

# Package ‘GulFM’

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**Type** Package

**Title** General Unilateral Load Estimator for Two-Layer Latent Factor Models

**Version** 0.1.2

**Description** Implements general unilateral loading estimator for two-layer latent factor models with smooth, element-wise factor transformations. We provide data simulation, loading estimation, finite-sample error bounds, and diagnostic tools for zero-mean and sub-Gaussian assumptions. A unified interface is given for evaluating estimation accuracy and cosine similarity. The philosophy of the package is described in Guo G. (2026) <[doi:10.1016/j.apm.2025.116280](https://doi.org/10.1016/j.apm.2025.116280)>.

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**Encoding** UTF-8

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**Depends** R (>= 3.5.0)

**Imports** MASS, matrixStats

**Suggests** testthat (>= 3.0.0), ggplot2

**NeedsCompilation** no

**Language** en-US

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**estimate\_gul\_loadings** *General unilateral load Estimator*

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### Description

General unilateral load Estimator

### Usage

```
estimate_gul_loadings(X, m)
```

### Arguments

- X n \*p data matrix (already centred and scaled if desired).
- m number of latent factors (both layers).

### Details

Step 1: PCA on X to get hat\_A1 Step 2: Regress X on hat\_A1 to get hat\_gF1 Step 3: PCA on hat\_gF1 to get hat\_A2 Step 4: hat\_Ag = hat\_A1

### Value

A list with hat\_A1 : p \* m 1st-layer loadings hat\_A2 : m \* m 2nd-layer loadings hat\_Ag : p \* m overall loadings Sigma1 : p \* p sample cov(X) (for diagnostics) Sigma2 : m \* m sample cov(hat\_gF1) hat\_gF1 : n \* m estimated transformed latent factors eig1 : eigen-values of Sigma1 eig2 : eigen-values of Sigma2

### Examples

```
dat <- generate_gfm_data(500, 50, 5, tanh, seed = 1)
est <- estimate_gul_loadings(dat$X, m = 5)
err <- sqrt(mean((est$hat_Ag - dat$Ag)^2)) # overall RMSE
```

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<code>generate_gfm_data</code>	<i>Generate general factor model with smooth latent transformation</i>
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## Description

Generate general factor model with smooth latent transformation

## Usage

```
generate_gfm_data(n, p, m, g_fun, seed = 1, sigma_V = 0.1)
```

## Arguments

<code>n</code>	Integer: sample size.
<code>p</code>	Integer: number of observed variables.
<code>m</code>	Integer: number of latent factors (both layers).
<code>g_fun</code>	Function: smooth, element-wise transformation applied to latent factors. Must be vectorised, e.g. ‘sin’, ‘tanh’, ‘scale’.
<code>seed</code>	1.
<code>sigma_V</code>	Numeric: standard deviation of the idiosyncratic noise (default 0.1 => Var = 0.01).

## Value

List with components X : n \* p matrix of standardised observations. A1 : p \* m first-layer loading matrix. A2 : m \* m second-layer loading matrix. Ag : p \* m overall loading matrix (Ag = A1 F1 : n \* m latent factors (before transformation). gF1: n \* m latent factors (after transformation). V1 : n \* p noise matrix (for diagnostics).

## Examples

```
dat <- generate_gfm_data(200, 50, 5, g_fun = tanh)
```

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<code>gul_simulation</code>	<i>Single-replication GUL simulation</i>
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## Description

Generates one synthetic data set, estimates loadings with the GUL, and evaluates estimation accuracy.

## Usage

```
gul_simulation(n, p, m, g_fun)
```

**Arguments**

<i>n</i>	Integer: sample size.
<i>p</i>	Integer: number of observed variables.
<i>m</i>	Integer: number of latent factors (both layers).
<i>g_fun</i>	Function: element-wise, smooth transformation applied to the latent factors (e.g. ‘tanh’, ‘sin’).

**Value**

Named numeric vector with components error\_F : Frobenius norm  $\|\hat{A}g\|_F$

**Examples**

```
gul_simulation(200, 50, 5, g_fun = tanh)
```

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*g\_fun*

*Smooth link functions compliant with Theorems 9&10*

---

**Description**

Returns a vectorised map  $g(\cdot)$  and its exact Lipschitz constant  $L_g$  for three increasingly nonlinear choices.

