Beyond Regular Languages

Exploring the Limits of Finite Automata

What We Know So Far

Kleene's Theorem: Three equivalent models for regular languages

1. **Regular Expressions** - Specification
2. **DFAs** - Deterministic recognition
3. **NFAs** - Nondeterministic recognition

Question: Are there languages that are NOT regular?

The Central Question

> Can we find languages that cannot be:
>
> - Specified by any regular expression?
> - Recognized by any DFA or NFA?

Answer: YES! Let's see an example...

Example: Equal 0s and 1s

Language Definition:

$$L = \{0^n 1^n \mid n \geq 0\}$$

In the language:

- ε (empty string)
- 01
- 0011
- 000111
- 00001111

Not in the language:

- 0
- 001
- 0111
- 1100

Why This Language Is Not Regular

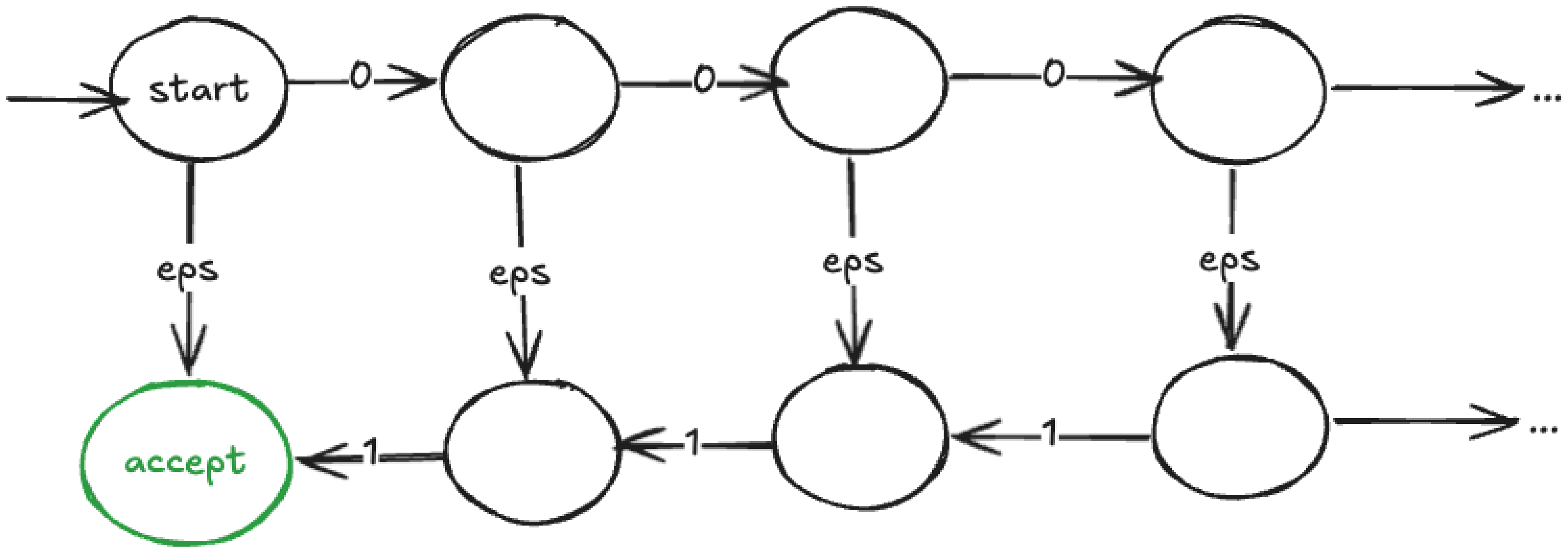
Intuitive Argument:

To recognize 0^n1^n, we need to:

1. Count the number of 0s
2. Then verify we have exactly the same number of 1s

The Problem:

- Need one state for each possible count of 0s
- But n is **unbounded** (can be any positive integer)
- Therefore need **infinite** states
- But DFAs/NFAs must have **finite** states!


start
0
0
0
...
eps
eps
eps
eps
accept
1
1
1
...

