Nondeterministic Finite Automata (NFA)

A useful generalization of a DFA

Learning Objectives

By the end of this lecture, you will be able to:

- **Define** what an NFA is and how it differs from a DFA
- **Identify** nondeterministic transitions in automata
- **Convert** between NFAs and DFAs
- **Implement** NFAs in code
- **Recognize** regular languages

What is an NFA?

Nondeterministic Finite Automata (NFA)

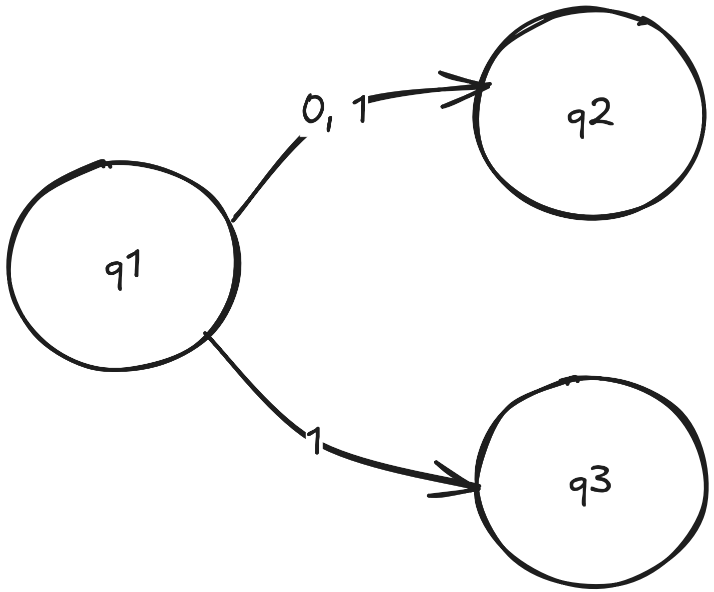
- A useful generalization of a DFA
- Allows more flexible state transitions
- Makes some automata simpler to design

Key Question: Does this added flexibility make NFAs more powerful than DFAs?

NFA Enhancement #1

Multiple Transitions on Same Symbol

A state can have transitions to **multiple states** on the same symbol



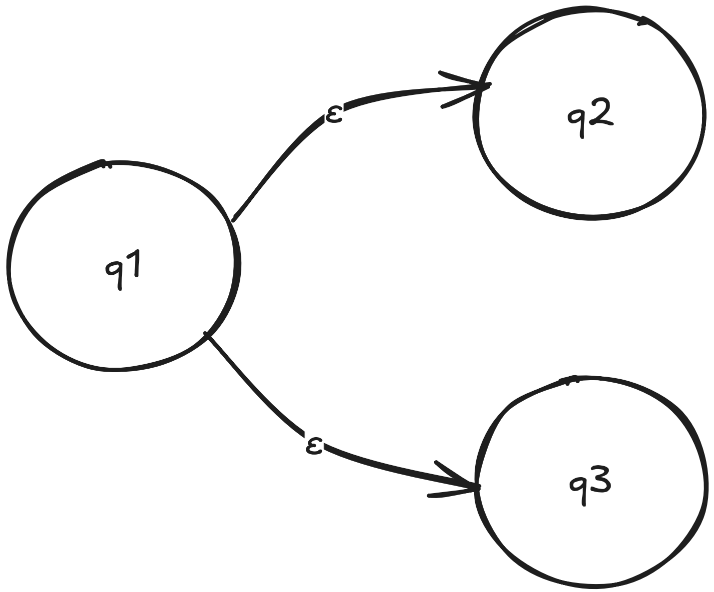
When reading '1' from state q1:

- Can go to q2 OR
- Can go to q3

NFA Enhancement #2

Null Transitions (ε-transitions)

