfrac{\sigma((1+x/\mu)\exp(-x/\mu) - (1+(x+1)/\mu)\exp(-(x+1)/\mu))}{(1-(1-\sigma)(1+x/\mu)\exp(-x/\mu))((1-(1-\sigma)(1+(x+1)/\mu)\exp(-(x+1)/\mu))} \\ \text{with } \mu &> 0 \text{ and } \sigma > 0 \end{split}
```

34 DMOLBE

Value

Returns a gamlss.family object which can be used to fit a DMOLBE distribution in the gamlss() function.

Author(s)

Olga Usuga, <olga.usuga@udea.edu.co>

References

Aljohani HM, Ahsan-ul-Haq M, Zafar J, Almetwally EM, Alghamdi AS, Hussam E, Muse AH (2023). "Analysis of Covid-19 data using discrete Marshall-Olkinin Length Biased Exponential: Bayesian and frequentist approach." *Scientific Reports*, **13**(1), 12243.

See Also

dDMOLBE.

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(1234)
y <- rDMOLBE(n=100, mu=10, sigma=7)
# Fitting the model
library(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=DMOLBE,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what='mu'))
exp(coef(mod1, what='sigma'))
# Example 2
# Generating random values under some model
# A function to simulate a data set with Y ~ DMOLBE
gendat <- function(n) {</pre>
  x1 <- runif(n, min=0.4, max=0.6)
  x2 <- runif(n, min=0.4, max=0.6)
       \leftarrow exp(1.21 - 3 * x1) # 0.75 approximately
  sigma \leftarrow exp(1.26 - 2 * x2) # 1.30 approximately
  y <- rDMOLBE(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}
set.seed(123)
dat <- gendat(n=350)</pre>
```

dPOISXL 35

dPOISXL

The Discrete Poisson XLindley

Description

These functions define the density, distribution function, quantile function and random generation for the Discrete Poisson XLindley distribution with parameter μ .

Usage

```
dPOISXL(x, mu = 0.3, log = FALSE)

pPOISXL(q, mu = 0.3, lower.tail = TRUE, log.p = FALSE)

qPOISXL(p, mu = 0.3, lower.tail = TRUE, log.p = FALSE)

rPOISXL(n, mu = 0.3)
```

Arguments

x, q vector of (non-negative integer) quantiles.

mu vector of the mu parameter.

log, log.p logical; if TRUE, probabilities p are given as log(p).

lower.tail logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x].

p vector of probabilities.

n number of random values to return

Details

The Discrete Poisson XLindley distribution with parameters μ has a support 0, 1, 2, ... and mass function given by

$$f(x|\mu) = \frac{\mu^2(x+\mu^2+3(1+\mu))}{(1+\mu)^{4+x}}$$
; with $\mu > 0$.

Note: in this implementation we changed the original parameters α for μ , we did it to implement this distribution within gamlss framework.

Value

dPOISXL gives the density, pPOISXL gives the distribution function, qPOISXL gives the quantile function, rPOISXL generates random deviates.

36 dPOISXL

Author(s)

Mariana Blandon Mejia, <mblandonm@unal.edu.co>

References

Ahsan-ul-Haq M, Al-Bossly A, El-Morshedy M, Eliwa MS, others (2022). "Poisson XLindley distribution for count data: statistical and reliability properties with estimation techniques and inference." *Computational Intelligence and Neuroscience*, **2022**.

