Package 'orthopolynom'

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Description A collection of functions to construct sets of orthogonal polynomials and their recurrence relations. Additional functions are provided to calculate the derivative, integral, value and roots of lists of polynomial objects.
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chebyshev.c.inner.products

Inner products of Chebyshev polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Chebyshev polynomial of the first kind, $C_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.c.inner.products(n)
```

Arguments

n integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle C_n | C_n \rangle = \begin{cases} 4\pi & n \neq 0 \\ 8\pi & n = 0 \end{cases}$$

Value

A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial

2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
gegenbauer.inner.products
```

```
###
### generate the inner products vector for the
### C Chebyshev polynomials of orders 0 to 10
###
h <- chebyshev.c.inner.products( 10 )
print( h )</pre>
```

chebyshev.c.polynomials

Create list of Chebyshev polynomials

Description

This function returns a list with n+1 elements containing the order k Chebyshev polynomials of the first kind, $C_k(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.c.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function chebyshev.c.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n+1 polynomial objects

- 1 order 0 Chebyshev polynomial
- 2 order 1 Chebyshev polynomial

•••

n+1 order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.c.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized C Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.c.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized C Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.c.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

chebyshev.c.recurrences

Recurrence relations for Chebyshev polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Chebyshev polynomial of the first kind, $C_k(x)$, and for orders $k=0,1,\ldots,n$.

Usage

```
chebyshev.c.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

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References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
chebyshev.c.inner.products
```

Examples

```
###
### generate the recurrences data frame for
### the normalized Chebyshev C polynomials
### of orders 0 to 10.
###
normalized.r <- chebyshev.c.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Chebyshev C polynomials
### of orders 0 to 10.
###
unnormalized.r <- chebyshev.c.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

chebyshev.c.weight

Weight function for the Chebyshev polynomial

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the first kind, $C_k(x)$.

Usage

```
chebyshev.c.weight(x)
```

Arguments

Х

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (-2, 2). The formula used to compute the weight function is as follows.

$$w\left(x\right) = \frac{1}{\sqrt{1 - \frac{x^2}{4}}}$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the C Chebyshev weight function for arguments between -3 and 3 ### x <- seq(-3, 3, .01) y <- chebyshev.c.weight( x ) plot( x, y )
```

chebyshev.s.inner.products

Inner products of Chebyshev polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Chebyshev polynomial of the second kind, $S_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.s.inner.products(n)
```

Arguments

n

integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle S_n | S_n \rangle = \pi.$$

Value

```
A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial
2 inner product of order 1 orthogonal polynomial
...

n+1 inner product of order n orthogonal polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### S Chebyshev polynomials of orders 0 to 10
###
h <- chebyshev.s.inner.products( 10 )
print( h )</pre>
```

chebyshev.s.polynomials

Create list of Chebyshev polynomials

Description

This function returns a list with n+1 elements containing the order k Chebyshev polynomials of the second kind, $S_k(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.s.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function chebyshev.s.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

```
A list of n+1 polynomial objects

order 0 Chebyshev polynomial

order 1 Chebyshev polynomial

order n Chebyshev polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.s.recurrences, orthogonal.polynomials, orthonormal.polynomials

```
###
### gemerate a list of normalized S Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.s.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized S Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.s.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

chebyshev.s.recurrences

```
chebyshev.s.recurrences
```

Recurrence relations for Chebyshev polynomials

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Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Chebyshev polynomial of the second kind, $S_k(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
chebyshev.s.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
chebyshev.s.inner.products
```

```
###
### generate the recurrences data frame for
### the normalized Chebyshev S polynomials
### of orders 0 to 10.
###
```

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```
normalized.r <- chebyshev.s.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Chebyshev S polynomials
### of orders 0 to 10.
###
unnormalized.r <- chebyshev.s.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

chebyshev.s.weight

Weight function for the Chebyshev polynomial

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the second kind, $S_k(x)$.

Usage

```
chebyshev.s.weight(x)
```

Arguments

Х

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (-2, 2). The formula used to compute the weight function is as follows.

$$w\left(x\right) = \sqrt{1 - \frac{x^2}{4}}$$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the S Chebyshev weight function for arguments between -2 and 2 ### x <- seq( -2, 2, .01 ) y <- chebyshev.s.weight( x ) plot( x, y )
```

chebyshev.t.inner.products

Inner products of Chebyshev polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Chebyshev polynomial of the first kind, $T_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.t.inner.products(n)
```

Arguments

n int

integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle T_n | T_n \rangle = \begin{cases} \frac{\pi}{2} & n \neq 0 \\ \pi & n = 0 \end{cases}$$

Value

A vector with n+1 elements

- 1 inner product of order 0 orthogonal polynomial
- 2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### T Chybyshev polynomials of orders 0 to 10
###
h <- chebyshev.t.inner.products( 10 )
print( h )</pre>
```

chebyshev.t.polynomials

Create list of Chebyshev polynomials

Description

This function returns a list with n+1 elements containing the order k Chebyshev polynomials of the first kind, $T_k(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.t.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function chebyshev.t.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

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Value

```
A list of n+1 polynomial objects

order 0 Chebyshev polynomial

order 1 Chebyshev polynomial

order n Chebyshev polynomial
```

Author(s)

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References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.u.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized T Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized T Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.t.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

chebyshev.t.recurrences

Recurrence relations for Chebyshev polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Chebyshev polynomial of the first kind, $T_k(x)$, for orders $k=0,1,\ldots,n$.

Usage

```
chebyshev.t.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
chebyshev.t.inner.products
```

```
###
### generate the recurrence relations for
### the normalized T Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- chebyshev.t.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the normalized T Chebyshev polynomials
### of orders 0 to 10
###
unnormalized.r <- chebyshev.t.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

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chebyshev.t.weight

Weight function for the Chebyshev polynomial

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the first kind, $T_k(x)$.

Usage

```
chebyshev.t.weight(x)
```

Arguments

Χ

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (-1,1). The formula used to compute the weight function is as follows.

$$w\left(x\right) = \frac{1}{\sqrt{1-x^2}}$$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

```
### ### compute the T Chebyshev function for argument values between -2 and 2 x <- seq( -1, 1, .01 ) y <- chebyshev.t.weight( x ) plot( x, y )
```

chebyshev.u.inner.products

Inner products of Chebyshev polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Chebyshev polynomial of the second kind, $U_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.u.inner.products(n)
```

Arguments

n

integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle U_n | U_n \rangle = \frac{\pi}{2}$$

Value

A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial

2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### U Chebyshev polynomials of orders 0 to 10
###
h <- chebyshev.u.inner.products( 10 )
print( h )</pre>
```

chebyshev.u.polynomials

Create list of Chebyshev polynomials

Description

This function returns a list with n+1 elements containing the order k Chebyshev polynomials of the second kind, $U_k(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.u.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function chebyshev.u.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n+1 polynomial objects

1 order 0 Chebyshev polynomial
2 order 1 Chebyshev polynomial
...
n+1 order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.u.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized U Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.u.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized T Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.u.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

chebyshev.u.recurrences

Recurrence relations for Chebyshev polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Chebyshev polynomial of the second kind, $U_k(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
chebyshev.u.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

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Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
chebyshev.u.inner.products
```

Examples

```
###
### generate the recurrence relations for
### the normalized U Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- chebyshev.u.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized U Chebyshev polynomials
### of orders 0 to 10
###
unnormalized.r <- chebyshev.u.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

chebyshev.u.weight

Weight function for the Chebyshev polynomial

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the second kind, $U_k(x)$.

