Package 'RND'

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Description

This package is a collection of various functions to extract the implied risk neutral density from option.

Details

Package: RND Type: Package Version: 1.2

Date: 2017-01-10 License: GPL (>= 2)

Author(s)

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References

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

approximate.max 3

Examples

```
###
### You should see that all methods extract the same density!
      = 0.05
r
     = 60/365
te
      = 1000
sigma = 0.25
      = 0.02
call.strikes.bsm
                 = seq(from = 500, to = 1500, by = 5)
market.calls.bsm = price.bsm.option(r =r, te = te, s0 = s0,
                     k = call.strikes.bsm, sigma = sigma, y = y)$call
                  = seq(from = 500, to = 1500, by = 5)
put.strikes.bsm
market.puts.bsm
                  = price.bsm.option(r = r, te = te, s0 = s0,
                     k = put.strikes.bsm, sigma = sigma, y = y)$put
###
### See where your results will be outputted to...
###
getwd()
###
###
    Running this may take a few minutes...
###
### MOE(market.calls.bsm, call.strikes.bsm, market.puts.bsm,
### put.strikes.bsm, s0, r , te, y, "bsm2")
###
```

approximate.max

Max Function Approximation

Description

approximate.max gives a smooth approximation to the max function.

Usage

```
approximate.max(x, y, k = 5)
```

Arguments

```
x the first argument for the max function
y the second argument fot the max function
```

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k

a tuning parameter. The larger this value, the closer the function output to a true max function.

Details

approximate.max approximates the max of x, and y as follows:

$$g(x,y) = \frac{1}{1 + \exp(-k(x-y))}, \quad \max(x,y) \approx xg(x,y) + y(1 - g(x,y))$$

Value

approximate maximum of x and y

Author(s)

Kam Hamidieh

References

Melick, W. R. and Thomas, C.P. (1997) Recovering an asset's implied pdf from option proces: An application to crude oil during the gulf crisis. *Journal of Financial and Quantitative Analysis*, 32(1), 91-115

Examples

```
#
# To see how the max function compares with approximate.max,
# run the following code.
#

i = seq(from = 0, to = 10, by = 0.25)
y = i - 5
max.values = pmax(0,y)
approximate.max.values = approximate.max(0,y,k=5)
matplot(i, cbind(max.values, approximate.max.values), lty = 1, type = "l",
col=c("black","red"), main = "Max in Black, Approximate Max in Red")
```

bsm.objective

BSM Objective Function

Description

bsm. objective is the objective function to be minimized in extract.bsm.density.

Usage

```
bsm.objective(s0, r, te, y, market.calls, call.strikes, call.weights = 1,
  market.puts, put.strikes, put.weights = 1, lambda = 1, theta)
```

bsm.objective 5

Arguments

s0	current asset value
r	risk free rate
te	time to expiration
У	dividend yield
market.calls	market calls (most expensive to cheapest)
call.strikes	strikes for the calls (smallest to largest)
call.weights	weights to be used for calls
market.puts	market calls (cheapest to most expensive)
put.strikes	strikes for the puts (smallest to largest)
put.weights	weights to be used for calls
lambda	Penalty parameter to enforce the martingale condition
theta	initial values for the optimization. This must be a vector of length 2: first component is μ , the lognormal mean of the underlying density, and the second component is $\sqrt{t}\sigma$ which is the time scaled volatility parameter of the underlying density.

Details

This function evaluates the weighted squared differences between the market option values and values predicted by the Black-Scholes-Merton option pricing formula.

Value

Objective function evalued at a specific set of values.

Author(s)

Kam Hamidieh

References

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

```
r = 0.05
te = 60/365
s0 = 1000
sigma = 0.25
y = 0.01

call.strikes = seq(from = 500, to = 1500, by = 25)
market.calls = price.bsm.option(r = r, te = te, s0 = s0, k = call.strikes, sigma = sigma, y = y)$call
```

```
put.strikes
              = seq(from = 510, to = 1500, by = 25)
market.puts
              = price.bsm.option(r =r, te = te, s0 = s0,
                k = put.strikes, sigma = sigma, y = y)$put
###
### perfect initial values under BSM framework
###
        = log(s0) + (r - y - 0.5 * sigma^2) * te
mu.0
       = sigma * sqrt(te)
zeta.0
mu.0
zeta.0
### The objective function should be *very* small
###
bsm.obj.val = bsm.objective(theta=c(mu.0, zeta.0), r = r, y=y, te = te, s0 = s0,
             market.calls = market.calls, call.strikes = call.strikes,
             market.puts = market.puts, put.strikes = put.strikes, lambda = 1)
bsm.obj.val
```

Description

compute.implied.volatility extracts the implied volatility for a call option.

Usage

```
compute.implied.volatility(r, te, s0, k, y, call.price, lower, upper)
```

Arguments

r	risk free rate
te	time to expiration
s0	current asset value
k	strike of the call option
У	dividend yield
call.price	call price
lower	lower bound of the implied volatility to look for
upper	upper bound of the implied volatility to look for

Details

The simple R uniroot function is used to extract the implied volatility.

Value

sigma extratced implied volatility

Author(s)

Kam Hamidieh

References

- J. Hull (2011) *Options, Futures, and Other Derivatives and DerivaGem Package* Prentice Hall, Englewood Cliffs, New Jersey, 8th Edition
- R. L. McDonald (2013) Derivatives Markets Pearson, Upper Saddle River, New Jersey, 3rd Edition

```
#
# Create prices from BSM with various sigma's
      = 0.05
      = 0.02
      = 60/365
te
      = 400
s0
sigma.range = seq(from = 0.1, to = 0.8, by = 0.05)
          = floor(seq(from = 300, to = 500, length.out = length(sigma.range)))
bsm.calls = numeric(length(sigma.range))
for (i in 1:length(sigma.range))
  bsm.calls[i] = price.bsm.option(r = r, te = te, s0 = s0, k = k.range[i],
                                  sigma = sigma.range[i], y = y)$call
}
bsm.calls
k.range
# Computed implied sigma's should be very close to sigma.range.
compute.implied.volatility(r = r, te = te, s0 = s0, k = k.range, y = y,
                          call.price = bsm.calls, lower = 0.001, upper = 0.999)
sigma.range
```

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dew

Edgeworth Density

Description

dew is the probability density function implied by the Edgeworth expansion method.

Usage

```
dew(x, r, y, te, s0, sigma, skew, kurt)
```

Arguments

X	value at which the denisty is to be evaluated
r	risk free rate
у	dividend yield
te	time to expiration
s0	current asset value
sigma	volatility
skew	normalized skewness
kurt	normalized kurtosis

Details

This density function attempts to capture deviations from lognormal density by using Edgeworth expansions.

Value

density value at x

Author(s)

Kam Hamidieh

References

- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London
- R. Jarrow and A. Rudd (1982) Approximate valuation for arbitrary stochastic processes. *Journal of Financial Economics*, 10, 347-369
- C.J. Corrado and T. Su (1996) S&P 500 index option tests of Jarrow and Rudd's approximate option valuation formula. *Journal of Futures Markets*, 6, 611-629

dgb

Examples

```
#
# Look at a true lognorma density & related dew
       = 0.05
       = 0.03
       = 1000
s0
sigma = 0.25
       = 100/365
strikes = seq(from=600, to = 1400, by = 1)
       = sqrt(exp(sigma^2 * te) - 1)
ln.skew = 3 * v + v^3
ln.kurt = 16 * v^2 + 15 * v^4 + 6 * v^6 + v^8
skew.4 = ln.skew * 1.50
kurt.4 = ln.kurt * 1.50
skew.5 = ln.skew * 0.50
kurt.5 = ln.kurt * 2.00
ew.density.4 = dew(x=strikes, r=r, y=y, te=te, s0=s0, sigma=sigma,
                     skew=skew.4, kurt=kurt.4)
ew.density.5
              = dew(x=strikes, r=r, y=y, te=te, s0=s0, sigma=sigma,
                     skew=skew.5, kurt=kurt.5)
bsm.density
              = dlnorm(x = strikes, meanlog = log(s0) + (r - y - (sigma^2)/2)*te,
                 sdlog = sigma*sqrt(te), log = FALSE)
matplot(strikes, cbind(bsm.density, ew.density.4, ew.density.5), type="1",
        lty=c(1,1,1), col=c("black","red","blue"),
        main="Black = BSM, Red = EW 1.5 Times, Blue = EW 0.50 & 2")
```

dgb

Generalized Beta Density

Description

dgb is the probability density function of generalized beta distribution.