A state can transition without reading any symbol

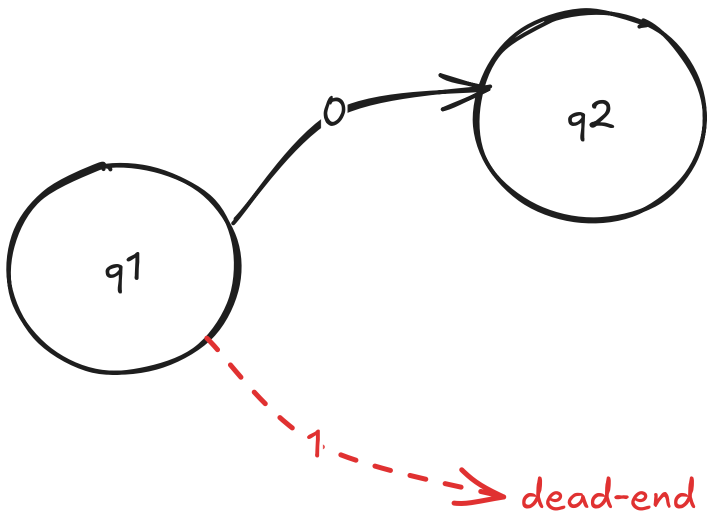


From q1, can spontaneously move to q2 or q3 without consuming input

NFA Enhancement #3

Missing Transitions

A state can have **no transitions** for some symbols (dead ends)



No transition defined for '1' from q1

- If '1' is read, this path dies

How to Run an NFA?

The Nondeterministic Approach:

1. **Follow all possible paths** simultaneously
2. If **any path** leads to an accept state → **ACCEPT**
3. If **all paths** fail → **REJECT**

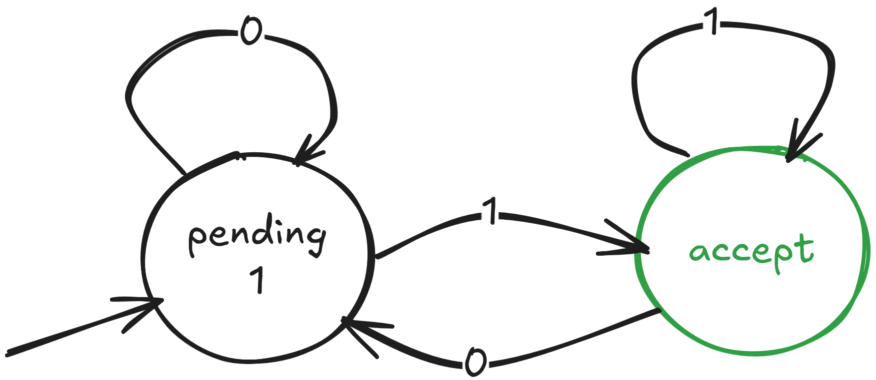
Think of it as:

- Exploring multiple parallel universes
- Success in any universe = overall success

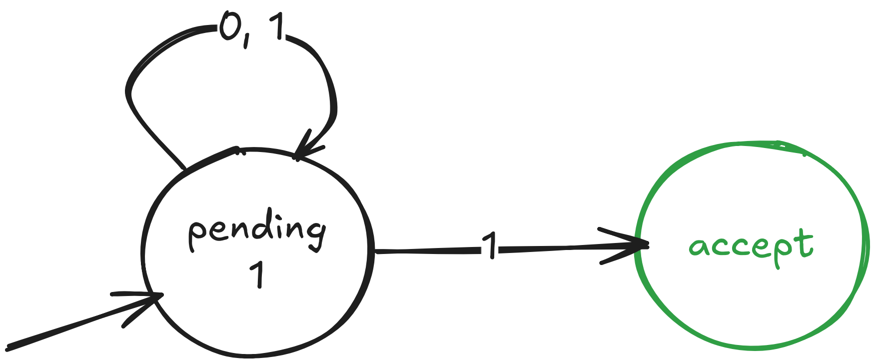
Example 1: Strings Ending in "1"

