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UNIT-4

EIGON VALUE AND EIGON VECTOR

$|A-\lambda I|=0$

$A=\cancel{\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}}$ $A=\begin{bmatrix} & \end{bmatrix}_{3\times3}$ $\lambda I=\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$|A-\lambda I| = \lambda^3 + (\ )\lambda^2 + (\ )\lambda + D = 0$

$\lambda = \lambda_1, \lambda_2, \lambda_3$ } Eigon Value

PROP

(1) $\lambda_1+\lambda_2+\lambda_3$ = Trance of Matrix [Sum of Diagonal Elements]

(2) $\lambda_1\lambda_2\lambda_3 = |A|$

(3) If, $\lambda_1, \lambda_2, \dots, \lambda_n$ are ideal values of A then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots \frac{1}{\lambda_n}$ will be ideal values of A^{-1}.

(4) $\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}, \frac{|A|}{\lambda_3}, \dots \frac{|A|}{\lambda_n}$ ideal values of adjoint (A).

(5) $\lambda_1^K, \lambda_2^K, \lambda_3^K, \dots \lambda_n^K$ ideal values of A^K.

(6) $K\lambda_1, K\lambda_2, K\lambda_n$, ideal valof K.A

(7) Eigon values of triangular matrix are its diagonal elements.

$\begin{bmatrix}5 & 7 & 9\\ 0 & -1 & 2\\ 0 & 0 & 7\end{bmatrix}$ $\lambda_1=5$, $\lambda_2=-1$, $\lambda_3=7$

EIGON VALUE

Eigon Vector: $AX = \lambda X$

$$[A - \lambda I]X = 0$$

$$\underbrace{\begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{bmatrix}}_{A-\lambda I} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

When, $\lambda_1, \lambda_2, \lambda_3$ then alg. multi $= 1$ & geo. multi $= 1$

Q. when 2 Eigon values are same the alg. mult. $= 2$ + geo multi $= 2$

When am = gm then we can diagonalise a matrix

Q

$$M = \underbrace{\begin{bmatrix} | & | & | \\ | & | & | \\ | & | & | \end{bmatrix}}_{\text{Eigon Vectors}} \qquad MAM^{-1} = D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

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Q. $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ Eigen values $= 6A^{-1} + A^{3} + 2I$

$\left\{ \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right\}$

$\begin{bmatrix} 2-\lambda & 4 \\ 0 & 3-\lambda \end{bmatrix}$

$(2-\lambda)(3-\lambda)$

$6 - 2\lambda - 3\lambda + \lambda^2 = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda^2 - 3\lambda - 2\lambda + 6$

Since, A is triangular in
Eigen values of A are
$\lambda = 2, 3$

Eigen value of $6A^{-1} + A^{3} + 2I$

$\lambda = 2$ $6 \times \frac{1}{2} + 2^3 + 2$

$= 3 + 8 + 2 = 13$

$\lambda = 3$ $= 6 \times \frac{1}{3} + 3^3 + 2$

$= 2 + 27 + 3$

$= 31$

Q. $A = \begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix}$ $A^2 + 2A + I$

$\begin{bmatrix} 1-\lambda & 8 \\ 2 & 1-\lambda \end{bmatrix}$

$(1-\lambda)(1-\lambda) - 16 = 0$

$1 - \lambda - \lambda + \lambda^2 - 16 = 0$

$\lambda^2 - 2\lambda - 15 = 0$

$\lambda^2 - 5\lambda + 3\lambda - 15 = 0$

$\lambda(\lambda - 5) + 3(\lambda - 5)$

$\lambda = 5, -3$

$\lambda = 5$

$25 + 10 + 1 = 36$

$\lambda = -3$

$9 - 6 + 1 = 4$

Q. $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ $A^3 - 5A^2 + 9A - I$

The characteristic matrix

$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 & -2 \\ -1 & 3-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{bmatrix}$

$= \lambda^3 - \lambda^2$ (sum of diagonal elements) $+ \lambda$ { sum of minors of diagonal element) $- |A| = 0$

$= \lambda^3 - \lambda^2(1+3+1) + \lambda[3+0+1+0+3+2] - 1 = 0$

$= \lambda^3 - 5\lambda^2 + 9\lambda - 1 = 0$

$\lambda = 0.12, \quad 2.44 \pm 1.56$

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Q. $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

$\lambda^3 - \lambda^2(6) + \lambda(4+5+2) - 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda = 1, 3, 2$

Q Find Eigen Value & Eigen Vector

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$\lambda^3 - 18\lambda^2 + \lambda(21-16+24-4+56-36)\lambda = 0$

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\lambda = 0, 15, 3$

To find Eigen Vector $AX = \lambda X$

$(A - \lambda I)X = 0$

A

$8x - 6y + 2z = 0$

$\Rightarrow 4x - 3y + z = 0$ (1)

$2x - 4y + 3z = 0$ (2)

Cramer's Rule

$$\frac{x}{+4+9} = \frac{-y}{12-2} = \frac{z}{-16+6}$$

$$X_1 = \begin{bmatrix} 5 \\ 10 \\ 10 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$\lambda = 3$

$$\left[\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-5x + -6y + 2z = 0$ (1)

$-6x + 4y - 4z = 0$

$3x - 2y + 2z = 0$ (2)

$$\frac{x}{-12+4} = \frac{-y}{10-6} = \frac{z}{10-8} \quad \frac{z}{10-18}$$

$$\begin{bmatrix} 8 \\ 4 \\ -8 \end{bmatrix}$$

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$\lambda = 15$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix}$$

$7x + 6y - 2z = 0$
$3x + 4y + 2z = 0$

$$\frac{x}{12+8} = \frac{-y}{14+6} = \frac{z}{28-18}$$

$$\begin{bmatrix} 20 \\ -20 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Q. $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\lambda^3 - 7\lambda^2 + \lambda($

$\lambda^3 - 7\lambda^2 + 11\lambda$

$\lambda = 1, 1, 5$

$\lambda = 1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$x + 2y + 3z = 0$
$x + 2y + z = 0$