Usage

```
dgb(x, a, b, v, w)
```

Arguments

```
    value at which the denisty is to be evaluated
    power parameter > 0
    scale parameter > 0
    first beta parameter > 0
    second beta parameter > 0
```

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Details

Let B be a beta random variable with parameters v and w, then $Z = b(B/(1-B))^{1/a}$ is a generalized beta with parameters (a,b,v,w).

Value

density value at x

Author(s)

Kam Hamidieh

References

R.M. Bookstaber and J.B. McDonald (1987) A general distribution for describing security price returns. *Journal of Business*, 60, 401-424

X. Liu and M.B. Shackleton and S.J. Taylor and X. Xu (2007) Closed-form transformations from risk-neutral to real-world distributions *Journal of Business*, 60, 401-424

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

Examples

```
#
# Just simple plot of the density
#

x = seq(from = 500, to = 1500, length.out = 10000)
a = 10
b = 1000
v = 3
w = 3
dx = dgb(x = x, a = a, b = b, v = v, w = w)
plot(dx ~ x, type="l")
```

dmln

Density of Mixture Lognormal

Description

mln is the probability density function of a mixture of two lognormal densities.

Usage

```
dmln(x, alpha.1, meanlog.1, meanlog.2, sdlog.1, sdlog.2)
```

dmln 11

Arguments

X	value at which the denisty is to be evaluated
alpha.1	proportion of the first lognormal. Second one is 1 - alpha.1
meanlog.1	mean of the log of the first lognormal
meanlog.2	mean of the log of the second lognormal
sdlog.1	standard deviation of the log of the first lognormal
sdlog.2	standard deviation of the log of the second lognormal

Details

mln is the density f(x) = alpha.1 * g(x) + (1 - alpha.1) * h(x), where g and h are densities of two lognormals with parameters (mean.log.1, sdlog.1) and (mean.log.2, sdlog.2) respectively.

Value

out density value at x

Author(s)

Kam Hamidieh

References

- B. Bahra (1996): Probability distribution of future asset prices implied by option prices. *Bank of England Quarterly Bulletin*, August 1996, 299-311
- P. Soderlind and L.E.O. Svensson (1997) New techniques to extract market expectations from financial instruments. *Journal of Monetary Economics*, 40, 383-429
- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

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dmln.am

Density of Mixture Lognormal for American Options

Description

mln.am is the probability density function of a mixture of three lognormal densities.

Usage

```
dmln.am(x, u.1, u.2, u.3, sigma.1, sigma.2, sigma.3, p.1, p.2)
```

Arguments

X	value at which the denisty is to be evaluated
u.1	log mean of the first lognormal
u.2	log mean of the second lognormal
u.3	log mean of the third lognormal
sigma.1	log standard deviation of the first lognormal
sigma.2	log standard deviation of the second lognormal
sigma.3	log standard deviation of the third lognormal
p.1	weight assigned to the first density
p.2	weight assigned to the second density

Details

mln is density f(x) = p.1 * f1(x) + p.2 * f2(x) + (1 - p.1 - p.2) * f3(x), where f1, f2, and f3 are lognormal densities with log means u.1,u.2, and u.3 and standard deviations sigma.1, sigma.2, and sigma.3 respectively.

Value

out density value at x

Author(s)

Kam Hamidieh

References

Melick, W. R. and Thomas, C. P. (1997). Recovering an asset's implied pdf from option prices: An application to crude oil during the gulf crisis. *Journal of Financial and Quantitative Analysis*, 32(1), 91-115.

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Examples

```
###
### Just look at a generic density and see if it integrates to 1.
###
       = 4.2
u.1
       = 4.5
u.2
       = 4.8
u.3
sigma.1 = 0.30
sigma.2 = 0.20
sigma.3 = 0.15
       = 0.25
p.1
     = 0.45
p.2
x = seq(from = 0, to = 250, by = 0.01)
y = dmln.am(x = x, u.1 = u.1, u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
            sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)
plot(y \sim x, type="l")
sum(y * 0.01)
###
### Yes, the sum is near 1.
```

dshimko

Density Implied by Shimko Method

Description

dshimko is the probability density function implied by the Shimko method.

Usage

```
dshimko(r, te, s0, k, y, a0, a1, a2)
```

Arguments

r	risk free rate
te	time to expiration
s0	current asset value
k	strike at which volatility to be computed
у	dividend yield
a0	constant term in the quadratic polynomial
a1	coefficient term of k in the quadratic polynomial
a2	coefficient term of k squared in the quadratic polynomial

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Details

The implied volatility is modeled as: $\sigma(k) = a_0 + a_1k + a_2k^2$

Value

density value at x

Author(s)

Kam Hamidieh

References

D. Shimko (1993) Bounds of probability. Risk, 6, 33-47

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

```
# a0, a1, a2 values come from Shimko's paper.
      = 0.05
     = 0.02
     = 0.892
a0
     = -0.00387
     = 0.00000445
te
     = 60/365
      = 400
      = seq(from = 250, to = 500, by = 1)
sigma = 0.15
# Does it look like a proper density and intergate to one?
dx = dshimko(r = r, te = te, s0 = s0, k = k, y = y, a0 = a0, a1 = a1, a2 = a2)
plot(dx \sim k, type="l")
# sum(dx) should be about 1 since dx is a density.
sum(dx)
```

ew.objective 15

ew.objective	Edgeworth Exapnsion Objective Function	

Description

ew.objective is the objective function to be minimized in ew.extraction.

Usage

```
ew.objective(theta, r, y, te, s0, market.calls, call.strikes, call.weights = 1,
  lambda = 1)
```

Arguments

theta	initial values for the optimization
r	risk free rate
у	dividend yield
te	time to expiration
s0	current asset value
market.calls	market calls (most expensive to cheapest)
call.strikes	strikes for the calls (smallest to largest)
call.weights	weights to be used for calls
lambda	Penalty parameter to enforce the martingale condition

Details

This function evaluates the weighted squared differences between the market option values and values predicted by Edgworth based expansion of the risk neutral density.

Value

Objective function evalued at a specific set of values

Author(s)

Kam Hamidieh

References

- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London
- R. Jarrow and A. Rudd (1982) Approximate valuation for arbitrary stochastic processes. *Journal of Financial Economics*, 10, 347-369
- C.J. Corrado and T. Su (1996) S&P 500 index option tests of Jarrow and Rudd's approximate option valuation formula. *Journal of Futures Markets*, 6, 611-629

Examples

```
= 0.05
У
       = 0.03
       = 1000
s0
sigma = 0.25
te
       = 100/365
       = seq(from=800, to = 1200, by = 50)
       = sqrt(exp(sigma^2 * te) - 1)
ln.skew = 3 * v + v^3
ln.kurt = 16 * v^2 + 15 * v^4 + 6 * v^6 + v^8
# The objective function should be close to zero.
# Also the weights are automatically set to 1.
market.calls.bsm = price.bsm.option(r = r, te = te, s0 = s0, k=k,
                   sigma=sigma, y=y)$call
ew.objective(theta = c(sigma, ln.skew, ln.kurt), r = r, y = y, te = te, s0=s0,
             market.calls = market.calls.bsm, call.strikes = k, lambda = 1)
```

extract.am.density

Mixture of Lognormal Extraction for American Options

Description

extract.am.density extracts the mixture of three lognormals from American options.

