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OR

Assignment-III

Q1.

a.

0	2	1	6
2	1	5	7
2	4	3	7
5	5	10	

NORTHWEST CORNER

Find If Balanced or Un Balanced

⇒ The Row & Col sum is 20, hence balanced.

Table

0 (5)	2 (1)	1	~~6~~ ~~1~~ ~~0~~
2	1 (4)	5 (3)	~~7~~ 3
2	4	3 (7)	~~7~~
~~5~~	5 ~~4~~	10 ~~7~~	

Now,

Find Deg or Non-Deg Soln. $= m+n-1 = 3+3-1 = 5$

$\therefore$ Non-Deg Soln. BFS

Optimal Cost $= 0\times5 + 2\times1 + 1\times4 + 5\times3 + 7\times3$

$= 2 + 4 + 15 + 21 = 42$

LEAST COST

Table.

0 (5)	2	1 (1)	~~6~~ ~~7~~
2	1 (5)	5 (2)	~~7~~ ~~2~~
2	4	3 (7)	~~7~~
~~5~~	~~5~~	~~10~~ ~~9~~ ~~2~~	

Now,
Find Deg or Non-Deg Soln. $= m+n-1 = 3+3-1 = 5$
$\therefore$ Non-Deg Soln. BF's

Optimal Cost $= 0\times5 + 1\times1 + 1\times5 + 2\times5 + 3\times7$
$= 0+1+5+10+21 = \underline{\underline{37}}$

VOGEL'S

Table.

					P1	P2	P3	P4
	0 (5)	2	1 (6)	~~6~~ ~~7~~ 0	1	–	–	–
	2 (2)	1 (5)	5	~~7~~ ~~2~~	~~4~~ 1	1	← 5	–
	2 (3)	4	3 (4)	~~7~~ ~~3~~	1	1	1	1
	~~5~~ ~~5~~ 3	~~5~~	~~10~~ 4					
P1	2	1	2 ↑					
P2	0	3 ↑	2					
P3	0	–	2					
P4	2	–	3 ↑					

Final Table.

0	2	(6) 1	6
(2) 2	(5) 1	5	7
(3) 2	4	(4) 3	7
5	5	10	

~~Optimal Soln.~~

Now,

Find Deg or ~~Deg~~ Non-Deg Soln. $= m+n-1 = 5$

$\therefore$ Non-Deg Soln BF's

Optimal Soln. $= 1\times6 + 2\times2 + 1\times5 + 2\times3 + 3\times4$

$= 6 + 4 + 5 + 6 + 12$

$= 33$

h. NORTHWEST CORNER

Table

1 (7)	2	6	~~7~~
0 (3)	4 (9)	2	~~12~~ ~~9~~
3	1 (1)	5 (10)	~~11~~ 10
~~10~~ ~~3~~	~~10~~ ~~1~~	10	

$m+n-1 = 3+3-1 = 5$ ∴ Non-Deg Soln.

Optimal Soln. $= 1\times7 + 0\times3 + 4\times9 + 1\times1 + 5\times10$
$= 7+0+36+1+50$
$= 94.$

LEAST COST ~~MET~~

Table

1	2	6 (7)	~~7~~
0 (10)	4	2 (2)	~~12~~ ~~2~~
3	1 (10)	5 (1)	~~11~~
10	10	10 [illegible]	

$m+n-1 = 5$ ∴ Non-Deg Soln.

Optimal Soln. $= 6\times7 + 0\times10 + 1\times10 + 5\times1 + 2\times2$
$= 42 + 10 + 5 + 4$
$= 61.$

VOGEL'S

				Supply	P_1	P_2	P_3
	1 (7)	2	$\cancel{6}$	$\cancel{7}$	1	1	1
	0 (2)	4	2 (10)	$\cancel{12}\ \cancel{2}$	2	4 ←	-
	3 (1)	1 (10)	5	$\cancel{11}\ \cancel{10}$	2	2	2
Demand	$\cancel{10}\ \cancel{8}\ \cancel{7}$	$\cancel{10}$	$\cancel{10}$				
P_1	1	1	3 ↑				
P_2	2 1	1	-				
P_3	2 ↑	1	-				

$m+n-1 = 5$ ∴ Non-Deg Soln.

Optimal Soln. $= 1\times7 + 0\times2 + 3\times1 + 1\times10 + 2\times10$
$= 7 + 0 + 3 + 10 + 20$
$= \cancel{8}40.$

c. NORTHWEST CORNER

5 (9)	1 (3)	8	~~12~~ ~~3~~
2	4 (7)	0 (7)	~~14~~ ~~7~~
3	6	7 (4)	~~4~~
~~9~~	~~10~~ ~~3~~	~~11~~ ~~4~~	

$m+n-1 = 5 \therefore$ Non-Deg Soln.