$\lambda = 5$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}$$

$3x - 2y + z = 0$
$x + 2y - 3z = 0$

$$\frac{x}{6+2} = \frac{-y}{-3-1}$$

$$x = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

Q. $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\lambda^3 - \cancel{\lambda^2}(7\lambda^2 + \lambda(6-2) + \cancel{2} - 1) + 6 - \cancel{2}) + 5 = 0$

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\lambda = 1, 1, 5$

$\lambda = 1$

$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$x + 2y + 3z = 0$

$x + 2y + z = 0$

$\frac{x}{2-2} = \frac{y}{1-1} = \frac{z}{2-2}$

$\lambda = 5$

$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}$

$3x - 2y + z = 0$

$x + 2y - 3z = 0$

$\frac{x}{\cancel{2}+2} = \frac{-y}{-9-1} = \frac{z}{6-2}$

$x = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$

For Equal Eigen & All Eqn same

$\lambda = 1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x + 2y + z = 0$$

Let, $z = K_1$; $x = K_2$

$$K_2 + 2y + K_1 = 0$$

$$y = \frac{-(K_1 + K_2)}{2}$$

Let, $z = 0$ $x = -2y$

$$\frac{x}{-2} = \frac{y}{1}$$

$$X = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

Let $y = 0$ & $x = -z$

$$\frac{x}{-1} = \frac{z}{1}$$

$$X = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Q. $A = \begin{bmatrix} 8 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

$\lambda^3 - 7\lambda^2 + \lambda(-21 + 20 + 21 - 15 - 9 + 20) - 12 = 0$

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

$\lambda = 3, 2, 2$

$\lambda = 3 \quad \begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix}$

$0x + 10y + 5z = 0$
$x + 3y + 2z = 0$

$\frac{x}{20-15} = \frac{-y}{-5} = \frac{z}{-10}$

$X = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

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$\lambda = 2$ AM = 2 GM = 1

$$\begin{bmatrix} 1 & 10 & 5 \\ -2 & -5 & -4 \\ 3 & 5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + 10y + 5z = 0$
$2x + 5y + 4z = 0$

$$\frac{x}{40-25} = \frac{-y}{4-10} = \frac{z}{5-20}$$

$$\begin{bmatrix} 15 \\ 6 \\ -15 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix}$$

Q For symmetric Matrix the Eigen Vectors orthogonal

A =

X_1, X_2, X_3

$X_1 X_1' = 0$
$X_2 X_3' = 0$
$X_3 X_3' = 0$

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Q. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\lambda^3 - 12\lambda^2 + \lambda(9 - 1 + 18 - 4 + 18 - 4) - 32 = 0$

$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$

$\lambda = 8, 2, 2$

$\lambda = 8$

$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}$

$x + y - z = 0$

$\cancel{-2x + 5y + z = 0}$ $\quad 2x + 5y + z = 0$

$\dfrac{x}{1+5} = \dfrac{-y}{1+2} = \dfrac{z}{5-2}$

$\begin{bmatrix} 6 \\ -3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

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$\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$2x - y + z = 0$

$2x - y + z = 0$

$\frac{x}{-1+1} = \frac{-y}{2-2} = \frac{z}{}$

$y = 0 \quad 2x = \pm z$

$$X_2 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \qquad X_3 = \begin{bmatrix} 1 & \\ -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$X_1 X_1' = 0$

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} = 0$$

$4 + 1 + 1 = 0$

$X_2 X_1'$

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \end{bmatrix}$$

$4 + 1 + 1 = 0$

$$X_1 X_3' = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$2 \quad -$

For Equal Eigen value compare diff

$$X_2 \quad X_2' = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

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Q. $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$\lambda^3 - 3\lambda^2 + \lambda(1-4) +$

$\lambda^3 - 3\lambda^2 - 9\lambda - 5 =$

$\lambda = 5, -1, -1$

$$\begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$X_1 X_3' = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$

$2 \quad -2 \quad = \boxed{0}$

For Equal Eigen value
compare diff Eigen vector & others complex

$X_2 X_1' = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$

1

Q. $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$\lambda^3 - 3\lambda^2 + \lambda(1-4+1-4+1-4) -$

$\lambda^3 - 3\lambda^2 + -9\lambda - 5 = 0$

$\lambda = 5, -1, -1$

$\begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix}$

$-2x + y + z = 0$

$x - 2y + z = 0$

$\frac{x}{1+2} = \frac{-y}{-2-1} = \frac{z}{4-1}$

$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

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$\lambda = -1$

$$\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$x + y + z = 0$

$\cancel{x} = 0$

$y = -\cancel{z}$

$\frac{x}{0} = \frac{-y}{1} = \frac{z}{0}$

$$X_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$y = 0$

$x = -z$

$\frac{x}{-1} = \frac{z}{1}$

$$X_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$X_1 X_2' = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [1\ 0\ 0]$$

$= 2\ 0$

$$X_1 X_3' = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [1\ 0\ -1]$$

$= 1 - 1$

$= 0$

$$X_2 X_2' \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} [0\ 1\ 1] = 1$$

$$X_1 X' = \begin{bmatrix} 0 \\ 1 \end{bmatrix} [0\ 1\ 1] = 2$$

$\therefore$ Orthogonal

Q. $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\cancel{\lambda^2} - \quad |A| = 1$

$$A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix}$$

$\lambda^2 + 1 = 0$

$\boxed{\lambda = \pm i}$

$\lambda = i$

$[A - \lambda I] X_1 = 0$

$$\begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$-ix + y = 0$

$x + iy = 0$

$x + iy = 0$

$x = -iy$

$\frac{x}{-i} = \frac{y}{1}$

$$\begin{bmatrix} -i \\ 1 \end{bmatrix}$$

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Q. $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$|A| = 1$

$A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix}$

$\lambda^2 + 1 = 0$

$\boxed{\lambda = \pm i}$

$\lambda = i$

$[A - \lambda I] X_1 = 0$

$\begin{bmatrix} -i & 1 \\ -1 & -i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$

$\left.\begin{array}{l} -ix + y = 0 \\ x + iy = 0 \end{array}\right\}$ Both are same