Language: Binary strings ending in "1"

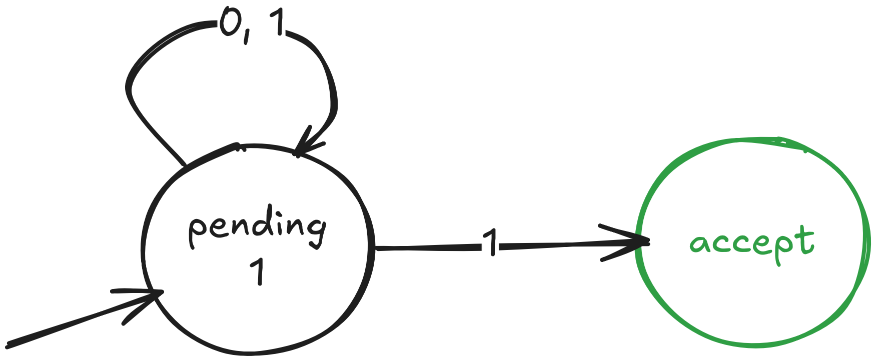
DFA Version



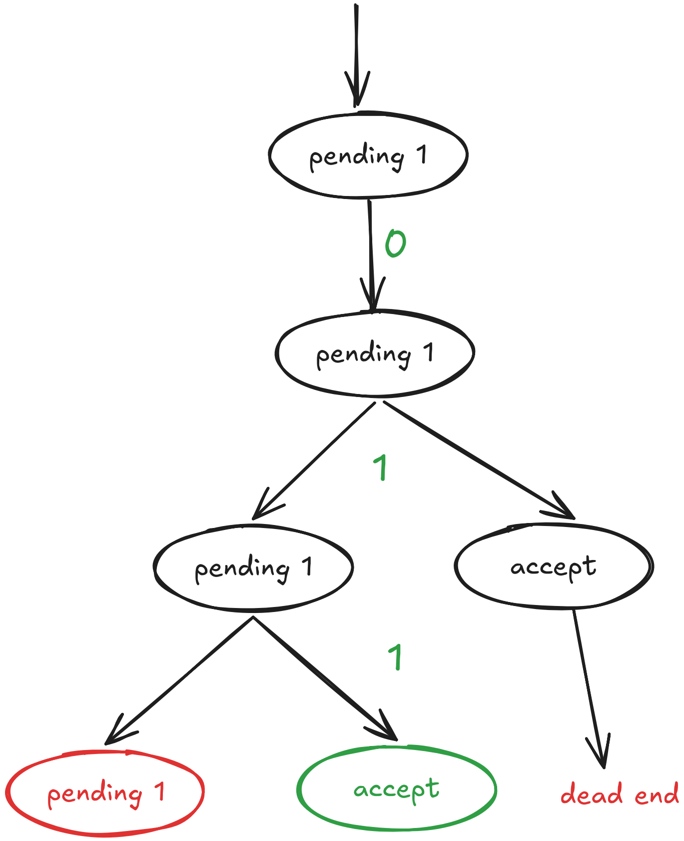
NFA Version (Simpler!)