Optimal Soln. $= 5\times9 + 1\times3 + 4\times7 + 0\times7 + 7\times4$
$= 45+3+28+0+28$
$= 104$

LEAST COST

Table

5 (2)	1 (10)	8	~~12~~ 2
2 (3)	4	0 (11)	~~14~~ ~~3~~
3 (4)	6	7	~~4~~
~~9~~ ~~6~~ ~~2~~	~~10~~	~~11~~	

$m+n-1 = 5 \therefore$ Non-Deg Soln.

Optimal Soln. $= 5\times2 + 1\times10 + 2\times3 + 0\times11 + 3\times4$
$= 10+10+6+12 = 38.$

VOGEL'S

				Supply	P_1	P_2	P_3
	5 (2)	1 (10)	8	~~12~~ 2	4	4	–
	2 (3)	4	0 (11)	~~14~~ 3	2	2	2
	3 (4)	6	7	~~4~~	3	3	3
	~~7~~ ~~5~~	~~10~~	~~11~~				
P_1	1	3	7 ↑				
P_2	1	3	–				
P_3	1	–	–				

$m+n-1 = 5$ ∴ Non-Deg Soln.

Optimal Soln. $= 5\times2 + 1\times10 + 2\times3 + 3\times4$

$= 10 + 10 + 6 + 12$

$= \underline{\underline{38}}$

Q2.

a. NORTHWEST CORNER

3 (200)	1 (50)	7	4	~~250~~ ~~50~~	$U_1 = 0$
2	6 (250)	5 (100)	9	~~350~~ ~~100~~	$U_2 = 5$
8	3	3 (250)	2 (150)	~~400~~ ~~150~~	$U_3 = 3$
~~200~~	~~300~~ ~~250~~	~~350~~ ~~250~~	~~150~~		

$V_1 = 3$ $V_2 = 1$ $V_3 = 0$ $V_4 = \cancel{2} -1$

$m+n-1 = 3+4-1 = 6$ $\therefore$ Non-Deg Soln.

Optimal Soln. $= 3\times200 + 1\times50 + 6\times250 + 5\times100 + 3\times250 + 2\times150$
$= 600 + 50 + 1500 + 500 + 750 + 300$
$= \underline{3700}$

Now,
$C_{ij} = U_i + V_j$
Let, $U_1 = 0$. So,
$\cancel{3 = 0 +}$ $C_{11} = U_1 + V_1 \Rightarrow 3 = 0 + V_1 \Rightarrow V_1 = 3$
$C_{12} = U_1 + V_2 \Rightarrow \cancel{6}\ 1 = 0 + V_2 \Rightarrow V_2 = 1$
$C_{22} = U_2 + V_2 \Rightarrow 6 = U_2 + 1 \Rightarrow U_2 = 5$
$C_{23} = U_2 + V_3 \Rightarrow 5 = \cancel{U_2}\ 5 + V_3 \Rightarrow V_3 = 0$
$C_{33} = U_3 + V_3 \Rightarrow 3 = U_3 + 0 \Rightarrow U_3 = 3$
$C_{34} = U_3 + V_4 \Rightarrow 2 = 3 + V_4 \Rightarrow V_4 = \cancel{2} -1$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [for Un-occ Cells]

So,

$\Delta_{13} = C_{13} - (U_1 + V_3) \Rightarrow 7 - (0+0) = 7$

$\Delta_{14} = C_{14} - (U_1 + V_4) \Rightarrow 4 - (0+(-1)) = 5$

$\Delta_{21} = C_{21} - (U_2 + V_1) \Rightarrow 2 - (5+3) = -6$

$\Delta_{24} = C_{24} - (U_2 + V_4) \Rightarrow 9 - (5 + \overset{(-1)}{2}) = 5$

$\Delta_{31} = C_{31} - (U_3 + V_1) \Rightarrow 8 - (3+3) = 2$

$\Delta_{32} = C_{32} - (U_3 + V_2) \Rightarrow 3 - (3+1) = -1$

Now,

We choose, $\Delta_{21} = -6$ [as most -ve]

$x = \min(200, 250)$

$= 200$

Modified Table.