Usage

```
chebyshev.u.weight(x)
```

Arguments

Χ

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (-1,1). The formula used to compute the weight function is as follows.

$$w\left(x\right) = \sqrt{1 - x^2}$$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the U Chebyshev function for argument values between -2 and 2 ### x \leftarrow seq(-1, 1, .01) y \leftarrow chebyshev.u.weight(x) plot(x, y)
```

gegenbauer.inner.products

Inner products of Gegenbauer polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Gegenbauer polynomial, $C_k^{(\alpha)}(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
gegenbauer.inner.products(n,alpha)
```

Arguments

n integer value for the highest polynomial order alpha numeric value for the polynomial parameter

Details

The formula used to compute the inner products is as follows.

$$h_n = \left\langle C_n^{(\alpha)} | C_n^{(\alpha)} \right\rangle = \left\{ \begin{array}{cc} \frac{\pi \ 2^{1-2 \ \alpha} \ \Gamma(n+2 \ \alpha)}{n! \ (n+\alpha) \left[\Gamma(\alpha)\right]^2} & \alpha \neq 0 \\ \frac{2 \ \pi}{n^2} & \alpha = 0 \end{array} \right..$$

Value

A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial

2 inner product of order 1 orthogonal polynomial

...

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
ultraspherical.inner.products
```

```
###
### generate the inner products vector for the
### Gegenbauer polynomials of orders 0 to 10
### the polynomial parameter is 1.0
###
h <- gegenbauer.inner.products( 10, 1 )
print( h )</pre>
```

gegenbauer.polynomials

Create list of Gegenbauer polynomials

Description

This function returns a list with n+1 elements containing the order k Gegenbauer polynomials, $C_{k}^{(\alpha)}(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
gegenbauer.polynomials(n, alpha, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

alpha polynomial parameter

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function gegenbauer.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n+1 polynomial objects

1 order 0 Gegenbauer polynomial 2 order 1 Gegenbauer polynomial

n+1 order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

gegenbauer.recurrences 25

See Also

gegenbauer.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Gegenbauer polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
normalized.p.list <- gegenbauer.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Gegenbauer polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
unnormalized.p.list <- gegenbauer.polynomials( 10, 1, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

gegenbauer.recurrences

Recurrence relations for Gegenbauer polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Gegenbauer polynomial, $C_k^{(\alpha)}(x)$, and for orders $k=0, 1, \ldots, n$.

Usage

```
gegenbauer.recurrences(n, alpha, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order alpha numeric value for polynomial parameter

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

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References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
gegenbauer.inner.products
```

Examples

```
###
### generate the recurrences data frame for
### the normalized Gegenbauer polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###
normalized.r <- gegenbauer.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Gegenbauer polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###
unnormalized.r <- gegenbauer.recurrences( 10, 1, normalized=FALSE )
print( unnormalized.r )</pre>
```

gegenbauer.weight

Weight function for the Gegenbauer polynomial

Description

This function returns the value of the weight function for the order k Gegenbauer polynomial, $C_k^{(\alpha)}(x)$.

Usage

```
gegenbauer.weight(x,alpha)
```

Arguments

x the function argument which can be a vector alpha polynomial parameter

Details

The function takes on non-zero values in the interval (-1,1). The formula used to compute the weight function is as follows.

$$w\left(x\right) = \left(1 - x^2\right)^{\alpha - 0.5}$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the Gegenbauer weight function for argument values between -1 and 1 ### x <- seq(-1, 1, .01) y <- gegenbauer.weight( x, 1 ) plot( x, y )
```

ghermite.h.inner.products

Inner products of generalized Hermite polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k generalized Hermite polynomial, $H_k^{(\mu)}(x)$, with itself (i.e. the norm squared) for orders $k=0,1,\ldots,n$.

Usage

```
ghermite.h.inner.products(n, mu)
```

Arguments

n n integer value for the highest polynomial order
mu mu polynomial parameter

Details

The parameter μ must be greater than -0.5. The formula used to compute the inner products is as follows

$$h_n(\mu) = \left\langle H_m^{(\mu)} | H_n^{(\mu)} \right\rangle = 2^{2n} \left[\frac{n}{2} \right]! \Gamma\left(\left[\frac{n+1}{2} \right] + \mu + \frac{1}{2} \right)$$

Value

A vector with n+1 elements

- 1 inner product of order 0 orthogonal polynomial
- 2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

```
###
### generate the inner products vector for the
### generalized Hermite polynomials of orders 0 to 10
### polynomial parameter is 1
###
h <- ghermite.h.inner.products( 10, 1 )
print( h )</pre>
```

ghermite.h.polynomials

```
ghermite.h.polynomials
```

Create list of generalized Hermite polynomials

Description

This function returns a list with n+1 elements containing the order k generalized Hermite polynomials, $H_k^{(\mu)}(x)$, for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
ghermite.h.polynomials(n, mu, normalized = FALSE)
```

Arguments

n integer value for the highest polynomial order
mu numeric value for the polynomial parameter

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Details

The parameter μ must be greater than -0.5. The function ghermite.h.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n + 1 polynomial objects

1 order 0 generalized Hermite polynomial

2 order 1 generalized Hermite polynomial

•••

n+1 order n generalized Hermite polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Alvarez-Nordase, R., M. K. Atakishiyeva and N. M. Atakishiyeva, 2004. A q-extension of the generalized Hermite polynomials with continuous orthogonality property on R, *International Journal of Pure and Applied Mathematics*, 10(3), 335-347.

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

ghermite.h.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized generalized Hermite polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
normalized.p.list <- ghermite.h.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized generalized Hermite polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
unnormalized.p.list <- ghermite.h.polynomials( 10, 1, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

ghermite.h.recurrences

Recurrence relations for generalized Hermite polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k generalized Hermite polynomial, $H_k^{(\mu)}(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
ghermite.h.recurrences(n, mu, normalized = FALSE)
```

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Arguments

n integer value for the highest polynomial order mu numeric value for the polynomial parameter

normalized normalized boolean value which, if TRUE, returns recurrence relations for nor-

malized polynomials

Details

The parameter μ must be greater than -0.5.

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Alvarez-Nordase, R., M. K. Atakishiyeva and N. M. Atakishiyeva, 2004. A q-extension of the generalized Hermite polynomials with continuous orthogonality property on R, *International Journal of Pure and Applied Mathematics*, 10(3), 335-347.

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
ghermite.h.inner.products
```

```
###
### generate the recurrences data frame for
### the normalized generalized Hermite polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###
normalized.r <- ghermite.h.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized generalized Hermite polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###</pre>
```

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```
unnormalized.r <- ghermite.h.recurrences( 10, 1, normalized=FALSE ) print( unnormalized.r )
```

ghermite.h.weight

Weight function for the generalized Hermite polynomial

Description

This function returns the value of the weight function for the order k generalized Hermite polynomial, $H_k^{(\mu)}(x)$.

Usage

```
ghermite.h.weight(x, mu)
```

Arguments

x a numeric vector function argument

mu polynomial parameter

Details

The function takes on non-zero values in the interval $(-\infty, \infty)$. The parameter μ must be greater than -0.5. The formula used to compute the generalized Hermite weight function is as follows.

$$w(x, \mu) = |x|^{2 \mu} \exp(-x^2)$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the generalized Hermite weight function for argument values ### between -3 and 3 ### x <- seq( -3, 3, .01 ) y <- ghermite.h.weight( x, 1 )
```

glaguerre.inner.products

Inner products of generalized Laguerre polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k generalized Laguerre polynomial, $L_n^{(\alpha)}(x)$, with itself (i.e. the norm squared) for orders $k=0,\,1,\,\ldots,\,n$.

Usage

```
glaguerre.inner.products(n,alpha)
```

Arguments

n integer highest polynomial order alpha polynomial parameter

Details

The formula used to compute the inner products is as follows.

$$h_n = \left\langle L_n^{(\alpha)} | L_n^{(\alpha)} \right\rangle = \frac{\Gamma(\alpha + n + 1)}{n!}.$$

Value

A vector with n+1 elements

inner product of order 0 orthogonal polynomial
 inner product of order 1 orthogonal polynomial
 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### generalized Laguerre polynomial inner products of orders 0 to 10
### polynomial parameter is 1.
###
h <- glaguerre.inner.products( 10, 1 )
print( h )</pre>
```

glaguerre.polynomials Create list of generalized Laguerre polynomials

Description

This function returns a list with n+1 elements containing the order n generalized Laguerre polynomials, $L_n^{(\alpha)}(x)$, for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
glaguerre.polynomials(n, alpha, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order alpha numeric value for the polynomial parameter