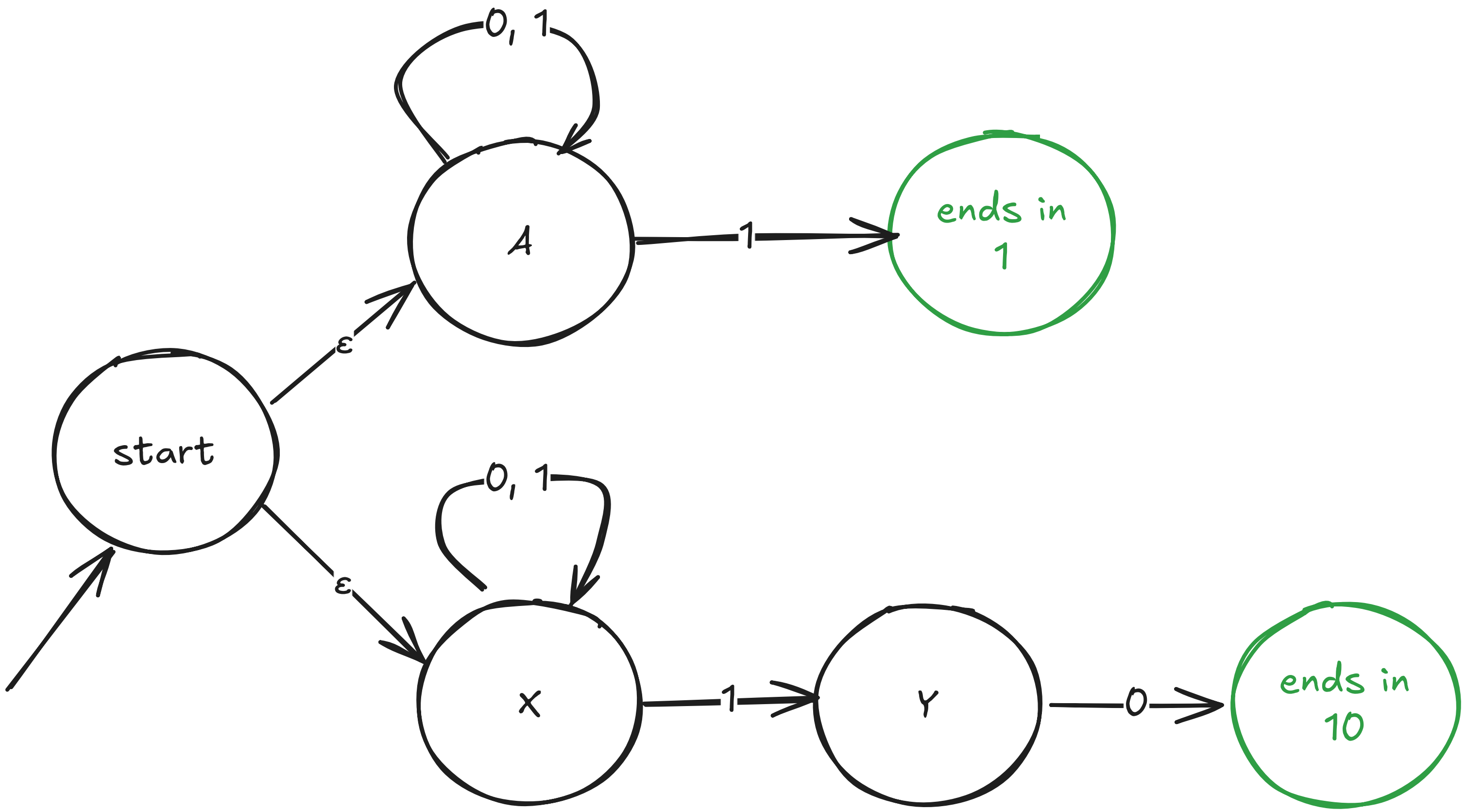
Example 1: Trace



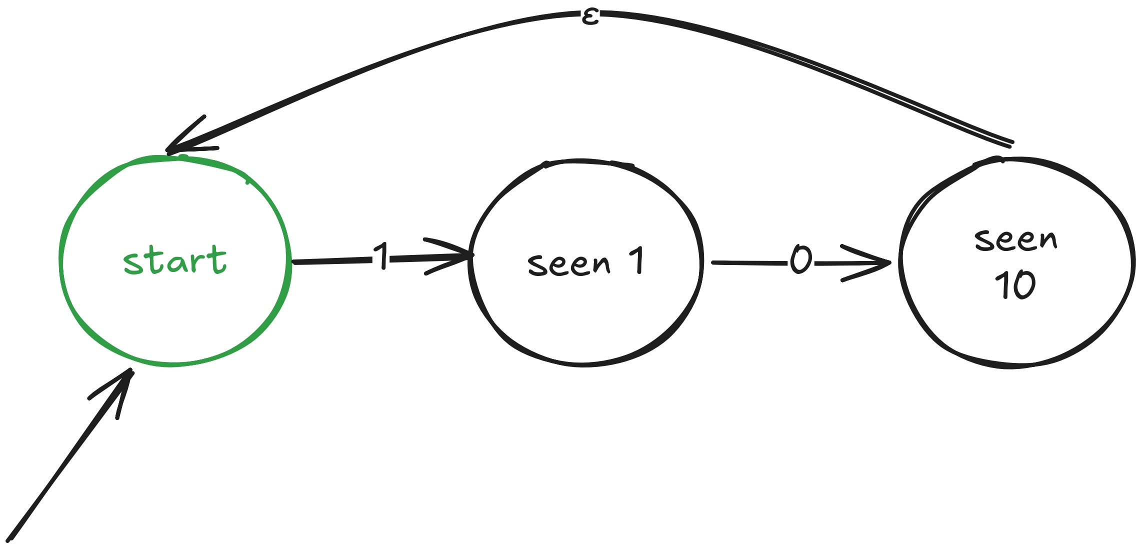