3	1 (250)	7	4	$U_1 = -5$
2 (200)	6 (50)	5 (100)	9	$U_2 = 0$
8	3	3 (250)	2 (150)	$U_3 = -2$
$V_1 = 2$	$V_2 = 6$	$V_3 = 5$	$V_4 = 9$	

Now, $C_{ij} = U_i + V_j$

Let, $U_2 = 0$. So,

$C_{24} = U_2 + V_4 \Rightarrow 9 = 0 + V_4 \Rightarrow V_4 = 9$

$C_{14} = U_1 + V_4 \Rightarrow 4 = U_1 + 9 \Rightarrow U_1 = -5$

$C_{11} = U_1 + V_1 \Rightarrow 3 = -5 + V_1 \Rightarrow V_1 = 8$

Now, $C_{ij} = U_i + V_j$

Let, $U_2 = 0$. So,

$C_{21} = U_2 + V_1 \Rightarrow 2 = 0 + V_1 \Rightarrow V_1 = 2$

$C_{22} = U_2 + V_2 \Rightarrow 6 = 0 + V_2 \Rightarrow V_2 = 6$

$C_{23} = U_2 + V_3 \Rightarrow 5 = 0 + V_3 \Rightarrow V_3 = 5$

$C_{33} = U_3 + V_3 \Rightarrow 3 = U_3 + 5 \Rightarrow U_3 = -2$

$C_{34} = U_3 + V_4 \Rightarrow 2 = -2 + V_4 \Rightarrow V_4 = 4$

$C_{12} = U_1 + V_2 \Rightarrow 1 = U_1 + 6 \Rightarrow U_1 = -5$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-occ Cells]

So,

$\Delta_{11} = C_{11} - (U_1 + V_1) \Rightarrow 3 - (-5 + 2) = 6$

$\Delta_{13} = C_{13} - (U_1 + V_3) \Rightarrow 7 - (-5 + 5) = 7$

$\Delta_{14} = C_{14} - (U_1 + V_4) \Rightarrow 4 - (-5 + 4) = 5$

$\Delta_{24} = C_{24} - (U_2 + V_4) \Rightarrow 9 - (0 + 4) = 5$

$\Delta_{31} = C_{31} - (U_3 + V_1) \Rightarrow 8 - (-2 + 2) = 8$

$\Delta_{32} = C_{32} - (U_3 + V_2) \Rightarrow 3 - (6 + (-2)) = -1$

Now, $\because \Delta_{32} \leq 0 \therefore$ Sol^n is not Optimal.

Now, We choose $\Delta_{32} = -1$ [as most -ve]

$\theta = \min(50, 250) = 50$

Modified Table

3	1 (250)	7	4	$U_1 = -2$
2 (200)	6	5 (150)	9	$U_2 = 2$
8	3 (50)	3 (200)	2 (150)	$U_3 = 0$
$V_1 = 0$	$V_2 = 3$	$V_3 = 3$	$V_4 = 2$	

Now, $C_{ij} = U_i + V_j$

Let, $U_3 = 0$, So,

$C_{32} = U_3 + V_2 \Rightarrow 3 = 0 + V_2 \Rightarrow V_2 = 3$

$C_{33} = U_3 + V_3 \Rightarrow 3 = 0 + V_3 \Rightarrow V_3 = 3$

$C_{34} = U_3 + V_4 \Rightarrow 2 = 0 + V_4 \Rightarrow V_4 = 2$

$C_{12} = V_2 + U_1 \Rightarrow 1 = U_1 + 3 \Rightarrow U_1 = -2$

$C_{23} = U_2 + V_3 \Rightarrow 5 = U_2 + 3 \Rightarrow U_2 = 2$

$C_{21} = U_2 + V_1 \Rightarrow 2 = 2 + V_1 \Rightarrow V_1 = 0$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

So,

$\Delta_{11} = C_{11} - (U_1 + V_1) = 3 - (-2+0) = 5$

$\Delta_{13} = C_{13} - (U_1 + V_3) = 7 - (-2+3) = 6$

$\Delta_{14} = C_{14} - (U_1 + V_4) = 4 - (-2+2) = 4$

$\Delta_{22} = C_{22} - (U_2 + V_2) = 6 - (2+3) = 1$

$\Delta_{24} = C_{24} - (U_2 + V_4) = 9 - (2+2) = 5$

$\Delta_{31} = C_{31} - (U_3 + V_1) = 8 - (0-0) = 8$

$\because$ All $\Delta_{ij} > 0$, $\therefore$ Optimal Sol^n is obtained.

