# Package 'calibrator'

October 12, 2022

Type Package
Title Bayesian Calibration of Complex Computer Codes
Version 1.2-8
<b>Depends</b> R (>= 2.0.0), emulator (>= 1.2-11), mvtnorm
Imports cubature
Maintainer Robin K. S. Hankin <a href="mailto:robin@gmail.com">hankin.robin@gmail.com</a>
<b>Description</b> Performs Bayesian calibration of computer models as per Kennedy and O'Hagan 2001. The package includes routines to find the hyperparameters and parameters; see the help page for stage1() for a worked example using the toy dataset. A tutorial is provided in the calex.Rnw vignette; and a suite of especially simple one dimensional examples appears in inst/doc/one.dim/.
License GPL-2
<pre>URL https://github.com/RobinHankin/calibrator.git</pre>
BugReports https://github.com/RobinHankin/calibrator/issues
NeedsCompilation no
<b>Author</b> Robin K. S. Hankin [aut, cre] ( <a href="https://orcid.org/0000-0001-5982-0415">https://orcid.org/0000-0001-5982-0415</a> )
Repository CRAN
<b>Date/Publication</b> 2019-03-07 06:52:58 UTC
R topics documented:
calibrator-package       1         beta1hat.fun       4         beta2hat.fun       6         betahat.fun.koh       7         blockdiag       9         C1       10         cov.p5.supp       1         create.new.toy.datasets       1
D1.fun

2 calibrator-package

•	W2	•		•	
	W1				
	W				
	Vd				
	V2				
	V1				
	V.fun				
	tt.fun				
i	toys				
1	tee				
	symmetrize				
	stagel				
	reality				
	prob.psi1				
	phi.fun.toy				
1	p.page4				
	p.eqn8.supp				
1	p.eqn4.supp				
	$\hat{ ext{MH}}$				
	is.positive.definite				
	hbar.fun.toy				
	h1.toy				
	H1.toy				
	H.fun				
	Ez.eqn9.supp				
	Ez.egn7.supp				
	extractor.toy		•	•	
	etahat		•	•	
	E.theta.toy			•	
	D2.fun				

calibrator-package

Bayesian Calibration of Complex Computer Codes

# Description

Performs Bayesian calibration of computer models as per Kennedy and O'Hagan 2001. The package includes routines to find the hyperparameters and parameters; see the help page for stage1() for a worked example using the toy dataset. A tutorial is provided in the calex.Rnw vignette; and a suite of especially simple one dimensional examples appears in inst/doc/one.dim/.

# **Details**

The DESCRIPTION file:

calibrator-package 3

Package: calibrator Type: Package

Title: Bayesian Calibration of Complex Computer Codes

Version: 1.2-8

Authors@R: person(given=c("Robin", "K. S."), family="Hankin", role = c("aut", "cre"), email="hankin.robin@gmail.com",

Depends: R (>= 2.0.0), emulator (>= 1.2-11), mvtnorm

Imports: cubature

Maintainer: Robin K. S. Hankin <a href="mailto:knakin.robin@gmail.com">hankin.robin@gmail.com</a>

Description: Performs Bayesian calibration of computer models as per Kennedy and O'Hagan 2001. The package includes

License: GPL-2

URL: https://github.com/RobinHankin/calibrator.git
BugReports: https://github.com/RobinHankin/calibrator/issues

Author: Robin K. S. Hankin [aut, cre] (<a href="https://orcid.org/0000-0001-5982-0415">https://orcid.org/0000-0001-5982-0415</a>)

# Index of help topics:

C1 Matrix of distances from D1 to D2

D1.fun Function to join x.star to t.vec to give matrix

D1

D2.fun Augments observation points with parameters E.theta.toy Expectation and variance with respect to theta

EK.eqn10.supp Posterior mean of K

Ez.eqn7.supp Expectation of z given y, beta2, phi
Ez.eqn9.supp Expectation as per equation 10 of KOH2001

H.fun H function

H1.toy Basis functions for D1 and D2
MH Very basic implementation of the Metropolis-Hastings algorithm

V.fun Variance matrix for observations

V1 Distance matrix

V2 distance between observation points

Vd Variance matrix for d
W covariance matrix for beta
W1 Variance matrix for beta1hat
W2 variance matrix for beta2

beta1hat.fun beta1 estimator beta2hat.fun estimator for beta2

betahat.fun.koh Expectation of beta, given theta, phi and d blockdiag Assembles matrices blockwise into a block

diagonal matrix

calibrator-package Bayesian Calibration of Complex Computer Codes cov.p5.supp Covariance function for posterior distribution

 $\quad \text{of } z$ 

create.new.toy.datasets

Create new toy datasets

dists.2frames Distance between two points etahat Expectation of computer output

4 beta1hat.fun

extractor.toy Extracts lat/long matrix and theta matrix from

D2.

h1.toy Basis functions

hbar.fun.toy
is.positive.definite
p.eqn4.supp

Toy example of hbar (section 4.2)
Is a matrix positive definite?
Apostiori probability of psi1

reality Reality

stage1 Stage 1,2 and 3 optimization on toy dataset symmetrize Symmetrize an upper triangular matrix tee Auxiliary functions for equation 9 of the

supplement

toys Toy datasets

tt.fun Integrals needed in KOH2001

Further information is available in the following vignettes:

calex Calex: a cookbook for the emulator package (source)

# Author(s)

NA

Maintainer: Robin K. S. Hankin <a href="mailto:rhankin.robin@gmail.com">hankin.robin@gmail.com</a>

### References

- M. C. Kennedy and A. O'Hagan 2001. "Bayesian calibration of computer models". Journal of the Royal Statistical Society, Series B, 63(3): 425–464
- R. K. S. Hankin 2005. "Introducing BACCO, an R bundle for Bayesian analysis of computer code output", Journal of Statistical Software, 14(16)

beta1hat.fun beta1 estimator

### **Description**

Least squares estimator for beta1

# Usage

```
beta1hat.fun(D1, H1, y, phi)
```

beta1hat.fun 5

# **Arguments**

D1 code run points

H1 regressor basis funs

y code outputs

phi hyperparameters

### Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
beta2hat.fun
```

```
data(toys)
y.toy <- create.new.toy.datasets(D1=D1.toy , D2=D2.toy)$y.toy
beta1hat.fun(D1=D1.toy, H1=H1.toy, y=y.toy, phi=phi.toy)

# now cheat: force the hyperparameters to have the correct psi1:
phi.fix <- phi.change(old.phi=phi.toy,psi1=c(1, 0.5, 1.0, 1.0, 0.5, 0.4),phi.fun=phi.fun.toy)

# The value for psi1 is obtained by cheating and #examining the source
# code for computer.model(); see ?phi.change

# Create a new toy dataset with 40 observations:
D1.big <- latin.hypercube(40,5)

jj <- create.new.toy.datasets(D1=D1.big , D2=D2.toy)

# We know that the real coefficients are 4:9 because we
# we can cheat and look at the source code for computer.model()

# Now estimate the coefficients without cheating:
beta1hat.fun(D1=D1.big, H1=H1.toy, jj$y, phi=phi.toy)

# Not bad!</pre>
```

6 beta2hat.fun

```
# We can do slightly better by cheating and using the
# correct value for the hyperparameters:
beta1hat.fun(D1=D1.big, H1=H1.toy, jj$y,phi=phi.true.toy(phi=phi.toy))
#marginally worse.
```

beta2hat.fun

estimator for beta2

# Description

estimates beta2 as per the equation of page 4 of the supplement. Used by p.page4()

# Usage

```
beta2hat.fun(D1, D2, H1, H2, V, z, etahat.d2, extractor, E.theta, Edash.theta, phi)
```

# Arguments

D1	Matrix of code run points
D2	Matrix of observation points
H1	regression basis functions
H2	regression basis functions
V	overall covariance matrix
z	vector of observations

etahat.d2 expectation as per etahat.vector

extractor extractor function

E.theta Expectation

Edash.theta Expectation wrt thetadash

phi hyperparameters

# Author(s)

Robin K. S. Hankin

betahat.fun.koh 7

#### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

W2

```
data(toys)
etahat.d2 <- etahat(D1=D1.toy, D2=D2.toy, H1=H1.toy, y=y.toy,
E.theta=E.theta.toy, extractor=extractor.toy, phi=phi.toy)
beta2hat.fun(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy, V=NULL,
z=z.toy, etahat.d2=etahat.d2, extractor=extractor.toy,
E.theta=E.theta.toy, Edash.theta=Edash.theta.toy, phi=phi.toy)
jj <- create.new.toy.datasets(D1.toy , D2.toy)</pre>
phi.true <- phi.true.toy(phi=phi.toy)</pre>
y.toy <- jj$y.toy</pre>
z.toy <- jj$z.toy</pre>
d.toy <- jj$d.toy</pre>
etahat.d2 <- etahat(D1=D1.toy, D2=D2.toy, H1=H1.toy, y=y.toy,
E.theta=E.theta.toy, extractor=extractor.toy, phi=phi.toy)
beta2hat <- beta2hat.fun(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy, V=NULL,
z=z.toy, etahat.d2=etahat.d2, extractor=extractor.toy,
E.theta=E.theta.toy, Edash.theta=Edash.theta.toy,
phi=phi.toy)
print(beta2hat)
plot(z.toy , H2.toy(D2.toy) %*% beta2hat)
```

8 betahat.fun.koh

# **Description**

Determines the mean of  $\beta$ , given parameters  $\theta$ , hyperparameters  $\phi$ , and the vector of code outputs and observations d. It is named so as to avoid conflict with function betahat.fun of package **emulator**.

#### Usage

```
betahat.fun.koh(D1, D2, H1, H2, theta, d, phi)
betahat.fun.koh.vector(D1, D2, H1, H2, theta, d, phi)
```

# **Arguments**

D1	Matrix whose rows are observation points and parameter values at which the code has been run
D2	Matrix whose rows are the observation points
H1	Regression function for D1
H2	Regression function for D2
theta	Parameters
d	Vector of code outputs and observations
phi	Hyperparameters

#### **Details**

This function is defined between equations 2 and 3 of the supplement. It is used in functions Ez.eqn9.supp() and p.eqn8.supp().

The user should always use betahat.fun.koh(), which is a wrapper for betahat.fun.koh.vector(). The forms differ in their treatment of  $\theta$ . In the former,  $\theta$  must be a vector; in the latter,  $\theta$  may be a matrix, in which case betahat.fun.koh.vector() is applied to the rows.

In betahat.fun.koh(), the rownames are assigned by a kludgy call to H.fun(), which itself uses a kludge to determine colnames.

The function returns

$$\hat{\beta}(\theta) = \mathbf{W}(\theta)^T \mathbf{H}(\theta)^T \mathbf{V}_d(\theta)^{-1} \mathbf{d}.$$

# Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

blockdiag 9

### **Examples**