See Also

POISXL.

```
# Example 1
# Plotting the mass function for different parameter values
x_max <- 20
probs1 <- dPOISXL(x=0:x_max, mu=0.2)</pre>
probs2 <- dPOISXL(x=0:x_max, mu=0.5)</pre>
probs3 <- dPOISXL(x=0:x_max, mu=1.0)</pre>
# To plot the first k values
plot(x=0:x_max, y=probs1, type="o", lwd=2, col="dodgerblue", las=1,
     ylab="P(X=x)", xlab="X", main="Probability for Poisson XLindley",
     ylim=c(0, 0.50))
points(x=0:x_max, y=probs2, type="o", lwd=2, col="tomato")
points(x=0:x_max, y=probs3, type="o", lwd=2, col="green4")
legend("topright", col=c("dodgerblue", "tomato", "green4"), lwd=3,
       legend=c("mu=0.2", "mu=0.5", "mu=1.0"))
# Example 2
# Checking if the cumulative curves converge to 1
x_max <- 20
plot_discrete_cdf(x=0:x_max,
                  fx=dPOISXL(x=0:x_max, mu=0.2), col="dodgerblue",
                  main="CDF for Poisson XLindley with mu=0.2")
plot_discrete_cdf(x=0:x_max,
                  fx=dPOISXL(x=0:x_max, mu=0.5), col="tomato",
                  main="CDF for Poisson XLindley with mu=0.5")
plot_discrete_cdf(x=0:x_max,
                  fx=dPOISXL(x=0:x_max, mu=1.0), col="green4",
                  main="CDF for Poisson XLindley with mu=1.0")
# Example 3
# Comparing the random generator output with
# the theoretical probabilities
```

f11_cpp 37

```
x_max <- 15
probs1 <- dPOISXL(x=0:x_max, mu=0.3)</pre>
names(probs1) <- 0:x_max</pre>
x \leftarrow rPOISXL(n=3000, mu=0.3)
probs2 <- prop.table(table(x))</pre>
cn <- union(names(probs1), names(probs2))</pre>
height <- rbind(probs1[cn], probs2[cn])</pre>
nombres <- cn
mp <- barplot(height, beside = TRUE, names.arg = nombres,</pre>
               col=c("dodgerblue3", "firebrick3"), las=1,
               xlab="X", ylab="Proportion")
legend("topright",
       legend=c("Theoretical", "Simulated"),
       bty="n", lwd=3,
       col=c("dodgerblue3","firebrick3"), lty=1)
# Example 4
# Checking the quantile function
mu <- 0.3
p <- seq(from=0, to=1, by = 0.01)
qxx <- qPOISXL(p, mu, lower.tail = TRUE, log.p = FALSE)</pre>
plot(p, qxx, type="s", lwd=2, col="green3", ylab="quantiles",
     main="Quantiles for Poisson XLindley mu=0.3")
```

f11_cpp

Function to obtain F11 with C++.

Description

Function to obtain F11 with C++.

Usage

```
f11_cpp(gamma, lambda, maxiter_series = 10000L, tol = 1e-10)
```

Arguments

```
gamma numeric value for gamma.
lambda numeric value for lambda.
maxiter_series numeric value.
tol numeric value.
```

Value

returns the F11 value.

38 GGEO

GGEO

The GGEO family

Description

The function GGEO() defines the Generalized Geometric distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
GGEO(mu.link = "logit", sigma.link = "log")
```

Arguments

mu.link defines the mu.link, with "log" link as the default for the mu parameter.

sigma.link defines the sigma.link, with "logit" link as the default for the sigma. Other links

are "probit" and "cloglog" '(complementary log-log)

Details

The GGEO distribution with parameters μ and σ has a support 0, 1, 2, ... and mass function given by

$$f(x|\mu,\sigma) = \frac{\sigma \mu^x (1-\mu)}{(1-(1-\sigma)\mu^{x+1})(1-(1-\sigma)\mu^x)}$$

with $0 < \mu < 1$ and $\sigma > 0$. If $\sigma = 1$, the GGEO distribution reduces to the geometric distribution with success probability $1 - \mu$.

Value

Returns a gamlss.family object which can be used to fit a GGEO distribution in the gamlss() function.

Author(s)

Valentina Hurtado Sepúlveda, <vhurtados@unal.edu.co>

References

Gómez-Déniz E (2010). "Another generalization of the geometric distribution." Test, 19, 399-415.

See Also

dGGEO.

GGEO 39

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(123)
y <- rGGEO(n=200, mu=0.95, sigma=1.5)
# Fitting the model
library(gamlss)
mod1 <- gamlss(y~1, family=GGEO,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
# Extracting the fitted values for mu and sigma
# using the inverse link function
inv_logit <- function(x) 1/(1 + exp(-x))
inv_logit(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))
# Example 2
# Generating random values under some model
\# A function to simulate a data set with Y \sim GGEO
gendat <- function(n) {</pre>
  x1 <- runif(n)</pre>
  x2 <- runif(n)</pre>
       <- inv_logit(1.7 - 2.8*x1)
  sigma <- exp(0.73 + 1*x2)
  y <- rGGEO(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
}
set.seed(78353)
datos <- gendat(n=100)</pre>
mod2 <- gamlss(y~x1, sigma.fo=~x2, family=GGEO, data=datos,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
# Example 3
# Number of accidents to 647 women working on H. E. Shells
# for 5 weeks. Taken from Gomez-Deniz (2010) page 411.
y \leftarrow rep(x=0:5, times=c(447, 132, 42, 21, 3, 2))
mod3 <- gamlss(y~1, family=GGEO,</pre>
               control=gamlss.control(n.cyc=500, trace=TRUE))
# Extracting the fitted values for mu and sigma
# using the inverse link function
inv_logit <- function(x) 1/(1 + exp(-x))
```

