Nondeterministic Finite Automata (NFA)

A useful generalization of a DFA

Learning Objectives

By the end of this lecture, you will be able to:

- **Define** what an NFA is and how it differs from a DFA
- **Identify** nondeterministic transitions in automata
- **Convert** between NFAs and DFAs
- **Implement** NFAs in code
- **Recognize** regular languages

What is an NFA?

Nondeterministic Finite Automata (NFA)

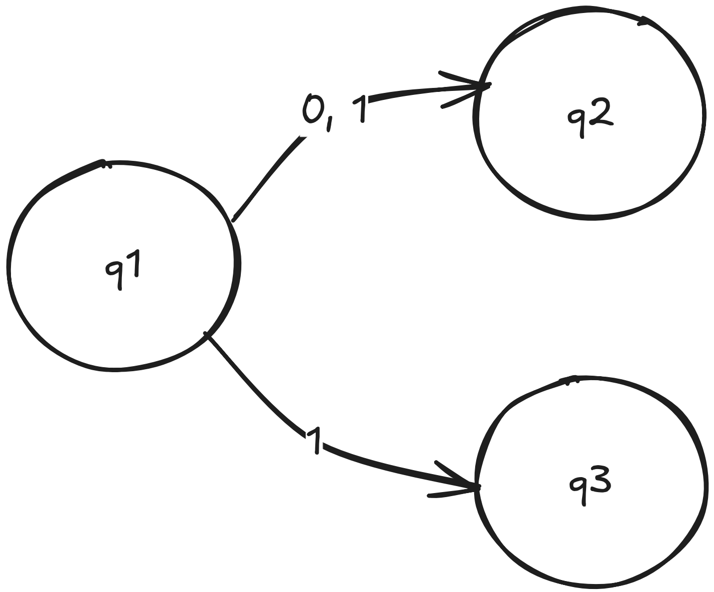
- A useful generalization of a DFA
- Allows more flexible state transitions
- Makes some automata simpler to design

Key Question: Does this added flexibility make NFAs more powerful than DFAs?

NFA Enhancement #1

Multiple Transitions on Same Symbol

A state can have transitions to **multiple states** on the same symbol



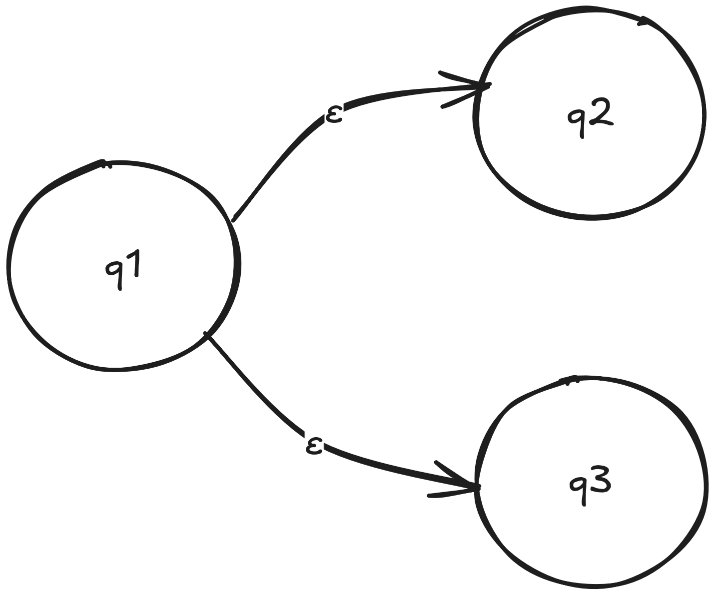
When reading '1' from state q1:

- Can go to q2 OR
- Can go to q3

NFA Enhancement #2

Null Transitions (ε-transitions)

