AKTU

B. Tech I-Year

Gateway Classes

GATEWAY CLASSES

Semester -I & II

Common to All Branches

BAS101 / BAS201: ENGINEERING PHYSICS

Unit-1: Quantum Mechanics

Hand Written Notes

Gateway Series for Engineering

- Topic Wise Entire Syllabus
- Long - Short Questions Covered
- AKTU PYQs Covered
- DPP
- Result Oriented Content

Download App

For Full Courses including Video Lectures

AKTU

B. Tech I-Year

Gateway Classes

GATEWAY CLASSES

BAS101 / BAS201: ENGINEERING PHYSICS

Hand Written Notes

Unit-1

Introduction to Quantum Mechanics

Syallbus

Inadequacy of classical mechanics, Planck's theory of black body radiation(qualitative), Compton effect, de-Broglie concept of matter waves, Davisson and Germer Experiment, Phase velocity and group velocity, Time-dependent and time-independent Schrodinger wave equations, Physical interpretation of wave function, Particle in a one-Dimensional box.

UNIT: Quantum Mechanics

Lecture-1

Today's Target

→ Inadequacy of classical mechanics

→ Black Body and Black Body Radiations

→ Black Body Spectrum

→ Stefan law

→ Wien's Displacement law of radiation or wien's first law

→ Wien's Second law of radiation OR Wien's Radiation formula

→ Rayleigh Jeans Law

→ Planck's theory of black body radiation (qualitative)

Inadequacy of classical Mechanics

1. It does not explain the behavior of microscopic particles such as atomic particles / subatomic particles / wave.

2. It could not explain the stability of atoms.

3. It could not explain the observed spectrum of black body radiation.

4. It could not explain the observed variation of specific heat of metal and gases.

5. It could not explain photo electric effect, Compton effect, Raman effect etc.

Note. The inadequacy of classical mechanics led to the development of quantum mechanics.

Black Body and Black Body Radiations

→ A perfectly black body is one which can absorb all the radiations of all wavelength incident on it.

→ It is not possible to make a perfectly black body but Ferry designed a black body which can absorb 97% of radiations incident on it.

→ When a black body is heated, it emits all types of radiations called black body radiations.

Black Body Spectrum

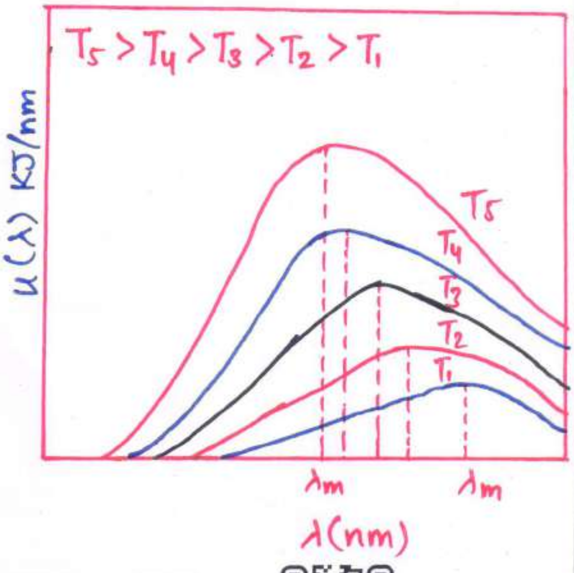
OR

Energy Distribution in the spectrum of Black Body Radiations

→ The energy distribution in the spectrum of black body radiation at different temperature is shown by graph between λ and E_λ.

From the graph following conclusion can be drawn

→ The intensity of radiation increases with wavelength and reaches a maximum value at a particular wavelength and decreases further as wavelength increases.



→ Wavelength corresponding to maximum intensity is λ_{max}.

→ At each temperature of black body, there is a wavelength (λ_{max}) for which E_λ is max.

→ As temperature of body increases, the λ_{max} shifts towards shorter wave-length.

→ There is non-uniform distribution of energy in the spectrum of black body radiation at a given temperature.

→ The amount of energy emitted per second increases with increase in temperature of black body.

→ The area under each curve represents the total energy emitted by black body at given temp.

Stefan law:

According to stefan's law, the total amount of ~~radiation~~ radiant energy by a black body per unit area per unit time due to all the wavelength is directly proportional to the fourth power of absolute temperature.

$$E \propto T^4$$

$$\boxed{E = \sigma T^4}$$

Where σ is called the 'stefan constant'.

$$\sigma = 5.56704 \times 10^{-8}\ W/m^2K^4$$

Wien's Displacement law of radiation or Wien's first law

According to wien's Displacement law, λ_m is inversely proportional to the absolute temperature (T) of the body

$$\lambda_m \propto \frac{1}{T}$$

$$\lambda_m = \frac{\text{constant}}{T}$$

$$\boxed{\lambda_m \times T = \text{constant}}$$

Wien's second law of radiation OR Wien's Radiation formula

According to wien's law, the amount of radiant energy emitted by a black body per unit area per unit time for a given wavelength range λ and $\lambda+d\lambda$ at a given temperature is expressed as

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{d\lambda}{e^{\frac{hc}{\lambda KT}}}\right)$$

where $h = 6.63 \times 10^{-34}$ J-sec

$c = 3\times 10^8$ m/sec

k = Boltzmann's constant $= 1.38\times 10^{-23}$ J/K

Limitation: This law is applicable for shorter wavelength only.

Rayleigh-Jeans law

Rayleigh used the classical theories of electro magnetism and thermodynamics to show that the black body spectral energy is given by

$$E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

Limitation: This law is applicable for longer wavelength only.

DPP

Q.1 What is Wien's law?

Q.2 State Wien's displacement law and Rayleigh-Jeans law.

Q.3 Describe energy distribution in black body radiation.

UNIT: Quantum Mechanics

Lecture-2

Today's Target

→ Planck's Quantum Hypothesis

→ Basic assumption of Planck's Hypothesis

→ Derivation of Planck's Radiation law / Planck's Radiation formula

Planck's Quantum Hypothesis

In 1900 Max Planck presented a paper to explain Black body radiation. According to Max Planck radiation behave like particles and consists of tiny packet of energy. Each packet of energy is called photon or Quanta and have energy.

$$E = h\nu$$

Max Planck proposed that the correct result can be obtained if the emission and absorption of energy is not taken as continuous process but it takes place in discrete amount. He derived radiation law by using following assumption and hypothesis.

Basic Assumptions of Planck's hypothesis

(1) He assumed that the atoms of the wall of a black body behave as oscillators and each has characteristics frequency of oscillation.

(2) The energy of an oscillator can have only discrete values

$$E_n = nh\nu$$

$E_1 = h\nu$

$E_2 = 2h\nu$

$E_3 = 3h\nu$

and so on

n = positive integer / Quantum Number

h = Planck's constant

ν = Frequency of oscillation

(3) The oscillator do not emit or absorb radiation energy continuously but only in a certain multiplies of packets of $h\nu$, while jumping from one state to another.

Planck's Radiation Formula

Max Planck developed a theory of black body radiation that leads to an equation for the energy density of the radiation. This equation is in complete agreement with experimental observations.

Derivation

We first calculate the average energy of Planck oscillator (E_{avg}) of frequency ν

Let N = Total number of oscillators

E = Total energy of oscillators

$$E_{avg} = \frac{E}{N} \quad -(1)$$

Let $N_0, N_1, N_2 \ldots N_n \ldots$ be the number of oscillators having energies 0, $h\nu, 2h\nu, \ldots n h\nu \ldots$ respectively

