# Package 'apc'

October 12, 2022

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# Description

The package includes functions for age-period-cohort analysis. The statistical model is a generalized linear model (GLM) allowing for age, period and cohort factors, or a sub-set of the factors. The canonical parametrisation of Kuang, Nielsen and Nielsen (2008a) is used. The outline of an analysis is described below.

## **Details**

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Package: apc
Type: Package
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The apc package uses the canonical parameters suggested by Kuang, Nielsen and Nielsen (2008a) and generalized by Nielsen (2014). These evolve around the second differences of age, period and cohort factors as well as an three parameters (level and two slopes) for a linear plane. The age, period and cohort factors themselves are not identifiable. They could be ad hoc identified by associating the levels and two slopes to the age, period and cohort factors in a particular way. This should be done with great care as such ad hoc identification easily masks which information is coming from the data and which information is coming from the choice of ad hoc identification scheme. An illustration is given below. A short description of the package can be found in Nielsen (2015).

A formal analysis of the identification of the age-period-cohort model can be found in Nielsen and Nielsen (2014). Forecasting is discussed in Kuang, Nielsen and Nielsen (2008b, 2011) and Martinez Miranda, Nielsen and Nielsen (2015). Methods for cross section data are introduced in Fannon, Monden and Nielsen (2019). Methods for panel data are introduced in Fannon (2020). For a recent overview see Fannon and Nielsen (2019).

The package covers age-period-cohort models for three types of data.

- 1. Tables of aggregate data.
- 2. Repeated cross sectional data.
- 3. Panel data.

The apc package can be used as follows.

- 1. **Aggregate data**. For a vignette with an introduction to analysis of aggregate data, see see IntroductionAggregateData.pdf, IntroductionAggregateData.R on Vignettes.
  - (a) Organize the data in as an apc.data.list. Data are included in matrix format. Information needs to be given about the original data format. Optionally, information can be given about the labels for the time scales.
  - (b) Construct descriptive plots using apc.plot.data.all. This gives a series of descriptive plots. The plots can be called individually through
    - i. Plot data sums using apc.plot.data.sums. Numerical values can be obtained through apc.data.sums.
    - ii. Sparsity plots of data using apc.plot.data.sparsity.
    - iii. Plot data using all combinations of two time scales using apc.plot.data.within.
  - (c) Get an deviance table for the age-period-cohort model through apc.fit.table.
  - (d) Estimate a particular (sub-model of) age-period-cohort model through apc.fit.model.
  - (e) Plot probability transforms of observed responses given fit using apc.plot.fit.pt.
  - (f) Plot estimated parameters through apc.plot.fit. Numerical values of certain transformations of the canonical parameter can be obtained through apc.identify.

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(g) Recursive analysis can be done by selecting a subset of the observations through apc.data.list.subset and then repeating analysis. This will reveal how sensitive the results are to particular age, period and cohort groups.

- (h) Forecasting. Some functions have been been added for forecasting in from a Poisson response-only model with an age-cohort parametrization apc.forecast.ac and with an age-period parametrization apc.forecast.ap. See also the overview on apc.forecast
- 2. **Repeated cross section** and **Panel Data**. For a vignette with an introduction to analysis of repeated cross section data and panel data, see <a href="IntroductionIndividualData.pdf">IntroductionIndividualData.R</a> on <a href="Vignettes">Vignettes</a> Further examples can be found in a second vignette, see <a href="IntroductionIndividualDataFurtherExample.R">IntroductionIndividualDataFurtherExample.R</a>.

#### Data examples include

# 1. Aggregate data

- (a) data.asbestos includes counts of deaths from mesothelioma in the UK. This dataset has no measure for exposure. It can be analysed using a Poisson model with an "APC" or an "AC" design. Source: Martinez Miranda, Nielsen and Nielsen (2015). Also used in Nielsen (2015).
- (b) data.Italian.bladder.cancer includes counts of deaths from bladder cancer in the Italy. This dataset includes a measure for exposure. It can be analysed using a Poisson model with an "APC" or an "AC" design. Source: Clayton and Schifflers (1987a).
- (c) data.Belgian.lung.cancer includes counts of deaths from lung cancer in the Belgium. This dataset includes a measure for exposure. It can be analysed using a Poisson model with an "APC", "AC", "AP" or "Ad" design. Source: Clayton and Schifflers (1987a).
- (d) data.Japanese.breast.cancer includes counts of deaths from breast cancer in the Japan. This dataset includes a measure for exposure. It can be analysed using a Poisson model with an "APC" design. Source: Clayton and Schifflers (1987b).

## Repeated cross section data

(a) Wage data from the package ISLR

## Repeated cross section data

(a) PSID7682 data from the package AER. These are panel data on earnings for 595 individuals for the years 1976-1982.

#### Author(s)

Bent Nielsen <br/> bent.nielsen@nuffield.ox.ac.uk> 29 Jan 2015 updated 26 Aug 2020.

#### References

Clayton, D. and Schifflers, E. (1987a) Models for temperoral variation in cancer rates. I: age-period and age-cohort models. *Statistics in Medicine* 6, 449-467.

Clayton, D. and Schifflers, E. (1987b) Models for temperoral variation in cancer rates. II: age-period-cohort models. *Statistics in Medicine* 6, 469-481.

Fannon, Z. (2020). D.Phil. thesis. University of Oxford.

Fannon, Z., Monden, C. and Nielsen, B. (2018) Age-period cohort modelling and covariates, with an application to obesity in England 2001-2014. *Download*: Nuffield DP. Supplement Code for replication: Nuffield DP supplement.

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Fannon, Z. and Nielsen, B. (2019) Age-period-cohort models. *Oxford Research Encyclopedia of Economics and Finance*. Oxford University Press. *Download*: doi.org/10.1093/acrefore/9780190625979.013.495; Earlier version Nuffield DP.

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008b) Forecasting with the age-period-cohort model and the extended chain-ladder model. Biometrika 95, 987-991. *Download*: Article; Earlier version Nuffield DP.

Kuang, D., Nielsen, B. and Nielsen, J.P. (2011) Forecasting in an extended chain-ladder-type model. Journal of Risk and Insurance 78, 345-359. *Download*: Article; Earlier version: Nuffield DP.

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Article, Nuffield DP.

Nielsen, B. (2015) apc: An R package for age-period-cohort analysis. *R Journal* 7, 52-64. *Download*: Open access.

Nielsen, B. (2014) Deviance analysis of age-period-cohort models. *Download*: Nuffield DP.

Nielsen, B. and Nielsen, J.P. (2014) Identification and forecasting in mortality models. The Scientific World Journal. vol. 2014, Article ID 347043, 24 pages. *Download*: Article.

## See Also

Vignettes are available on Vignettes.

Further information, including minor upgrades and a python version can be found on apc development web page.

# Examples

# see vignettes

apc-internal

Internal apc Functions

#### **Description**

Internal apc functions

#### **Details**

These are not to be called by the user.

#### Author(s)

Bent Nielsen <br/> bent.nielsen@nuffield.ox.ac.uk> 1 Feb 2016

apc.data.list

Arrange data as an apc.data.list

## **Description**

This is step 1 of the apc analysis.

The apc package is aimed at range of data types. This analysis and labelling of parameters depends on the choice data type. In order to keep track of this choice the data first has to be arranged as an apc.data.list. The function purpose of this function is to aid the user in constructing a list with the right information.

Age period cohort analysis is used in two situations. A dose-response situation, where both doses (exposure, risk set, cases) and responses (counts of deaths, outcomes) are available. And a response situation where only a response is available. If the aim is to directly model mortality ratios (counts of death divided by exposure) this will be thought of a response

The apc.data.list gives sufficient information for the further analysis. It is sufficient to store this information. It has 2 obligatory arguments, which are a response matrix and a character indicating the data format. It also has some further optional arguments, which have certain default values. Some times it may be convenient to add further arguments to the apc.data.list. This will not affect the apc analysis.

apc.data.list generates default row and column names for the response and dose matrices when these are not provided by the user.

#### **Usage**

```
apc.data.list(response, data.format, dose=NULL,
age1=NULL, per1=NULL, coh1=NULL, unit=NULL,
per.zero=NULL, per.max=NULL,
time.adjust=NULL, label=NULL,
n.decimal=NULL)
```

## **Arguments**

response

matrix (or vector). Numbers of responses. It should have a format matching data.format. Time should be increasing with the row/column index of the matrix. For instance, consider a 10x5 matrix in "AP" format: Then the row index is for age, and it should be increasing in age. Thus, higher ages are further down the rows of the matrix. In the same way, the column index is for period.

data.format

character. The following options are implemented:

"AC" has age/cohort as increasing row/column index.

"AP" has age/period as increasing row/column index.

"CA" has cohort/age as increasing row/column index.

"CL" has cohort/age as increasing row/column index, triangular.

"CP" has cohort/period as increasing row/column index.

"PA" has period/age as increasing row/column index.

'' <b>PC</b> '' h	nas period	/cohort as	increasing	row/co	lumn index.
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<sup>&</sup>quot;trapezoid" has age/period as increasing row/column index, period-diagonals are NA for period <= per.zero and >per.zero+per.max.

dose Optional. matrix or NULL. Numbers of doses. It should have same format as

response.

age1 Optional. Numeric or NULL. Time label for youngest age group. Used if

 ${\tt data.format\ is\ "AC",\ "AP",\ "CA",\ "CL",\ "PA",\ "trapezoid".\ If\ NULL\ default\ and\ trapezoid\ default\ defau$ 

is unit.

per1 Optional. Numeric or NULL. Time label for oldest period group. Used if

data.format is "AP", "CP", "PA", "PC". If NULL default is unit.

coh1 Optional. Numeric or NULL. Time label for youngest age group. Used if

data.format is "AC", "CA", "CL", "CL.vector.by.row", "CP", "PC", "trape-

zoid". If NULL default is unit.

unit Optional. Numeric or NULL. Common time steps for age, period and cohort.

For quarterly data use 1/4. For monthly data use 1/12. If NULL default is 1.

per.zero Optional. Numeric or NULL. Needed if data format is "trapezoid".

per.max Optional. Numeric or NULL. Needed if data format is "trapezoid".

time.adjust Optional. Numeric. Time labels are based on two of age1, per1 and coh1. The

third time label is computed according to the formula age 1+coh 1=per1+time.adjust.

Default is 0. If age1=coh=1 it is natural to choose time.adjust=1.

label Optional. Character. Useful when working with multiple data sets. Some inter-

nal functions use the first three characters of the label for identification of the

two datasets.

n.decimal Optional. Numeric or NULL. The labels for parameters involves a date. This is

found by converting a number into a character. If the value is set to d package uses sprintf. If the value is set to NULL and unit==1/4 for quarterly data or unit==1/12 for monthly data or 1/20<=unit && unit<1 then package uses sprintf. If the value is set to NULL and 1/20>unit || unit>=1 then package uses as.character, which looks nice for integers, but can be messy otherwise.

# **Details**

If the user does not set values for any of age1, per1, coh1, unit then the value is set to unit.

The user can set values of age1, per1, coh1 that are incongruent. The functions only use two these that are relevant for the chosen data.format. Example: the data.format may be "AC" and the user sets age1, per1, but age1, coh1 are relevant for this data format. The apc.data.list then sets coh1=unit, by default, while ignoring the value for per1. Other commands such as apc.data.list.subset or apc.fit.table, will internally, as default option, call the function apc.get.index. That function will, in this example, set per1 according to the values of age1 and coh1.

If the user does not set a value for time.adjust this is set equal to unit when the user does not specify at least two age1, per1, coh1. Otherwise it is set to 0. The former choice matches the values in the theory papers, where indices count 1,2,... to follow standard notation for row/column indices for matrices, so that age+coh=per+unit. The latter choice seeks to match a real time scale the user sets according to age+coh=per.

#### Value

response matrix (or vector). Numbers of responses.

dose matrix (or NULL). Numbers of doses.

data.format character.

age1 Numeric. Default is NULL.
per1 Numeric. Default is NULL.
coh1 Numeric. Default is NULL.

unit Numeric. Default is NULL. For monthly data one use unit=1/12.

per.zero Numeric. If data.format is not "trapezoid" the value is NULL. If data.format is

"trapezoid" the coordinate system is in age-cohort format and this value counts

how many periods are cut off. The default is per.zero=0.

per.max Numeric. If data.format is not "trapezoid" the value is NULL. If data.format is

"trapezoid" the coordinate system is in age-cohort format and this value counts

how many periods are included in the data array. The default is per.max=nrow(response)+ncol(respon

time.adjust Numeric. Default is NULL.
label Character. Default of NULL.

n.decimal Numeric or NULL.

#### Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 17 Nov 2016

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Nielsen, B. (2014) Deviance analysis of age-period-cohort models. *Download*: Nuffield DP.

Nielsen, B. (2015) apc: An R package for age-period-cohort analysis. *R Journal* 7, 52-64. *Download*: Open access.

#### See Also

The below example shows how the data. Japanese.breast.cancer data.list was generated. Other provided data sets include data.asbestos data.Belgian.lung.cancer data.Italian.bladder.cancer.

A subset of the data can be selected using apc.data.list.subset.

## **Examples**

```
age1=25,per1=1955,coh1=NULL,unit=5,
per.zero=NULL,per.max=NULL)
data.list
# (2) Chain Ladder data
k <- 5
v.response \leftarrow seq(1:(k*(k+1)/2))
data.list <- apc.data.list(response=vector.2.triangle(v.response,k),</pre>
data.format="CL.vector.by.row",age1=2001)
data.list
################
# Japanese breast cancer
# This is the code used to generate the data.Japanese.breast.cancer
v.rates <- c( 0.44, 0.38, 0.46, 0.55, 0.68,
    1.69, 1.69, 1.75, 2.31, 2.52,
 4.01, 3.90, 4.11, 4.44, 4.80,
 6.59, 6.57, 6.81, 7.79, 8.27,
 8.51, 9.61, 9.96, 11.68, 12.51,
 10.49, 10.80, 12.36, 14.59, 16.56,
 11.36,11.51,12.98,14.97,17.79,
 12.03, 10.67, 12.67, 14.46, 16.42,
 12.55, 12.03, 12.10, 13.81, 16.46,
 15.81,13.87,12.65,14.00,15.60,
 17.97, 15.62, 15.83, 15.71, 16.52)
v.cases <- c( 88, 78, 101, 127, 179, 299, 330, 363, 509, 588,
   596, 680, 798, 923, 1056,
   874, 962, 1171, 1497, 1716,
 1022, 1247, 1429, 1987, 2398,
 1035, 1258, 1560, 2079, 2794,
   970, 1087, 1446, 1828, 2465,
   820, 861, 1126, 1549, 1962,
   678, 738, 878, 1140, 1683,
   640, 628, 656, 900, 1162,
   497, 463, 536, 644, 865)
# see also example below for generating labels
rates <- matrix(data=v.rates,nrow=11, ncol=5,byrow=TRUE)</pre>
cases <- matrix(data=v.cases,nrow=11, ncol=5,byrow=TRUE)</pre>
# A data list is now constructed as follows
# note that list entry rates is redundant,
# but included since it represents original data
data.Japanese.breast.cancer <- apc.data.list(response=cases,</pre>
dose=cases/rates,data.format="AP",
age1=25,per1=1955,coh1=NULL,unit=5,
per.zero=NULL,per.max=NULL,time.adjust=0,
label="Japanese breast cancer")
# or when exploiting the default values
```

10 apc.data.list.subset

apc.data.list.subset *Cut age, period and cohort groups from data set.* 

## **Description**

For a recursive analysis it is useful to be able to cut age, period and cohort groups from a data set. Function returns an apc.data.list with data.format "trapezoid".

When used with default values the function turns an apc.data.list into a new apc.data.list with data.format "trapezoid" without reducing dataset.

#### Usage

```
apc.data.list.subset(apc.data.list,
age.cut.lower=0,age.cut.upper=0,
per.cut.lower=0,per.cut.upper=0,
coh.cut.lower=0,coh.cut.upper=0,
apc.index=NULL,
suppress.warning=FALSE)
```

## Arguments

apc.data.list	List. See apc.data.list for a description of the format.
age.cut.lower	<i>Optional</i> . Numeric. Specifies how many age groups to cut at lower end. Default is zero.
per.cut.lower	<i>Optional</i> . Numeric. Specifies how many period groups to cut at lower end. Default is zero.
coh.cut.lower	<i>Optional</i> . Numeric. Specifies how many cohort groups to cut at lower end. Default is zero.
age.cut.upper	<i>Optional</i> . Numeric. Specifies how many age groups to cut at upper end. Default is zero.
per.cut.upper	Optional. Numeric. Specifies how many period groups to cut at upper end. Default is zero.

apc.data.list.subset

coh.cut.upper	<i>Optional</i> . Numeric. Specifies how many cohort groups to cut at upper end. Default is zero.	
apc.index	<i>Optional</i> . List. See apc.get.index for a description of the format. If not provided this is computed internally.	
suppress.warning		

*Optional*. Logical. Suppresses warnings. This is useful when generating data sums using apc.data.sums but reducing the data set so much that models cannot be fitted.

## Value

response	matrix (or vector). Numbers of responses.
dose	matrix (or NULL). Numbers of doses.
data.format	"trapezoid"
age1	Numeric.
per1	Numeric.
coh1	Numeric.
unit	Numeric.
per.zero	Numeric.
per.max	Numeric.

## **Arguments: Notes**

If apc.index is supplied then the input can be simplified. It suffices to write apc.data.list = list(response=response,data.format=data.format,dose=dose), where dose could be dose=NULL. Likewise apc.index does not need to be a full apc.index list. It suffices to construct a list with entries age.max, per.max, coh.max, age1, per1, coh1, unit, per.zero, index.trap, index.data.

#### Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 4 Dec 2013 recoded 26 Apr 2017

#### See Also

The below example uses artificial data. For an example using data.asbestos see apc.plot.fit.

## **Examples**

12 apc.data.sums

```
apc.data.list.subset(data.list,1,1,0,0,0,0)
```

apc.data.sums

Computes age, period and cohort sums of a matrix

## **Description**

Computes age, period and cohort sums of a matrix. This is the same as taking column, row and diagonal sums. The match between the age, period and cohort sums and column, row and diagonal sums depends on the data format

## Usage

```
apc.data.sums(apc.data.list,data.type="r",
average=FALSE,keep.incomplete=TRUE,apc.index=NULL)
```

## **Arguments**

apc.data.list List. See apc.data.list for a description of the format.

data.type Optional. Character. "r","d","m" if sums are computed for responses,dose,(mortality)

rates. Rates are computed as responses/doses. "r" is default.

average Optional. Logical. If TRUE/FALSE reports averages/sums. Default is FALSE.

keep.incomplete

Optional. Logical. If true perform calculation for incomplete sequences by removing NA. If false incomplete sequences are NA. See example. Default=TRUE.

apc.index Optional. List. See apc.get.index for a description of the format. If not

provided this is computed.

# Value

sums.age	Vector. Sums/Averages over data.matrix by age.
sums.per	Vector. Sums/Averages over data.matrix by period.
sums.coh	Vector. Sums/Averages over data.matrix by cohort.

## **Arguments: Notes**

If apc.index is supplied then the input can be simplified. For instance if data.type="r" then, for the first argument, it suffices to write apc.data.list = list(response=response). Likewise apc.index does not need to be a full apc.index list. It suffices to construct a list with entries age.max, per.max, coh.max, index.trap, index.data, per.zero.

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 15 Aug 2018 (15 Dec 2013)

apc.data.sums

## See Also

The example below uses Japanese breast cancer data, see data. Japanese.breast.cancer

## **Examples**

```
# EXAMPLE with artificial data
# generate a 3x4 matrix in "AP" data.format with the numbers 1..12
m.data <- matrix(data=seq(length.out=12),nrow=3,ncol=4)</pre>
m.data
data.list <- apc.data.list(m.data,"AP")</pre>
apc.data.sums(data.list)
# $sums.age
# [1] 22 26 30
# $sums.per
# [1] 6 15 24 33
# $sums.coh
# [1] 3 8 15 24 18 10
apc.data.sums(data.list,average=TRUE)
# $sums.age
# [1] 5.5 6.5 7.5
# $sums.per
#[1] 2 5 8 11
# $sums.coh
#[1] 3 4 5 8 9 10
apc.data.sums(data.list,keep.incomplete=FALSE)
# $sums.age
# [1] 22 26 30
# $sums.per
# [1] 6 15 24 33
# $sums.coh
# [1] NA NA 15 24 NA NA
##############################
# EXAMPLE with Japanese breast cancer data
data.list <- data.Japanese.breast.cancer() # function gives data list</pre>
apc.data.sums(data.list)
# $sums.age
# [1] 573 2089 4053 6220 8083 8726 7796 6318 5117 3986 3005
# $sums.per
# [1] 7519 8332 10064 13183 16868
# $sums.coh
# [1] 497 1103 1842 2858 4474 5550 6958 7471 7531 6931 5111 3080 1666 715 179
# Compare with the response matrix
```

#### data.list\$response

#		1955-1959	1960-1964	1965-1969	1970-1974	1975-1979
#	25-29	88	78	101	127	179
#	30-34	299	330	363	509	588
#	35-39	596	680	798	923	1056
#	40-44	874	962	1171	1497	1716
#	45-49	1022	1247	1429	1987	2398
#	50-54	1035	1258	1560	2079	2794
#	55-59	970	1087	1446	1828	2465
#	60-64	820	861	1126	1549	1962
#	65-69	678	738	878	1140	1683
#	70-74	640	628	656	900	1162
#	75-79	497	463	536	644	865

apc.fit.model

Fits an age period cohort model

## **Description**

apc.fit.model fits the age period cohort model as a Generalized Linear Model using glm.fit. The model is parametrised in terms of the canonical parameter introduced by Kuang, Nielsen and Nielsen (2008), see also the implementation in Martinez Miranda, Nielsen and Nielsen (2015). This parametrisation has a number of advantages: it is freely varying, it is the canonical parameter of a regular exponential family, and it is invariant to extentions of the data matrix. Other parametrizations can be computed using apc.identify.

apc.fit.model can be be used for all three age period cohort factors, or for submodels with fewer of these factors.

apc.fit.model can be used either for mortality rates through a dose-response model or for mortality counts through a pure response model without doses/exposures.