A state can transition without reading any symbol

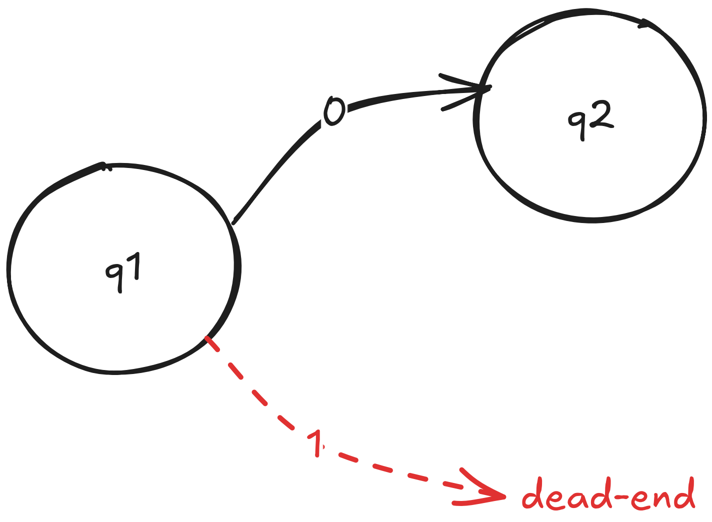


From q1, can spontaneously move to q2 or q3 without consuming input

NFA Enhancement #3

Missing Transitions

A state can have **no transitions** for some symbols (dead ends)



No transition defined for '1' from q1

- If '1' is read, this path dies

How to Run an NFA?

The Nondeterministic Approach:

1. **Follow all possible paths** simultaneously
2. If **any path** leads to an accept state → **ACCEPT**
3. If **all paths** fail → **REJECT**

Think of it as:

- Exploring multiple parallel universes
- Success in any universe = overall success

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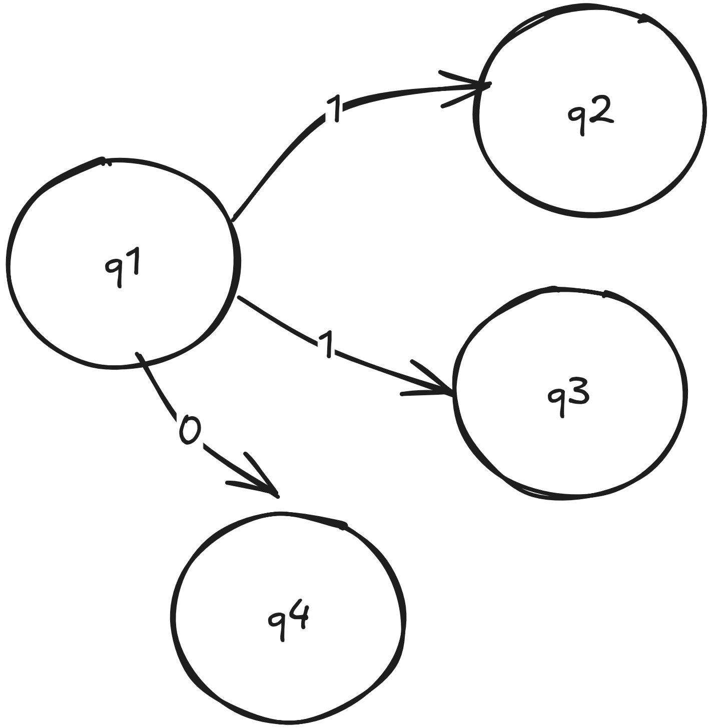
Quick Check #1

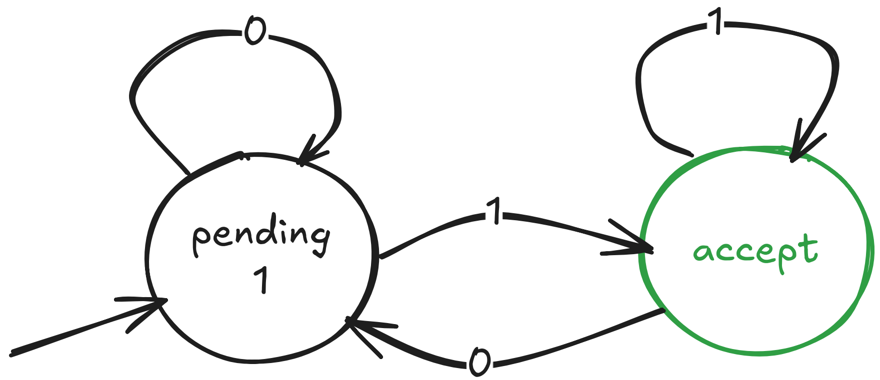
Given this NFA fragment, what happens when we read '1' from q1?