Total number of oscillators

$$N = N_0 + N_1 + N_2 + N_3 + \ldots \quad (2)$$

According to Boltzmann's law, the number of oscillators having $nh\nu$ energy at energy at temperature T will be

$$N_n = N_0 e^{-\frac{nh\nu}{KT}}$$

$N_0 = N_0$

$N_1 = N_0 e^{\frac{-h\nu}{KT}}$

$N_2 = N_0 e^{\frac{-2h\nu}{KT}}$

$N_3 = N_0 e^{\frac{-3h\nu}{KT}}$

and so on

Put above values in (2)

$$N = N_0 \left[1 + e^{\frac{-h\nu}{KT}} + e^{\frac{-2h\nu}{KT}} + e^{\frac{-3h\nu}{KT}} + \ldots \right]$$

$$N = N_0 \left[1 + e^{\frac{-h\nu}{KT}} + \left(e^{\frac{-h\nu}{KT}}\right)^2 + \left(e^{\frac{-h\nu}{K}}\right)^3 + \ldots \right] =$$

$$N = N_0 \left(1 - e^{\frac{-h\nu}{KT}} \right)^{-1} \quad \left\{ \begin{array}{c} 1 + x + x^2 + \ldots = (1-x)^{-1} \\ \text{OR} \\ S_\infty = \frac{a}{1-r} \end{array} \right\}$$

$$N = \frac{N_0}{\left(1-e^{\frac{-h\nu}{K}}\right)} \quad -(3)$$

Total Energy

$$E = N_0 \times 0 + N_1 h\nu + N_2 \times 2h\nu + N_3 \times 3h\nu + \text{-----}$$

$$E = N_1 h\nu + 2N_2 h\nu + 3N_3 h\nu + \text{-----}$$

$$E = h\nu\,(N_1 + 2N_2 + 3N_3 + \text{------})$$

$$E = h\nu\left[N_0 e^{\frac{-h\nu}{KT}} + 2N_0 e^{\frac{-2h\nu}{KT}} + 3N_0 e^{\frac{-3h\nu}{KT}} + \text{---}\right]$$

$$E = h\nu \times N_0 \times e^{\frac{-h\nu}{KT}}\left[1 + 2e^{\frac{-h\nu}{KT}} + 3e^{\frac{-2h\nu}{KT}} + \text{---}\right]$$

$$E = h\nu \times N_0 \times e^{\frac{-h\nu}{KT}}\left[1 + 2e^{\frac{-h\nu}{KT}} + 3\left(e^{-\frac{h\nu}{KT}}\right)^2 + \text{---}\right]$$

$$E = h\nu \times N_0 \times e^{\frac{-h\nu}{KT}}\left(1-e^{\frac{-h\nu}{KT}}\right)^{-2} \quad \left\{1+2x+3x^2+4x^3+ \text{....} = (1-x)^{-2}\right\}$$

$$E = \frac{h\nu \times N_0 \times e^{\frac{-h\nu}{KT}}}{\left(1-e^{\frac{-h\nu}{KT}}\right)^2} \quad -(4)$$

Put E and N in (1)

$$E_{avg} = \frac{h\nu \times \cancel{N_0} \times e^{-\frac{h\nu}{KT}}}{\left(1-e^{-\frac{h\nu}{KT}}\right)^{\cancel{2}}} \times \frac{\cancel{\left(1-e^{-\frac{h\nu}{KT}}\right)}}{\cancel{N_0}}$$

$$E_{avg} = \frac{h\nu\, e^{-\frac{h\nu}{KT}}}{\left(1-e^{-\frac{h\nu}{KT}}\right)} = \frac{h\nu}{e^{\frac{h\nu}{KT}}\left(1-e^{-\frac{h\nu}{KT}}\right)}$$

$$E_{avg} = \frac{h\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)}$$

This is the avg. energy of oscillators

The number of oscillators per unit volume lying in the frequency range of ν to $\nu+d\nu$ as obtain by Rayleigh and Jeans

$$= \frac{8\pi\nu^2 d\nu}{c^3}$$

Energy of radiation per unit volume
OR
Energy density $(u_\nu d\nu)$

~~$u_\nu d\nu$~~ $u_\nu d\nu = \frac{h\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)} \times \frac{8\pi\nu^2 d\nu}{c^3}$

$$u_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \times \frac{d\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)}$$

This is the plank's radiation formula in terms of ν.

In terms of wavelength

$$\nu = \frac{c}{\lambda}$$

$$d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$u_\lambda d\lambda = -u_\nu d\nu$$

$$= -\frac{8\pi h\nu^3}{c^3} \times \frac{d\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)}$$

$$= \frac{-8\pi h c^3}{c^3 \times \lambda^3} \times \frac{(-c)}{\lambda^2} \times \frac{d\lambda}{\left(e^{\frac{hc}{\lambda KT}}-1\right)}$$

$$\boxed{u_\lambda d_\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{d\lambda}{\left(e^{\frac{hc}{\lambda KT}}-1\right)}}$$

Planck's Radiation Formulae

(i) Planck's radiation formula for the black body radiation spectrum in terms of frequency

$$u_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \frac{d\nu}{\left(e^{\frac{h\nu}{KT}}-1\right)}$$

(ii) Planck's radiation formula for the black body radiation spectrum in terms of wavelength

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\left(e^{\frac{hc}{\lambda KT}}-1\right)}$$

Deduction of Various laws using Planck's Radiation Formula

(i) Wien's law from Planck's Formula

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\left(e^{\frac{hc}{\lambda KT}}-1\right)}$$

For short wavelength or when λ is small

$$e^{\frac{hc}{\lambda KT}} >>> 1$$

So, we can be neglected

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{\frac{hc}{\lambda KT}}}$$

This is the Wien's law agree with experimental curve at short wavelengths region.

Deduction of Various laws using Planck's Radiation Formula

(ii) Rayleigh – Jeans law from Planck's Radiation Formula

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{\left(e^{\frac{hc}{\lambda KT}} - 1\right)}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \text{-----}$$

For longs wavelength or when λ is large

$$e^{\frac{hc}{\lambda KT}} = 1 + \frac{hc}{\lambda KT} + \underbrace{\left(\frac{hc}{\lambda KT}\right)^2 \times \frac{1}{2!} + \text{---}}_{\text{neglect}}$$

$$e^{\frac{hc}{\lambda KT}} = 1 + \frac{hc}{\lambda KT}$$

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{\left(1 + \frac{hc}{\lambda KT} - 1\right)}$$

$$\boxed{u_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

This is the Rayleingh – Jeans law ~~or~~ agree with experimental curve at short wavelengths region

DPP

Q.1 Write down the Basic Assumptions of Planck's hypothesis ?

Q.2 Derive the Planck's Radiation Formula.

UNIT: Quantum Mechanics

Lecture-3

Today's Target

→ Compton effect

→ Derivation of compton shift

→ Numerical Problems

Compton Effect

→ In 1921, Professor A.H Compton discovered that when a monochromatic beam of high frequency radiation (or X-ray) is scattered by a substance (or electron).

→ The scattered radiations contain not only of the same wavelength or frequency as that of incident ray but also the radiation of greater wavelength or lower frequency.

→ The radiation of unchanged wavelength in the scattered ray are called Unmodified radiation.

→ The radiation of greater wavelength (changed wavelength) in the scattered ray are called modified radiation.

→ The phenomenon of change in wavelength of scattered X-ray is known as Compton effect.

→ The change in wavelength of high frequency radiation during scattering process is called compton shift.

$$\Delta\lambda = \lambda' - \lambda$$

where $\Delta\lambda$ = compton shift

λ' = Wavelength of scattered radiation

λ = wavelength of incident radiation

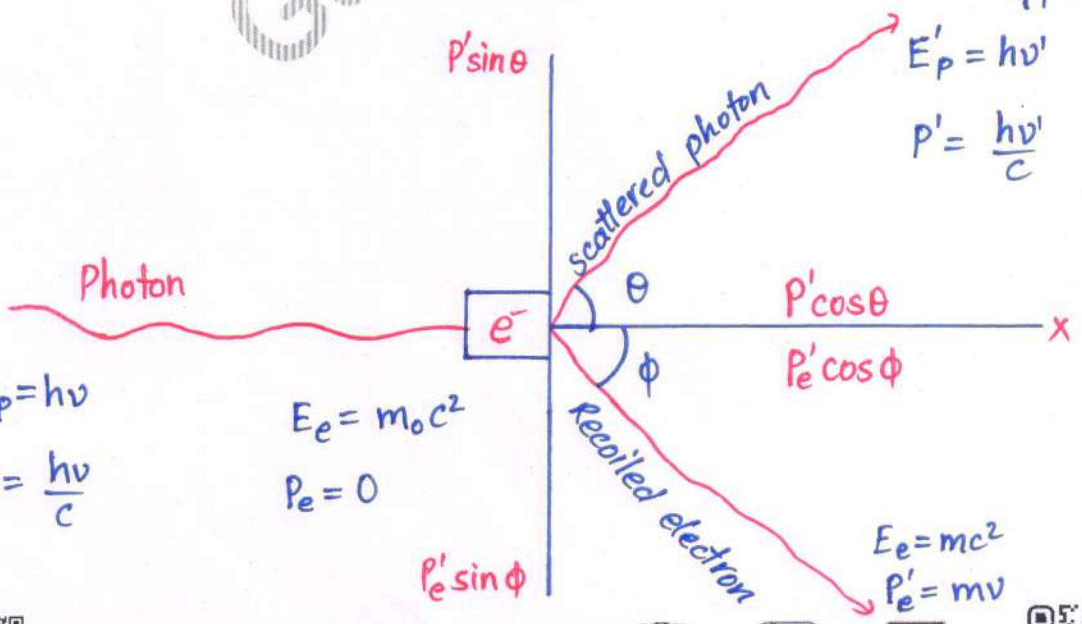
Derivation

Following assumption were made to explain the Compton effect.