Trace of "011":



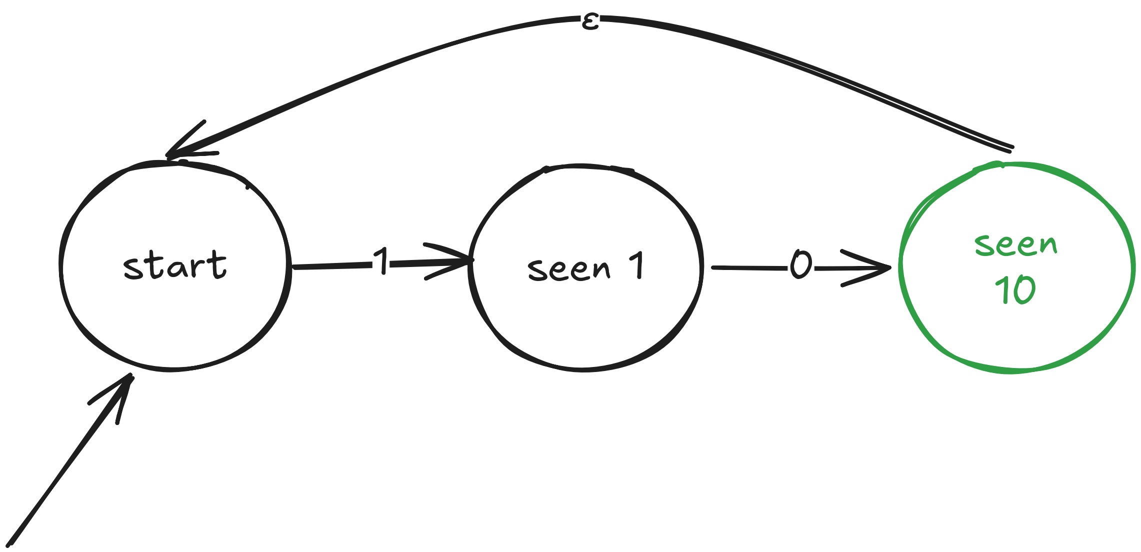
Example 2: Strings Ending in "1" OR "10"