```
betahat.fun.koh(theta=theta.toy, d=d.toy, D1=D1.toy, D2=D2.toy,
      H1=H1.toy, H2=H2.toy, phi=phi.toy)
betahat.fun.koh.vector(theta=theta.toy, d=d.toy, D1=D1.toy,
      D2=D2.toy, H1=H1.toy, H2=H2.toy, phi=phi.toy)
## should be identical
jj.theta <- rbind(theta.toy,theta.toy+1,theta.toy+2,theta.toy*0)</pre>
betahat.fun.koh(theta=jj.theta, d=d.toy, D1=D1.toy, D2=D2.toy,
     H1=H1.toy, H2=H2.toy, phi=phi.toy)
## Now try with true hyperparameters:
phi.true <- phi.true.toy(phi=phi.toy)</pre>
## And magically create the REAL parameters:
theta.REAL <- create.new.toy.datasets(export=TRUE)$REAL.PARAMS
jj.theta <- rbind(jj.theta, theta.REAL)</pre>
## Generate some data:
jj <- create.new.toy.datasets(D1.toy , D2.toy)</pre>
d.toy <- jj$d.toy</pre>
## And finally, observe that the estimated values for beta are pretty
## close to the real values (which omniscient beings can extract using
## reality() and computer.model()):
betahat.fun.koh(theta=jj.theta, d=d.toy, D1=D1.toy, D2=D2.toy,
       H1=H1.toy, H2=H2.toy, phi=phi.true)
## [
## that is, compare the last column of the above with
## c(computer.model(ex=T)$REAL.COEFFS, reality(ex=T)$REAL.BETA2)
```

blockdiag

Assembles matrices blockwise into a block diagonal matrix

# **Description**

Assembles matrices blockwise into a block diagonal matrix with optional padding value

# Usage

```
blockdiag(m1, m2, p.tr = 0, p.ll = 0)
```

10 C1

# Arguments

m1	Upper left matrix
m2	Lower right matrix
p.tr	Padding value for top right quadrant. Defaults to zero.
p.11	Padding value for lower left quadrant. Defaults to zero.

# Note

The function documented here is a subset of adiag of package magic

# Author(s)

Robin K. S. Hankin

# **Examples**

```
data(toys)
blockdiag(D1.toy,D2.toy)
```

C1

Matrix of distances from D1 to D2

# Description

Returns a matrix of distances from the code run points to the augmented observation points. A wrapper for c1.fun().

# Usage

```
C1(D1, D2, theta, phi)
```

# Arguments

D1D1D2D2

theta Parameters

phi Hyperparameters

# Author(s)

Robin K. S. Hankin

cov.p5.supp

### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

### See Also

t.fun

### **Examples**

```
data(toys)
C1(D1=D1.toy, D2=D2.toy, theta=theta.toy, phi=phi.toy)
```

cov.p5.supp

Covariance function for posterior distribution of z

# Description

Covariance function for posterior distribution of  $z(\cdot)$  conditional on estimated hyperparameters and calibration parameters  $\theta$ .

### Usage

```
Cov.eqn9.supp(x, xdash=NULL, theta, d, D1, D2, H1, H2, phi) cov.p5.supp (x, xdash=NULL, theta, d, D1, D2, H1, H2, phi)
```

# Arguments

X	first point, or (Cov.eqn9.supp()) a matrix whose rows are the points of interest
xdash	The second point, or (Cov.eqn9.supp()) a matrix whose rows are the points of interest. The default of NULL means to use xdash=x
theta	Parameters. For Cov.eqn9.supp(), supply a vector which will be interpreted as a single point in parameter space. For cov.p5.supp(), supply a matrix whose rows will be interpreted as points in parameter space
d	Observed values
D1	Code run design matrix
D2	Observation points of real process
H1	Basis function for D1
H2	Basis function for D2
phi	Hyperparameters

12 cov.p5.supp

#### **Details**

Evaluates the covariance function: the last formula on page 5 of the supplement. The two functions documented here are vectorized differently.

Function Cov.eqn9.supp() takes matrices for arguments x and xdash and a single vector for theta. Evaluation is thus taken at a single, fixed value of theta. The function returns a matrix whose rows correspond to rows of x and whose columns correspond to rows of xdash.

Function cov.p5.supp() takes a vector for arguments x and xdash and a matrix for argument theta whose rows are the points in parameter space. A vector V, with elements corresponding to the rows of argument theta is returned:

$$V[i] = \operatorname{cov}(z(x), z(x')|\theta_i)$$

### Value

Returns a matrix of covariances

#### Note

May return the transpose of the desired object

#### Author(s)

Robin K. S. Hankin

# References

- M. C. Kennedy and A. O'Hagan 2001. Bayesian calibration of computer models. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

create.new.toy.datasets 13

```
# Now try a sequence of thetas:
cov.p5.supp(x=x.toy,theta=t.vec.toy,d=d.toy,D1=D1.toy,D2=D2.toy,
H1=H1.toy,H2=H2.toy,phi=phi.toy)
```

create.new.toy.datasets

Create new toy datasets

# **Description**

Creates new toy datasets, by sampling from an explicitly specified multivariate Gaussian distribution whose covariance matrix is that required for a Gaussian process.

# Usage

```
create.new.toy.datasets(D1,D2,export=FALSE)
```

# Arguments

export Boolean, with default FALSE meaning to return toy datasets and TRUE meaning

to return, instead, a list of the true values of the parameters

D1; set of code run points

D2; set of field observation points

# Value

Returns a list of three elements:

y.toy

z.toy

d.toy

#### Note

Because function create.new.toy.datasets() calls computer.model() and model.inadequacy(), the datasets returned are drawn from a multivariate Gaussian distribution which **is** a Gaussian process

14 D1.fun

### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# See Also

```
toys, reality, latin.hypercube
```

# Examples

```
data(toys)
create.new.toy.datasets(D1=D1.toy , D2=D2.toy)
```

D1.fun

Function to join x.star to t.vec to give matrix D1

# **Description**

Function to join x. star to t. vec to give matrix D1 with correct row- and column- names.

#### **Usage**

```
D1.fun(x.star, t.vec)
```

### **Arguments**

x.star Matrix of code run points

t.vec Matrix of parameter theta values

### **Details**

Note that the matrix returned is a D1 matrix: it is a design matrix for code observations as it contains both x and theta

#### Author(s)

Robin K. S. Hankin

D2.fun 15

### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

### See Also

toys

# **Examples**

```
data(toys)
jj <- extractor.toy(D1.toy)
x.star.toy <- jj$x.star
t.vec.toy <- jj$t.vec
D1.fun(x.star.toy , t.vec.toy) # both dataframes
D1.fun(x.star.toy , theta.toy) # one dataframe, one vector
D1.fun(x.toy , t.vec.toy) # one vector, one dataframe
D1.fun(x.toy,theta.toy) # two vectors</pre>
```

D2.fun

Augments observation points with parameters

# **Description**

Augments observation points with parameters; will recycle if necessary

# Usage

```
D2.fun(D2, theta)
```

# Arguments

D2 Observation points

theta Parameters

#### Author(s)

Robin K. S. Hankin

16 dists.2frames

#### References

M. C. Kennedy and A. O'Hagan 2001. "Bayesian calibration of computer models". Journal of the Royal Statistical Society B, 63(3) pp425-464

M. C. Kennedy and A. O'Hagan 2001. "Supplementary details on Bayesian calibration of computer models", Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps

R. K. S. Hankin 2005. "Introducing BACCO, an R bundle for Bayesian analysis of computer code output", Journal of Statistical Software, 14(16)

#### See Also

```
D1.toy, theta.toy
```

# **Examples**

```
data(toys)
D2.fun(D2=D2.toy, theta=theta.toy)
D2.fun(D2=t(x.toy), theta=theta.toy)
D2.fun(D2=D2.toy[1,,drop=FALSE], theta=theta.toy)
```

dists.2frames

Distance between two points

# **Description**

Distance between points specified by rows of two matrices, according to a positive definite matrix. If not specified, the second matrix used is the first.

# Usage

```
dists.2frames(a, b=NULL, A=NULL, A.lower=NULL, test.for.symmetry=TRUE)
```

# **Arguments**

a First dataframe whose rows are the points

b Second dataframe whose rows are the points; if NULL, use a

A Positive definite matrix; if NULL, a value for A. lower is needed. If a value for A

is supplied, use a clear but possibly slower method

A. lower The lower triangular Cholesky decomposition of A (only needed if A is NULL).

If a value for A. lower is specified, this means that a relatively opaque but possibly faster method will be used. The time saving ought to be negligible unless nrow(a) (or nrow(b) if supplied), is huge. **Note that this option does not test** 

for symmetry of matrix A

test.for.symmetry

Boolean, with default TRUE meaning to calculate all element arrays (elegantly), and FALSE meaning to calculate only the upper triangular elements (using loops), which ought to be faster. The value of this argument should not affect the returned value, up to numerical accuracy

E.theta.toy

### Author(s)

Robin K. S. Hankin

#### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
dists.2frames
```

# **Examples**

```
data(toys)
dists.2frames(a=D2.toy,A=diag(2))
A <- diag(2) + matrix(0.2,2,2)
A.lower <- t(chol(A))
jj.1 <- dists.2frames(a=D2.toy, A=A, test=TRUE)
jj.2 <- dists.2frames(a=D2.toy, A=A, test=FALSE)

jj.3 <- dists.2frames(a=D2.toy, A.lower=A.lower, test=FALSE)

jj.4 <- dists.2frames(a=D2.toy, A.lower=A.lower, test=TRUE)</pre>
```

E.theta.toy

Expectation and variance with respect to theta

# Description

Function E.theta.toy returns expectation of  $H_1(D)$  with respect to  $\theta$ ; Edash.theta.toy returns expectation with respect to E'. Function E.theta.toy also returns information about nonlinear behaviour of h1(x,theta).