Cost $= 1 \times 250 + 2 \times 200 + 5 \times 150 + 3 \times 50 + 3 \times 200 + 2 \times 150$
$= 250 + 400 + 750 + 150 + 600 + 300$
$= 2450$

LEAST COST

3	1 (250)	7	4	~~250~~	$U_1 = -5$
2 (200)	6 (50)	5 (100)	9	~~350~~ ~~150~~ ~~50~~	$U_2 = 0$
8	3	3 (250)	2 (150)	~~400~~ ~~250~~	$U_3 = -2$
~~200~~	~~300~~ ~~50~~	~~350~~ ~~100~~	~~150~~		

$V_1 = 2$ $V_2 = 6$ $V_3 = 5$ $V_4 = 4$

Now, Δ_{ij} for Non-Occ Cells [$\Delta_{ij} = C_{ij} - (U_i + V_j)$]

$\Delta_{11} = 6$ $\Delta_{21} = 5$

$\Delta_{13} = 10$ $\Delta_{31} = 8$

$\Delta_{14} = 7$ $\Delta_{32} = -1$

$Cost = 1\times250 + 2\times200 + 6\times50 + 5\times100 + 3\times250 + 2\times150$
$= 250 + 400 + 300 + 500 + 750 + 300$
$= 2500$

We choose $\Delta_{32} = -1$ [as most -ve]
$x = \min(50, 250) = 50$

Modified Table

3	1 (250)	7	4	250	$U_1 = -2$
2 (200)	6	5 (150)	9	350	$U_2 = 2$
8	3 (50)	3 (200)	2 (150)	400	$U_3 = 0$
200	300	350	150		
$V_1 = 0$	$V_2 = 3$	$V_3 = 3$	$V_4 = 2$		

Now, Δ_{ij} for Non-Occ Cells $[\Delta_{ij} = C_{ij} - (U_i + V_j)]$

$\Delta_{11} = 5$ $\quad\Delta_{22} = 1$
$\Delta_{13} = 6$ $\quad\Delta_{24} = 5$
$\Delta_{14} = 4$ $\quad\Delta_{31} = 8$

$\because$ All $\Delta_{ij} \geq 0$, $\therefore$ Optimal Soln is obtained
Cost $= 1\times250 + 2\times200 + 5\times150 + 3\times50 + 3\times50 + 3\times200 + 2\times150$
$= 2450$

VOGEL'S

				Supply	P_1	P_2	P_3	
3	1 (250)	7	4	~~250~~	2	3	3	$u_1 = -2$
2 (200)	6	5 (150)	9	~~150~~ ~~350~~	3	–	–	$u_2 = 2$
8	3 (50)	3 (200)	2 (150)	~~50~~ ~~200~~ ~~400~~	1	1	1	$u_3 = 0$
~~200~~	~~300~~ 50	~~350~~ 200	~~150~~					
P_1: 1	2	2	2					
P_2: –	2	4 ↑	2					
P_3: –	2	–	2					
$v_1 = 0$	$v_2 = 3$	$v_3 = 3$	$v_4 = 2$					

$m+n-1 = 3+4-1 = 6$ $\therefore$ Non-Deg Soln

Cost $= 1\times250 + 2\times200 + 5\times150 + 3\times50 + 3\times200 + 2\times150$

$= 250 + 400 + 750 + 150 + 600 + 300$

$= 2450$

Δ_{ij} For Un-Occ Cells. $[\Delta_{ij} = C_{ij} - (u_i + v_j)]$

$\Delta_{11} = 5$ $\quad \Delta_{14} = 4$ $\quad \Delta_{24} = 5$

$\Delta_{13} = 6$ $\quad \Delta_{22} = 1$ $\quad \Delta_{31} = 8$

$\therefore$ All $\Delta_{ij} > 0$ $\therefore$ Optimal Solⁿ is obtained.

Cost $= 1\times250 + 2\times200 + 5\times150 + 3\times50 + 3\times200 + 2\times150$

$= 250 + 400 + 750 + 150 + 600 + 300$

$= 2450$

c. NORTHWEST CORNER

10 (100)	16 (100)	9	12	~~200~~ 100	$U_1 = 0$
12	12 (100)	13 (200)	5	~~300~~ ~~200~~	$U_2 = -4$
14	8	13 (250)	4 (50)	~~300~~ 50	$U_3 = \cancel{13} -4$
0	0	0	0 (200)	~~200~~	$U_4 = -8$
100	~~200~~ 100	~~450~~ 250	~~250~~ 200		
$V_1 = 10$	$V_2 = 16$	$V_3 = 17$	$V_4 = 8$		

$m+n-1 = 4+4-1 = 7$ ∴ Non-Deg soln.