Usage

```
extract.am.density(initial.values = rep(NA, 10), r, te, s0, market.calls,
  call.weights = NA, market.puts, put.weights = NA, strikes, lambda = 1,
  hessian.flag = F, cl = list(maxit = 10000))
```

Arguments

```
initial.values initial values for the optimization

r risk free rate

te time to expiration

s0 current asset value

market.calls market calls (most expensive to cheapest)

call.weights weights to be used for calls. Set to 1 by default.

market.puts market calls (cheapest to most expensive)
```

put.weights weights to be used for puts. Set to 1 by default.

strikes strikes (smallest to largest)

lambda Penalty parameter to enforce the martingale condition

hessian.flag If FALSE then no Hessian is produced

cl List of parameter values to be passed to the optimization function

Details

The extracted density is in the form of f(x) = p.1 * f1(x) + p.2 * f2(x) + (1 - p.1 - p.2) * f3(x), where f1, f2, and f3 are lognormal densities with log means u.1,u.2, and u.3 and standard deviations sigma.1, sigma.2, and sigma.3 respectively.

For the description of w.1 and w.2 see equations (7) & (8) of Melick and Thomas paper below.

Value

w.1	First weight, a number between 0 and 1, to weigh the option price bounds				
w.2	Second weight, a number between 0 and 1, to weigh the option price bounds				
u.1	log mean of the first lognormal				
u.2	log mean of the second lognormal				
u.3	log mean of the third lognormal				
sigma.1	log sd of the first lognormal				
sigma.2	log sd of the second lognormal				
sigma.3	log sd of the third lognormal				
p.1	weight assigned to the first density				
p.2	weight assigned to the second density				
converge.result					
	Captures the convergence result				
hessian	Hessian Matrix				

Author(s)

Kam Hamidieh

References

Melick, W. R. and Thomas, C. P. (1997). Recovering an asset's implied pdf from option prices: An application to crude oil during the gulf crisis. *Journal of Financial and Quantitative Analysis*, 32(1), 91-115.

```
###
### Try with synthetic data first.
###
       = 0.01
te
       = 60/365
       = 0.4
w.1
       = 0.25
w.2
       = 4.2
u.1
u.2
       = 4.5
u.3
       = 4.8
sigma.1 = 0.30
sigma.2 = 0.20
sigma.3 = 0.15
       = 0.25
p.1
p.2
       = 0.45
theta = c(w.1, w.2, u.1, u.2, u.3, sigma.1, sigma.2, sigma.3, p.1, p.2)
       = 1 - p.1 - p.2
### Generate some synthetic American calls & puts
###
expected.f0
             = sum(c(p.1, p.2, p.3) * exp(c(u.1,u.2,u.3) +
                    (c(sigma.1, sigma.2, sigma.3)^2)/2) )
expected.f0
strikes = 50:150
market.calls = numeric(length(strikes))
market.puts = numeric(length(strikes))
for (i in 1:length(strikes))
  if ( strikes[i] < expected.f0) {</pre>
   market.calls[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.1, u.1 = u.1,
                      u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                      sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$call.value
   market.puts[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.2, u.1 = u.1,
                      u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                      sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)put.value
  } else {
   market.calls[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.2, u.1 = u.1,
                      u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                      sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$call.value
   market.puts[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.1, u.1 = u.1,
```

```
u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                      sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)put.value
     }
}
### ** IMPORTANT **: The code that follows may take 1-2 minutes.
                      Copy and paste onto R console the commands
###
###
                      that follow the greater sign >.
###
### Try the optimization with exact inital values.
### They should be close the actual initials.
###
#
# > s0
            = expected.f0 * exp(-r * te)
\# > s0
#
# > extract.am.density(initial.values = theta, r = r, te = te, s0 = s0,
               market.calls = market.calls, market.puts = market.puts, strikes = strikes,
#
                   lambda = 1, hessian.flag = FALSE)
#
# > theta
#
###
### Now try without our the correct initial values...
###
#
\# > \text{optim.obj.no.init} = \text{extract.am.density}( r = r, te = te, s0 = s0,
#
               market.calls = market.calls, market.puts = market.puts, strikes = strikes,
#
                     lambda = 1, hessian.flag = FALSE)
#
# > optim.obj.no.init
# > theta
#
###
### We do get different values but how do the densities look like?
###
#
### plot the two densities side by side
###
#
\# > x = 10:190
\# > y.1 = dmln.am(x = x, p.1 = theta[9], p.2 = theta[10],
            u.1 = theta[3], u.2 = theta[4], u.3 = theta[5],
           sigma.1 = theta[6], sigma.2 = theta[7], sigma.3 = theta[8] )
# > o = optim.obj.no.init
\# > y.2 = dmln.am(x = x, p.1 = o$p.1, p.2 = o$p.2,
```

extract.bsm.density

Extract Lognormal Density

Description

bsm. extraction extracts the parameters of the lognormal density as implied by the BSM model.

Usage

```
extract.bsm.density(initial.values = c(NA, NA), r, y, te, s0, market.calls,
  call.strikes, call.weights = 1, market.puts, put.strikes, put.weights = 1,
  lambda = 1, hessian.flag = F, cl = list(maxit = 10000))
```

Arguments

```
initial.values initial values for the optimization
                  risk free rate
                   dividend yield
У
                   time to expiration
te
                  current asset value
50
market.calls
                   market calls (most expensive to cheapest)
call.strikes
                   strikes for the calls (smallest to largest)
call.weights
                   weights to be used for calls
market.puts
                   market calls (cheapest to most expensive)
put.strikes
                   strikes for the puts (smallest to largest)
                   weights to be used for puts
put.weights
lambda
                  Penalty parameter to enforce the martingale condition
hessian.flag
                   if F, no hessian is produced
cl
                   list of parameter values to be passed to the optimization function
```

Details

If initial.values are not specified then the function will attempt to pick them automatically. cl is a list that can be used to pass parameters to the optim function.

Value

Let S_T with the lognormal random variable of the risk neutral density.

```
\begin{array}{ccc} \text{mu} & \text{mean of log}(S\_T) \\ \text{zeta} & \text{sd of log}(S\_T) \\ \text{converge.result} \\ & & \text{Did the result converge?} \\ \text{hessian} & & \text{Hessian matrix} \end{array}
```

Author(s)

Kam Hamidieh

References

- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London
- J. Hull (2011) *Options, Futures, and Other Derivatives and DerivaGem Package* Prentice Hall, Englewood Cliffs, New Jersey, 8th Edition
- R. L. McDonald (2013) Derivatives Markets Pearson, Upper Saddle River, New Jersey, 3rd Edition

Examples

#

```
#
# Create some BSM Based options
      = 0.05
      = 60/365
te
      = 1000
sigma = 0.25
      = 0.01
call.strikes
              = seq(from = 500, to = 1500, by = 25)
market.calls
              = price.bsm.option(r =r, te = te, s0 = s0,
                 k = call.strikes, sigma = sigma, y = y)$call
put.strikes
               = seq(from = 510, to = 1500, by = 25)
               = price.bsm.option(r =r, te = te, s0 = s0,
market.puts
                 k = put.strikes, sigma = sigma, y = y)$put
  Get extract the parameter of the density
#
#
extract.bsm.density(r = r, y = y, te = te, s0 = s0, market.calls = market.calls,
               call.strikes = call.strikes, market.puts = market.puts,
               put.strikes = put.strikes, lambda = 1, hessian.flag = FALSE)
```

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```
# The extracted parameters should be close to these actual values: # actual.mu = log(s0) + (r - y - 0.5 * sigma^2) * te \\ actual.zeta = sigma * sqrt(te) \\ actual.mu \\ actual.zeta
```

extract.ew.density

Extract Edgeworth Based Density

Description

ew.extraction extracts the parameters for the density approximated by the Edgeworth expansion method

Usage

```
extract.ew.density(initial.values = c(NA, NA, NA), r, y, te, s0, market.calls,
  call.strikes, call.weights = 1, lambda = 1, hessian.flag = F,
  cl = list(maxit = 10000))
```

Arguments

