# Package 'SBMSplitMerge'

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Title Inference for a Generalised SBM with a Split Merge Sampler

Version 1.1.1

Description Inference in a Bayesian framework for a generalised stochastic block model. The generalised stochastic block model (SBM) can capture group structure in network data without requiring conjugate priors on the edge-states. Two sampling methods are provided to perform inference on edge parameters and block structure: a split-merge Markov chain Monte Carlo algorithm and a Dirichlet process sampler. Green, Richardson (2001) <doi:10.1111/1467-9469.00242>; Neal (2000) <doi:10.1080/10618600.2000.10474879>; Ludkin (2019) <arXiv:1909.09421>.

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# R topics documented:

ассерт	 	$\cdot$ $\cdot$ $\cdot$ $\cdot$	-
addblock	 		4
ARI	 		5
blockmat	 		5
blockmat.blocks	 		6
blockmat.numeric	 		6

olockmat.sbm	7
olockmod	7
olocks	8
olocktrace	9
стр	9
ldirichlet	10
ledges	10
ledges.numeric	11
ledges.sbm	12
lelblock	12
lma	13
lrawblock.dp	14
lrawblock.gibbs	14
lrawblocks.dp	15
lrawblocks.gibbs	16
Irawparams	16
edgemod	17
edges	17
edges_bern	18
edges_nbin	19
edges_norm	19
edges_pois	20
Enron	20
eval_plots	21
s.sbm	21
Macaque	22
narglike_bern	22
narglike_norm	23
narglike_pois	23
nergeavg	24
nergeblocks	24
nergeparams	25
nergeparams.default	25
nergeparams.numeric	26
nodeblocks	26
nultinom	27
nutunom	27
numblockstrace	
parammat	
parammat.blocks	
parammat.matrix	
parammat.params	
parammat.sbm	
parammod	
params	
paramtrace	
param_beta	
naram gamma	34

accept 3

param_nbin	35
param_norm	35
plot.blocks	36
plot.edges	37
plot.sbm	38
plotpostpairs	38
postpairs	39
rcat	39
rdirichlet	40
redges	40
rw	41
sampler	41
sampler.conj	43
sampler.dp	44
sampler.gibbs	44
sampler.rj	45
sbm	46
sbmmod	46
splitavg	47
splitblocks	48
splitparams	48
splitparams.numeric	49
splitparams.params	49
StackOverflow	50
updateblock	50
updateblock.blocks	51
updateblock.sbm	51
vmeasure	52
	53
	param_norm plot.blocks plot.edges plot.sbm plotpostpairs postpairs reat rdirichlet redges rw sampler sampler.conj sampler.dp sampler.dp sampler.gibbs sampler.rj sbm sbmmod splitavg splitblocks splitparams splitparams splitparams.numeric splitparams.params StackOverflow updateblock updateblock.blocks updateblock.sbm

accept

accept propsbm with the acceptance probability alpha

# Description

accept propsbm with the acceptance probability alpha

# Usage

```
accept(currsbm, propsbm, edges, sbmmod, logjac = 0, logu = 0, ...)
```

4 addblock

## Arguments

currsbm current sbm state
propsbm proposed sbm state
edges an edges object
sbmmod an sbmmod model

logjac log Jacobian of transformation of variables

logu log density for auxiliary variables

... additional arguments to pass to dedges

#### Value

updated sbm object

addblock Add a block move

# Description

proposes adding an empty block labelled kappa+1 to sbm

#### Usage

```
addblock(sbm, edges, sbmmod, rho = 1)
```

# Arguments

sbm the current state of the sampler

edges an edges object sbmmod an sbmmod model

rho probability of choosing to add a block

## Value

an updated sbm object

ARI 5

ARI

Adjusted Rand Index

## Description

Calculate the Adjusted Rand Index between two clusterings

#### Usage

```
ARI(z, truez)
```

## Arguments

z input vector truez reference vector

## Value

Adjusted Rand Index of z against truez

# **Examples**

```
ARI(c(1,1,2,2,3,3), c(2,2,1,1,3,3)) ## 1 - doesn't care for labels ARI(c(1,1,2,2,3,3), c(1,1,1,1,2,2)) ## 0.444
```

blockmat

Block matrix

## **Description**

converts x to a matrix of block assignments

#### Usage

```
blockmat(x, ...)
```

## Arguments

x object for dispatch

... additional arguments for method

#### Value

matrix of block assignment indicators

#### See Also

```
blockmat.sbm blockmat.blocks blockmat.numeric
```

6 blockmat.numeric

blockmat.blocks

Block matrix

## Description

converts block assignments of a blocks object to a matrix of block assignments

#### Usage

```
## S3 method for class 'blocks'
blockmat(blocks, kappa)
```

#### **Arguments**

blocks a blocks object

kappa number of blocks in matrix

#### Value

matrix with kappa rows and a 1 at (k, i) if node i is in block k under blocks

blockmat.numeric

Block matrix

## Description

converts a vector of block assignments to a matrix of block assignments

## Usage

```
## S3 method for class 'numeric'
blockmat(x, kappa)
## S3 method for class 'factor'
blockmat(x, kappa)
```

## **Arguments**

x a numeric-vector of node-to-block assignments

kappa number of blocks

#### Value

matrix with kappa rows and a 1 at (k,i) if node i is in block k under x

blockmat.sbm 7

|--|

#### **Description**

converts block assignments of an sbm object to a matrix of block assignments

#### Usage

```
## S3 method for class 'sbm'
blockmat(SBM, kappa)
```

## Arguments

SBM an sbm object

kappa number of blocks in matrix

#### Value

matrix with kappa rows and a 1 at (k,i) if node i is in block k under SBM

blockmod	Block Model	

#### **Description**

create a blockmod object

## Usage

```
blockmod(fixkappa, logd, dcond, r, ...)
```

#### Arguments

fixkappa Logical - is kappa fixed or can it vary under the model?