The GLM families include Poisson regressions (with log link) and Normal/Gaussian least squares regressions.

apc.fit.table produces a deviance table for 15 combinations of the three factors and linear trends: "APC", "AP", "AC", "PC", "Ad", "Pd", "Cd", "A", "P", "C", "t", "tA", "tP", "tC", "1".

#### Usage

```
apc.fit.model(apc.data.list,model.family,model.design,apc.index=NULL,
replicate.version.1.3.1=FALSE)
apc.fit.table(apc.data.list,model.family,model.design.reference="APC",
apc.index=NULL)
```

## Arguments

apc.data.list List. See apc.data.list for a description of the format.

model.family Character. The following options are implemented. These are used internally when calling glm.fit.

- "poisson.response" This sets family=poisson(link="log"). Only responses are used. Inference is done in a multinomial model, conditioning on the overall level as documented in Martinez Miranda, Nielsen and Nielsen (2015).
- "od.poisson.response" This sets family=quasipoisson(link="log") in the estimation step, but then reverts to family=poisson(link="log") when computing standard errors, which are then corrected. Only responses are used. Inference is done in an over-dispersed Poisson model as documented in Harnau and Nielsen (2016). Note that limit distributions are t and F not normal and chi2.
- "poisson.dose.response" This sets family=poisson(link="log"). Doses are used as offset.
- "binomial.dose.response" This sets family=binomial(link="logit") and gives a logistic regression.
- **"gaussian.rates"** This sets family=gaussian(link="identity"). The dependent variable is the mortality rates, which are computed as response/dose.
- "gaussian.response" This sets family=gaussian(link="identity"). Only responses are used. The dependent variable is the responses.
- "log.normal.rates" Gaussian regression for log(rates) and with identity link (Least Squares).
- "log.normal.response" Gaussian regression for log(response) and with identity link (Least Squares).

## model.design

Character. This indicates the design choice. The following options are possible.

- "APC" Age-period-cohort model.
- "AP" Age-period model. Nested in "APC"
- "AC" Age-cohort model. Nested in "APC"
- "PC" Period-cohort model. Nested in "APC"
- "Ad" Age-trend model, including age effect and two linear trends. Nested in "AP", "AC".
- "Pd" Period-trend model, including period effect and two linear trends. Nested in "AP", "PC".
- "Cd" Cohort-trend model, including cohort effect and two linear trends. Nested in "AC", "PC".
- "A" Age model. Nested in "Ad".
- "P" Period model. Nested in "Pd".
- "C" Cohort model. Nested in "Cd".
- "t" Trend model, with two linear trends. Nested in "Ad", "Pd", "Cd".
- "tA" Single trend model in age index. Nested in "A", "t".
- "tP" Single trend model in period index. Nested in "P", "t".
- "tC" Single trend model in cohort index. Nested in "C", "t".
- "1" Constant model. Nested in "tA", "tP", "tC".

#### model.design.reference

Character. This indicates the reference design choice for the deviance table. Choices are "APC", "AP", "AC", "PC", "Ad", "Pd", "Cd", "A", "P", "C", "t". Default is "APC".

apc.index

*Optional.* List. See apc.get.index for a description of the format. If not provided this is computed internally. If apc.fit.model is used in a simulation study computational effort can be saved when using this option.

replicate.version.1.3.1

*Optional*. Logical. Replicate error in covariance calculation for "poisson.response", "od.poisson.response in versions 1.2.3-1.3.1. Default=FALSE

#### Value

*apc.fit.table* produces a deviance table. There are 15 rows corresponding to all possible design choices. The columns are as follows.

"-2logL"

-2 log Likelihood up to some constant. If the model family is Poisson or binomial (logistic) this is the same as the glm deviance: That is the difference in -2 log likelihood value between estimated model and the saturated model. If the model family is Gaussian it is different from the traditional glm deviance. Here the -2 log likelihood value is measured in a model with unknown variance, which is the standard in regression analysis, whereas in the glm package the deviance is the residual sum of squares, which can be interpreted as the -2 log likelihood value in a model with variance set to one.

"df.residual"

Degrees of freedom of residual: nrow x ncol - dim(parameter). If the model.family="poisson.response" the degrees of freedom is one lower.

"prob(>chi\_sq)"

p-value of the deviance, -2logL. Left out in Gaussian case which has no saturated model

"LR vs APC"

the likelihood ratio statistic against the "APC" model.

"df"

Degrees of freedom against the "APC" model.

"prob(>chi\_sq)"

p-value of log likelihood ratio statistic.

"aic"

Akaike's "An Information Criterion", minus twice the maximized log-likelihood plus twice the number of parameters upto a constant. It is take directly from the glm function. For the "poisson.dose.response" and "binomial.dose.response" model families the dispersion is fixed at one and the number of parameters is the number of coefficients. The "poisson.response" model is conditional on the level. The number of parameters should therefore be adjusted by subtracting 2 to take this into account to get the proper AIC. However, in practice this does not matter, since we are only interested in relative effects. For the "gaussian.response" and "gaussian.dose.response" model families the dispersion is estimated from the residual deviance.

"F"

Only for "od.poisson.response". F statistic: Ratio of deviance for submodel divided by degrees of freedom to deviance of apc model divided by degrees of freedom.

"prof(>F)"

Only for "od.poisson.response". F statistic: with degrees of freedom given by differences between sub-model and apc model and between apc model and saturated model.

apc.fit.model returns a list. The entries are as follows.

fit List. Values from glm. fit.

apc.index List. Values from apc.get.index.

coefficients.canonical

Matrix. For each coordinate of the canonical parameters is reported coefficient, standard deviation, z-value, which is the ratio of those, and asymptotically normal p-values. Note, for "od.poisson.response" the reported standard errors corrected by the deviance and p-values are asymptotically t distributed, see Harnau and Nielsen (2016). Other parametrizations can be computed using apc.identify.

covariance.canonical

Matrix. Estimated covariance matrix for canonical parameters.

slopes Vector. Length three. The design matrix found by apc.get.design.collinear

has age, period, and cohort linear trends. slopes indicates which of these are

actually used in estimation.

difdif Vector. Length three. The design matrix found by apc.get.design.collinear

has age, period, and cohort double differences. slopes indicates which of these

are actually used in estimation.

index.age Vector. Indices for age double difference parameters within coefficients.canonical.

NULL if age double differences are not estimated.

index.per Vector. Indices for period double difference parameters within coefficients.canonical.

NULL if period double differences are not estimated.

index.coh Vector. Indices for cohort double difference parameters within coefficients.canonical.

NULL if cohort double differences are not estimated.

dates Vector. Indicates the dates for the double difference parameters within coefficients.canonical.

model.family Character. Argument.
model.design Character. Argument.

RSS Numeric. Residual sum of squares. NULL for non-gaussian families.

sigma2 Numeric. Maximum likelihood estimator for variance: RSS/n. NULL for non-

gaussian families.

s2 Numeric. Least squares estimator for variance: RSS/df. NULL for non-gaussian

families.

n.decimal Numeric. From apc.data.list.

predictors Vector. Design\*Estimates. Same as the glm.fit value linear.predictors

when there is no offset.

#### Note

For gaussian families *deviance* is defined differently in apc and glm. Here it is -2 log likelihood. In glm it is RSS.

The values for apc.fit.model include the apc.data.list and the apc.index returned by apc.get.index.

For the poisson.response the inference is conditional on the level, see Martinez Miranda, Nielsen and Nielsen (2015). The coefficients.canonical computed by apc are therefore different from the default coefficients computed by glm.

For the od.poisson.response an asymptotic theory is used that mimics the conditioning for poisson.response. The asymptotic distribution are, however, asymptotically t or F distributed, see Harnau and Nielsen (2017).

For the log.normal.response standard normal theory applies for quantities on the log scale including estimators. An asymptotic theory for quantities on the original scale is provided in Kuang and Nielsen (2018).

For coefficients the 3rd and 4th columns have headings t value and Pr(>|t|) for od. poisson. response to indicate an asymptotic t theory and otherwise z value and Pr(>|z|) to indicate an asymptotic normal theory. The labels are inherited from glm. fit.

#### Author(s)

Bent Nielsen <br/> bent.nielsen@nuffield.ox.ac.uk> 15 Aug 2018 (27 Aug 2014)

#### References

Harnau, J. and Nielsen (2016) Over-dispersed age-period-cohort models. To appear in *Journal of the American Statistical Association*. *Download*: Nuffield DP

Kuang, D, Nielsen B (2018) Generalized log-normal chain-ladder. mimeo Nuffield Collge.

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Article, Nuffield DP.

#### See Also

The fit is done using glm. fit.

The examples below use Italian bladder cancer data, see data.Italian.bladder.cancer and Belgian lung cancer data, see data.Belgian.lung.cancer.

In example 3 the design matrix is called is called using apc.get.design.

## **Examples**

#### 

```
# EXAMPLE 1 with Italian bladder cancer data
```

data.list <- data.Italian.bladder.cancer() # function gives data list
apc.fit.table(data.list,"poisson.dose.response")</pre>

#		-2logL	${\sf df.residual}$	<pre>prob(&gt;chi_sq)</pre>	LR.vs.APC	df.vs.APC	<pre>prob(&gt;chi_sq)</pre>	aic
#	APC	33.179	27	0.191	NA	NA	NA	487.624
#	AP	512.514	40	0.000	479.335	13	0.000	940.958
#	AC	39.390	30	0.117	6.211	3	0.102	487.835
#	PC	1146.649	36	0.000	1113.470	9	0.000	1583.094
#	Ad	518.543	43	0.000	485.364	16	0.000	940.988
#	Pd	4041.373	49	0.000	4008.194	22	0.000	4451.818

```
# Cd 1155.629 39 0.000 1122.450 12 0.000 1586.074
# A 2223.800 44 0.000 2190.621 17 0.000 2644.245
# P 84323.944 50 0.000 84290.765 23 0.000 84732.389
# C 23794.205 40 0.000 23761.026 13 0.000 24222.650
# t 4052.906 52 0.000 4019.727 25 0.000 4457.351
# tA 5825.158 53 0.000 5791.979 26 0.000 6227.602
# tP 84325.758 53 0.000 84292.579 26 0.000 84728.203
# tC 33446.796 53 0.000 33413.617 26 0.000 33849.241
# 1 87313.678 54 0.000 87280.499 27 0.000 87714.123
# Table suggests that "APC" and "AC" fit equally well. Try both
 fit.apc <- apc.fit.model(data.list,"poisson.dose.response","APC")</pre>
fit.ac <- apc.fit.model(data.list,"poisson.dose.response","AC")</pre>
 # Compare the estimates: They are very similar
 fit.apc$coefficients.canonical
 fit.ac$coefficients.canonical
 # EXAMPLE 2 with Belgian lung cancer data
 # This example illustrates how to find the linear predictors
data.list <- data.Belgian.lung.cancer()</pre>
 # Get an APC fit
fit.apc <- apc.fit.model(data.list,"poisson.dose.response","APC")</pre>
# The linear predictor of the fit is a vector.
 # But, we would like it in the same format as the data.
 # Thus create matrix of same dimension as response data
 # This can be done in two ways
m.lp <- data.list$response # using original information</pre>
m.lp <- fit.apc$response # using information copied when fitting</pre>
 # the fit object index.data is used to fill linear predictor in
 # vector format into matrix format
m.lp[fit.apc$index.data] <-fit.apc$linear.predictors</pre>
exp(m.lp)
#######################
 # EXAMPLE 3 with Belgian lung cancer data
 # This example illustrates how apc.fit.model works.
data.list <- data.Belgian.lung.cancer()</pre>
 # Vectorise data
 index <- apc.get.index(data.list)</pre>
 v.response <- data.list$response[index$index.data]</pre>
```

20 apc.forecast

```
v.dose <- data.list$dose[index$index.data]</pre>
# Get design
m.design <- apc.get.design(index, "APC")$design</pre>
# Fit using glm.fit from stats package
fit.apc.glm <- glm.fit(m.design,v.response,family=poisson(link="log"),offset=log(v.dose))</pre>
# Get canonical coefficients
v.cc <- fit.apc.glm$coefficients</pre>
# Find linear predictors and express in matrix form
m.fit <- data.list$response # create matrix</pre>
m.fit[index$index.data] <- m.design</pre>
m.fit.offset <- m.fit + log(data.list$dose) # add offset</pre>
exp(m.fit.offset)
# Compare with linear.predictors from glm.fit
# difference should be zero
sum(abs(m.fit.offset[index$index.data]-fit.apc.glm$linear.predictors))
# EXAMPLE 4 with Taylor-Ashe loss data
# This example illustrates the over-dispersed poisson response model.
data <- data.loss.TA()</pre>
fit.apc.od <- apc.fit.model(data, "od.poisson.response", "APC")</pre>
fit.apc.od$coefficients.canonical[1:5,]
fit.apc.no.od <- apc.fit.model(data, "poisson.response", "APC")</pre>
fit.apc.no.od$coefficients.canonical[1:5,]
```

apc.forecast

Forecasts from age-period-cohort models.

# Description

In general forecasts from age-period-cohort models require extrapolation of the estimated parameters. This has to be done without introducing identifications problems, see Kuang, Nielsen and Nielsen (2008b,2011). There are many different possibilities for extrapolation for the different sub-models. The extrapolation results in point forecasts. Distribution forecasts should be build on top of these, see Martinez Miranda, Nielsen and Nielsen (2015) and Harnau and Nielsen (2016). At present three experimental functions apc.forecast.ac, apc.forecast.apc and apc.forecast.ap are available.

#### Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 10 Sep 2016 (1 Feb 2016)

#### References

Harnau, J. and Nielsen (2016) Over-dispersed age-period-cohort models. To appear in *Journal of* the American Statistical Association. Download: Nuffield DP

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008b) Forecasting with the age-period-cohort model and the extended chain-ladder model. Biometrika 95, 987-991. Download: Article; Earlier version Nuffield DP.

Kuang, D., Nielsen B. and Nielsen J.P. (2011) Forecasting in an extended chain-ladder-type model. Journal of Risk and Insurance 78, 345-359. Download: Article; Earlier version: Nuffield DP.

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. Journal of the Royal Statistical Society A 178, 29-55. Download: Article, Nuffield DP.

apc.forecast.ac

Forecast for responses model with AC or CL structure.

#### **Description**

Computes forecasts for a model with AC or Chain Ladder structure. Forecasts of the linear predictor are given for all models. Distributions forecasts are provided for a Poisson response model (using Martinez Miranda, Nielsen and Nielsen, 2015), for an over-dispersed Poisson response model (using Harnau and Nielsen, 2017) and for a log normal response model (using Kuang and Nielsen, 2018) This is done for the triangle which shares age and cohort indices with the data.

## **Usage**

```
apc.forecast.ac(apc.fit,sum.per.by.age=NULL,
sum.per.by.coh=NULL, quantiles=NULL, suppress.warning=TRUE)
```

#### **Arguments**

apc.fit

List. Output from apc.fit.model. Note: apc.fit.model should be run with AC structure so that apc.fit\$model.design=="AC". Distribution forecasts are only provided for a Poisson response model where apc.fit\$model.family=="poisson.response" for an over-disperse Poisson response model where apc.fit\$model.family=="poisson.response" and for a log normal response model where apc.fit\$model.family=="log.normal.response". For other models only point forecasts of the linear predictor are provided, that is the first two values linear.predictors.forecast and index.trap.J.

sum.per.by.age Optional. Vector. If not NULL it will generate forecasts by period, where, for each period, the point forecasts are cummulated over certain age groups. Indicates which age groups. If sum.per.by.age is a scalar or vector of length one it represents a single age group. Point forecasts are made for the indicated age group. If sum.per.by.age is a vector of length two it represents lower and upper values of an range of age groups. Point forecasts are cummulated over the indicated age groups.

sum.per.by.coh Optional. Vector. Same as sum.per.by.age, but for cohort instead of age.

quantiles *Optional*. Vector. Generates forecast quantiles for indicated quantiles. Example: quantiles=c(0.05,0.50,0.95). Default is NULL.

suppress.warning

Logical. If true, suppresses warnings from apc.data.list.subset, which is called internally. Default is "TRUE".

#### **Details**

The default output only reports standard errors. By setting the argument quantiles to, for instance, quantiles=c(0.05, 0.50, 0.95) forecast quantiles are reported.

Poisson response forecast errors. The asymptotic theory for the Poisson forecast standard errors is presented in Martinez Miranda, Nielsen and Nielsen (2015). The sampling theory is based on multinomial model, conditional on the total number of outcomes. Asymptotically this gives a normal theory. There are two independent contributions to the forecast error: a process error and an estimation error. The empirical example of that paper uses the data data.asbestos. The results of that paper are reproduced in the vignette ReproducingMMNN2015.pdf, ReproducingMMNN2015.R on Vignettes.

Overdispersed Poisson response forecast errors. The asymptotic theory for the overdispersed Poission forecast standard errors is presented in Harnau and Nielsen (2018). The sampling theory is based on infinitely devisible distributions, with the compound Poisson distribution as a special case. This results in scale nuisance parameter, which is estimated by the deviance of the AC model divided by the degrees of freedom. Asymptotically this gives a t/F theory. There are three independent contributions to the forecast error: a process error, an estimation error and a sampling error for the overall mean.

Generalized log normal forecast errors. Uses the asymptotic theory present in Kuang and Nielsen (2018). The sampling theory is based on infinitely devisible distributions, using small sigma asymptotics. There are two independent contributions to the forecast error: a process error and an estimation error.

The examples below are based on the smaller data reserving sets data.loss.VNJ, data.loss.TA. See also data.loss.XL.

## Value

linear.predictors.forecast

Vector. Linear predictors for forecast area.

index.trap.J Matrix. age-coh coordinates for vector. Similar structure to index.trap in apc.index, see apc.get.index.

trap.response.forecast

Matrix. Includes data and point forecasts. Forecasts in lower right triangle. Trapezoid format.

response.forecast.cell

Matrix. 4 columns. 1: Point forecasts. 2: corresponding forecast standard errors 3: process standard errors 4: estimation standard errors Note that the square of column 2 equals the sums of squares of columns 3 and 4 Note that index.trap.J gives the age-coh coordinates for each entry.

response.forecast.age

Same as response.forecast.cell, but point forecasts by age cumulated over period/cohort.

response.forecast.per

Same as response.forecast.cell, but point forecasts by per cumulated over age/cohort.

response.forecast.per.ic

Same as response.forecast.cell, but point forecasts cumulated by per and intercept corrected by multiplying column 1 of response.forecast.per by intercept.correction.per.

response.forecast.coh

Same as response.forecast.cell, but point forecasts by coh cumulated over age/period.

response.forecast.all

Same as response.forecast.cell, but point forecasts cumulated by age and coh.

response.forecast.per.by.age

Only if sum.per.by.age!=NULL. Same as response.forecast.per, but point forecasts cumulated over ages indicated by sum.per.by.age.

response.forecast.per.by.age.ic

Only if sum.per.by.age!=NULL. Same as response.forecast.per.by.age, but intercept corrected using intercept.correction.per.by.age.

response.forecast.per.by.coh

Only if sum.per.by.coh!=NULL. Same as response.forecast.per, but point forecasts cumulated over cohorts indicated by sum.per.by.coh.

response.forecast.per.by.coh.ic

Only if sum.per.by.coh!=NULL. Same as response.forecast.per.by.coh, but intercept corrected using intercept.correction.per.by.coh.

intercept.correction.per

Numeric. The intercept correction is constructed as the ratio of the sum of data entries for the last period and the sum of the corresponding fitted values.

intercept.correction.per.by.age

Numeric. Only if sum.per.by.age!=NULL.

intercept.correction.per.by.coh

Numeric. Only if sum.per.by.coh!=NULL.

## Author(s)

Bent Nielsen <a href="mailto:bent.nielsen@nuffield.ox.ac.uk">bent.nielsen@nuffield.ox.ac.uk</a> 18 November 2019 (2 Mar 2016)

#### References

Harnau, J. and Nielsen (2018) Over-dispersed age-period-cohort models. *Journal of the American Statistical Association* 113, 1722-1732. *Download*: Nuffield DP

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Article, Nuffield DP.

Martinez Miranda, M.D., Nielsen, B., Nielsen, J.P. and Verrall, R. (2011) Cash flow simulation for a model of outstanding liabilities based on claim amounts and claim numbers. *ASTIN Bulletin* 41, 107-129.

Kuang, D, Nielsen B (2018) Generalized log-normal chain-ladder. mimeo Nuffield Collge.