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function glaguerre.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

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Value

A list of n+1 polynomial objects

```
order 0 generalized Laguerre polynomial
order 1 generalized Laguerre polynomial

n+1 order n generalized Laguerre polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

glaguerre.recurrences, orthogonal.polynomials, orthonormal.polynomials

```
###
### gemerate a list of normalized generalized Laguerre polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
normalized.p.list <- glaguerre.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized generalized Laguerre polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
unnormalized.p.list <- glaguerre.polynomials( 10, 1, normalized=FALSE )</pre>
```

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glaguerre.recurrences Recurrence relations for generalized Laguerre polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k generalized Laguerre polynomial, $L_n^{(\alpha)}(x)$ and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
glaguerre.recurrences(n, alpha, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

alpha numeric value for the polynomial parameter

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
glaguerre.inner.products
```

glaguerre.weight 37

Examples

```
###
### generate the recurrences data frame for
### the normalized generalized Laguerre polynomials
### of orders 0 to 10. the polynomial parameter value is 1.0.
###
normalized.r <- glaguerre.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized generalized Laguerre polynomials
### of orders 0 to 10. the polynomial parameter value is 1.0.
###
unnormalized.r <- glaguerre.recurrences( 10, 1, normalized=FALSE )
print( unnormalized.r )</pre>
```

glaguerre.weight

Weight function for the generalized Laguerre polynomial

Description

This function returns the value of the weight function for the order k generalized Laguerre polynomial, $L_n^{(\alpha)}(x)$.

Usage

```
glaguerre.weight(x,alpha)
```

Arguments

x the function argument which can be a vector alpha polynomial parameter

Details

The function takes on non-zero values in the interval $(0, \infty)$. The formula used to compute the weight function is as follows.

```
w(x) = e^{-x} x^{\alpha}
```

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the generalized Laguerre weight function for argument values
### between -3 and 3
### polynomial parameter value is 1.0
###

x <- seq( -3, 3, .01 )
y <- glaguerre.weight( x, 1 )</pre>
```

hermite.h.inner.products

Inner products of Hermite polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Hermite polynomial, $H_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
hermite.h.inner.products(n)
```

Arguments

n

integer value for highest polynomial order

Details

The formula used to compute the innner product is as follows.

$$h_n = \langle H_n | H_n \rangle = \sqrt{\pi} \ 2^n \ n!.$$

hermite.h.polynomials 39

Value

```
A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial

2 inner product of order 1 orthogonal polynomial

...

n+1 inner product of order n orthogonal polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### Hermite polynomials of orders 0 to 10
###
h <- hermite.h.inner.products( 10 )
print( h )</pre>
```

hermite.h.polynomials Create list of Hermite polynomials

Description

This function returns a list with n+1 elements containing the order k Hermite polynomials, $H_k(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
hermite.h.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function hermite.h.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct list of orthonormal polynomial objects.

Value

```
A list of n+1 polynomial objects

order 0 Hermite polynomial

order 1 Hermite polynomial

order n Hermite polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

hermite.h.recurrences, orthogonal.polynomials, orthonormal.polynomials

```
###
### gemerate a list of normalized Hermite polynomials of orders 0 to 10
###
normalized.p.list <- hermite.h.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Hermite polynomials of orders 0 to 10
###
unnormalized.p.list <- hermite.h.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

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hermite.h.recurrences Recurrence relations for Hermite polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Hermite polynomial, $H_k(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
hermite.h.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
hermite.h.inner.products,
```

```
###
### generate the recurrences data frame for
### the normalized Hermite H polynomials
### of orders 0 to 10.
###
normalized.r <- hermite.h.recurrences( 10, normalized=TRUE )
print( normalized.r )</pre>
```

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```
###
### generate the recurrences data frame for
### the unnormalized Hermite H polynomials
### of orders 0 to 10.
###
unnormalized.r <- hermite.h.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

hermite.h.weight

Weight function for the Hermite polynomial

Description

This function returns the value of the weight function for the order k Hermite polynomial, $H_k(x)$.

Usage

```
hermite.h.weight(x)
```

Arguments

Х

the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-\infty, \infty)$. The formula used to compute the weight function.

$$w\left(x\right) = \exp\left(-x^2\right)$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Hermite weight function for argument values
### between -3 and 3
x <- seq( -3, 3, .01 )
y <- hermite.h.weight( x )
plot( x, y )</pre>
```

hermite.he.inner.products

Inner products of Hermite polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Hermite polynomial, $He_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
hermite.he.inner.products(n)
```

Arguments

n

integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle He_n | He_n \rangle = \sqrt{2\pi} \ n!.$$

Value

A vector with n+1 elements

- 1 inner product of order 0 orthogonal polynomial
- 2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### scaled Hermite polynomials of orders 0 to 10
###
h <- hermite.he.inner.products( 10 )
print( h )</pre>
```

hermite.he.polynomials

Create list of Hermite polynomials

Description

This function returns a list with n+1 elements containing the order k Hermite polynomials, $He_k(x)$, for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
hermite.he.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for thehighest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function hermite.he.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

hermite.he.recurrences 45

Value

```
A list of n+1 polynomial objects

order 0 Hermite polynomial

order 1 Hermite polynomial

order n Hermite polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

hermite.he.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Hermite polynomials of orders 0 to 10
###
normalized.p.list <- hermite.he.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Hermite polynomials of orders 0 to 10
###
unnormalized.p.list <- hermite.he.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

hermite.he.recurrences

Recurrence relations for Hermite polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Hermite polynomial, $He_k(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

46 hermite.he.recurrences

Usage

```
hermite.he.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
hermite.he.inner.products
```

```
###
### generate the recurrences data frame for
### the normalized Hermite H polynomials
### of orders 0 to 10.
###
normalized.r <- hermite.he.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized Hermite H polynomials
### of orders 0 to 10.
###
unnormalized.r <- hermite.he.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

hermite.he.weight 47

hermite.he.weight

Weight function for the Hermite polynomial

Description

This function returns the value of the weight function for the order k Hermite polynomial, $He_k(x)$.

Usage

```
hermite.he.weight(x)
```

Arguments

Χ

the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-\infty, \infty)$. The formula used to compute the weight function is as follows.

$$w\left(x\right) = \exp\left(-\frac{x^2}{2}\right)$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

```
###
### compute the scaled Hermite weight function for argument values
### between -3 and 3
###

x <- seq( -3, 3, .01 )
y <- hermite.he.weight( x )</pre>
```

```
jacobi.g.inner.products
```

Inner products of Jacobi polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Jacobi polynomial, $G_k(p,q,x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
jacobi.g.inner.products(n,p,q)
```

Arguments

n	integer value for the highest polynomial order
p	numeric value for the first polynomial parameter
q	numeric value for the first polynomial parameter

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle G_n | G_n \rangle = \frac{n! \; \Gamma(n+q) \; \Gamma(n+p) \; \Gamma(n+p-q+1)}{(2\; n+p) \; [\Gamma(2\; n+p)]^2}.$$

Value

A vector with n+1 elements

inner product of order 0 orthogonal polynomial
 inner product of order 1 orthogonal polynomial
 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

49 jacobi.g.polynomials

Examples

```
###
### generate the inner products vector for the
### G Jacobi polynomials of orders 0 to 10
### parameter p is 3 and parameter q is 2
h <- jacobi.g.inner.products( 10, 3, 2 )</pre>
print( h )
```

Create list of Jacobi polynomials jacobi.g.polynomials

Description

This function returns a list with n+1 elements containing the order k Jacobi polynomials, $G_k(p,q,x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
jacobi.g.polynomials(n, p, q, normalized=FALSE)
```