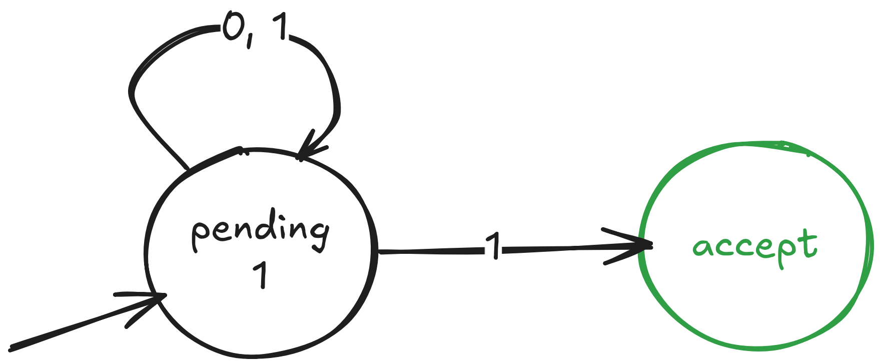
Example 1: Strings Ending in "1"

Language: Binary strings ending in "1"

DFA Version

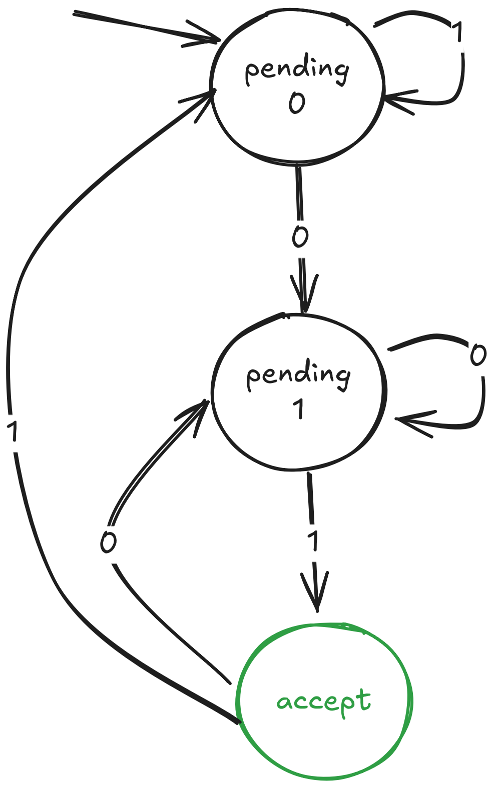


NFA Version (Simpler!)

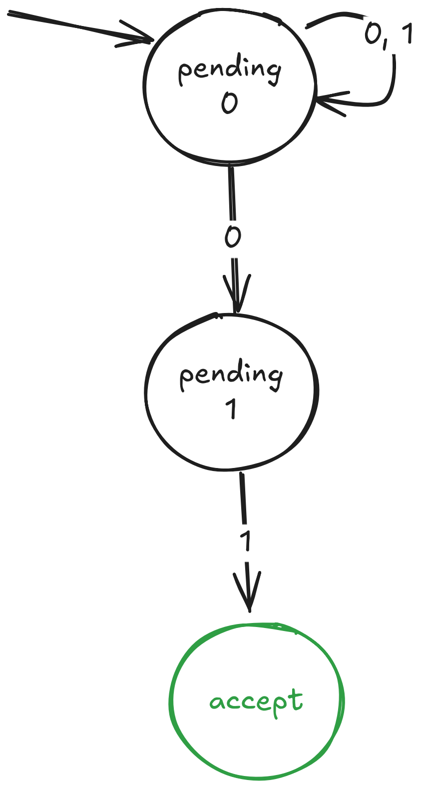


Example 2: Strings Ending in "01"