#### See Also

The example below uses Japanese breast cancer data, see data. Japanese.breast.cancer

# **Examples**

```
######################
# EXAMPLE with reserving data: data.loss.VNJ()
# Data used in Martinez Miranda, Nielsen, Nielsen and Verrall (2011)
# Point forecasts are the Chain-Ladder forecasts
# *NOTE* Data are over-dispersed,
# so distribution forecast are *NOT* reliable
# The same could be done data.asbestos(),
# which are not over-dispersed
# see vignette.
data <- data.loss.VNJ()</pre>
fit.ac <- apc.fit.model(data, "poisson.response", "AC")</pre>
forecast <- apc.forecast.ac(fit.ac)</pre>
# forecasts by "policy-year"
forecast$response.forecast.coh
         forecast se
                             se.proc
                                         se.est
# coh_2
         1684.763 57.69067 41.04586 40.53949
# coh_3 29379.085 220.53214 171.40328 138.76362
# coh_4 60637.929 313.33867 246.24770 193.76066
# coh_5 101157.697 385.69930 318.05298 218.18857
# coh_6 173801.522 501.42184 416.89510 278.60786
# coh_7 249348.589 595.21937 499.34816 323.94060
# coh_8 475991.739 864.06580 689.92155 520.20955
# coh_9 763918.643 1182.70450 874.02440 796.78810
# coh_10 1459859.526 2216.80272 1208.24647 1858.58945
# forecasts of "cash-flow"
forecast$response.forecast.per
# reproduces Table 6 of MMNNV (2011)
          forecast se se.proc se.est
# per_11 1353858.32 1456.92459 1163.55417 876.7958
# per_12 754180.12 1017.37629 868.43544 529.9758
# per_13 488612.42 816.62860 699.00817 422.2202
# per_14 318043.00 664.36135 563.95302 351.1880
# per_16 115022.56 414.64945 339.14976 238.5615
# per_17 63145.15 320.93564 251.28700 199.6360
2494.27 78.10439 49.94266 60.0502
# per_19
# forecast of "total reserve"
# reproduces Table 6 of MMNNV (2011)
forecast$response.forecast.all
     forecast se se.proc se.est
# all 3315779 3182.737 1820.928 2610.371
```

```
# Forecast of cashflows for 7th cohort (policy year)
# Note a series of warnings are given because
# this is done by truncating the data
# which generates the warnings associated
# with apc.data.list.subset()
forecast<- apc.forecast.ac(fit.ac,sum.per.by.coh=7)</pre>
forecast$response.forecast.per.by.coh
          forecast
                         se se.proc
                                          se.est
# per_11 102975.337 355.97444 320.89771 154.08590
# per_12 58061.306 267.24671 240.95914 115.58329
# per_13 40466.866 226.40049 201.16378 103.87646
# per_14 21615.765 170.90637 147.02301 87.13910
# per_15 24410.927 194.70158 156.23997 116.17994
# per_16 1818.389 61.09857 42.64257 43.75668
# This can also be intercept corrected
# Such intercept corrections are useful when
# analysing data.asbestos().
# Unclear if they are useful for
# reserving.
forecast$intercept.correction.per.by.coh
# > [1] 1.241798
forecast$response.forecast.per.by.coh.ic
          forecast
                    se se.proc
# per_11 127874.573 355.97444 320.89771 154.08590
# per_12 72100.417 267.24671 240.95914 115.58329
# per_13 50251.675 226.40049 201.16378 103.87646
# per_14 26842.415 170.90637 147.02301 87.13910
# per_15 30313.441 194.70158 156.23997 116.17994
# per_16 2258.071 61.09857 42.64257 43.75668
# Forecast of cashflows cumulated for
# 6th and 7th cohort (policy year)
forecast<- apc.forecast.ac(fit.ac,sum.per.by.coh=c(6,7))</pre>
forecast$response.forecast.per.by.coh.ic
          forecast
                       se se.proc
                                          se.est
# per_11 226219.380 460.52781 414.62816 200.42295
# per_12 139628.153 366.48699 325.74697 167.93339
# per_13 87022.435 295.86605 257.16360 146.29970
# per_14 66584.160 277.64858 224.94656 162.75067
# per_15 34962.678 206.77289 163.00324 127.22018
# per_16 2392.759 61.09857 42.64257 43.75668
########################
# EXAMPLE with reserving data: data.loss.TA()
# Data used in Harnau and Nielsen (2016)
data <- data.loss.TA()</pre>
fit.ac <- apc.fit.model(data, "od.poisson.response", "AC")</pre>
forecast \leftarrow apc.forecast.ac(fit.ac,quantiles=c(0.01,0.05,0.5,0.95,0.99))
forecast$response.forecast.all
     forecast
                 se se.proc se.est tau.est
```

apc.forecast.ap

Forecast for Poisson response model with AP structure.

## **Description**

Computes forecasts for a model with AP structure. The data can have any form allowed in, see <a href="mailto:apc.data.list">apc.data.list</a>. These are all special cases of generalised trapezoids. If the "lower triangle" with the largest (age,coh) values are not observed, they can be forecast using this function. The function extrapolates the AP model to the lower triangle where per.zero+per.max < per <= age.max+coh.max-1. The estimates of the age parameters can be used for the lower triangle. The estimates of the period parameters need to be extrapolated for the lower triangle. Thus, the function extrapolates per.forecast.J=age.max+coh.max-1-per.zero-per.max period values. The extrapolation method has to chosen so as not to introduce an identification problem, see Kuang, Nielsen and Nielsen (2008b,2011). Two such extrapolation methods are implemented in this function: "IO" and "I1". The default is to report the linear predictor.

If the model.family="binominal.dose.response", that is a logistic model, then forecasts of dose, response and survival probability are given for lower triangle.

#### Usage

```
apc.forecast.ap(apc.fit,extrapolation.type="I0",suppress.warning=TRUE)
```

## **Arguments**

apc.fit

List. Output from apc.fit.model. Note: apc.fit.model should be run with AP structure so that apc.fit\$model.design=="AP". Only point forecasts of the linear predictor are provided.

extrapolation.type

Character. Choices for extrapolating the differenced period parameter ("Delta.beta\_per"). Default is "I0".

"IO" extrapolates the first out-of-sample differenced period parameter by the average of cumulated sums of the in-sample estimated differenced period parameters. The subsequent out-of-sample differenced period parameters are zero.

"I1" extrapolates all out-of-sample differenced period parameters by zero.

Both methods are invariant to ad hoc identification of the implied period time effect, by following the ideas put forward in Kuang, Nielsen and Nielsen (2008b). Internally, the extrapolation is done as follows. The estimated differenced period parameters are found from "apc.fit\$coefficients.canonical" using apc.identify with type="dif". These imply period time effects by ad hoc identification: choose an arbitrary value for the first period time effect and add partial sums of the differenced period parameter. Fit a time series model: an intercept model with "I0" and a random walk model for "I1". Then extrapolate and take differences. These extrapolation methods are invariant to the actual choice of the arbitrary value for the first period time effect.

suppress.warning

Logical. If true, suppresses warnings from apc.data.list.subset, which is called internally. Default is "TRUE".

#### **Details**

When model.family=binomial.dose.response forecasts are made by the component method, see Cox (1976). It is intended to be used for a population analysis situation where the response equals cohort-decrease of dose. For cell in forecast array with index (age,cohort) then: Survival probability is survival=1/(1+exp(predictor\_(a,c))). Dose is dose\_(a,c)=max(0,dose\_(a-1,c)-response\_(a-1,c)-Response is response\_(a,c)=dose\_(a,c)\*(1-survival\_(a,c)).

#### Value

trap.predictors.forecast

Matrix. Includes estimates and point forecasts of linear predictor. That is design\*coefficient. Same as the glm.fit value linear.predictors when there is no offset. Forecasts in lower right triangle. Trapezoid format.

index.trap.J Matrix. age-coh coordinates for forecast area. Similar structure to index.trap in apc.index, see apc.get.index.

D.xi.per.extrapolated

Matrix. Extrapolated parameters. Dimension per.forecast. J=age.max+coh.max-1-per.zero-per.marows, 1 column.

trap.dose.forecast

Matrix. Includes data and point forecasts. Forecasts in lower right triangle. Dose in cell age,coh equal to dose in cell age-1,coh minus response in cell age-1,coh. Only implemented for model.family="binomial.dose.response". See details.

trap.response.forecast

Matrix. Includes data and point forecasts. Forecasts in lower right triangle. Response in cell age,coh equal to dose in cell age,coh times 1 minus probability of surviving in that cell. Only implemented for model.family="binomial.dose.response". See details.

trap.survival.forecast

Matrix. Point forecasts. Forecasts in lower right triangle Probability of surviving computed from trap.predictors.forecast using logistic link function. Only implemented for model.family="binomial.dose.response". See details.

#### Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 2 May 2016 (2 Mar 2016)

#### References

Cox, P.R. (1976) Demography. 5th Edition. Cambridge: Cambridge University Press. (page 324).

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008b) Forecasting with the age-period-cohort model and the extended chain-ladder model. Biometrika 95, 987-991. *Download*: Article; Earlier version Nuffield DP.

Kuang, D., Nielsen B. and Nielsen J.P. (2011) Forecasting in an extended chain-ladder-type model. Journal of Risk and Insurance 78, 345-359. *Download*: Article; Earlier version: Nuffield DP.

apc.forecast.apc

Forecast models with APC structure.

## **Description**

Computes forecasts for a model with APC structure. Forecasts of the linear predictor are given for all models. This is done for the triangle which shares age and cohort indices with the data.

#### Usage

```
apc.forecast.apc(apc.fit,extrapolation.type="I0",
suppress.warning=TRUE)
```

## **Arguments**

apc.fit

List. Output from apc.fit.model. Note: apc.fit.model should be run with APC structure so that apc.fit\$model.design=="APC". Point forecasts of the response are only provided for a Poisson response model where apc.fit\$model.family=="poisson.response and for an over-disperse Poisson response model where apc.fit\$model.family=="od.poisson.response response models only point forecasts of the linear predictor are provided, that is the first two values linear.predictors.forecast and index.trap.J.

extrapolation.type

Character. Choices for extrapolating the differenced period parameter ("Delta.beta\_per"). Default is "I0".

"I2" Extrapolates future DDbeta by 0.

"I1" Extrapolates future DDbeta as follows. Compute Dbeta=cumsum(DDbeta) for j=3,...,J. This determines Dbeta upto arbitrary level. Compute average mean(Dbeta). Forecast DDbeta[J+1]=mean(Dbeta)-Dbeta[J]. Forecast DDbeta[J+h]=0 for h>1. This forecast is invariant to arbitrary level.

"I0" Extrapolates future DDbeta as follows. Compute beta=cumsum(cumsum(DDbeta)) for j=3,...,J. This determines beta upto arbitrary linear trend. Regress on 1 and demeaned trend=j-(n+1)/2 giving estimates mu1 and mu2. Forecast beta[J+1]=mu1 + mu2\*(n+1-(n+1)/2). Forecast beta[J+2]=mu1 + mu2\*(n+2-(n+1)/2). Forecast DDbeta[J+h]=beta[J+h]-2\*beta[J+h-1]+beta[J+h-2] for

h=1,2. Forecast DDbeta[J+h]=0 for h>2. This forecast is invariant to arbitrary linear trend.

All methods are invariant to ad hoc identification of the implied period time effect, by following the ideas put forward in Kuang, Nielsen and Nielsen (2008b).

suppress.warning

Logical. If true, suppresses warnings from apc.data.list.subset, which is called internally. Default is "TRUE".

#### **Details**

The example below is based on the smaller data reserving sets data.loss.TA.

#### Value

linear.predictors.forecast

Vector. Linear predictors for forecast area.

index.trap.J Matrix. age-coh coordinates for vector. Similar structure to index.trap in apc.index, see apc.get.index.

trap.response.forecast

Matrix. Includes data and point forecasts. Forecasts in lower right triangle. Trapezoid format.

response.forecast.cell

Matrix. 4 columns. 1: Point forecasts. 2: corresponding forecast standard errors 3: process standard errors 4: estimation standard errors Note that the square of column 2 equals the sums of squares of columns 3 and 4 Note that index.trap.J gives the age-coh coordinates for each entry.

response.forecast.age

Same as response.forecast.cell, but point forecasts by age cumulated over period/cohort.

response.forecast.per

Same as response. forecast.cell, but point forecasts by per cumulated over age/cohort.

response.forecast.coh

Same as response.forecast.cell, but point forecasts by coh cumulated over age/period.

response.forecast.all

Same as response.forecast.cell, but point forecasts cumulated by age and coh.

xi.per.dd.extrapolated

The extrapolated double differences.

xi.extrapolated

The extrapolated parameters.

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 10 Sep 2016

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008b) Forecasting with the age-period-cohort model and the extended chain-ladder model. Biometrika 95, 987-991. *Download*: Article; Earlier version Nuffield DP.

#### See Also

The example below uses Taylor and Ashe reserving see data.loss.TA

#### **Examples**

```
# EXAMPLE with reserving data: data.loss.TA()
data <- data.loss.TA()</pre>
fit.apc <- apc.fit.model(data, "poisson.response", "APC")</pre>
forecast <- apc.forecast.apc(fit.apc)</pre>
# forecasts by "policy-year"
forecast$response.forecast.coh
          forecast
# coh_2
         91718.82
# coh_3 464661.38
# coh_4 704591.94
# coh_5 1025337.23
# coh_6 1503253.81
# coh_7 2330768.44
# coh_8 4115906.56
# coh_9 4257958.30
# coh_10 4567231.84
# forecasts of "cash-flow"
forecast$response.forecast.per
         forecast
# per_11 5274762.58
# per_12 4213526.23
# per_13 3188451.80
# per_14 2210649.45
# per_15 1644203.06
# per_16 1236495.32
# per_17 764552.75
# per_18 444205.71
# per_19 84581.44
# forecast of "total reserve"
forecast$response.forecast.all
      forecast
# all 19061428
```

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apc.get.design

Create design matrices

#### **Description**

Functions to create the apc design matrix for the canonical parameters. Based on Nielsen (2014b), which generalises introduced by Kuang, Nielsen and Nielsen (2008). In normal use these function are needed for internal use by apc.fit.model.

The resulting function design matrix is collinear, so a sub-set of the columns have to be selected. The columns are: intercept, age/period/cohort slopes, age/period/cohort double differences. Thus, there are three slopes instead of two. Before use, one has to select which parameters are needed. This should include at either one/two of age/cohort slopes or period slope or no slope.

# Usage

```
apc.get.design(apc.index,model.design)
apc.get.design.collinear(apc.index)
```

#### **Arguments**

apc.index

List. See apc.get.index for a description of the format. Note, apc.index can be replace by an apc.fit list. This is extended version of apc.index is the output from apc.fit.model.

model.design

Character. This indicates the design choice. The following options are possible.

- "APC" Age-period-cohort model.
- "AP" Age-period model. Nested in "APC"
- "AC" Age-cohort model. Nested in "APC"
- "PC" Period-cohort model. Nested in "APC"
- "Ad" Age-trend model, including age effect and two linear trends. Nested in "AP", "AC".
- "Pd" Period-trend model, including period effect and two linear trends. Nested in "AP", "PC".
- "Cd" Cohort-trend model, including cohort effect and two linear trends. Nested in "AC", "PC".
- "A" Age model. Nested in "Ad".
- "P" Period model. Nested in "Pd".
- "C" Cohort model. Nested in "Cd".
- "t" Trend model, with two linear trends. Nested in "Ad", "Pd", "Cd".
- "tA" Single trend model in age index. Nested in "A", "t".
- "tP" Single trend model in period index. Nested in "P", "t".
- "tC" Single trend model in cohort index. Nested in "C", "t".
- "1" Constant model. Nested in "tA", "tP", "tC".
- **NULL** The function then looks for information on model design in the first argument.

The model.design argument is not needed if the first argument is of type apc.fit. If given, the model.design argument is used.

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#### Value

apc.get.design returns a list with

design Matrix. The design matrix. The number of rows is the number of observations,

that is apc.index\$n.data. The order of the observations corresponds to the

internal choice made in apc.get.index.

slopes Vector. For internal use. Length 3 of logicals, indicate presence of age/period/cohort

linear slopes at most two slopes can be present if neither age/cohort present then

period may be presents, which is the case for model.design "P","tP"

difdif Vector. For internal use. Length 3 of logicals

apc.get.design.collinear returns a collinear design matrix for the unrestricted "APC" model. It has an extra column. The columns 2-4 are linear trends in age, period and cohort directions. At most two of these should be used. They are selected by slopes.

#### Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 1 Mar 2015

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Nielsen, B. (2014b) Deviance analysis of age-period-cohort models.

#### See Also

The vignette NewDesign.pdf, NewDesign.R on Vignettes.

## **Examples**

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```
# Compare with standard output from apc.fit.model
apc.fit.model(data.list,"poisson.dose.response","APC")$deviance
# EXAMPLE 2 with Belgian lung cancer data
# The age-drift model gives a good fit.
# This fit can be refined to a cubic or quadratic age effect.
# The latter is not precoded so one will have to work directly with the design matrix.
# SEE ALSO VIGNETTE
data.list <- data.Belgian.lung.cancer()</pre>
# Vectorise data
index <- apc.get.index(data.list)</pre>
v.response <- data.list$response[index$index.data]</pre>
v.dose <- data.list$dose[index$index.data]</pre>
# Get design matrix for "Ad"
m.design.ad <- apc.get.design(index,"Ad")$design</pre>
# Modify design matrix for cubic or quadratic age effect
# Note this implies a linear or constant double difference
# Quadractic age effect: restrict double differences to be equal
p <- ncol(m.design.ad)</pre>
m.rest.q <- matrix(data=0,nrow=p,ncol=4)</pre>
m.rest.q[1,1] <- 1
m.rest.q[2,2] <- 1
m.rest.q[3,3] <- 1
m.rest.q[4:p,4] <- 1
m.design.adq <- m.design.ad %*% m.rest.q</pre>
# Cubic age effect: restrict double differences to be linear
m.rest.c <- matrix(data=0,nrow=p,ncol=5)</pre>
m.rest.c[1,1] <- 1
m.rest.c[2,2] <- 1
m.rest.c[3,3] <- 1
m.rest.c[4:p,4] <- 1
m.rest.c[4:p,5] \leftarrow seq(1,p-3)
m.design.adc <- m.design.ad %*% m.rest.c</pre>
# Poisson regression for dose-response and with log link
fit.ad <- glm.fit(m.design.ad,v.response,family=poisson(link="log"),offset=log(v.dose))</pre>
fit.adc <- glm.fit(m.design.adc,v.response,family=poisson(link="log"),offset=log(v.dose))</pre>
fit.adq <- glm.fit(m.design.adq,v.response,family=poisson(link="log"),offset=log(v.dose))</pre>
# Deviance tests
fit.adc$deviance - fit.ad$deviance
fit.adq$deviance - fit.ad$deviance
# Degrees of freedom
ncol(m.design.ad) - ncol(m.design.adc)
ncol(m.design.ad) - ncol(m.design.adq)
```

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apc.get.index	Get indices for mapping data into trapezoid formation

# Description

This function does the internal book keeping between the original data format and the trapezoid format. It creates index matrices to transform data between original format, trapezoid format and a vector, as well as values to keep track of the labels for the time scales.

The generalized trapezoids are introduced in Kuang, Nielsen and Nielsen (2008), see also Nielsen (2014).

# Usage

```
apc.get.index(apc.data.list)
```

# Arguments

```
apc.data.list See apc.data.list for a description of the format
```

#### Value

A list containing the following values.

response	Matrix. An argument
dose	Matrix or NULL. An argument
data.format	Character. An argument
unit	Numeric. An argument.
data.xmax	Numeric. Number of rows of response matrix.
data.ymax	Numeric. Number of columns of response matrix.
data.xlab	Character. Label for row index of response matrix. Derived from data. format.
data.ylab	Character. Label for column index of response matrix. Derived from data. format.
data.xlab1	Numeric. Year for smallest row index of response matrix.
data.ylab1	Numeric. Year for smallest column index of response matrix.
n.data	Numeric. Number of observations.
index.data	Matrix of dimension n.datax2. Index pairs for observations in the original coordinate system as given by data. format. Same order as in index.trap.
index.trap	Matrix of dimension n.datax2. Index pairs for observations in an age/cohort system. Hence the coordinates of a trapezoid matrix. Same order as in index.data.
age.max	Numeric. Number of age groups.
per.max	Numeric. Number of period groups.
coh.max	Numeric. Number of cohort groups.
per.zero	Numeric. Anchor for period index, so that period starts from per.zero+1.

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per.odd	Logic. TRUE if per.zero is odd.
U	Numeric. Integer value of (per.zero+3)/2.
age1	Numeric. Year for smallest age index. Derived for data.format="CP", "PC", otherwise an argument.
per1	Numeric. Year for smallest period index. Derived for data.format="AC","CA","CL","CL.vector.by.row"," otherwise an argument.
coh1	Numeric. Year for smallest cohort index. Derived for data.format="AP", "PA", otherwise an argument.

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 31 Mar 2015

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Nielsen, B. (2014) Deviance analysis of age-period-cohort models. Nuffield DP.

## **Examples**

apc.hypothesis

Imposing hypotheses on age-period-cohort models.

## **Description**

apc has a set of standard hypotheses that can be imposed on the age-period-cohort model. A deviance table can be found on apc.fit.table, while fits of restricted models can be found using apc.fit.model.

Other linear hypotheses can be imposed using a little bit of coding, see the vignette NewDesign.pdf, NewDesign.R on Vignettes.

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For over-dispersed Poisson models for responses and no doses the theory is worked out in Harnau and Nielsen (2017).

In general forecasts from age-period-cohort models require extrapolation of the estimated parameters. This has to be done without introducing identifications problems, see Kuang, Nielsen and Nielsen (2008b,2011). There are many different possibilities for extrapolation for the different sub-models. The extrapolation results in point forecasts. Distribution forecasts should be build on top of these, see Martinez Miranda, Nielsen and Nielsen (2015) and Harnau and Nielsen (2016). At present three experimental functions apc.forecast.ac, apc.forecast.apc and apc.forecast.ap are available.

#### Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 10 Sep 2016 (1 Feb 2016)

#### References

Harnau, J. and Nielsen (2016) Over-dispersed age-period-cohort models. To appear in *Journal of the American Statistical Association*. *Download*: Nuffield DP

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008b) Forecasting with the age-period-cohort model and the extended chain-ladder model. Biometrika 95, 987-991. *Download*: Article; Earlier version Nuffield DP.

Kuang, D., Nielsen B. and Nielsen J.P. (2011) Forecasting in an extended chain-ladder-type model. Journal of Risk and Insurance 78, 345-359. *Download*: Article; Earlier version: Nuffield DP.

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Article, Nuffield DP.

apc.identify

Identification of time effects

#### **Description**

Computes ad hoc identified time effects.

#### Usage