40 HYPERPO

```
inv_logit(coef(mod3, what="mu"))
exp(coef(mod3, what="sigma"))
```

HYPERPO

The hyper Poisson family

Description

The function HYPERPO() defines the hyper Poisson distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
HYPERPO(mu.link = "log", sigma.link = "log")
```

Arguments

mu.link defines the mu.link, with "log" link as the default for the mu parameter.

sigma.link defines the sigma.link, with "log" link as the default for the sigma.

Details

The hyper-Poisson distribution with parameters μ and σ has a support 0, 1, 2, ... and density given by

$$f(x|\mu,\sigma) = \frac{\mu^x}{{}_1F_1(1;\mu;\sigma)} \frac{\Gamma(\sigma)}{\Gamma(x+\sigma)}$$

where the function ${}_{1}F_{1}(a;c;z)$ is defined as

$$_{1}F_{1}(a;c;z) = \sum_{r=0}^{\infty} \frac{(a)_{r}}{(c)_{r}} \frac{z^{r}}{r!}$$

and $(a)_r = \frac{\gamma(a+r)}{\gamma(a)}$ for a>0 and r positive integer.

Note: in this implementation we changed the original parameters λ and γ for μ and σ respectively, we did it to implement this distribution within gamlss framework.

Value

Returns a gamlss.family object which can be used to fit a hyper-Poisson distribution in the gamlss() function.

Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

References

Sáez-Castillo AJ, Conde-Sánchez A (2013). "A hyper-Poisson regression model for overdispersed and underdispersed count data." *Computational Statistics & Data Analysis*, **61**, 148–157.

HYPERPO2 41

See Also

dHYPERPO.

Examples

```
# Example 1
# Generating some random values with
# known mu and sigma
set.seed(1234)
y <- rHYPERPO(n=200, mu=10, sigma=1.5)
# Fitting the model
library(gamlss)
mod1 <- gamlss(y~1, sigma.fo=~1, family=HYPERPO,</pre>
               control=gamlss.control(n.cyc=500, trace=FALSE))
# Extracting the fitted values for mu and sigma
# using the inverse link function
exp(coef(mod1, what="mu"))
exp(coef(mod1, what="sigma"))
# Example 2
# Generating random values under some model
# A function to simulate a data set with Y ~ HYPERPO
gendat <- function(n) {</pre>
  x1 <- runif(n)</pre>
  x2 \leftarrow runif(n)
       \leftarrow exp(1.21 - 3 * x1) # 0.75 approximately
  sigma <- exp(1.26 - 2 * x2) # 1.30 approximately
  y <- rHYPERPO(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
set.seed(1235)
datos <- gendat(n=150)</pre>
mod2 <- NULL
mod2 <- gamlss(y~x1, sigma.fo=~x2, family=HYPERPO, data=datos,</pre>
                  control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
```

HYPERP02

The hyper Poisson family (with mu as mean)

Description

The function HYPERPO2() defines the hyper Poisson distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

42 HYPERPO2

Usage