(i) Compton effect is the result of interaction of X-ray photon and free electron of the target.

(ii) The collision is relativistic and elastic.

(iii) The laws of conservation of energy and momentum are applicable.



Law of conservation of momentum

Momentum before collision = Momentum after collision

Along X-axis

$$p + p_e = p'\cos\theta + p_e'\cos\phi$$

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$

$$h\nu = h\nu'\cos\theta + mcv\cos\phi$$

$$mcv\cos\phi = h\nu - h\nu'\cos\theta \quad \text{— (1)}$$

Along Y-axis

$$0 + 0 = p'\sin\theta - p_e'\sin\phi$$

$$p_e'\sin\phi = p'\sin\theta$$

$$mv\sin\phi = \frac{h\nu'}{c}\sin\theta$$

$$mcv\sin\phi = h\nu'\sin\theta \quad \text{— (2)}$$

Squaring and adding (1) and (2)

$$m^2c^2v^2\cos^2\phi + m^2c^2v^2\sin^2\phi = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$

$$m^2c^2v^2(\cos^2\phi + \sin^2\phi) = h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h^2\nu\nu'\cos\theta + h^2\nu'^2\sin^2\theta$$

$$m^2c^2v^2 = h^2\nu^2 + h^2\nu'^2(\cos^2\theta + \sin^2\theta) - 2h^2\nu\nu'\cos\theta$$

$$m^2c^2v^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta \quad \text{— (3)}$$

Apply Law of conservation of energy

Energy before collision = Energy after collision.

$$E_p + E_e = E_p' + E_e'$$

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = (h\nu - h\nu') + m_0c^2$$

Squaring both side

$$m^2c^4 = \left[(h\nu - h\nu') + m_0c^2\right]^2$$

$$m^2c^4 = (h\nu - h\nu')^2 + m_0^2c^4 + 2(h\nu - h\nu')m_0c^2$$

$$m^2c^4 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + m_0^2c^4 + 2(h\nu - h\nu')m_0c^2 \quad \text{— (4)}$$

Subtract eqn (3) From (4)

$$m^2c^4 - m^2c^2v^2 = -2h^2\nu\nu' + m_0^2c^4 + 2(h\nu - h\nu')m_0c^2 + 2h^2\nu\nu'\cos\theta$$

$$m^2c^4\left(1 - \frac{v^2}{c^2}\right) = -2h^2\nu\nu'(1-\cos\theta) + m_0^2c^4 + 2(h\nu - h\nu')m_0c^2 \quad \text{— (5)}$$

According to theory of Relativity

$$\boxed{m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad m^2 = \frac{m_0^2}{\left(1 - \frac{v^2}{c^2}\right)} \Rightarrow m^2\left(1 - \frac{v^2}{c^2}\right) = m_0^2$$

$$\cancel{m_0^2c^4} = 2h^2\nu\nu'(1-\cos\theta) + \cancel{m_0^2c^4} + 2(h\nu - h\nu')m_0c^2$$

$$2h^2\nu\nu'(1-\cos\theta) = 2h(\nu - \nu')m_0c^2$$

$$h\nu\nu'(1-\cos\theta) = (\nu - \nu')m_0c^2$$

$$\frac{h(1-\cos\theta)}{m_0 c^2} = \frac{\nu-\nu'}{\nu\nu'}$$

$$\frac{h(1-\cos\theta)}{m_0 c^2} = \frac{1}{\nu'} - \frac{1}{\nu} \quad -(6)$$

Multiply both side by c

$$\frac{h\cancel{c}(1-\cos\theta)}{m_0 c^{\cancel{2}}} = \lambda' - \lambda \Rightarrow \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\boxed{\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)}$$

Special cases

Case-1

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos 0^\circ)$$

$$\Delta\lambda = 0$$

$$\lambda' - \lambda = 0$$

$$\boxed{\lambda' = \lambda}$$

Case-2

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos 90)$$

$$\Delta\lambda = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$$\boxed{\Delta\lambda = 0.0243\,\text{Å}}$$

$1\,\text{Å} = 10^{-10}\,m$

Case-3

When $\theta = 180$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos 180)$$

$$\Delta\lambda = \frac{2h}{m_0 c}$$

$$\Delta\lambda = \frac{2\times6.63\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}$$

$$\boxed{\Delta\lambda = 0.0486\,\text{Å}}$$

Kinetic energy of recoiled electron

$$E_K = h\nu - h\nu'$$
$$= h(\nu - \nu')$$
$$= h\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right)$$

$$E_K = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$E_K = hc\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right)$$

Direction of recoiled electron

$$\tan\phi = \frac{\lambda \sin\theta}{\lambda' - \lambda\cos\theta}$$

Conclusion from Compton scattering Experiment

Compton shift is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

It is concluded from above equation that

(i) Wavelength of scattered photon is greater than the wavelength of incident photon.

(ii) Compton shift only depends on the scattering angle.

(iii) Compton shift have the same value for all the substance containing free electrons.

Why compton effect is not observed in visible spectrum?

The maximum change in wavelength λ_{max} is approx. 0.05Å. This is very small therefore can not be observed for wavelength longer than few angstrom units.

Example

(i) For X-ray,

Wavelength of incident radiation is about 1Å.

$$\lambda_{max} = 0.05\text{Å}$$

which is 5% (detectable) of incident radiation.

(ii) For visible radiation

Wavelength of incident radiation is about 5000Å.

$$\lambda_{max} = 0.05\text{ Å}$$

which is 0.001% (Undetectable) of incident radiation.

Q.1 In a Compton scattering experiment X-Ray of wavelength 0.015 Å is scattered at 60°, find the wavelength of scattered X-ray.

Given

$\lambda = 0.015$ Å

$\theta = 60°$

$\lambda' = ?$

$h = 6.63 \times 10^{-34}$ J-sec

$m_0 = 9.1 \times 10^{-31}$ kg

$c = 3 \times 10^8$ m/s

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8}(1-\cos 60°)$$

$$\lambda' - \lambda = 0.012 \times 10^{-10}\, m$$

$$\lambda' = 0.015 + 0.012$$

$$\lambda' = 0.027 \text{ Å}$$

Q.2 In an X-ray photon is found to have its wavelength doubled on being scattered through 90° from a material. Find the wavelength and energy of incident photon.

Given

$\lambda' = 2\lambda$

$\theta = 90°$

$\lambda = ?$

$E = ?$

$h = 6.63 \times 10^{-34}$ J-sec

$m_0 = 9.1 \times 10^{-31}$ kg

$c = 3 \times 10^8$ m/s

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda' - \lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8}(1-\cos\theta)$$

$$2\lambda - \lambda = 0.0245 \text{ Å}$$

$$\boxed{\lambda = 0.0245 \text{ Å}}$$

$$E = h\nu$$

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.0245 \times 10^{-10}}$$

$$\boxed{E = 8.106 \times 10^{-14} \text{ J}}$$

Q.3 X-ray of wavelength 2Å are scattered from a black body and x-ray are scattered at angle of 45°. Calculate the compton shift, wavelength of scattered photon.

Given

$\lambda = 2Å$

$\theta = 45°$

$\lambda' = ?$

$h = 6.63 \times 10^{-34}$ J-sec

$c = 3 \times 10^8$ m/s

$m_0 = 9.1 \times 10^{-31}$ kg

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8}(1-\cos 45°)$$

$$\Delta\lambda = 0.007Å$$

$$\Delta\lambda = 0.007Å$$

$$\lambda' - \lambda = 0.007Å$$

$$\lambda' - 2 = 0.007Å$$

$$\boxed{\lambda' = 2.007Å}$$

DPP

Q.1 Explain modified and unmodified radiation in compton effect/scattering.

Q.2 What is Compton effect and Compton shift? Derive the necessary expression for compton shift.

Q.3 Calculate Compton shift if x-ray of wavelength of 1.00Å are scattered from a carbon block. The scattered radiation is viewed at 90° to the incident.

UNIT: Quantum Mechanics

Lecture-4

Today's Target

→ De-Broglie concept of matter waves

De-Broglie Hypothesis

→ According to de-Broglie Hypothesis (1924), "Every moving particle has a wave associated with it"

→ The wave associated with a moving particle is called matter waves or de-Broglie waves.