### Usage

```
E.theta.toy(D2=NULL, H1=NULL, x1=NULL, x2=NULL, phi, give.mean=TRUE) Edash.theta.toy(x, t.vec, k, H1, fast.but.opaque=FALSE, a=NULL, b=NULL, phi=NULL)
```

18 E.theta.toy

#### **Arguments**

D2	Observation points
H1	Regression function for D1
phi	hyperparameters. Default value of NULL only to be used in Edash.theta.toy() when fast.but.opaque is TRUE
X	lat/long point (for Edash.theta.toy)
t.vec	Matrix whose rows are parameter values (for Edash.theta.toy)
k	Integer specifying column (for Edash.theta.toy)
give.mean	In E. theta.toy(), Boolean, with default TRUE meaning to return the mean (expectation), and FALSE meaning to return the "variance"
fast.but.opaque	
	In Edash.theta.toy(), Boolean, with default FALSE meaning to use a slow but clear method. If TRUE, use faster code but parameters a and b must then be specified
a	Constant term, needed if fast.but.opaque is TRUE: $\left(V_{\theta}^{-1}+2\Omega_{t}\right)^{-1}V_{\theta}^{-1}m_{\theta}$ . Specifying a in advance saves execution time
b	Linear term, needed if fast.but.opaque is TRUE: $2\left(V_{\theta}^{-1}+2\Omega_{t}\right)^{-1}\Omega_{t}$ (multiplied by t[k,] in Edash.theta.toy()).
x1	In E. theta. toy(g=F,), the value of x in $h_1(x,\theta)$ . The default value is NULL because in simple cases such as that implemented here, the output is independent of x1 and x2
x2	In E. theta. toy(g=F, $\dots$ ), the value of x in $h_1(x, \theta)$

# Note

A terse discussion follows; see the calex.pdf vignette and the 1D case study in directory inst/doc/one/dim/ for more details and examples.

Function E. theta.toy(give.mean=FALSE,...) does **not** return the variance! The matrix returned is a **different size** from the variance matrix!

It returns the thing that must be added to  $crossprod(E_theta(h1(x,theta)),t(E_theta(h1(x,theta))))$  to give  $E_theta(h1(x,theta),t(h1(x,theta)))$ .

In other words, it returns  $E_{theta}(h1(x,theta).t(h1(x,theta)))$ - crossprod( $E_{theta}(h1(x,theta)),t(E_{theta}(h1(x,theta)))$ 

If the terms of h1() are of the form c(o, theta) (where o is a vector that is a function of x alone, and independent of theta), then the function will include the variance matrix, in the lower right corner (zeroes elsewhere).

Function E. theta() must be updated if h1.toy() changes: unlike E. theta() and Edash. theta(), it does not "know" where the elements that vary with theta are, nor their (possibly x-dependent) coefficients.

This form of the function requires x1 and x2 arguments, for good form's sake, even though the returned value is independent of x in the toy example. To see why it is necessary to include x, consider a simple case with  $h_1(x,\theta) = (1,x\theta)^T$ . Now  $E_{\theta}(h(x,\theta))$  is just  $(1,x\overline{\theta})^T$  but

$$E_{\theta}\left(h_1(x,\theta)h_1(x,\theta)^T\right)$$

EK.eqn10.supp

is a 2-by-2 matrix (M, say) with  $E_{\theta}(M) = h_1(x, \overline{\theta})h_1(x, \overline{\theta})^T + \text{variance terms.}$ 

$$E_{\theta} \left( \begin{array}{cc} 1 & x\theta \\ x\theta & x^2\theta^2 \end{array} \right)$$

All three functions here are intimately connected to the form of h1.toy() and changing it (or indeed H1.toy()) will usually require rewriting all three functions documented here. Look at the definition of E.theta.toy(give=F), and you will see that even changing the meat of h1.toy() from c(1,x) to c(x,1) would require a redefinition of E.theta.toy(g=F).

The only place that E. theta. toy(g=F) is used is internally in hh.fun().

### Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# See Also

toys

# **Examples**

```
data(toys)
E.theta.toy(D2=D2.toy, H1=H1.toy,phi=phi.toy)
E.theta.toy(D2=D2.toy[1,], H1=H1.toy,phi=phi.toy)
E.theta.toy(D2=x.toy, H1=H1.toy,phi=phi.toy)
Edash.theta.toy(x=x.toy,t.vec=t.vec.toy,k=1, H1=H1.toy,phi=phi.toy)
```

EK.eqn10.supp

Posterior mean of K

# Description

Estimates the posterior mean of K as per equation 10 of KOH2001S, section 4.2

# Usage

```
EK.eqn10.supp(X.dist, D1, D2, H1, H2, d, hbar.fun,
    lower.theta, upper.theta, extractor, give.info=FALSE,
    include.prior=FALSE, phi, ...)
```

20 EK.eqn10.supp

# **Arguments**

X.dist	Probability distribution of X, in the form of a two-element list. The first element is the mean (which should have name "mean"), and the second element is the variance matrix, which should be a positive definite matrix of the correct size, and have name "var"
D1	Matrix whose rows are the code run points
D2	Matrix whose rows are field observation points
H1	Regression function for D1
H2	Regression function for D2
d	Vector of code outputs and field observations
include.prior	Boolean; passed to function p.eqn8.supp() (qv)
hbar.fun	Function that gives expectation (with respect to X) of $h1(x, theta)$ and $h2(x)$ as per section 4.2
lower.theta	Lower integration limit for theta (NB: a vector)
upper.theta	Lower integration limit for theta (NB: a vector)
extractor	Extractor function; see extractor.toy() for an example
give.info	Boolean, with default FALSE meaning to return just the answer and TRUE to return the answer along with all output from both integrations as performed by adaptIntegrate()
phi	Hyperparameters
•••	Extra arguments passed to the integration function. If multidimensional (ie length(theta)>1), then the arguments are passed to adaptIntegrate(); if one dimensional, they are passed to integrate()

### **Details**

This function evaluates a numerical approximation to equation 10 of section 4.2 of the supplement.

Equation 10 integrates over the prior distribution of theta. If theta is a vector, multidimensional integration is necessary.

In the case of multidimensional integration, function adaptIntegrate() is used.

In the case of one dimensional integration—theta being a scalar—function integrate() of the stats package is used.

Note that equation 10 is conditional on the observed data **and** the hyperparameters

# Value

Returns a scalar

# Note

The function was not reviewed by the Journal of Statistical Software.

The package formely used adapt package, but this is no longer available on CRAN. The package now uses the cubature package.

etahat 21

### Author(s)

Robin K. S. Hankin

#### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# **Examples**

etahat

Expectation of computer output

# Description

Returns the apostiori expectation of the computer program at a particular point with a particular set of parameters, given the code output.

### Usage

```
etahat(D1, D2, H1, y, E.theta, extractor, phi)
```

# Arguments

D1	Matrix of code observation points and parameters
D2	Matrix of field observation points
H1	Basis functions
у	Code observations corresponding to rows of D1

22 etahat

E. theta expectation wrt theta; see details

extractor Extractor function

theta Parameters

phi Hyperparameters

#### **Details**

Argument E. theta is officially a function that, given x, y returns  $E_{\theta}(h_1(x,\theta))$ .

However, if supplied a non-function (this is tested by is.function() in the code), E. theta is interpreted as values of  $\theta$  to use. Recycling is carried out by function D1.fun()

# Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# See Also

```
p.page4
```

extractor.toy 23

extractor.toy

Extracts lat/long matrix and theta matrix from D2.

# **Description**

Extracts x.star.toy and t.vec.toy from D2; toy example needed because the extraction differs from case to case.

# Usage

```
extractor.toy(D1)
```

# Arguments

D1

Matrix of code run points

#### **Details**

The first two columns give the elements of x.star and columns 3 through 5 give the elements of t.vec.

Function extractor. toy is the inverse of function D1. fun, in the sense that extractor. toy splits up D1 into x.star and t.vec, while D1. fun joins them up again

#### Value

Returns a list with two elements:

x.star A matrix containing the lat/longs of the code run pointst.vec A matrix containing the parameters used for the code runs

# Author(s)

Robin K. S. Hankin

### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# See Also

```
toys, D1.fun
```

24 Ez.eqn7.supp

# **Examples**

```
data(toys)
extractor.toy(D1.toy)
extractor.toy(D1.toy[1,,drop=FALSE])
(jj <- extractor.toy(D1.fun(x.star=x.toy , t.vec=theta.toy)))
D1.fun(jj$x.star,jj$t.vec)</pre>
```

Ez.eqn7.supp

Expectation of z given y, beta2, phi

### **Description**

Expectation as per equation 7 on the supplement

### Usage

```
Ez.eqn7.supp(z, D1, H1, D2, H2, extractor, beta2, y, E.theta, phi)
```

# Arguments

Z	Vector of observations
D1	Matrix whose rows are code run points
H1	Regressor basis functions
D2	Matrix whose rows are observation points
H2	Regressor basis functions
extractor	Function to split D1
beta2	coefficients
У	Code outputs at points corresponding to rows of D1
E.theta	Expectation function to use
phi	hyperparameters

# Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

Ez.eqn9.supp 25

# See Also

V.fun

# **Examples**

Ez.eqn9.supp

Expectation as per equation 10 of KOH2001

# **Description**

Expectation as per equation 10 of KOH2001 (not the supplement)

# Usage

```
Ez.eqn9.supp(x, theta, d, D1, D2, H1, H2, phi)
Ez.eqn9.supp.vector(x, theta, d, D1, D2, H1, H2, phi)
```

# Arguments

x	point at which expectation is needed
theta	parameters
d	observations and code outputs
D1	code run points
D2	observation points
H1	regression function for D1
H2	regression function for D2
phi	hyperparameters

26 Ez.eqn9.supp

#### **Details**

The user should always use Ez.eqn9.supp(), which is a wrapper for Ez.eqn9.supp.vector(). The forms differ in their treatment of  $\theta$ . In the former,  $\theta$  must be a vector; in the latter,  $\theta$  may be a matrix, in which case Ez.eqn9.supp.vector() is applied to the rows.

