Regular Expression Evaluation

Converting REs to NFAs for Pattern Matching

Motivating Question

How would you implement your own `grep`?

- If an RE specifies a regular language, and an NFA recognizes it...
- We need a way to **convert an RE to an NFA** to implement RE matching
- But how do we do this systematically?

RE → NFA: The Approach

Key Idea: Evaluate an RE by treating it as an expression

- **Operands** are NFAs corresponding to sub-expressions
- **Operators** act on the NFAs to produce new NFAs

Analogous to an arithmetic expression like $1 + (4 - 1) \times 2$
- operands are numbers (1, 4, 2, ...)
- operators are arithmetic ops (+, -, $\times$, ...)

Postfix Notation

Challenge: Expression evaluation is more convenient when the expression is in **postfix notation** (rather than the usual infix notation)

Postfix: Operator comes after its operands

Example:

$$\underbrace{1 + (4 - 1) \times 2}_{\text{infix}} \iff \underbrace{1\ 4\ 1\ -\ 2\ *\ +}_{\text{postfix}}$$

Why Postfix?

1. **Removes complexity** of operator precedence
    - No need for parentheses to override precedence
2. **Easier to write code** to evaluate
    - Simple stack-based algorithm
    - No need to handle precedence rules in code

Infix vs Postfix: Examples

Arithmetic (infix)	Arithmetic (postfix)
11	11
11 + 3	11 3 +
11 - 3	11 3 -
(11 + 3) $\times$ 2	11 3 + 2 $\times$
11 + 3 $\times$ 2	11 3 2 $\times$ +
11 + 3 - 2 $\div$ 4	11 3 + 2 4 $\div$ -

RE (infix)	RE (postfix)
0	0
1	1
0|1	01|
(0|1).(0|1)	01|01|.
(0|1)*	01|*
0|(1.(0|1)*)	101|*.0|

Note: `.` here is an explicit concatenation operator, not an extended regular expression symbol

Postfix Evaluation Algorithm

1. **While** the expression has tokens left:
   i. Read next token (operand or operator)
   ii. **If operand:** push to stack
   iii. **If operator:**
      a. Pop the required number of operands from stack
      b. Operate on the popped operands
      c. Push the result to stack
2. **If** the stack has a single operand:
   i. Pop and return it as the result
3. **Otherwise:** The expression was malformed

Evaluate: `1 4 1 - 2 * +` (equivalent to $1 + (4 - 1) \times 2$)

Token	Stack (rightmost is top)
1 4 1 - 2 * +	1
1 **4** 1 - 2 * +	1, 4
1 4 **1** - 2 * +	1, 4, 1
1 4 1 **-** 2 * +	1, 3
1 4 1 - **2** * +	1, 3, 2
1 4 1 - 2 ***** +	1, 6
1 4 1 - 2 * **+**	7

RE Postfix Evaluation

Same algorithm, but:

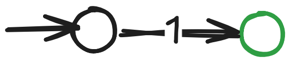
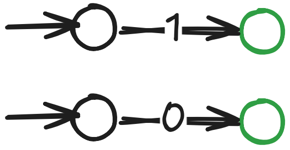
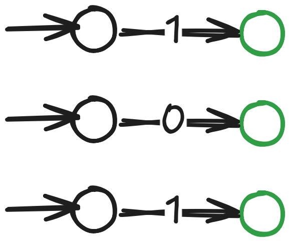
- **Operands** are NFAs for individual symbols
- **Operators** are NFA operations (union `|`, concatenation `.`, Kleene star `*`)

Example: Evaluate `101|*.0|`

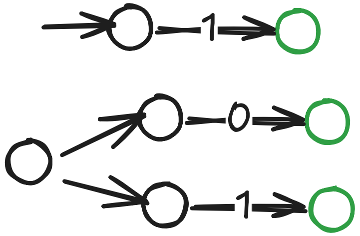
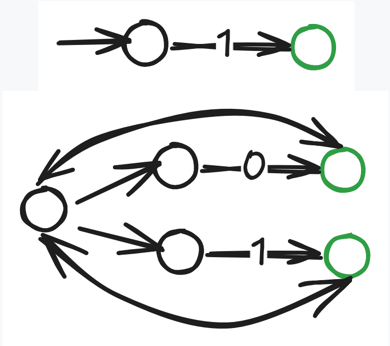
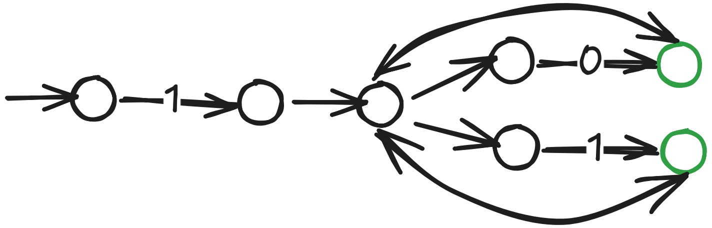
This represents the RE: `0|(1.(0|1)*)`

RE Example: Step-by-Step (1/3)