$x + iy = 0$ $\qquad$ $-ix + y = 0$

$x = -iy$ $\qquad$ $ix = y$

$\frac{x}{-i} = \frac{y}{1}$ $\qquad$ $\frac{x}{1} = \frac{y}{i}$

$\begin{bmatrix} -i \\ 1 \end{bmatrix}$ $\qquad$ $X_1 = \begin{bmatrix} 1 \\ i \end{bmatrix}$

$\lambda = -i$

$$\begin{bmatrix} i & 1 \\ -1 & i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$ix + y = 0$

$-x + iy = 0$

$ix + y = 0$

$ix = -y$

$\frac{x}{-1} = \frac{y}{i}$

$$X_2 = \begin{bmatrix} -1 \\ i \end{bmatrix}$$

Q $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

$$\begin{bmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{bmatrix}$$

$4 - 2\lambda - 2\lambda + \lambda^2 + 1 = 0$

$\lambda^2 - 4\lambda + 5 = 0$

$\lambda = 2 \pm i$

$\lambda = 2 + i$

$\lambda = 2 - i$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$x + iy = 0$$

$$x = -iy$$

$$X_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$\cancel{X_1 X_2' \quad \begin{bmatrix} i \\ 1 \end{bmatrix}\begin{bmatrix} -i & 1 \end{bmatrix}}$$

$$\cancel{-1 + 1 = 0}$$

~~$\therefore$ Orthogonal~~

Diagonalisation of a Matrix

Cond. for diag $\rightarrow$ am = gm

$$A = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$\lambda = \lambda_1, \lambda_2, \lambda_3$$
$$X_1, X_2, X_3$$

Modal Matrix

$$\begin{array}{ccc} \begin{bmatrix} \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix} \\ \uparrow\ X_1 \quad \uparrow\ X_2 \quad \uparrow\ X_3 \end{array}$$

$$M^{-1}AM = D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Q. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$\lambda^3 - 12\lambda^2 + (18+14+4)\lambda - 32 = 0$$
$$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$\underline{\lambda = 8, 2, 2}$$

$\lambda = 8$

$$\begin{bmatrix} -2 & -2 & \\ -2 & -5 & \\ 2 & -1 & \end{bmatrix}$$

$x + y - z = 0$
$2x + 5y + z$

$\frac{x}{1+5} = \frac{-y}{1+}$

$$X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$2x - y + z = 0$

$y = 0$
$2x = z$
$\frac{x}{-1} =$

$$\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$\lambda = 8$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$x + y - z = 0$
$2x + 5y + z = 0$

$$\frac{x}{1+5} = \frac{-y}{1+2} = \frac{z}{5-2}$$

$$X_1 :- \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$2x - y + z = 0$

$y = 0$
$2x = z$
$$\frac{x}{-1} = \frac{z}{2}$$

$z = 0$
$2x = y$
$$\frac{x}{1} = \frac{y}{2}$$

$$\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

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$$M = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Q. $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ Is not similar to diagonal Matrix

$\lambda = 2, 2, 1$ [∵ Triangular Matrix]

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$0x + 3y + 4z = 0$ $\qquad z = 0$

$\cancel{3y = -4z}$ $\qquad 0x + 0y - z = 0$

$\cancel{\frac{y}{-4} = \frac{z}{3}}$ $\qquad \frac{x}{-3} = \frac{-y}{0} = \frac{z}{0}$

$$\cancel{\begin{bmatrix} 0 \\ -4 \\ 3 \end{bmatrix}} \qquad \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$$

A

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$\lambda = 1$

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix};$$

$x + 3y + 4z = 0$

$0x + y - z = 0$

$$\frac{x}{-3-4} = \frac{-y}{-1} = \frac{z}{1}$$

$$X_1 = \begin{bmatrix} -7 \\ 1 \\ 1 \end{bmatrix}$$

For $\lambda = 2$, $am = 2$, $gm = 1$

$\because am \neq gm$

Matrix is not diagonalisable

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Q.

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

$$\lambda^3 - 2\lambda^2 + \lambda[-12+12+-3+4] = 0$$

$$\lambda^3 - 2\lambda^2 + \lambda = 0$$

$$\lambda = 1, 1, 0$$

$\lambda = 1$

$$\begin{bmatrix} 0 & -6 & -4 \\ 0 & 3 & 2 \\ 0 & -6 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$$0x + 3y + 2z = 0$$

$x = 0$

$3y = -2z$

$$\frac{y}{-2} = \frac{z}{3}$$

$y = 0$

$0x = -2z$

$$\frac{x}{-2} = \frac{z}{0}$$

$$\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = 0$

$$\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x - 6y - 4z = 0$$

$$0x + 2z + z = 0$$

$$\frac{x}{-6+8} = \frac{-y}{1} = \frac{z}{2}$$

$$\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & -2 & 2 \\ -2 & +20 & -1 \\ 3 & 30 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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When, $\lambda = 1$

$0x + 6y + 2z = 0$

Let, $x = k_1,\ y = k_2$

$z = -\frac{3}{2} k_2$

$$\begin{bmatrix} k_1 \\ k_2 \\ -\frac{3}{2}k_2 \end{bmatrix}$$

Put, $k_1 = 1,\ k_2 = -2$

$$\begin{bmatrix} 1 \\ -2 \\ +3 \end{bmatrix}$$

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Q. $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$

$\lambda^3 - \lambda^2 + \lambda[21 - 32 - 63 + 64 - 27 + 32] - 3 = 0$

$\lambda^3 - \lambda^2 - 5\lambda - 3 = 0$

$\lambda = 3, -1, -1$

$\lambda = 3$ $\begin{bmatrix} -12 & 4 & 4 \\ -8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$-3x + y + z = 0$

$-2x + 0y + z = 0$

$\frac{x}{1} = \frac{-y}{-3+2} = \frac{z}{2}$

$X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

[redacted]

$\lambda = -3$

$$= \begin{bmatrix} -8 & 4 & 4 \\ -8 & 4 & 4 \\ -16 & 8 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$-2x + y + z =$

$x = 0$

$y = -z$

$\frac{y}{-1} = \frac{z}{1}$

$$\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$y = 0$

$2x = z$

$\frac{x}{1} = \frac{z}{2}$

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$M \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix} \quad D. \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

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Cayley Hamilton Theorem

Every square Matrix satisfies its own characteristic eqn.

