# Package 'PhaseTypeR'

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```
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Title General-Purpose Phase-Type Functions
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Description General implementation of core function from phase-type
     theory. 'PhaseTypeR' can be used to model continuous and discrete
     phase-type distributions, both univariate and multivariate. The
     package includes functions for outputting the mean and (co)variance of
     phase-type distributions; their density, probability and quantile
     functions; functions for random draws; functions for
     reward-transformation; and functions for plotting the distributions as
     networks. For more information on these functions please refer to
     Bladt and Nielsen (2017, ISBN: 978-1-4939-8377-3) and Campillo Navarro
     (2019)
     <https://orbit.dtu.dk/en/publications/
     order-statistics-and-multivariate-discrete-phase-type-distributio>.
License GPL-3
URL https://rivasiker.github.io/PhaseTypeR/,
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```

DPH\_functions

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## Description

Density, distribution function, quantile function and random generation for the univariate discrete phase-type distribution.

## Usage

```
dDPH(x, obj)
qDPH(p, obj)
pDPH(q, obj)
rDPH(n, obj)
rFullDPH(obj)
```

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## **Arguments**

x,q	vector of quantiles.
obj	an object of class disc_phase_type.
р	vector of probabilities.
n	number of observations. If $length(n) > 1$ , the length is taken to be the number required.

#### Value

dDPH gives the density, pDPH gives the distribution function, qDPH gives the quantile function, and rDPH generates random deviates. rFullDPH returns the full path of a random draw from the distribution.

The length of the result is determined by n for rDPH, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

## **Functions**

- dDPH: Density function for the univariate continuous phase-type distribution.
- qDPH: Quantile function for the univariate discrete phase-type distribution.
- pDPH: Distribution function for the univariate discrete phase-type distribution.
- rDPH: Random number generator for the univariate discrete phase-type distribution.
- rFullDPH: Simulation of the full path for the univariate discrete phase-type distribution.

## See Also

Distributions for other standard distributions.

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## **Description**

Generator functions for the S3 classes cont\_phase\_type, disc\_phase\_type, mult\_cont\_phase\_type, mult\_disc\_phase\_type, which represent the different phase-type distributions.

## Usage

```
PH(subint_mat = NULL, init_probs = NULL)

DPH(subint_mat = NULL, init_probs = NULL)

MPH(subint_mat = NULL, init_probs = NULL, reward_mat = NULL)

MDPH(subint_mat = NULL, init_probs = NULL, reward_mat = NULL)
```

of the initial probabilities.

## Arguments

subint_mat	a square matrix containing the transition rates or probabilities between transient states for continuous or discrete phase-type respectively. If the phase-type is continuous, the sub-intensity matrix diagonal should only contain negative values and the row sums should be non-positive. If the phase-type is discrete, the sub-intensity matrix should only contain values between 0 and 1.
init_probs	a vector, a one-row matrix or NULL which gives the probabilities to start in each state. If init_probs is NULL, the probability to start on the first state will be 1 and 0 otherwise.
reward_mat	a matrix NULL(default) where each row is a reward vector, and each column corresponds to a state. It should have the same number of columns as the length

#### **Details**

PH, DPH, MPH and MDPH are the generator functions for the four types of phase-type distribution classes, this is, the continuous univariate, the discrete univariate, the continuous multivariate and the discrete multivariate respectively. The class is generated by supplying a sub-intensity matrix and an optional initial probability vector plus a reward matrix in the case of multivariate phase-type. If the initial probabilities are not specified, then the initial probability will be  $init_probs = c(1, 0, 0, ...)$  with the same length as the number of transient states.

#### Value

A phase-type object of class cont\_phase\_type for PH, disc\_phase\_type for DPH, mult\_cont\_phase\_type for MPH, and mult\_disc\_phase\_type for MDPH. All these classes inherit from list.