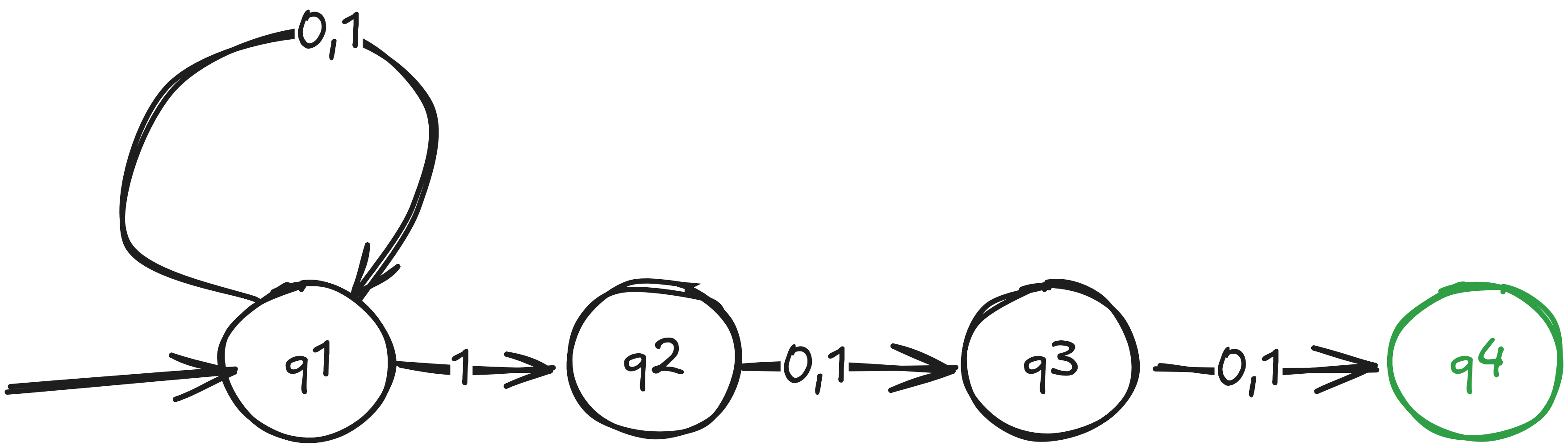
DFA Version



NFA Version

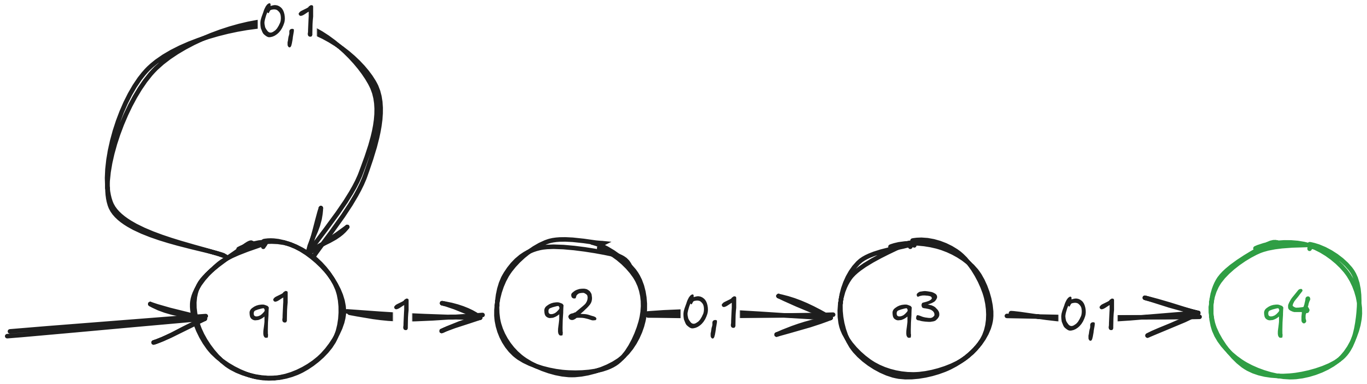


Example 3: Third Position from End



Question: Where's the nondeterminism?

Example 3: Analysis



Nondeterminism: q1 has multiple transitions on '1'

- q1 → q1 (stay)
- q1 → q2 (guess this is 3rd from end)