$$\lambda^3 - (\,)\lambda^2 + (\,)\lambda + (\,) = 0$$

Replace by λ by Matrix itself

$$A^3 - (\,)A^2 + (\,)A + (\,)I = 0$$

Q. Find characteristic eqn & verify the theorem

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 0 & -3 \\ -2 & 1 & 0 \end{bmatrix}$$

$$\lambda^3 - \lambda^2 + \lambda[\cancel{+3} + 6 \cancel{+} - 1] - 12 = 0$$

$$\lambda^2 - \lambda^2 + 8\lambda - 12 = 0$$

By CaHThe

$$A^3 - A^2 + 8A - 12I = 0$$

$$\begin{bmatrix} 0 & -4 & -24 \\ -1 & 10 & 27 \\ 15 & -10 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -4 & 0 \\ -7 & 2 & -3 \\ 1 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 8 & 1 & 24 \\ 8 & 0 & -24 \\ -16 & 8 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} -12 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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find A^{-1}

$\cancel{A^2 \ge A}$ $\quad A^2 - A + 8I - 12A^{-1} =$

$$A^{-1} = \frac{1}{12}\left[A^2 - A + 8I\right]$$

$$= \frac{1}{12}\begin{bmatrix} 4 & -4 & 0 \\ -7 & 2 & -3 \\ 1 & 2 & 9 \end{bmatrix} - \begin{bmatrix} -1 & -1 & -8 \\ -1 & 0 & 3 \\ 2 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= \frac{1}{12}\begin{bmatrix} 13 & -5 & -8 \\ -8 & 10 & 0 \\ 3 & 1 & 17 \end{bmatrix}$$

Q. $\cancel{A = \begin{bmatrix} \end{bmatrix}}$ $\quad A^5 - 4A^4 - 7A^3 + 11A^2 - 8A - 10I = 0$

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$|A - \lambda I| = (1-\lambda)(3-\lambda) - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

$$\cancel{A \Rightarrow}$$

$$A^2 - 4A - 5I = 0$$

$$A^5 - 5A^4 - 7A^3 + 11A^2 - A - 10I = 0$$

$$A^3(A^2 - 4A - 5I) - 2A^3 + 11A^2 - A - 10I =$$

$$-2A^3 + 11A^2 - A - 10I =$$

$$-2A\left[A^2 - 4A - 5I\right] + 3A^2 - 11A - 10I$$

$$3A^2 - 11A - 10I$$

$$3\left[A^2 - 4A - 5I\right] + A + 5I$$

$$A + 5I =$$

$$\begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}.$$

Q2 $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$|A-\lambda I| = 0 \quad \begin{vmatrix} 1-\lambda & 2 \\ 2 & -1-\lambda \end{vmatrix} = 0$

$-1 - \lambda + \lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 5 = 0$

$A^2 - 5I = 0$

Q7. $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\lambda^3 - 6\lambda^2 + \lambda[4-1+4-1+4-1] - 4 = 0$

$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

$\lambda = A$ apply

Q. $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$\lambda^3 - 5\lambda^2 + \lambda[2 + 4 - 1 + 2] - 3 = 0$

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

$A^3 - 5A^2 + 7A - 3I = 0$

$A^8 + 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$A^5[A^3 - 5A^2 + 7A - 3I] + \cancel{10A^7} + A^4 - 5A^3 + 8A^2 - 2A + I$

$\cancel{10A^7} + A^4 - 5A^3 + 8A^2 - 2A + I$

$\cancel{A^4[10A^3 + I\,A^3} - 5A^2$

$\cancel{10A^2} + A[A^3 - 5A^2 + 7A - 3I] + A^2 + A + I$

$\cancel{10A^2} + A^2 + A + I$

$\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$

$$\begin{array}{r} -A^5 \\ A^3 - 5A^2 + 7A - 3I \,\big)\, A^8 - 5A^7 + 7A^6 - 3 \\ A^8 - 5A^7 + 7A^6 - 3 \end{array}$$

Q. $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$\lambda^3 - 4\lambda^2 + \lambda[2 - 6 + 1 - 7 + 2 -$

$\lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0$

$A^3 - 4A^2 - 20A - 35I = 0$

$(A^7 - 4A^6 - 20A^5 - 34A^4) + 4A^3 - 20A^2 - 3$

$A^4[A^3 - 4A^2 - 20A - 35I] +$

$A^4 - 4A^3 - 20A^2 - 33A + I$

$A[A^3 - 4A^2 - 20A - 35I] + 2A + I$

$2A + I$

$\begin{bmatrix} 3 & 6 & 14 \\ 8 & 5 & 6 \\ 2 & 4 & 3 \end{bmatrix}$

$$A^3 - 5A^2 + 7A - 35 \overline{\smash{)}\, A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I}$$

$-A^5$

$A^8 - 5A^7 + 7A^6 - 3A^5$

Q. $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

$A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$

$\lambda^3 - 4\lambda^2 + \lambda[2-6+1-7+2-12] - 35 = 0$

$\lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0$

$A^3 - 4A^2 - 20A - 35I = 0$

$A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$

$A^4[A^3 - 4A^2 - 20A - 35I] + A^4 - 4A^3 - 20A^2 - 33A + I$

$A^4 - 4A^3 - 20A^2 - 33A + I$

$A[A^3 - 4A^2 - 20A - 35I] + 2A + I$

$2A + I$

$\begin{bmatrix} 3 & 6 & 14 \\ 5 & 5 & 6 \\ 2 & 4 & 3 \end{bmatrix}$

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Quadratic Forms

$Q(x) = X'AX$
$\quad\quad\quad\downarrow$
symm. Mat.

$\cancel{2x_1^2 + 3x_2^2 - 2x_1x_2}$

$= [x_1 \ \ x_2] \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\begin{bmatrix} 2x_1 & -x_1 \\ -x_2 & 3x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$2x_1^2 - x_1x_2 - x_1x_2 + 3x_2^2$

Eg.