```
apc.identify(apc.fit.model)
```

#### **Arguments**

```
apc.fit.model List. See apc.fit.model for a description of the format.
```

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## **Details**

Forms ad hoc identified time effects from the canonical parameter. These are used either indirectly by apc.plot.fit or they are computed directly with this command.

The ad hoc identifications are based on Nielsen (2014b). For details see also the vignette Identification.pdf, Identification.R on Vignettes or in the notes below.

For model designs of any type two ad hoc identified time effects.

- (1) The type "sum.sum" (same as "ss.dd") gives double sums anchored in the middle of the first period diagonal.
- (2) The type "detrend" gives double sums that start in zero and end in zero.

For model designs with only two time effects, that is "AC", "AP", "PC" there is a further ad hoc identification.

- (3) The type "demean" gives single sums of single differences. Derived from "detrend" where the linear trends are attributed to the double sums of double differences. Level unchanged.
- (4) The type "dif" gives the single differences derived from "demean". Could also have been chosen as canonical parametrisation for these models.

### Value

index.age.max	Vector. Indices for age parameters when using coefficients.ssdd or coefficients.detrend. The length is two longer that that of apc.model.fit\$index.age if model.design is "APC. NULL if age double differences are not estimated.
index.per.max	Vector. Indices for period parameters when using coefficients.ssdd or coefficients.detrend. The length is two longer that that of apc.model.fit\$index.per if model.design is "APC. NULL if age double differences are not estimated.
index.coh.max	Vector. Indices for cohort parameters when using coefficients.ssdd or coefficients.detrend. The length is two longer that that of apc.model.fit\$index.coh if model.design is "APC. NULL if age double differences are not estimated.
dates.max	Vector. Indicates the dates for the parameters when using coefficients.ssdd or coefficients.detrend. The length is six longer that that of apc.model.fit\$index.coh if model.design is "APC.
index.age.sub	* Vector. Indices for age parameters when using coefficients.demean. The length is two longer that that of apc.model.fit\$index.age if model.design is "APC. NULL if age double differences are not estimated.
index.per.sub	* Vector. Indices for period parameters when using coefficients.demean. The length is two longer that that of apc.model.fit\$index.per if model.design is "APC. NULL if age double differences are not estimated.
index.coh.sub	* Vector. Indices for cohort parameters when using coefficients.demean. The length is two longer that that of apc.model.fit\$index.coh if model.design is "APC. NULL if age double differences are not estimated.
dates.sub	* Vector. Indicates the dates for the parameters when using coefficients.demean. The length is six longer that that of apc.model.fit\$index.coh if model.design is "APC.

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\* Vector. Indices for age parameters when using coefficients.dif. The length is one longer that that of apc.model.fit\$index.age if model.design is "APC. NULL if age double differences are not estimated.

index.per.dif \* Vector. Indices for period parameters when using coefficients.dif. The length
is one longer that that of apc.model.fit\$index.per if model.design is "APC.
NULL if age double differences are not estimated.

index.coh.dif \* Vector. Indices for cohort parameters when using coefficients.dif. The length
is one longer that that of apc.model.fit\$index.cohif model.design is "APC.
NULL if age double differences are not estimated.

\* Vector. Indicates the dates for the parameters when using coefficients.dif. The length is three longer that that of apc.model.fit\$index.coh if model.design is "APC.

coefficients.ssdd

Matrix. Coefficients of the double sum of double differences. Normalised to be zero at two values chosen so age=cohort and period is at the minimal value. For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.ssdd

Matrix. Estimated covariance matrix for double sums.

coefficients.detrend

Matrix. Coefficients of the double sum of double differences. Normalised to be zero for first and last value. For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.detrend

Matrix. Estimated covariance matrix for detrended double sums.

coefficients.demean

\* Matrix. Coefficients of the sum of differences. Normalised to be zero for first value. Does not apply is design is "APC" For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.demean

\* Matrix. Estimated covariance matrix for demeaned sums.

coefficients.dif

\* Matrix. Coefficients of the differences. Does not apply is design is "APC" For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.dif

\* Matrix. Estimated covariance matrix for differences.

## Note

\* indicates that values only implemented for designs "AC", "AP", "PC".

The differences are not identified for design "APC". An arbitrary level can be moved between differences for age, period and cohort.

The differences are not identified for designs "Ad", "Pd", "Cd". These models have two linear trends and one set of double differences. In the model "Ad", as an example, one linear trend will be associated with age, but it is arbitrary whether the second linear trend should be associated with

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period or cohort. The slope of the age trend will depend on that arbitrary choice. In turn the level of the age differences will be arbitrary.

- (1) The type "sum.sum" (same as "ss.dd") gives double sums anchored to be zero in the three points where age=cohort=U, age=U+1, cohort=U age=U, cohort=U+1 with apc.fit.model\$U and where U is the integer value of (per.zero+3)/2 This corresponds to the representation in Nielsen (2014b). The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=U; an age slope, which is the difference of the values of the predictor at age=U+1, cohort=U and age=cohort=U; an cohort slope, which is the difference of the values of the predictor at age=U, cohort=U+1 and age=cohort=U.
- (2) The type "detrend" gives double sums that start in zero and end in zero. The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=1, which is usually outside the index set for the data; while age and cohort slopes are adjusted for the ad hoc identification of the time effects.
- (3) Subsumes var.apc.identify from apc.indiv (25 Sep 2020)

### Author(s)

Bent Nielsen <br/> <br/> bent.nielsen@nuffield.ox.ac.uk> & Zoe Fannon 25 Sep 2020 (12 Apr 2015)

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Nielsen, B. (2014b) Deviance analysis of age-period-cohort models. Work in progress.

### See Also

The vignette Identification.pdf.

```
id.ap$coefficients.detrend
id.ap$coefficients.demean
id.ap$coefficients.dif
```

apc.indiv.compare.direct

Implements direct tests between APC models

# Description

This function allows the user to directly compare any of the APC model, its submodels, or the TS model to any smaller model. For example, the function can be used to compare the TS to the Ad model or the Ad model to the A model. Comparisons are by likelihood ratio or Wald tests.

### Usage

```
apc.indiv.compare.direct(data, big.model, small.model, unit=1,
         dep.var, covariates=NULL, model.family,
                    n.coh.excl.start=0, n.coh.excl.end=0,
                    n.age.excl.start=0, n.age.excl.end=0,
                    n.per.excl.start=0, n.per.excl.end=0,
                    NR.controls=NULL, test, dist,
                    wt.var=NULL, plmmodel="notplm", id.var=NULL)
        apc.indiv.waldtest.fullapc(data, dist="F", big.model="APC",
                    small.model, dep.var, covariates=NULL,
                    model.family="gaussian", unit=1,
                    n.coh.excl.start=0, n.coh.excl.end=0,
                    n.age.excl.start=0, n.age.excl.end=0,
                    n.per.excl.start=0, n.per.excl.end=0,
                    existing.big.model.fit=NULL,
                    existing.small.model.fit=NULL,
                    existing.collinear=NULL,
plmmodel = "notplm", id.var=NULL, wt.var=NULL)
       apc.indiv.waldtest.TS(data, dist="F", small.model="APC",
                    dep.var, covariates=NULL,
                    model.family="gaussian", unit=1,
                    n.coh.excl.start=0, n.coh.excl.end = 0,
                    n.age.excl.start=0, n.age.excl.end = 0,
                    n.per.excl.start=0, n.per.excl.end = 0,
                    existing.small.model.fit=NULL,
                    existing.big.model.fit=NULL,
                    existing.collinear=NULL)
       apc.indiv.LRtest.fullapc(data, big.model="APC",
                    small.model,
                    dep.var, covariates=NULL,
                    model.family="binomial", unit=1,
```

```
n.coh.excl.start=0, n.coh.excl.end=0,
    n.age.excl.start=0, n.age.excl.end=0,
    n.per.excl.start=0, n.per.excl.end=0,
    existing.big.model.fit=NULL,
    existing.small.model.fit=NULL,
    existing.collinear=NULL)

apc.indiv.LRtest.TS(data, small.model="APC", dep.var, covariates=NULL,
    model.family="binomial", unit=1,
    n.coh.excl.start=0, n.coh.excl.end=0,
    n.age.excl.start=0, n.age.excl.end=0,
    n.per.excl.start=0, n.per.excl.end=0,
    existing.small.model.fit=NULL,
    existing.big.model.fit=NULL,
    existing.collinear=NULL)
```

## **Arguments**

data The data.frame in use. big.model The name of the larger of the two models to be tested. small.model The name of the smaller of the two models to be tested. unit The interval at which age, period, and cohort are recorded (must be the same for each). Default 1. dep.var The name of the dependent variable as it appears in the data covariates A vector of the names of covariates as they appear in the data. Default NULL. model.family Either "gaussian" or "binomial" n.coh.excl.start If any cohorts have been censored (AP data only). Default 0. n.coh.excl.end If any cohorts have been censored (AP data only). Default 0. n.per.excl.start If any periods have been censored (AC data only). Default 0. n.per.excl.end If any periods have been censored (AC data only). Default 0. n.age.excl.start If any ages have been censored (PC data only). Default 0. n.age.excl.end If any ages have been censored (PC data only). Default 0. Optional list to modify aspects of the Newton-Rhapson iteration for binomial NR.controls TS model. See details in apc.indiv.est.model. test The type of test. One of "LR", "Wald". dist The distribution against which the test statistic is compared. One of "F", "Chisq". Only if using survey weights. The name of the weights variable. wt.var plmmodel Used to indicate whether a panel data model is to be estimated and if so what type. Default is "notplm", for not panel data. Other values are "pooling", "within", "random". Further details in plm.

id.var Only if using panel data. The name of the individual ID variable.

existing.big.model.fit

Optional specify the output of apc.indiv.fit.model, if already run for the big model.

existing.small.model.fit

Optional specify the output of apc.indiv.fit.model, if already run for the small model.

existing.collinear

Optional specify the output of apc.indiv.design.collinear, if already run.

## **Details**

These functions are designed to facilitate direct comparison between sub-models. The functions are used to construct the rows of tables in apc.indiv.model.table but can also more helpfully be used to compare nested sub-models that gain similar levels of suport from such a table, e.g. PC to P.

### Value

test.type	The type of test, one of "LR", "Wald".
dist.type	The distribution against which the test statistic is compared. One of "F", "Chisq".
test.stat	The value of the test statistic.
df	Degrees of freedom.
df.num	Gaussian models only. Degrees of freedom used in the numerator of the F-statistic.
df.denom	Gaussian models only. Degrees of freedom used in the denominator of the F-statistic.
p.value	P-value from testing against a chi-square or F distribution.
aic.big	AIC of the big model.
aic.small	AIC of the small model.
lik.big	Log-likelihood of the big model.
lik.small	Log-likelihood of the small model.
NR.report	Binomial TS model only. Report on the Newton-Rhapson algorithm.

## Author(s)

Zoe Fannon <zoe.fannon@economics.ox.ac.uk> 26 Jun 2020

## References

Fannon, Z. (2018) apc.indiv: R tools to estimate age-period-cohort models with repeated cross section data. Mimeo. University of Oxford.

Fannon, Z., Monden, C. and Nielsen, B. (2018) Age-period-cohort modelling and covariates, with an application to obesity in England 2001-2014. Mimeo. University of Oxford.

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#### See Also

For model estimation: apc.indiv.est.model. The data in these examples are the Wage data from the package ISLR and the PSID7682 data from the package AER.

For examples, see the vignette IntroductionIndividualData.pdf, IntroductionIndividualData.R on Vignettes. Further examples in the vignette IntroductionIndividualDataFurtherExamples.pdf, IntroductionIndividualDataFurtherExamples.R.

## **Examples**

```
#### see vignettes
```

```
apc.indiv.est.model Estimate a single APC model
```

## Description

The function apc.indiv.est.model is used to estimate any of: the APC model, any APC submodel, or the time-saturated model. To estimate the APC model or a submodel, it calls apc.indiv.design.collinear, apc.indiv.design.model, and apc.indiv.fit.model in that order. To estimate the time-saturated (TS) model it calls either apc.indiv.estimate.TS or apc.indiv.logit.TS, depending on the selected model.family. These functions can also be called directly by the user.

## Usage

```
apc.indiv.est.model(data, unit = 1,
                    n.coh.excl.start=0, n.coh.excl.end=0,
                    n.per.excl.start=0, n.per.excl.end=0,
                    n.age.excl.start=0, n.age.excl.end=0,
                    model.design = "APC", dep.var = NULL,
                     covariates = NULL, model.family = "gaussian",
                    NR.controls = NULL,
                    existing.collinear = NULL,
                    existing.design = NULL,
plmmodel = "notplm", id.var = NULL,
                    wt.var = NULL)
apc.indiv.design.collinear(data, unit = 1,
                            n.coh.excl.start = 0, n.coh.excl.end = 0,
                            n.per.excl.start = 0, n.per.excl.end = 0,
                            n.age.excl.start = 0, n.age.excl.end = 0)
apc.indiv.design.model(apc.indiv.design.collinear,
                        model.design = "APC", dep.var = NULL,
                        covariates = NULL, plmmodel = "notplm",
                        wt.var = NULL, id.var = NULL)
apc.indiv.fit.model(apc.indiv.design.model, model.family = "gaussian", DV = NULL)
apc.indiv.estimate.TS(data, dep.var, covariates = NULL)
apc.indiv.logit.TS(data, dep.var, covariates = NULL, NR.controls = NULL)
```

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### **Arguments**

data The data.frame in use

unit The interval at which age, period, and cohort are recorded (must be the same for

each). Default 1.

n.coh.excl.start

If any cohorts have been censored (AP data only). Default 0.

n.coh.excl.end If any cohorts have been censored (AP data only). Default 0.

n.per.excl.start

If any periods have been censored (AC data only). Default 0.

n.per.excl.end If any periods have been censored (AC data only). Default 0.

n.age.excl.start

If any ages have been censored (PC data only). Default 0.

n.age.excl.end If any ages have been censored (PC data only). Default 0.

model.design The name of the model to be estimated. One of "TS", "APC", "AC", etc.

dep.var The name of the dependent variable as it appears in the data

DV apc.indiv.fit.model only. Optional. Vector containing dependent variable.

covariates A vector of the names of covariates as they appear in the data. Default NULL.

plmmodel Used to indicate whether a panel data model is to be estimated and if so what

type. Default is "notplm", for not panel data. Other values are "pooling",

"within", "random". Further details in plm.

id.var Only if using panel data. The name of the individual ID variable. wt.var Only if using survey weights. The name of the weights variable.

model.family Either "gaussian" or "binomial". Default "gaussian".

NR. controls Optional list to modify aspects of the Newton-Rhapson iteration for binomial

TS model. Further information in "Details", below.

existing.collinear

Optional specify the output of apc.indiv.design.collinear, if already run.

existing.design

Optional specify the output of apc.indiv.design.model, if already run.

apc.indiv.design.collinear

Output from the command apc.indiv.design.collinear.

apc.indiv.design.model

Output from the command apc.indiv.design.model.

### **Details**

The casual user should start with the general function apc.indiv.est.model for analysis. The underlying functions should be employed if the user needs to run many models using the same relatively large dataset, in which case time can be saved by running apc.indiv.design.collinear just once and using apc.indiv.design.model and apc.indiv.fit.model to estimate each of the models.

The time-saturated (TS) binomial model is estimated by a customized Newton-Rhapson iteration. Aspects of this iteration can be controlled by specifying the NR. controls option of apc.indiv.est.model

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or of apc.indiv.logit.TS. NR.controls is a named list of length 8. In order, the elements are: maxit.loop, maxit.linesearch, tolerance, init, inv.tol, d1.tol, custom.kappa, custom.zeta. maxit.loop sets the maximum number of Newton-Rhapson iterations, and has a default of 10. maxit.linesearch sets the maximum number of linesearch iterations within each Newton-Rhapson iteration, and has a default of 20. tolerance sets the condition for convergence, i.e. the tolerated difference between likelihoods from one Newton-Rhapson iteration to the next; the default is .002. init sets the starting values for the iteration. The default is "ols", meaning that estimates from the linear probability model are the starting values; one can also use "zero" to set the starting values to zero, or use "custom" and specify custom starting values using custom.kappa and custom.zeta. inv.tolsets the tolerance of small values when inverting a matrix (using solve), and the default is the machine precision. d1.tol sets the magnitude of norm of first derivative to be tolerated in Newton-Rhapson iteration, and has a default of .002. custom.kappa is used to specify custom starting values for the TS indicator parameters, while custom.zeta is used to specify custom starting values for parameters on any covariates.

#### Value

fit

The output of either glm, svyglm, or plm for repeated cross-section, repeated cross-section with survey weights, or panel models respectively. Can be used directly with follow-on functions like waldtest

.

coefficients.canonical

Matrix of estimates, standard error, t-statistic, and p-value of canonical parame-

coefficients.covariates

Matrix of estimates, standard error, t-statistic, and p-value of covariates.

coefficients.TS

TS model only: matrix of estimates, standard error, t-statistic, and p-value of TS

indicators.

aic TS model only: Akaike Information Criterion.

likelihood model likelihood.

model.design which APC submodel has been estimated.

fixef When plmmodel = "within", estimated individual fixed effects. Otherwise NULL.

full.design.collinear

from apc.indiv.design.collinear only. The collinear design matrix.

full.design from apc.indiv.design.model only. The design matrix used to estimate the model.

DV from apc.indiv.design.model only, if dep.var specified. A vector of the outcome

variable.

ID from apc.indiv.design.model only, if panel model. A vector of the individual ID

variable.

PER from apc.indiv.design.model only, if panel model. A vector of the period vari-

able.

WT from apc.indiv.design.model only, if wt.var specified. A vector of the survey

weight variable.

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model.formula	from apc.indiv.design.model only, the implied model formula. NULL if dep.var not specified.
model.string	from apc.indiv.design.model only, the implied model formula as a character string. RHS only if dep.var not specified.

### Author(s)

Zoe Fannon <zoe.fannon@economics.ox.ac.uk> 26 Jun 2020

#### References

Fannon, Z. (2018) apc.indiv: R tools to estimate age-period-cohort models with repeated cross section data. Mimeo. University of Oxford.

Fannon, Z., Monden, C. and Nielsen, B. (2018) Age-period-cohort modelling and covariates, with an application to obesity in England 2001-2014. Mimeo. University of Oxford.

### See Also

For model estimation: glm, svyglm, plm For model testing: apc.indiv.model.table, codeapc.indiv.compare.direct, waldtest, linearHypothesis For plotting: apc.plot.fit. The data in these examples are the Wage data from the package ISLR and the PSID7682 data from the package AER.

For examples, see the vignette IntroductionIndividualData.pdf, IntroductionIndividualData.R on Vignettes. Further examples in the vignette IntroductionIndividualDataFurtherExamples.pdf, IntroductionIndividualDataFurtherExamples.R.

## **Examples**

```
#### see vignettes

apc.indiv.model.table Generate table to select APC submodel
```

## **Description**

These functions test, for a given choice of dependent variable and covariates, which of the TS, APC, and APC submodels provides the best fit to the data. Comparison is by Wald or likelihood ratio test and where appropriate by Akaike Information Criterion. A table is generated with these statistics for each model considered.

# Usage

```
apc.indiv.model.table(data, dep.var, covariates = NULL,
unit = 1, n.coh.excl.start = 0, n.coh.excl.end = 0,
n.age.excl.start = 0, n.age.excl.end = 0,
n.per.excl.start = 0, n.per.excl.end = 0,
model.family, NR.controls = NULL,
test, dist,
```

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```
TS=FALSE, wt.var=NULL, plmmodel="notplm",
id.var=NULL)
  apc.indiv.waldtable(data, dep.var, covariates = NULL,
  dist="F", unit = 1, model.family,
    n.coh.excl.start = 0, n.coh.excl.end = 0,
n.age.excl.start = 0, n.age.excl.end = 0,
n.per.excl.start = 0, n.per.excl.end = 0,
wt.var=NULL, plmmodel="notplm",
id.var=NULL)
apc.indiv.waldtable.TS(data, dep.var, covariates=NULL, dist = "F",
                                unit=1, model.family = "gaussian",
                                n.coh.excl.start=0, n.coh.excl.end=0,
                                n.age.excl.start=0, n.age.excl.end=0,
                                n.per.excl.start=0, n.per.excl.end=0)
    apc.indiv.LRtable(data,
                              dep.var, covariates=NULL,
                              model.family, unit=1,
                              n.coh.excl.start=0, n.coh.excl.end=0,
                              n.age.excl.start=0, n.age.excl.end=0,
                              n.per.excl.start=0, n.per.excl.end=0)
  apc.indiv.LRtable.TS(data,
                               dep.var, covariates=NULL,
                                 model.family, unit=1,
                                 n.coh.excl.start=0, n.coh.excl.end=0,
                                 n.age.excl.start=0, n.age.excl.end=0,
                                 n.per.excl.start=0, n.per.excl.end=0,
                                 NR.controls=NR.controls)
```

## **Arguments**

The data frame in use data The name of the dependent variable as it appears in the data dep.var covariates A vector of the names of covariates as they appear in the data. Default NULL. unit The interval at which age, period, and cohort are recorded (must be the same for each). Default 1. n.coh.excl.start If any cohorts have been censored (AP data only). Default 0. n.coh.excl.end If any cohorts have been censored (AP data only). Default 0. n.age.excl.start If any ages have been censored (PC data only). Default 0. n.age.excl.end If any ages have been censored (PC data only). Default 0. n.per.excl.start If any periods have been censored (AC data only). Default 0. n.per.excl.end If any periods have been censored (AC data only). Default 0. model.family Either "gaussian" or "binomial" NR.controls Optional list to modify aspects of the Newton-Rhapson iteration for binomial TS model. See details in apc.indiv.est.model. The type of test. One of "LR", "Wald". test

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TS ...

dist The distribution against which the test statistic is compared. One of "F", "Chisq".

wt.var Only if using survey weights. The name of the weights variable.

plmmodel Used to indicate whether a panel data model is to be estimated and if so what

type. Default is "notplm", for not panel data. Other values are "pooling",

"within", "random". Further details in plm.

id.var Only if using panel data. The name of the individual ID variable.

### **Details**

Each row of the table corresponds to a single sub-model of the APC model. The first three columns test the sub-model in question against the time-saturated model. The next three columns test the sub-model against the full APC model. The final two columns report the likelihood and AIC of the estimated sub-model. The model with the lowest AIC value which is also not rejected in tests against the APC and TS models should be selected.

#### Value

table contains the table of comparison statistics.

NR. report for logit models only, a report on the Newton-Rhapson algorithm used to esti-

mate the time-saturated model.

### Author(s)

Zoe Fannon <zoe.fannon@economics.ox.ac.uk> 26 Jun 2020

### References

Fannon, Z. (2018) apc.indiv: R tools to estimate age-period-cohort models with repeated cross section data. Mimeo. University of Oxford.