Note that Ez.eqn9. supp.vector() is vectorized in x but not  $\theta$  (if given a multi-row object, apply(theta,1,...) is used to evaluate the function for each row supplied).

Function Ez.eqn9.supp() will take multiple-row arguments for x and theta. The output will be a matrix, with rows corresponding to the rows of x and columns corresponding to the rows of theta. See the third example below.

Note that function Ez.eqn9. supp() determines whether there are multiple values of  $\theta$  by is.vector(theta). If this returns TRUE, it is assumed that  $\theta$  is a single point in multidimensional parameter space; if FALSE, it is assumed to be a matrix whose rows correspond to points in parameter space.

So if  $\theta$  is one dimensional, calling Ez.eqn9.supp() with a vector-valued  $\theta$  will fail because the function will assume that  $\theta$  is a single, multidimensional, point. To get round this, use as .matrix(theta), which is not a vector; the rows are the (1D) parameter values.

# Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# See Also

tee

H.fun 27

H.fun	H function	

# **Description**

H. See front page of KOHsupp.

# Usage

```
H.fun(theta, D1, D2, H1, H2, phi)
```

# **Arguments**

theta	parameters
D1	matrix of code run points
D2	matrix of observation points
H1	Regressor function for D1
H2	Regressor function for D2
phi	hyperparameters

# Author(s)

Robin K. S. Hankin

# References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

28 H1.toy

H1.toy

Basis functions for D1 and D2

# Description

Applies basis functions to rows of D1 and D2

# Usage

```
H1.toy(D1)
H2.toy(D2)
```

# **Arguments**

D1	Matrix of code run points
D2	Matrix of observation points

### Value

Returns a matrix whose rows are the basis functions of the code run points or observation points. Function H1.toy() operates on datasets like D1.toy (latlong and parameters) and function H2.toy() operates on datasets like D2.toy (latlong only)

#### Note

See package **goldstein** for a less trivial example of h().

### Author(s)

Robin K. S. Hankin

### References

- M. C. Kennedy and A. O'Hagan 2001. Bayesian calibration of computer models. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
D1.toy,
```

h1.toy 29

### **Examples**

```
data(toys)
jj <- extractor.toy(D1.toy)
x.star.toy <- jj$x.star
t.vec.toy <- jj$t.vec

H1.toy(D1=D1.toy)
H1.toy(D1.toy[1,,drop=FALSE])
H1.toy(D1.fun(x.star.toy , theta.toy)[1,,drop=FALSE])
H1.toy(D1.fun(x.star=x.toy,t.vec=theta.toy))
H1.toy(D1.fun(x.star=x.star.toy[1,],t.vec=t.vec.toy[1,]))
H1.toy(D1.fun(x.star=x.star.toy[1,],t.vec=t.vec.toy[1:2,]))
H2.toy(D2.toy)
H2.toy(t(x.toy))</pre>
```

h1.toy

Basis functions

# Description

Basis functions for D1 and D2 respectively.

### Usage

```
h1.toy(x)
h2.toy(x)
```

#### **Arguments**

Х

Vector of lat/long or lat/long and theta

### **Details**

Note that h1() operates on a vector: for dataframes, use H1.toy() which is a wrapper for apply(D1, 1, h1).

**NB** If the definition of h1.toy() or h2.toy() is changed, then function hbar.toy() must be changed to match. This cannot be done automatically, as the form of hbar.toy() depends on the distribution of X. The shibboleth is whether  $E_X()$  commutes with h\_1(); it does in this case but does not in general (for example, consider  $h(x,\theta) = c(1,x,x^2)$  and  $X \sim N(m,V)$ . Then  $E_X(h(x,\theta))$  will be  $(1,m,m^2+V,\theta)$ ; note the V)

#### Value

Returns basis functions of a vector; in the toy case, just prepend a 1.

# Author(s)

Robin K. S. Hankin

30 hbar.fun.toy

### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
H1.toy
```

# **Examples**

```
data(toys)
h1.toy(D1.toy[1,])
```

hbar.fun.toy

*Toy example of hbar (section 4.2)* 

# **Description**

A toy example of the expectation of h as per section 4.2

# Usage

```
hbar.fun.toy(theta, X.dist, phi)
```

#### **Arguments**

theta Parameter set

X.dist Distribution of variable inputs X as per section 4.2

phi Hyperparameters

# **Details**

Note that if h1.toy() or h2.toy() change, then hbar.fun.toy() will have to change too; see ?h1.toy for an example in which nonlinearity changes the form of E. theta.toy()

#### Value

Returns a vector as per section 4.2 of KOH2001S

# Author(s)

Robin K. S. Hankin

is.positive.definite 31

### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
h1.toy
```

# **Examples**

```
data(toys)
hbar.fun.toy(theta=theta.toy, X.dist=X.dist.toy, phi=phi.toy)
```

# **Description**

Returns TRUE if and only if a matrix is positive definite.

### Usage

```
is.positive.definite(a, ...)
```

### **Arguments**

- a Matrix to be tested
- ... Extra arguments passed to eigen(), such as symmetric.

#### **Details**

A wrapper for eigen() (a matrix is positive definite if all its eigenvalues are positive). This function is included for convenience only.

# Author(s)

Robin K. S. Hankin

```
is.positive.definite(diag(3),sym=TRUE)
is.positive.definite(diag(6)-0.1)
```

32 MH

MH

Very basic implementation of the Metropolis-Hastings algorithm

# **Description**

Very basic implementation of the Metropolis-Hastings algorithm using a multivariate Gaussian proposal distribution. Useful for sampling from p.eqn8.supp().

### Usage

```
MH(n, start, sigma, pi)
```

### **Arguments**

n	Number of samples to take
start	Start value
sigma	Variance matrix for kernel
pi	Functional proportional to the desired sampling pdf

### **Details**

This is a **basic** implementation. The proposal distribution  $\sim q(X|Y)$  is  $q(\cdot|X) = N(X, \sigma^2)$ 

### Value

Returns a matrix whose rows are samples from  $\pi()$ . Note that the first few rows will be "burn-in", so should be ignored

# Note

This function is a little slow because it is not vectorized.

# Author(s)

Robin K. S. Hankin

# References

- W. R. Gilks et al 1996. *Markov Chain Monte Carlo in practice*. Chapman and Hall, 1996. ISBN 0-412-05551-1
- N. Metropolis and others 1953. *Equation of state calculations by fast computing machines*. The Journal of Chemical Physics, volume 21, number 6, pages 1087-1092

# See Also

```
p.eqn8.supp
```

p.eqn4.supp 33

### **Examples**

```
# First, a bivariate Gaussian:
A < - diag(3) + 0.7
quad.form <- function(M,x){drop(crossprod(crossprod(M,x),x))}</pre>
pi.gaussian \leftarrow function(x) \{ exp(-quad.form(A/2,x)) \}
x.gauss \leftarrow MH(n=1000, start=c(0,0,0), sigma=diag(3), pi=pi.gaussian)
cov(x.gauss)/solve(A) # Should be a matrix of 1s.
# Now something a bit weirder:
pi.triangle <- function(x){</pre>
  1*as.numeric( (abs(x[1])<1.0) & (abs(x[2])<1.0) ) +
  5*as.numeric( (abs(x[1])<0.5) & (abs(x[2])<0.5) ) *
    as.numeric(x[1]>x[2])
x.tri <- MH(n=100,start=c(0,0),sigma=diag(2),pi=pi.triangle)</pre>
plot(x.tri,main="Try with a higher n")
# Now a Gaussian mixture model:
pi.2gauss <- function(x){</pre>
  exp(-quad.form(A/2,x)) +
  exp(-quad.form(A/2,x+c(2,2,2)))
}
x.2 \leftarrow MH(n=100, start=c(0,0,0), sigma=diag(3), pi=pi.2gauss)
## Not run: p3d(x.2, theta=44,d=1e4,d0=1,main="Try with more points")
```

p.eqn4.supp

Apostiori probability of psi1

# Description

Gives the probability of  $\psi_1$ , given observations. Equation 4 of the supplement

# Usage

```
p.eqn4.supp(D1, y, H1, include.prior=TRUE, lognormally.distributed, return.log, phi)
```

# **Arguments**

D1	Matrix of code run points
у	Vector of code outputs
H1	Regression function
include.prior	Boolean with default TRUE meaning to return the likelihood multiplied by the aprior probability and FALSE meaning to return the likelihood without the prior.

p.eqn8.supp

lognormally.distributed

Boolean; see ?prob. theta for details

return.log Boolean, with default FALSE meaning to return the probability and TRUE mean-

ing to return the logarithm of the probability

phi hyperparameters

# Author(s)

Robin K. S. Hankin

### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

W1

# **Examples**

```
data(toys)
p.eqn4.supp(D1=D1.toy, y=y.toy , H1=H1.toy, lognormally.distributed=TRUE,
phi=phi.toy)
```

p.eqn8.supp

A postiori probability of hyperparameters

# **Description**

Function to determine the a-postiori probability of hyperparameters  $\rho$ ,  $\lambda$  and  $\psi_2$ , given observations and  $\psi_1$ .

# Usage

```
p.eqn8.supp(theta, D1, D2, H1, H2, d, include.prior=FALSE,
lognormally.distributed=FALSE, return.log=FALSE, phi)
p.eqn8.supp.vector(theta, D1, D2, H1, H2, d, include.prior=FALSE,
lognormally.distributed=FALSE, return.log=FALSE, phi)
```

p.eqn8.supp 35

### **Arguments**

theta	Parameters	
D1	Matrix of code run points	
D2	Matrix of observation points	
H1	Regression function for D1	
H2	Regression function for D2	
d	Vector of code output values and observations	
include.prior	Boolean, with TRUE meaning to include the prior PDF for $\theta$ and default FALSE meaning return the likelihood, multiplied by an undetermined constant	
lognormally.distributed		
	Boolean, with TRUE meaning to assume prior is lognormal (see prob.theta() for more info)	
return.log	Boolean, with default FALSE meaning to return the probability; TRUE means to return the (natural) logarithm of the answer	
phi	Hyperparameters	

#### **Details**

The user should always use p.eqn8.supp(), which is a wrapper for p.eqn8.supp.vector(). The forms differ in their treatment of  $\theta$ . In the former,  $\theta$  must be a vector; in the latter,  $\theta$  may be a matrix, in which case p.eqn8.supp.vector() is applied to the rows

# Author(s)

Robin K. S. Hankin

### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# See Also

W2,stage1