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```
##======##
## For continuous univariate ##
##=======##
subintensity_matrix <- matrix(c(-1.5, 1.5, 0,
                             0, -1, 1,
0, 0, -0.5),
                          ncol = 3,
                          byrow = TRUE)
PH(subintensity_matrix)
#---
subintensity_matrix <- matrix(c(-1.5, 1.5, 0,</pre>
                             0, -1, 1,
                             0, 0, -0.5),
                          ncol = 3,
                          byrow = TRUE)
initial_probabilities <- c(0.9, 0.1, 0)</pre>
PH(subintensity_matrix, initial_probabilities)
##======##
## For discrete univariate ##
##======##
subintensity_matrix <- matrix(c(0.4, 0.24, 0.12,</pre>
                            0, 0.4, 0.2,
                            0, 0, 0.5),
                          ncol = 3,
                          byrow = TRUE)
DPH(subintensity_matrix)
subintensity_matrix <- matrix(c(0.4, 0.24, 0.12,
                           0, 0.4, 0.2,
                            0, 0, 0.5),
                          ncol = 3,
                          byrow = TRUE)
initial_probabilities <- c(0.9, 0.1, 0)
DPH(subintensity_matrix, initial_probabilities)
##======##
## For continuous multivariate ##
##======##
subintensity_matrix <- matrix(c(-3, 2, 0,
```

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```
0, -2, 1,
                               0, 0, -1),
                            nrow = 3,
                            byrow = TRUE)
reward_matrix = matrix(sample(seq(0, 10, 0.1), 6), nrow = 3, ncol = 2)
initial_probabilities = c(1, 0, 0)
MPH(subintensity_matrix,
    initial_probabilities,
   reward_matrix)
##======##
## For discrete multivariate ##
##======##
subintensity_matrix <- matrix(c(0.4, 0.24, 0.12,</pre>
                              0, 0.4, 0.2,
                              0, 0,
                                        0.5),
                            ncol = 3,
                            byrow = TRUE)
reward_matrix <- matrix(sample(seq(0, 10), 6), nrow = 3, ncol = 2)</pre>
initial_probabilities = c(1, 0, 0)
MDPH(subintensity_matrix,
    initial_probabilities,
    reward_mat = reward_matrix)
```

MDPH\_functions

The Multivariate Discrete Phase-Type Distribution

## **Description**

Density, distribution function, quantile function and random generation for the multivariate discrete phase-type distribution.

## Usage

```
dMDPH(x, obj)
qMDPH(p, obj)
pMDPH(q, obj)
rMDPH(n, obj)
rFullMDPH(obj)
```

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#### **Arguments**

x, q	vector of quantiles.
obj	an object of class mult_disc_phase_type.
р	vector of probabilities.
n	number of observations. If $length(n) > 1$ , the length is taken to be the number required.

#### Value

dMDPH gives the density, pMDPH gives the distribution function, qMDPH gives the quantile function, and rMDPH generates random deviates. rFullMDPH returns the full path of a random draw from the distribution.

Each row of the result of For dMDPH, pMDPH, qMDPH, and rMDPH corresponds to each univariate reward transformation. For dMDPH, qMDPH and pMDPH, the inputs x, p and q can be matrices where in row i the i\_th reward transformation and in col j the j\_th value of x, p or q tested.

The length of the result is determined by n for rMDPH, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

## **Functions**

- dMDPH: Density function for the multivariate discrete phase-type distribution.
- qMDPH: Quantile function for the multivariate discrete phase-type distribution.
- pMDPH: Distribution function for the multivariate discrete phase-type distribution.
- rMDPH: Random number generator for the multivariate discrete phase-type distribution.
- rFullMDPH: Simulation of the full path for the multivariate discrete phase-type distribution.

#### See Also

Distributions for other standard distributions.

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```
set.seed(0)
rMDPH(6, Y)
rFullMDPH(Y)
```

mean

Mean of Phase-Type Distributions

## **Description**

Calculates the mean of continuous, discrete and multivariate phase-type distributions, represented by the cont\_phase\_type, disc\_phase\_type and mult\_cont\_phase\_type classes.

## Usage

```
## S3 method for class 'cont_phase_type'
mean(x, ...)
## S3 method for class 'disc_phase_type'
mean(x, ...)
## S3 method for class 'mult_cont_phase_type'
mean(x, v = NULL, ...)
## S3 method for class 'mult_disc_phase_type'
mean(x, v = NULL, ...)
```

## Arguments

```
x a cont_phase_type, disc_phase_type, mult_cont_phase_type or mult_disc_phase_type object
... other arguments passed to methods
v NULL, integer or vector.
```

#### **Details**

For the univariate case (cont\_phase\_type and disc\_phase\_type), the mean of the distribution is returned.

In the case of multivariate phase-type distributions three different usages can be distinguished:

- $\bullet$  If v = NULL (default), the means of all the variables defined by the sub-intensity matrix are returned
- If v is an integer, then the mean of the variable with the specified index in the reward matrix is returned.
- If v is a vector, then the means of the variables defined by those indices will be returned.

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## Value

This function returns a single value for the mean of univariate phase-type distributions, or a vector of means for each reward-transformed distribution of the multivariate phase-type distributions.

## **Examples**

MPH\_functions

The Multivariate Continuous Phase-Type Distribution

## Description

Density, distribution function, quantile function and random generation for the multivariate continuous phase-type distribution.

## Usage

```
dMPH(x, obj)
qMPH(p, obj)
pMPH(q, obj)
rMPH(n, obj)
rFullMPH(obj)
```

## Arguments