→ If a particle of mass 'm' has momentum 'p' and λ is the wave-length of wave associated with it, then according to de-Broglie Hypothesis.

$$\boxed{\lambda = \frac{h}{p}}$$

h → Planck's constant

λ → De-Broglie wavelength

$p = mv$

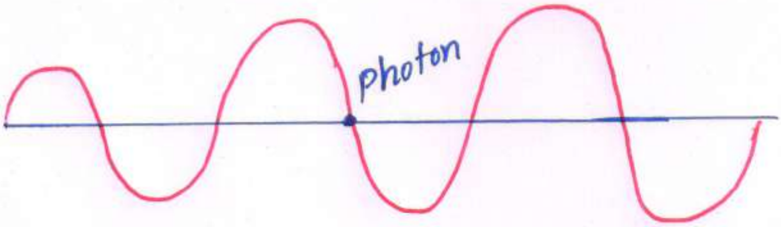
m = mass of particle

v = velocity of particle

$$\lambda = \frac{h}{mv}$$

Expression for De-Broglie wavelength of photon

$$\nu = \frac{c}{\lambda}$$



According to planck's theory of Radiation

$$E = h\nu \quad \text{—} ①$$

According to Einstein Mass energy relation

$$E = mc^2 \quad \text{—} ②$$

Where

E = energy of photon

From ① & ②

$$h\nu = mc^2$$

$$\frac{h\cancel{c}}{\lambda} = mc^{\cancel{2}}$$

$$\boxed{\lambda = \frac{h}{mc}}$$

Some other important Expression for de-Broglie wavelength

1. In terms of kinetic energy

$$E = \frac{1}{2} m v^2$$

$$E = \frac{1}{2} \frac{m^2 v^2}{m}$$

$$E = \frac{1}{2m} (mv)^2$$

$$E = \frac{1}{2m} p^2$$

$$p^2 = 2mE$$

$$p = \sqrt{2mE}$$

de-Broglie wavelength

$$\lambda = \frac{h}{p}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

2. In terms of temperature

We know that

$$E = \frac{3}{2} kT$$

k = Boltzmann's constant

T = Temperature

E = kinetic energy

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2m \times \frac{3}{2} kT}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

3. For any particle carrying charge q accelerated by potential difference V volts.

We know that

$$E = qV$$

where

E = kinetic energy

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$$

4. De-Broglie wavelength of electron wave.

$1Å = 10^{-10} m$

We know that

$M_e = 9.1 \times 10^{-31}$ kg

$q = e = 1.6 \times 10^{-19}$ C

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\boxed{\lambda = \frac{12.28}{\sqrt{V}} Å}$$

Note: (i) Mass of Electron

$m_e = 9.1 \times 10^{-31}$ kg

(ii) Mass of proton

$m_p = 1.67 \times 10^{-27}$ kg

(iii) Mass of Neutron

$m_n = 1.67 \times 10^{-27}$ kg

(iv) Planck's constant

$h = 6.63 \times 10^{-34}$ J-sec

(v) Boltzmann's constant

$k = 1.38 \times 10^{-23}$ J/K

Q.1 Find the de-Broglie wavelength for an electron of energy V eV.

$E = V eV$

$E = V \times 1.6 \times 10^{-9}$ J

$E = 1.6 \times 10^{-19} V$ J

We know that

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$$

$$\boxed{\lambda = \frac{12.28}{\sqrt{V}} Å}$$

$1 eV = 1.6 \times 10^{-19}$ J

Q.2 Calculate the de-Broglie wavelength associated with an electron subjected to a potential difference of 50 volts.

Given

$V = 50V$

de-Broglie wavelength

$$\lambda = \frac{12.28}{\sqrt{V}} \text{Å}$$

$$\lambda = \frac{12.28}{\sqrt{50}} \text{Å}$$

$$\boxed{\lambda = 1.7366 \text{Å}}$$

Q.3 Calculate wavelength of an α particle accelerated through a potential difference of 200 volts.

Given

$V = 200V$

$q = 2e = 2 \times 1.6 \times 10^{-19} C$

$m = 4m_p = 4 \times 1.67 \times 10^{-27} kg$

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 4.1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$$\boxed{\lambda = 0.00716 \text{Å}}$$

$1\text{Å} = 10^{-10} m$

Q.4 Calculate the velocity and kinetic energy of a neutron having de-Broglie wavelength 1 Å.

Given

$\lambda = 1\text{Å} = 10^{-10} m$

$m_n = 1.67 \times 10^{-27} kg$

de-Broglie wavelength

$$\lambda = \frac{h}{mv}$$

$$10^{-10} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times V}$$

$$V = \frac{6.63 \times 10^{-34}}{10^{-10} \times 1.67 \times 10^{-27}}$$

$$\boxed{V = 3.96 \times 10^{3} \text{ m/sec}}$$

kinetic energy

$$E = \frac{1}{2}mv^2 = 1.67 \times 10^{-27} \times (3.96 \times 10^3)^2$$

$$\boxed{E = 1.309 \times 10^{-20} J}$$

Q.5 Calculate the de-Broglie wavelength associated with a proton moving with velocity equal to $\frac{1}{20}$th velocity of light.

Given

$$v = \frac{1}{20}c = \frac{1}{20} \times 3\times10^{8} m/s$$

de-Broglie wavelength

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{h}{\frac{m_0 c}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$\lambda = \frac{h}{m_0 v}\sqrt{1-\frac{v^2}{c^2}}$$

$$\lambda = \frac{6.63\times10^{-34}}{1.67\times10^{-27}\times\frac{1}{20}\times3\times10^{8}}\sqrt{1-\left(\frac{1}{20}c\right)^2\frac{1}{c^2}}$$

$$\lambda = \frac{6.63\times10^{-34}\times 20}{1.67\times10^{-27}\times3\times10^{8}}\sqrt{1-\frac{1}{400}}$$

$$\boxed{\lambda = 0.000264\ \mathring{A}}$$

Q.6 Calculate the de-Broglie wavelength of neutron having kinetic energy 1eV.

Given

$$E = 1\,eV = 1.67\times10^{-19} J$$

Rest mass energy of neutron

$$E = mc^2$$

$$E = 1.67\times10^{-27}\times(3\times10^{8})^2 J$$

$$E = \frac{1.67\times10^{-27}\times(3\times10^{8})^2}{1.6\times10^{-19}}$$

$$E = 939375000\ eV$$

So, ignored Relativistic concept

de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.67\times10^{-19}}}$$

$$\boxed{\lambda = 0.287\ \mathring{A}}$$

Properties of Matter waves

→ The de-Broglie wavelength of a wave associated with Lighter particle greater than the wavelength associated with heavier particle.

→ The de-Broglie wavelength of a wave associated with slow moving particle is greater than the wavelength associated with fast moving particle.

→ If $v=0$, then $\lambda=\infty$, Wave become indeterminate and if $v=\infty$, then $\lambda=0$, This shows that the matter waves are generated only when the material particles are in motion.

→ The matter waves are not electromagnetic in nature.

→ The matter waves are generated by moving charged particles as well as by moving neutral particles.

Difference between Electromagnetic wave and Matter wave

Matter Wave	Electromagnetic wave
(i) Matter waves are associated with moving particle and does not depend on charge	(i) Electromagnetic waves are associated with accelerated charge particle.
(ii) Matter waves do not have any associated electric and magnetic field.	(ii) Electromagnetic waves have electric and magnetic fields associated with them
(iii) Matter waves do not travel through the vacuum.	(iii) Electromagnetic waves travel through the vacuum.
(iv) The velocity of matter waves is generally less than the velocity of light.	(iv) The velocity of electromagnetic waves is equal to the velocity of light
(v) Example:- Electron beam	(v) Example:- Radio waves, UV light

DPP

Q.1 Calculate the de-Broglie wavelength associated with an electron subjected to a potential difference of 100 volts.

Q.2 The kinetic energy of an electron is 4.55×10^{-25} J. Calculate the velocity, momentum and wavelength of electron.

Q.3 A proton is moving with a velocity 2×10^8 m/s. Calculate the wavelength of matter wave associated with it.