Cost $= 10\times100 + 16\times100 + 12\times100 + 13\times200 + 13\times250 + 4\times50 + 0\times200$
$= 1000 + 1600 + 1200 + 2600 + 3250 + 200 + 0$
$= \underline{9850}$

Now,
$C_{ij} = U_i + V_j$
Let, $U_1 = 0$, So,
$C_{11} = U_1 + V_1 \Rightarrow 10 = 0 + V_1 \Rightarrow V_1 = 10$
$C_{12} = U_1 + V_2 \Rightarrow 16 = 0 + V_2 \Rightarrow V_2 = 16$
$C_{22} = U_2 + V_2 \Rightarrow 12 = U_2 + 16 \Rightarrow U_2 = -4$
$C_{23} = U_2 + V_3 \Rightarrow 13 = \cancel{0} -4 + V_3 \Rightarrow V_3 = 17$
$C_{33} = U_3 + V_3 \Rightarrow 13 = U_3 + 17 \Rightarrow U_3 = -4$
$C_{34} = U_3 + V_4 \Rightarrow 4 = -4 + V_4 \Rightarrow V_4 = 8$
$C_{44} = U_4 + V_4 \Rightarrow 0 = U_4 + 8 \Rightarrow U_4 = -8$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-occ Cell]
$\Delta_{13} = C_{13} - (U_1 + V_3) = 9 - (0+17) = -8$
$\Delta_{14} = C_{14} - (U_1 + V_4) = 12 - (0+8) = 4$
$\Delta_{21} = C_{21} - (U_2 + V_1) = 12 - (-4+10) = 6$
$\Delta_{24} = C_{24} - (U_2 + V_4) = 5 - (-4+8) = 1$
$\Delta_{31} = C_{31} - (U_3 + V_1) = 14 - (-4+10) = 8$
$\Delta_{32} = C_{32} - (U_3 + V_2) = 8 - (-4+16) = -4$
$\Delta_{41} = C_{41} - (U_4 + V_1) = 0 - (-8+10) = -2$
$\Delta_{42} = C_{42} - (U_4 + V_2) = 0 - (-8+16) = -8$
$\Delta_{43} = C_{43} - (U_4 + V_3) = 0 - (-8+17) = -9$

Now,

We Choose, $\Delta_{43} = -9$ [as Most -ve]

$x = \min(200, 250)$

$= 200$

Modified Table.

10 (100)	16 (100)	9	12	200	$U_1 = 17$
12	12 (100)	13 (200)	5	300	$U_2 = 13$
14	8	13 (250)	4 (250)	300	$U_3 = 13$
0	0	0 (200)	0	200	$U_4 = 0$
100	200	450	250		
$V_1 = -7$	$V_2 = -1$	$V_3 = 0$	$V_4 = -9$		

Now, $C_{ij} = U_i + V_j$

Let $V_3 = 0$. So,

$C_{33} = U_3 + V_3 \Rightarrow 13 = U_3 + 0 \Rightarrow U_3 = 13$

$C_{43} = U_4 + V_3 \Rightarrow 0 = U_4 + 0 \Rightarrow U_4 = 0$

$C_{23} = U_2 + V_3 \Rightarrow 13 = U_2 + 0 \Rightarrow U_2 = 13$

$C_{34} = U_3 + V_4 \Rightarrow 4 = 13 + V_4 \Rightarrow V_4 = -9$

$C_{22} = U_2 + V_2 \Rightarrow 12 = 13 + V_2 \Rightarrow V_2 = -1$

$C_{12} = U_1 + V_2 \Rightarrow 16 = U_1 - 1 \Rightarrow U_1 = 17$

$C_{11} = U_1 + V_1 \Rightarrow 10 = 17 + V_1 \Rightarrow V_1 = -7$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [for Un-occ Cells]

$\Delta_{13} = C_{13} - (U_1 + V_3) = 9 - (17 + 0) = -8$

$\Delta_{14} = 12 - (17 - 9) = 4$

$\Delta_{21} = 12 - (13 - 7) = 6$

$\Delta_{24} = 5 - (13 - 9) = 1$

$\Delta_{31} = 14 - (13 - 7) = 8$

$\Delta_{32} = 8 - (13 - 1) = -4$

$\Delta_{41} = 0 - (0 - 7) = 7$

$\Delta_{42} = 0 - (0 - 1) = 1$

$\Delta_{44} = 0 - (0 - 9) = 9$

Now,
We Choose, $\Delta_{13} = -8$ [as Most -ve]
$x = \min(200, 100)$
$= 100$

Modified Table

10 (100)	16	9 (100)	12	200	$u_1 = 9$
12	12 (200)	13 (100)	5	300	$u_2 = 13$
14	8	13 (50)	4 (250)	300	$u_3 = 13$
0	0	0 (200)	0	200	$u_4 = 0$
100	200	450	250		
$v_1 = 1$	$v_2 = -1$	$v_3 = 0$	$v_4 = -9$		