Language: Binary strings with '1' in 3rd position from end

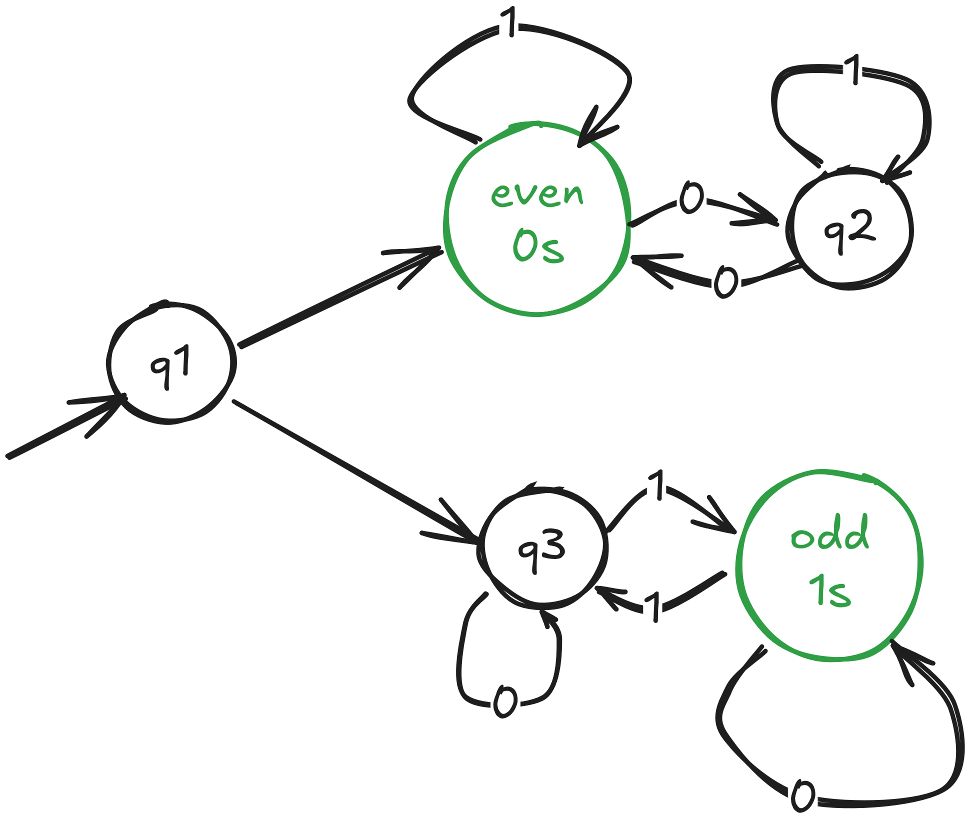
Active Learning: Trace Example

Trace the string "10110" through Example 3 NFA

Work with a partner to:

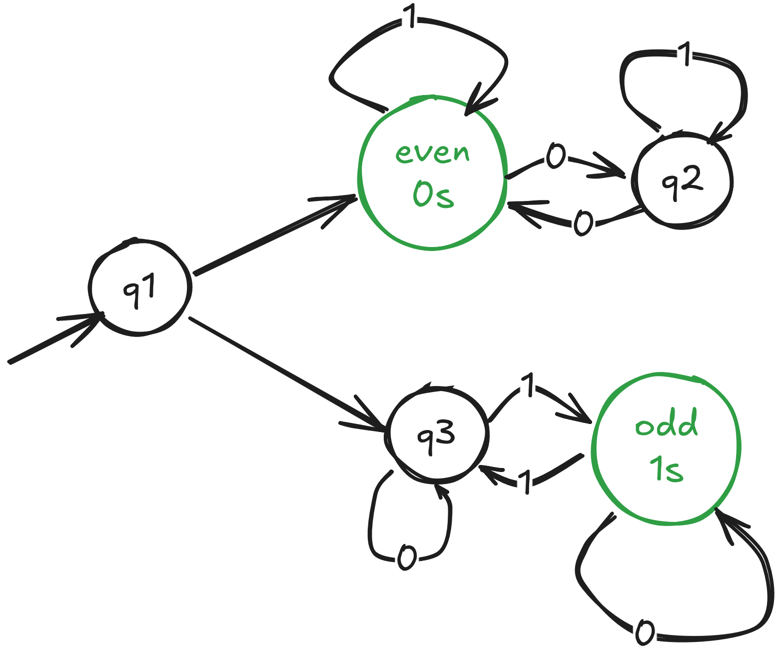
1. List all possible paths
2. Identify which paths accept
3. Determine if string is accepted

Example 4: "OR" with null-transitions



Where's the nondeterminism?

Example 4: Analysis



Nondeterminism: q1 has ε-transitions to two branches

Language: Strings with even number of 0s OR odd number of 1s

The Big Question

Does nondeterminism make NFAs more powerful than DFAs?

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Think about it...

Can NFAs recognize languages that DFAs cannot?

The Surprising Answer

NO!

> **An NFA is equivalent to a DFA**

Every NFA can be converted to an equivalent DFA!