Fannon, Z., Monden, C. and Nielsen, B. (2018) Age-period-cohort modelling and covariates, with an application to obesity in England 2001-2014. Mimeo. University of Oxford.

## See Also

For model estimation: apc.indiv.est.model For pairwise model comparison: apc.indiv.model.table, waldtest, linearHypothesis. The data in these examples are the Wage data from the package ISLR and the PSID7682 data from the package AER.

For examples, see the vignette IntroductionIndividualData.pdf, IntroductionIndividualData.R on Vignettes. Further examples in the vignette IntroductionIndividualDataFurtherExamples.pdf, IntroductionIndividualDataFurtherExamples.R.

## **Examples**

#### see vignettes

apc.plot.data.all 49

apc.plot.data.all Make all descriptive plots.

## **Description**

Plots data sums using apc.plot.data.sums. Sparsity plots of data using apc.plot.data.sparsity. Plots data using all combinations of two time scales using apc.plot.data.within. Level plots of data using apc.plot.data.level. The latter plot is done for responses and if applicable also for doses and mortality rates.

### **Usage**

```
apc.plot.data.all(apc.data.list,log ="",rotate=FALSE)
```

## **Arguments**

apc.data.list List. See apc.data.list for a description of the format.

Optional plot argument. Character. "y" if y-scale is logarithmic, otherwise "".

Default is "y".

Optional. Logical. If TRUE rotates apc.plot.data.level 90 degrees clock-

wise (or anti-clockwise if data.format is "CL"). Default is FALSE.

## Warning

A warning is produced if dimension is not divisible by thin, so that one group is smaller than other groups.

### Author(s)

Bent Nielsen <br/> <bent.nielsen@nuffield.ox.ac.uk> 25 Apr 2015

### See Also

The example below uses Italian bladder cancer data, see data. Italian.bladder.cancer

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```
#
# get data list, then make all descriptive plots.
# Note that warnings are given in relation to the data chosen thinning
# This can be avoided by working with the individual plots, and in particular
# with apc.plot.data.within where the thinning happens.
#
# data.list <- data.Italian.bladder.cancer()
# apc.plot.data.all(data.list)</pre>
```

apc.plot.data.level Level plot of data matrix.

## **Description**

This plot shows level plot of data matrix based on levelplot in the package lattice.

# Usage

```
apc.plot.data.level(apc.data.list,data.type="r",
    rotate=FALSE,apc.index=NULL,
main=NULL,lab=NULL,
contour=FALSE,colorkey=TRUE)
```

## **Arguments**

apc.data.list	List. See apc.data.list for a description of the format.
data.type	Optional. Character. "r"="response" / "d"="dose" / "m"="mortality"="rates" if sums are computed for responses/dose/rates, where rates are found through division response/dose. It also takes data types "residual" / "fitted.values" / "linear.predictors" when the argument apc.data.list is the output of the fitting function apc.fit.model, which is an extended apc.data.list. "r" is default.
rotate	Optional. Logical. If TRUE rotates plot 90 degrees clockwise (or anti-clockwise if data.format is "CL"). Default is FALSE.
apc.index	Optional. List. See apc.get.index for a description of the format. If not provided this is computed.
main	Optional. Character. Main title.
lab	Optional plot parameter. A numerical vector of the form $c(x, y, len)$ which modifies the default way that axes are annotated. The values of x and y give the (approximate) number of tickmarks on the x and y axes. len is not implemented.
contour	Optional levelplot (lattice) parameter. Logical. Contour lines drawn if TRUE. Default FALSE.
colorkey	Optional levelplot (lattice) parameter. Logical or list. Determines color key. Default TRUE.

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 26 Apr 2015

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## See Also

data. Japanese. breast. cancer for information on the data used in the example.

## **Examples**

```
# EXAMPLE with Japanese breast cancer data
# Clayton and Shifflers (1987b) use APC design
# Make a data list
# Then plot data.
# Note: No plot appears to have approximately parallel lines.
data.list <- data.Japanese.breast.cancer()</pre>
apc.plot.data.level(data.list,"r")
dev.new()
apc.plot.data.level(data.list,"d",contour=TRUE)
# It also works with a single argument, but then a default log scale is used.
# Note that warnings are given in relation to the data chosen thinning
apc.plot.data.within(data.list)
######################
# EXAMPLE with Italian bladder cancer data
# Clayton and Shifflers (1987a) use AC design
# Note: plot of within cohort against age appears to have approximately parallel lines.
# This is Figure 2 in Clayton and Shifflers (1987a)
# Note: plot of within age against cohort appears to have approximately parallel lines.
   Indicates that interpretation should be done carefully.
data.list <- data.Italian.bladder.cancer()</pre>
apc.plot.data.within(data.list,"m",1,log="y")
# EXAMPLE with asbestos data
# Miranda Martinex, Nielsen and Nielsen (2014).
# This is Figure 1d
data.list <- data.asbestos()</pre>
apc.plot.data.within(data.list,type="l",lty=1)
```

apc.plot.data.sparsity

This plot shows heat map of the sparsity of a data matrix.

## **Description**

The plot shows where the data matrix is sparse.

## Usage

```
apc.plot.data.sparsity(apc.data.list,
data.type="a",swap.axes=FALSE,
apc.index=NULL,
sparsity.limits=c(1,2),
cex=NULL,pch=15,
main.outer=NULL)
```

### Arguments

apc.data.list List. See apc. data.list for a description of the format. Optional. Character. "r"/"d"/"m" if sums are computed for responses/dose/all. data.type "r" is default. swap.axes Optional. Logical. If true swap axes in plot. Default is FALSE unless data.format="CL" Optional. List. See apc.get.index for a description of the format. If not apc.index provided this is computed. sparsity.limits Optional. vector with two values in increasing order. Default is c(1,2). The sparsity plot is a heat map with three colours: black if the observation is smaller than first index (default 1), grey if the observation is smaller than the second index (default 2) and otherwise white. cex Optional plot argument. A numerical value giving the amount by which plotting text and symbols should be magnified. Default is NULL in which case program chooses. pch Optional, vector with two values. Either integers specifying a symbol or characters. See points for possible values and their interpretation. Default is c(15,15), which is filled square.

### **Details**

main.outer

The default values is used to highlight where a matrix of counts has values of zero and one. Estimation can be very noise in those areas.

NULL, in which case a title is generated internally.

Optional. Character. Main title for plot, to be shown in outer margin. Default is

### Note

Note that the axes for plots grow from bottom left while axes for matrices grow from top left. The exception is when data.format="CL", in which case both grow from top left.

### Author(s)

Bent Nielsen <br/> bent.nielsen@nuffield.ox.ac.uk> 25 Apr 2015 updated 27 Apr 2015

## See Also

The example below uses asbestos data, see data.asbestos

apc.plot.data.sums 53

## **Examples**

apc.plot.data.sums

This plot shows sums of data matrix by age, period or cohort.

# Description

Produces plots showing age, period and cohort sums. As a default this is done both for responses and dose, giving a total of six plots.

## Usage

```
apc.plot.data.sums(apc.data.list,data.type="a",
average=FALSE,keep.incomplete=TRUE,apc.index=NULL,
type="o",log="",main.outer=NULL,main.sub=NULL)
```

## **Arguments**

apc.data.list	List. See apc.data.list for a description of the format.
data.type	Optional. Character. "r", "d", "m", "a" if sums are computed for responses, dose, (mortality rates), all. Rates are computed as responses/doses. Default is "a".
average	Optional. Logical. Sums are reported if FALSE, Averages are reported if TRUE. Default is FALSE.
keep.incomplete	
	Optional. Logical. If true perform calculation for incomplete sequences by removing NA. If false incomplete sequences are NA. See example in apc.data.sums. Default=TRUE.
apc.index	Optional. List. See apc.get.index for a description of the format. If not provided this is computed.
type	Optional plot argument. Character. "o" if overlaid points and lines. "l" if lines. "p" if points. Default is "o".

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log	Optional plot argument. Character. "y" if y-scale is logarithmic, otherwise "". Default is "".
main.outer	Optional. Character. Main title for plot, to be shown in outer margin. Default is NULL, in which case a title is generated internally.
main.sub	Optional. Titles for sub plots. Use with data.type "r","d","m". For data.type "a" use default. Default is NULL, in which case a title is generated internally.

## **Details**

The data sums are computed using apc.data.sums. Then plotted as requested.

#### Note

Use apc.data.sums if numerical values needed.

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 15 Aug 2018 (15 Dec 2013)

### References

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Article, Nuffield DP.

### See Also

The example below uses Japanese breast cancer data, see data. Japanese.breast.cancer

```
#######################
   EXAMPLE with artificial data
    Generate a 3x4 matrix in "AP" data.format with the numbers 1..12
# Then make a data list
# Then plot data sums.
# Note only 3 plots are made as there are no doses
         <- matrix(data=seq(length.out=12),nrow=3,ncol=4)</pre>
data.list <- apc.data.list(m.data, "AP")</pre>
apc.plot.data.sums(data.list)
apc.plot.data.sums(data.list,average=TRUE)
apc.plot.data.sums(data.list,keep.incomplete=FALSE)
##########################
# EXAMPLE with Japanese breast cancer data
# Make a data list
# Then plot data sums for both responses and doses.
data.list <- data.Japanese.breast.cancer()</pre>
```

apc.plot.data.within 55

apc.plot.data.within This plot shows time series of matrix within age, period or cohort.

## **Description**

apc.plot.data.within produces plot showing time series of matrix within age, period or cohort against one of the other two indices. apc.plot.data.within.all.six produces all six plots in one panel plot.

These plots are sometimes used to gauge how many of the age, period, cohort factors are needed: If lines are parallel when dropping one index the corresponding factor may not be needed. In practice these plots should possibly be used with care, see Italian bladder cancer example below.

## Usage

```
apc.plot.data.within(apc.data.list,
data.type="r",plot.type="awc",
average=FALSE,
thin=NULL,apc.index=NULL,
ylab=NULL,type="o",log="",legend=TRUE,
lty=1:5,col=1:6,bty="n",main=NULL,
x="topleft",return=FALSE)
apc.plot.data.within.all.six(apc.data.list,
data.type="r",
average=FALSE,
thin=NULL,apc.index=NULL,
ylab=NULL,type="o",log="",legend=TRUE,
lty=1:5,col=1:6,bty="n",main.outer=NULL,
x="topleft")
```

## **Arguments**

```
apc.data.list List. See apc.data.list for a description of the format.
```

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data.type	Optional. Character. "r"="response" / "d"="dose" / "m"="mortality"="rates" if sums are computed for responses/dose/rates, where rates are found through division response/dose. "r" is default.
plot.type	Optional. "awp", "pwa" "awc", "cwa, "cwp", "pwc": for example: "awp" gives time series in age within each period level: for an AP data-array these are the column sums.
average	Optional. Logical. If TRUE/FALSE reports averages/sums. Default is FALSE.
thin	Optional. Numerical. age/periods/cohorts are grouped in groups of size thin. Default is computed from dimensions of data. A warning is produced if dimension is not divisible by thin, so that one group is smaller than other groups.
apc.index	Optional. List. See apc.get.index for a description of the format. If not provided this is computed.
ylab	Optional plot argument. Character. Common label for y-axes. Default is "".
type	Optional plot argument. Character. "o" if overlaid points and lines. "l" if lines. "p" if points. Default is "o".
log	Optional plot argument. Character. "y" if y-scale is logarithmic, otherwise "". Default is ""
legend	Optional plot argument. Logical. Should legends be drawn? Default is TRUE.
lty	
ity	Optional plot argument. Vector of line types. The first element is for the first column, the second element for the second column, etc., even if lines are not plotted for all columns. Line types will be used cyclically until all plots are drawn. Default is 1:5
col	column, the second element for the second column, etc., even if lines are not plotted for all columns. Line types will be used cyclically until all plots are
·	column, the second element for the second column, etc., even if lines are not plotted for all columns. Line types will be used cyclically until all plots are drawn. Default is 1:5  Optional plot argument. Vector of colors. The first element is for the first column, the second element for the second column, etc., even if lines are not plotted for all columns. Colors will be used cyclically until all plots are drawn.
col	column, the second element for the second column, etc., even if lines are not plotted for all columns. Line types will be used cyclically until all plots are drawn. Default is 1:5  Optional plot argument. Vector of colors. The first element is for the first column, the second element for the second column, etc., even if lines are not plotted for all columns. Colors will be used cyclically until all plots are drawn. Default is 1:6.  Optional plot argument. Character. The type of box to be drawn around the
col	column, the second element for the second column, etc., even if lines are not plotted for all columns. Line types will be used cyclically until all plots are drawn. Default is 1:5  Optional plot argument. Vector of colors. The first element is for the first column, the second element for the second column, etc., even if lines are not plotted for all columns. Colors will be used cyclically until all plots are drawn. Default is 1:6.  Optional plot argument. Character. The type of box to be drawn around the legend. The allowed values are "n" and "o". Default is "n".  Optional. Character. Main title for single plot. Default is NULL, in which case
col bty main	column, the second element for the second column, etc., even if lines are not plotted for all columns. Line types will be used cyclically until all plots are drawn. Default is 1:5  Optional plot argument. Vector of colors. The first element is for the first column, the second element for the second column, etc., even if lines are not plotted for all columns. Colors will be used cyclically until all plots are drawn. Default is 1:6.  Optional plot argument. Character. The type of box to be drawn around the legend. The allowed values are "n" and "o". Default is "n".  Optional. Character. Main title for single plot. Default is NULL, in which case a title is generated internally.

# Warning

A warning is produced if dimension is not divisible by thin, so that one group is smaller than other groups.

# Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 17 Nov 2016 (25 Apr 2015)

apc.plot.data.within 57

### References

Clayton, D. and Schifflers, E. (1987a) Models for temperoral variation in cancer rates. I: age-period and age-cohort models. *Statistics in Medicine* 6, 449-467.

Clayton, D. and Schifflers, E. (1987b) Models for temperoral variation in cancer rates. II: age-period-cohort models. *Statistics in Medicine* 6, 469-481.

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Article, Nuffield DP.

### See Also

data. Japanese. breast. cancer, data. Italian. bladder. cancer and data. asbestos for information on the data used in the example.

```
EXAMPLE with artificial data
   Generate a 3x4 matrix in "AP" data.format with the numbers 1..12
# Then make a data list
# Then plot data.
   Note: this deterministic matrix has neither age, period, or cohort factors,
# only linear trends. Thus all 6 plots have parallel lines.
m.data
        <- matrix(data=seq(length.out=12),nrow=3,ncol=4)</pre>
m.data
data.list <- apc.data.list(m.data, "AP")</pre>
apc.plot.data.within(data.list,log="")
# It also works with a single argument, but then a default log scale is used.
apc.plot.data.within(data.list)
# EXAMPLE with Japanese breast cancer data
# Clayton and Shifflers (1987b) use APC design
# Make a data list
# Then plot data.
# Note: No plot appears to have approximately parallel lines.
data.list <- data.Japanese.breast.cancer()</pre>
apc.plot.data.within(data.list, "m", 1, log="y")
# It also works with a single argument, but then a default log scale is used.
# Note that warnings are given in relation to the data chosen thinning
apc.plot.data.within(data.list)
# EXAMPLE with Italian bladder cancer data
# Clayton and Shifflers (1987a) use AC design
```

apc.plot.fit

Plots of apc estimates

## **Description**

Functions to plot the apc estimates found by apc.fit.model. The function apc.plot.fit detects the type of model.design and model.family from the fit values and makes appropriate plots.

Depending on the model.design the plot has up to 9 sub plots. The type of these can be chosen using type

**Model designs of any type.** If type is "detrend" or "sum.sum" the canonical age period cohort parametrisation is used. This involves double differences of the time effects. The first row of plots are double differences of the time effects. The next two rows of plots illustrate the representation theorem depending on the choice of type. In both cases the sum of the plots add up to the predictor.

"detrend" The last row of plots are double sums of double differences detrend so that that each series starts in zero and ends in zero. The corresponding level and (up to) two linear trends are shown in the middle row of plots. The linear trends are identified to be 0 for age, period or cohort equal to its smallest value. See note 2 below.

"sum.sum" The last row of plots are double sums of double differences anchored as in the derivation of Nielsen (2014b). The corresponding level and (up to) two linear trends are shown in the middle row of plots. The linear trends are identified to be 0 for the anchoring point U of age, period or cohort as described in Nielsen (2014b). See note 1 below.

Model designs with 2 factors. If type is "dif" the canonical two factor parametrisation is used. This involves single differences. It is only implemented for model.design of "AC", "AP", "PC". It does not apply for model.design of "APC" because single differences are not identified. It does not apply for the drift models where model.design is "Ad", "Pd", "Cd", "t" because it is not clear which time scale the second linear trend should be attributed to. It is not implemented for model.design of "tA, "tP", "tC", "1". The first row of plots are single differences of the time effects. The next two rows of plots illustrate the representation theorem. In the second row the level is given and in the third row plots of single sums of single differences are given, normalised to start in zero.

**Appearance may vary.** Note, the plots "detrend" and "dif" can give very different appearance of the time effects. The "dif" plots are dominated by linear trends. They can therefore be more difficult to interpret than the "detrend" plots, where linear trends are set aside.

**Standard deviations.** All plots include plots of 1 and 2 standard deviations. The only exception is the intercept in the case model.family is "poisson.response" as this uses a multinomial sampling scheme, where the intercept is set to increase in the asymptotic experiment. The default is to plot standard deviations around zero, so that they represent a test for zero values of the parameters. Using the argument sdv.at.zero the standard deviations can be centered around the estimates. This can give a very complicated appearance.

Values of coefficients. These can be found using apc.identify.

## Usage

```
apc.plot.fit(apc.fit.model,scale=FALSE,
sdv.at.zero=TRUE,type="detrend",
include.linear.plane=TRUE,
include.double.differences=TRUE,
sub.plot=NULL,main.outer=NULL,main.sub=NULL,
cex=NULL,cex.axis=NULL,cex.lab=NULL,cex.main=NULL,
cex.main.outer=1.2,
line.main=0.5,line.main.outer=NULL,
las=NULL,mar=NULL,oma=NULL,mgp=c(2,1,0),
vec.xlab=NULL)
```

## **Arguments**

apc.fit.model	List. See apc.fit.model for a description of the format.
scale	Optional. Logical. If (TRUE) FALSE use scale of (inverse) link function. Default is FALSE.
sdv.at.zero	Optional. Logical. If FALSE/TRUE standard deviations are plotted around estimates/zero. Default is TRUE.
type	Optional. Character. If "detrend" double sums start and end in zero. If "sum.sum" double sums anchored as discussed in Nielsen (??). Default is "detrend".
include.linear.	plane
	Optional. Logical. If true include plots of linear plane. Default TRUE
include.double.	differences
	Optional. Logical. If true include plots of double differences. Default TRUE
sub.plot	Optional. Character: "a", "b",, "i". Only the indicated sub plot is plotted. Default is NULL so all plots shown.
main.outer	Optional. Character. Main title in outer margin. Default is generated internally.
main.sub	Optional. Vector of 9 characters. Main titles for individual plots. Default is generated internally, see note 3 below.
cex	Optional. Plot parameter, see par. Controls size of text. Default is NULL so that R default is used.
cex.axis	Optional. Plot parameter, see par. Controls magnification of axis annotations. Default is NULL so that R default is used.

cex.lab	Optional. Plot parameter, see par. Controls magnification of axis labels. Default is NULL so that R default is used.
cex.main	Optional. Plot parameter, see par. Controls magnification of main title. Default is NULL so that R default is used.
cex.main.outer	Optional. Controls magnification of outer main title if an array of plots is shown. Default is 1.2 (same as cex.main).
line.main	Optional. Specifies the line position of main title in individual plots. Default is 0.5.
line.main.outer	
	Optional. Specifies the line position of outer main title if an array of plots is shown. Default is NULL so that R default is used.
las	Optional. Plot parameter, see par. Numeric. The style of axis labels. Default is NULL so that R default is used.
mar	Optional. Gives the number of lines of margin to be specified on the four sides of the plot. Default: $c(4,3,2,0)$ for array of plots, $c(4,4,3,1)$ for a single plot.
oma	Optional. Gives the size of the outer margins in lines of text. Default: $c(0,0,5,1)$ for array of plots, $c(0,0,0,0)$ for a single plot.
mgp	Optional. Plot parameter, see par. The margin line for the axis title, axis label and axis line. Defauls is $c(2,1,0)$ , different from R default.
vec.xlab	Optional. Controls title for xaxis. Should be a 9-vector of characters for an array of plots and a character for a single plot. As R recycles entries if a vector is too short, then vec.xlab="" will remove titles on x-axis. Default: NULL.

### Note

- (1) The type "sum.sum" (same as "ss.dd") gives double sums anchored to be zero in the three points where age=cohort=U, age=U+1, cohort=U age=U, cohort=U+1 with apc.fit.model\$U and where U is the integer value of (per.zero+3)/2 This corresponds to the representation in Nielsen (2014b). The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=U; an age slope, which is the difference of the values of the predictor at age=U+1, cohort=U and age=cohort=U; an cohort slope, which is the difference of the values of the predictor at age=U, cohort=U+1 and age=cohort=U.
- (2) The type "detrend" gives double sums that start in zero and end in zero. The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=1, which is usually outside the index set for the data; while age and cohort slopes are adjusted for the ad hoc identification of the time effects.
- (3) The default of the titles main.sub are generated internally depending on model specification. In the case of model.design="APC" and a dose-response model family the default value is c(expression(paste("(a) ",Delta^2,alpha)),expression(paste("(b) ",Delta^2,beta)),expression(paste("(",Delta^2,gamma)),"(d) first linear trend","(e) level","(f) second linear trend",expression(paste("(g) detrended ",Sigma^2,Delta^2,alpha)),expression(paste("(h) detrended ",Sigma^2,Delta^2,beta)),expression(paste("(h) detrended ",Sigma^2,Delta^2,gamma)))
- (4) Default values of parameters changed (28 Sep 2020). The old appearance can be reproduced by setting cex.lab=1.5. For example:

```
data.list <- data.Italian.bladder.cancer()
fit.apc <- apc.fit.model(data.list,"poisson.dose.response","APC")
apc.plot.fit(fit.apc,cex.lab=1.5)
The code subsumes var.apc.plot.fit by Zoe Fannon.</pre>
```

### Author(s)

Bent Nielsen <br/> <br/> dent.nielsen@nuffield.ox.ac.uk> & Zoe Fannon 28 September 2020 (12 Apr 2015).

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Nielsen, B. (2014b) Deviance analysis of age-period-cohort models. Work in progress.

### See Also

data.asbestos and data.Italian.bladder.cancer for information on the data used in the example.

Values of coefficients can be found using apc.identify.

Further information on the identification in the vignette Identification.pdf, Identification.R on Vignettes.