Now, $C_{ij} = U_i + V_j$

Let, $V_3 = 0$ so,

$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 0 \Rightarrow U_1 = 9$

$C_{23} = U_2 + V_3 \Rightarrow 13 = U_2 + 0 \Rightarrow U_2 = 13$

$C_{33} = U_3 + V_3 \Rightarrow 13 = U_3 + 0 \Rightarrow U_3 = 13$

$C_{43} = U_4 + V_3 \Rightarrow 0 = U_4 + 0 \Rightarrow U_4 = 0$

$C_{11} = U_1 + V_1 \Rightarrow 10 = 9 + V_1 \Rightarrow V_1 = 1$

$C_{22} = U_2 + V_2 \Rightarrow 12 = 13 + V_2 \Rightarrow V_2 = -1$

$C_{34} = U_3 + V_4 \Rightarrow 4 = 13 + V_4 \Rightarrow V_4 = -9$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{12} = C_{12} - (U_1 + V_2) = 16 - (9 - 1) = 8$

$\Delta_{14} = 12 - (9 - 9) = 12$

$\Delta_{21} = 12 - (13 + 1) = 2$

$\Delta_{24} = 5 - (13 - 9) = 1$

$\Delta_{31} = 14 - (13 - 1) = 2$

$\Delta_{32} = 8 - (13 - 1) = -4$

$\Delta_{41} = 0 - (0 + 1) = -1$

$\Delta_{42} = 0 - (0 - 1) = 1$

$\Delta_{44} = 0 - (0 - 9) = 9$

Now, We Choose $\Delta_{32} = -4$ [As Most -ve]

$x = \min(50, 200)$

$= 50$

Modified Table.

10 (100)	16	9 (100)	12	200	$u_1 = 9$
12	12 (150)	13 (150)	5	300	$u_2 = 13$
14	8 (50)	13	4 (250)	300	$u_3 = 9$
0	0	0 (200)	0	200	$u_4 = 0$
10	200	450	250		
$v_1 = 1$	$v_2 = -1$	$v_3 = 0$	$v_4 = -5$		

Now, $C_{ij} = u_i + v_j$

Let, $v_3 = 0$, So

$C_{13} = u_1 + v_3 \Rightarrow 9 = u_1 + 0 \Rightarrow u_1 = 9$

$C_{23} = u_2 + v_3 \Rightarrow 13 = u_2 + 0 \Rightarrow u_2 = 13$

$C_{43} = u_4 + v_3 \Rightarrow 0 = u_4 + 0 \Rightarrow u_4 = 0$

$C_{11} = u_1 + v_1 \Rightarrow 10 = 9 + v_1 \Rightarrow v_1 = 1$

$C_{22} = u_2 + v_2 \Rightarrow 12 = 13 + v_2 \Rightarrow v_2 = -1$

$C_{32} = u_3 + v_2 \Rightarrow 8 = u_3 - 1 \Rightarrow u_3 = 9$

$C_{34} = u_3 + v_4 \Rightarrow 4 = 9 + v_4 \Rightarrow v_4 = -5$

Now, $\Delta_{ij} = C_{ij} - (u_i + v_j)$ [For Un-Occ Cells]

$\Delta_{12} = C_{12} - (u_1 + v_2) = 16 - (9 - 1) = 8$

$\Delta_{14} = 12 - (9 - 5) = 8$

$\Delta_{21} = 12 - (13 + 1) = 2$

$\Delta_{24} = 5 - (13 - 5) = \cancel{8}\, -3$

$\Delta_{31} = 14 - (9 + 1) = 4$

$\Delta_{33} = 13 - (9 + 0) = 4$

$\Delta_{41} = 0 - (0 + 1) = -1$

$\Delta_{44} = 0 - (0 - 5) = 5$

$\Delta_{42} = 0 - (0 - 1) = 1$

Now, We Choose $\Delta_{24} = -3$ [As Most -ve]

$x = \min(150, 250)$

$= 150$

Modified Table

10 (100)	16	9 (100)	12	200	$U_1 = 9$
12	12	13 (150)	5 (150)	300	$U_2 = 13$
14	8 (200)	13	4 (250)	300	$U_3 = 12$
0	0	0 (200)	0	200	$U_4 = 0$
10	200	450	250		
$V_1 = 1$	$V_2 = -4$	$V_3 = 0$	$V_4 = -8$		