$Q(x) = 5x_1^2 - 7x_2^2 + x_3^2 - 5x_1x_2 + 8x_2x_3 - 4x_1x_3$

$A = \begin{bmatrix} 5 & -5/2 & -2 \\ -5/2 & -7 & 4 \\ -2 & 4 & 3 \end{bmatrix}$

Q $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3 + 5x_2x_3$

$\begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 5/2 \\ 4 & 5/2 & -7 \end{bmatrix}$

Q2. $x_1^2 - 2x_2^2 + 4x_3^3 - 4x_4^2 - 2x_1x_2$

$\begin{bmatrix} 1 & -1 & 3/2\ 0 & 3/2 \\ -1 & -2 & 2 & 0 \\ 0 & 2 & 4 & -5/2 \\ 3/2 & 0 & -5/2 & 4 \end{bmatrix}$

Q3.

$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

$\begin{bmatrix} x_1 + 2x_2 + 5x_3 \\ 2x_1 + 3x_3 \\ 5x_1 + 3x_2 + 4x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\cancel{x_1^2 + 2x_2^2 + 5x_3^2 + 2x_1^2}$

$x_1^2 + 4x_3^2 + 4x_1x_2 +$

$x_1^2 - 2x_2^2 + 4x_3^2 - 4x_4^2 - 2x_1x_2 + 3x_1$

$\begin{bmatrix} 1 & -1 & 0 & 3/2 \\ -1 & -2 & 2 & 0 \\ 0 & 2 & 4 & -5/2 \\ 3/2 & 0 & -5/2 & -4 \end{bmatrix}$

Q2. $x_1^2 - 2x_2^2 + 4x_3^3 - 4x_4^2 - 2x_1x_2 + 3x_1x_4 + 4x_2x_3 - 5x_3x_4$

$$\begin{bmatrix} 1 & -1 & 3/2\ 0 & 3/2 \\ -1 & -2 & 2 & 0 \\ 0 & 2 & 4 & -5/2 \\ 3/2 & 0 & -5/2 & -4 \end{bmatrix}$$

Q3. $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

$$\begin{bmatrix} x_1 + 2x_2 + 5x_3 \\ 2x_1 + 3x_3 \\ 5x_1 + 3x_2 + 4x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\cancel{x_1^2 + 2x_2^2 + 5x_3^2 + 2x_1^2}$

$x_1^2 + 4x_3^2 + 4x_1x_2 + 10x_1x_3 + 6x_2x_3$

$x_1^2 - 2x_2^2 + 4x_3^2 - 4x_4^2 - 2x_1x_2 + 3x_1x_4 + 4x_2x_3 - 5x_3x_4$

$$\begin{bmatrix} 1 & -1 & 0 & 3/2 \\ -1 & -2 & 2 & 0 \\ 0 & 2 & 4 & -5/2 \\ 3/2 & 0 & -5/2 & -4 \end{bmatrix}$$

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Linear Transformation

Consider 2 set of variables $x_1, x_2 \dots x_n$ & $(y_1, y_2, \dots y_n)$
Let these variables are related by following eqn.

$y_1 = a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n$

$y_2 = a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n$

$\vdots$

$y_n = a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n$

These above n eqn can be expressed as a single matrix expression eqn.

$$\boxed{Y = AX}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & \dots \\ a_{21} & a_{22} & \dots \\ \dots & & a_{nn} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The transformation from y to x by above eqn is called linear transformation.
And linear transformation depends singular or non-singular that depends on A

Singular $|A| = 0$

Non Singular $|A| \neq 0$

Q. For linear transformation

$$X = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

coordinates (y_1, y_2, y_3) in y corresponding to $(1, 2, -1)$ in x

$$\cancel{1 = 2x_1 + x_2 + x_3}$$
$$\cancel{2 = x_1 + x_2 + 2x_3}$$
$$\cancel{-1 = x_1 - 2x_3}$$

$$\cancel{\begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}}$$

$R_2 \to 2R_2 - R_1$; $R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

$y_3 = 0$
$y_2 + 3y_3 = 3$
$y_2 = 3$

$2y_1 + y_2 + y_3 = 1$
$2y_1 + 3 = -2$
$\underline{y_1 = -1}$

Q. $Y = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$R_2 \to R_2 - R_1 ; R_3 \to R_3 - R_2$

$\begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$\begin{bmatrix} 2 \\ -2 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$x_3 = -7/3$

$x_2 + 3x_3 = -2$

$x_2 = 5$

$x_1 + x_2 + 2x_3 = 2$

$x_1 + 5 - \frac{14}{3} = 2$

$x_1 = \frac{1}{3}$

$x_1 + x_2 + 2x_3 = 2$

$x + 2x_2 + 5 \cdots$

$x_1 = \frac{13}{5} \quad x_2 = \frac{4}{5} \quad x_3 = \frac{-7}{5}$

Q. Express each of fo...

$x_1 = 2y_1$ ...

$x_2 = 4$ ...

Find composite ...

of z_1, z_2

$x_1 = 2z_1 - 4$ ...

$x_2 = 4z_1 - 8$ ...

$x_1 = -4z_1$ ...

$x_2 = 6z_1$ ...

$X = \begin{bmatrix} -4 & - \\ 6 & -5 \end{bmatrix}$ ...

$x_1 = -4z$ ...

$x_2 = 6z_1 -$ ...

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Q. Express each of following transformation

$x_1 = 2y_1 - 3y_2$ & $y_1 = z_1 - 2z_2$

$x_2 = 4y_1 + y_2$ $\quad$ $y_2 = 2z_1 + 3z_2$

Find composite transformation which expresses x_1 x_2 in terms of z_1, z_2

$x_1 = 2z_1 - 4z_2 - 6z_1 - 9z_2 = -4z_1 - 13z_2$

$x_2 = 4z_1 - 8z_2 + 2z_1 + 3z_2 = 6z_1 - 5z_2$

$\begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{matrix} -4z_1 & -13z_2 \\ 6z_1 & -5z_2 \end{matrix}$

$X = \begin{bmatrix} -4 & -13 \\ 6 & -5 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$x_1 = -4z_1 - 13z_2$

$x_2 = 6z_1 - 5z_2$

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The quadratic form of linear transformation

Consider a quadratic form $X'AX$ where A is symmetric and are non-singular linear transformation