```
#######################
# Example with Italian bladder cancer data
# Note that the model.design "AC" cannot be rejected against "APC"
# so there is little difference between the two plots of those fits.
data.list <- data.Italian.bladder.cancer()</pre>
apc.fit.table(data.list, "poisson.dose.response")
fit.apc <- apc.fit.model(data.list,"poisson.dose.response","APC")</pre>
apc.plot.fit(fit.apc)
# now try an AC model
# can use dev.new() to see both
fit.ac <- apc.fit.model(data.list,"poisson.dose.response","AC")</pre>
apc.plot.fit(fit.ac)
# to check the numerical values for the last two rows of plots use
apc.identify(fit.ac)$coefficients.detrend
# to get only a sub plot and playing with titles
# main.outer not used with individual plot
apc.plot.fit(fit.ac,sub.plot="a",main.outer="My outer title",main.sub="My sub title")
# to play with
# titles (main.outer/main.sub),
# label orientation (las),
```

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```
# axis titles (vec.xlab)
apc.plot.fit(fit.ac,main.outer="My outer title",
main.sub=c("1","2","3","4","5","6","7","8","9"),
las=1,
vec.xlab=c("a","b","c","d","e","f","g","h","i"))
```

apc.plot.fit.all

Make all fit plots.

### **Description**

Plots estimates using apc.plot.fit. Probability transform plot of residuals using apc.plot.fit.pt. Level plot of residuals using apc.plot.fit.residuals. Level plot of fitted values using apc.plot.fit.fitted.values. Level plot of linear predictors using apc.plot.fit.linear.predictors. Level plots of responses and rates (if dose is availble) using apc.plot.data.level.

## Usage

```
apc.plot.fit.all(apc.fit.model,log ="",rotate=FALSE)
```

### **Arguments**

apc.fit.model List. Output from apc.fit.model. See there for a description of the format.

log Optional plot argument. Character. "y" if y-scale is logarithmic, otherwise "".

Default is "".

rotate Optional. Logical. If TRUE rotates level plots 90 degrees clockwise (or anti-

clockwise if data.format is "CL"). Default is FALSE.

## Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 2t Apr 2015

### See Also

The example below uses Italian bladder cancer data, see data.Italian.bladder.cancer

apc.plot.fit.pt 63

# Description

Constructs probability transforms of responses given fitted values from apc.fit.model. The plot is given in the original coordinate system. Colours and symbols are used to indicate whether responses are central to the fitted distribution or in the tails of the fitted distribution.

## Usage

```
apc.plot.fit.pt(apc.fit.model,
   do.plot=TRUE,do.value=FALSE,
   pch=c(21,24,25),
   col=c("black","green","blue","red"),
   bg=NULL,cex=NULL,main=NULL)
```

# Arguments

apc.fit.model	List. See apc.fit.model for a description of the format.
do.plot	Optional. Logical. If FALSE plot is not produced. Default is TRUE.
do.value	Optional. Logical. If TRUE value is produced. Default is FALSE.
pch	Optional points argument. Numeric. Default is 21/24/25. 21 is a circle used for the central 80% of distribution. 24/25 are triangle point up/down used for right tail and left tail.
col	Optional plot argument. Character or Numeric. Default is "black"/"green"/"blue"/"red". Black is use for central 80%, Green is used for 90-95% and 5-10%, Blue is used for 95-99% and 1-5%, Red is used for tails.
bg	Optional plot argument. Character or Numeric. Default is bg=col.
cex	Optional plot argument. Numeric. Magnification. Default is internally computed.
main	Optional plot argument. Character. Main title. Default is internally computed.

## Value

Vector of probability transforms. Only produced if do. value is set to TRUE. See example below.

# Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 2 Dec 2013

## See Also

data.Italian.bladder.cancer for information on the data used in the example.

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## **Examples**

```
# Example with Italian bladder cancer data
# HOW TO USE VALUE
data.list <- data.Italian.bladder.cancer()</pre>
fit <- apc.fit.model(data.list, "poisson.dose.response", "APC")</pre>
v.pt <- apc.plot.fit.pt(fit,do.value=TRUE)</pre>
m.pt <- matrix(data=NA,nrow=fit$data.xmax,ncol=fit$data.ymax)</pre>
m.pt[fit$index.data] <- v.pt</pre>
m.pt
                        [,2]
                                   [,3]
                                                         [.5]
  [1,] 0.63782311 0.5651585 0.33982477 0.91299734 0.5759652
 [2,] 0.82676269 0.8992667 0.26378120 0.28795884 0.3708787
# [3,] 0.54139571 0.2445995 0.51923747 0.63451773 0.7955547
# [4,] 0.87364488 0.8228499 0.07219437 0.38789788 0.5938305
  [5,] 0.86797473 0.3934085 0.34525271 0.38955656 0.5097203
  [6,] 0.65027598 0.8377994 0.29018594 0.03694977 0.7990229
  [7,] 0.43769468 0.1099946 0.50261364 0.56777485 0.8916552
  [8,] 0.67518708 0.5519831 0.67817803 0.19793887 0.5354669
# [9,] 0.02717016 0.2066092 0.77035122 0.89047749 0.5017919
# [10,] 0.71037782 0.9464356 0.36897847 0.41790169 0.2080577
# [11,] 0.50922468 0.3085978 0.55261186 0.77592343 0.3597815
```

```
apc.plot.fit.residuals
```

Level plots of residuals / fitted values / linear predictors

### **Description**

Level plots of residuals / fitted values / linear predictors. Returns residuals / fitted values / linear predictors as matrices when requested. The plots use apc.plot.data.level. They plot are given in the original coordinate system.

# Usage

```
apc.plot.fit.residuals(apc.fit.model,
rotate=FALSE,main=NULL,lab=NULL,
contour=FALSE,colorkey=TRUE,return=FALSE)
    apc.plot.fit.fitted.values(apc.fit.model,
rotate=FALSE,main=NULL,lab=NULL,
contour=FALSE,colorkey=TRUE,return=FALSE)
    apc.plot.fit.linear.predictors(apc.fit.model,
rotate=FALSE,main=NULL,lab=NULL,
contour=FALSE,colorkey=TRUE,return=FALSE)
```

apc.plot.fit.residuals 65

## **Arguments**

apc.fit.model	List. Output from apc.fit.model. See there for a description of the format.
rotate	Optional. Logical. If TRUE rotates plot 90 degrees clockwise (or anti-clockwise if data.format is "CL"). Default is FALSE.
main	Optional. Character. Main title.
lab	Optional plot parameter. A numerical vector of the form $c(x, y, len)$ which modifies the default way that axes are annotated. The values of x and y give the (approximate) number of tickmarks on the x and y axes. len is not implemented.
contour	Optional levelplot (lattice) parameter. Logical. Contour lines drawn if TRUE. Default FALSE.
colorkey	Optional levelplot (lattice) parameter. Logical or list. Determines color key. Default TRUE.
return	Optional. Logical. If TRUE returns matrix with values. Default is FALSE.

### Value

Matrix of the original format with residuals / fitted values /linear predictors as entries. Only produced if return is set to TRUE.

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 26 Apr 2015

## See Also

data.Italian.bladder.cancer for information on the data used in the example.

```
# Example with Italian bladder cancer data
data.list <- data.Italian.bladder.cancer()</pre>
fit <- apc.fit.model(data.list, "poisson.dose.response", "APC")</pre>
apc.plot.fit.fitted.values(fit,return=TRUE)
                 1960-1964 1965-1969 1970-1974
       1955-1959
                                                    1975-1979
# 25-29 3.04200
                                         2.327538
                                                    12.000000
                  3.368944
                              2.261518
# 30-34 13.11980 12.835733
                              13.955859
                                         10.416142
                                                     9.672462
# 35-39 24.15536 33.591644
                              33.388355
                                         37.542301
                                                    26.322340
# 40-44 69.89262 68.842728
                             96.652963
                                         98.478793 113.132896
# 45-49 217.97285 189.375728 189.115063 272.281239
                                                    285.255119
# 50-54 450.44864 529.823519 462.504305
                                        469.869189 701.354350
# 55-59 724.88451 904.298410 1069.452434 969.346982 966.017661
# 60-64 877.17820 1226.088350 1532.521380 1877.331703 1807.880364
# 65-69 950.36106 1296.011123 1798.196048 2336.012274 3028.419493
# 70-74 903.94495 1187.708772 1598.021907 2302.605072 3222.719298
# 75-79 831.00000 953.055049 1280.930166 1755.788768 2678.226017
```

apc.polygon

# Description

Draws a line for point forecasts and adds shaded region for forecast distribution around it. This is added to a plot in the same way as lines and polygon add lines and polygons to a plot.

# Usage

```
apc.polygon(m.forecast,x.origin=1,
plot.se=TRUE,plot.se.proc=FALSE,plot.se.est=FALSE,
unit=1,
col.line=1,lty.line=1,lwd.line=1,
q.se=c(2,2,2),
angle.se=c(45,45,45),
border.se=c(NA,NA,NA),
col.se=gray(c(0.50,0.80,0.90)),
density.se=c(NULL,NULL,NULL),
lty.se=c(1,1,1))
```

## **Arguments**

m.forecast	Matrix. Up to 4 columns. Column 1: point forecasts. Column 2: forecast standard errors. Column 3: process standard errors. Column 4: estimation standard errors.
x.origin	<i>Optional.</i> Numerical. x-coordinate for last observation. The first point forecast is made at x.origin+unit, where unit (with default 1) is defined in apc.data.list. Default: 1.
plot.se	Optional. Logical. Should forecast standard errors be plotted? Default: TRUE.
plot.se.proc	Optional. Logical. Should process standard errors be plotted? Default: FALSE.
plot.se.est	Optional. Logical. Should estimation standard errors be plotted? Default: FALSE.
unit	Optional. Numerical. step length for point forecasts. Default=1.
col.line	Optional. Point forecasts: Colour of line. Same as col for lines. Default: 1.
lty.line	Optional. Point forecasts: Type of line. Same as 1ty for lines. Default: 1.
lwd.line	Optional. Point forecasts: Width of line. Same as 1wd for lines. Default: 1.
q.se	<i>Optional</i> . Vector of length 3. Multiplication factors for standard errors. Default: c(2,2,2).
angle.se	<i>Optional</i> . Standard error polygon: 3-vector: Angle of shading. Same as angle for polygon. Default: =c(45, 45, 45).
border.se	<i>Optional.</i> Standard error polygon: 3-vector: Border of polygon. Same as border for polygon. Default: =c(NA,NA,NA).

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col.se	<i>Optional.</i> Standard error polygon: 3-vector: Colour of polygon. Same as col for polygon. Default: gray(c(0.50,0.80,0.90)).
density.se	<i>Optional</i> . Standard error polygon: 3-vector: Density of shading. Same as density for polygon. Default: =c(NULL, NULL, NULL).
lty.se	<i>Optional.</i> Standard error polygon: 3-vector: Type of shading. Same as 1ty for polygon. Default: =c(1,1,1).

### **Details**

The empirical example of Martinez Miranda, Nielsen and Nielsen (2015) uses the data data.asbestos. The results of that paper are reproduced in the vignette ReproducingMMNN2015.pdf, ReproducingMMNN2015.R on Vignettes. The function is used there.

### Author(s)

Bent Nielsen <br/> bent.nielsen@nuffield.ox.ac.uk> 6 Jan 2016

#### References

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Article, Nuffield DP.

data.aids	UK aids data	

## **Description**

Function that organises UK aids data in apc.data.list format.

The data set is taken from table 1 of De Angelis and Gilks (1994). The data are also analysed by Davison and Hinkley (1998, Example 7.4). The data are reporting delays for AIDS counting the number of cases by the date of diagnosis and length of reporting delay, measured by quarter.

The data set is in "trapezoid"-format. The original data set is unbalanced in various ways: first column covers a reporting delay of less than one month (or should it be less than one quarter?); last column covers a reporting delay of at least 14 quarters; last diagonal include incomplete counts. The default data set excludes the incomplete counts in the last diagonal, but includes the unbalanced first and last columns.

## Usage

```
data.aids(all.age.groups = FALSE)
```

#### **Arguments**

all.age.groups logical. If FALSE (default), the last calendar year with incomplete counts is ignored.

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### Value

The value is a list in apc. data. list format.

response matrix of cases

data.format logical equal to "trapezoid".

age1 numeric equal to 0. This is the label for the reporting delay.

per1 NULL. Not needed when data.format="trapezoid"

coh1 numeric equal to 1983.5. This is the label for the diagnosis quarter (1983, third

quarter).

unit numeric equal to 1/4. This is the width of the age and period groups.

per.zero numeric equal to 0. per.max numeric equal to 38. time.adjust numric equal to 0.

label character. Default data has "UK AIDS - clean".

### Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 7 Feb 2016

#### Source

Table 1 of De Angelis and Gilks (1994). Also analysed by Davison and Hinkley (1998, Example 7.4).

## References

De Angelis, D. and Gilks, W.R. (1994) Estimating acquired immune deficiency syndrome incidence accounting for reporting delay. *Journal of the Royal Statistical Society* A 157, 31-40.

Davison, A.C. and Hinkley, D.V. (1998) *Bootstrap methods and their application*. Cambridge: Cambridge University Press.

#### See Also

General description of apc.data.list format.

## **Examples**

```
#####################################
```

```
## It is convient to construct a data variable
data <- data.Belgian.lung.cancer()
## To see the content of the data
data</pre>
```

### 

```
# Forecast AIDS incidences by diagonsis year (cohort).
# uses as poisson response model with an AC structure
# although there is evidence of overdispersion and the
```

data.asbestos 69

```
# period effect appears significant.
# The omission of the period effect follows
# Davison and Hinkley and a parsimoneous model may be
# advantageous when forecasting.
apc.fit.table(data.aids(), "poisson.response")
fit <- apc.fit.model(data.aids(), "poisson.response", "AC")</pre>
forecast <- apc.forecast.ac(fit)</pre>
data.sums.coh <- apc.data.sums(data.aids())$sums.coh</pre>
forecast.total <- forecast$response.forecast.coh</pre>
forecast.total[,1] <- forecast.total[,1]+data.sums.coh[25:38]</pre>
x \leftarrow seq(1983.5, 1992.75, by=1/4)
y <- data.sums.coh
xlab<- "diagnosis year (cohort)"</pre>
ylab<- "diagnoses"
main<- "Davison and Hinkley, Fig 7.6, parametric version"
\verb|plot(x,y,xlim=c(1988,1993),ylim=c(200,600),xlab=xlab,ylab=ylab,main=main)|\\
apc.polygon(forecast.total,x.origin=1989.25,unit=1/4)
```

data.asbestos

Ashestos data

### **Description**

Function that organises asbestos data in apc.data.list format.

Counts of mesothelioma deaths in the UK by age and period. Mesothelioma is most often caused by exposure to asbestos.

The data set is in "PA"-format.

data.asbestos is for men 1967-2012 data.asbestos.2013 is the same as data.asbestos.2013.men and is for men 1968-2013. data.asbestos.2013.women and is for women 1968-2013.

The primary data set includes ages 25-89, which is obtained when using the function without arguments or with argument all.age.groups=FALSE. The secondary data includes younger and older age groups, which is obtained when using the function with argument all.age.groups=TRUE. The apc.package is at present not aimed at such unbalanced data.

## Usage

```
data.asbestos(all.age.groups = FALSE)
data.asbestos.2013(all.age.groups = FALSE)
data.asbestos.2013.women(all.age.groups = FALSE)
data.asbestos.2013.men(all.age.groups = FALSE)
```

## **Arguments**

all.age.groups logical. If FALSE (default), only age groups 25-89 are included.

70 data.asbestos

### Value

The value is a list in apc.data.list format.

response matrix of cases. Numbers of mesothelioma deaths by period and age. Period

runs 1967-2007. Age runs 25-89 when all.age.groups=FALSE. "PA"-format.

dose NULL

data.format logical equal to "PA". Data organised with period-groups in rows and age-

groups in columns.

age1 numeric equal to 25. This is the label for the first age group of 25.

per1 numeric equal to 1967. This is the label for the first period group of 1967.

coh1 NULL. Not needed when data.format="PA"

unit numeric equal to 1. This is the width of the age and period groups.

per.zero NULL. Not needed when data.format="PA"
per.max NULL. Not needed when data.format="PA"

time.adjust 0. Thus age=89 in period=1967 corresponds to cohort=1967-89+0=1878.

label character. "UK asbestos".

### Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 30 April 2016

## Source

Data were prepared for the Asbestos Working Party by the UK Health and Safety Executive. An APC analysis of these data can be found in Martinez Miranda, Nielsen and Nielsen (2015). The results of that paper are reproduced in the vignette ReproducingMMNN2015.pdf, ReproducingMMNN2015.R on Vignettes. These data are also used in Nielsen (2015).

The updated data set data.asbestos.2013 is for 1968-2013 and has the same structure. This is analysed in Martinez-Miranda, Nielsen and Nielsen (2016).

### References

Martinez Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2015) Inference and forecasting in the age-period-cohort model with unknown exposure with an application to mesothelioma mortality. *Journal of the Royal Statistical Society* A 178, 29-55. *Download*: Nuffield DP.

Martinez-Miranda, M.D., Nielsen, B. and Nielsen, J.P. (2016) A simple benchmark for mesothelioma projection for Great Britain. To appear in *Occupational and Environmental Medicine*. *Download*: Nuffield DP.

Nielsen, B. (2015) apc: An R package for age-period-cohort analysis. R Journal 7, 52-64. *Download*: Open access.

### See Also

General description of apc.data.list format.

## **Examples**

```
data.Belgian.lung.cancer
```

Belgian lung cancer data

## **Description**

Function that organises Belgian lung cancer data in apc.data.list format.

The data set is taken from table VIII of Clayton and Schifflers (1987a), which contains age-specific incidence rates (per 100,000 person-years observation) of lung cancer in Belgian females during the period 1955-1978. Numerators are also available. The original source was the WHO mortality database.

The data set is in "AP"-format. The original data set is unbalanced since the first four period groups cover 5 years, while the last covers 4 years. The primary data set has 4 period groups, which is obtained when using the function without arguments or with argument unbalanced=FALSE. The secondary data set has 5 uneven sized period groups, which is obtained when using the function with argument unbalanced=TRUE. The apc.package is at present not aimed at such unbalanced data.

### **Usage**

```
data.Belgian.lung.cancer(unbalanced = FALSE)
```

## **Arguments**

unbalanced logical. If TRUE (default), the last 4-year group column of the data is ignored.

### Value

The value is a list in apc.data.list format.

matrix of mortality rates. This is not needed for the apc.data.list format, but

included as this is the original data formats

matrix of cases response dose matrix of cases/rates

logical equal to "AP". Data organised with age-groups in rows and perioddata.format

groups in columns.

age1 numeric equal to 25. This is the label for the first age group covering ages 25-29. per1

numeric equal to 1955. This is the label for the first period group covering period

1955-1959.

NULL. Not needed when data.format="AP" coh1

numeric equal to 5. This is the width of the age and period groups. unit

NULL. Not needed when data.format="AP" per.zero NULL. Not needed when data.format="AP" per.max

time.adjust 0. Thus age=25 in period=1955 corresponds to cohort=1955-25+0=1930, and

indeed the centers of the age and period groups, that is age=27 and period=1957

translate into cohort=1957-27+0=1930.

label character. "Belgian lung cancer".

## Author(s)

Bent Nielsen <br/> bent.nielsen@nuffield.ox.ac.uk> 8 Sep 2015 (24 Oct 2013)

#### Source

Table VIII of Clayton and Schifflers (1987a).

### References

Clayton, D. and Schifflers, E. (1987a) Models for temperoral variation in cancer rates. I: age-period and age-cohort models. Statistics in Medicine 6, 449-467.

## See Also

General description of apc.data.list format.

```
## It is convient to construct a data variable
data <- data.Belgian.lung.cancer()</pre>
## To see the content of the data
data
```

data.Italian.bladder.cancer 73

data.Italian.bladder.cancer

Italian bladder cancer data

## **Description**

Function that organises Italian bladder data in apc.data.list format.

The data set is taken from table IV of Clayton and Schifflers (1987a), which contains age-specific incidence rates (per 100,000 person-years observation) of bladder cancer in Italian males during the period 1955-1979. Numerators are also available. The original source was the WHO mortality database.

The data set is in "AP"-format.

# Usage

```
data.Italian.bladder.cancer()
```

#### Value

The value is a list in apc.data.list format.

rates matrix of mortality rates. This is not needed for the apc.data.list format, but

included as this is the original data formats

response matrix of cases
dose matrix of cases/rates

data.format logical equal to "AP". Data organised with age-groups in rows and period-

groups in columns.

age1 numeric equal to 25. This is the label for the first age group covering ages 25-29.

per1 numeric equal to 1955. This is the label for the first period group covering period

1955-1959.

coh1 NULL. Not needed when data.format="AP"

unit numeric equal to 5. This is the width of the age and period groups.

per.zero NULL. Not needed when data.format="AP"
per.max NULL. Not needed when data.format="AP"

time.adjust 0. Thus age=25 in period=1955 corresponds to cohort=1955-25+0=1930, and

indeed the centers of the age and period groups, that is age=27 and period=1957

translate into cohort=1957-27+0=1930.

label character. "Italian bladder cancer".

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 8 Sep 2015 (24 Oct 2013)

#### **Source**

Table IV of Clayton and Schifflers (1987a).

#### References

Clayton, D. and Schifflers, E. (1987a) Models for temperoral variation in cancer rates. I: age-period and age-cohort models. *Statistics in Medicine* 6, 449-467.