Key Insight

> **There exist formal languages that are NOT regular languages**

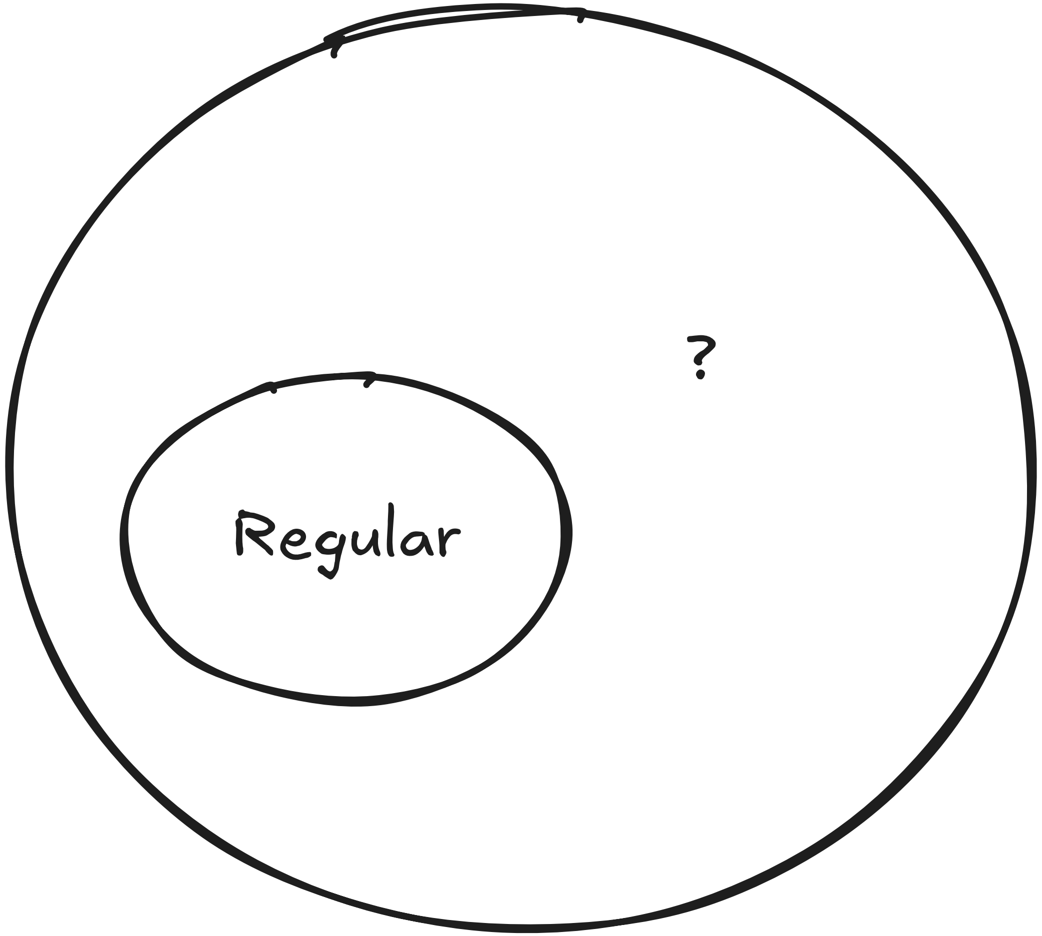
This is a fundamental limitation of finite automata!

Implications:

- Regular expressions cannot describe all languages
- DFAs and NFAs are limited in power
- We need more powerful computational models

The Language Hierarchy

Venn diagram of formal languages



More Powerful Machines

To recognize non-regular languages, we need enhanced computational models:

1. **Pushdown Automata (PDA)**
2. **Linear Bounded Automata (LBA)**
3. **Turing Machines (TM)**

Each adds more capability than the previous one

Pushdown Automata (PDA)

Enhancement: Add a **stack** to a DFA

What is a Stack?

- LIFO (Last In, First Out)
- Can push symbols
- Can pop symbols
- Unbounded capacity

How It Helps

- Count 0s by pushing
- Match 1s by popping
- Accept when stack empty

Recognizes: Context Free Languages (CFLs)

Context Free Languages (CFLs)