$X = PY$

$$X'AX = (PY)'A(PY)$$
$$= Y'P'APY$$
$$= Y'(P'AP)Y$$
$$= Y'BY$$

$$B = (P'AP)' = P'A'(P')' = P'AP = B$$

Q. Obtain linear transformation of quad form
$2x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 4x_1x_3 - 4x_2x_3$

$x_1 = y_1 - y_2 + 2y_3$
$x_2 = 2y_2 + 2y_3$
$x_3 = 3y_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \overset{P}{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$X'AX$ when,

$$A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix}$$

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The given quad form $Y'BY$ where $B = P'AP$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

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Q. $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$

$x_1 = y_1 + 2y_2 + 4y_3$

$x_2 = y_2 + 4y_3$

$x_3 = y_3$

$$A = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & -7 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \quad P' \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 4 & 1 \end{bmatrix}$$

$$B = P'AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$Y'BY = y_1^2 - 2y_2^2 + 9y_3^2$$

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Congruence Of A Quadratic Form

- A square matrix B of order n is said to be congruent to A of ~~order~~ same order if there exist a non-singular matrix P such that $\boxed{B = P'AP}$

- Two quadratic forms $X'AX$ and $Y'BY$ is said to be congruent if A and B are congruent.

RESULT: If $X'AX$ is a quadratic form where A is matrix of rank R then there exist a non-singular linear transformation $X = PY$ which transforms given quad form to sum of R terms.

$$b_1 y_1^2 + b_2 y_2^2 + \cdots + b_r y_r^2$$

Q. Reduce the given quadratic form to diagonal form through congruent transformation

$$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 + 4x_1x_3 - 2x_2x_3$$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A = IAI$$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to 3R_2 + R_1 \cdot R_3 \to 3R_3 - R_1$

$$\begin{bmatrix} 6 & -2 & 2 \\ 0 & 7 & -1 \\ 0 & -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 6 & -2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to 3C_2 + C_1$; $C_3 \to 3C_3 - C_1$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & -3 \\ 0 & -3 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 0 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$R_3 \to 7R_3 + R_2$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & -3 \\ 0 & 0 & 144 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -6 & 3 & 21 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$C_3 \to 7C_3 + C_2$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 1008 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -6 & 3 & 21 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -6 \\ 0 & 3 & 3 \\ 0 & 0 & 21 \end{bmatrix}$$

$R_2 \to \frac{1}{\sqrt{7}} R_2$; $C_2 \to \frac{1}{\sqrt{7}} C_2$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1008 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/\sqrt{7} & 3/\sqrt{7} & 0 \\ -6 & 3 & 21 \end{bmatrix} A \begin{bmatrix} 1 & 1/\sqrt{7} & -6 \\ 0 & 3/\sqrt{7} & 3 \\ 0 & 0 & 21 \end{bmatrix}$$

$R_3 \to \frac{1}{\sqrt{7}} R_3$; $C_3 \to \frac{1}{\sqrt{7}} C_3$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 144 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/\sqrt{7} & 3/\sqrt{7} & 0 \\ -6/\sqrt{7} & 3/\sqrt{7} & 21/\sqrt{7} \end{bmatrix} A \begin{bmatrix} 1 & 1/\sqrt{7} & -6 \\ 0 & 3/\sqrt{7} & 3 \\ 0 & 0 & 21/\sqrt{7} \end{bmatrix}$$

$R_3 \to \frac{1}{3} R_3$; $C_3 \to \frac{1}{3} C_3$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/\sqrt{7} & 3 & 0 \\ -2/\sqrt{7} & 0 & 7/\sqrt{7} \end{bmatrix} A \begin{bmatrix} 1 & 1/\sqrt{7} & -2/\sqrt{7} \\ 0 & 3/\sqrt{7} & 1/\sqrt{7} \\ 0 & 0 & 7/\sqrt{7} \end{bmatrix}$$

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$B = P'AP$

$B = Y'BY = 6y_1^2 + 3y_2^2 + 16y_3^2$

Under, $X = PY = \cancel{8y_1^2}$ $x_1 = y_1 + \frac{1}{\sqrt{7}}y_2 - \frac{2}{\sqrt{7}}y_3$

$x_2 = \frac{3}{\sqrt{7}}y_2 + \frac{1}{\sqrt{7}}y_3$

$x_3 = \frac{7}{\sqrt{7}}y_3$

Q. $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_1x_2 + 8x_1x_3 + 4x_2x_3$

$$A = \begin{bmatrix} 6 & 2 & 4 \\ 2 & 3 & 2 \\ 4 & 2 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 & 4 \\ 2 & 3 & 2 \\ 4 & 2 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to 3R_2 - R_1$; $R_3 \to 3R_3 - 2R_1$

$$\begin{bmatrix} 6 & 2 & 4 \\ 0 & 7 & 2 \\ 0 & 2 & 34 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to 3C_2 - C_1$; $C_3 \to 3C_3 - 2R_1$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 6 \\ 0 & \cancel{326} & 102 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$R_3 \to 7R_3 - 2R_2$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 6 \\ 0 & 0 & \cancel{786}702 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -12 & \cancel{2}-2 & 21 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Scanned with CamScanner

$-\frac{2}{\sqrt{7}}y_3$

$x_3 + 4x_2x_3$

$C_3 \to 7C_3 - 2C_2$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & \cancel{5502}\ 4914 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -12 & -2 & 7 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -12 \\ 0 & 1 & -2 \\ 0 & 0 & 7 \end{bmatrix}$$

$R_3 \to \frac{1}{\sqrt{21}} R_3 ;\ C_3 \to \frac{1}{\sqrt{21}} C_3$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & \cancel{262}\ 234 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \frac{-12}{\sqrt{21}} & \frac{-2}{\sqrt{21}} & \frac{7}{\sqrt{21}} \end{bmatrix} A \begin{bmatrix} 1 & -1 & -12/\sqrt{21} \\ 0 & 1 & -2/\sqrt{21} \\ 0 & 0 & 7/\sqrt{21} \end{bmatrix}$$

$R_2 \to \frac{1}{\sqrt{21}} R_2 ;\ C_2 \to \frac{1}{\sqrt{21}} C_2$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cancel{262}\ 234 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -1/\sqrt{21} & 1/\sqrt{21} & 0 \\ -12/\sqrt{21} & -2/\sqrt{21} & 7/\sqrt{21} \end{bmatrix} A \begin{bmatrix} 1 & -1/\sqrt{21} & -12/\sqrt{21} \\ 0 & 1/\sqrt{21} & -2/\sqrt{21} \\ 0 & 0 & 7/\sqrt{21} \end{bmatrix}$$

$B = Y'BY = 6y_1^2 + y_2^2 + 262$

Ans : $6y_1^2 + 7/3\, y_2^2 + \frac{1}{14} y_3^2$

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Q. Canonical Form

$X'AX$.