But how?
🤯

NFA to DFA Conversion: The Idea

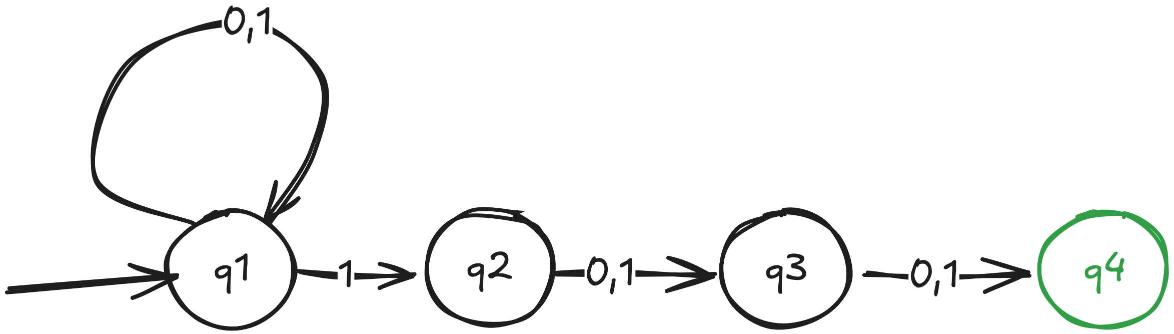
Subset Construction:

- If NFA has N states
  - There are 2^N possible subsets of states
- Each subset becomes a DFA state

Example: NFA with 3 states {q1, q2, q3}

- DFA states: {}, {q1}, {q2}, {q3}, {q1,q2}, {q1,q3}, {q2,q3}, {q1,q2,q3}

Conversion Example: Building the Table



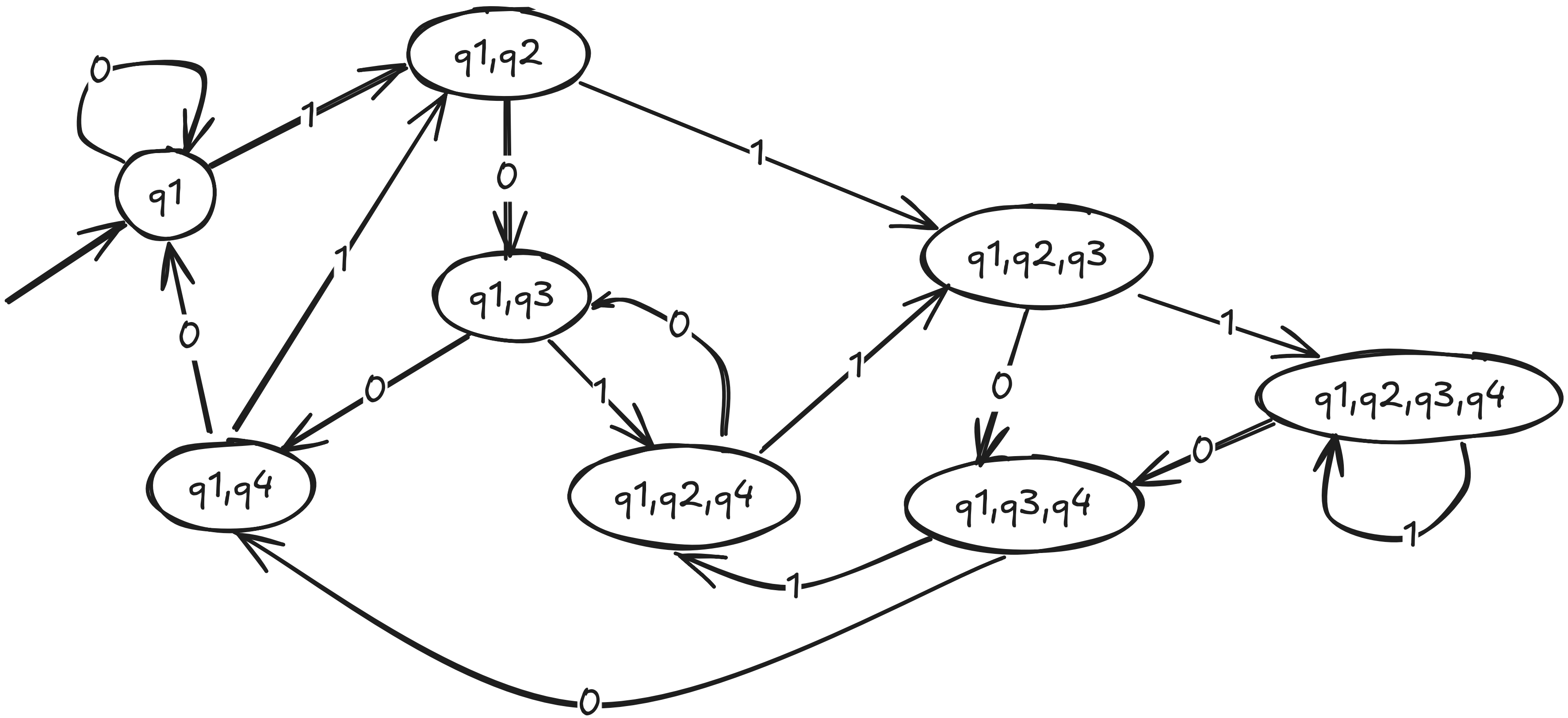
Let's convert this NFA to a DFA...