logd function(blocks) - log density for blocks

dcond function(blocks, i) - conditional density for the block assignment i in blocks

r function(n), sorted=FALSE - samples a blocks object from the model

... parameters of the model for use in r, logd, dcond

#### **Details**

A block model is a probability model for a blocks object. This class creates a closure with three functions: - a random method for sampling block a structure from the model with n nodes; a - a log-density method for computing the log-density of a given block structure in a blocks object - a conditional density function that takes a blocks object and a node i

8 blocks

#### Value

```
a blockmod object
```

## See Also

multinom dma crp blocks

blocks

Blocks object

## Description

create a blocks object

## Usage

```
blocks(z, kappa)
```

## Arguments

z vector of block labels for each node

kappa maximum number of blocks

#### **Details**

stores the block allocations and total number of blocks for a stochastic block model

## Value

a blocks object

```
## Assign six nodes to four blocks:
b <- blocks(c(1,1,2,3,4,4), 4)
print(b)
plot(b) ## shows id two nodes are members of the same block</pre>
```

blocktrace 9

blocktrace

plot a trace of the blocks from MCMC samples

## Description

plot a trace of the blocks from MCMC samples

## Usage

```
blocktrace(postz, burnin)
```

## **Arguments**

postz output from sampler

burnin which iterations to plot? defaults to all.

#### Value

'ggplot2' object

crp

Chinese Restaurant Process

#### **Description**

A blockmod for the Chinese restaurant process (CRP)

#### Usage

```
crp(gamma)
```

## Arguments

gamma

concentration parameter

## **Details**

The CRP posits that each node arrives in turn. The first node joins the first block. Each subsequent node starts a new block with probability 'gamma' or joins an existing block proportional to the block size.

#### Value

a block model representing a CRP(gamma) distribution

10 dedges

#### **Examples**

```
## simulate from a CRP(5) prior
m <- crp(5)
print(m)
m$r(10)</pre>
```

ddirichlet

Dirichlet distribution

#### **Description**

Density of Dirichlet distribution

## Usage

```
ddirichlet(x, gam, log = FALSE)
```

# Arguments

x random variable in the d-dimensional simplex

gam a length K concentration parameter log return the log-probability instead?

#### Value

the density

## **Examples**

```
g <- rep(2,5)
p <- rdirichlet(1, g) ## a length-5 probability vector
ddirichlet(p, g)</pre>
```

dedges

Density of edges

## Description

Compute the probability density for an edges object

## Usage

```
dedges(x, edges, edgemod, na.rm = TRUE, ...)
```

dedges.numeric 11

#### **Arguments**

```
x an R object for dispatch
edges an edges object
edgemod an edgemod object
na.rm remove NAs when calculating?
```

# ... additional arguments

#### Value

matrix same size as edges\$E with density of each edge

#### See Also

```
dedges.sbm dedges.sbm
```

dedges.numeric

likelihood of edges

## Description

likelihood of edges

## Usage

```
## S3 method for class 'numeric'
dedges(x, edges, edgemod, na.rm = na.rm, ...)
```

## **Arguments**

```
    a matrix of parameters (with same size as edges$E)
    edges
    edges object
    edgemod
    an edgemod object
    ra.rm
    remove NAs when calculating?
    additional arguments passed to edgemod$logd
```

#### Value

likelihood of edges under the edgemod using parameters in matrix pmat

12 delblock

dedges.sbm

Density of edges

#### **Description**

Compute the probability density for an edges object under an sbm object

## Usage

```
## S3 method for class 'sbm'
dedges(x, edges, edgemod, na.rm = TRUE, ...)
```

#### **Arguments**

```
x an sbm objectedges an edges objectedgemod an edgemod object
```

na.rm remove NAs when calculating?

... additional arguments for dedges.params

#### Value

matrix same size as edges\$E with density of each edge

#### **Examples**

```
## make an sbm model, sample data then plot and print:
model <- sbmmod(dma(2,5), param_beta(1,1,1,1), edges_bern())
s <- model$r(100)
e <- redges(s, model$edge)
dedges(s, e, model$edge)</pre>
```

delblock

Delete a block move

#### **Description**

proposes deleting an empty block (chosen at random among empty Blocks)

## Usage

```
delblock(sbm, edges, sbmmod, rho = 1)
```

dma 13

#### **Arguments**

sbm the current state of the sampler

edges an edges object sbmmod an sbmmod model

rho probability of choosing to add a block

#### Value

an updated sbm object

dma

Dirichlet Multinomial Allocation

## **Description**

A blockmod for Dirichlet Multinomial Allocation (DMA)

## Usage

```
dma(gamma, delta)
```

#### **Arguments**

gamma parameter for Dirichlet component delta parameter for Poison component

#### **Details**

This model posits:

 $kappa-1\ Pois(delta)$ 

 $omega|kappa, gamma\ Dirichlet(gamma)$ 

 $Z_i|omega\ Multinomial(omega)fori=1..n$ 

#### Value

a block model representing a dma(gamma, delta) distribution

```
## simulate from a DMA(2, 5) prior
## This models the `number of blocks-1` as Poisson(5)
## and block assignments as Dirichlet-Multinomial(2, 2, ...)
m <- dma(2, 5)
print(m)
m$r(10)</pre>
```

14 drawblock.gibbs

drawblock.dp

Draw block membership

#### **Description**

Draw block membership in a Dirichlet process sampler

#### Usage

```
drawblock.dp(i, currsbm, edges, sbmmod)
```

## Arguments

i node to update
currsbm current sbm object
edges an edges object
sbmmod an sbmmod object

#### **Details**

sample a new block assignment for i under a Dirichlet process. Care needs to be taken with singleton blocks to update the parameter model in currsbm.