```
HYPERPO2(mu.link = "log", sigma.link = "log")
```

Arguments

mu.link defines the mu.link, with "log" link as the default for the mu parameter. sigma.link defines the sigma.link, with "log" link as the default for the sigma.

Details

The hyper-Poisson distribution with parameters μ and σ has a support 0, 1, 2, ...

Note: in this implementation the parameter μ is the mean of the distribution and σ corresponds to the dispersion parameter. If you fit a model with this parameterization, the time will increase because an internal procedure to convert μ to λ parameter.

Value

Returns a gamlss.family object which can be used to fit a hyper-Poisson distribution version 2 in the gamlss() function.

Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

References

Sáez-Castillo AJ, Conde-Sánchez A (2013). "A hyper-Poisson regression model for overdispersed and underdispersed count data." *Computational Statistics & Data Analysis*, **61**, 148–157.

See Also

```
dHYPERPO2, HYPERPO.
```

mean_var_hp 43

```
# Example 2
# Generating random values under some model
\# A function to simulate a data set with Y \sim HYPERPO2
gendat <- function(n) {</pre>
  x1 <- runif(n)</pre>
  x2 <- runif(n)</pre>
        <- \exp(1.21 - 3 * x1) # 0.75 approximately
  sigma <- exp(1.26 - 2 * x2) # 1.30 approximately
  y <- rHYPERPO2(n=n, mu=mu, sigma=sigma)
  data.frame(y=y, x1=x1, x2=x2)
set.seed(1234)
datos <- gendat(n=500)</pre>
mod2 <- NULL
mod2 <- gamlss(y~x1, sigma.fo=~x2, family=HYPERPO2, data=datos,</pre>
                control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
```

mean_var_hp

Mean and variance for hyper-Poisson distribution

Description

This function calculates the mean and variance for the hyper-Poisson distribution with parameters μ and σ .

Usage

```
mean_var_hp(mu, sigma)
mean_var_hp2(mu, sigma)
```

Arguments

mu value of the mu parameter. sigma value of the sigma parameter.

Details

The hyper-Poisson distribution with parameters μ and σ has a support 0, 1, 2, ... and density given by

$$f(x|\mu,\sigma) = \frac{\mu^x}{{}_1F_1(1;\mu;\sigma)} \frac{\Gamma(\sigma)}{\Gamma(x+\sigma)}$$

44 mean_var_hp

where the function ${}_1F_1(a;c;z)$ is defined as

$$_1F_1(a;c;z)=\sum_{r=0}^{\infty}\frac{(a)_r}{(c)_r}\frac{z^r}{r!}$$
 and $(a)_r=\frac{\gamma(a+r)}{\gamma(a)}$ for $a>0$ and r positive integer.

This function calculates the mean and variance of this distribution.

Value

the function returns a list with the mean and variance.

Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

References

Sáez-Castillo AJ, Conde-Sánchez A (2013). "A hyper-Poisson regression model for overdispersed and underdispersed count data." *Computational Statistics & Data Analysis*, **61**, 148–157.

See Also

HYPERPO.

```
# Example 1
# Theoretical values
mean_var_hp(mu=5.5, sigma=0.1)
# Using simulated values
y <- rHYPERPO(n=1000, mu=5.5, sigma=0.1)
mean(y)
var(y)
# Example 2
# Theoretical values
mean_var_hp2(mu=5.5, sigma=1.9)
# Using simulated values
y <- rHYPERPO2(n=1000, mu=5.5, sigma=1.9)
mean(y)
var(y)</pre>
```

plot_discrete_cdf 45

plot_discrete_cdf

Draw the CDF for a discrete random variable

Description

Draw the CDF for a discrete random variable

Usage