```
initial.values initial values for the optimization
                   risk free rate
                   dividend yield
У
te
                   time to expiration
s0
                   current asset value
market.calls
                   market calls (most expensive to cheapest)
call.strikes
                   strikes for the calls (smallest to largest)
call.weights
                   weights to be used for calls
lambda
                   Penalty parameter to enforce the martingale condition
hessian.flag
                   if F, no hessian is produced
c1
                   list of parameter values to be passed to the optimization function
```

Details

If initial.values are not specified then the function will attempt to pick them automatically. cl in form of a list can be used to pass parameters to the optim function.

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Value

```
sigma volatility of the underlying lognormal skew normalized skewness kurt normalized kurtosis converge.result Did the result converge? hessian Hessian matrix
```

Author(s)

Kam Hamidieh

References

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

R. Jarrow and A. Rudd (1982) Approximate valuation for arbitrary stochastic processes. *Journal of Financial Economics*, 10, 347-369

C.J. Corrado and T. Su (1996) S&P 500 index option tests of Jarrow and Rudd's approximate option valuation formula. *Journal of Futures Markets*, 6, 611-629

```
ln.skew & ln.kurt are the normalized skewness and kurtosis of a true lognormal.
       = 0.05
       = 0.03
       = 1000
sigma
       = 0.25
       = 100/365
strikes = seq(from=600, to = 1400, by = 1)
       = sqrt(exp(sigma^2 * te) - 1)
ln.skew = 3 * v + v^3
ln.kurt = 16 * v^2 + 15 * v^4 + 6 * v^6 + v^8
# Now "perturb" the lognormal
new.skew = ln.skew * 1.50
new.kurt = ln.kurt * 1.50
# new.skew & new.kurt should not be extracted.
# Note that weights are automatically set to 1.
#
```

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extract.gb.density

Generalized Beta Extraction

Description

extract.gb.density extracts the generalized beta density from market options.

Usage

```
extract.gb.density(initial.values = c(NA, NA, NA, NA), r, te, y, s0, market.calls,
  call.strikes, call.weights = 1, market.puts, put.strikes, put.weights = 1,
  lambda = 1, hessian.flag = F, cl = list(maxit = 10000))
```

Arguments

```
initial.values initial values for the optimization
                   risk free rate
                  time to expiration
te
                   dividend yield
У
s0
                   current asset value
market.calls
                   market calls (most expensive to cheapest)
call.strikes
                   strikes for the calls (smallest to largest)
call.weights
                   weights to be used for calls
market.puts
                  market calls (cheapest to most expensive)
put.strikes
                   strikes for the puts (smallest to largest)
put.weights
                   weights to be used for puts
lambda
                  Penalty parameter to enforce the martingale condition
hessian.flag
                  if F, no hessian is produced
                  list of parameter values to be passed to the optimization function
cl
```

Details

This function extracts the generalized beta density implied by the options.

extract.gb.density 25

Value

```
a extracted power parameter
b extracted scale paramter
v extracted first beta paramter
w extracted second beta parameter
converge.result
Did the result converge?
hessian Hessian matrix
```

Author(s)

Kam Hamidieh

References

- R.M. Bookstaber and J.B. McDonald (1987) A general distribution for describing security price returns. *Journal of Business*, 60, 401-424
- X. Liu and M.B. Shackleton and S.J. Taylor and X. Xu (2007) Closed-form transformations from risk-neutral to real-world distributions *Journal of Business*, 60, 401-424
- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

```
# create some GB based calls and puts
#
r = 0.03
te = 50/365
k = seq(from = 800, to = 1200, by = 10)
  = 10
b = 1000
v = 2.85
w = 2.85
y = 0.01
s0 = \exp((y-r)*te) * b * beta(v + 1/a, w - 1/a)/beta(v,w)
call.strikes = seq(from = 800, to = 1200, by = 10)
market.calls = price.gb.option(r = r, te = te, y = y, s0 = s0,
                               k = call.strikes, a = a, b = s0, v = v, w = w)$call
put.strikes = seq(from = 805, to = 1200, by = 10)
market.puts = price.gb.option(r = r, te = te, y = y, s0 = s0,
                               k = put.strikes, a = a, b = s0, v = v, w = w)$put
```

extract.mln.density

Extract Mixture of Lognormal Densities

Description

mln.extraction extracts the parameters of the mixture of two lognormals densities.

Usage

```
extract.mln.density(initial.values = c(NA, NA, NA, NA, NA), r, y, te, s0,
  market.calls, call.strikes, call.weights = 1, market.puts, put.strikes,
  put.weights = 1, lambda = 1, hessian.flag = F, cl = list(maxit = 10000))
```

Arguments

```
initial.values initial values for the optimization
                  risk free rate
                  dividend yield
y
                  time to expiration
te
s0
                  current asset value
market.calls
                  market calls (most expensive to cheapest)
call.strikes
                   strikes for the calls (smallest to largest)
call.weights
                  weights to be used for calls
market.puts
                  market calls (cheapest to most expensive)
                  strikes for the puts (smallest to largest)
put.strikes
                  weights to be used for puts
put.weights
lambda
                  Penalty parameter to enforce the martingale condition
hessian.flag
                  if F, no hessian is produced
cl
                  list of parameter values to be passed to the optimization function
```

Details

mln is the density f(x) = alpha.1 * g(x) + (1 - alpha.1) * h(x), where g and h are densities of two lognormals with parameters (mean.log.1, sdlog.1) and (mean.log.2, sdlog.2) respectively.

Value

```
alpha.1 extracted proportion of the first lognormal. Second one is 1 - alpha.1 meanlog.1 extracted mean of the log of the first lognormal extracted mean of the log of the second lognormal sdlog.1 extracted standard deviation of the log of the first lognormal extracted standard deviation of the log of the second lognormal converge.result Did the result converge?

hessian Hessian matrix
```

Author(s)

Kam Hamidieh

References

- F. Gianluca and A. Roncoroni (2008) Implementing Models in Quantitative Finance: Methods and Cases
- B. Bahra (1996): Probability distribution of future asset prices implied by option prices. *Bank of England Quarterly Bulletin*, August 1996, 299-311
- P. Soderlind and L.E.O. Svensson (1997) New techniques to extract market expectations from financial instruments. *Journal of Monetary Economics*, 40, 383-4
- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

```
# Create some calls and puts based on mln and
# see if we can extract the correct values.
          = 0.05
У
         = 0.02
         = 60/365
te
meanlog.1 = 6.8
meanlog.2 = 6.95
sdlog.1 = 0.065
sdlog.2 = 0.055
alpha.1
         = 0.4
call.strikes = seq(from = 800, to = 1200, by = 10)
market.calls = price.mln.option(r = r, y = y, te = te, k = call.strikes,
               alpha.1 = alpha.1, meanlog.1 = meanlog.1, meanlog.2 = meanlog.2,
                                sdlog.1 = sdlog.1, sdlog.2 = sdlog.2)$call
```

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extract.rates

Extract Risk Free Rate and Dividend Yield

Description

extract.rates extracts the risk free rate and the dividend yield from European options.

Usage

```
extract.rates(calls, puts, s0, k, te)
```

Arguments

calls	market calls (most expensive to cheapest)
puts	market puts (cheapest to most expensive)
s0	current asset value
k	strikes for the calls (smallest to largest)
te	time to expiration

Details

The extraction is based on the put-call parity of the European options. Shimko (1993) - see below - shows that the slope and intercept of the regression of the calls minus puts onto the strikes contains the risk free and the dividend rates.

extract.shimko.density 29

Value

```
risk.free.rate
extracted risk free rate
dividend.yield
extracted dividend rate
```

Author(s)

Kam Hamidieh

References

D. Shimko (1993) Bounds of probability. Risk, 6, 33-47

Examples