#### Value

updated sbm object

## See Also

For full algorithm details see http://doi.org/10.17635/lancaster/thesis/296

drawblock.gibbs

Gibbs-like reassignment of nodes to the current set of blocks

#### **Description**

Reassign node 'i' to the current set of blocks given the current number of blocks and the other block assignments

## Usage

```
drawblock.gibbs(i, currsbm, edges, sbmmod)
```

drawblocks.dp 15

## **Arguments**

i the node to reassign

currsbm an sbm object
edges an edges object
sbmmod an sbmmod object

#### Value

updated sbm object with new block assignment for i

drawblocks.dp

Draw block memberships

## Description

Draw block memberships in a Dirichlet process sampler

## Usage

```
drawblocks.dp(currsbm, edges, sbmmod)
```

## Arguments

currsbm current sbm object
edges an edges object
sbmmod an sbmmod object

#### **Details**

iteratively updates the block assignment of each node using a Dirichlet process update move

## Value

updated sbm object

16 drawparams

drawblocks.gibbs

Gibbs-like reassignment of nodes to the current set of blocks

## Description

Sweep through the set of nodes and reassign to the current set of blocks given the current number of blocks

#### Usage

```
drawblocks.gibbs(currsbm, edges, sbmmod)
```

### **Arguments**

currsbm an sbm object
edges an edges object
sbmmod an sbmmod object

#### Value

updated sbm object with new block assignments

drawparams

Metropolis updates by drawing parameters

## **Description**

Simulate parameters for the given model with a Metropolis-Hastings step

#### Usage

```
drawparams(sbm, edges, sbmmod, sigma = 0.1)
```

## Arguments

sbm current sbm object

edges an edges sbmmod an sbmmod

sigma parameter for drawparam

## **Details**

iterate through the parameters in currsbm and update.

#### Value

updated sbm object

edgemod 17

edgemod

Class for edge models

#### **Description**

A class with a random and density method for edges objects

## Usage

```
edgemod(logd, r, ...)
```

## **Arguments**

logd function(e, p) to calculate likelihood of edge an edge e given parameter array p

r function(p) - simulate an edge given a parameter p (optional) . . . . additional arguments to append to edgemod internal list

#### Value

an edgemod object

#### Note

the parameter for logd is an array of c(dimension of theta, dim(E)) e.g. from parammat

## See Also

```
edges_bern edges_pois edges_norm
```

edges

Class for edge data

#### **Description**

A class to hold edge data

## Usage

```
edges(e, sym, loops, ...)
```

#### **Arguments**

e a matrix or array representing the raw edge-state data

sym is the network symmetric? (e[ji] = e[ji])

loops does the network contain self-loops? (edges from node i to i)

... additional arguments to append to edges internal list

18 edges\_bern

#### Value

```
an edges object
```

# Examples

```
## make an sbm model, sample data then plot and print:
model <- sbmmod(dma(2,5), param_beta(1,1,1,1), edges_bern())
s <- model$r(100)
e <- redges(s, model$edge)
plot(e)
plot(e, s)
print(e)</pre>
```

edges\_bern

Bernoulli edge model

## Description

Make an edgemod model with Bernoulli edge-states

#### Usage

```
edges_bern(...)
```

## Arguments

... additional parameters to pass to rbinom

## Value

an edgemod

```
eb <- edges_bern() ## makes `eb` an edgemod for Bernoulli edge-states</pre>
```

edges\_nbin 19

edges\_nbin

Negative-Binomial edge model

## Description

Make an edgemod model with Negative-Binomial edge-states

#### Usage

```
edges_nbin(...)
```

## **Arguments**

... additional parameters to pass to rnbinom

#### Value

an edgemod

## **Examples**

```
enb <- edges_nbin() ## makes `enb` an edgemod for Negative-Binomial edge-states</pre>
```

edges\_norm

Normal edge model

## Description

Make an edgemod model with Normal edge-states

## Usage

```
edges_norm(...)
```

## Arguments

... additional parameters to pass to rnorm

#### Value

an edgemod

```
en <- edges_norm() ## makes `en` an edgemod for Normal edge-states</pre>
```

20 Enron

edges\_pois

Poisson edge model

#### **Description**

Make an edgemod model with Poisson edge-states

## Usage

```
edges_pois(...)
```

#### **Arguments**

... additional parameters to pass to rpois

#### Value

an edgemod

#### **Examples**

```
ep <- edges_pois() ## makes 'ep' an edgemod for Poisson edge-states
```

Enron

The Enron data set as extracted from igraph using the script in data-raw

#### **Description**

A data set of counts of emails between email addresses This is a non-symmetric network. Nodes represent email address. The edge-state ij between two email addresses i and j is the number of emails sent from i to j The Groups vector is the node label from the igraph attribute "notes"

#### Usage

Enron

#### **Format**

A list containing

**Edges** an edges object with each edge-state representing the number of emails between two email addresses

**Groups** A vector giving a group name to which the email address belong. The order matches the edges such that Edges[i,j] is the edge-state between the nodes i and nodes j who are members of Groups[i] and Groups[j] respectively

eval\_plots 21

#### Source

https://cran.r-project.org/package=igraphdata

eval\_plots

get a set of evaluation plots from MCMC samples

## Description

get a set of evaluation plots from MCMC samples

#### Usage

```
eval_plots(output, burnin, theta_index)
```

## Arguments

output from sampler

burnin burn-in period (a vector of iteration numbers to subset outputs)

theta\_index which set of thetas to plot?

#### Value

list of ggplot objects (with descriptive names)

is.sbm

is.sbm

## Description

Logical check if an object is an sbm object

#### Usage

is.sbm(x)

## Arguments

Χ

an R object

# Value

Logical indicating if x is an sbm object

22 marglike\_bern

Macaque	The Macaque data set as extracted from igraph using the script in
	data-raw