Now, $C_{ij} = U_i + V_j$

Let, $V_3 = 0$. So,

$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 0 \Rightarrow U_1 = 9$

$C_{23} = U_2 + V_3 \Rightarrow 13 = U_2 + 0 \Rightarrow U_2 = 13$

$C_{43} = U_4 + V_3 \Rightarrow 0 = U_4 + 0 \Rightarrow U_4 = 0$

$C_{11} = U_1 + V_1 \Rightarrow 10 = 9 + V_1 \Rightarrow V_1 = 1$

$C_{24} = U_2 + V_4 \Rightarrow 5 = 13 + V_4 \Rightarrow V_4 = -8$

$C_{34} = U_3 + V_4 \Rightarrow 4 = U_3 - 8 \Rightarrow U_3 = 12$

$C_{32} = U_3 + V_2 \Rightarrow 8 = 12 + V_2 \Rightarrow V_2 = -4$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [Unocc]

$\Delta_{12} = C_{12} - (U_1 + V_2) = 16 - (9 - 4) = 11$

$\Delta_{14} = 12 - (9 - 8) = 11$

$\Delta_{21} = 12 - (13 + 1) = -2$

$\Delta_{22} = 12 - (13 - 4) = 3$

$\Delta_{31} = 14 - (12 + 1) = 1$

$\Delta_{33} = 13 - (12 + 0) = 1$

$\Delta_{41} = 0 - (0 + 1) = -1$

$\Delta_{42} = 0 - (0 - 4) = 4$

$\Delta_{44} = 0 - (0 - 8) = 8$

Now, We Choose $\Delta_{21} = -2$ [As Most -ve]

$x = \min(100, 150)$
$= 100$

Modified Table

10	16	9 (200)	12	200	$U_1 = 9$
12 (100)	12	13 (50)	5 (150)	300	$U_2 = 13$
14	8 (200)	13	4 (100)	300	$U_3 = 12$
0	0	0 (200)	0	200	$U_4 = 0$
100	200	450	250		

$V_1 = -1$ $\quad V_2 = -4$ $\quad V_3 = 0$ $\quad V_4 = -8$

Now, $C_{ij} = U_i + V_j$
Let, $V_3 = 0$. So,
$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 0 \Rightarrow \underline{U_1 = 9}$
$C_{23} = U_2 + V_3 \Rightarrow 13 = U_2 + 0 \Rightarrow U_2 = 13$
$C_{43} = U_4 + V_3 \Rightarrow 0 = U_4 + 0 \Rightarrow U_4 = 0$
$C_{21} = U_2 + V_1 \Rightarrow 12 = 13 + V_1 \Rightarrow V_1 = -1$
$\cancel{C_{43} = U_4 + V_3 \Rightarrow 0 = 0}$.
$C_{24} = U_2 + V_4 \Rightarrow 5 = 13 + V_4 \Rightarrow V_4 = -8$
$C_{34} = U_3 + V_4 \Rightarrow 4 = U_3 - 8 \Rightarrow U_3 = 12$
$C_{32} = U_3 + V_2 \Rightarrow 8 = 12 + V_2 \Rightarrow V_2 = -4$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ]
$\Delta_{11} = C_{11} - (U_1 + V_1) = 10 - (9 - 1) = 2$
$\Delta_{12} = 16 - (9 - 4) = 11$
$\Delta_{31} = 14 - (12 - 1) = 3$
$\Delta_{22} = 12 - (13 - 4) = 3$
$\Delta_{33} = 13 - (12 - 0) = 1$
$\Delta_{41} = 0 - (0 - 1) = 1$
$\Delta_{42} = 0 - (0 - 4) = 4$
$\Delta_{44} = 0 - (0 - 8) = 8$

$\because$ All $\Delta_{ij} > 0$ $\therefore$ Optimal Solⁿ is obtained

Cost $= 9\times200 + 12\times100 + 13\times50 + 5\times150 + 8\times200 + 4\times100 + 0\times200$
$= 1800 + 1200 + 650 + 750 + 1600 + 400$
$= 6400$

LEAST COST

10	16	200 9	12	~~200~~	$U_1 = 8$
12	50 12	250 13	5	250 ~~300~~	$U_2 = 12$
14	50 8	13	250 4	~~50~~ ~~300~~	$U_3 = 8$
100 0	100 0	0	0	~~100~~ ~~200~~	$U_4 = 0$
~~100~~	~~100~~ 50 ~~200~~	250 ~~450~~	~~250~~		

$V_1 = 0$ $\quad V_2 = 0$ $\quad V_3 = \cancel{4}1$ $\quad V_4 = -4$

$m+n-1 = 4+4-1 = 7$ $\therefore$ Non-Deg soln.