Q.4 Calculate the de-Broglie wavelength associated with nitrogen at 3.0 atmospheric pressure and 27°C mass of N_2 atom $= 4.65 \times 10^{-26}$

UNIT: Quantum Mechanics

Lecture- 5

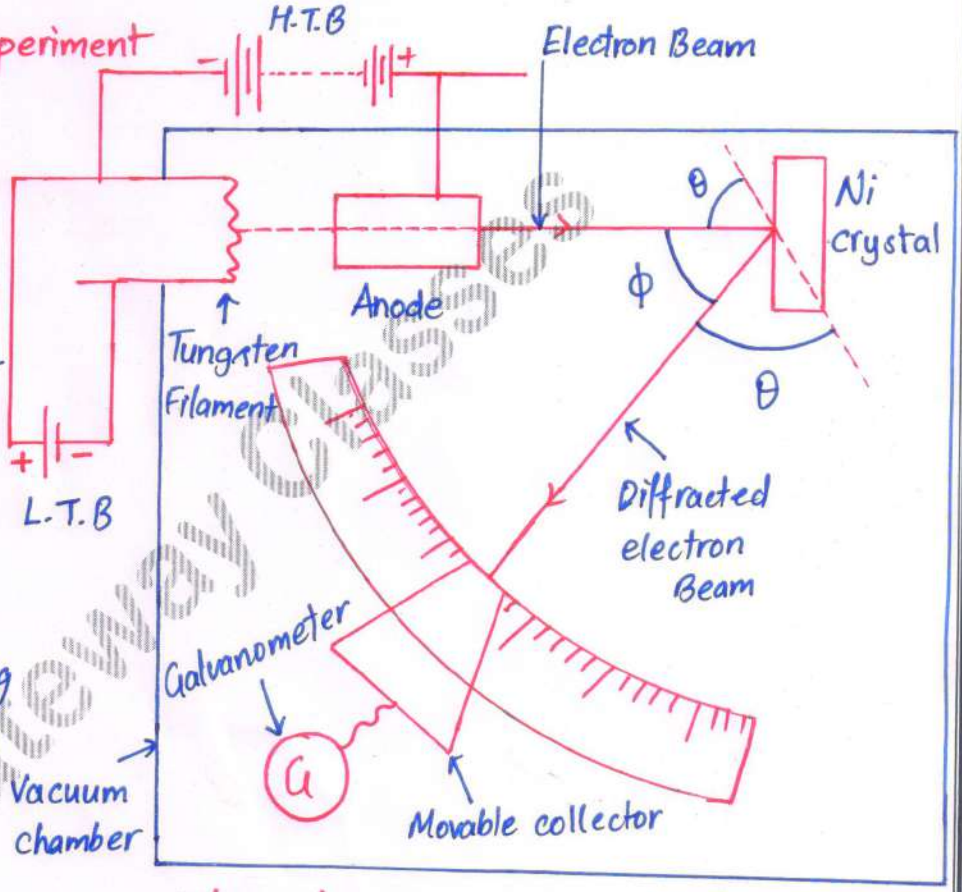
Today's Target

Davisson- Germer Experiment

Davisson and Germen Experiment

→ In 1927, Davisson and Germer gave first experi--mental evidence of matter waves predicted by de-Brog--lie in 1924.

→ They not only confirmed the existence of waves associated with fast moving electrons but also measured the wavelength of matter wave. $\lambda = \frac{h}{p}$



Set-up of Davisson-Germer Experiment

→ The experimental setup consist of an electron gun having tungsten filament F which is heated by passing electric current ~~flow~~ from low tension battery (L.T.B).

→ Due to thermionic emission electrons are emitted by the filament and these electrons are then accelerated under suitable potential difference provided by high tension battery (H.T.B.).

→ The narrow beam of electrons comes out from cylinder is allowed to fall on the surface of a nickel crystal.

→ The Electrons, scattered in different directions by the atoms of nickel crystal, are received by a movable detector which is just an electron collector.

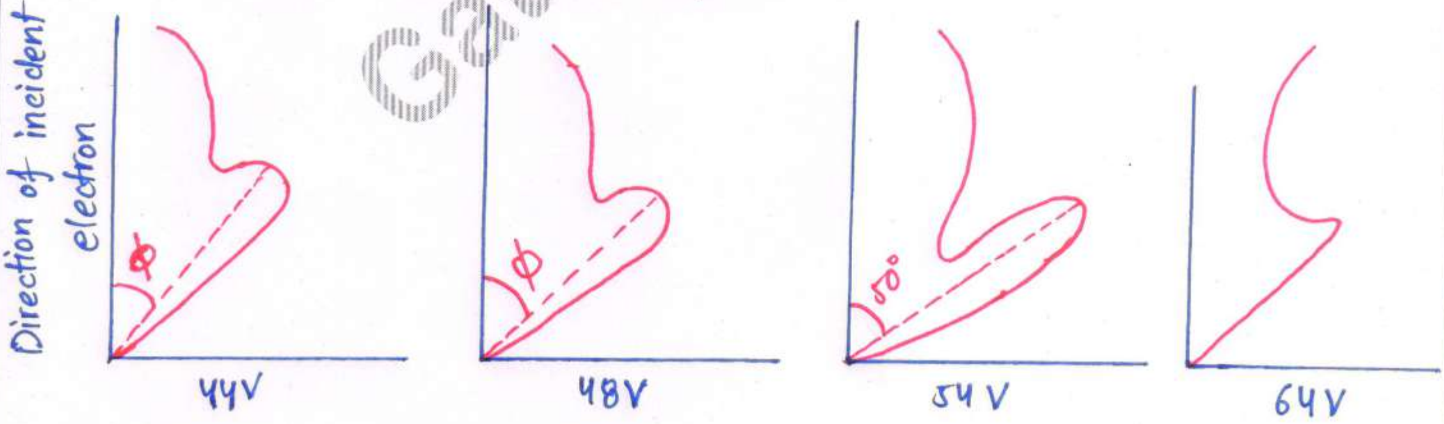
→ This electron collector is connected with a galvanometer (G).

→ The deflection in G is directly proportional to intensity of electron beam entering the collector.

→ The entire apparatus is enclosed in vacuum chamber.

→ This experiment was performed at different accelerating voltage (V) but significant intensity of electron was observed around 44V – 68V.

Result of Davisson and Germer Experiment



Above Graphs show the intensity of electrons as a function of scattering angle ϕ.

(i) Intensity of scattered electron depends upon angle of scattering ϕ.

(ii) A bump begins to appear in curve at 44 V.

(iii) It was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54 V at a scattering angle 50°.

(iv) The size of bump start decreasing with further increase in accelerating voltage.

(v) From the electron diffraction measurements, the wavelength of matter waves was found to be 0.165 nm.

Conclusion: The maximum intensity obtained in a particular direction shows the constructive interference.

So, for crystal constructive interference, Bragg's law stated that

$$2d\sin\theta = n\lambda \quad \text{—(1)}$$

where

d = interatomic seperation for Ni crystal

$d = 0.914\,Å$

$n = 1$

from figure

$\theta + \theta + \phi = 180$

$2\theta + 50 = 180$

$\theta = 65°$

Put these values in (1)

$2 \times 0.914\,Å \times \sin 65 = \lambda$

$\lambda = 1.66\,Å$

By de-Broglie Hypothesis

$$\lambda = \frac{12.28}{\sqrt{V}}$$

$$\lambda = \frac{12.28}{\sqrt{54}} \Rightarrow \boxed{\lambda = 1.67\,Å}$$

DPP

Q-1 What was the objective of conducting Davission - Germer experiment?

Q-2 Describe the experiment of Davission and Germer to demonstrate the wave character of electrons.

UNIT : Quantum Mechanics

Lecture-6

Today's Target

→ Phase Velocity and Group Velocity

Phase Velocity

→ The velocity with which a monochromatic wave (single frequency and wavelength) travel through a medium is known as phase velocity or wave velocity.

→ It is the characteristic of individual wave.