## Description

The Macaque data set as extracted from igraph using the script in data-raw

## Usage

Macaque

#### **Format**

An edges object of activation counts between brain regions in a Macaque

#### See Also

igraph

marglike\_bern

Marginal likelihood model for Bernoulli distributed edges

## Description

calculate the marginal likelihood for a node for samplers using conjugate models

## Usage

```
marglike_bern(znoi, ei, parammod)
```

## **Arguments**

znoi a matrix of block assignments without node i

ei edge-states incident to i

parammod a parammod object representing the Bernoulli-Beta model

#### Value

log-probability of node i belonging to each block

marglike\_norm 23

marglike\_norm

Marginal likelihood model for Normal distributed edges

## **Description**

calculate the marginal likelihood for a node for samplers using conjugate models

## Usage

```
marglike_norm(znoi, ei, parammod)
```

## **Arguments**

znoi a matrix of block assignments without node i

ei edge-states incident to i parammod a parammod object

#### Value

log-probability of node i belonging to each block

marglike\_pois

Marginal likelihood model for Poisson distributed edges

## **Description**

calculate the marginal likelihood for a node for samplers using conjugate models

## Usage

```
marglike_pois(znoi, ei, parammod)
```

#### **Arguments**

znoi a matrix of block assignments without node i

ei edge-states incident to i parammod a parammod object

#### Value

log-probability of node i belonging to each block

24 mergeblocks

mergeavg	Merge	blocks
	1,10,00	0 00 0.00

#### **Description**

Merge-move using an average to merge parameters

#### Usage

```
mergeavg(sbm, edges, sbmmod, ...)
```

#### **Arguments**

sbm the current state of the sampler

edges an edges object sbmmod an sbmmod model

... additional parameter to 'accept'

#### **Details**

the blocks are chosen at random, the nodes reassigned to the block with the smallest index, then the parameters are combined using the average on the transformed scale

#### Value

an updated sbm object

mergeblocks

merge move block merging

#### **Description**

merge move block merging

#### Usage

```
mergeblocks(currblocks, propparams, edges, sbmmod, k, 1)
```

#### **Arguments**

currblocks current blocks

propparams proposed parameters
edges an edges object
sbmmod an sbmmod model
k Blocks to merge
1 Blocks to merge

mergeparams 25

#### Value

list(proposed block structure, log-acceptance-prob)

mergeparams

merge parameters

## Description

merge parameters

## Usage

```
mergeparams(x, ...)
```

#### **Arguments**

x an object to dispatch on

... additional arguments for methods

#### Value

merged parameters from x

## See Also

mergeparams.default mergeparams.numeric

```
mergeparams.default Merge step: parameters
```

## Description

Merge step: parameters

## Usage

```
## Default S3 method:
mergeparams(params, k, 1, parammod)
```

## Arguments

```
params a params object
k Blocks to merge
l Blocks to merge
parammod a parammod object
```

26 modeblocks

#### Value

```
list(proposed_params, log-acceptance-prob)
```

mergeparams.numeric

Merge step - parameter merging

# Description

Merge step - parameter merging

## Usage

```
## S3 method for class 'numeric'
mergeparams(thetak, thetal, x, parammod)
```

#### **Arguments**

thetak, thetal parameters to merge x auxiliary parameter parammod a parammod object

#### Value

list(proposed\_params, log-acceptance-prob)

modeblocks

modal block assignments from MCMC samples

#### **Description**

modal block assignments from MCMC samples

## Usage

```
modeblocks(postz)
```

# Arguments

postz

output from sampler

#### Value

a blocks object with the modal block assignments under postz

multinom 27

multinom

Multinomial block assignment

## Description

A blockmod for Multinomial allocation

#### Usage

```
multinom(gamma, kappa)
```

#### **Arguments**

 ${\tt gamma} \qquad \qquad {\tt parameter \ for \ Dirichlet} \ component \ Dirichlet} (gamma,...,gamma)$ 

kappa the number of blocks

#### **Details**

This model posits that: for i=1:n

 $Z_i Multinomial(omega)$ 

where

 $omega\ Dirichlet(gamma)$ 

#### Value

a block model representing a Multinomial (gamma) distribution

## **Examples**

```
## A fixed number of blocks with multinomial assignment of nodes m \leftarrow multinom(1, 4) print(m) m r(10) ## simulate a blocks object with 10 nodes
```

nodelike

Likelihood of node assignment

## Description

Calculate the likelihood of a nod belonging to each of block

## Usage

```
nodelike(blocks, params, edges, i, sbmmod, ...)
```

28 numblockstrace

## **Arguments**

blocks an blocks object
params an params object
edges an edges object
i the node of interest
sbmmod an sbmmod object

... additional arguments for nodelike.blocks

#### **Details**

the number of blocks considered is either the number of blocks in sbm (kappa) or kappa+1 when sbmmod has a variable number of blocks. care is taken for data which is directed and with loops.

#### Value

likelihood of edges emanating from node i

numblockstrace

plot a trace of the number of blocks from MCMC samples

## Description

plot a trace of the number of blocks from MCMC samples

#### Usage

```
numblockstrace(postk, burnin)
```

# Arguments

postk output from sampler

burnin which iterations to plot? defaults to all.

#### Value

```
'ggplot2' object
```

parammat 29

parammat

Parameter Matrix

# Description

Make a matrix of parameters

## Usage

```
parammat(x, ...)
```

#### **Arguments**

x object for dispatch

... additional arguments for method

#### Value

a parameter matrix object

parammat.blocks

Parameter Matrix

## Description

Make a matrix of parameters from a blocks and params object

## Usage

```
## S3 method for class 'blocks'
parammat(x, params, ...)
```

#### **Arguments**

```
x a blocks object params a params object ... (unused)
```

#### Value

```
an NxN matrix P, with P[i, j] = the parameter governing edge ij
```

30 parammat.params

parammat.matrix

Parameter Matrix

## Description

Make a matrix of parameters from a matrix of block assignments

# Usage

```
## S3 method for class 'matrix'
parammat(zleft, zright, params, ...)
```

#### **Arguments**

zleft block assignment matrix on the left zright block assignment matrix on the right

params the parameters object

... (unused)