Example 3a: Strings that repeat "10" zero or more times



Example 3b: Strings that repeat "10" one or more times



🎯

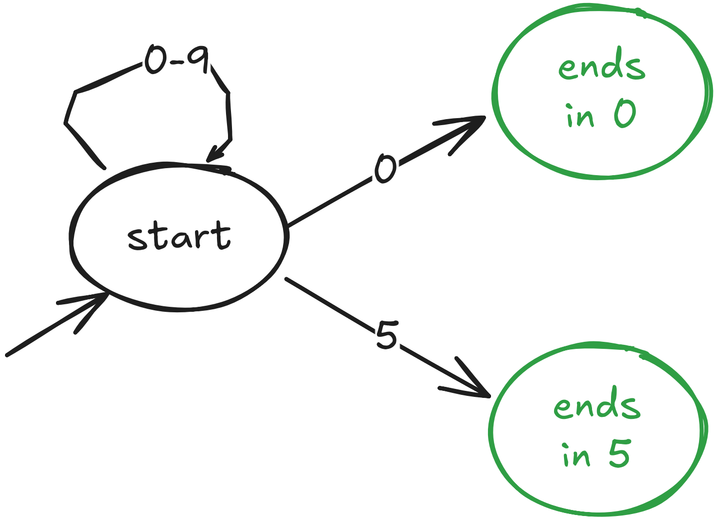
Active Learning: NFA Design

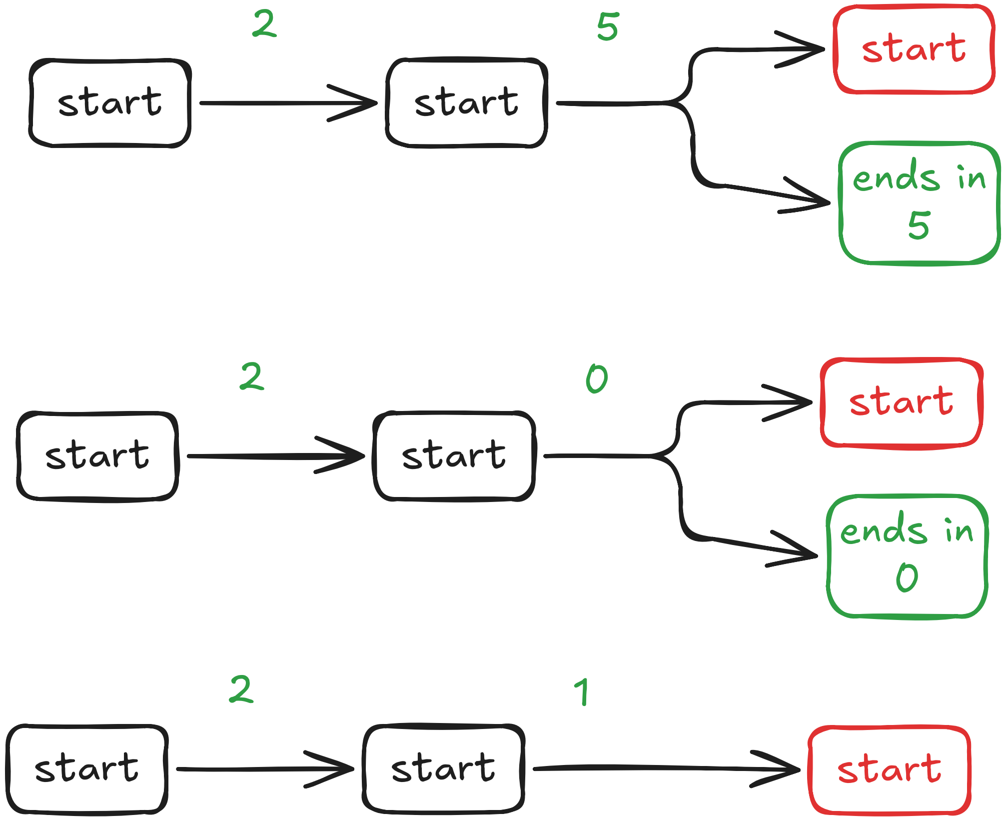
Alphabet: {0, 1, ..., 9}

Language: Whole numbers divisible by 5

1. Design an NFA that recognizes the above language
2. Trace sample inputs "25", "20", "21"

Active Learning: NFA Design Solution





The Big Question

Does nondeterminism make NFAs more powerful than DFAs?