```
x, q vector of quantiles.
obj an object of class mult_cont_phase_type.
```

- p vector of probabilities.
- n number of observations. If length(n) > 1, the length is taken to be the number required.

## Value

dMPH gives the density, pMPH gives the distribution function, qMPH gives the quantile function, and rMPH generates random deviates. rFullMPH returns the full path of a random draw from the distribution.

Each row of the result of For dMPH, pMPH, qMPH, and rMPH corresponds to each univariate reward transformation. For dMDPH, qMDPH and pMDPH, the inputs x, p and q can be matrices where in row i the i\_th reward transformation and in col j the j\_th value of x, p or q tested.

The length of the result is determined by n for rMPH, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

#### **Functions**

- dMPH: Density function for the multivariate continuous phase-type distribution.
- qMPH: Quantile function for the multivariate continuous phase-type distribution.
- pMPH: Distribution function for the multivariate continuous phase-type distribution.
- rMPH: Random number generator for the multivariate continuous phase-type distribution.
- rFullMPH: Simulation of the full path for the multivariate continuous phase-type distribution.

## See Also

Distributions for other standard distributions.

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```
phase_type_to_network Phase-type distribution to network
```

## Description

This function converts a phase-type distribution into an igraph graph object.

## Usage

```
phase_type_to_network(phase_type, t = NULL)
```

## **Arguments**

```
phase_type an object of class disc_phase_type or cont_phase_type

t NULL or numeric. Sampling time for the continuous phase-type distribution.
```

#### Value

An igraph graph object of the phase-type distribution.

## **Examples**

PH\_functions

The Univariate Continuous Phase-Type Distribution

## **Description**

Density, distribution function, quantile function and random generation for the univariate continuous phase-type distribution.

PH\_functions

## Usage

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```
dPH(x, obj)
qPH(p, obj)
pPH(q, obj)
rPH(n, obj)
rFullPH(obj)
```

## **Arguments**

x, q	vector of quantiles.
obj	an object of class cont_phase_type.
p	vector of probabilities.
n	number of observations. If $length(n) > 1$ , the length is taken to be the number required.

## Value

dPH gives the density, pPH gives the distribution function, qPH gives the quantile function, and rPH generates random deviates. rFullPH returns the full path of a random draw from the distribution.

The length of the result is determined by n for rPH, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than n are recycled to the length of the result. Only the first elements of the logical arguments are used.

#### **Functions**

- dPH: Density function for the univariate continuous phase-type distribution.
- qPH: Quantile function for the univariate continuous phase-type distribution.
- pPH: Distribution function for the univariate continuous phase-type distribution.
- rPH: Random number generator for the univariate continuous phase-type distribution.
- rFullPH: Simulation of the full path for the univariate continuous phase-type distribution.

#### See Also

Distributions for other standard distributions.

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```
dPH(3:4, Y)
pPH(1.45, Y)
qPH(0.5, Y)
set.seed(0)
rPH(6, Y)
rFullPH(Y)
```

```
print.cont_phase_type Print method for phase-type objects
```

## **Description**

Print method for cont\_phase\_type, disc\_phase\_type, mult\_cont\_phase\_type and mult\_disc\_phase\_type classes.

## Usage

```
## S3 method for class 'cont_phase_type'
print(x, ...)
## S3 method for class 'disc_phase_type'
print(x, ...)
## S3 method for class 'mult_cont_phase_type'
print(x, ...)
## S3 method for class 'mult_disc_phase_type'
print(x, ...)
```

## **Arguments**

```
x phase-type object... other arguments not used by this method
```

## Value

Prints the phase-type object as a list.

```
subintensity_matrix <- matrix(c(-1.5, 1.5, 0, 0, -1, 1, 0, 0, -0.5), ncol = 3, byrow = TRUE)
```

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```
ph1 <- PH(subintensity_matrix)
print(ph1)</pre>
```

reward\_phase\_type

Transformation of Phase-Type Distributions via Rewards

## **Description**

Transform a variable following a phase-type distribution according to a non-negative reward vector.

## Usage

```
reward_phase_type(phase_type, reward)
```

## **Arguments**

phase\_type

an object of class cont\_phase\_type or disc\_phase\_type.

reward

a vector of the same length as the number of states. The vector should contain non-negative values. Rewards for the discrete phase-type distribution can only

be integers.

#### Details

For the reward transformation for continuous phase-type distribution, the transformation will be performed as presented in the book of Bladt and Nielsen (2017).

For the discrete phase\_type distribution is based on the PhD of Navarro (2018) and Hobolth, Bladt and Andersen (2021).

#### Value

An object of class disc\_phase\_type or cont\_phase\_type.

#### References

Bladt, M., & Nielsen, B. F. (2017). \*Matrix-exponential distributions in applied probability\* (Vol. 81). New York: Springer.

Campillo Navarro, A. (2018). \*Order statistics and multivariate discrete phase-type distributions\*. DTU Compute. DTU Compute PHD-2018, Vol.. 492

Hobolth, A., Bladt, M. & Andersen, L.A. (2021). \*Multivariate phase-type theory for the site frequency spectrum\*. ArXiv.

## See Also

PH, DPH

## **Examples**