#### Value

a matrix of parameters of size |z|eft| x |zright|

parammat.params

Parameter Matrix

#### **Description**

Make a matrix of parameters from a params object

#### Usage

```
## S3 method for class 'params'
parammat(x, kappa, ...)
```

## Arguments

```
x a params objectkappa - number of blocks to compute for matrix (optional)... (unused)
```

#### Value

a matrix of parameters

parammat.sbm 31

parammat.sbm

Parameter Matrix

# Description

Make a matrix of parameters from an sbm object

# Usage

```
## S3 method for class 'sbm'
parammat(x, ...)
```

#### **Arguments**

```
x an sbm object
... (unused)
```

#### Value

a matrix of parameters

parammod

Parameter Model

# Description

```
create a parammod object
```

## Usage

```
parammod(logd, r, t, invt, loggradt, ...)
```

# Arguments

logd	function(params) - log-density function for parameters
r	function(kappa) - random function to draw parameters
t	mapping parameter space to real line
invt	mapping real line to parameter space
loggradt	log of the gradient of mapping t
	additional arguments to store in the parammod object

32 params

#### **Details**

A parameter model is a probability model for a params object. This class creates a closure with five functions: - a random method for sampling a params object - a log-density method for computing the log-density of a given params object - a transformation function t that maps a parameter value to the real line - the inverse of t - the log-gradient of t

#### Value

a parammod object

#### See Also

param\_beta param\_gamma param\_nbin param\_norm

params

params S3 object

## Description

make a params object from the between-block parameter theta0 and a vector of within block parameters thetak

#### Usage

```
params(theta0, thetak)
```

#### **Arguments**

theta0 between block parameters - a vector of length 'dimension of theta'

thetak within block parameters - a matrix with ncol=kappa and nrow=dimension of

theta

## Value

a params object

```
p \leftarrow params(0.1, c(0.2, 0.4, 0.5)) p
```

paramtrace 33

paramtrace

plot a trace of parameter values from MCMC samples

## Description

plot a trace of parameter values from MCMC samples

## Usage

```
paramtrace(theta, range, burnin)
```

## **Arguments**

theta output from sampler

range which thetas to plot? defaults to all. burnin which iterations to plot? defaults to all.

#### Value

'ggplot2' object

param\_beta

Beta parameter model

## **Description**

A parammod with beta-distributed parameters

#### Usage

```
param_beta(a0, a1, b0, b1)
```

## Arguments

a0	theta_0 ~ Beta(a0,a1)
a1	theta_0 ~ Beta(a0,a1)
b0	theta_k ~ Beta(b0,b1)
b1	theta_k ~ Beta(b0,b1)

#### **Details**

This model represents a prior on theta with:

```
theta<sub>0</sub> Beta(a0, a1)
theta<sub>k</sub> Beta(b0, b1)
```

```
for k = 1 \dots kappa
```

param\_gamma

#### Value

```
a parammod
```

## **Examples**

```
## theta0 ^{\circ} Beta(1,9); thetak ^{\circ} Beta(9,1) pb <- param_beta(1,9,9,1) pb$r(5) ## a draw with 5 within-block parameters
```

param\_gamma

Gamma parameter model

# Description

A parammod with gamma-distributed parameters

#### Usage

```
param_gamma(a0, a1, b0, b1)
```

#### **Arguments**

a0	theta_0 ~ Gamma(a0,a1)
a1	theta_0 ~ Gamma(a0,a1)
b0	theta_k ~ Gamma(b0,b1)
b1	theta_k ~ Gamma(b0,b1)

#### **Details**

This model represents a prior on theta with:

```
theta_0 \; Gamma(a0,a1)
```

 $theta_k \ Gamma(b0, b1)$ 

```
for k = 1 ... kappa
```

## Value

a parammod

```
## theta0 ~ Gamma(1,1); thetak ~ Gamma(5,5)
pg <- param_gamma(1,1,5,5)
pg$r(5) ## a draw with 5 within-block parameters</pre>
```

param\_nbin 35

param\_nbin

Parameter model for Negative Binomial

#### **Description**

Negative Binomial parameter model: theta\_0 = (mu0, sigma0) theta\_k = (muk, sigmak)

#### Usage

```
param_nbin(a0, a1, b0, b1, c0, c1, d0, d1)
```

#### **Arguments**

```
a0, a1 mu0 \sim Gamma(a0,a1)

b0, b1 sig_0 \sim Beta(b0,b1)

c0, c1 muk \sim Gamma(c0,c1)

d0, d1 sig_k \sim Beta(d0,d1)
```

## Value

parammod representing Negative-Binomial distributed parameters

#### **Examples**

```
## theta0 = (r0, p0); r0^{Gamma}(1,1); p0^{Gamma}(1,1); ## thetak = (rk, pk); rk^{Gamma}(3,3); pk^{Gamma}(3,3); pk^{Gamma}(5,5); pn^{Gamma}(5,5); pn^{Gamma}(5,5) ## a draw with 5 within-block parameters
```

param\_norm

Parameter model for Normal Model

#### **Description**

```
Normal parameter model: theta_0 = (mu0, sigma0) theta_k = (muk, sigmak)
```

## Usage

```
param_norm(a0, a1, b0, b1, c0, c1, d0, d1)
```

#### **Arguments**

```
a0, a1 mu0 \sim Normal(a0,a1)
b0, b1 sig_0 \sim Gamma(b0,b1)
c0, c1 muk \sim Normal(c0,c1)
d0, d1 sig_k \sim Gamma(d0,d1)
```

36 plot.blocks

#### Value

parammod representing Normal distributed parameters

#### **Examples**

```
## theta0 = (mu0, sigma0); mu0\sim Normal(0,5); sigma0 \sim Gamma(1,1); ## thetak = (muk, sigmak); muk\sim Normal(0,3); sigmak \sim Gamma(5,2); pn <- param_norm(0,5,1,1,0,3,5,2) pn$r(5) ## a draw with 5 within-block parameters
```

plot.blocks

Plot blocks

#### **Description**

plots a block object

#### Usage

```
## S3 method for class 'blocks'
plot(x, col, xaxt = "n", yaxt = "n", xlab = "Nodes", ylab = "Nodes", ...)
## S3 method for class 'blocks'
image(x, col, xaxt = "n", yaxt = "n", xlab = "Nodes", ylab = "Nodes", ...)
```

## Arguments

X	a blocks object to plot
col	colours for the plot
xaxt	override image parameters
yaxt	override image parameters
xlab	override image parameters
ylab	override image parameters
	additional parameters for image