```
#
# Create calls and puts based on BSM
#

r = 0.05
te = 60/365
s0 = 1000
k = seq(from = 900, to = 1100, by = 25)
sigma = 0.25
y = 0.01

bsm.obj = price.bsm.option(r = r, te = te, s0 = s0, k = k, sigma = sigma, y = y)

calls = bsm.obj$call
puts = bsm.obj$put

#
# Extract rates should give the values of r and y above:
#

rates = extract.rates(calls = calls, puts = puts, k = k, s0 = s0, te = te)
rates
```

extract.shimko.density

Extract Risk Neutral Density based on Shimko's Method

Description

shimko.extraction extracts the implied risk neutral density based on modeling the volatility as a quadratic function of the strikes.

extract.shimko.density

Usage

```
extract.shimko.density(market.calls, call.strikes, r, y, te, s0, lower, upper)
```

Arguments

market.calls	market calls (most expensive to cheapest)
call.strikes	strikes for the calls (smallest to largest)
r	risk free rate
У	dividend yield
te	time to expiration
s0	current asset value
lower	lower bound for the search of implied volatility
upper	upper bound for the search of implied volatility

Details

The correct values for range of search must be specified.

Value

Author(s)

Kam Hamidieh

References

- D. Shimko (1993) Bounds of probability. Risk, 6, 33-47
- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

```
#
# Test the function shimko.extraction. If BSM holds then a1 = a2 = 0.
#

r = 0.05
y = 0.02
```

fit.implied.volatility.curve

Fit Implied Quadratic Volatility Curve

Description

fit.implied.volatility.curve estimates the coefficients of the quadratic equation for the implied volatilities.

Usage

```
fit.implied.volatility.curve(x, k)
```

Arguments

x a set of implied volatilities

k range of strikes

Details

This function estimates volatility σ as a quadratic function of strike k with the coefficients a_0, a_1, a_2 : $\sigma(k) = a_0 + a_1 k + a_2 k^2$

Value

a0 constant term in the quadratic ploynomial

a1 coefficient term of k in the quadratic ploynomial

a2 coefficient term of k squared in the quadratic polynomial

summary.obj statistical summary of the fit

Author(s)

Kam Hamidieh

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References

- D. Shimko (1993) Bounds of probability. Risk, 6, 33-47
- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

Examples

```
#
# Suppose we see the following implied volatilities and strikes:
#
implied.sigma = c(0.11, 0.08, 0.065, 0.06, 0.05)
strikes = c(340, 360, 380, 400, 410)
tmp = fit.implied.volatility.curve(x = implied.sigma, k = strikes)
tmp

strike.range = 340:410
plot(implied.sigma ~ strikes)
lines(strike.range, tmp$a0 + tmp$a1 * strike.range + tmp$a2 * strike.range^2)
```

gb.objective

Generalized Beta Objective

Description

gb.objective is the objective function to be minimized in extract.gb.density.

Usage

```
gb.objective(theta, r, te, y, s0, market.calls, call.strikes, call.weights = 1,
    market.puts, put.strikes, put.weights = 1, lambda = 1)
```

Arguments

initial values for optimization
risk free rate
time to expiration
dividend yield
current asset value
market calls (most expensive to cheapest)
strikes for the calls (smallest to largest)
weights to be used for calls
market calls (cheapest to most expensive)

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```
put.strikes strikes for the puts (smallest to largest)
put.weights weights to be used for puts
lambda Penalty parameter to enforce the martingale condition
```

Details

This is the function minimized by extract.gb.desnity function.

Value

obj value of the objective function

Author(s)

Kam Hamidieh

References

R.M. Bookstaber and J.B. McDonald (1987) A general distribution for describing security price returns. *Journal of Business*, 60, 401-424

X. Liu and M.B. Shackleton and S.J. Taylor and X. Xu (2007) Closed-form transformations from risk-neutral to real-world distributions *Journal of Business*, 60, 401-424

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

34 get.point.estimate

get.point.estimate

Point Estimation of the Density

Description

get.point.estimate estimates the risk neutral density by center differentiation.

Usage

```
get.point.estimate(market.calls, call.strikes, r, te)
```

Arguments

market.calls market calls (most expensive to cheapest)
call.strikes strikes for the calls (smallest to largest)
r risk free rate
te time to expiration

Details

This is a non-parametric estimate of the risk neutral density. Due to center differentiation, the density values are not estimated at the highest and lowest strikes.

Value

```
point.estimates
```

values of the estimated density at each strike

Author(s)

Kam Hamidieh

References

J. Hull (2011) *Options, Futures, and Other Derivatives and DerivaGem Package* Prentice Hall, Englewood Cliffs, New Jersey, 8th Edition

mln.am.objective 35

Examples

```
###
### Recover the lognormal density based on BSM
      = 0.05
r
     = 60/365
te
s0
     = 1000
     = seq(from = 500, to = 1500, by = 1)
sigma = 0.25
      = 0.01
bsm.calls = price.bsm.option(r = r, te = te, s0 = s0, k = k, sigma = sigma, y = y)$call
density.est = get.point.estimate(market.calls = bsm.calls,
              call.strikes = k, r = r, te = te)
len = length(k)-1
### Note, estimates at two data points (smallest and largest strikes) are lost
plot(density.est ~ k[2:len], type = "1")
```

mln.am.objective

Objective function for the Mixture of Lognormal of American Options

Description

mln.am.objective is the objective function to be minimized in extract.am.density.

Usage

```
mln.am.objective(theta, s0, r, te, market.calls, call.weights = NA, market.puts,
   put.weights = NA, strikes, lambda = 1)
```

Arguments

theta	initial values for the optimization
s0	current asset value
r	risk free rate
te	time to expiration
market.calls	market calls (most expensive to cheapest)
call.weights	weights to be used for calls
market.puts	market calls (cheapest to most expensive)
put.weights	weights to be used for calls
strikes	strikes for the calls (smallest to largest)
lambda	Penalty parameter to enforce the martingale condition

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Details

mln is density f(x) = p.1 * f1(x) + p.2 * f2(x) + (1 - p.1 - p.2) * f3(x), where f1, f2, and f3 are lognormal densities with log means u.1,u.2, and u.3 and standard deviations sigma.1, sigma.2, and sigma.3 respectively.

Value

obj Value of the objective function

Author(s)

Kam Hamidieh

References

Melick, W. R. and Thomas, C. P. (1997). Recovering an asset's implied pdf from option prices: An application to crude oil during the gulf crisis. *Journal of Financial and Quantitative Analysis*, 32(1), 91-115.

```
= 0.01
r
te
        = 60/365
       = 0.4
w.1
        = 0.25
w.2
        = 4.2
u.1
u.2
        = 4.5
u.3
        = 4.8
sigma.1 = 0.30
sigma.2 = 0.20
sigma.3 = 0.15
p.1
       = 0.25
p.2
       = 0.45
theta = c(w.1, w.2, u.1, u.2, u.3, sigma.1, sigma.2, sigma.3, p.1, p.2)
p.3 = 1 - p.1 - p.2
p.3
expected.f0 = sum(c(p.1, p.2, p.3) * exp(c(u.1,u.2,u.3) +
                     (c(sigma.1, sigma.2, sigma.3)^2)/2) )
expected.f0
strikes = 30:170
market.calls = numeric(length(strikes))
market.puts = numeric(length(strikes))
for (i in 1:length(strikes))
{
  if ( strikes[i] < expected.f0) {</pre>
```

mln.am.objective 37

```
market.calls[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.1, u.1 = u.1,
                                               u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                               sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$call.value
       market.puts[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.2, u.1 = u.1,
                                               u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                               sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)put.value
   } else {
       market.calls[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.2, u.1 = u.1,
                                               u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                               sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$call.value
       market.puts[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.1, u.1 = u.1, u.1, u.1 = u
                                               u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                               sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)put.value
          }
}
### Quickly look at the option values...
###
par(mfrow=c(1,2))
plot(market.calls ~ strikes, type="l")
plot(market.puts ~ strikes, type="1")
par(mfrow=c(1,1))
###
### ** IMPORTANT **: The code that follows may take a few seconds.
###
                                               Copy and paste onto R console the commands
###
                                               that follow the greater sign >.
###
###
### Next try the objective function. It should be zero.
### Note: Let weights be the defaults values of 1.
###
\# > s0
                         = expected.f0 * exp(-r * te)
\# > s0
#
# > mln.am.objective(theta, s0 =s0, r = r, te = te, market.calls = market.calls,
#
                                      market.puts = market.puts, strikes = strikes, lambda = 1)
#
###
### Now directly try the optimization with perfect initial values.
###
#
#
# > optim.obj.with.synthetic.data = optim(theta, mln.am.objective, s0 = s0, r=r, te=te,
#
                                market.calls = market.calls, market.puts = market.puts, strikes = strikes,
#
                                      lambda = 1, hessian = FALSE , control=list(maxit=10000) )
```