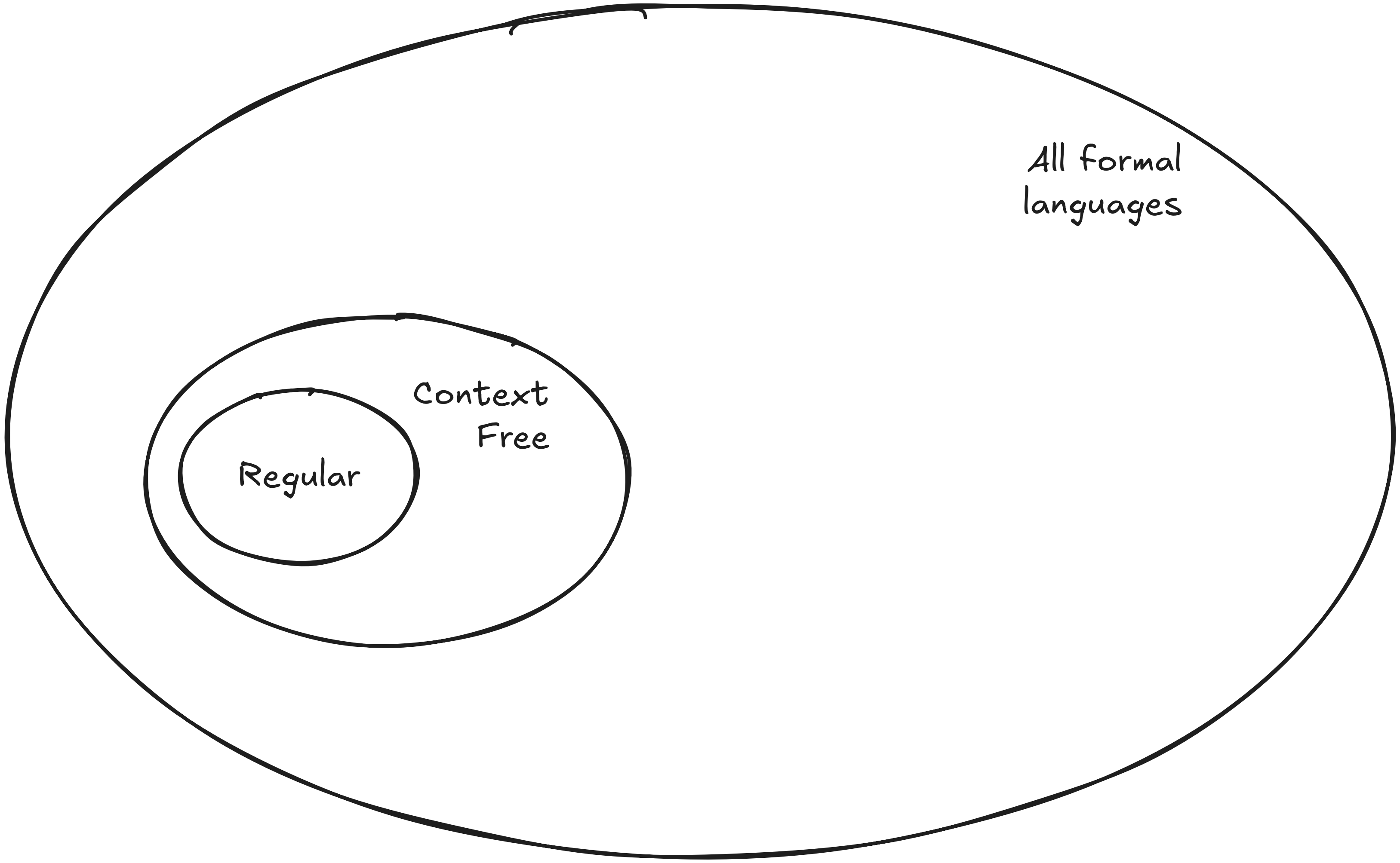
Definition: Languages recognized by PDAs

Examples:

- 0^n1^n (equal 0s followed by 1s)
- Balanced parentheses
- Programming language syntax (mostly)
- Arithmetic expressions

Key Property: All regular languages are CFLs, but not all CFLs are regular

Venn diagram of formal languages



Turtles All The Way Down?