```
data(toys)
p.eqn8.supp(theta=theta.toy, D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy,
d=d.toy, phi=phi.toy)
```

p.page4

p.page4

A postiori probability of hyperparameters

# Description

Function to determine a postiori probability of hyperparameters  $\rho$ ,  $\lambda$  and  $\psi_2$ , given observations and  $\psi_1$ .

# Usage

p.page4(D1, D2, H1, H2, V, y, z, E.theta, Edash.theta, extractor, include.prior=FALSE, lognormally.distributed=FALSE, return.log=FALSE, phi)

# Arguments

	D1	Matrix of code run points
	D2	Matrix of observation points
	H1	Basis function (vectorized)
	H2	Regression function for D2
	V	Covariance matrix; default value of NULL results in the function evaluating it (but this takes a long time, so supply V if known)
	У	Vector of code outputs
	z	Vector of observation values
	E.theta	Expectation over theta
	Edash.theta	Expectation over theta WRT $E'$
	extractor	Function to extract independent variables and parameters from D1
	include.prior	Boolean, with TRUE meaning to include the prior PDF for $\theta$ and default value of FALSE meaning to return the likelihood multiplied by an undetermined constant
lognormally.distributed		
		D 1 11 TOUT 1 1 11 C 1 11 11

Boolean with TRUE meaning to assume lognormality. See prob.psi1 for details

return.log Boolean, with default FALSE meaning to return the probability, and TRUE mean-

ing to return the (natural) logarithm of the probability (which is useful when

considering very small probabilities)

phi Hyperparameters

## Author(s)

Robin K. S. Hankin

#### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

W2

## **Examples**

```
data(toys)
p.page4(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy, V=NULL, y=y.toy,
z=z.toy,E.theta=E.theta.toy, Edash.theta=Edash.theta.toy, extractor=extractor.toy, phi=phi.toy)
## Now compare the above value with p.page4() calculated with phi
## differing only in psi2:
phi.toy.new <- phi.change(phi.fun=phi.fun.toy, old.phi = phi.toy, psi2=c(8,8,8))
p.page4(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy, V=V.toy, y=y.toy, z=z.toy,
E.theta=E.theta.toy, Edash.theta=Edash.theta.toy,
extractor=extractor.toy, phi=phi.toy.new)
## different!</pre>
```

phi.fun.toy

Functions to create or change hyperparameters

#### **Description**

Function to create (phi.fun.toy) or modify (phi.change) toy hyperparameters  $\phi$  in a form suitable for passing to the other functions in the library.

The user should never make  $\phi$  by hand; always use one of these functions

#### Usage

```
phi.fun.toy(rho, lambda, psi1, psi1.apriori, psi2, psi2.apriori,
    theta.apriori)
phi.change(phi.fun, old.phi = NULL, rho = NULL, lambda = NULL,
        psi1 = NULL, psi1.apriori=NULL, psi1.apriori.mean=NULL,
        psi1.apriori.sigma=NULL, psi2 = NULL, psi2.apriori=NULL,
        psi2.apriori.mean=NULL, psi2.apriori.sigma=NULL,
        theta.apriori=NULL, theta.apriori.mean=NULL,
        theta.apriori.sigma=NULL)
```

#### **Arguments**

phi.fun In phi.change(), the name of the function that creates the hyperparameters.

Use phi.fun.toy() for the toy dataset

old.phi In function phi.change(), the hyperparameter object  $\phi$  to be modified

rho Correlation hyperparameter appearing in main equation

lambda Noise hyperparameter

psi1 Roughness lengths hyperparameter for design matrix D1. Internal function pdm. maker.psi1()

takes psi1 as an argument and returns omega\_x, omega\_t and sigma1squared. Recall that  $\Omega_x$  and  $Omega_t$  are arbitrary functions of  $\psi_1$ . In this case, the values are omega\_x=psi1[1:2], omega\_t=psi1[3:4] and sigma1squared=psi1[6]

psi1.apriori A priori PDF for  $\psi_1$ . In the form of a two element list with first element (mean)

the mean and second element (sigma) the covariance matrix; distribution of the logarithms is assumed to be multivariate normal. In the toy example, the mean is a vector of length six (the first five are  $\psi_1$  and the sixth is for  $\sigma_1^2$ ), and the variance is the corresponding six-by-six matrix. Use function prob.psi1() to

calculate the apriori probability density for a particular value of  $\psi_1$ 

psi1.apriori.mean

In function phi.change.toy(), use this argument to change just the mean of psi1 (and leave the value of sigma unchanged)

psi1.apriori.sigma

In function phi.change.toy(), use this argument to change just the variance matrix of psi1

psi2 Roughness lengths hyperparameter for D2.

Internal function pdm.maker.psi2() takes psi2 as an argument and returns

 $omegastar\_x\ and\ sigma 2 squared.\ In\ phi.\ fun.\ toy(), the\ values\ are\ omegastar\_x=psi2[1:2]$ 

and sigma2squared=psi2[3].

NB: function stage2() optimizes more than just psi2. It simultaneously opti-

mizes psi2 and lambda and rho

psi2.apriori A priori PDF for  $\psi_2$  and hyperparameters  $\rho$  and  $\lambda$  (in that order).

As for psi1.apriori, this is in the form of a list with the first element (mean) the mean and second element (sigma) the covariance matrix; the logs are multivariate normal. In the toy example, the mean is a vector of length five. The first and second elements of the mean are the apriori mean of  $\rho$  and  $\lambda$  respectively; the third and fourth elements are the apriori mean of  $\psi_2$  (that is, x and y

respectively); and the fifth is the mean of  $\sigma_2^2$ .

The second element of phi.toy\$psi2.apriori, sigma, is the corresponding four-by-four variance matrix. Use function prob.psi2() to calculate the apriori probability density of a particular value of  $\psi_2$ 

psi2.apriori.mean

In phi. change. toy(), use to change just the mean of psi2

psi2.apriori.sigma

In phi.change.toy(), use to change just the variance matrix of psi2

theta.apriori

Apriori PDF for  $\theta$ . As above, in the form of a list with elements for the mean and covariance. The distribution is multivariate normal (NB: The distribution is multivariate normal and NOT lognormal! To be explicit:  $\log(\theta)$  is lognormally distributed). Use function prob.theta() to calculate the apriori probability density of a particular value of  $\theta$ 

theta.apriori.mean

In phi.change.toy(), use to change just the mean of theta

theta.apriori.sigma

In phi.change.toy(), use to change just the variance matrix of theta

#### **Details**

Note that this toy function contains within itself pdm.maker.toy() which extracts omega\_x and omega\_t and sigma1squared from psi1. This will need to be changed for real-world applications.

Earlier versions of the package had pdm.maker.toy() defined separately.

## Value

Returns a list of several elements:

rho	Correlation hyperparameter
lambda	Noise hyperparameter
psi1	Roughness lengths hyperparameter for D1
psi1.apriori	Apriori mean and variance matrix for psi1
psi2	Roughness lengths hyperparameter for D2
psi2.apriori	Apriori mean and variance matrix for psi2
theta.apriori	Apriori mean and variance matrix for the parameters
omega_x	Positive definite matrix for the lat/long part of D1, whose diagonal is psi1[1:2]
omega_t	Positive definite matrix for the code parameters theta, whose diagonal is psi1[3:5]
omegastar_x	Positive definite matrix for use in equation 13 of the supplement; represents distances between rows of D2
sigma1squared	variance
sigma2squared	variance
omega_x.upper	Upper triangular Cholesky decomposition for omega_x
omega_x.lower	Lower triangular Cholesky decomposition for omega_x
omega_t.upper	Upper triangular Cholesky decomposition for omega_t

omega_t.lower	Lower triangular Cholesky decomposition for omega_t
a	$Precalculated\ matrix\ for\ use\ in\ {\tt Edash.theta(,fast.but.opaque=TRUE)}$
b	$Precalculated\ matrix\ for\ use\ in\ {\tt Edash.theta} (\ldots, {\tt fast.but.opaque=TRUE})$
С	Precalculated scalar for use in ht.fun(,fast.but.opaque=TRUE)
A	Precalculated scalarfor use in tt.fun()
A.upper	Upper triangular Cholesky decomposition for A
A.lower	Lower triangular Cholesky decomposition for A

## Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
toys, H1.toy
```

```
phi.fun.toy(100,101,1:6,list(mean=rep(1,6),sigma=1+diag(6)),50:55,
list(mean=rep(0,4),sigma=0.1+diag(4)),
list(mean=0.1+(1:3), sigma=2.1+diag(3)))
phi.fun.toy(rho=1, lambda=1,
    psi1 = structure(c(1.1, 1.2, 1.3, 1.4, 1.5, 0.7),
            .Names = c("x", "y", "A", "B", "C", "s1sq")),
    psi1.apriori = list(
             mean=rep(0,6), sigma=0.4+diag(6)),
             psi2=structure(c(2.1, 2.2), .Names = c("x", "y")),
             psi2.apriori = list(mean=rep(0,5), sigma=0.2+diag(5)),
             theta.apriori = list(mean=0.1+(1:3), sigma=2.1+diag(3))
)
data(toys)
phi.change(phi.fun=phi.fun.toy, old.phi = phi.toy, rho = 100)
phi.change(phi.fun=phi.fun.toy, old.phi = phi.toy,
     theta.apriori.sigma = 4*diag(3))
identical(phi.toy, phi.change(phi.fun=phi.fun.toy, old.phi=phi.toy))
```

prob.psi1 41

prob.psi1	A priori probability of psi1, psi2, and theta	

## **Description**

Function to determine the a-priori probability of  $\psi_1$  and  $\psi_2$  of the hyperparameters, and  $\theta$ , given the apriori means and standard deviations.

Function sample. theta() samples  $\theta$  from its prior distribution.