$A = I A I$

$$\begin{bmatrix} \ddots & & \\ & & \\ & & \end{bmatrix} = \begin{bmatrix} & \end{bmatrix} A \begin{bmatrix} & \end{bmatrix}$$

+1 -1] Diagonal elements either then canonical

Rank : 2 No. of Non zero Row.

Index : No. of +ve 1

Sign : No. of +ve − No. of −ves

Positive definite : if all λ_i's are +ve

Semi Positive definite : +1, +1, 0, 0, : Some of +ve & some are 0

Negative definite : All all λ_i's are −ve

Semi Negative definite : −1, −1, 0, 0 ; some are −ve & some are 0

Indefinite : some −ve, some +ve, some 0.

Q. Reduce quad form

$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 8x_1x_3 + 12x_1x_2$

$$\begin{bmatrix} 2 & -4 & -4 \\ -4 & 1 & 6 \\ -4 & 6 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$; $R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 2 & -4 & -4 \\ 0 & -7 & -8 \\ 0 & -2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + 2R_1$; $C_3 \to C_3 + 2C_1$

$$\cancel{\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 8 \\ 0 & -2 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

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$R_3 \to R_3 \mp 2R_2$

$$\cancel{\begin{bmatrix} 2 & 0 & \\ 0 & & \\ 0 & & \end{bmatrix}}$$

-1,-6 and

$$\begin{bmatrix} 2 & 6 & -4 \\ 6 & 1 & -4 \\ -4 & -4 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$R_2 \to R_2 - 3R_1$; $R_3 \to R_3 + 2R_1$; $C_2 \to C_2 - 3C_1$; $C_3 \to C_3 + 2C_1$

$R_3 \to R_3 + \frac{8}{17} R_2$

,2 and

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 8 \\ 0 & 0 & \frac{-123}{17} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 10/17 & 8/17 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 + \frac{8}{17} C_2$

is. 1,1,1

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 0 \\ 0 & 0 & \frac{-123}{17} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 10/17 & 8/17 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 10/17 \\ 0 & 1 & 8/17 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to \frac{1}{\sqrt{2}} R_1$; $R_2 \to \frac{1}{\sqrt{17}} R_2$; $R_3 \to \frac{\sqrt{17}}{\sqrt{123}} R_3$ } Same for Col

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 \\ -3/\sqrt{17} & 1/\sqrt{17} & 0 \\ 10/\sqrt{123 \times 17} & 8/\sqrt{17 \times 123} & \frac{\sqrt{17}}{\sqrt{123}} \end{bmatrix} A \begin{bmatrix} 1/\sqrt{2} & -3/\sqrt{17} & \frac{10}{\sqrt{123 \times 17}} \\ 0 & 1/\sqrt{17} & 8/\sqrt{123 \times 17} \\ 0 & 0 & \sqrt{17}/\sqrt{123} \end{bmatrix}$$

Rank = 3 ; Index = 1 ; Signature = −1

Canonical F.

$$Y'BY = y_1^2 - y_2^2 - y_3^2$$

Q. $x^2 + 2y^2 + 2z^2 - 2xy - 2yz + 2zx$

$$\begin{bmatrix} 1 & -1 & 1/2 \\ -1 & 2 & -1 \\ -1/2 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$; $R_3 \to 2R_3 \mp R_1$

$$\begin{bmatrix} 1 & -1 & 1/2 \\ 0 & 1 & -1/2 \\ 0 & \cancel{3}-1 & 7/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + C_1$; $C_3 \to 2C_3 - C_1$

$$\begin{bmatrix} 1 & \cancel{1/2}\, 0 & 0 \\ 0 & \cancel{1/2}\, 1 & -1 \\ 0 & \cancel{13/2}\, -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$C_3 \to C_3 + C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Rank 3; Index: 3; Sig: 3 Value class: +ve definite.

0. $$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to 5R_2 - 3R_1$; $R_3 \to 5R_3 - 7R_1$

$$\begin{bmatrix} 5 & 3 & 7 \\ 0 & 121 & -11 \\ 0 & -11 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 5 & 0 \\ -7 & 0 & 5 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to 5C_2 - 3C_1$; $C_3 \to 5C_3 - 7C_1$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 121 & -11 \\ 0 & -11 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 5 & 0 \\ -7 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -3 & -7 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + \frac{1}{11} R_2$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 121 & -11 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 5 & 0 \\ -80/11 & 5/11 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 89/11 \\ 0 & 5 & 5/11 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank = 2; Index = 2, Signature = 2

Value Class: Semi-definite

$$\begin{bmatrix} 2 & -6 & 1/2 \\ -6 & 1 & -4 \\ 1/2 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$; $R_3 \to 4R_3 - R_1$

$$\begin{bmatrix} 2 & -6 & 1/2 \\ 0 & -17 & -7/2 \\ 0 & -10 & -25/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & 0 & 4 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + 3C_1$; $C_3 \to 4C_3 - C_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 3 \\ 0 & -10 & -40 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 1/2 \\ 6 & 1 & -4 \\ 1/2 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$; $R_3 \to 4R_3 - R_1$

$$\begin{bmatrix} 2 & 6 & 1/2 \\ 0 & -17 & -11/2 \\ 0 & -22 & -25/2 \end{bmatrix}$$

$C_2 \to C_2 - 3C_1$; $C_3 \to 4C_3 - C_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & -22 \\ 0 & -22 & -50 \end{bmatrix}$$

Unitary And Hermitian Matrix

Hermitian Matrix

$A^* = \bar{A}' = A$ [Transpose conjugate]