## See Also

General description of apc.data.list format.

## **Examples**

```
data.Japanese.breast.cancer
```

Japanese breast cancer data

# **Description**

Function that organises Japanese breast data in apc.data.list format.

The data set is taken from table I of Clayton and Schifflers (1987b), which contains age-specific mortality rates (per 100,000 person-years observation) of breast cancer in Japan, during the period 1955-1979. Reported in 5 year age groups and 5 year period groups. Numbers of cases on which rates are based are also available. The original source was WHO mortality data base.

The data set is in "AP"-format.

## Usage

```
data.Japanese.breast.cancer()
```

#### Value

The value is a list in apc.data.list format.

rates matrix of mortality rates. This is not needed for the apc.data.list format, but

included as this is the original data formats

response matrix of cases

dose	matrix of cases/rates
data.format	logical equal to "AP". Data organised with age-groups in rows and period-groups in columns.
age1	numeric equal to 25. This is the label for the first age group covering ages 25-29.
per1	numeric equal to 1955. This is the label for the first period group covering period 1955-1959.
coh1	NULL. Not needed when data.format="AP"
unit	numeric equal to 5. This is the width of the age and period groups.
per.zero	NULL. Not needed when data.format="AP"
per.max	NULL. Not needed when data.format="AP"
time.adjust	0. Thus age=25 in period=1955 corresponds to cohort=1955-25+0=1930, and indeed the centers of the age and period groups, that is age=27 and period=1957 translate into cohort=1957-27+0=1930.

# Author(s)

label

Bent Nielsen <br/> <br/> dent.nielsen@nuffield.ox.ac.uk> 8 Sep 2015 (24 Oct 2013)

character. "Japanese breast cancer".

## **Source**

Table I of Clayton and Schifflers (1987b)

## References

Clayton, D. and Schifflers, E. (1987b) Models for temperoral variation in cancer rates. II: age-period-cohort models. *Statistics in Medicine* 6, 469-481.

# See Also

General description of apc.data.list format.

76 data.loss.BZ

data.loss.BZ	Motor data

## Description

Function that organises loss data in apc.data.list format.

The data set is taken from table 3.5 of Barnett & Zehnwirth (2000). Source of data unclear. It includes a run-off triangle: "response" (X) is paid amounts (units not reported) along with measures of exposure.

Data also analysed in e.g. Kuang, Nielsen, Nielsen (2011).

The data set is in "CL"-format.

At present apc.package does not have functions for either forecasting or for exploiting the counts. For this one can with advantage use the DCL.package.

## Usage

```
data.loss.BZ
```

#### Value

The value is a list in apc.data.list format.

response vector of paid amounts, X

counts vector of number of reported claims, N

dose NULL.

data.format logical. Equal to "CL.vector.by.row". Data organised in vectors.

age1 numeric. Equal to 1.

per1 NULL. Not needed when data.format="CL"

coh1 numeric. Equal to 1.
unit numeric. Equal to 1.

per.zero NULL. Not needed when data.format="CL"
per.max NULL. Not needed when data.format="CL"

time.adjust 0. Thus age=1 in cohort=1 corresponds to period=1+1-1+0=1.

label character. "loss BZ".

## Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 8 Sep 2015 (18 Mar 2015)

## Source

Tables 1,2 of Verrall, Nielsen and Jessen (2010).

data.loss.TA 77

## References

Barnett G, Zehnwirth B (2000) Best estimates for reserves. Proc. Casualty Actuar. Soc. 87, 245–321

Kuang D, Nielsen B, Nielsen JP (2011) Forecasting in an extended chain-ladder-type model *Journal* of Risk and Insurance 78, 345-359

#### See Also

General description of apc.data.list format.

# **Examples**

data.loss.TA

Motor data

# **Description**

Function that organises loss data in apc.data.list format.

The data set is taken from Table 1 of Verrall (1991), who attributes the data to Taylor and Ashe (1983). It includes a run-off triangle: "response" (X) is paid amounts (units not reported).

Data also analysed in various papers, e.g. England and Verrall (1999).

The data set is in "CL"-format.

At present apc.package does not have functions for either forecasting or for exploiting the counts. For this one can with advantage use the DCL.package.

## Usage

```
data.loss.TA
```

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## Value

The value is a list in apc. data. list format.

response vector of paid amounts, X

dose NULL.

data. format logical. Equal to "CL.vector.by.row". Data organised in vectors.

age1 numeric. Equal to 1.

per1 NULL. Not needed when data.format="CL"

coh1 numeric. Equal to 1.
unit numeric. Equal to 1.

per.zero NULL. Not needed when data.format="CL"
per.max NULL. Not needed when data.format="CL"

time.adjust 0. Thus age=1 in cohort=1 corresponds to period=1+1-1+0=1.

label character. "loss TA".

#### Author(s)

Bent Nielsen <br/> <br/>bent.nielsen@nuffield.ox.ac.uk> 8 Sep 2015 (18 Mar 2015)

#### Source

Tables 1 of Verrall (1991).

# References

England, P., Verrall, R.J. (1999) Analytic and bootstrap estimates of prediction errors in claims reserving Insurance: Mathematics and Economics 25, 281-293

Taylor, G.C., Ashe, F.R. (1983) Second moments of estimates of outstanding claims Journal of Econometrics 23, 37-61

Verrall, R.J. (1991) On the estimation of reserves from loglinear models Insurance: Mathematics and Economics 10, 75-80

#### See Also

General description of apc.data.list format.

data.loss.VNJ 79

data.loss.VNJ

Motor data

## **Description**

Function that organises motor data in apc.data.list format.

The data set is taken from tables 1,2 of Verrall, Nielsen and Jessen (2010). Data from Codan, Danish subsiduary of Royal & Sun Alliance. It is a portfolio of third party liability from motor policies. The time units are in years. There are two run-off triangles: "response" (X) is paid amounts (units not reported) "counts" (N) is number of reported claims.

Data also analysed in e.g. Martinez Miranda, Nielsen, Nielsen and Verrall (2011) and Kuang, Nielsen, Nielsen (2015).

The data set is in "CL"-format.

At present apc.package does not have functions for either forecasting or for exploiting the counts. For this one can with advantage use the DCL.package.

# Usage

data.loss.VNJ

## Value

The value is a list in apc.data.list format.

response vector of paid amounts, X

counts vector of number of reported claims, N

dose NULL

data.format logical. Equal to "CL.vector.by.row". Data organised in vectors.

age1 numeric. Equal to 1.

per1 NULL. Not needed when data.format="CL"

coh1 numeric. Equal to 1.
unit numeric. Equal to 1.

per.zero NULL. Not needed when data.format="CL"

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```
per.max NULL. Not needed when data.format="CL"
```

time.adjust 0. Thus age=1 in cohort=1 corresponds to period=1+1-1+0=1.

label character. "loss VNJ".

## Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 18 Mar 2015 updated 4 Jan 2016

#### **Source**

Tables 1,2 of Verrall, Nielsen and Jessen (2010).

#### References

Verrall R, Nielsen JP, Jessen AH (2010) Prediction of RBNS and IBNR claims using claim amounts and claim counts *ASTIN Bulletin* 40, 871-887

Martinez Miranda, M.D., Nielsen, B., Nielsen, J.P. and Verrall, R. (2011) Cash flow simulation for a model of outstanding liabilities based on claim amounts and claim numbers. *ASTIN Bulletin* 41, 107-129

Kuang D, Nielsen B, Nielsen JP (2015) The geometric chain-ladder *Scandinavian Acturial Journal* 2015, 278-300.

#### See Also

General description of apc.data.list format.

```
## It is convient to construct a data variable
data <- data.loss.VNJ()</pre>
## To see the content of the data
data
#############################
# Fit chain-ladder model
fit.ac <- apc.fit.model(data, "poisson.response", "AC")</pre>
fit.ac$coefficients.canonical
id.ac <- apc.identify(fit.ac)</pre>
id.ac$coefficients.dif
#############################
# Compare output with table 7.2 in
# Kuang D, Nielsen B, Nielsen JP (2015)
                Estimate Std. Error
                                         z value
                                                       Pr(>|z|)
             13.07063963 0.0000000000
# level
                                             Inf 0.000000e+00
```

data.loss.VNJ 81

```
# D_age_2
            -0.06543495 0.0006018694 -108.71950 0.000000e+00
# D_age_3
            -0.80332424 0.0008757527 -917.29576 0.000000e+00
# D_age_4
            -0.41906516 0.0012294722 -340.84965 0.000000e+00
           -0.29097802 0.0015627740 -186.19329 0.000000e+00
# D_age_5
# D_age_6
           -0.57299006 0.0021628918 -264.91850 0.000000e+00
# D_age_7
           -0.36101594 0.0030016569 -120.27222 0.000000e+00
# D_age_8
           -0.62706059 0.0046139466 -135.90547 0.000000e+00
           0.12160793 0.0061126021 19.89463 4.529830e-88
# D_age_9
# D_age_10 -2.59708012 0.0245028290 -105.99103 0.000000e+00
# D_cohort_2 -0.02591843 0.0009037977 -28.67724 7.334840e-181
# D_cohort_3  0.18973130 0.0011301184 167.88621 0.000000e+00
# D_cohort_4   0.12354693   0.0010508785   117.56539   0.000000e+00
# D_cohort_5 -0.10114701 0.0010566534 -95.72392 0.000000e+00
# D_cohort_7 -0.17175409 0.0011676536 -147.09336 0.000000e+00
# D_cohort_8  0.20671145 0.0012098255 170.86055 0.000000e+00
# D_cohort_9 0.04056617 0.0012325163 32.91329 1.418555e-237
# D_cohort_10 0.06876759 0.0015336998 44.83771 0.000000e+00
###################################
# Get deviance table.
# APC strongly rejected => overdispersion?
   AC (Chain-ladder) rejected against APC (inference invalid anyway)
    => one should be careful with distribution forecasts
apc.fit.table(data, "poisson.response")
-2logL df.residual prob(>chi_sq) LR.vs.APC df.vs.APC prob(>chi_sq)
                                                                           aic
# APC
      176030.0 28
                                   0
                                          NA NA NA
                                                                     176841.7
                     36
# AP
      305784.6
                                    0 129754.6
                                                     8
                                                                  0 306580.3
# AC
      374155.2
                     36
                                  0 198125.2
                                                     8
                                                                 0 374950.9
# PC
      553555.1
                     36
                                  0 377525.0
                                                     8
                                                                 0 554350.7
# Ad
      486013.4
                     44
                                  0 309983.4
                                                    16
                                                                 0 486793.0
                                                                 0 710789.3
# Pd
                    44
                                  0 533979.6
      710009.6
                                                    16
# Cd
      780859.4
                     44
                                  0 604829.4
                                                                 0 781639.1
                                                    16
# A
      575389.6
                     45
                                   0 399359.6
                                                    17
                                                                  0 576167.3
                                   0 9307658.0
                                                                  0 9484465.7
# P
      9483688.1
                      45
                                                     17
# C
                                   0 7793004.0
                                                                  0 7969811.7
      7969034.0
                      45
                                                     17
      898208.1
                      52
                                   0
                                      722178.1
                                                     24
                                                                     898971.7
# t
                                                                   0
                      53
                                   0
                                                     25
# tA
      987389.4
                                       811359.4
                                                                      988151.1
# tP
      9690623.4
                      53
                                   0 9514593.4
                                                     25
                                                                   0 9691385.1
# tC
      8079187.6
                      53
                                   0 7903157.6
                                                     25
                                                                   0 8079949.3
                                                     26
# 1
     10815443.5
                      54
                                   0 10639413.5
                                                                   0 10816203.2
#####################################
# Fit geometric chain-ladder model
fit.ac <- apc.fit.model(data,"log.normal.response","AC")</pre>
fit.ac$coefficients.canonical
id.ac <- apc.identify(fit.ac)</pre>
```

id.ac\$coefficients.dif

```
###################################
# Compare output with table 7.2 in
# Kuang D, Nielsen B, Nielsen JP (2015)
              Estimate Std. Error
                                 t value
                                           Pr(>|t|)
# level
          # D_age_2
          -0.0721758004   0.1291053   -0.55904595   5.761304e-01
          # D_age_3
          # D_age_4
          -0.3354312554  0.1538274  -2.18056918  2.921530e-02
# D_age_5
          # D_age_6
          # D_age_7
# D_age_8
           # D_age_9
           0.0078494549
          -2.5601846890 0.3415805 -7.49511273 6.624141e-14
# D_age_10
# D_cohort_2 -0.1025686798 0.1291053 -0.79445748 4.269292e-01
           # D_cohort_3
           # D_cohort_4
# D_cohort_5 -0.0920821506 0.1538274 -0.59860701 5.494350e-01
# D_cohort_6 -0.0530061052 0.1673396 -0.31675768 7.514275e-01
# D_cohort_7 -0.2053813051 0.1854134 -1.10769405 2.679940e-01
           0.2705853742  0.2112982  1.28058555  2.003393e-01
# D_cohort_8
# D_cohort_9 -0.0009224552 0.2531172 -0.00364438 9.970922e-01
# D_cohort_10 0.0736954734 0.3415805 0.21574845 8.291838e-01
# Get deviance table.
# AC marginally rejected against APC
apc.fit.table(data,"log.normal.response")
#####################################
     -2logL df.residual LR.vs.APC df.vs.APC prob(>chi_sq)
                                                  aic
# APC -28.528
                                            NA 27.472
                 28
                                 NA
                      24.530
                                  8
# AP
     -3.998
                 36
                                          0.002 36.002
# AC
     -9.686
                 36
                      18.842
                                  8
                                          0.016
                                               30.314
# PC
     31.722
                 36
                      60.250
                                  8
                                          0.000
                                               71.722
     6.251
# Ad
                      34.779
                                               30.251
                 44
                                 16
                                          0.004
# Pd
     41.338
                 44
                      69.866
                                 16
                                          0.000
                                               65.338
# Cd
     38.919
                 44
                      67.447
                                 16
                                          0.000
                                               62.919
                 45
# A
     12.765
                      41.292
                                 17
                                          0.001 34.765
# P
    171.283
                 45
                     199.811
                                 17
                                          0.000 193.283
# C
    162.451
                 45
                     190.979
                                 17
                                          0.000 184.451
# t.
     46.300
                 52
                      74.827
                                 24
                                          0.000 54.300
```

53

53

53

78.069

200.298

191.808

210.694

25

25

25

26

0.000 55.541

0.000 177.770

0.000 169.280

0.000 186.166

# tA

# tP

# tC

49.541

171.770

163.280

182.166

## **Description**

Function that organises US Casualty data from XL Group in apc.data.list format.

The data set is taken from table 1.1 Kuang and Nielsen (2020). Data are for US Casualty data from the XL Group. They are gross paid and reported loss and allocated loss adjustment expense in 1000 USD.

The data set is in "CL"-format.

## Usage

```
data.loss.XL
```

#### Value

The value is a list in apc.data.list format.

response matrix of paid amounts, incremental

dose NULL.

data.format logical. Equal to "CL". age1 numeric. Equal to 1.

per1 NULL. Not needed when data.format="CL"

coh1 numeric. Equal to 1997. unit numeric. Equal to 1997.

per.zero NULL. Not needed when data.format="CL"
per.max NULL. Not needed when data.format="CL"

time.adjust -1996. Thus age=1 in cohort=1997 corresponds to period=1997+1997-1+(-

1996)=1997.

label character. "loss, US casualty, XL Group".

#### Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 26 August 2020 (10 Mar 2018)

#### Source

Table 1.1 of Kuang and Nielsen (2020) and in turn download: xls file from XL Group files.

# References

Kuang, D. and Nielsen B. (2020) Generalized log-normal chain-ladder. *Scandinavian Actuarial Journal* 2020, 553-576. *Download*: Open access. Earlier version: Nuffield DP.

#### See Also

General description of apc.data.list format.

For explanation for Chain Ladder forecast, see apc. forecast.ac.

The analysis in Kuang and Nielsen (2020) is reproduced in the vignette ReproducingKN2020.pdf, ReproducingKN2020.R on Vignettes.

```
## It is convenient to construct a data variable for paid data
data <- data.loss.XL()</pre>
## To see the content of the data
data
#############################
# Get deviance table.
# reproduce Table 4.1 in Kuang and Nielsen (2018).
apc.fit.table(data, "log.normal.response")
apc.fit.table(data, "log.normal.response", model.design.reference="AC")
# > apc.fit.table(data,"log.normal.response")
      -2logL df.residual LR vs.APC df vs.APC prob(>chi_sq) F vs.APC prob(>F)
# APC 170.003
                   153
                            NaN
                                     NaN
                                                  NaN
                                                          NaN
                                                                     NaN 286.003
# AP 243.531
                    171
                           73.527
                                       18
                                                  0.000
                                                           3.564
                                                                   0.000 323.531
# AC 179.873
                    171
                           9.869
                                        18
                                                  0.936
                                                          0.409
                                                                   0.984 259.873
                    171
                         463.428
# PC 633.432
                                        18
                                                  0.000 68.736
                                                                   0.000 713.432
# Ad 258.570
                    189
                          88.567
                                        36
                                                  0.000
                                                                   0.000 302.570
                                                         2.230
# Pd 643.892
                    189 473.888
                                        36
                                                  0.000 36.340
                                                                   0.000 687.892
# Cd 649.142
                    189 479.139
                                        36
                                                  0.000 37.368
                                                                   0.000 693.142
# A
     357.359
                    190 187.355
                                       37
                                                  0.000
                                                         5.956
                                                                   0.000 399.359
# P
     644.176
                    190
                         474.172
                                       37
                                                  0.000 35.412
                                                                   0.000 686.176
     672.392
                    190
                                       37
                                                                   0.000 714.392
# C
                          502.388
                                                  0.000 41.099
# t 664.488
                    207
                          494.484
                                        54
                                                  0.000
                                                        27.015
                                                                   0.000 672.488
# tA 681.993
                    208
                          511.989
                                        55
                                                        29.072
                                                                   0.000 687.993
                                                  0.000
# tP 664.746
                    208
                          494.742
                                        55
                                                  0.000
                                                         26.560
                                                                   0.000 670.746
# tC 686.181
                                                                   0.000 692.181
                    208
                          516.178
                                        55
                                                          29.713
                                                  0.000
# 1
    690.399
                    209
                          520.396
                                        56
                                                  0.000
                                                          29.830
                                                                   0.000 694.399
# > apc.fit.table(data, "log.normal.response", model.design.reference="AC")
     -2logL df.residual LR vs.AC df vs.AC prob(>chi_sq) F vs.AC prob(>F)
                                                                         aic
# AC 179.873
                   171
                           NaN
                                    NaN
                                               NaN
                                                        NaN
                                                              NaN 259.873
# Ad 258.570
                   189
                        78.698
                                    18
                                                 0
                                                      4.319
                                                                  0 302.570
# Cd 649.142
                   189 469.269
                                     18
                                                  0 79.257
                                                                   0 693.142
# A 357.359
                   190 177.486
                                     19
                                                  0 11.955
                                                                   0 399.359
# C 672.392
                   190 492.519
                                                  0 84.930
                                                                   0 714.392
                                     19
# t 664.488
                   207 484.615
                                     36
                                                 0 42.993
                                                                   0 672.488
# tA 681.993
                   208 502.120
                                     37
                                                 0 45.869
                                                                   0 687.993
                   208 506.308
                                     37
                                                 0 46.886
# tC 686.181
                                                                   0 692.181
                   209 510.526
                                                  0 46.670
# 1 690.399
                                     38
                                                                   0 694.399
#####################################
# Fit log normal chain-ladder model
# reproduce Table 4.2 in Kuang and Nielsen (2018).
fit.ac <- apc.fit.model(data, "log.normal.response", "AC")</pre>
```

id.ac <- apc.identify(fit.ac)
id.ac\$coefficients.dif
fit.ac\$s2
fit.ac\$RSS</pre>

#### #############################

# > fit.ac\$RSS
# [1] 28.9557
# > fit.ac\$RSS

```
# > id.ac$coefficients.dif
           Estimate Std. Error t value
                                Pr(>|t|)
# level
         7.660055032 0.1377951 55.59016605 0.000000e+00
# D_age_1998 2.272100342 0.1335080 17.01846386 5.992216e-65
# D_age_1999
         0.932530550 0.1362610 6.84370899 7.716860e-12
       # D_age_2000
        # D_age_2001
# D_age_2002
         # D_age_2003
# D_age_2004
       # D_age_2005
# D_age_2006 -0.399729999 0.1721838 -2.32153023 2.025824e-02
# D_age_2007 -0.189656058 0.1802245 -1.05233225 2.926471e-01
# D_age_2008
       -0.242063670 0.1895226 -1.27722853 2.015216e-01
# D_age_2009 -0.260459607 0.2004421 -1.29942545 1.937980e-01
# D_age_2010
       -0.555317528 0.2135164 -2.60081872 9.300158e-03
# D_age_2011
       0.405830766 0.2499291 1.62378389 1.044219e-01
# D_age_2012
       # D_age_2013
         # D_age_2014
# D_age_2015
```

-0.273419402 0.5083832 -0.53782152 5.907003e-01 # D\_age\_2016 # D\_cohort\_1998 0.288755900 0.1335080 2.16283663 3.055375e-02 # D\_cohort\_1999 0.163424236 0.1362610 1.19934721 2.303930e-01 # D\_cohort\_2001 0.149829430 0.1438733 1.04139815 2.976908e-01 # D\_cohort\_2002 -0.374386828 0.1483681 -2.52336417 1.162380e-02 # D\_cohort\_2003 -0.198735893 0.1533567 -1.29590632 1.950078e-01 # D\_cohort\_2004 -0.008807130 0.1589136 -0.05542087 9.558032e-01 # D\_cohort\_2005 -0.005337953 0.1651428 -0.03232325 9.742143e-01 # D\_cohort\_2006 -0.132272851 0.1721838 -0.76820710 4.423642e-01 # D\_cohort\_2007 -0.021862643 0.1802245 -0.12130783 9.034472e-01 # D\_cohort\_2008 -0.472602270 0.1895226 -2.49364600 1.264386e-02 # D\_cohort\_2009 -0.437572798 0.2004421 -2.18303804 2.903301e-02 # D\_cohort\_2010 0.295511564 0.2135164 1.38402260 1.663515e-01 # D\_cohort\_2011 0.310545832 0.2295651 1.35275725 1.761332e-01 # D\_cohort\_2012 -0.268692406 0.2499291 -1.07507473 2.823413e-01 # D\_cohort\_2013 0.142131410 0.2769988 0.51311192 6.078730e-01 # D\_cohort\_2014 0.201777590 0.3156054 0.63933494 5.226051e-01 # D\_cohort\_2015 -0.092672697 0.3777268 -0.24534320 8.061907e-01 # D\_cohort\_2016 0.872997251 0.5083832 1.71720334 8.594203e-02 # > fit.ac\$s2 # [1] 0.1693316

86 data.RH.mortality