```
plot_discrete_cdf(x, fx, col = "blue", lwd = 3, ...)
```

Arguments

X	vector with the values of the random variable \boldsymbol{X} .
fx	vector with the probabilities of X .

col color for the line.

lwd line width.

... further arguments and graphical parameters.

Value

A plot with the cumulative distribution function.

Author(s)

Freddy Hernandez, <fhernanb@unal.edu.co>

46 POISXL

POISXL

The Discrete Poisson XLindley

Description

The function POISXL() defines the Discrete Poisson XLindley distribution, one-parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

Usage

```
POISXL(mu.link = "log")
```

Arguments

mu.link

defines the mu.link, with "log" link as the default for the mu parameter.

Details

The Discrete Poisson XLindley distribution with parameters μ has a support 0, 1, 2, ... and mass function given by

$$f(x|\mu) = \frac{\mu^2(x+\mu^2+3(1+\mu))}{(1+\mu)^{4+x}}$$
; with $\mu > 0$.

Note: in this implementation we changed the original parameters α for μ , we did it to implement this distribution within gamlss framework.

Value

Returns a gamlss.family object which can be used to fit a Discrete Poisson XLindley distribution in the gamlss() function.

Author(s)

Mariana Blandon Mejia, <mblandonm@unal.edu.co>

References

Ahsan-ul-Haq M, Al-Bossly A, El-Morshedy M, Eliwa MS, others (2022). "Poisson XLindley distribution for count data: statistical and reliability properties with estimation techniques and inference." *Computational Intelligence and Neuroscience*, **2022**.

See Also

dPOISXL.

POISXL 47