## Usage

```
prob.psi1(phi,lognormally.distributed=TRUE)
prob.psi2(phi,lognormally.distributed=TRUE)
prob.theta(theta,phi,lognormally.distributed=FALSE)
sample.theta(n=1,phi)
```

## **Arguments**

phi Hyperparameters theta Parameters lognormally.distributed

Boolean variable with FALSE meaning to assume a Gaussian distribution and

TRUE meaning to use a lognormal distribution.

n In function sample. theta(), the number of observations to take

#### **Details**

These functions use package mvtnorm to calculate the probability density under the assumption that the PDF is lognormal. One implication would be that phi\$psi2.apriori\$mean and phi\$psi1.apriori\$mean are the means of the **logarithms** of the elements of psi1 and psi2 (which are thus assumed to be positive). The sigma matrix is the covariance matrix of the logarithms as well.

In these functions, interpretation of argument phi depends on the value of Boolean argument lognormally.distributed. Take prob.theta() as an example. If lognormally.distributed is TRUE, then log(theta) is normally distributed with mean phi\$theta.aprior\$mean and variance phi\$theta.apriori\$sigma. If FALSE, theta is normally distributed with mean phi\$theta.aprior\$mean and variance phi\$theta.apriori\$sigma.

Interpretation of phi\$theta.aprior\$mean depends on the value of lognormally.distributed: if TRUE it is the expected value of log(theta); if FALSE, it is the expectation of theta.

The reason that prob.theta has a different default value for lognormally.distributed is that some elements of theta might be negative, contraindicating a lognormal distribution

## Author(s)

Robin K. S. Hankin

42 reality

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
p.eqn4.supp, stage1, p.eqn8.supp
```

## **Examples**

```
data(toys)
prob.psi1(phi=phi.toy)
prob.psi2(phi=phi.toy)

prob.theta(theta=theta.toy,phi=phi.toy)
sample.theta(n=4,phi=phi.toy)
```

reality

Reality

## **Description**

Function to compute reality, gratis *deus ex machina*. Includes a simple computer model that substitutes for a complex climate model, and a simple function that substitutes for the base system, in this case the climate.

#### Usage

## **Arguments**

Boolean, with the default value of TRUE meaning to set the RNG seed to zero

reality 43

draw.from.prior

Boolean, with default FALSE meaning to generate obsevations from the "true" values of the parameters, and TRUE meaning to draw from the relevant apriori distribution.

export.true.hyperparameters

Boolean, with default value of FALSE meaning to return the observed scalar. Set to TRUE to exercise omniscience and access the *true* values of the parameters and hyperparameters. Only the omnipotent should set this variable, and only the omniscient may see its true value.

phi

In function phi.true.toy() the hyperparameters  $\phi$ . Note that apriori distributions are unchanged (they are irrelevant to omniscient beings).

In functions reality() and computer.model(), the prior distributions of the hyperparameters is passed via phi (so it only elements psi1.apriori and psi2.apriori need to be set).

#### **Details**

Function reality() provides *the* scalar value observed at a point x. Evaluation expense is zero; there is no overhead.

(However, it does not compute "reality": the function returns a value subject to observational error  $N(0, \lambda)$  as per equation 5. It might be better to call this function observation())

Function computer.model() returns the output of a simple, nonlinear computer model.

Both functions documented here return a random variable drawn from an appropriate (correlated) multivariate Gaussian distribution, and are thus Gaussian processes.

The approach is more explicit in the help pages of the emulator package. There, Gaussian processes are generated by directly invoking rmvnorm() with a suitable correlation matrix

#### Author(s)

Robin K. S. Hankin

## References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

computer.model

44 stage1

#### **Examples**

```
data(toys)
 computer.model(X=D2.toy,params=theta.toy)
 computer.model(D1.toy)
 computer.model(X=x.toy, params=extractor.toy(D1.toy)$t.vec)
 phi.fix <- phi.change(old.phi=phi.toy,</pre>
           psi1=c(1, 0.5, 1, 1, 0.5, 0.4),phi.fun=phi.fun.toy)
      #The values come from c(REAL.SCALES, REAL.SIGMA1SQUARED) as
     #seen in the sourcecode for computer.model().
 computer.model(D1.toy)
                           # use debug(computer.model) and examine
                           # var.matrix directly. It should match the
                           # output from V1():
          # first fix phi so that it has the correct values for psi1 (see the
          # section on psi1 in ?phi.fun.toy for how to get this):
  phi.fix <- phi.change(old.phi=phi.toy,psi1=c(1, 0.5, 1.0, 1.0, 0.5,
  0.4), phi.fun=phi.fun.toy)
  V1(D1.toy,phi=phi.fix)
# What are the hyperparameters that were used to create reality?
phi.true.toy(phi=phi.toy)
computer.model(X=D2.toy,params=theta.toy,draw.from.prior=TRUE,phi=phi.toy)
```

stage1

Stage 1,2 and 3 optimization on toy dataset

## Description

Perform O'Hagan's three stage optimization on the toy dataset. Function stage1() and stage2() find the optimal values for the hyperparameters and stage3() finds the optimal values for the three parameters.

stage1 45

#### Usage

#### **Arguments**

maxit	Maximum number of iterations as passed to optim()	
trace	Amount of information displayed, as passed to optim()	
D1	Matrix whose rows are points at which code output is known	
D2	Matrix whose rows are points at which observations were made	
H1,H2	Regressor basis functions for D1 and D2	
У	Code outputs. Toy example is y. toy	
Z	Observations. Toy example is z.toy	
d	Data vector consisting of the code runs and observations	
extractor E.theta,Edash.	extractor function for D1 theta	
	Expectation WRT theta, and dashed theta. Toy examples are E.theta.toy() and Edash.theta.toy()	
phi.fun	Function to create hyperparameters; passed (in stage1() and stage2()) to phi.change(). Toy version is phi.fun.toy()	
method	Method argument passed to optim(); qv	
include.prior	Boolean variable with default TRUE meaning to include the prior distribution in the optimization process and FALSE meaning to use an uniformative prior (effectively uniform support). This variable is passed to p.eqn4.supp() for stage1(), p.page4() for stage2(), and p.eqn8.supp() for stage3()	
lognormally.distributed		
	Boolean with TRUE meaning to use a lognormal distn. See prob. theta for details	
do.filewrite	Boolean, with TRUE meaning to save a loadable file stage[123]. <date>, containing the interim value of phi and the corresponding optimand to directory at each evalution of the optimizer. If FALSE, don't write the files</date>	
directory	The directory to write files to; only matters if do.filewrite is TRUE	
isotropic	In function stage2(), Boolean with default FALSE meaning to carry out a full optimization, and TRUE meaning to restrict the scope to isotroic roughness ma-	

trices. See details section below

46 stage1

do.print	Boolean, with default TRUE meaning to print interim values of phi at each evaluation
use.standin	In stage2(), a Boolean argument, with default FALSE meaning to use the real value for matrix V.temp, and TRUE meaning to use a standing that is the same size but contains fictitious values. The only time to set use.standin to TRUE is when debugging as it runs several orders of magnitude faster
rho.eq.1	Boolean, with default TRUE meaning to hold the value of rho constant at one (1)
theta.start	In stage3(), the starting point of the optimization with default NULL meaning to use the maximum likelihood point of the apriori distribution (ie phi\$theta.apriori\$mean)
phi	Hyperparameters. Used as initial values for the hyperparameters in the optimization routines

#### **Details**

The three functions documented here carry out the multi-stage optimization detailed in KOH2001 (actually, KOH2001 only defined stage 1 and stage 2, which estimated the hyperparameters. What is here called "stage3()" estimates the true value of  $\theta$  given the hyperparameters).

stage1() carries out stage 1 of KOH2001 which is used to estimate  $\psi_1$  using optimization.

In function stage2(), setting argument isotropic to TRUE will force phi\$omegastar\_x to be a function of a length one scalar. The value of phi\$omegastar\_x used will depend on pdm.maker.psi2() (an internal function appearing in hpa.fun.toy()). In stage2(), several kludges are made. The initial conditions are provided by argument phi. The relevant part of this is phi\$psi2.

Function stage2() estimates  $\psi_2$  and  $\rho$  and  $\lambda$ , using optimization. Note that  $\psi_2$  includes  $\sigma_2^2$  in addition to omegastar\_X (in the toy case,  $\psi_2$  has three elements: the first two are the diagonal of omegastar\_x and the third is  $\sigma_2^2$  but this information is encoded in phi.fun.toy(), which changes from application to application).

Function stage3() attempts to find the maximum likelihood estimate of  $\theta$ , given hyperparameters and observations, using optimization

#### Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. Bayesian calibration of computer models. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

## See Also

toys, phi.fun.toy

symmetrize 47

#### **Examples**

```
stage1(D1=D1.toy,y=y.toy,H1=H1.toy, maxit=5, phi.fun=phi.fun.toy, phi=phi.toy)
##now try with a slightly bigger dataset:
##Examples below take a few minutes to run:
set.seed(0)
data(toys)
jj <- create.new.toy.datasets(D1.toy , D2.toy)</pre>
y.toy <- jj$y.toy</pre>
z.toy <- jj$z.toy</pre>
d.toy <- jj$d.toy</pre>
phi.toy.stage1 <- stage1(D1=D1.toy, y=y.toy, H1=H1.toy, maxit=10, phi.fun=phi.fun.toy, phi=phi.toy)
phi.toy.stage2 <- stage2(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy,</pre>
y=y.toy, z=z.toy, extractor=extractor.toy,
phi.fun=phi.fun.toy, E.theta=E.theta.toy, Edash.theta=Edash.theta.toy,
maxit=3, phi=phi.toy.stage1)
stage3(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy, d=d.toy, maxit=3, phi=phi.toy.stage2)
# Now try with the true values of the hyperparameters:
phi.true <- phi.true.toy(phi=phi.toy)</pre>
stage3(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy, d=d.toy, maxit=3, phi=phi.true)
```

symmetrize

Symmetrize an upper triangular matrix

#### **Description**

Symmetrize an upper triangular matrix by copying the upper triangular elements into the lower triangular places

#### Usage

```
symmetrize(a)
```

## **Arguments**

а

Upper triangular matrix to be symmetrized

## **Details**

Also works for lower triangular matrices

48 tee

#### Author(s)

Robin K. S. Hankin

## **Examples**

tee

Auxiliary functions for equation 9 of the supplement

# Description

Returns a vector whose elements are the "distances" from a point to the observations and code run points (tee()); and basis functions for use in Ez.eqn9.supp()

## Usage

```
tee(x, theta, D1, D2, phi)
h.fun(x, theta, H1, H2, phi)
```

#### **Arguments**

X	Point from which distances are calculated
theta	Value of parameters
D1,D2	Design matrices of code run points and field observation points respectively (tee())
H1,H2	Basis functions for eta and model inadequacy term respectively (h.fun())
phi	Hyperparameters

## **Details**

Equation 9 of the supplement is identical to equation 10 of KOH2001.

Function h.fun() returns the first of the subsidiary equations in equation 9 of the supplement and function tee() returns the second (NB: do not confuse this with functions t1bar() and t2bar() which are internal to EK.eqn10.supp())

## Author(s)

Robin K. S. Hankin

toys 49

#### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
Ez.eqn9.supp
```

# Examples

```
data(toys)
tee(x=x.toy, theta=theta.toy, D1=D1.toy, D2=D2.toy, phi=phi.toy)
# Now some vectorized examples:
jj <- rbind(x.toy , x.toy , x.toy+0.01,x.toy+1,x.toy*10)
tee(x=jj, theta=theta.toy, D1=D1.toy, D2=D2.toy, phi=phi.toy)
h.fun(x=jj, theta=theta.toy, H1=H1.toy, H2=H2.toy, phi=phi.toy)</pre>
```

toys

Toy datasets

# **Description**

Toy datasets that illustrate the package.