Arguments

integer value for the highest polynomial order n numeic value for the first polynomial parameter p numeric value for the second polynomial parameter normalized a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function jacobi.g.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n+1 polynomial objects

```
order 0 Jacobi polynomial
1
2
                  order 1 Jacobi polynomial
                  order n Jacobi polynomial
n+1
```

50 jacobi.g.recurrences

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
jacobi.g.recurrences, orthogonal.polynomials, orthonormal.polynomials
```

Examples

```
###
### gemerate a list of normalized Jacobi G polynomials of orders 0 to 10
### first parameter value p is 3 and second parameter value q is 2
###
normalized.p.list <- jacobi.g.polynomials( 10, 3, 2, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of normalized Jacobi G polynomials of orders 0 to 10
### first parameter value p is 3 and second parameter value q is 2
###
unnormalized.p.list <- jacobi.g.polynomials( 10, 3, 2, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

jacobi.g.recurrences Recurrence relations for Jacobi polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Jacobi polynomial, $G_k(p,q,x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.g.recurrences(n, p, q, normalized=FALSE)
```

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Arguments

n	integer value for the highest polynomial order
p	numeric value for the first polynomial parameter
q	numeric value for the second polynomial parameter
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
jacobi.g.inner.products, pochhammer
```

```
###
### generate the recurrences data frame for
### the normalized Jacobi G polynomials
### of orders 0 to 10.
### parameter p is 3 and parameter q is 2
###
normalized.r <- jacobi.g.recurrences( 10, 3, 2, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Jacobi G polynomials
### of orders 0 to 10.
### parameter p is 3 and parameter q is 2
###
unnormalized.r <- jacobi.g.recurrences( 10, 3, 2, normalized=FALSE )
print( unnormalized.r )</pre>
```

52 jacobi.g.weight

jacobi.g.weight

Weight function for the Jacobi polynomial

Description

This function returns the value of the weight function for the order k Jacobi polynomial, $G_k(p,q,x)$.

Usage

```
jacobi.g.weight(x,p,q)
```

Arguments

x the function argument which can be a vector

p the first polynomial parameter

q the second polynomial parameter

Details

The function takes on non-zero values in the interval (0,1). The formula used to compute the weight function is as follows.

$$w(x) = (1-x)^{p-q} x^{q-1}$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

jacobi.matrices 53

Examples

```
###
### compute the Jacobi G weight function for argument values
### between 0 and 1
### parameter p is 3 and q is 2
###
x <- seq( 0, 1, .01 )
y <- jacobi.g.weight( x, 3, 2 )</pre>
```

jacobi.matrices

Create list of Jacobi matrices from monic recurrence parameters

Description

Return a list of $n\$ real symmetric, tri-diagonal matrices which are the principal minors of the $n \times n$ Jacobi matrix derived from the monic recurrence parameters, a and b, for orthogonal polynomials.

Usage

```
jacobi.matrices(r)
```

Arguments

r

a data frame containing the parameters a and b

Value

A list of symmetric, tri-diagnonal matrices

```
1 a 1 \times 1 matrix
2 a 2 \times 2 matrix
... n an n \times n matrix
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

```
r <- chebyshev.t.recurrences( 5 )
m.r <- monic.polynomial.recurrences( r )
j.m <- jacobi.matrices( m.r )</pre>
```

jacobi.p.inner.products

Inner products of Jacobi polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Jacobi polynomial, $P_k^{(\alpha,\beta)}(x)$, with itself (i.e. the norm squared) for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.p.inner.products(n,alpha,beta)
```

Arguments

n integer value for the highest polynomial order
alpha numeric value for the first polynomial parameter
beta numeric value for the first polynomial parameter

Details

The formula used to compute the innser products is as follows.

$$h_n = \left\langle P_n^{(\alpha,\beta)} | P_n^{(\alpha,\beta)} \right\rangle = \frac{2^{\alpha+\beta+1}}{2\,n+\alpha+\beta+1} \frac{\Gamma(n+\alpha+1)\,\Gamma(n+\beta+1)}{n!\,\Gamma(n+\alpha+\beta+1)}.$$

Value

A vector with n+1 elements

inner product of order 0 orthogonal polynomial
 inner product of order 1 orthogonal polynomial
 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

jacobi.p.polynomials 55

Examples

```
###
### generate the inner product vector for the P Jacobi polynomials of orders 0 to 10
###
h <- jacobi.p.inner.products( 10, 2, 2 )
print( h )</pre>
```

jacobi.p.polynomials

Create list of Jacobi polynomials

Description

```
This function returns a list with n+1 elements containing the order k Jacobi polynomials, P_k^{(\alpha,\beta)}(x), for orders k=0,\ 1,\ \ldots,\ n.
```

Usage

```
jacobi.p.polynomials(n, alpha, beta, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order
alpha numeric value for the first polynomial parameter
beta numeric value for the second polynomial parameter

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function jacobi.p.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n + 1 polynomial objects

order 0 Jacobi polynomial
 order 1 Jacobi polynomial

•••

n+1 order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

56 jacobi.p.recurrences

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
jacobi.p.recurrences, orthogonal.polynomials, orthonormal.polynomials
```

Examples

```
###
### gemerate a list of normalized Jacobi P polynomials of orders 0 to 10
### first parameter value a is 2 and second parameter value b is 2
###
normalized.p.list <- jacobi.p.polynomials( 10, 2, 2, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Jacobi P polynomials of orders 0 to 10
### first parameter value a is 2 and second parameter value b is 2
###
unnormalized.p.list <- jacobi.p.polynomials( 10, 2, 2, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

jacobi.p.recurrences Recurrence relations for Jacobi polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Jacobi polynomial, $P_k^{(\alpha,\beta)}(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.p.recurrences(n, alpha, beta, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order
alpha numeric value for the first polynomial parameter
beta numeric value for the second polynomial parameter

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

jacobi.p.weight 57

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
jacobi.p.inner.products, pochhammer
```

Examples

```
###
### generate the recurrences data frame for
### the normalized Jacobi P polynomials
### of orders 0 to 10.
### parameter a is 2 and parameter b is 2
###
normalized.r <- jacobi.p.recurrences( 10, 2, 2, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized Jacobi P polynomials
### of orders 0 to 10.
### parameter a is 2 and parameter b is 2
###
unnormalized.r <- jacobi.p.recurrences( 10, 2, 2, normalized=FALSE )
print( unnormalized.r )</pre>
```

jacobi.p.weight

Weight function for the Jacobi polynomial

Description

This function returns the value of the weight function for the order k Jacobi polynomial, $P_k^{(\alpha,\beta)}(x)$.

Usage

```
jacobi.p.weight(x,alpha,beta)
```

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Arguments

Х	the function argument which can be a vector
alpha	the first polynomial parameter
beta	the second polynomial parameter

Details

The function takes on non-zero values in the interval (-1,1). The formula used to compute the weight function is as follows.

$$w(x) = (1-x)^{\alpha} (1+x)^{\beta}$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

```
###
### compute the Jacobi P weight function for argument values
### between -1 and 1
###

x <- seq( -1, 1, .01 )
y <- jacobi.p.weight( x, 2, 2 )</pre>
```

laguerre.inner.products 59

laguerre.inner.products

Inner products of Laguerre polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Laguerre polynomial, $L_n(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
laguerre.inner.products(n)
```

Arguments

n integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle L_n | L_n \rangle = 1.$$

Value

A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial

2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### Laguerre polynomial inner products of orders 0 to 10
###
h <- laguerre.inner.products( 10 )
print( h )</pre>
```

laguerre.polynomials Create list of Laguerre polynomials

Description

This function returns a list with n+1 elements containing the order k Laguerre polynomials, $L_n(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
laguerre.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function laguerre.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n + 1 polynomial objects

```
order 0 Laguerre polynomial
order 1 Laguerre polynomial
...
```

n+1 order *n* Laguerre polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

laguerre.recurrences 61

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

laguerre.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Laguerre polynomials of orders 0 to 10
###
normalized.p.list <- laguerre.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Laguerre polynomials of orders 0 to 10
###
unnormalized.p.list <- laguerre.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

laguerre.recurrences Recurrence relations for Laguerre polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Laguerre polynomial, $L_n(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
laguerre.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order
normalized boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

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Author(s)

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References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
glaguerre.recurrences
```

Examples

```
###
### generate the recurrences data frame for
### the normalized Laguerre polynomials
### of orders 0 to 10.
###
normalized.r <- laguerre.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Laguerre polynomials
### of orders 0 to 10.
###
unnormalized.r <- laguerre.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

laguerre.weight

Weight function for the Laguerre polynomial

Description

This function returns the value of the weight function for the order k Laguerre polynomial, $L_n(x)$.