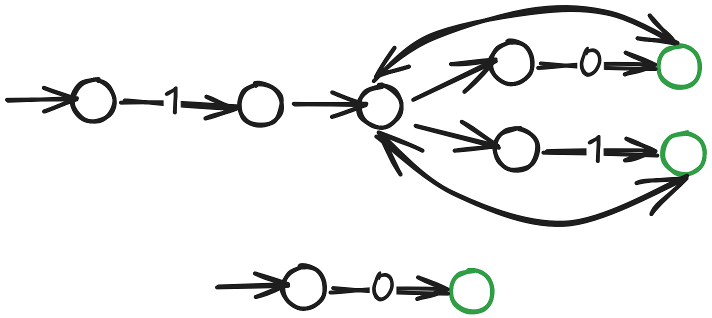
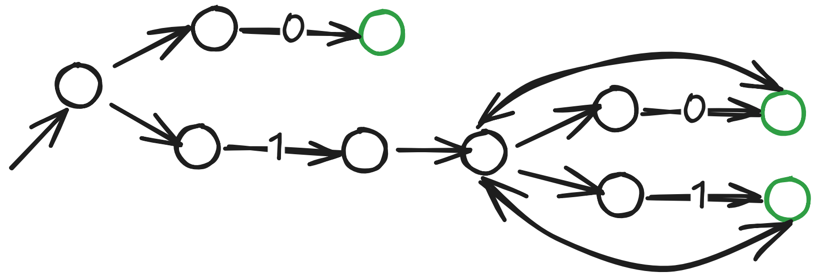
Evaluate: `101|*.0|`

Token	Stack
101|*.0|	
1**0**1|*.0|	
10**1**|*.0|	

RE Example: Step-by-Step (2/3)

Token	Stack
101 | *.0|	
101| *.0|	
101|* . 0|	

RE Example: Step-by-Step (3/3)

<table>
<tr><th>Token</th><th>Stack</th></tr>
<tr><td>101|*.0|</td><td>
</td></tr>
<tr><td>101|*.0|</td><td>
</td></tr>
</table>

Result: Final NFA that recognizes the language of the original RE!

🎯

Active Learning Activity

Practice: Convert to postfix and trace the evaluation

1. RE (infix): `(a|b)*c`
   - What is the postfix form?
   - Trace the first 3 steps of evaluation
2. RE (infix): `a(b|c)*`
   - What is the postfix form?
   - What does this language accept?

Implementation: Generic Postfix Evaluation

1. Operands of type `T`
    - e.g. `Double` for arithmetic expressions
    - e.g. `NFA` for regular expressions
2. Operand creator of type `Function<String, T>`
    - Converts a `String` symbol for an operand into an operand of type `T`
    - e.g. arithmetic: `s -> Double.parseDouble(s)`
    - e.g. RE: `s -> NFAOps.createForSymbol(s.charAt(0))`

Implementation: Generic Postfix Evaluation (contd.)

3. Operators registry of type `Map<String, Consumer<Stack<T>>>`
    - key: operator symbol
    - value: operator implementation
        - pops the appropriate number of operands from the stack, applies the operator on those operands, and pushes the result back on to the stack
    - e.g. arithmetic plus: `"+" ->  s -> s.push(s.pop() + s.pop())`
    - e.g. RE union: `"|" ->  s -> s.push(NFAOps.union(s.pop(), s.pop()))`

Implementation: Generic Postfix Evaluation (contd.)

4. `PostfixEvaluator<T>`
   - `T` : operand type (e.g. `Double` or `NFA`)
   - State:
     - `Function<String, T> operandCreator`
     - `Map<String, Consumer<Stack<T>>> operators`
   - Behavior:
     - `T evaluate(String[] expressionTokens) {...}`
       - implementation: the stack-based postfix evaluation algo

Implementation: Arithmetic Postfix Evaluation

```
Function<String, Double> operandCreator = s -> Double.parseDouble(s);
Map<String, Consumer<Stack<Double>>> operators = Map.of(
    "+", s -> s.push(s.pop() + s.pop()),
    "*", s -> s.push(s.pop() * s.pop()),
    ...
);
PostfixEvaluator<Double> arithmeticPostfixEvaluator = new PostfixEvaluator<>(operandCreator, operators);
double result = arithmeticPostfixEvaluator.evaluate("1 4 1 - 2 * +".split(" "));
```

Implementation: RE Postfix Evaluation

```
Function<String, NFA> operandCreator = s -> NFAOps.createForSymbol(s.charAt(0));
Map<String, Consumer<Stack<NFA>>> operators = Map.of(
    "|", s -> s.push(NFAOps.union(s.pop(), s.pop())),
    ...
);
PostfixEvaluator<NFA> regularPostfixEvaluator = new PostfixEvaluator<>(operandCreator, operators);
NFA nfa = regularPostfixEvaluator.evaluate("101|*.0|".split(""));
bool accepts = nfa.accepts("101");
```

Summary

1. **RE → NFA** conversion enables RE matching implementation
2. **Postfix notation** simplifies expression evaluation
3. **Stack-based algorithm** evaluates postfix expressions
4. **Generic design** allows reuse (arithmetic, NFAs, etc.)

Next: You can now implement your own `grep`! 🎉