**Usage**

```
g_fun(type = c("linear", "weak_nonlinear", "strong_nonlinear"))
```

**Arguments**

<i>type</i>	Character string selecting the map: “linear”, “weak_nonlinear”, or “strong_nonlinear”.
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**Value**

Named list with components

<i>g_fun</i>	vectorised function $g(\cdot)$
<i>L_g</i>	scalar Lipschitz constant of $g$

**Examples**

```
## pick a link with L_g = 1
tmp <- g_fun("linear")
dat <- generate_gfm_data(n = 500, p = 200, m = 5, g_fun = tmp$g_fun)
est <- estimate_gul_loadings(dat$X, m = 5)
err <- norm(est$hat_Ag - dat$Ag, "F")
sprintf("F-error (L_g = %d) = %.3f", tmp$L_g, err)
```

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**g\_theorem***Simulation wrapper for Theorems 9 & 10*

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**Description**

One Monte-Carlo replicate; returns empirical error, exceedance indicator, theoretical bounds, and assumption-check flags.

**Usage**

```
g_theorem(n, p, m, g_type, epsilon, zero_tol = 0.02)
```

**Arguments**

n	sample size
p	number of observed variables
m	number of latent factors
g_type	character: "linear", "weak_nonlinear", "strong_nonlinear"
epsilon	error threshold
zero_tol	zero-mean tolerance (default 0.02)

**Value**

one-row data-frame

**Examples**

```
df <- g_theorem(500, 200, 5, "linear", 0.6)
```

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**loading\_metrics***Multi-metric evaluation of factor loading matrix estimation error*

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**Description**

Multi-metric evaluation of factor loading matrix estimation error

**Usage**

```
loading_metrics(A_true, A_hat)
```

**Arguments**

A_true	True loading matrix (p x m)
A_hat	Estimated loading matrix (p x m)

**Value**

data.frame with MSE, RMSE, MAE, MaxDev, and Cosine similarity

**Examples**

```
## simulated example
p <- 100; m <- 5
Ag_true <- matrix(rnorm(p*m), p, m)
Ag_hat <- Ag_true + matrix(rnorm(p*m, 0, 0.1), p, m)
metrics <- loading_metrics(Ag_true, Ag_hat)
print(metrics)
```

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verify\_mean

*Verify zero-mean preservation (Theorem 10 assumption 2a)*

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**Description**

Draws n i.i.d.  $N(0, I_m)$  latent factors, applies g component-wise, and checks whether  $|E[g(x)]| < tol$  on every coordinate.

**Usage**

```
verify_mean(g_fun, m = 5, n = 10000, tol = 0.001)
```

**Arguments**

g_fun	vectorised map g: R -> R
m	latent dimension
n	Monte-Carlo sample size
tol	numerical tolerance (default 1e-3)

**Value**

logical TRUE if  $|mean| < tol$  on all coords

**Examples**

```
tmp <- g_fun("weak_nonlinear")
verify_mean(tmp$g_fun, m = 5)
```

---

verify\_subgaussian      *Verify sub-Gaussian preservation*

---

### Description

Draws  $n$  i.i.d.  $N(0, I_m)$  latent factors, applies  $g$  component-wise, and checks whether  $E[\exp(g(x))]$  remains below an empirical cut-off. This is a quick proxy for finite sub-Gaussian norm.

### Usage

```
verify_subgaussian(g_fun, m = 5, n = 1000, cut = exp(2))
```

### Arguments

g_fun	vectorised map $g: \mathbb{R} \rightarrow \mathbb{R}$
m	latent dimension
n	Monte-Carlo sample size
cut	empirical threshold (default $\exp(2) \approx 7.389$ )

### Value

logical TRUE if  $E[\exp(g)] < \text{cut}$  on all coords

### Examples

```
tmp <- g_fun("strong_nonlinear")
verify_subgaussian(tmp$g_fun, m = 5)
```

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