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```
#
# > optim.obj.with.synthetic.data
#
# > theta
#
###
###
It does take a few seconds but the optim converges to exact theta values.
###
```

mln.objective

Objective function for the Mixture of Lognormal

Description

mln.objective is the objective function to be minimized in extract.mln.density.

Usage

```
mln.objective(theta, r, y, te, s0, market.calls, call.strikes, call.weights,
    market.puts, put.strikes, put.weights, lambda = 1)
```

Arguments

theta	initial values for the optimization
r	risk free rate
у	dividend yield
te	time to expiration
s0	current asset value
market.calls	market calls (most expensive to cheapest)
call.strikes	strikes for the calls (smallest to largest)
call.weights	weights to be used for calls
market.puts	market calls (cheapest to most expensive)
put.strikes	strikes for the puts (smallest to largest)
put.weights	weights to be used for puts
lambda	Penalty parameter to enforce the martingale condition

Details

mln is the density f(x) = alpha.1 * g(x) + (1 - alpha.1) * h(x), where g and h are densities of two lognormals with parameters (mean.log.1, sdlog.1) and (mean.log.2, sdlog.2) respectively.

Value

obj value of the objective function

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Author(s)

Kam Hamidieh

References

F. Gianluca and A. Roncoroni (2008) Implementing Models in Quantitative Finance: Methods and Cases

B. Bahra (1996): Probability distribution of future asset prices implied by option prices. *Bank of England Quarterly Bulletin*, August 1996, 299-311

P. Soderlind and L.E.O. Svensson (1997) New techniques to extract market expectations from financial instruments. *Journal of Monetary Economics*, 40, 383-429

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

```
# The mln objective function should be close to zero.
# The weights are automatically set to 1.
r = 0.05
te = 60/365
y = 0.02
meanlog.1 = 6.8
meanlog.2 = 6.95
sdlog.1 = 0.065
sdlog.2 = 0.055
alpha.1 = 0.4
# This is the current price implied by parameter values:
s0 = 981.8815
call.strikes = seq(from = 800, to = 1200, by = 10)
market.calls = price.mln.option(r=r, y = y, te = te, k = call.strikes,
               alpha.1 = alpha.1, meanlog.1 = meanlog.1, meanlog.2 = meanlog.2,
               sdlog.1 = sdlog.1, sdlog.2 = sdlog.2)$call
put.strikes = seq(from = 805, to = 1200, by = 10)
market.puts = price.mln.option(r = r, y = y, te = te, k = put.strikes,
               alpha.1 = alpha.1, meanlog.1 = meanlog.1, meanlog.2 = meanlog.2,
               sdlog.1 = sdlog.1, sdlog.2 = sdlog.2)$put
mln.objective(theta=c(alpha.1, meanlog.1, meanlog.2, sdlog.1, sdlog.2),
               r = r, y = y, te = te, s0 = s0,
              market.calls = market.calls, call.strikes = call.strikes,
              market.puts = market.puts, put.strikes = put.strikes, lambda = 1)
```

40 MOE

MOE	Mother of All Extractions	

Description

MOE function extracts the risk neutral density based on all models and summarizes the results.

Usage

```
MOE(market.calls, call.strikes, market.puts, put.strikes, call.weights = 1,
   put.weights = 1, lambda = 1, s0, r, te, y, file.name = "myfile")
```

Arguments

market.calls	market calls (most expensive to cheapest)
call.strikes	strikes for the calls (smallest to largest)
market.puts	market calls (cheapest to most expensive)
put.strikes	strikes for the puts (smallest to largest)
call.weights	Weights for the calls (must be in the same order of calls)
put.weights	Weights for the puts (must be in the same order of puts)
lambda	Penalty parameter to enforce the martingale condition
s0	Current asset value
r	risk free rate
te	time to expiration
У	dividend yield
file.name	File names where analysis is to be saved. SEE DETAILS!

Details

The MOE function in a few key strokes extracts the risk neutral density via various methods and summarizes the results.

This function should only be used for European options.

NOTE: Three files will be produced: filename will have the pdf version of the results. file.namecalls.csv will have the predicted call values. file.nameputs.csv will have the predicted put values.

Value

bsm.mu	mean of $log(S(1))$, when $S(1)$ is lognormal
bsm.sigma	SD of $log(S(T))$, when $S(T)$ is lognormal
gb.a	extracted power parameter, when S(T) is assumed to be a GB rv
gb.b	extracted scale paramter, when S(T) is assumed to be a GB rv
gb.v	extracted first beta paramter, when S(T) is assumed to be a GB rv

MOE 41

gb.w	extracted second beta parameter, when S(T) is assumed to be a GB rv
mln.alpha.1	extracted proportion of the first lognormal. Second one is 1 - alpha.1 in mixture of lognormals
mln.meanlog.1	extracted mean of the log of the first lognormal in mixture of lognormals
mln.meanlog.2	extracted mean of the log of the second lognormal in mixture of lognormals
mln.sdlog.1	extracted standard deviation of the log of the first lognormal in mixture of lognormals
mln.sdlog.2	extracted standard deviation of the log of the second lognormal in mixture of lognormals
ew.sigma	volatility when using the Edgeworth expansions
ew.skew	normalized skewness when using the Edgeworth expansions
ew.kurt	normalized kurtosis when using the Edgeworth expansions
a0	extracted constant term in the quadratic polynomial of Shimko method
a1	extracted coefficient term of k in the quadratic polynomial of Shimko method
a2	extracted coefficient term of k squared in the quadratic polynomial of Shimko method

Author(s)

Kam Hamidieh

References

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

42 oil.2012.10.01

```
### See where your results will go...
getwd()
###
###
    Running this may take 1-2 minutes...
###
### MOE(market.calls = calls, call.strikes = strikes, market.puts = puts,
       put.strikes = strikes, call.weights = 1, put.weights = 1,
###
       lambda = 1, s0 = s0, r = r, te = te, y = y, file.name = "myfile")
###
###
### You may get some warning messages. This happens because the
       automatic initial value selection sometimes picks values
###
###
       that produce NaNs in the generalized beta density estimation.
###
       These messages are often inconsequential.
###
```

oil.2012.10.01

West Texas Intermediate Crude Oil Options on 2013-10-01

Description

This dataset contains West Texas Intermediate (WTI) crude oil options with 43 days to expiration at the end of the business day October 1, 2012. On October 1, 2012, WTI closed at 92.44.

Usage

```
data(oil.2012.10.01)
```

Format

A data frame with 332 observations on the following 7 variables.