🤔
Think about it...

Can NFAs recognize languages that DFAs cannot?

The Surprising Answer

NO!

> **An NFA is equivalent to a DFA**

Every NFA can be converted to an equivalent DFA!

But how?
🤯

NFA to DFA Conversion: The Idea

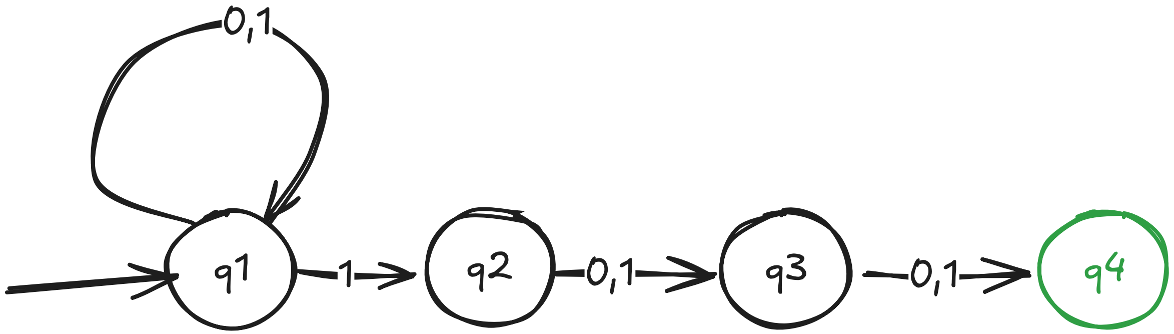
Subset Construction:

- If NFA has N states
  - There are 2^N possible subsets of states
- Each subset becomes a DFA state