Usage

```
laguerre.weight(x)
```

Arguments

Х

the function argument which can be a vector

legendre.inner.products

Details

The function takes on non-zero values in the interval $(0,\infty)$. The formula used to compute the weight function is as follows. $w(x) = e^{-x}$

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Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the Laguerre weight function for argument values ### between 0 and 3 x <- seq(-0, 3, .01) y <- laguerre.weight( x ) plot( x, y )
```

legendre.inner.products

Inner products of Legendre polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k Legendre polynomial, $P_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
legendre.inner.products(n)
```

Arguments

n integer value for the highest polynomial order

Details

The formula used compute the inner products is as follows.

$$h_n = \langle P_n | P_n \rangle = \frac{2}{2n+1}.$$

Value

A vector with n+1 elements

- 1 inner product of order 0 orthogonal polynomial
- 2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
spherical.inner.products
```

```
###
### compute the inner product for the
### Legendre polynomials of orders 0 to 1
###
h <- legendre.inner.products( 10 )
print( h )</pre>
```

legendre.polynomials 65

legendre.polynomials Create list of Legendre polynomials

Description

This function returns a list with n+1 elements containing the order k Legendre polynomials, $P_k(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

legendre.polynomials(n, normalized=FALSE)

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function legendre.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n+1 polynomial objects

order 0 Legendre polynomial
order 1 Legendre polynomial

•••

n+1 order n Legendre polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

66 legendre.recurrences

See Also

legendre.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Laguerre polynomials of orders 0 to 10
###
normalized.p.list <- legendre.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Laguerre polynomials of orders 0 to 10
###
unnormalized.p.list <- legendre.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

legendre.recurrences Recurrence relations for Legendre polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k Legendre polynomial, $P_k(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
legendre.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

legendre.weight 67

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
legendre.inner.products
```

Examples

```
###
### generate the recurrences data frame for
### the normalized Legendre polynomials
### of orders 0 to 10.
###
normalized.r <- legendre.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Legendre polynomials
### of orders 0 to 10.
###
unnormalized.r <- legendre.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

legendre.weight

Weight function for the Legendre polynomial

Description

This function returns the value of the weight function for the order k Legendre polynomial, $P_k(x)$.

Usage

```
legendre.weight(x)
```

Arguments

Х

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (-1,1). The formula used to compute the weight function is as follows.

```
w\left(x\right) = 1
```

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Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Legendre weight function for argument values
### between -1 and 1
###

x <- seq( -1, 1, .01 )
y <- legendre.weight( x )
plot( x, y )</pre>
```

1pochhammer

Calculate the logarithm of Pochhammer's symbol

Description

1pochhammer returns the value of the natural logarithm of Pochhammer's symbol calculated as

$$\ln \left[(z)_n \right] = \ln \Gamma \left(z + n \right) - \ln \Gamma \left(z \right)$$

where $\Gamma(z)$ is the Gamma function

Usage

```
lpochhammer(z, n)
```

Arguments

z argument of the symbol

n integer number of terms in the symbol

Value

The value of the logarithm of the symbol

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

See Also

pochhammer

Examples

lpochhammer(pi, 5)

monic.polynomial.recurrences

Create data frame of monic recurrences

Description

This function returns a data frame with parameters required to construct monic orthogonal polynomials based on the standard recurrence relation for the non-monic polynomials. The recurrence relation for monic orthogonal polynomials is as follows.

$$q_{k+1}(x) = (x - a_k) \ q_k(x) - b_k \ q_{k-1}(x)$$

We require that $q_{-1}\left(x\right)=0$ and $q_{0}\left(x\right)=1$. The recurrence for non-monic orthogonal polynomials is given by

$$c_k p_{k+1}(x) = (d_k + e_k x) p_k(x) - f_k p_{k-1}(x)$$

We require that $p_{-1}(x) = 0$ and $p_0(x) = 1$. The monic polynomial recurrence parameters, **a** and **b**, are related to the non-monic polynomial parameter vectors **c**, **d**, **e** and **f** in the following manner.

$$a_k = -\frac{d_k}{e_k}$$

$$b_k = \frac{c_{k-1} f_k}{e_{k-1} e_k}$$

with $b_0 = 0$.

Usage

monic.polynomial.recurrences(recurrences)

Arguments

recurrences the data frame of recurrence parameter vectors **c**, **d**, **e** and **f**

70 monic.polynomials

Value

A data frame with n + 1 rows and two named columns, **a** and **b**.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
orthogonal.polynomials,
```

Examples

```
###
### construct a list of the recurrences for the T Chebyshev polynomials of
### orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=TRUE )
###
### construct the monic polynomial recurrences from the above list
###
m.r <- monic.polynomial.recurrences( r )</pre>
```

monic.polynomials

Create list of monic orthogonal polynomials

Description

This function returns a list with n+1 elements containing the order k monic polynomials for orders $k=0,\ 1,\ \ldots,\ n.$

Usage

```
monic.polynomials(monic.recurrences)
```

monic.polynomials 71

Arguments

```
monic.recurrences
```

a data frame containing the parameters a and b

Value

```
A list with n+1 polynomial objects

order 0 monic orthogonal polynomial

order 1 monic orthogonal polynomial
```

•••

n+1 order n monic orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

See Also

```
monic.polynomial.recurrences
```

```
###
### generate the recurrences for the T Chebyshev polynomials
### of orders 0 to 10
###

r <- chebyshev.t.recurrences( 10, normalized=TRUE )
###
### get the corresponding monic polynomial recurrences
###
m.r <- monic.polynomial.recurrences( r )
###
### obtain the list of monic polynomials
###
p.list <- monic.polynomials( m.r )</pre>
```

orthogonal.polynomials

Create orthogonal polynomials

Description

Create list of orthogonal polynomials from the following recurrence relations for $k = 0, 1, \ldots, n$.

$$c_k p_{k+1}(x) = (d_k + e_k x) p_k(x) - f_k p_{k-1}(x)$$

We require that $p_{-1}(x) = 0$ and $p_0(x) = 1$. The coefficients are the column vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} .

Usage

orthogonal.polynomials(recurrences)

Arguments

recurrences a data frame containing the parameters of the orthogonal polynomial recurrence

relations

Details

The argument is a data frame with n+1 rows and four named columns. The column names are c, d, e and f. These columns correspond to the column vectors described above.

Value

A list of n + 1 polynomial objects

- 1 Order 0 orthogonal polynomial
- 2 Order 1 orthogonal polynomial

•••

n+1 Order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the recurrence relations for T Chebyshev polynomials of orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=FALSE )
print( r )
###
### generate the list of orthogonal polynomials
###
p.list <- orthogonal.polynomials( r )
print( p.list )</pre>
```

orthonormal.polynomials

Create orthonormal polynomials

Description

Create list of orthonormal polynomials from the following recurrence relations for $k = 0, 1, \ldots, n$.

$$c_k p_{k+1}(x) = (d_k + e_k x) p_k(x) - f_k p_{k-1}(x)$$

We require that $p_{-1}(x) = 0$ and $p_0(x) = 1$. The coefficients are the column vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} .

Usage

```
orthonormal.polynomials(recurrences, p.0)
```

Arguments

recurrences a data frame containing the parameters of the orthonormal polynomial recurrence relations

p.0 a polynomial object for the order 0 orthonormal polynomial

Details

The argument is a data frame with n+1 rows and four named columns. The column names are c, d, e and f. These columns correspond to the column vectors described above.