```
type a factor with levels C for call option P for put option
strike option strike
settlement option settlement price
openint option open interest
volume trading volume
delta option delta
impliedvolatility option implied volatility
```

Source

CME posts sample data at: http://www.cmegroup.com/market-data/datamine-historical-data/endofday.html

```
data(oil.2012.10.01)
```

pgb 43

pgb

CDF of Generalized Beta

Description

pgb is the cumulative distribution function (CDF) of a genaralized beta random variable.

Usage

```
pgb(x, a, b, v, w)
```

Arguments

X	value at which the CDF is to be evaluated
а	power parameter > 0
b	scale paramter > 0
V	first beta paramter > 0
W	second beta parameter > 0

Details

Let B be a beta random variable with parameters v and w. Then $Z = b *(B/(1-B))^{(1/a)}$ is a generalized beta random variable with parameters (a,b,v,w).

Value

out

CDF value at x

Author(s)

Kam Hamidieh

References

- R.M. Bookstaber and J.B. McDonald (1987) A general distribution for describing security price returns. *Journal of Business*, 60, 401-424
- X. Liu and M.B. Shackleton and S.J. Taylor and X. Xu (2007) Closed-form transformations from risk-neutral to real-world distributions *Journal of Business*, 60, 401-424
- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

price.am.option

Examples

```
#
# What does the cdf of a GB look like?
#

a = 1
b = 10
v = 2
w = 2

x = seq(from = 0, to = 500, by = 0.01)
y = pgb(x = x, a = a, b = b, v = v, w = w)
plot(y ~ x, type = "1")
abline(h=c(0,1), lty=2)
```

price.am.option

Price American Options on Mixtures of Lognormals

Description

price.am.option gives the price of a call and a put option at a set strike when the risk neutral density is a mixture of three lognormals.

Usage

```
price.am.option(k, r, te, w, u.1, u.2, u.3, sigma.1, sigma.2, sigma.3, p.1, p.2)
```

Arguments

k	Strike
r	risk free rate
te	time to expiration
W	Weight, a number between 0 and 1, to weigh the option price bounds
u.1	log mean of the first lognormal
u.2	log mean of the second lognoral
u.3	log mean of the second lognoral
sigma.1	log sd of the first lognormal
sigma.2	log mean of the second lognormal
sigma.3	log mean of the third lognormal
p.1	weight assigned to the first density
p.2	weight assigned to the second density

price.am.option 45

Details

mln is density f(x) = p.1 * f1(x) + p.2 * f2(x) + (1 - p.1 - p.2) * f3(x), where f1, f2, and f3 are lognormal densities with log means u.1,u.2, and u.3 and standard deviations sigma.1, sigma.2, and sigma.3 respectively.

Note: Different weight values, w, need to be assigned to whether the call or put is in the money or not. See equations (7) & (8) of Melick and Thomas paper below.

Value

```
call.value American call value

put.value American put value

expected.f0 Expected mean value of asset at expiration

prob.f0.gr.k Probability asset values is greater than strike

prob.f0.ls.k Probability asset value is less than strike

expected.f0.f0.gr.k

Expected value of asset given asset exceeds strike

expected.f0.f0.ls.k

Expected value of asset given asset is less than strike
```

Author(s)

Kam Hamidieh

References

Melick, W. R. and Thomas, C. P. (1997). Recovering an asset's implied pdf from option prices: An application to crude oil during the gulf crisis. *Journal of Financial and Quantitative Analysis*, 32(1), 91-115.

```
###
### Set a few parameters and create some
### American options.
###
        = 0.01
       = 60/365
te
       = 0.4
w.1
w.2
       = 0.25
u.1
        = 4.2
u.2
        = 4.5
u.3
       = 4.8
sigma.1 = 0.30
sigma.2 = 0.20
sigma.3 = 0.15
       = 0.25
p.1
```

46 price.bsm.option

```
p.2
theta
                             = c(w.1, w.2, u.1, u.2, u.3, sigma.1, sigma.2, sigma.3, p.1, p.2)
p.3 = 1 - p.1 - p.2
p.3
expected.f0 = sum(c(p.1, p.2, p.3) * exp(c(u.1,u.2,u.3) +
                                                                                 (c(sigma.1, sigma.2, sigma.3)^2)/2) )
expected.f0
strikes = 30:170
market.calls = numeric(length(strikes))
market.puts = numeric(length(strikes))
for (i in 1:length(strikes))
        if ( strikes[i] < expected.f0) {</pre>
               market.calls[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.1, u.1 = u.1, 
                                                                                        u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                                                                         sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$call.value
               market.puts[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.2, u.1 = u.1, respectively)
                                                                                        u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                                                                         sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$put.value
        } else {
               market.calls[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.2, u.1 = u.1, respectively)
                                                                                        u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                                                                         sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$call.value
               market.puts[i] = price.am.option(k = strikes[i], r = r, te = te, w = w.1, u.1 = u.1, u.1, u.1 = u
                                                                                        u.2 = u.2, u.3 = u.3, sigma.1 = sigma.1, sigma.2 = sigma.2,
                                                                                         sigma.3 = sigma.3, p.1 = p.1, p.2 = p.2)$put.value
                    }
}
###
 ### Quickly look at the option values...
 ###
par(mfrow=c(1,2))
plot(market.calls ~ strikes, type="1")
plot(market.puts ~ strikes, type="l")
par(mfrow=c(1,1))
```

price.bsm.option 47

Description

bsm.option.price computes the BSM European option prices.

Usage

```
price.bsm.option(s0, k, r, te, sigma, y)
```

Arguments

s0	current asset value
k	strike
r	risk free rate
te	time to expiration
sigma	volatility
у	dividend yield

Details

This function implements the classic Black-Scholes-Merton option pricing model.

Value

Author(s)

Kam Hamidieh

References

- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London
- J. Hull (2011) *Options, Futures, and Other Derivatives and DerivaGem Package* Prentice Hall, Englewood Cliffs, New Jersey, 8th Edition
- R. L. McDonald (2013) Derivatives Markets Pearson, Upper Saddle River, New Jersey, 3rd Edition

```
#
# call should be 4.76, put should be 0.81, from Hull 8th, page 315, 316
#

r = 0.10
te = 0.50
```

48 price.ew.option

```
s0 = 42
k = 40
sigma = 0.20
y = 0

bsm.option = price.bsm.option(r = r, te = te, s0 = s0, k = k, sigma = sigma, y = y)
bsm.option

# 
# Make sure put-call parity holds, Hull 8th, page 351
#

(bsm.option$call - bsm.option$put) - (s0 * exp(-y*te) - k * exp(-r*te))
```

price.ew.option

Price Options with Edgeworth Approximated Density

Description

price.ew.option computes the option prices based on Edgeworth approximated densities.

Usage

```
price.ew.option(r, te, s0, k, sigma, y, skew, kurt)
```

Arguments

risk free rate r time to expiration te current asset value s0 strike k volatility sigma dividend rate У normalized skewness skew normalized kurtosis kurt

Details

Note that this function may produce negative prices if skew and kurt are not well estimated from the data.

Value

call Edgeworth based call put Edgeworth based put

price.ew.option 49

Author(s)

Kam Hamidieh

References

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

R. Jarrow and A. Rudd (1982) Approximate valuation for arbitrary stochastic processes. *Journal of Financial Economics*, 10, 347-369

C.J. Corrado and T. Su (1996) S&P 500 index option tests of Jarrow and Rudd's approximate option valuation formula. *Journal of Futures Markets*, 6, 611-629

```
#
# Here, the prices must match EXACTLY the BSM prices:
       = 0.05
r
       = 0.03
s0
       = 1000
sigma = 0.25
       = 100/365
       = seq(from=800, to = 1200, by = 50)
       = sqrt(exp(sigma^2 * te) - 1)
ln.skew = 3 * v + v^3
ln.kurt = 16 * v^2 + 15 * v^4 + 6 * v^6 + v^8
ew.option.prices = price.ew.option(r = r, te = te, s0 = s0, k=k, sigma=sigma,
                                     y=y, skew = ln.skew, kurt = ln.kurt)
bsm.option.prices = price.bsm.option(r = r, te = te, s0 = s0, k=k, sigma=sigma, y=y)
ew.option.prices
bsm.option.prices
###
### Now ew prices should be different as we increase the skewness and kurtosis:
new.skew = ln.skew * 1.10
new.kurt = ln.kurt * 1.10
new.ew.option.prices = price.ew.option(r = r, te = te, s0 = s0, k=k, sigma=sigma,
                                        y=y, skew = new.skew, kurt = new.kurt)
new.ew.option.prices
bsm.option.prices
```

price.gb.option

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Generalized Beta Option Pricing

Description

price.gb.option computes the price of options.

Usage