Example: NFA with 3 states {q1, q2, q3}

- DFA states: {}, {q1}, {q2}, {q3}, {q1,q2}, {q1,q3}, {q2,q3}, {q1,q2,q3}

Conversion Example: Building the Table



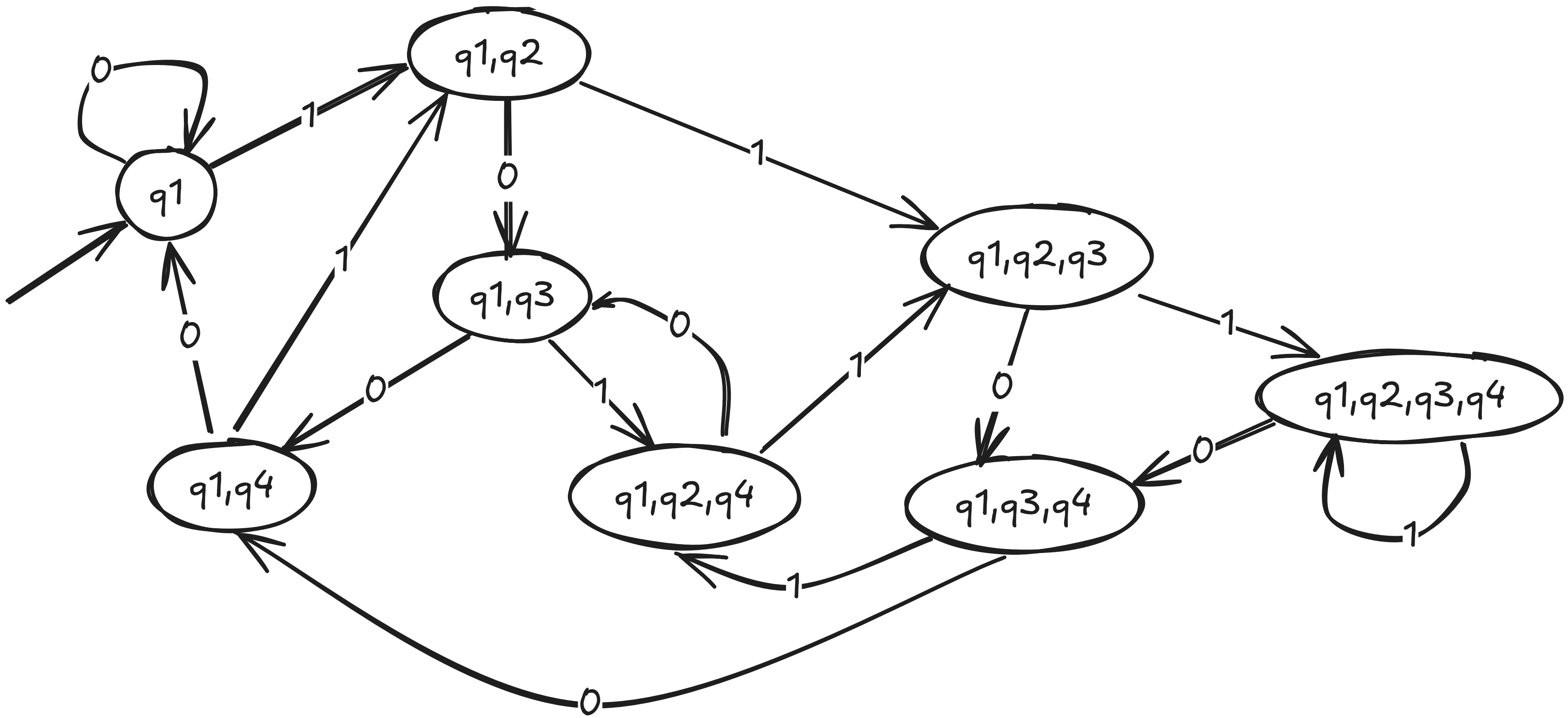
Let's convert this NFA to a DFA...

DFA State Transition Table

DFA State	0	1
{q1}	{q1}	**{q1,q2}**
{q1,q2}	**{q1,q3}**	**{q1,q2,q3}**
{q1,q3}	**{q1,q4}**	**{q1,q2,q4}**
{q1,q4}	{q1}	{q1,q2}

(Partial table shown - 8 more rows needed for complete DFA)

The Resulting DFA



Which looks simpler to you?

🤔

Implementing NFAs in Java

Key Differences from DFA

```java
public class NFA {
  public static class State {
    // Symbol can be NULL ('\0')!
    void addTransition(Character symbol, State to) {...}

    // Returns SET of states, not single state!
    Set<State> getTransition(Character symbol) {...}
  }

  // 1. Set current state to the start state
  // 2. Find all null-closure states i.e. reachable from the current state via null transitions
  // 3. If the input is exhausted, return whether any null-closure state is accept
  // 3. For each null-closure state, read the next symbol and the set of next states
  // 4. For each next state, set it as the current state and repeat from step 2 (recursion helps)
  public boolean accepts(String input) {...}
}
```

Regular Languages

Definition

> A formal language is called a **regular language** if some DFA **or NFA** recognizes it.

Key insight: Since DFA ≡ NFA

- Language regular if DFA recognizes it
- Language regular if NFA recognizes it
- Same expressive power!

Why Use NFAs Then?

If NFAs = DFAs in power, why bother?

Advantages of NFAs:

- **Simpler** to design
- **Fewer states** needed
- **More intuitive** for certain patterns
- **Easier** to combine (union, concatenation)

Trade-off:

- Harder to implement/simulate

Key Takeaways

1. NFAs add two types of nondeterminism:
    i. Multiple transitions per symbol
    ii. Epsilon transitions
2. NFAs are equivalent to DFAs in power
3. NFAs often simpler to design but harder to simulate
4. Regular languages = Languages recognized by DFA or NFA