Expression for Phase Velocity

Consider a wave whose displacement y is expressed as

$$y = a\sin(\omega t - kx)$$

where

a = Amplitude

ω = Angular Frequency

$= 2\pi\nu$

K = Propagation constant

OR

Wave Vector

$$= \frac{2\pi}{\lambda}$$

For the plane of constant phase

$$\omega t - kx = \text{constant}$$

differentiate w.r.t t

$$\omega - k\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = \frac{\omega}{k}$$

$\boxed{V_p = \frac{\omega}{k}}$ known as phase velocity

Phase velocity (or wave velocity) of De-Broglie wave

A particle has two velocities

(i) Particle velocity (v)

(ii) Associated wave velocity (V_p)

we know that

$$V_p = \frac{\omega}{K} \quad \text{—} \ (1)$$

where

$$\omega = 2\pi\nu$$

$$\omega = 2\pi\frac{E}{h} \left\{ \because E = h\nu \right\}$$

$$\omega = \frac{2\pi mc^2}{h} \left\{ \because E = mc^2 \right\}$$

Also $K = \frac{2\pi}{\lambda} = \frac{2\pi}{h/mv} \left\{ \because \lambda = \frac{h}{mv} \right\}$

$$K = \frac{2\pi mv}{h}$$

Put ω and K in (1)

$$V_p = \frac{\cancel{2\pi}\cancel{m}c^2}{\cancel{h}} \times \frac{\cancel{h}}{\cancel{2\pi}\cancel{m}v}$$

$$\boxed{V_p = \frac{c^2}{v}}$$

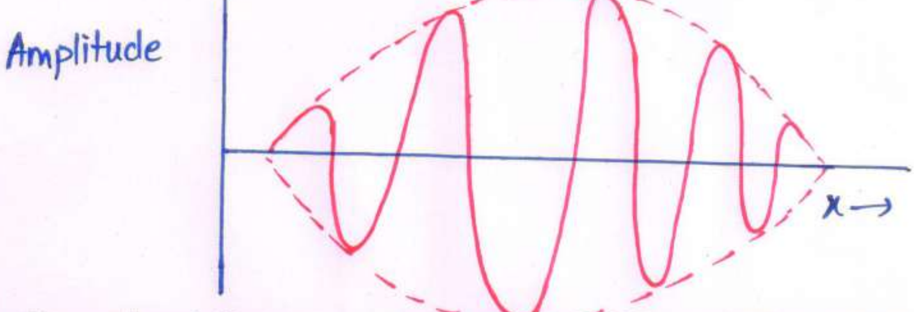
$V_p > c$ {violation of theory of Relativity}

But $v = V_g$

$$V_p = \frac{c^2}{V_g} \Rightarrow \boxed{V_p \times V_g = c^2}$$

Wave Packet

A wave packet is a group of waves with slightly different wavelength and velocity such that they interfere constructively over a small region of space where the particle can be located and outside this space they interfere destructively so that the amplitude reduces to zero.



Group velocity

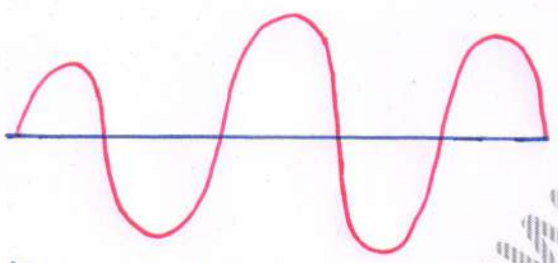
→ The velocity with which a wave packet (group of waves) travel in a medium is known as group velocity.

$$\boxed{V_g = \frac{d\omega}{dK}}$$

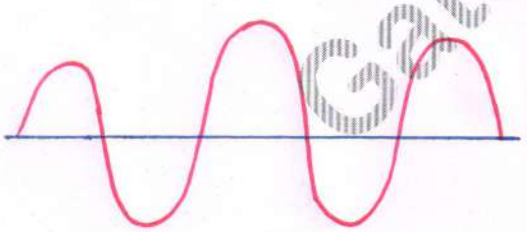
→ It is the characteristics of group of waves

Expression for Group Velocity

Let us consider two waves whose having same amplitude and different angular frequency or wavelength

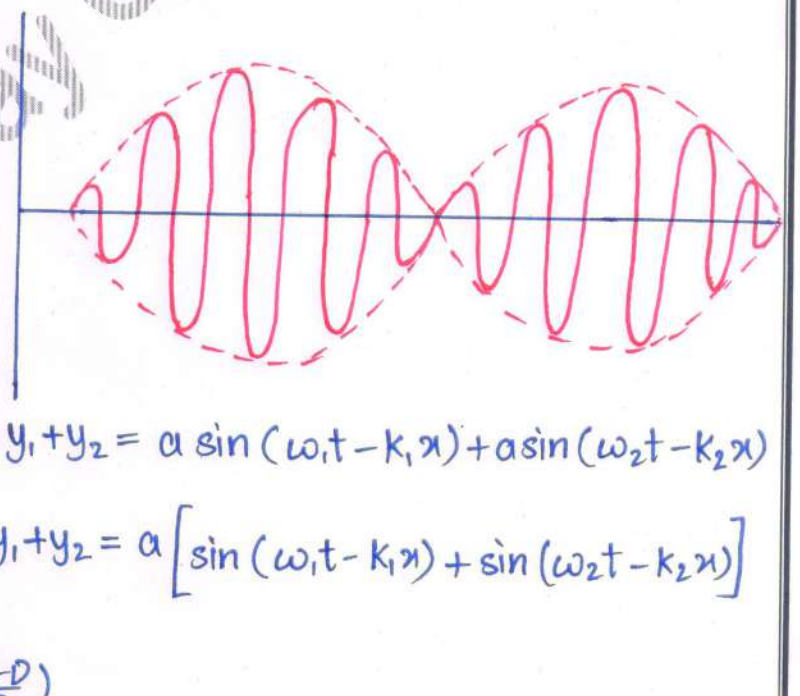


$y_1 = a\sin(\omega_1 t - K_1 x)$



$y_2 = a\sin(\omega_2 t - K_2 x)$

Apply principle of superposition



$$y_1 + y_2 = a\sin(\omega_1 t - K_1 x) + a\sin(\omega_2 t - K_2 x)$$

$$y_1 + y_2 = a\left[\sin(\omega_1 t - K_1 x) + \sin(\omega_2 t - K_2 x)\right]$$

$$\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$y_1 + y_2 = a\left[2\sin\left(\frac{\omega_1 t - K_1 x + \omega_2 t - K_2 x}{2}\right)\cos\left(\frac{\omega_1 t - K_1 x - \omega_2 t + K_2 x}{2}\right)\right]$$

$$y_1 + y_2 = 2a\left[\sin\left\{\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{K_1+K_2}{2}\right)x\right\} \cos\left\{\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{K_1-K_2}{2}\right)x\right\}\right]$$

$$y_1 + y_2 = y, \quad \frac{\omega_1+\omega_2}{2} = \omega, \quad \frac{K_1+K_2}{2} = K, \quad \frac{\omega_1-\omega_2}{2} = \frac{d\omega}{2}, \quad \frac{K_1-K_2}{2} = \frac{dK}{2}$$

$$y = 2a\sin(\omega t - Kx)\cos\left(\frac{d\omega}{2}t - \frac{dK}{2}x\right)$$

$$\boxed{y = 2a\cos\left(\frac{d\omega}{2}t - \frac{dK}{2}x\right)\sin(\omega t - Kx)}$$

Where

$$A = 2a\cos\left(\frac{d\omega}{2}t - \frac{dK}{2}x\right)$$

$$y = A\sin(\omega t - Kx)$$

Group Velocity (V_g)

$$V_g = \frac{\frac{d\omega}{2}}{\frac{dK}{2}} \Rightarrow \boxed{V_g = \frac{d\omega}{dK}}$$

Relation between Phase Velocity and Group Velocity

For dispersive medium

We know that

Group velocity (V_g)

$$V_g = \frac{d\omega}{dK} \quad \text{—(1)}$$

Phase velocity

$$V_p = \frac{\omega}{K}$$

$$\omega = K V_p$$

diff. w.r.t. K

$$\frac{d\omega}{dK} = K\frac{dV_p}{dK} + V_p$$

Put $\frac{d\omega}{dK}$ in (1)

$$V_g = V_p + K\frac{dV_p}{dK} \quad \text{—(3)}$$

We know that

$$K = \frac{2\pi}{\lambda}$$

differentiate w.r.t λ

$$\frac{dK}{d\lambda} = -\frac{2\pi}{\lambda^2}$$

$$dK = -\frac{2\pi}{\lambda^2} d\lambda$$

Put K and dK in (3)

$$V_g = V_p + \frac{\cancel{2\pi}}{\lambda} \times \frac{dV_p}{\left(\frac{-\cancel{2\pi}}{\lambda^{\cancel{2}}}\right) d\lambda}$$

$$\boxed{V_g = V_p - \frac{\lambda dV_p}{d\lambda}}$$

Required Relation

For non-dispersive medium

$$dV_p = 0$$

$$V_g = V_p = 0$$

$$\boxed{V_p = V_g}$$

DPP

Q.1 What do you mean by group velocity?

Q.2 Define wave packet with neat digram.

Q.3 Distinguish between group velocity and phase velocity. Establish a relation between them in a dispersive medium. What will be relation between these velocities in non-dispersive medium?