Value

A list of n + 1 polynomial objects

1 Order 0 orthonormal polynomial
2 Order 1 orthonormal polynomial
...

n+1 Order n orthonormal polynomial

74 pochhammer

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate a data frame with the recurrences parameters for normalized T Chebyshev
### polynomials of orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=TRUE )
print( r )
norm <- sqrt( pi )
###
### create the order 0 orthonormal polynomial
###
library("polynom")
p.0 <- polynomial( c( 1 / norm ) )
###
### generate a list of orthonormal polynomial objects
###
p.list <- orthonormal.polynomials( r, p.0 )
print( p.list )</pre>
```

pochhammer

Calculate the value of Pochhammer's symbol

Description

pochhammer returns the value of Pochhammer's symbol calculated as

$$(z)_n = z \ (z+1) \ \dots \ (z+n-1) = \frac{\Gamma(z+n)}{\Gamma(z)}$$

where $\Gamma(z)$ is the Gamma function

Usage

```
pochhammer(z, n)
```

polynomial.coefficients 75

Arguments

numeric value for the argument of the symbol Z

integer value for the number of terms in the symbol n

Value

The value of Pochhammer's symbol

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### compute the Pochhamer's symbol fo z equal to 1 and
### n equal to 5
pochhammer( 1, 5 )
```

polynomial.coefficients

Create list of polynomial coefficient vectors

Description

This function returns a list with n+1 elements containing the vector of coefficients of the order kpolynomials for orders $k = 0, 1, \ldots, n$. Each element in the list is a vector.

Usage

```
polynomial.coefficients(polynomials)
```

Arguments

n+1

```
list of polynomial objects
polynomials
```

Value

A list of n+1 polynomial objects where each element is a vector of coefficients.

```
Coefficient(s) of the order 0 polynomial
1
                   Coefficient(s) of the order 1 polynomial
2
                   Coefficient(s) of the order n polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of normalized T Chebyshev polynomials
### of orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### obtain the list of coefficients for these polynomials
###
p.coef <- polynomial.coefficients( p.list )</pre>
```

polynomial.derivatives

Create list of polynomial derivatives

Description

This function returns a list with n+1 elements containing polynomial objects which are the derivatives of the order k polynomials for orders $k=0, 1, \ldots, n$.

Usage

```
polynomial.derivatives(polynomials)
```

Arguments

```
polynomials list of polynomial objects
```

Details

The polynomial objects in the argument polynomials are as follows

- 1order 0 polynomial
- 2order 1 polynomial ...
- \bullet n+1order n polynomial

Value

```
List of n + 1 polynomial objects
```

polynomial.functions 77

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of normalized T Chebyshev polynomials of
### orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### generate the corresponding list of polynomial derivatives
###
p.deriv <- polynomial.derivatives( p.list )</pre>
```

polynomial.functions

Coerce polynomials to functions

Description

This function returns a list with n+1 elements containing the functions of the order $k \ge 0, 1, \ldots, n$ and for the given argument x.

Usage

```
polynomial.functions(polynomials, ...)
```

Arguments

```
polynomials a list of polynomial objects
... further arguments to be passed to or from methods
```

Details

The function uses the method as.function.polynomial to coerce each polynomial object to a function object.

Value

A list of n+1 polynomial objects where each element is the function for the polynomial.

```
Function for the order 0 polynomial
Function for the order 1 polynomial

run

Function for the order n polynomial
```

78 polynomial.integrals

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of T Chebyshev polynomials of
### orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=FALSE )
###
### create the list of functions for each polynomial
###
f.list <- polynomial.functions( p.list )</pre>
```

```
polynomial.integrals Create list of polynomial integrals
```

Description

This function returns a list with n+1 elements containing polynomial objects which are the indefinite integrals of the order k polynomials for orders $k=0, 1, \ldots, n$.

Usage

```
polynomial.integrals(polynomials)
```

Arguments

```
polynomials list of polynomial objects
```

Details

The polynomial objects in the argument polynomials are as follows

- 1 order 0 polynomial
- 2order 1 polynomial ...
- n+1order n polynomial

Value

```
List of n + 1 polynomial objects
```

```
integral of polynomials[[1]]
integral of polynomials[[2]]
...
n+1 integral of polynomials[[n+1]]
```

polynomial.orders 79

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of normalized T Chebyshev polynomials
### of orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### generate the corresponding list of polynomial integrals
###
p.int <- polynomial.integrals( p.list )</pre>
```

polynomial.orders

Create vector of polynomial orders

Description

This function returns a vector with n elements containing the orders of the polynomials

Usage

```
polynomial.orders(polynomials)
```

Arguments

polynomials list of \$n\$ polynomial objects

Value

A vector of n values

```
1 Order of polynomials[[1]]
```

2 Order of polynomials[[2]]

•••

n Order of polynomials[[n]]

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

80 polynomial.powers

Examples

```
###
### generate a list of normalized T Chebyshev polynomials
### of orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### get the vector of polynomial orders
###
p.order <- polynomial.orders( p.list )</pre>
```

polynomial.powers

Create a list of polynomial linear combinations

Description

This function returns a list with n+1 elements containing the vector of linear combinations of the order k polynomials for orders $k=0,\ 1,\ \ldots,\ n$. Each element in the list is a vector.

Usage

```
polynomial.powers(polynomials)
```

Arguments

polynomials A list of polynomials

Details

The j-th component in the list is a vector of linear combinations of the order k polynomials for orders $k = 0, 1, \ldots, j-1$ equal to the monomial x raised to the power j-1.

Value

A list of n + 1 elements where each element is a vector of linear combinations.

- 1 Linear combination(s) of polynomials up to order 0
- 2 Linear combination(s) of polynomials up to order 1

•••

n+1 Linear combination(s) of polynomials up to order n

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

polynomial.roots 81

Examples

```
###
### generate Legendre polynomials of orders 0 to 10
###
polynomials <- legendre.polynomials( 10 )
###
### generate list of linear combinations of these polynomials
###
alphas <- polynomial.powers( polynomials )
print( alphas )</pre>
```

polynomial.roots

Create a list of polynomial roots

Description

This function returns a list with n elements containing the roots of the order k monic orthogonal polynomials for orders $k = 0, 1, \ldots, n$ using a data frame with the monic polynomial recurrence parameter vectors \mathbf{a} and \mathbf{b}

Usage

```
polynomial.roots(m.r)
```

Arguments

m.r

monic recurrence data frame with parameters a and b

Details

The parameter m.r is a data frame with n\$+1 rows and two names columns. The columns which are names a and b correspond to the above referenced vectors. Function jacobi.matrices is used to create a list of symmetric, tridiagonal Jacobi matrices from these named columns. The eigenvalues of the $k \times k$ Jacobi matrix are the roots or zeros of the order k monic orthogonal polynomial.

Value

A list with n elements each of which is a vector of polynomial roots

```
1 roots of the order 1 monic polynomial 2 roots of the order 2 monic polynomial ... n roots of the order n monic polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

82 polynomial.values

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
monic.polynomial.recurrences, jacobi.matrices
```

Examples

```
###
### generate the recurrences data frame for
### the normalized Chebyshev polynomials of
### orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=TRUE )
###
### construct the corresponding monic polynomial
### recurrences
###
m.r <- monic.polynomial.recurrences( r )
###
### obtain the polynomial roots from the monic polynomial
### recurrences
p.roots <- polynomial.roots( m.r )</pre>
```

polynomial.values

Create vector of polynomial values

Description

This function returns a list with n+1 elements containing the values of the order k polynomials for orders $k=0,\ 1,\ \ldots,\ n$ and for the given argument x.

Usage

```
polynomial.values( polynomials, x )
```

Arguments

```
polynomials list of polynomial objects

x the argument which can be any numeric object
```

scaleX 83

Value

A list of n+1 polynomial objects where each element is the value of the polynomial.