#### **Details**

plot the block assignments in a blocks object as a matrix, color-coded by block membership

```
## Assign six nodes to four blocks:
b <- blocks(c(1,1,2,3,4,4), 4)
plot(b)
## note that the lower left corner has one 2x2 red square
## indicating node 1 and 2 belong to the same block</pre>
```

plot.edges 37

plot.edges Plot

# **Description**

```
plots an edges objects
```

#### Usage

```
## S3 method for class 'edges'
plot(x, Blocks, sorted = TRUE, xlab = "Node", ylab = "Node", ...)
## S3 method for class 'edges'
image(x, Blocks, sorted = TRUE, xlab = "Node", ylab = "Node", ...)
```

# **Arguments**

```
x an edges object

Blocks a blocks object or sbm object

sorted sort by block membership in sbm before plotting?

xlab label for x-axis

ylab label for y-axis

... parameters for image
```

#### Value

```
ggplot2 plot of edges in a raster
```

```
## make an sbm model, sample data then plot and print:
model <- sbmmod(dma(2,5), param_beta(1,1,1,1), edges_bern())
s <- model$r(100)
e <- redges(s, model$edge)
plot(e)
plot(e, s)
print(e)</pre>
```

38 plotpostpairs

plot.sbm

Plot for sbm object

# Description

```
plot an sbm object as an image
```

# Usage

```
## S3 method for class 'sbm'
plot(x, col, ...)
## S3 method for class 'sbm'
image(x, col, ...)
```

# Arguments

an sbm object

col colours for each block - if missing, rainbow is used

... additional arguments for plot

# See Also

plot.default

plotpostpairs

helper function for trace plots

# Description

helper function for trace plots

# Usage

```
plotpostpairs(mat)
```

# **Arguments**

mat

matrix to plot as an image using ggplot2

# Value

```
'ggplot2' plot objecy
```

postpairs 39

postpairs

mean proportion of times two nodes were in the same block under MCMC samples

# **Description**

mean proportion of times two nodes were in the same block under MCMC samples

# Usage

```
postpairs(postz)
```

# **Arguments**

postz

output from sampler

#### Value

matrix P with P[i,j] = proportion of times i and j are in the same block under postz

rcat

Draw draw Categorical distribution

# Description

Draw draw Categorical distribution

# Usage

```
rcat(n, p, replace = TRUE)
```

# **Arguments**

n number of draws

p a length-d probability vector

replace should the categories be replaced? If so n < p required

#### Value

```
a draw from Categorical(p)
```

```
rcat(1, 1) ## returns 1 with probability 1
rcat(1, rep(1/6,6)) ## a dice roll
```

40 redges

rdirichlet

Dirichlet distribution

# **Description**

Draw from Dirichlet distribution

# Usage

```
rdirichlet(n, gam)
```

# Arguments

n number of variates to draw

gam a vector of concentration parameters of length K

#### Value

matrix dimension n\*k of samples

# **Examples**

```
rdirichlet(1, rep(2,5)) ## a length-5 probability vector
```

redges

Simulate edges

# Description

Simulate edges from an sbm object with a given edgemod

# Usage

```
redges(SBM, edgemod, sym = TRUE, loops = FALSE, ...)
```

# Arguments

SBM an sbm object edgemod an edgemod object

sym should the network be symmetric?

loops should the network have self-loops?

... additional arguments passed to edgemod\$r

# **Details**

None

rw 41

# Value

```
an edges object
```

# **Examples**

```
## make an sbm model, sample data then plot and print:
model <- sbmmod(dma(2,5), param_beta(1,1,1,1), edges_bern())
s <- model$r(100)
e <- redges(s, model$edge)
plot(e)
plot(e, s)
print(e)</pre>
```

rw

Random Walk

# Description

performs a random walk on a parameter value with a given parameter model

# Usage

```
rw(p, pm, sigma)
```

# **Arguments**

```
p a parameterpm a parammod objectsigma - scale of random walk
```

#### Value

```
ist(proposed parameter, locjacobian)
```

sampler

top level sampler function

# Description

top level sampler function

42 sampler

# Usage

```
sampler(
  edges,
  sbmmod,
  nSteps = 1000,
  algorithm = "rj",
  sigma = 0.5,
  statusfreq,
  currsbm,
  ...
)
```

# Arguments

edges	an edges object
sbmmod	an sbmmod model
nSteps	number of steps to run sampler
algorithm	choice of algorithm options are: "conjugate", "gibbs", "dp", "rj"
sigma	random walk parameter for theta
statusfreq	print the elapsed number of iterations every statusfreq iterations
currsbm	initial state for sbm object (optional - one is drawn from sbmmod if not supplied)
• • •	additional parameters to pass to step

# Value

```
postz traces for block assignments z

postt traces for theta

postk traces for number of blocks kappa

postn traces for number of occupied blocks

nsteps number of iterations of chain

algorithm choice
```

```
## see vignette("Weibull-edges")
```

sampler.conj 43

sam			

Conjugate model sampler

# Description

Conjugate model sampler

#### Usage

```
sampler.conj(currsbm, edges, sbmmod, sigma = NULL, ...)
```

# Arguments

currsbm the current state of the sampler
edges an edges object
sbmmod an sbmmod model
sigma unused