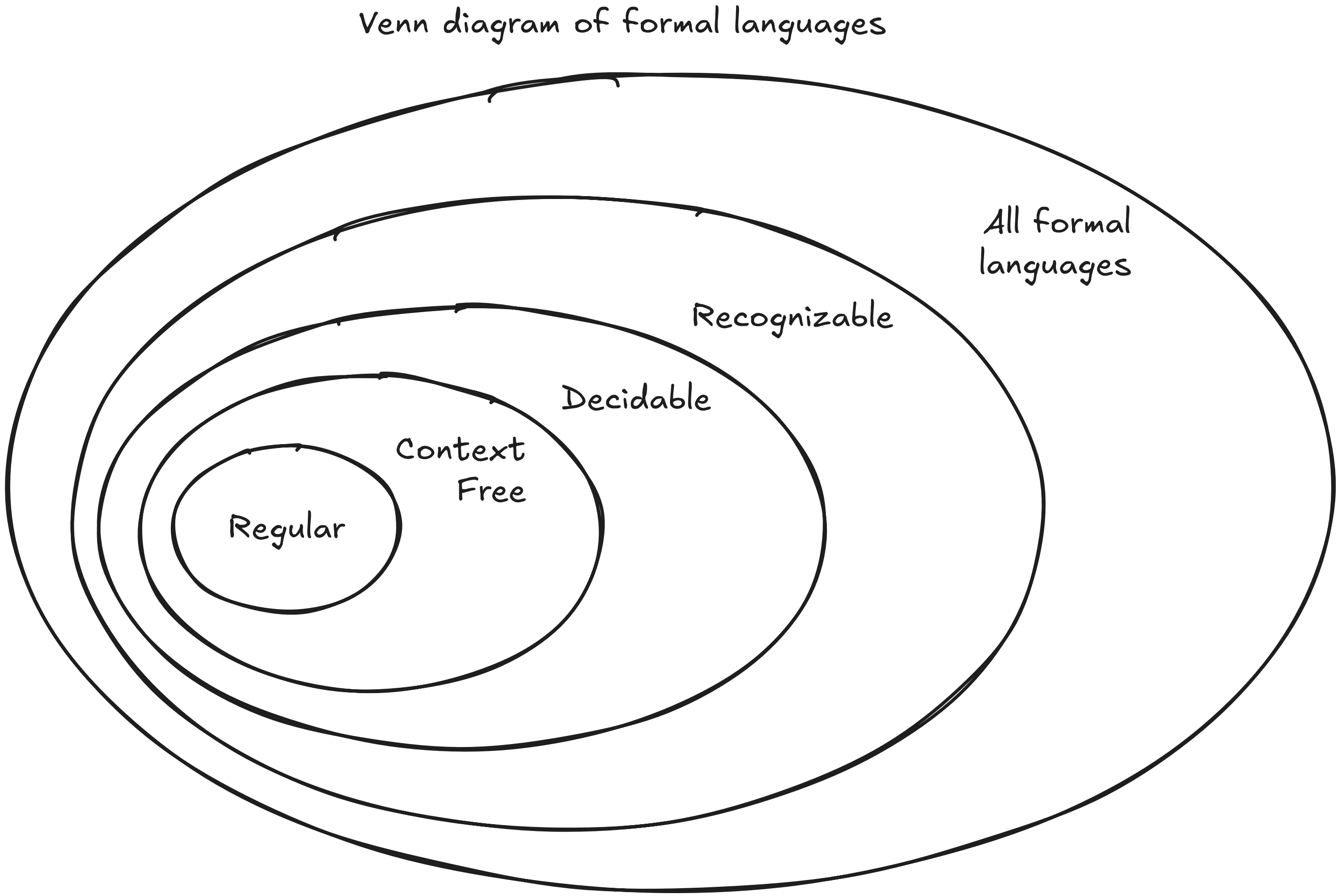
> Are we doomed to an endless sequence of increasingly powerful machines?

Answer: NO!

There exists a **pinnacle** of abstract machines:

- **Turing Machines** and equivalent models
- Called **Turing complete** or **Turing universal**
- As powerful as any possible machine
- We'll study these next!

Complete Language Hierarchy



Language Hierarchy Table

Language Type	Specification	Recognition
Regular	Regular Expression	NFA/DFA
Context Free	Context Free Grammar	PDA (DFA + stack)
Decidable	Set description	Turing Machine
Recognizable	Set description	Turing Machine
Unrecognizable	Set description	**No machine possible!**

Why This Matters

Practical Implications:

- **Compiler Design:** Programming language syntax is (mostly) context-free
- **Pattern Matching:** Know when regex won't work
- **Algorithm Design:** Understand what's computable vs. what's not
- **Complexity Theory:** Some solvable problems are intractable

Theoretical Foundation: Understanding limits is crucial for computer science

Optional Deep Dives

Want to explore further?

1. **Context Free Languages** - Study CFGs and PDAs in detail
2. **Context Sensitive Languages** - Examine CSGs and LBAs
3. **Pumping Lemma** - Formal proof technique for non-regularity

See the course notes for some coverage of these topics