```
forecast <- apc.forecast.ac(fit.ac,quantiles=c(0.995)) forecast$response.forecast.coh
```

## 

# > forecast\$response.forecast.coh

#	# forecast		se se.p	roc se	.est t-0	. 995
#	coh_2	1871.073	1026.463	707.4405	743.7428	4544.891
#	coh_3	5099.330	1874.681	1375.8435	1273.3744	9982.659
#	coh_4	7171.317	2123.128	1622.5220	1369.3412	12701.822
#	coh_5	11699.350	2984.949	2274.8292	1932.6338	19474.801
#	coh_6	13717.388	3345.138	2654.4080	2035.6984	22431.090
#	coh_7	14343.522	3188.410	2471.3130	2014.5886	22648.964
#	coh_8	18377.001	3834.057	2910.9751	2495.2390	28364.281
#	coh_9	25488.052	5241.618	3976.5389	3414.9225	39141.867
#	coh_10	30524.942	6213.652	4662.3320	4107.5694	46710.794
#	coh_11	40078.245	8115.990	5976.5789	5490.8835	61219.471
#	coh_12	32680.319	6603.511	4727.4210	4610.6241	49881.712
#	coh_13	28509.077	5895.265	4143.1332	4193.8760	43865.568
#	coh_14	51760.526	11013.030	7540.3989	8026.7807	80448.208
#	coh_15	98747.731	22063.641	14798.3216	16365.0210	156220.991
#	coh_16	100330.677	23254.845	14704.7084	18015.5316	160906.889
#	coh_17	149813.314	36629.836	21310.2885	29792.8931	245229.846
#	coh_18	221549.649	58610.037	29815.3239	50459.7158	374222.093
#	coh_19	229480.904	69931.745	29102.9866	63588.2473	411645.102
#	coh 20	575343.178	235016.967	70362.1087	224236.8135	1187535.497

data.RH.mortality

2-sample mortality data.

# Description

Function that organises mortality data from Riebler and Held (2010) in apc.data.list format.

The data set is taken from the supplementary data of Riebler and Held (2010). Mortality data for women in Denmark and Norway

The original source was Jacobsen et al. (2004).

The data set is in "AP"-format.

# Usage

```
data.RH.mortality.dk()
data.RH.mortality.no()
```

# Value

The value is a list in apc.data.list format.

response matrix of cases

data.RH.mortality 87

dose matrix of cases/rates

data.format logical equal to "AP". Data organised with age-groups in rows and period-

groups in columns.

age1 numeric equal to 0.
per1 numeric equal to 1960.

coh1 NULL. Not needed when data.format="AP"

unit numeric equal to 5. This is the width of the age and period groups.

per.zero NULL. Not needed when data.format="AP"
per.max NULL. Not needed when data.format="AP"

time.adjust 0. Thus age=0 in period=1960 corresponds to cohort=1960-0+0=1960, and in-

deed the centers of the age and period groups, that is age=2 and period=1962

translate into cohort=1962-2+0=1960.

label character. "RH mortality Denmark" or "RH mortality Norway".

## Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 17 Sep 2016

#### Source

Riebler and Held (2010), supplementary material.

#### References

Jacobsen, R, von Euler, M, Osler, M, Lynge, E and Keiding, N (2004) Women's death in Scandinavia - what makes Denmark different? *European Journal of Epidemiology* 19, 117-121.

Riebler, A and Held, L. (2010) The analysis of heterogeneous time trends in multivariate age-period-cohort models. *Biostatistics* 11, 57–59. *Download*: Open access, Supplementary material.

## See Also

General description of apc.data.list format.

data.US.prostate.cancer

Japanese breast cancer data

#### **Description**

Function that organises US prostate data in apc.data.list format.

The data set is taken from table 2 of Holford (1983), which contains age-specific counts of deaths and midperiod population measured in 1000s, during the period 1935-1969. Reported in 5 year age groups and 5 year period groups.

The original source was Cancer deaths: National Center for Health Statistics, 1937-1973 Population 1935-60: Grove and Hetzel, 1968 Population 1960-69: Bureau of the Census, 1974

The data set is in "AP"-format.

# Usage

```
data.US.prostate.cancer()
```

#### Value

The value is a list in apc.data.list format.

matrix of cases response dose matrix of cases/rates data.format logical equal to "AP". Data organised with age-groups in rows and periodgroups in columns. numeric equal to 50. This is the label for the first age group covering ages 25-29. age1 numeric equal to 1935. This is the label for the first period group covering period per1 1955-1959. coh1 NULL. Not needed when data.format="AP" numeric equal to 5. This is the width of the age and period groups. unit NULL. Not needed when data.format="AP" per.zero NULL. Not needed when data.format="AP" per.max time.adjust 0. Thus age=50 in period=1935 corresponds to cohort=1935-50+0=1885, and indeed the centers of the age and period groups, that is age=52 and period=1937 translate into cohort=1937-52+0=1885. label character. "US prostate cancer".

## Author(s)

Bent Nielsen <bent.nielsen@nuffield.ox.ac.uk> 8 Sep 2015 (28 Apr 2015)

## **Source**

```
Table 2 of Holford (1983)
```

#### References

Holford, T.R. (1983) The estimation of age, period and cohort effects for vital rates. *Biometrics* 39, 311-324.

# See Also

General description of apc.data.list format.

## **Examples**

new.apc.identify

Identification of time effects

# **Description**

Computes ad hoc identified time effects.

# Usage

```
new.apc.identify(apc.fit.model)
```

## **Arguments**

```
apc.fit.model List. See apc.fit.model for a description of the format.
```

#### **Details**

Forms ad hoc identified time effects from the canonical parameter. These are used either indirectly by apc.plot.fit or they are computed directly with this command.

The ad hoc identifications are based on Nielsen (2014b). For details see also the vignette Identification.pdf, Identification.R on Vignettes or in the notes below.

For model designs of any type two ad hoc identified time effects.

(1) The type "sum.sum" (same as "ss.dd") gives double sums anchored in the middle of the first period diagonal.

(2) The type "detrend" gives double sums that start in zero and end in zero.

For model designs with only two time effects, that is "AC", "AP", "PC" there is a further ad hoc identification.

- (3) The type "demean" gives single sums of single differences. Derived from "detrend" where the linear trends are attributed to the double sums of double differences. Level unchanged.
- (4) The type "dif" gives the single differences derived from "demean". Could also have been chosen as canonical parametrisation for these models.

## Value

index.age.max	Vector. Indices for age parameters when using coefficients.ssdd or coefficients.detrend. The length is two longer that that of apc.model.fit\$index.age if model.design is "APC. NULL if age double differences are not estimated.
index.per.max	Vector. Indices for period parameters when using coefficients.ssdd or coefficients.detrend. The length is two longer that that of apc.model.fit\$index.per if model.design is "APC. NULL if age double differences are not estimated.
index.coh.max	Vector. Indices for cohort parameters when using coefficients.ssdd or coefficients.detrend. The length is two longer that that of apc.model.fit\$index.coh if model.design is "APC. NULL if age double differences are not estimated.
dates.max	Vector. Indicates the dates for the parameters when using coefficients.ssdd or coefficients.detrend. The length is six longer that that of apc.model.fit\$index.coh if model.design is "APC.
index.age.sub	* Vector. Indices for age parameters when using coefficients.demean. The length is two longer that that of apc.model.fit\$index.age if model.design is "APC. NULL if age double differences are not estimated.
index.per.sub	* Vector. Indices for period parameters when using coefficients.demean. The length is two longer that that of apc.model.fit\$index.per if model.design is "APC. NULL if age double differences are not estimated.
index.coh.sub	* Vector. Indices for cohort parameters when using coefficients.demean. The length is two longer that that of apc.model.fit\$index.coh if model.design is "APC. NULL if age double differences are not estimated.
dates.sub	* Vector. Indicates the dates for the parameters when using coefficients.demean. The length is six longer that that of apc.model.fit\$index.coh if model.design is "APC.
index.age.dif	* Vector. Indices for age parameters when using coefficients.dif. The length is one longer that that of apc.model.fit\$index.age if model.design is "APC. NULL if age double differences are not estimated.
index.per.dif	* Vector. Indices for period parameters when using coefficients.dif. The length is one longer that that of apc.model.fit\$index.perifmodel.design is "APC. NULL if age double differences are not estimated.
index.coh.dif	* Vector. Indices for cohort parameters when using coefficients.dif. The length is one longer that that of apc.model.fit\$index.coh if model.design is "APC. NULL if age double differences are not estimated.

dates.dif

\* Vector. Indicates the dates for the parameters when using coefficients.dif. The length is three longer that that of apc.model.fit\$index.coh if model.design is "APC.

coefficients.ssdd

Matrix. Coefficients of the double sum of double differences. Normalised to be zero at two values chosen so age=cohort and period is at the minimal value. For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.ssdd

Matrix. Estimated covariance matrix for double sums.

coefficients.detrend

Matrix. Coefficients of the double sum of double differences. Normalised to be zero for first and last value. For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.detrend

Matrix. Estimated covariance matrix for detrended double sums.

coefficients.demean

\* Matrix. Coefficients of the sum of differences. Normalised to be zero for first value. Does not apply is design is "APC" For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.demean

\* Matrix. Estimated covariance matrix for demeaned sums.

coefficients.dif

\* Matrix. Coefficients of the differences. Does not apply is design is "APC" For each parameter is reported coefficient, standard deviation, z-value, which is the ratio of those, and p-value.

covariance.dif

\* Matrix. Estimated covariance matrix for differences.

#### Note

\* indicates that values only implemented for designs "AC", "AP", "PC".

The differences are not identified for design "APC". An arbitrary level can be moved between differences for age, period and cohort.

The differences are not identified for designs "Ad", "Pd", "Cd". These models have two linear trends and one set of double differences. In the model "Ad", as an example, one linear trend will be associated with age, but it is arbitrary whether the second linear trend should be associated with period or cohort. The slope of the age trend will depend on that arbitrary choice. In turn the level of the age differences will be arbitrary.

(1) The type "sum.sum" (same as "ss.dd") gives double sums anchored to be zero in the three points where age=cohort=U, age=U+1, cohort=U age=U, cohort=U+1 with apc.fit.model\$U and where U is the integer value of (per.zero+3)/2 This corresponds to the representation in Nielsen (2014b). The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=U; an age slope, which is the difference of the values of the predictor at age=U+1, cohort=U and age=cohort=U; an cohort slope, which is the difference of the values of the predictor at age=U, cohort=U+1 and age=cohort=U.

(2) The type "detrend" gives double sums that start in zero and end in zero. The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=1, which is usually outside the index set for the data; while age and cohort slopes are adjusted for the ad hoc identification of the time effects.

(3) Subsumes var.apc.identify from apc.indiv (25 Sep 2020)

#### Author(s)

Bent Nielsen <br/> Sep 2020 (12 Apr 2015)

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Nielsen, B. (2014b) Deviance analysis of age-period-cohort models. Work in progress.

#### See Also

The vignette Identification.pdf.

```
##########################
# Belgian lung cancer
# first an example with APC design, note that demean and dif not defined.
data.list <- data.Belgian.lung.cancer()</pre>
fit.apc <- apc.fit.model(data.list,"poisson.dose.response","APC")</pre>
fit.apc$coefficients.canonical
id.apc <- apc.identify(fit.apc)</pre>
id.apc$coefficients.ssdd
id.apc$coefficients.detrend
id.apc$coefficients.demean
id.apc$coefficients.dif
fit.ap <- apc.fit.model(data.list, "poisson.dose.response", "AP")</pre>
fit.ap$coefficients.canonical
id.ap <- apc.identify(fit.ap)</pre>
id.ap$coefficients.ssdd
id.ap$coefficients.detrend
id.ap$coefficients.demean
id.ap$coefficients.dif
```

new.apc.plot.fit 93

new.apc.plot.fit

Plots of apc estimates

## Description

Functions to plot the apc estimates found by apc.fit.model. The function apc.plot.fit detects the type of model.design and model.family from the fit values and makes appropriate plots.

Depending on the model.design the plot has up to 9 sub plots. The type of these can be chosen using type

**Model designs of any type.** If type is "detrend" or "sum.sum" the canonical age period cohort parametrisation is used. This involves double differences of the time effects. The first row of plots are double differences of the time effects. The next two rows of plots illustrate the representation theorem depending on the choice of type. In both cases the sum of the plots add up to the predictor.

"detrend" The last row of plots are double sums of double differences detrend so that that each series starts in zero and ends in zero. The corresponding level and (up to) two linear trends are shown in the middle row of plots. The linear trends are identified to be 0 for age, period or cohort equal to its smallest value. See note 2 below.

"sum.sum" The last row of plots are double sums of double differences anchored as in the derivation of Nielsen (2014b). The corresponding level and (up to) two linear trends are shown in the middle row of plots. The linear trends are identified to be 0 for the anchoring point U of age, period or cohort as described in Nielsen (2014b). See note 1 below.

**Model designs with 2 factors.** If type is "dif" the canonical two factor parametrisation is used. This involves single differences. It is only implemented for model.design of "AC", "AP", "PC". It does not apply for model.design of "APC" because single differences are not identified. It does not apply for the drift models where model.design is "Ad", "Pd", "Cd", "t" because it is not clear which time scale the second linear trend should be attributed to. It is not implemented for model.design of "tA, "tP", "tC", "1". The first row of plots are single differences of the time effects. The next two rows of plots illustrate the representation theorem. In the second row the level is given and in the third row plots of single sums of single differences are given, normalised to start in zero.

**Appearance may vary.** Note, the plots "detrend" and "dif" can give very different appearance of the time effects. The "dif" plots are dominated by linear trends. They can therefore be more difficult to interpret than the "detrend" plots, where linear trends are set aside.

**Standard deviations.** All plots include plots of 1 and 2 standard deviations. The only exception is the intercept in the case model.family is "poisson.response" as this uses a multinomial sampling scheme, where the intercept is set to increase in the asymptotic experiment. The default is to plot standard deviations around zero, so that they represent a test for zero values of the parameters. Using the argument sdv.at.zero the standard deviations can be centered around the estimates. This can give a very complicated appearance.

**Values of coefficients.** These can be found using apc.identify.

## Usage

```
new.apc.plot.fit(apc.fit.model,scale=FALSE,
```

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```
sdv.at.zero=TRUE,type="detrend",
include.linear.plane=TRUE,
include.double.differences=TRUE,
sub.plot=NULL,main.outer=NULL,main.sub=NULL,
cex=NULL,cex.axis=NULL,cex.lab=NULL,cex.main=NULL,
cex.main.outer=1.2,
line.main=0.5,line.main.outer=NULL,
mar=NULL,oma=NULL,mgp=c(2,1,0))
```

## **Arguments**

apc.fit.model List. See apc. fit.model for a description of the format. Optional. Logical. If (TRUE) FALSE use scale of (inverse) link function. Descale fault is FALSE. sdv.at.zero Optional. Logical. If FALSE/TRUE standard deviations are plotted around estimates/zero. Default is TRUE. Optional. Character. If "detrend" double sums start and end in zero. If "sum.sum" type double sums anchored as discussed in Nielsen (??). Default is "detrend". include.linear.plane Optional. Logical. If true include plots of linear plane. Default TRUE include.double.differences Optional. Logical. If true include plots of double differences. Default TRUE Optional. Character: "a", "b",..., "i". Only the indicated sub plot is plotted. Desub.plot fault is NULL so all plots shown. main.outer Optional. Character. Main title in outer margin. Default is generated internally. main.sub Optional. Vector of 9 characters. Main titles for individual plots. Default is generated internally, see note 3 below. Optional. Plot parameter, see par. Controls size of text. Default is NULL so cex that R default is used. Optional. Plot parameter, see par. Controls magnification of axis annotations. cex.axis Default is NULL so that R default is used. cex.lab Optional. Plot parameter, see par. Controls magnification of axis labels. Default is NULL so that R default is used. cex.main Optional. Plot parameter, see par. Controls magnification of main title. Default is NULL so that R default is used. Optional. Controls magnification of outer main title if an array of plots is shown. cex.main.outer Default is 1.2 (same as cex.main). line.main Optional. Specifies the line position of main title in individual plots. Default is line.main.outer Optional. Specifies the line position of outer main title if an array of plots is shown. Default is NULL so that R default is used.

mar

Optional. Gives the number of lines of margin to be specified on the four sides of the plot. Default: c(4,3,2,0) for array of plots, c(4,4,3,1) for a single plot.

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oma	Optional. Gives the size of the outer margins in lines of text. Default: $c(0,0,5,1)$ for array of plots, $c(0,0,0,0)$ for a single plot.
mgp	Optional. Plot parameter, see par. The margin line for the axis title, axis label and axis line. Defauls is $c(2,1,0)$ , different from R default.

#### Note

- (1) The type "sum.sum" (same as "ss.dd") gives double sums anchored to be zero in the three points where age=cohort=U, age=U+1, cohort=U age=U, cohort=U+1 with apc.fit.model\$U and where U is the integer value of (per.zero+3)/2 This corresponds to the representation in Nielsen (2014b). The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=U; an age slope, which is the difference of the values of the predictor at age=U+1, cohort=U and age=cohort=U; an cohort slope, which is the difference of the values of the predictor at age=U, cohort=U+1 and age=cohort=U.
- (2) The type "detrend" gives double sums that start in zero and end in zero. The linear plane is parametrised in terms of a level, which is the value of the predictor at age=cohort=1, which is usually outside the index set for the data; while age and cohort slopes are adjusted for the ad hoc identification of the time effects.
- (3) The default of the titles main.sub are generated internally depending on model specification. In the case of model.design="APC" and a dose-response model family the default value is c(expression(paste("(a) ",Delta^2,alpha)),expression(paste("(b) ",Delta^2,beta)),expression(paste("(",Delta^2,gamma)),"(d) first linear trend","(e) level","(f) second linear trend",expression(paste("(g) detrended ",Sigma^2,Delta^2,alpha)),expression(paste("(h) detrended ",Sigma^2,Delta^2,beta)),expression(paste("(h) detrended ",Sigma^2,Delta^2,gamma)))
- (4) Default values of parameters changed (25 Sep 2020). The old appearance can be reproduced by setting cex.lab=1.5. For example:

```
data.list <- data.Italian.bladder.cancer()
fit.apc <- apc.fit.model(data.list,"poisson.dose.response","APC")
apc.plot.fit(fit.apc,cex.lab=1.5)</pre>
```

#### Author(s)

Bent Nielsen <br/> sent.nielsen@nuffield.ox.ac.uk> 12 Apr 2015 updated 24 September 2020 vs 2.0.0. Subsumes var.apc.plot.fit by Zoe Fannon.

#### References

Kuang, D., Nielsen, B. and Nielsen, J.P. (2008a) Identification of the age-period-cohort model and the extended chain ladder model. Biometrika 95, 979-986. *Download*: Article; Earlier version Nuffield DP.

Nielsen, B. (2014b) Deviance analysis of age-period-cohort models. Work in progress.

#### See Also

data.asbestos and data.Italian.bladder.cancer for information on the data used in the example.

Values of coefficients can be found using apc.identify.

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Further information on the identification in the vignette Identification.pdf, Identification.R on Vignettes.

# **Examples**

```
######################
# Example with Italian bladder cancer data
# Note that the model.design "AC" cannot be rejected against "APC"
# so there is little difference between the two plots of those fits.
data.list <- data.Italian.bladder.cancer()</pre>
apc.fit.table(data.list, "poisson.dose.response")
fit.apc <- apc.fit.model(data.list,"poisson.dose.response","APC")</pre>
new.apc.plot.fit(fit.apc)
# now try an AC model
# can use dev.new() to see both
fit.ac <- apc.fit.model(data.list, "poisson.dose.response", "AC")</pre>
new.apc.plot.fit(fit.ac)
# to check the numerical values for the last two rows of plots use
new.apc.identify(fit.ac)$coefficients.detrend
# to get only a sub plot and playing with titles
# main.outer not used with individual plot
new.apc.plot.fit(fit.ac,sub.plot="a",main.outer="My outer title",main.sub="My sub title")
# to get only a all plots and playing with titles
new.apc.plot.fit(fit.ac,main.outer="My outer title",main.sub=c("1","2","3","4","5","6","7","8","9"))
```

triangle

Triangular matrices used in reserving

# Description

Triangular matrices are used for reserving in general insurance. A matrix is triangular if it is square and it has NAs in lower triangle where row+col>dim. The apc package uses incremental triangles.

The function is.triangle tests if an object is a triangular matrix.

The function triangle.cumulative forms the cumulative version of an incremental matrix by taking partial sums in each row.

The function triangle.incremental forms the incremental version of an cumulative matrix by taking differences in each row.

The function vector . 2. triangle turns a k\*(k+1)/2 vector into a triangular matrix of dimension k.

## Usage

```
is.triangle(m)
triangle.cumulative(m)
triangle.incremental(m)
vector.2.triangle(v,k)
```

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# Arguments

```
\begin{array}{lll} v & & \text{vector. Length } k*(k+1)\!/\!2 \\ k & & \text{integer. Dimension} \\ m & & \text{matrix. Square matrix} \end{array}
```

# Author(s)

Bent Nielsen <br/> bent.nielsen@nuffield.ox.ac.uk> 21 Nov 2019 (7 Feb 2015)

```
m <- vector.2.triangle(1:10,4)
m
is.triangle(m)
triangle.cumulative(m)
triangle.incremental(m)</pre>
```

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