```
1 Value(s) for the order 0 polynomial
2 Value(s) for the order 1 polynomial
...
n+1 Value(s) for the order n polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of T Chebyshev polynomials of
### orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=FALSE )
x <- seq( -2, 2, .01 )
###
### compute the value of the polynomials for the given range of values in x
###
y <- polynomial.values( p.list, x )
print( y )</pre>
```

scaleX

Scale values from [a,b] to [u.v]

Description

This function returns a vector of values that have been mapped from the interval [a,b] to the interval [u.v].

Usage

```
scaleX(x, a = min(x, na.rm = TRUE), b = max(x, na.rm = TRUE), u, v)
```

Arguments

X	A numerical vector of values to be mapped into a target interval
а	A numerical lower bound for the domain interval with $min(x)$ as the default value
b	A numerical upper bound for the domain interval with $\max(x)$ as the default value
u	A numerical lower bound for the target interval
V	A numerical upper bound for the target interval

Details

Target lower and/or upper bounds can be $-\infty$ and ∞ , respectively. This accommodates finite target intervals, semi-infinite target intervals and infinite target intervals.

Value

A vector of transformed values with four attributes. The first attribute is called "a" and it is the domain interval lower bound. The second attribute is called "b" and it is the domain interval upper bound. The third attribute is called "u" and it is the target interval lower bound. The fourth attribute is called "v" and it is the target interval upper bound.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>, Gregor Gorjanc <gregor.gorjanc@bfro-uni-lj.si>

References

Seber, G. A. F. (1997) Linear Regression Analysis, New York.

Examples

```
x <- rnorm( 1000, 0, 10 )
y0 <- scaleX( x, u=0 , v=1 )
y1 <- scaleX( x, u=-1, v=1 )
y2 <- scaleX( x, u=-Inf, v=0 )
y3 <- scaleX( x, u=0, v=Inf )
y4 <- scaleX( x, u=-Inf, v=Inf )</pre>
```

```
schebyshev.t.inner.products
```

Inner products of shifted Chebyshev polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k shifted Chebyshev polynomial of the first kind, $T_k^*(x)$, with itself (i.e. the norm squared) for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.t.inner.products(n)
```

Arguments

n

integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle T_n^* | T_n^* \rangle = \left\{ \begin{array}{ll} \frac{\pi}{2} & n \neq 0 \\ \pi & n = 0 \end{array} \right.$$

Value

A vector with n+1 elements

- 1 inner product of order 0 orthogonal polynomial
- 2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., NY.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### shifted T Chebyshev polynomials of orders 0 to 10
###
h <- schebyshev.t.inner.products( 10 )
print( h )</pre>
```

 ${\it schebyshev.t.polynomials}$

Create list of shifted Chebyshev polynomials

Description

This function returns a list with n+1 elements containing the order k shifted Chebyshev polynomials of the first kind, $T_k^*(x)$, for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.t.polynomials(n, normalized)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function schebyshev.t.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n + 1 polynomial objects

- 1 order 0 shifted Chebyshev polynomial
- 2 order 1 shifted Chebyshev polynomial

...

n+1 order n shifted Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

schebyshev.u.recurrences, orthogonal.polynomials, orthonormal.polynomials

schebyshev.t.recurrences 87

Examples

```
###
### gemerate a list of normalized shifted T Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- schebyshev.t.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized shifted T Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- schebyshev.t.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

schebyshev.t.recurrences

Recurrence relations for shifted Chebyshev polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k shifted Chebyshev polynomial of the first kind, $T_k^*(x)$, and for orders $k=0, 1, \ldots, n$.

Usage

```
schebyshev.t.recurrences(n, normalized)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

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See Also

```
schebyshev.t.inner.products
```

Examples

```
###
### generate the recurrence relations for
### the normalized shifted T Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- schebyshev.t.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized shifted T Chebyshev polynomials
### of orders 0 to 10
###
unnormalized.r <- schebyshev.t.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

schebyshev.t.weight

Weight function for the shifted Chebyshev polynomial

Description

This function returns the value of the weight function for the order k shifted Chebyshev polynomial of the first kind, $T_k^*(x)$.

Usage

```
schebyshev.t.weight(x)
```

Arguments

Х

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (0,1). The formula used to compute the weight function is as follows.

$$w\left(x\right) = \frac{1}{\sqrt{x - x^2}}$$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the shifted T Chebyshev weight function for argument values
### between 0 and 1
x \leftarrow seq(0, 1, .01)
y \leftarrow schebyshev.t.weight(x)
plot(x, y)
```

schebyshev.u.inner.products

Inner products of shifted Chebyshev polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k shifted Chebyshev polynomial of the second kind, $U_k^*(x)$, with itself (i.e. the norm squared) for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.u.inner.products(n)
```

Arguments

n

integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle U_n^* | U_n^* \rangle = \frac{\pi}{8}.$$

Value

A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial

2 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
h <- schebyshev.u.inner.products( 10 )</pre>
```

schebyshev.u.polynomials

Create list of shifted Chebyshev polynomials

Description

This function returns a list with n+1 elements containing the order k shifted Chebyshev polynomials of the second kind, $U_k^*(x)$, for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.u.polynomials(n, normalized)
```

Arguments

n integer value for highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function schebyshev.u.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

```
A list of n+1 polynomial objects

order 0 shifted Chebyshev polynomial

order 1 shifted Chebyshev polynomial

order n shifted Chebyshev polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
schebyshev.u.recurrences, orthogonal.polynomials, orthonormal.polynomials
```

Examples

```
###
### gemerate a list of normalized shifted U Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- schebyshev.u.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized shifted U Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- schebyshev.u.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

schebyshev.u.recurrences

Recurrence relations for shifted Chebyshev polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k shifted Chebyshev polynomial of the second kind, $U_k^*(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.u.recurrences(n, normalized)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
schebyshev.u.inner.products
```

schebyshev.u.weight 93

Examples

```
###
### generate the recurrence relations for
### the normalized shifted U Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- schebyshev.u.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized shifted T Chebyshev polynomials
### of orders 0 to 10
unnormalized.r <- schebyshev.u.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

schebyshev.u.weight

Weight function for the shifted Chebyshev polynomial

Description

This function returns the value of the weight function for the order k shifted Chebyshev polynomial of the second kind, $U_k^*(x)$.

Usage

```
schebyshev.u.weight(x)
```

Arguments

Χ

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (0,1). The formula used to compute the weight function is as follows.

$$w\left(x\right) = \sqrt{x - x^2}$$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the shifted U Chebyshev weight function for argument values
### between 0 and 1
###

x <- seq( 0, 1, .01 )
y <- schebyshev.u.weight( x )
plot( x, y )</pre>
```

slegendre.inner.products

Inner products of shifted Legendre polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k shifted Legendre polynomial, $P_k^*(x)$, with itself (i.e. the norm squared) for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
slegendre.inner.products(n)
```

Arguments

n

integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle P_n^* | P_n^* \rangle = \frac{1}{2n+1}.$$

slegendre.polynomials 95

Value

A vector with \$n\$+1 elements

inner product of order 0 orthogonal polynomial
 inner product of order 1 orthogonal polynomial
 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the inner products vector for the
### shifted Legendre polynomials of orders 0 to 10
###
h <- slegendre.inner.products( 10 )
print( h )</pre>
```

slegendre.polynomials Create list of shifted Legendre polynomials

Description

This function returns a list with n+1 elements containing the order k shifted Legendre polynomials, $P_k^*(x)$, for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
slegendre.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function slegendre.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

```
A list of n+1 polynomial objects

order 0 shifted Legendre polynomial

order 1 shifted Legendre polynomial

order n shifted Legendre polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
slegendre.recurrences, orthogonal.polynomials, orthonormal.polynomials
```

Examples

```
###
### gemerate a list of normalized shifted Legendre polynomials of orders 0 to 10
###
normalized.p.list <- slegendre.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized shifted Legendre polynomials of orders 0 to 10
###
unnormalized.p.list <- slegendre.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

slegendre.recurrences 97

slegendre.recurrences Recurrence relations for shifted Legendre polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k shifted Legendre polynomial, $P_k^*(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

slegendre.recurrences(n, normalized=FALSE)

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

slegendre.inner.products,

98 slegendre.weight

Examples

```
###
### generate the recurrence relations for normalized shifted Legendre polynomials
### of orders 0 to 10
###
normalized.r <- slegendre.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for normalized shifted Legendre polynomials
### of orders 0 to 10
###
unnormalized.r <- slegendre.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

slegendre.weight

Weight function for the shifted Legendre polynomial

Description

This function returns the value of the weight function for the order k shifted Legendre polynomial, $P_k^*(x)$.