```
# Example 1
# Generating some random values with
# known mu
y \leftarrow rPOISXL(n=1000, mu=1)
# Fitting the model
library(gamlss)
mod1 <- gamlss(y~1, family=POISXL,</pre>
                control=gamlss.control(n.cyc=500, trace=FALSE))
# Extracting the fitted values for mu
# using the inverse link function
exp(coef(mod1, what='mu'))
# Example 2
# Generating random values under some model
# A function to simulate a data set with Y ~ POISXL
gendat <- function(n) {</pre>
  x1 <- runif(n, min=0.4, max=0.6)</pre>
  mu \leftarrow exp(1.21 - 3 * x1) # 0.75 approximately
  y <- rPOISXL(n=n, mu=mu)
 data.frame(y=y, x1=x1)
dat <- gendat(n=1500)
# Fitting the model
mod2 <- NULL
mod2 <- gamlss(y~x1, family=POISXL, data=dat,</pre>
                control=gamlss.control(n.cyc=500, trace=FALSE))
summary(mod2)
```

Index

DBH, 3, 6 dDBH, 4, 5 dDGEII, 7, 19 dDGEII, 7, 19 dDIKUM, 10, 30 dDLD, 13, 32 dDMOLBE, 16, 34 DGEII, 8, 18 dGGEO, 20, 38 dHYPERPO, 23, 41 dHYPERPO2, 26, 42 dHYPERPO, 20, 38 dHYPERPO, 27, 41 DDLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO (dHYPERPO), 26 plot_discrete_cdf, 45 POISXL, 36, 46	add, 2	qDBH (dDBH), 5 qDGEII (dDGEII), 7
dDGEII, 7, 19 dDIKUM, 10, 30 dDLD, 13, 32 dDMOLBE, 16, 34 DGEII, 8, 18 dGGEO, 20, 38 dHYPERPO, 23, 41 dHYPERPO2, 26, 42 dHYPERPO2, 26, 42 dHYPERPO2, 26, 42 dHYPERPO2, 26, 43 dHYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp, 43 mean_var_hp, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp, 27 dHYPERPO2, 26 pDISKUM, (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE, (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE, (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		qDIKUM (dDIKUM), 10
dDIKUM, 10, 30 dDLD, 13, 32 dDMOLBE, 16, 34 DGEII, 8, 18 dGGEO, 20, 38 dHYPERPO, 23, 41 dHYPERPO, 26, 42 dHYPERPO, 26, 42 dHYPERPO_single, 28 dHYPERPO_single, 28 dHYPERPO_single, 28 dHYPERPO_single, 11 DDLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO, 24, 27, 40, 42, 44 HYPERPO, 27, 41 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO), 26 plot_discrete_cdf, 45 POISXL, 36, 46		• •
dDLD, 13, 32 dDMOLBE, 16, 34 DGEII, 8, 18 dGGEO, 20, 38 dHYPERPO, 23, 41 dHYPERPO_2, 26, 42 dHYPERPO_single, 28 dHYPERPO_wec, 29 DIKUM, 11, 30 DLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 rOLD (dHYPERPO2), 26 rPOISXL (dPOISXL), 35		
dDMOLBE, 16, 34 DGEII, 8, 18 dGGEO, 20, 38 dHYPERPO, 23, 41 dHYPERPO, 26, 42 dHYPERPO_single, 28 dHYPERPO_vec, 29 DIKUM, 11, 30 DLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE, 17, 33 pDMOLBE, 17, 33 pDMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		* * * * * * * * * * * * * * * * * * * *
DGEII, 8, 18		
dGGEO, 20, 38 dHYPERPO, 23, 41 dHYPERPO2, 26, 42 dHYPERPO_single, 28 dHYPERPO_wec, 29 DIKUM, 11, 30 DLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO2, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO2, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO2 (dHYPERPO2), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		
dHYPERPO, 23, 41 dHYPERPO2, 26, 42 dHYPERPO_single, 28 dHYPERPO_wec, 29 DIKUM, 11, 30 DLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (ddDIKUM), 10 pDLD (ddLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		q. 010/12 (d. 010/12), 80
dHYPERPO_single, 28 dHYPERPO_vec, 29 DIKUM, 11, 30 DLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO_2, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 rHYPERPO2, 27, 41 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO2), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		rDBH (dDBH), 5
dHYPERPO_vec, 29 DIKUM, 11, 30 DLD, 14, 31 DNOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO2, 26 plot_discrete_cdf, 45 POISXL, 36, 46	dHYPERP02, 26, 42	* * * * * * * * * * * * * * * * * * * *
DIKUM, 11, 30 DLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	dHYPERPO_single, 28	
DLD, 14, 31 DMOLBE, 17, 33 dPOISXL, 35, 46 f11_cpp, 37 GGEO, 21, 38 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		' ''
DMOLBE, 17, 33 dPOISXL, 35, 46 rHYPERPO2 (dHYPERPO2), 26 rPOISXL (dPOISXL), 35 f11_cpp, 37 GGEO, 21, 38 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		
dPOISXL, 35, 46 rHYPERPO2 (dHYPERPO2), 26 rPOISXL (dPOISXL), 35 f11_cpp, 37 GGEO, 21, 38 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		
rPOISXL (dPOISXL), 35 f11_cpp, 37 GGEO, 21, 38 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		*
f11_cpp, 37 GGEO, 21, 38 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	uroisal, 33, 40	•
GGEO, 21, 38 HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	f11_cpp, 37	, ,,
HYPERPO, 24, 27, 40, 42, 44 HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		
HYPERPO2, 27, 41 mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	GGEO, 21, 38	
mean_var_hp, 43 mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	HYPERPO, 24, 27, 40, 42, 44	
mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	HYPERP02, 27, 41	
mean_var_hp2 (mean_var_hp), 43 pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		
pDBH (dDBH), 5 pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	• •	
pDGEII (dDGEII), 7 pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	mean_var_np2 (mean_var_np), +3	
pDIKUM (dDIKUM), 10 pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	pDBH (dDBH), 5	
pDLD (dDLD), 13 pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	pDGEII (dDGEII), 7	
pDMOLBE (dDMOLBE), 16 pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		
pGGEO (dGGEO), 20 pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	• • •	
pHYPERPO (dHYPERPO), 23 pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46	•	
pHYPERPO2 (dHYPERPO2), 26 plot_discrete_cdf, 45 POISXL, 36, 46		
plot_discrete_cdf, 45 POISXL, 36, 46		
POISXL, 36, 46	• • • • • • • • • • • • • • • • • • • •	
pPOISXL (dPOISXL), 35	pPOISXL (dPOISXL), 35	