```
##======##
## For continuous phase-type ##
##=======##
subint_mat <- matrix(c(-3, 1, 1,
                   2, -3, 0,
                   1, 1, -3), ncol = 3)
init_probs <- c(0.9, 0.1, 0)
ph <- PH(subint_mat, init_probs)</pre>
reward <- c(0.5, 0, 4)
reward_phase_type(ph, reward)
##======##
## For discrete phase-type ##
##=======##
subint_mat <- matrix(c(0.4, 0, 0,
                   0.24, 0.4, 0,
                   0.12, 0.2, 0.5), ncol = 3)
init_probs <- c(0.9, 0.1, 0)
ph <- DPH(subint_mat, init_probs)</pre>
reward <- c(1, 0, 4)
reward_phase_type(ph, reward)
```

```
summary.cont_phase_type
```

Pretty summary of the cont\_phase\_type class.

## Description

Pretty summary of the cont\_phase\_type class.

## Usage

```
## S3 method for class 'cont_phase_type'
summary(object, ...)
```

## **Arguments**

```
object a cont_phase_type object
... other arguments passed to methods
```

## Value

This function prints a nicely-formatted summary of a cont\_phase\_type object. The summary includes the sub-intensity matrix, the initial probabilities, the defect, the mean and the variance of the phase-type object.

## **Examples**

```
summary.disc_phase_type
```

Pretty summary of the disc\_phase\_type class.

## **Description**

Pretty summary of the disc\_phase\_type class.

## Usage

```
## S3 method for class 'disc_phase_type'
summary(object, ...)
```

## **Arguments**

```
object a disc_phase_type object
... other arguments passed to methods
```

## Value

This function prints a nicely-formatted summary of a disc\_phase\_type object. The summary includes the sub-intensity matrix, the initial probabilities, the defect, the mean and the variance of the phase-type object.

```
summary.mult_cont_phase_type
```

Pretty summary of the mult\_cont\_phase\_type class.

## **Description**

Pretty summary of the mult\_cont\_phase\_type class.

## Usage

```
## S3 method for class 'mult_cont_phase_type'
summary(object, ...)
```

## Arguments

```
object a mult_cont_phase_type object
... other arguments passed to methods
```

#### Value

This function prints a nicely-formatted summary of a mult\_cont\_phase\_type object. The summary includes the sub-intensity matrix, the initial probabilities, the defect, the reward matrix, the mean and the (co)variance of the phase-type object.

## **Examples**

```
summary.mult_disc_phase_type
```

Pretty summary of the mult\_dist\_phase\_type class.

## **Description**

Pretty summary of the mult\_dist\_phase\_type class.

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## Usage

```
## S3 method for class 'mult_disc_phase_type'
summary(object, ...)
```

## Arguments

```
object a mult_dist_phase_type object
... other arguments passed to methods
```

## Value

This function prints a nicely-formatted summary of a mult\_dist\_phase\_type object. The summary includes the sub-intensity matrix, the initial probabilities, the defect, the reward matrix, the mean and the (co)variance of the phase-type object.

## **Examples**

var

Variance and Covariance of Phase-Type Distributions

## **Description**

Calculates the (co)variance of continuous, discrete and multivariate phase-type distributions generated by PH, DPH, MPH and MDPH.

## Usage

```
var(obj, ...)
## S3 method for class 'cont_phase_type'
var(obj, ...)
## S3 method for class 'disc_phase_type'
var(obj, ...)
```

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```
## S3 method for class 'mult_cont_phase_type'
var(obj, v = NULL, ...)
## S3 method for class 'mult_disc_phase_type'
var(obj, v = NULL, ...)
```

#### **Arguments**

```
obj a cont_phase_type, disc_phase_type, mult_cont_phase_type or mult_disc_phase_type object
... other arguments passed to methods
v NULL, integer or vector of length 2.
```

#### **Details**

For the univariate case (cont\_phase\_type and disc\_phase\_type), the variance of the distribution is returned.

In the case of multivariate phase-type distributions three different usages can be distinguished:

- If v = NULL (default), then a variance-covariance matrix of all the variables specified in the reward matrix are returned, where variances are in the diagonal and covariances in the rest of the matrix element.
- If v is an integer, then the variance of the variable encoded by the v index in the reward matrix is returned.
- If v is a vector of length 2, then the covariance between the two variables encoded by the v indices in the reward matrix is returned.

## Value

The value returned is either the variance (for univariate distributions) or the variance-covariance matrix (for multivariate distributions).

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