UNIT : Quantum Mechanics

Lecture-7

Today's Target

→ Schrodinger wave equation

(i) Time In-depen Schrodinger wave equation

(ii) Time Dependent Schrodinger wave equation

→ Born interpretation of wave function

Schrodinger wave equation

→ Wave function

The state of the particle at any instant is represented by a wave function(Ψ) psi

OR

The quantity whose variation builts up matter waves is called wave function (Ψ).

OR

The wave function (Ψ) describe the behaviour of wave associated with moving particle.

→ Schrodinger wave equations describe the wave motion of matter wave associated with particle and also known as Equation of Motion of Matter waves.

Gateway Classes

Engg. Physics (BAS-101/201) By Gulshan Sir

(i) Time In-dependent Schrodinger wave equation

(ii) Time Dependent schrodinger wave equation.

(i) Time dependent Schrodinger wave equation

The differential equation of a wave motion of a particle in one dimension can be written as

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{— (1)}$$

The general solution of (1)

$$\psi(x,t) = Ae^{-i\omega(t-\frac{x}{v})}$$

Put

$$\omega = 2\pi\nu$$

$$V = \nu\lambda$$

$$\psi(x,t) = Ae^{-i2\pi\nu(t-\frac{x}{\nu\lambda})}$$

$$\psi(x,t) = Ae^{-2\pi i(\nu t-\frac{x}{\lambda})}$$

Put $\nu = \frac{E}{h}$ $\{\because E = h\nu\}$

$$\lambda = \frac{h}{p}$$

$$\psi(x,t) = Ae^{-2\pi i\left(\frac{E}{h}t - \frac{px}{h}\right)}$$

$$\psi(x,t) = Ae^{\frac{-2\pi}{h}i(Et-px)}$$

$$\psi(x,t) = Ae^{\frac{-i}{h/2\pi}(Et-px)}$$

Put $\frac{h}{2\pi} = \hbar$

$$\psi(x,t) = Ae^{\frac{-i}{\hbar}(Et-px)} \quad \text{— (2)}$$

Diff. w.r.t. x

$$\frac{\partial\psi}{\partial x} = Ae^{\frac{-i}{\hbar}(Et-px)} \times \left(\frac{i}{\hbar}p\right)$$

Again diff. w.r.t. x

$$\frac{\partial^2\psi}{\partial x^2} = A\left(\frac{ip}{\hbar}\right)e^{\frac{-i}{\hbar}(Et-px)} \times \left(\frac{i}{\hbar}p\right)$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{-Ap^2}{\hbar^2}e^{\frac{-i}{\hbar}(Et-px)}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{-p^2}{\hbar^2} A e^{\frac{-i}{\hbar}(Et-px)}$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{-p^2\psi}{\hbar^2} \quad (\text{from } 2)$$

$$p^2 = -\frac{\hbar^2}{\psi}\frac{\partial^2\psi}{\partial x^2} \quad — (3)$$

diff. eqn (2) w.r.t. 't'

$$\frac{\partial\psi}{\partial t} = A e^{\frac{-i}{\hbar}(Et-px)} \times \left(\frac{-i}{\hbar}E\right)$$

$$\frac{\partial\psi}{\partial t} = \psi\left(\frac{-i}{\hbar}E\right) \quad \text{From } (2)$$

$$E = -\frac{\hbar}{\psi i}\frac{d\psi}{\partial t}$$

$$E = \frac{\hbar i}{\psi}\frac{\partial\psi}{\partial t} \quad — (4)$$

Apply conservation of Energy

$$E = K.E + PE$$

$$E = \frac{p^2}{2m} + V \quad — (5)$$

Put p^2 and E in (5)

$$\frac{\hbar i}{\psi}\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m\psi}\frac{\partial^2\psi}{\partial x^2} + V$$

Multiply both side by ψ

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

Required Equation

In 3-Dimension

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\psi = \left(i\hbar\frac{\partial}{\partial t}\right)\psi$$

where

$\frac{-\hbar^2}{2m}\nabla^2 + V \rightarrow$ Hamiltonian operator

$i\hbar\frac{\partial}{\partial t} \rightarrow$ Energy operator

(ii) Time In-dependent Schrodinger wave equation

The differential equation of a wave motion of a Particle in one dimension can be written as

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad -(1)$$

The General solution of eq^n (1)

$$\psi(x,t) = Ae^{-i\omega(t-\frac{x}{v})}$$

Put $\omega = 2\pi\nu$

$$v = \nu\lambda$$

$$\psi(x,t) = Ae^{-i2\pi\nu(t-\frac{x}{\nu\lambda})}$$

$$\psi(x,t) = Ae^{-2\pi i(\nu t-\frac{x}{\lambda})}$$

Put $\nu = \frac{E}{h}$ $\{\because E = h\nu\}$

$$\lambda = \frac{h}{p}$$

$$\psi(x,t) = Ae^{-2\pi i\left(\frac{Et}{h}-\frac{px}{h}\right)}$$

$$\psi(x,t) = Ae^{-\frac{2\pi}{h}i(Et-px)}$$

$$\psi(x,t) = Ae^{\frac{-i}{h/2\pi}(Et-px)}$$

Put $\frac{h}{2\pi} = \hbar$

$$\psi(x,t) = Ae^{\frac{-i}{\hbar}(Et-px)} \quad -(2)$$

Diff. w.r.t. x

$$\frac{\partial \psi}{\partial x} = Ae^{\frac{-i}{\hbar}(Et-px)} \times \frac{i}{\hbar}p$$

Again diff. w.r.t. x

$$\frac{\partial^2 \psi}{\partial x^2} = Ae^{\frac{-i}{\hbar}(Et-px)} \times \left(\frac{i}{\hbar}p\right)^2$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2}Ae^{\frac{-i}{\hbar}(Et-px)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-p^2}{\hbar^2}\psi \quad (\text{from eq}^n\ (2))$$

$$p^2 = -\frac{\hbar^2 \partial^2 \psi}{\psi\, \partial x^2} \quad -(3)$$

Apply Principle of conservation of energy

$$E = K.E + P.E$$

$$E = \frac{p^2}{2m} + V$$

Put p^2 in above equation

$$E = -\frac{\hbar^2}{2m\psi}\frac{\partial^2\psi}{\partial x^2} + V$$

$$E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + (E-V)\psi = 0$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

Required equation

In 3-D

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

For Free particle

$$V = 0$$

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}E\psi = 0}$$

Physical interpretation of wave function or Born interpretation of wave function.

→ The state of the particle at any instant is represented by a wave function

OR

The quantity whose variation builts up matter waves is called wave function.

→ The wave function is a complex function and the satisfactory interpretation of the wave function 'Ψ' was given by Max Born in 1926.

→ According to him the probability of finding a particle at any point in space (x,y,z) at a given time t is proportional to $|\psi|^2$, which is called probability density and 'ψ' is probability amplitude.

→ Since the particle is certainly somewhere in space, the total probability of finding the particle in space is unity, i.e;

$$\iiint |\psi|^2 dx\,dy\,z = 1$$

→ This is called normalization condition and 'ψ' is said to be normalized.

→ The value of energy of which schrodinger equation (time independent) can be solved are called 'eigen values' and the corresponding wave function are called "eigen function".

→ Characteristics of wave function.

(i) It must be normalized

(ii) It must be finite everywhere

(iii) It must be single - valued.

(iv) It must be continuous and its first derivative should also be continuous.

DPP

Q.1 Derive time-dependent schrodinger's wave equation.

Q.2 Derive time-independent schrodinger's wave equation.

Q.3 Explain the physical significance of wave function given by Max Born.

UNIT: Quantum Mechanics

Lecture-8

Today's Target

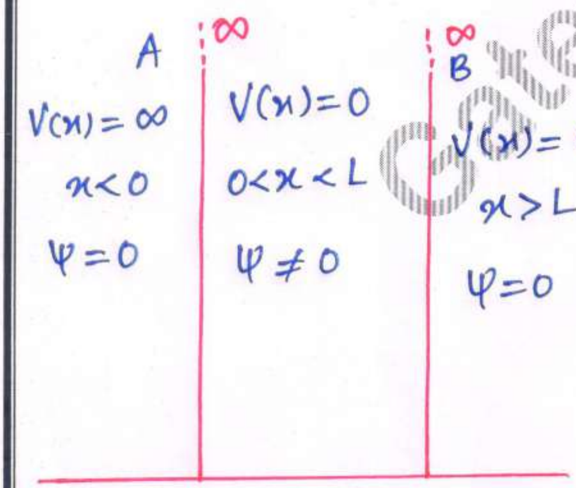
→ Particle in one dimensional box

Particle in one dimensional box

(Application of schrodinger wave equation)

Let us consider a particle of mass 'm' moving along x-axis between two rigid walls A and B at $x=0$ and $x=L$.