... additional arguments for sbmmod\$marglike

#### Value

next state of currsbm object

#### Note

If using the CRP as the block model, then this is the IRM sampler of Schmidt or Morup (Schmidt, M.N. and Morup, M., 2013. Nonparametric Bayesian modeling of complex networks: An introduction. IEEE Signal Processing Magazine, 30(3), pp.110-128.)

```
model <- sbmmod(crp(3), param_beta(1,1,1,1), edges_bern(), marglike=marglike_bern)
trueSBM <- model$r(100)
Edges <- redges(trueSBM, model$edge)
out <- sampler(Edges, model, 10, "conjugate")</pre>
```

sampler.gibbs

sampler.dp

Dirichlet process sampler

# Description

Dirichlet process sampler

# Usage

```
sampler.dp(currsbm, edges, sbmmod, sigma)
```

# Arguments

currsbm the current state of the sampler

edges an edges object sbmmod an sbmmod model

sigma random walk parameter for theta

#### Value

next state of currsbm object

#### See Also

For full algorithm details see http://doi.org/10.17635/lancaster/thesis/296

# **Examples**

```
model <- sbmmod(crp(4), param_norm(0,0,1,1,3,3,1,1), edges_norm())
trueSBM <- model$r(100)
Edges <- redges(trueSBM, model$edge)
dp_out <- sampler(Edges, model, 25, "dp", sigma=0.1)</pre>
```

sampler.gibbs

Gibbs sampling for node assignments

# Description

Gibbs sampling for node assignments

# Usage

```
sampler.gibbs(currsbm, edges, sbmmod, sigma)
```

sampler.rj 45

# Arguments

currsbm the current state of the sampler

edges an edges object sbmmod an sbmmod model

sigma random walk parameter for theta

#### Value

next state of currsbm object

#### Note

This requires a block model with a fixed kappa

# Examples

```
model <- sbmmod(multinom(1, 3), param_gamma(1,1,1,1), edges_pois())
trueSBM <- model$r(10)
Edges <- redges(trueSBM, model$edge)
gibbs_out <- sampler(Edges, model, algorithm="gibbs", 10, sigma=0.1)
eval_plots(gibbs_out)</pre>
```

sampler.rj

reversible jump Markov chain Monte Carlo split-merge sampler

# **Description**

reversible jump Markov chain Monte Carlo split-merge sampler

#### **Usage**

```
sampler.rj(currsbm, edges, sbmmod, sigma, rho = 10)
```

#### **Arguments**

currsbm the current state of the sampler

edges an edges object sbmmod an sbmmod model

sigma random walk parameter for theta

rho propensity to add a block

#### Value

next state of currsbm object

46 sbmmod

#### See Also

For full algorithm details see http://doi.org/10.17635/lancaster/thesis/296

#### **Examples**

```
model <- sbmmod(dma(1,10), param_nbin(1,1,4,4,0.5,0.5,0.5,0.5), edges_nbin())
trueSBM <- model$r(100)
Edges <- redges(trueSBM, model$edge)
rj_out <- sampler(Edges, model, 10, "rj", sigma=0.1)</pre>
```

sbm

 ${\it Class}$  sbm

#### Description

Class sbm

# Usage

```
sbm(blocks, params)
```

# **Arguments**

blocks a blocks object params a params object

#### Value

an sbm object

# **Examples**

```
sbm(blocks(c(1,1,2,2,3,3)), params(0.1, c(0.4,0.5,0.6)))
```

 ${\tt sbmmod}$ 

Stochastic block model object

# Description

A wrapper for a block and parameter model

# Usage

```
sbmmod(blockmod, parammod, edgemod, ...)
```

splitavg 47

# **Arguments**

```
blockmod a blockmod object
parammod a parammod object
edgemod an edgemod object
```

... additional arguments to store in the sbmmod object

#### **Details**

Simple wrapper for the block and parameter model for an sbm object

#### Value

an sbmmod object with a method r(n) sampling an sbm object with n nodes from the model and a method logd(sbm) computing the log-density of sbm under the model

# Author(s)

Matthew Ludkin

# See Also

blockmod parammod edgemod

splitavg

split move using average to merge parameters

# Description

split move using average to merge parameters

#### Usage

```
splitavg(sbm, edges, sbmmod, ...)
```

# **Arguments**

sbm the current state of the sampler

edges an edges object sbmmod an sbmmod model

... additional parameter to 'accept'

#### Value

an updated sbm object

48 splitparams

splitblocks

split move: blocks

# Description

```
split move: blocks
```

# Usage

```
splitblocks(currblocks, propparams, edges, sbmmod, k)
```

# **Arguments**

currblocks current blocks

propparams proposed parameters
edges an edges object
sbmmod a model list
k block to split

# Value

list(proposed block structure, log-acceptance-prob)

splitparams

split move: parameters

# Description

```
split move: parameters
```

# Usage

```
splitparams(x, ...)
```

# Arguments

x object for dispatch

... additional arguments for method

# Value

```
list(proposed_params, log-acceptance-prob)
```

splitparams.numeric 49

```
splitparams.numeric split move: params
```

# Description

```
split move: params
```

# Usage

```
## S3 method for class 'numeric'
splitparams(theta, u, x, parammod)
```

# Arguments

theta a parameter to split
u auxiliary variable
x auxiliary variable
parammod parammod object

#### Value

```
list(proposed_params, log-acceptance-prob)
```

```
splitparams.params split move: params
```

# **Description**

```
split move: params
```

# Usage

```
## S3 method for class 'params'
splitparams(params, k, parammod)
```

# Arguments

params a params object to split

k block to split parammod object

#### Value

```
list(proposed_params, log-acceptance-prob)
```

50 updateblock

StackOverflow

The Stack-Overflow data set as extracted from igraph
using the script in data-raw Extracted on 27/8/2019
from Kaggle (login required) using: library(rvest)
read\_html("https://www.kaggle.com/stackoverflow/stack-overflow-tag-network/dow

#### **Description**

The Stack-Overflow data set as extracted from igraph using the script in data-raw Extracted on 27/8/2019 from Kaggle (login required) using: library(rvest) read\_html("https://www.kaggle.com/stackoverflow/