Usage

```
slegendre.weight(x)
```

Arguments

Х

the function argument which can be a vector

Details

The function takes on non-zero values in the interval (0,1). The formula used to compute the weight function is as follows.

```
w\left( x\right) =1
```

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

spherical.inner.products

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

99

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the shifted Legendre weight function for argument values ### between 0 and 1 ###  x <- seq( \ 0, \ 1, \ .01 \ )   y <- slegendre.weight( \ x \ )
```

spherical.inner.products

Inner products of spherical polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k spherical polynomial, $P_k(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
spherical.inner.products(n)
```

Arguments

n

integer value for the highest polynomial order

Details

The formula used to compute the inner products of the spherical orthogonal polynomials is the same as that used for the Legendre orthogonal polynomials.

Value

```
A vector with n+1 elements

1 inner product of order 0 orthogonal polynomial
2 inner product of order 1 orthogonal polynomial
...

n+1 inner product of order n orthogonal polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
legendre.inner.products
```

Examples

```
###
### generate the inner products vector for the spherical polynomals
### of orders 0 to 10.
###
h <- spherical.inner.products( 10 )
print( h )</pre>
```

spherical.polynomials Create list of spherical polynomials

Description

This function returns a list with n+1 elements containing the order k spherical polynomials, $P_k(x)$, for orders $k=0, 1, \ldots, n$.

Usage

```
spherical.polynomials(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function spherical.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

spherical.recurrences 101

Value

```
A list of n+1 polynomial objects

order 0 spherical polynomial

order 1 spherical polynomial

order n Chebyshev polynomial
```

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

spherical.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### generate a list of spherical orthonormal polynomials of orders 0 to 10
###
normalized.p.list <- spherical.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### generate a list of spherical orthogonal polynomials of orders 0 to 10
###
unnormalized.p.list <- spherical.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

spherical.recurrences Recurrence relations for spherical polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k spherical polynomial, $P_k(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

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Usage

```
spherical.recurrences(n, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
spherical.inner.products
```

Examples

```
###
### generate the recurrence relations for
### the normalized spherical polynomials
### of orders 0 to 10
###
normalized.r <- spherical.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized spherical polynomials
### of orders 0 to 10
###
unnormalized.r <- spherical.recurrences( 10, normalized=FALSE )
print( unnormalized.r )</pre>
```

spherical.weight 103

spherical.weight

Weight function for the spherical polynomial

Description

This function returns the value of the weight function for the order k spherical polynomial, $P_k(x)$.

Usage

```
spherical.weight(x)
```

Arguments

Χ

the function argument which can be a vector or matrix

Details

The function takes on non-zero values in the interval (-1,1). The formula used to compute the weight function is as follows.

```
w\left(x\right) = 1
```

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H. S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C.*

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
### ### compute the spherical weight function for a sequence of values between -2 and 2 ### x \leftarrow seq(-2, 2, .01) y \leftarrow spherical.weight(x) plot(x, y)
```

ultraspherical.inner.products

Inner products of ultraspherical polynomials

Description

This function returns a vector with n+1 elements containing the inner product of an order k ultraspherical polynomial, $C_k^{(\alpha)}(x)$, with itself (i.e. the norm squared) for orders $k=0, 1, \ldots, n$.

Usage

```
ultraspherical.inner.products(n,alpha)
```

Arguments

n integer value for the highest polynomial order alpha numeric value for the polynomial parameter

Details

This function uses the same formula as the function gegenbauer.inner.products.

Value

A vector with n+1 elements

inner product of order 0 orthogonal polynomial
 inner product of order 1 orthogonal polynomial

•••

n+1 inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., NY.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY. Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
gegenbauer.inner.products
```

Examples

```
###
### generate the inner products vector for the
### ultraspherical polynomials of orders 0 to 10.
### the polynomial parameter is 1.0
###
h <- ultraspherical.inner.products( 10, 1 )
print( h )</pre>
```

ultraspherical.polynomials

Create list of ultraspherical polynomials

Description

This function returns a list with n+1 elements containing the order k ultraspherical polynomials, $C_k^{(\alpha)}(x)$, for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
ultraspherical.polynomials(n, alpha, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order

alpha polynomial parameter

normalized a boolean value which, if TRUE, returns a list of normalized orthogonal poly-

nomials

Details

The function ultraspherical.recurrences produces a data frame with the recurrence relation parameters for the polynomials. If the normalized argument is FALSE, the function orthogonal.polynomials is used to construct the list of orthogonal polynomial objects. Otherwise, the function orthonormal.polynomials is used to construct the list of orthonormal polynomial objects.

Value

A list of n+1 polynomial objects

order 0 ultraspherical polynomial
order 1 ultraspherical polynomial

•••

n+1 order n ultraspherical polynomial

Author(s)

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References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
gegenbauer.recurrences, orthogonal.polynomials, orthonormal.polynomials
```

Examples

```
###
### gemerate a list of normalized ultra spherical polynomials
### of orders 0 to 10
###
normalized.p.list <- ultraspherical.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized ultra spherical polynomials
### of orders 0 to 10
###
unnormalized.p.list <- ultraspherical.polynomials( 10, 1, normalized=FALSE )
print( unnormalized.p.list )</pre>
```

ultraspherical.recurrences

Recurrence relations for ultraspherical polynomials

Description

This function returns a data frame with n+1 rows and four named columns containing the coefficient vectors \mathbf{c} , \mathbf{d} , \mathbf{e} and \mathbf{f} of the recurrence relations for the order k ultraspherical polynomial, $C_k^{(\alpha)}(x)$, and for orders $k=0,\ 1,\ \ldots,\ n$.

Usage

```
ultraspherical.recurrences(n, alpha, normalized=FALSE)
```

Arguments

n integer value for the highest polynomial order
alpha numeric value for the polynomial parameter

normalized boolean value which, if TRUE, returns recurrence relations for normalized poly-

nomials

Value

A data frame with the recurrence relation parameters.

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Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

```
ultraspherical.recurrences
```

Examples

```
###
### generate the recurrence relations for
### the normalized ultraspherical polynomials
### of orders 0 to 10
### polynomial parameter value is 1.0
###
normalized.r <- ultraspherical.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the normalized ultraspherical polynomials
### of orders 0 to 10
### polynomial parameter value is 1.0
###
unnormalized.r <- ultraspherical.recurrences( 10, 1, normalized=FALSE )
print( unnormalized.r )</pre>
```

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ultraspherical.weight Weight function for the ultraspherical polynomial

Description

This function returns the value of the weight function for the order k ultraspherical polynomial, $C_k^{(\alpha)}(x)$.

Usage

ultraspherical.weight(x,alpha)

Arguments

x the function argument which can be a vector

alpha polynomial parameter

Details

The function takes on non-zero values in the interval (-1,1). The formula used to compute the weight function is as follows.

$$w(x) = (1 - x^2)^{\alpha - 0.5}$$

Value

The value of the weight function

Author(s)

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References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. Methods of Mathematical Physics, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

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Examples

```
###
### compute the ultraspherical weight function for arguments between -2 and 2
### polynomial parameter is 1.0
###

x <- seq( -2, 2, .01 )
y <- ultraspherical.weight( x, 1 )
plot( x, y )</pre>
```

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