The particle is free to move between walls.



Potential Function

$$V(x) = \begin{cases} 0 & 0<x<L \\ \infty & x<0 \text{ and } x>L \end{cases}$$

using schrodinger time independent wave equation for one-dimension

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

OR

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

For Free particle ($V=0$)

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

Put $\frac{2mE}{\hbar^2} = k^2$ — (1)

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{— (2)}$$

General solution of eqn (2)

$$\psi = A\sin kx + B\cos kx \quad \text{— (3)}$$

Apply First Boundary condition

At $x=0$, $\psi=0$

$0 = B + 0$

$B = 0$

Put $B=0$ in (3)

$$\boxed{\psi = A\sin kx} \quad \text{— (4)}$$

Eigen Functions

Apply second Boundary condition

At $x=L$, $\psi=0$

$0 = A\sin KL + 0$

$A\sin KL = 0$

$\because A \neq 0$

$\therefore \sin KL = 0$

$\Rightarrow KL = \pm n\pi$

$$\boxed{K = \pm \frac{n\pi}{L}} \quad \text{— (5)}$$

Put k in (1)

$$\frac{2mE}{\hbar^2} = \left(\frac{n\pi}{L}\right)^2 \Rightarrow \frac{2mE}{\left(\frac{h}{2\pi}\right)^2} = \frac{n^2\pi^2}{L^2}$$

$$\frac{2mE \times 4\cancel{\pi^2}}{h^2} = \frac{n^2\cancel{\pi^2}}{L^2}$$

$$E_n = \frac{n^2h^2}{8mL^2}$$

where, $n = 1, 2, 3, \ldots$

Hence

Energy of the particle is in discrete form

$$E_1 = \frac{h^2}{8mL^2}$$

$$E_2 = \frac{4h^2}{8mL^2}$$

$$E_3 = \frac{9h^2}{8mL^2}$$

and so on

Here E_1, E_2, E_3 ----

are known as eigen values

Put K in (4)

$$\psi = A \sin \frac{n\pi x}{L} \quad \text{— (6)}$$

(Eigen Function)

Using Normalisation

$$\int_0^L |\psi|^2 dx = 1$$

$$\int_0^L A^2 \sin^2 \frac{n\pi x}{L} dx = 1$$

$$\frac{A^2}{2} \int_0^L \left(1 - \cos \frac{2n\pi x}{L}\right) dx = 1 \quad \left\{ \sin^2\theta = \frac{1-\cos 2\theta}{2} \right\}$$

$$\frac{A^2}{2} \left(x - \frac{\sin \frac{2n\pi x}{L}}{\frac{2n\pi}{L}} \right)_0^L = 1$$

$$\frac{A^2}{2} \left[L - 0 - (0 - 0) \right] = 1 \Rightarrow \frac{A^2 L}{2} = 1$$

$$A^2 L = 2 \Rightarrow A^2 = \frac{2}{L} \Rightarrow A = \sqrt{\frac{2}{L}}$$

Put A in (6)

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Normalised wave function OR Eigen function

Graph of energy Level, Wave function (ψ) and probability density ($|\psi|^2$).

We know that

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$E_1 = \frac{h^2}{8mL^2}$$

$$E_2 = \frac{4h^2}{8mL^2} = 4E_1$$

$$E_3 = \frac{9h^2}{8mL^2} = 9E_1$$

$$E_4 = \frac{16h^2}{8mL^2} = 16E_1$$

$$E_5 = \frac{25h^2}{8mL^2} = 25E_1$$

$$E_2 - E_1 = 3E_1$$

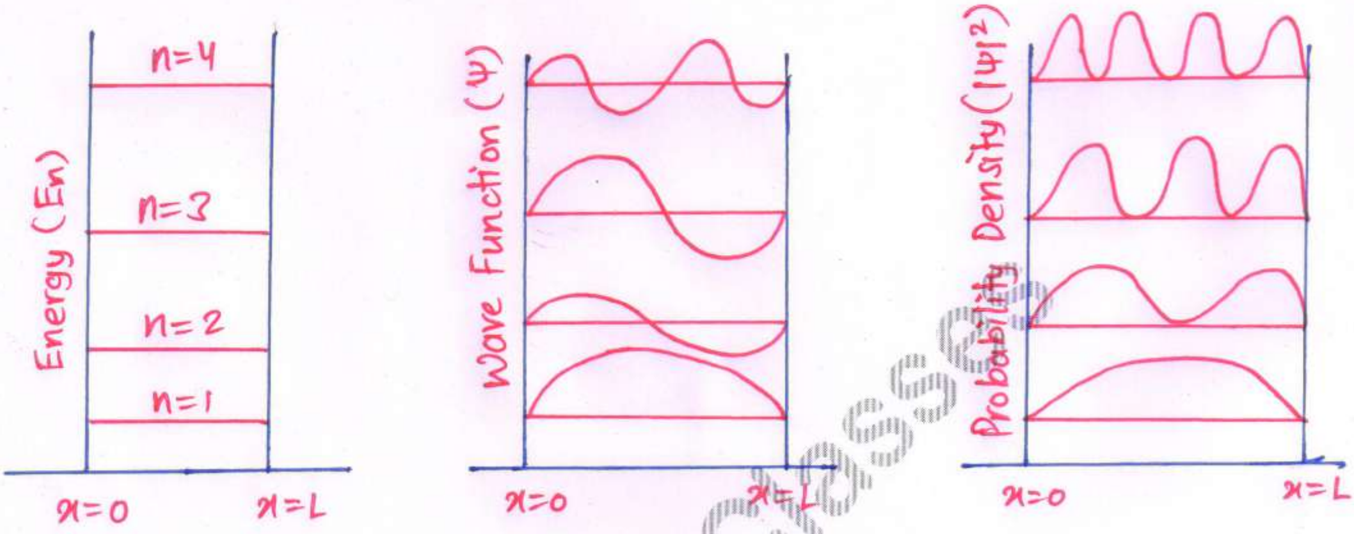
$$E_3 - E_2 = 5E_1$$

$$E_4 - E_3 = 7E_1$$

$$E_5 - E_4 = 9E_1$$

and so on

It is clear that the difference between energy of consecutive state is not constant. Therefore it is concluded that the energy level are not equally spaced.



$$\boxed{\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}$$

Q.1 An electron is bound in one dimensional potential box which has width 2.5×10^{-10} m. Assuming the height of the box to be infinite, calculate the lowest two permitted energy values of the electron.

Given

$L = 2.5 \times 10^{-10}$ m

$m = 9.1 \times 10^{-31}$ kg

$h = 6.63 \times 10^{-34}$ J-sec

we know that

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$E_n = \frac{n^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$E_n = \frac{0.0966 \times 10^{-17} n^2 \text{ J}}{1.6 \times 10^{-19}}$$

$$E_n = 6.04 n^2 \text{ eV}$$

$E_1 = 6.04$ eV

$E_2 = 24.15$ eV

Q.2 What is the lowest energy that a neutron can have if ~~confirmed~~ confined to move along the edge of an impenetrable box of length 10^{-14} m.

Given $L = 10^{-14}$ m

$m = 1.67 \times 10^{-27}$ kg

$h = 6.63 \times 10^{-34}$ J-sec

we know that

$$E_n = \frac{n^2h^2}{8mL^2}$$

For Lowest energy

$n = 1$

$$E_1 = \frac{(1)^2 \times (6.63 \times 10^{-34})^2}{8 \times 1.67 \times 10^{-27} \times (10^{-14})^2}$$

$$E_1 = \frac{3.28 \times 10^{-13}\ \text{J}}{1.6 \times 10^{-19}} \Rightarrow \boxed{E_1 = 2.05\ \text{MeV}}$$

DPP

Q.1 Write down the time independent schrodinger wave equation for a particle in one-dimensional box (infinitely deep potential well) and find out energy eigen values (energy wall) and corresponding energy eigen functions (normalized wave functions) of the particle.

Q.2 Find the two lowest permissible energy states for an electron which is confined in a one dimensional ~~fi~~ infinite potential box width 3.5×10^{-9} m.

Q.3 An electron is trapped in one dimensional region of length 1Å. Find the amount of energy that must be supplied to excite the electron from ground ~~staff~~ state to first excited state.

Q.4 Calculate the energy difference between the ground state and the first excited state for an electron in a one-dimensional rigid box of length 10^{-8} cm.

Thank You!