```
price.gb.option(r, te, s0, k, y, a, b, v, w)
```

Arguments

r	risk free interest rate
te	time to expiration
s0	current asset value
k	strike
У	dividend yield
a	power parameter > 0
b	scale paramter > 0
V	first beta paramter > 0
W	second beta parameter > 0

Details

This function is used to compute European option prices when the underlying has a generalized beta (GB) distribution. Let B be a beta random variable with parameters v and w. Then $Z = b *(B/(1-B))^{(1/a)}$ is a generalized beta random variable with parameters with (a,b,v,w).

Value

prob.1	Probability that a GB random variable with parameters (a,b,v+1/a,w-1/a) will be above the strike
prob.2	Probability that a GB random variable with parameters (a,b,v,w) will be above the strike
call	call price
put	put price

Author(s)

Kam Hamidieh

price.mln.option 51

References

R.M. Bookstaber and J.B. McDonald (1987) A general distribution for describing security price returns. *Journal of Business*, 60, 401-424

X. Liu and M.B. Shackleton and S.J. Taylor and X. Xu (2007) Closed-form transformations from risk-neutral to real-world distributions *Journal of Business*, 60, 401-424

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

Examples

```
# # A basic GB option pricing....
#

r = 0.03
te = 50/365
s0 = 1000.086
k = seq(from = 800, to = 1200, by = 10)
y = 0.01
a = 10
b = 1000
v = 2.85
w = 2.85
price.gb.option(r = r, te = te, s0 = s0, k = k, y = y, a = a, b = b, v = v, w = w)
```

price.mln.option

Price Options on Mixture of Lognormals

Description

mln.option.price gives the price of a call and a put option at a strike when the risk neutral density is a mixture of two lognormals.

Usage

```
price.mln.option(r, te, y, k, alpha.1, meanlog.1, meanlog.2, sdlog.1, sdlog.2)
```

Arguments

```
r risk free rate
te time to expiration
y dividend yield
k strike
```

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alpha.1	proportion of the first lognormal. Second one is 1 - alpha.1
meanlog.1	mean of the log of the first lognormal
meanlog.2	mean of the log of the second lognormal
sdlog.1	standard deviation of the log of the first lognormal
sdlog.2	standard deviation of the log of the second lognormal

Details

mln is the density f(x) = alpha.1 * g(x) + (1 - alpha.1) * h(x), where g and h are densities of two lognormals with parameters (mean.log.1, sdlog.1) and (mean.log.2, sdlog.2) respectively.

Value

call	call price
put	put price
s0	current value of the asset as implied by the mixture distribution

Author(s)

Kam Hamidieh

References

- F. Gianluca and A. Roncoroni (2008) Implementing Models in Quantitative Finance: Methods and Cases
- B. Bahra (1996): Probability distribution of future asset prices implied by option prices. *Bank of England Quarterly Bulletin*, August 1996, 299-311
- P. Soderlind and L.E.O. Svensson (1997) New techniques to extract market expectations from financial instruments. *Journal of Monetary Economics*, 40, 383-429
- E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

```
#
# Try out a range of options
#

r = 0.05
te = 60/365
k = 700:1300
y = 0.02
meanlog.1 = 6.80
meanlog.2 = 6.95
sdlog.1 = 0.065
sdlog.2 = 0.055
alpha.1 = 0.4
```

price.shimko.option 53

```
mln.prices = price.mln.option(r = r, y = y, te = te, k = k, alpha.1 = alpha.1,
    meanlog.1 = meanlog.1, meanlog.2 = meanlog.2, sdlog.1 = sdlog.1, sdlog.2 = sdlog.2)

par(mfrow=c(1,2))
plot(mln.prices$call ~ k)
plot(mln.prices$put ~ k)
par(mfrow=c(1,1))
```

price.shimko.option

Price Option based on Shimko's Method

Description

price. shimko. option prices a European option based on the extracted Shimko volatility function.

Usage

```
price.shimko.option(r, te, s0, k, y, a0, a1, a2)
```

Arguments

r	risk free rate
te	time to expiration
s0	current asset value
k	strike
У	dividend yield
a0	constant term in the quadratic polyynomial
a1	coefficient term of k in the quadratic polynomial
a2	coefficient term of k squared in the quadratic polynomial

Details

This function may produce negative option values when nonsensical values are used for a0, a1, and a2.

Value

```
call call price put put price
```

Author(s)

Kam Hamidieh

54 sp500.2013.04.19

References

D. Shimko (1993) Bounds of probability. Risk, 6, 33-47

E. Jondeau and S. Poon and M. Rockinger (2007): Financial Modeling Under Non-Gaussian Distributions Springer-Verlag, London

Examples

```
r
       = 0.05
       = 0.02
       = 60/365
       = 1000
       = 950
sigma = 0.25
a0
       = 0.30
a1
       = -0.00387
a2
       = 0.00000445
# Note how Shimko price is the same when a0 = sigma, a1=a2=0 but substantially
  more when a0, a1, a2 are changed so the implied volatilies are very high!
#
price.bsm.option(r = r, te = te, s0 = s0, k = k, sigma = sigma, y = y)$call
price.shimko.option(r = r, te = te, s0 = s0, k = k, y = y,
                   a0 = sigma, a1 = 0, a2 = 0)$call
price.shimko.option(r = r, te = te, s0 = s0, k = k, y = y,
                   a0 = a0, a1 = a1, a2 = a2)$call
```

sp500.2013.04.19

S&P 500 Index Options on 2013-04-19

Description

This dataset contains S&P 500 options with 62 days to expiration at the end of the business day April 19, 2013. On April 19, 2013, S&P 500 closed at 1555.25.

Usage

```
data(sp500.2013.04.19)
```

Format

A data frame with 171 observations on the following 19 variables.

```
bidsize.c call bid size
bid.c call bid price
ask.c call ask price
```

sp500.2013.06.24 55

```
asksize.c call ask size
chg.c change in call price
impvol.c call implied volatility
vol.c call volume
openint.c call open interest
delta.c call delta
strike option strike
bidsize.p put bid size
bid.p put bid price
ask.p put ask price
asksize.p put ask size
chg.p change in put price
impvol.p put implied volatility
vol.p put volume
openint.p put open interest
delta.p put delta
```

Source

http://www.cboe.com/DelayedQuote/QuoteTableDownload.aspx

Examples

```
data(sp500.2013.04.19)
```

sp500.2013.06.24

S&P 500 Index Options on 2013-06-24

Description

This dataset contains S&P 500 options with 53 days to expiration at the end of the business day June 24, 2013. On June 24, 2013, S&P 500 closed at 1573.09.

Usage

```
data(sp500.2013.06.24)
```

56 vix.2013.06.25

Format

```
A data frame with 173 observations on the following 9 variables.
```

```
bid.c call bid price
ask.c call ask price
vol.c call volume
openint.c call open interest
strike option strike
bid.p put bid price
ask.p put ask price
vol.p put volume
openint.p put open interest
```

Source

http://www.cboe.com/DelayedQuote/QuoteTableDownload.aspx

Examples

```
data(sp500.2013.06.24)
```

vix.2013.06.25

VIX Options on 2013-06-25

Description

This dataset contains VIX options with 57 days to expiration at the end of the business day June 25, 2013. On June 25, 2013, VIX closed at 18.21.

Usage

```
data(vix.2013.06.25)
```

Format

A data frame with 35 observations on the following 13 variables.

```
last.c closing call price
change.c change in call price from previous day
bid.c call bid price
ask.c call ask price
vol.c call volume
openint.c call open interest
strike option strike
```

vix.2013.06.25

```
last.p closing put price
change.p change in put price from previous day
bid.p put bid price
ask.p put ask price
vol.p put volume
openint.p put open interest
```

Source

http://www.cboe.com/DelayedQuote/QuoteTableDownload.aspx

```
data(vix.2013.06.25)
```

Index

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