DFA State Transition Table

DFA State	0	1
{q1}	{q1}	**{q1,q2}**
{q1,q2}	**{q1,q3}**	**{q1,q2,q3}**
{q1,q3}	**{q1,q4}**	**{q1,q2,q4}**
{q1,q4}	{q1}	{q1,q2}

(Partial table shown - 8 more rows needed for complete DFA)

The Resulting DFA



Which looks simpler to you?

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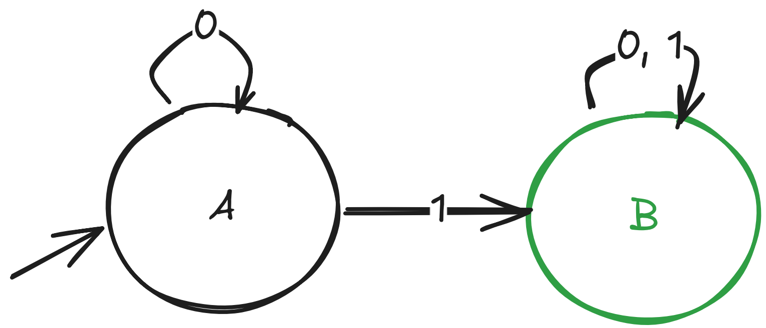
🎯 Active Learning: Conversion Practice

Given this simple NFA:



1. Identify all subset states needed
2. Build first 3 rows of transition table
3. Which subsets are accept states?

Implementing NFAs in Java

Key API Difference from DFA

```java
public class NFA {
  public static class State {
    void addTransition(Character symbol, State to) {...}

    // Returns SET of states, not single state!
    Set<State> getTransition(Character symbol) {...}
  }

  public boolean accepts(String input) {...}
}
```

Implementation Strategy

Processing the i-th symbol:

1. **Before reading:** Check ε-reachable states
    - Follow all null transitions
    - Beware of cycles!
2. **After reading:** Follow all symbol transitions
    - Explore all possible next states

Recommendation: Use recursion!

Supporting Null Transitions

Practical approach:

```
// Use special character for epsilon
char EPSILON = '\0';

// Add epsilon transitions
state1.addTransition(EPSILON, state2);

// Check epsilon transitions
Set<State> epsilonStates =
    currentState.getTransition(EPSILON);
```

Regular Languages

Definition

> A formal language is called a **regular language** if some DFA **or NFA** recognizes it.

Key insight: Since DFA ≡ NFA

- Language regular if DFA recognizes it
- Language regular if NFA recognizes it
- Same expressive power!

Why Use NFAs Then?

If NFAs = DFAs in power, why bother?

Advantages of NFAs:

- **Simpler** to design
- **Fewer states** needed
- **More intuitive** for certain patterns
- **Easier** to combine (union, concatenation)

Trade-off:

- Harder to implement/simulate

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Design Challenge

Your Turn: Design an NFA for:

"Binary strings containing the substring '101'"

1. Sketch your NFA
2. Compare with a partner
3. How many states did you use?

Would the DFA be simpler or more complex?

Key Takeaways

1. NFAs add two types of nondeterminism:
   i. Multiple transitions per symbol
   ii. Epsilon transitions
2. NFAs are equivalent to DFAs in power
3. NFAs often simpler to design but harder to simulate
4. Regular languages = Languages recognized by DFA or NFA