#### Usage

```
data(toys)
D1.toy
D2.toy
d.toy
phi.toy
theta.toy
V.toy
X.dist.toy
```

50 toys

#### **Format**

The D1. toy matrix is 8 rows of code run points, with five columns. The first two columns are the lat and long and the next three are parameter values.

The D2. toy matrix is five rows of observations on two variables, x and y which are styled "latitude and longitude".

d. toy is the "data" vector consisting of length 13: elements 1-8 are code runs and elements 9-13 are observations.

theta.toy is a vector of length three that is a working example of  $\theta$ . The parameters are designed to work with computer.model().

- t.vec.toy is a matrix of eight rows and three columns. Each row specifies a value for  $\theta$ . The eight rows correspond to eight code runs.
- x. toy and x. toy2 are vectors of length two that gives a sample point at which observations may be made (or the code run). The gloss of the two elements is latitude and longitude.
- x.vec is a matrix whose rows are reasonable x values but *not* those in D2.toy.
- y. toy is a vector of length eight. Each element corresponds to the output from a code run at each of the rows of D1. toy.
- z. toy is a vector of length five. Each element corresponds to a measurement at each of the rows of D2. toy.
- V. toy is a five by five variance-covariance matrix for the toy datasets.
- X.dist.toy is a toy example of a distribution of X for use in calibrated uncertainty analysis, section 4.2.

# Brief description of toy functions fully documented under their own manpage

Function create.new.toy.datasets() creates new toy datasets with any number of observations and code runs.

Function E.theta.toy() returns expectation of H(D) with respect to  $\theta$ ; Edash.theta.toy() returns expectation with respect to E'.

Function extractor.toy() extracts x.star.toy and t.vec.toy from D2; toy example needed because the extraction differs from case to case.

Function H1.toy() applies basis functions to rows of D1 and D2

Function phi.fun.toy() creates a hyperparameter object such as phi.toy in a form suitable for passing to the other functions in the library.

Function phi.change.toy() modifies the hyperparameter object.

#### See the helpfiles listed in the "see also" section below

#### Details

All toy datasets are documented here. There are also several toy functions that are needed for a toy problem; these are documented separately (they are too diverse to document fully in a single manpage). Nevertheless a terse summary for each toy function is provided on this page. All toy functions in the package are listed under "See Also".

tt.fun 51

#### Author(s)

Robin K. S. Hankin

#### References

• M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464

- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
create.new.toy.datasets, E.theta.toy, extractor.toy, H1.toy, phi.fun.toy, stage1
```

## **Examples**

```
data(toys)
D1.toy
extractor.toy(D1.toy)

D2.fun(theta=theta.toy , D2=D2.toy)
D2.fun(theta=theta.toy,D2=D2.toy[1,,drop=FALSE])

library("emulator")
corr.matrix(D1.toy,scales=rep(1,5))
corr.matrix(D1.toy, pos.def.matrix=diag(5))
```

tt.fun

Integrals needed in KOH2001

#### **Description**

Calculates the three integrals needed for V, under the restrictions specified in the KOH2001 supplement

#### Usage

```
tt.fun(D1, extractor, x.i, x.j, test.for.symmetry=FALSE, method=1, phi)
ht.fun(x.i, x.j, D1, extractor, Edash.theta, H1, fast.but.opaque=TRUE,
x.star=NULL, t.vec=NULL, phi)
hh.fun(x.i, x.j, H1, E.theta, phi)
t.fun(x, D1, extractor, phi)
```

52 tt.fun

#### **Arguments**

D1 Matrix of code run points

H1 regression basis functions for D1

extractor Function to extract x.star and t.vec from D1 x Lat and long of a point in t.fun() (eg D2[1,])

x.i Lat and long of first point (eg D2[1,])x.j Lat and long of second point (eg D2[2,])

theta parameters

Edash. theta Function to return expectation of H with respect to the alternative distribution of

 $\theta$ ; Edash. theta. toy is the example for the toy dataset

E. theta Function to return expectation of H with respect to  $\theta$ 

test.for.symmetry

In tt.fun(), Boolean with TRUE meaning to calculate each element of C explicitly. If FALSE, then calculate only the elements of C that lie on or over the diagonal and use the fact that C is symmetric to calculate the other matrix elements. For n observations, this means n(n+1)/2 evaluations, compared with  $n^2$  for the full case.

Set this argument to TRUE only when debugging, or testing accuracy.

fast.but.opaque

In ht.fun(), Boolean with default TRUE meaning to pass some precalculated results as arguments, to save time. Set this argument to FALSE only when debugging.

x.star In ht.fun(), value of  $x^*$  (required only if fast.but.opaque is TRUE) t.vec In ht.fun(), value of t (required only if fast.but.opaque is TRUE)

method In tt.fun(), zero means use the old method and nonzero means use the new

method. The new method is faster, but the code is harder to understand. The

two methods should give identical results.

phi Hyperparameters

#### **Details**

The four functions return integrals representing means taken over theta. To wit:

• Function tt.fun() evaluates

$$\int t(x_j, \theta) t(x_i, \theta)^T p(\theta) d\theta$$

and is used in V. fun(). Note that this function is symmetric in  $x_i$  and  $x_j$ .

• Function ht.fun() evaluates

$$\int h_1(x_j,\theta)t(x_i,\theta)^T p(\theta)d\theta$$

and is used in V. fun(). Note that this function is **not** symmetric in  $x_i$  and  $x_j$ .

tt.fun 53

• Function hh.fun() evaluates

$$\int h_1(x_j,\theta)h_1(x_i,\theta)^T p(\theta)d\theta$$

and is used in V. fun(). Note that this function is symmetric in  $x_i$  and  $x_j$ .

• Function t.fun() evaluates

$$\int t(x_i, \theta)^T p(\theta) d\theta = \int c_1 ((x_i, \theta), (x_j^*, t_j)) p(\theta) d\theta$$

using the formula

$$\sigma_{1}^{2}\left|I+2V_{\theta}\Omega_{x}\right|^{-1/2}\exp\left\{-\left(x_{i}-x_{j}^{*}\right)^{T}\Omega_{x}\left(x_{i}-x_{j}^{*}\right)\right\}\times\exp\left\{-\left(m_{\theta}-t_{j}\right)^{T}\left(2V_{\theta}+\Omega_{t}^{-1}\right)^{-1}\left(m_{\theta}-t_{j}\right)\right\}.$$

It is used in Ez\_eq7. supp(). NB: do not confuse this function with tee(), which is different.

These functions are not generally of much interest to the end user; they are called by V. fun(). They are defined separately as a debugging aid, and to simplify the structure of V. fun().

#### Value

Each function returns a matrix as described in KOH2001

#### Author(s)

Robin K. S. Hankin

## References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

V.fun

```
data(toys)
```

```
tt.fun(D1=D1.toy, extractor=extractor.toy, x.i=D2.toy[1,],
    x.j=D2.toy[2,], phi=phi.toy)
ht.fun(x.i=D2.toy[1,], x.j=D2.toy[2,], D1=D1.toy,
    extractor=extractor.toy,
```

54 V.fun

V.fun

Variance matrix for observations

#### **Description**

Determines the variance/covariance matrix for the observations and code run points.

#### Usage

```
V.fun(D1, D2, H1, H2, extractor,
E.theta, Edash.theta, give.answers=FALSE, test.for.symmetry=FALSE, phi)
```

Matrix of code run points

# **Arguments**D1

test.for.symmetry

	1
D2	Matrix of observation points
H1	Regression function for D1
H2	Regression function for D2
extractor	Function to extract x.star and t.vec from D1
Edash.theta	Function to return expectation of H with respect to the alternative distribution of $\theta$ ; Edash. theta. toy is the example for the toy dataset
E.theta	Expectation of h WRT theta over the apriori distribution. Note that this function must be updated if h1() changes.
give.answers	Boolean (defaulting to FALSE) with TRUE meaning to return a list whose elements are V and its constituent parts, viz line1 to line6. This argument is used mainly for debugging.

Boolean with TRUE meaning to calculate each element of C explicitly, and default FALSE meaning to calculate only the elements of C that lie on or over the diagonal and use the fact that C is symmetric to calculate the other matrix elements. For n observations, this means n(n+1)/2 evaluations, compared with  $n^2$  for the full case. The time saving is considerable, even for small matrices. Set this argument to TRUE only when debugging, or testing accuracy

V.fun 55

phi Hyperparameters

#### **Details**

See KOH2001 for full details on page 3 of the supplement

#### Value

If give . answers is the default value of FALSE, returns a matrix of covariances for use in p. page4().

If give answers is TRUE, returns a named list of (currently) 17 elements. Elements one to six are lines one to six respectively from page 3 of the supplement; subsequent lines give intermediate steps in the calculation. The final element is the matrix is the covariances as returned when give answers is FALSE.

#### Note

This function takes a long time to run

#### Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
tt.fun,p.page4
```