If A is real, $A^* = \bar{A}$

Result: Let U & V be complex matrices & let $K \in C$

(a) $(U^*)^* = U$

(b) $(KU+V)^* = \bar{K}\, U^* + V^*$

(c) $(UV)^* = V^* U^*$

(d) If U & $V \in C$, then dot product $U \cdot V = V^* U$

Soln.: a. $(U^*)^* = \overline{(\overline{U}^T)}^T = \overline{(\overline{U})^T}^T = \overline{\overline{U}} = U$

b. $(KU+V)^* = \overline{(KU+V)}^T = \overline{(KU)}^T + \overline{V}^T = \bar{K}\,\overline{U}^T + \overline{V}^T$
$= \bar{K}\, U^* + V^*$

c. $(UV)^* = \overline{(UV)}^T = \overline{U}^T \overline{V}^T = U^* V^*$

d. Dot product of complex vectors $U = (U_1, U_2, \ldots, U_n)$ & $V = (V_1, V_2, \ldots, V_n)$ is given by

$U \cdot V = U_1 \bar{V}_1 + U_2 \bar{V}_2 + U_3 \bar{V}_3 + \ldots U_n \bar{V}_n$

$$= [\bar{V}_1, \bar{V}_2, \bar{V}_3, \ldots \bar{V}_n] \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_n \end{bmatrix}$$

$= V^* U$

Linear Algebra by Gilbert.

Q. $(1+3i, 2+i)\cdot(4-5i, 2+3i)$

$(4+5i, 2-3i)\begin{bmatrix}1+3i\\2+i\end{bmatrix}$

$(4+5i)(1+3i)+(2-3i)(2+i)$

$4+12i+5i+15i^2+4+2i-6i-3i^2$

$\cancel{4+12+16}-4+13i$

Q. $\|2+i, 3-5i\| \rightarrow$ Norm [Length of Matrix]

$\|2+i, 3-5i\|^2 = (2+i)(3-5i)\cdot(2+i, 3-5i)$

$(2-i, 3+5i)\begin{bmatrix}2+i\\3-5i\end{bmatrix}$

$(2-i)(2+i) + (3+5i)(3-5i)$

$= 39$

$\therefore \|2+i, 3-5i\| = \sqrt{39}$

$$\boxed{\|(a+ib, c+id)\| = \sqrt{a^2+b^2+c^2+d^2}}$$

Q. Find non zero vector $\overset{x_1}{(a,b)}$ which is orthogonal to $(1+8i, 2-3i)$ x_2

$x_1' x_2 = 0$

$\begin{bmatrix} a \\ b \end{bmatrix} [1+8i, 2+3i] = 0$

$a + 8ai + 2b + 3ai = 0$

$a[1+8i]$ $b[2+3i]$

$a = 2+3i$

$[1-8i, 2+3i] \begin{bmatrix} a \\ b \end{bmatrix} = 0$

$(1-8i)a + (2+3i)b = 0$

$a = 2+3i$; $b = -1+8i$

$(a,b) = (2+3i, -1+8i)$

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Matrix is Unitary

If $\boxed{A \cdot A^* = I}$

Q. Find c & d so that following matrix is unitary

$$\begin{bmatrix} \frac{1}{\sqrt{7}}(1+2i) & c \\ \frac{1}{\sqrt{7}}(1-i) & d \end{bmatrix}$$

Columns of matrix to be orthogonal rather complex dot product should. So we will find vector which is orthogonal to 1st vector

$$\begin{bmatrix} \cancel{\frac{1}{\sqrt{7}}(1+2i)} \\ \frac{1}{\sqrt{7}}(1-i) \end{bmatrix}$$

$$\begin{bmatrix} 1+2i \\ 1-i \end{bmatrix} \quad (a,b) \quad (1+2i, 1-i)$$

$a = (1+i), \; b = (-1+2i)$

$(a,b) = (1+i, -1+2i)$

$\| 1+i, -1+2i \| = \sqrt{1+1+1+4} = \sqrt{7}$

$$(c,d) = \left(\frac{1+i}{\sqrt{7}} , \frac{-1+2i}{\sqrt{7}} \right)$$

Put this in matrix $A \cdot A^*$

We then divide the vector by norm to get identity m

Q. $A = \begin{bmatrix} 1 & 2-i \\ 2+i & -3 \end{bmatrix}$

show that eigen values are real & eigen vectors for different eigen value are real

$|A-\lambda I| = 0$

$\begin{bmatrix} 1-\lambda & 2-i \\ 2+i & -3-\lambda \end{bmatrix} =$

$(1-\lambda)(-3-\lambda) - (2+i)(2-i) =$

$-3 - \lambda + 3\lambda + \lambda^2 - (4 - \cancel{2i} + \cancel{2i} - i^2) = 0$

$-3 + 2\lambda + \lambda^2 + i^2 - 4 = 0$

$\lambda^2 + 2\lambda - 8 = 0$

$\lambda = 2, -4$

$\lambda = 2$

$\begin{bmatrix} -1 & 2-i \\ 2+i & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

~~$5 - 5 = 0$~~

$-x + (2-i)y = 0$

$(2+i)x - 5y = 0$

$\lambda = 2 \begin{bmatrix} 2-i \\ 1 \end{bmatrix}$ $\lambda = -4 \begin{bmatrix} 2-i \\ -5 \end{bmatrix}$

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$$x_1' x_2 = [2+i \quad -5]\begin{bmatrix} 2-i \\ -5 \end{bmatrix}$$

$(2+i)(2-i)-5$

$4+1-5=0$

$\therefore$ Orthogonal

Q. A 2×2 real symmetric matrix A has Eigen values 1 & 3 (2, −3) is an eigen vector to $\lambda=1$. Find Find:
① Eigen vector for $\lambda=3$
② Find matrix A.

~~$[2, -3]$~~

Since Eigen vectors of real symmetric matrix are orthogonal so let a, b

$(2,-3)(a,b)=0$
$2a-3b=0$
$(a,b)=(3,2)$

$\lambda=1 \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ $\lambda=3\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$P^{-1}AP = D$

$$D=\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$A = PDP^{-1}$

$$P = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2/13 & -3/13 \\ 3/13 & 2/13 \end{bmatrix}$$

$$\begin{bmatrix} 31/13 & 12/13 \\ 12/13 & 21/13 \end{bmatrix}$$