# Usage

StackOverflow

#### **Format**

An edges object of activation counts between brain regions in a Macaque

# **Source**

https://www.kaggle.com/stackoverflow/stack-overflow-tag-network/

#### See Also

igraph

updateblock Update the block assignment of a node

# **Description**

change the block assignment in x of a node to a new block

# Usage

```
updateblock(x, ...)
```

#### **Arguments**

x object for dispatch

... additional arguments for method

# Value

object like 'x' with updated block structure

updateblock.blocks 51

#### See Also

updateblock.blocks updateblock.sbm

updateblock.blocks

Update the block assignment of a node

# **Description**

change the block assignment in an blocks object to a new block

# Usage

```
## S3 method for class 'blocks'
updateblock(blocks, i, newblock)
```

# **Arguments**

blocks a blocks object i the node to update

newblock the new block for node i

#### Value

new blocks object

updateblock.sbm

Update the block assignment of a node

# Description

change the block assignment in an sbm object to a new block

# Usage

```
## S3 method for class 'sbm'
updateblock(currsbm, i, newblock, model)
```

# **Arguments**

currsbm an sbm object
i the node to update
newblock the new block for node i
model an sbmmod object

52 vmeasure

#### Value

new sbm object

#### Note

If adding a new block, this draws from the prior

vmeasure

V-measure

#### **Description**

Calculate the V-measure of two clusterings

# Usage

```
vmeasure(z, truez, beta = 1)
```

# **Arguments**

z input vector truez reference vector

beta parameter beta=1 gives equal weight to homogeneity and completeness

#### **Details**

An information based measure of similarity between two clusterings

#### Value

v-measure of z against truez

#### See Also

Rosenberg, A., & Hirschberg, J. (2007, June). V-measure: A conditional entropy-based external cluster evaluation measure. In Proceedings of the 2007 joint conference on empirical methods in natural language processing and computational natural language learning (EMNLP-CoNLL) (pp. 410-420).

```
vmeasure(c(1,1,2,2,3,3), c(2,2,1,1,3,3)) ## 1 - doesn't care for labels vmeasure(c(1,1,2,2,3,3), c(1,1,2,2,2,2)) ## 0.7333 vmeasure(c(1,1,2,2,3,3), c(1,1,2,2,3,4)) ## 0.904
```

# **Index**

* datasets Enron, 20 Macaque, 22 StackOverflow, 50	<pre>image.blocks(plot.blocks), 36 image.edges(plot.edges), 37 image.sbm(plot.sbm), 38 is.sbm, 21</pre>
accept, 3 addblock, 4 ARI, 5 blockmat, 5	Macaque, 22 marglike_bern, 22 marglike_norm, 23 marglike_pois, 23 mergeavg, 24
blockmat.blocks, 5, 6 blockmat.factor(blockmat.numeric), 6 blockmat.numeric, 5, 6 blockmat.sbm, 5, 7 blockmod, 7, 9, 13, 27, 47 blocks, 8, 8, 28, 51	mergeblocks, 24 mergeparams, 25 mergeparams.default, 25, 25 mergeparams.numeric, 25, 26 modeblocks, 26 multipom 8, 27
blocktrace, 9  crp, 8, 9	multinom, 8, 27 nodelike, 27 numblockstrace 28
	numblockstrace, 28
ddirichlet, 10	param_beta, 32, 33
dedges, 10	param_gamma, 32, 34
dedges.numeric, 11	param_nbin, 32, 35
dedges.sbm, 11, 12	param_norm, <i>32</i> , <i>35</i>
delblock, 12	parammat, <i>17</i> , 29
dma, 8, 13	parammat.blocks, 29
drawblock.dp, 14	parammat.matrix, 30
drawblock.dp, 14 drawblock.gibbs, 14	parammat.matrix, 30 parammat.params, 30
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16 edgemod, 11, 12, 17, 40, 47	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33 plot.blocks, 36
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16 edgemod, 11, 12, 17, 40, 47 edges, 4, 10–16, 17, 24, 28, 37, 42–45, 47	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33 plot.blocks, 36 plot.edges, 37
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16 edgemod, 11, 12, 17, 40, 47 edges, 4, 10–16, 17, 24, 28, 37, 42–45, 47 edges_bern, 17, 18	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33 plot.blocks, 36 plot.edges, 37 plot.sbm, 38
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16 edgemod, 11, 12, 17, 40, 47 edges, 4, 10–16, 17, 24, 28, 37, 42–45, 47 edges_bern, 17, 18 edges_nbin, 19	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33 plot.blocks, 36 plot.edges, 37 plot.sbm, 38 plotpostpairs, 38
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16 edgemod, 11, 12, 17, 40, 47 edges, 4, 10–16, 17, 24, 28, 37, 42–45, 47 edges_bern, 17, 18 edges_nbin, 19 edges_norm, 17, 19	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33 plot.blocks, 36 plot.edges, 37 plot.sbm, 38
drawblock.dp, 14 drawblocks.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16 edgemod, 11, 12, 17, 40, 47 edges, 4, 10–16, 17, 24, 28, 37, 42–45, 47 edges_bern, 17, 18 edges_nbin, 19 edges_norm, 17, 19 edges_pois, 17, 20	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33 plot.blocks, 36 plot.edges, 37 plot.sbm, 38 plotpostpairs, 38 postpairs, 39
drawblock.dp, 14 drawblock.gibbs, 14 drawblocks.dp, 15 drawblocks.gibbs, 16 drawparams, 16 edgemod, 11, 12, 17, 40, 47 edges, 4, 10–16, 17, 24, 28, 37, 42–45, 47 edges_bern, 17, 18 edges_nbin, 19 edges_norm, 17, 19	parammat.matrix, 30 parammat.params, 30 parammat.sbm, 31 parammod, 31, 47 params, 28, 32 paramtrace, 33 plot.blocks, 36 plot.edges, 37 plot.sbm, 38 plotpostpairs, 38

54 INDEX

```
redges, 40
rw, 41
sampler, 41
sampler.conj, 43
sampler.dp, 44
sampler.gibbs, 44
sampler.rj, 45
sbm, 12, 15, 16, 21, 37, 38, 40, 46, 51
sbmmod, 4, 13–16, 24, 28, 42–45, 46, 47, 51
splitavg, 47
{\tt splitblocks}, 48
{\tt splitparams}, {\tt 48}
splitparams.numeric, 49
splitparams.params, 49
{\tt StackOverflow}, {\tt 50}
updateblock, 50
updateblock.blocks, 51, 51
updateblock.sbm, 51,51
vmeasure, 52
```