56 V1

```
Edash.theta=Edash.theta.toy,
  E.theta=E.theta.toy, phi=phi.different.theta)
## different!
## Now compare jj above with V.fun() calculated with
## different phi2:
phi.toy.new <- phi.change(phi.fun=phi.fun.toy, old.phi = phi.toy, psi2=c(8,8,8))</pre>
V.fun(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy,
  extractor=extractor.toy,
  Edash.theta=Edash.theta.toy,
  E.theta=E.theta.toy, phi=phi.toy.new)
## different!
## Not run:
data(toys)
set.seed(0)
jj <- create.new.toy.datasets(D1=D1.toy , D2=D2.toy)</pre>
y.toy <- jj$y.toy</pre>
z.toy <- jj$z.toy</pre>
d.toy <- jj$d.toy</pre>
v.fun <- function(...){V.fun(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy,
     \verb|extractor=extractor.toy, Edash.theta=Edash.theta.toy,|\\
     E.theta=E.theta.toy, phi=phi.toy, give=TRUE)}
Rprof(file="~/f.txt");ignore <- v.fun();Rprof(file=NULL)</pre>
system("cd ; R CMD Rprof ~/f.txt > ~/ff.txt")
## End(Not run)
```

۷1

Distance matrix

# Description

Gives the distance matrix between rows of D1 and D1 (or, if supplied, another matrix)

#### Usage

```
V1(D1, other = NULL, phi)
```

## **Arguments**

D1

Matrix of code run points

V2 57

other Second matrix to compute distances to. If NULL, use the first supplied matrix phi Hyperparameters

#### Value

Returns a matrix

## Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

٧2

## **Examples**

```
data(toys)
V1(D1=D1.toy, other=NULL, phi=phi.toy)
V1(D1=D1.toy[1,,drop=FALSE], other=NULL, phi=phi.toy)
V1(D1=D1.toy, other=D1.toy[1:3,], phi=phi.toy)
V1(D1=D1.toy,other=D1.fun(x.star=x.vec,t.vec=theta.toy),phi=phi.toy)
```

٧2

distance between observation points

# Description

distance between observation points

## Usage

```
V2(x, other = NULL, phi)
```

58 Vd

## **Arguments**

x Matrix whose rows are observation points
--

other Second matrix; if NULL, use x

phi Hyperparameters

## **Details**

This function returns the variance matrix of observations of the real process z at points  $D_2 = \{x_1, \ldots, x_n\}$ .

It appears in the lower right corner of the variance matrix on the bottom of page 1 of the supplement, calculated by function Vd().

It is also used in functions Cov.eqn9.supp() and V.fun().

#### Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. Bayesian calibration of computer models. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

## See Also

۷1

# Examples

```
data(toys)
V2(D2.toy,other=NULL, phi=phi.toy)
V2(D2.toy,x.vec,phi=phi.toy)
```

۷d

Variance matrix for d

## Description

Variance matrix for d, as per the bottom of page 1 of the supplement

W 59

#### Usage

```
Vd(D1, D2, theta, phi)
```

## **Arguments**

D1 matrix of code run points
D2 matrix of observation points

theta Parameters

phi hyperparameters

#### Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. Supplementary details on Bayesian calibration of computer models, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
H. fun, V1, V2, C1
```

## **Examples**

```
data(toys)
Vd(D1=D1.toy, D2=D2.toy, theta=theta.toy, phi=phi.toy)
```

W

covariance matrix for beta

## **Description**

Covariance matrix of beta given theta, phi, d

## Usage

```
W(D1, D2, H1, H2, theta, det=FALSE, phi)
```

60

## **Arguments**

D1	Matrix whose rows are code run points
D2	Matrix whose rows are observation points
H1	regression function
H2	regression function
theta	parameters
det	Boolean, with default FALSE meaning to return the covariance matrix, and TRUE meaning to return its determinant.
phi	Hyperparameters

#### **Details**

This function is defined between equations 2 and 3 of the supplement. It is used in functions betahat.fun.koh(), p.eqn8.supp(), and p.joint().

Returns

$$\mathbf{W}(\theta) = \left(\mathbf{H}(\theta)^T \mathbf{V}_d(\theta)^{-1} \mathbf{H}(\theta)\right)^{-1}$$

If only the determinant is required, setting argument det to TRUE is faster than using det(W(..., det=FALSE)), as the former avoids an unnecessary use of solve().

#### Author(s)

Robin K. S. Hankin

# References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

betahat.fun.koh

```
data(toys)
W(D1=D1.toy, D2=D2.toy, H1=H1.toy, H2=H2.toy, theta=theta.toy, phi=phi.toy)
```

W1 61

## **Description**

returns the variance-covariance matrix for the estimate of beta1hat

## Usage

```
W1(D1, H1, det=FALSE, phi)
```

# Arguments

D1	matrix of code points
H1	Basis function generator
phi	Hyperparameters
det	Boolean, with default FALSE meaning to return the matrix, and TRUE meaning to return its determinant only

#### **Details**

If only the determinant is required, setting argument det to TRUE is faster than using det(W1(...,det=FALSE)), as the former avoids an unnecessary use of solve().

# Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

#### See Also

```
beta1hat.fun
```

```
data(toys)
W1(D1=D1.toy, H1=H1.toy, phi=phi.toy)
```

62 W2

W2

variance matrix for beta2

## **Description**

Variance matrix for beta2 as per page 4 of the supplement

## Usage

```
W2(D2, H2, V, det=FALSE)
```

# Arguments

D2	matrix of observation points
H2	regression function
V	Overall covariance matrix
det	Boolean, with default FALSE meaning to return the matrix, and TRUE meaning to return its determinant only

#### **Details**

If only the determinant is required, setting argument det to TRUE is faster than using det(W2(...,det=FALSE)), as the former avoids an unnecessary use of solve()

## Author(s)

Robin K. S. Hankin

#### References

- M. C. Kennedy and A. O'Hagan 2001. *Bayesian calibration of computer models*. Journal of the Royal Statistical Society B, 63(3) pp425-464
- M. C. Kennedy and A. O'Hagan 2001. *Supplementary details on Bayesian calibration of computer models*, Internal report, University of Sheffield. Available at http://www.tonyohagan.co.uk/academic/ps/calsup.ps
- R. K. S. Hankin 2005. *Introducing BACCO, an R bundle for Bayesian analysis of computer code output,* Journal of Statistical Software, 14(16)

# See Also

V.fun

```
data(toys)
W2(D2=D2.toy, H2=H2.toy, V=V.toy)
```

# **Index**

* array	* datasets
beta1hat.fun,4	create.new.toy.datasets, 13
beta2hat.fun, 6	toys, 49
betahat.fun.koh,7	* package
blockdiag, 9	calibrator-package, 2
C1, 10	
cov.p5.supp, 11	beta1hat.fun, 4, <i>61</i>
D1. fun, 14	beta2hat.fun, 5, 6
D2. fun, 15	betahat.fun.koh, 7, 60
dists.2frames, 16	blockdiag, 9
E.theta.toy, 17	C1 10 50
EK.eqn10.supp, 19	C1, 10, 59 calibrator (calibrator-package), 2
etahat, 21	calibrator (calibrator -package), 2
extractor.toy, 23	computer.model, 43
Ez.eqn7.supp, 24	computer.model(reality), 42
Ez.eqn9.supp, 25	Cov.eqn9.supp (cov.p5.supp), 11
H. fun, 27	cov.p5.supp, 11
H1.toy, 28	create.new.toy.datasets, 13, 51
h1.toy, 29	or cate mem. toy . aa tabe to, 15, 51
hbar.fun.toy, 30	d.toy (toys), 49
is.positive.definite, 31	D1.fun, 14, 23
MH, 32	D1.toy, <i>16</i> , 28
p.eqn4.supp, 33	D1.toy(toys), 49
p.eqn8.supp, 34	D2. fun, 15
p.page4, 36	D2.toy(toys), 49
phi.fun.toy, 37	dists.2frames, 16, <i>17</i>
prob.psi1,41	5 (1 ) 17 51
reality, 42	E. theta. toy, 17, 51
stage1,44	Edash. theta. toy (E. theta. toy), 17
symmetrize, 47	EK.eqn10.supp, 19
tee, 48	etahat, 21 extractor.toy, 23, <i>51</i>
tt.fun, 51	Ez.eqn7.supp, 24
V. fun, 54	Ez.eqn7.supp, 24 Ez.eqn9.supp, 25, 49
V1, 56	L2. eq113. Supp, 23, 49
V2, 57	H. fun, 27, 59
Vd, 58	h.fun(tee),48
W, 59	H1.toy, 28, 30, 40, 51
W1, 61	h1.toy, 29, <i>31</i>
W2, 62	H2.toy (H1.toy), 28

INDEX

```
h2.toy(h1.toy), 29
                                                   X.dist.toy(toys), 49
hbar.fun.toy, 30
                                                   x.toy (toys), 49
hh.fun (tt.fun), 51
                                                   x. toy2 (toys), 49
ht.fun(tt.fun), 51
                                                   x.vec (toys), 49
is.positive.definite, 31
                                                   y. toy (toys), 49
                                                   z. toy (toys), 49
latin.hypercube, 14
MH, 32
model.inadequacy(reality), 42
p.eqn4.supp, 33, 42
p.eqn8.supp, 32, 34, 42
p.equationn4.supp (p.eqn4.supp), 33
p.page4, 22, 36, 55
phi.change (phi.fun.toy), 37
phi.fun.toy, 37, 46, 51
phi.toy(toys), 49
phi.true (reality), 42
prob.psi1,41
prob.psi2 (prob.psi1), 41
prob.theta(prob.psi1), 41
reality, 14, 42
sample.theta(prob.psi1),41
stage1, 35, 42, 44, 51
stage2 (stage1), 44
stage3 (stage1), 44
symmetrize, 47
t.fun, 11
t.fun(tt.fun), 51
t.vec.toy(toys), 49
tee, 26, 48
theta.toy, 16
theta.toy (toys), 49
toys, 14, 15, 19, 23, 40, 46, 49
tt.fun, 51, 55
V. fun, 25, 53, 54, 62
V. toy (toys), 49
V1, 56, 58, 59
V2, 57, 57, 59
Vd, 58
W, 59
W1, 34, 61
W2, 7, 35, 37, 62
```