Cost = $9\times200 + 12\times50 + 13\times250 + 8\times50 + 0\times100 + 4\times250 + 0\times100$
$= 1800 + 600 + 3250 + 400 + 1000$
$= 7050$

Now, $C_{ij} = U_i + V_j$
& Let, $V_2 = 0$, So,
$C_{22} = U_2 + V_2 \Rightarrow 12 = U_2 + 0 \Rightarrow U_2 = 12$
$C_{23} = U_2 + V_3 \Rightarrow \cancel{8}^{13} = \cancel{0}\,12 + V_3 \Rightarrow V_3 = \cancel{4}1$
$C_{32} = U_3 + V_2 \Rightarrow 8 = U_3 + 0 \Rightarrow U_3 = 8$
$C_{42} = U_4 + V_2 \Rightarrow 0 = U_4 + 0 \Rightarrow U_4 = 0$
$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 1 \Rightarrow U_1 = 8$
$C_{41} = U_4 + V_1 \Rightarrow 0 = 0 + V_1 \Rightarrow V_1 = 0$
$C_{34} = U_3 + V_4 \Rightarrow 4 = 8 + V_4 \Rightarrow V_4 = -4$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]
$\Delta_{11} = C_{11} - (U_1 + V_1) = 10 - (8+0) = 2$
$\Delta_{12} = 16 - (8+0) = 8$
$\Delta_{14} = 12 - (8-4) = 8$
$\Delta_{21} = 12 - (12+0) = 0$
$\Delta_{24} = 5 - (12-4) = -3$
$\Delta_{31} = 14 - (8+0) = 6$
$\Delta_{33} = 13 - (8+1) = 4$
$\Delta_{43} = 0 - (0+1) = -1$
$\Delta_{44} = 0 - (0-4) = 4$

Now, We Choose $\Delta_{24} = -3$ [As Most -ve]

$x = \min(50, 250)$

$= 50$

Modified Table

10	16	9 (200)	12	200	$U_1 = -4$
12	12	13 (250)	5 (50)	300	$U_2 = 0$
14	8 (100)	13	4 (200)	300	$U_3 = -1$
0 (100)	0 (100)	0	0	200	$U_4 = -9$
100	200	450	250		

$V_1 = 9 \quad V_2 = 9 \quad V_3 = 13 \quad V_4 = 5$

Now, $C_{ij} = U_i + V_j$

Let, $U_2 = 0$, so,

$C_{23} = U_2 + V_3 \Rightarrow 13 = 0 + V_3 \Rightarrow V_3 = 13$

$C_{24} = U_2 + V_4 \Rightarrow 5 = 0 + V_4 \Rightarrow V_4 = 5$

$C_{34} = U_3 + V_4 \Rightarrow 4 = U_3 + 5 \Rightarrow U_3 = -1$

$C_{32} = U_3 + V_2 \Rightarrow 8 = -1 + V_2 \Rightarrow V_2 = 9$

$C_{42} = U_4 + V_2 \Rightarrow 0 = U_4 + 9 \Rightarrow U_4 = -9$

$C_{41} = U_4 + V_1 \Rightarrow 0 = -9 + V_1 \Rightarrow V_1 = 9$

$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 13 \Rightarrow U_1 = -4$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [Non-Occ Cells]

$\Delta_{11} = C_{11} - (U_1 + V_1) = 10 - (-4 + 9) = 5$

$\Delta_{12} = 16 - (-4 + 9) = 11$

$\Delta_{14} = 12 - (-4 + 5) = 11$

$\Delta_{21} = 12 - (0 + 9) = 3$

$\Delta_{22} = 12 - (0 + 9) = 3$

$\Delta_{31} = 14 - (-1 + 9) = 6$

$\Delta_{33} = 13 - (-1 + 13) = 1$

$\Delta_{43} = 0 - (-9 + 13) = -4$

$\Delta_{44} = 0 - (-9 + 5) = 4$

~~Now, we Choose $\Delta_{43} = -4$~~

LEAST COST

10	16	9 (200)	12	~~200~~	$u_1 = 9$
12 (100)	12	13 (200)	5	~~300~~ ~~200~~	$u_2 = 13$
14	8 (d)	13 (50)	4 (250)	~~300~~ ~~50~~	$u_3 = 13$
0	0 (200)	0	0	~~200~~	$u_4 = 5$
~~100~~	~~200~~	~~450~~ ~~250~~ 50	~~250~~		

$V_1 = -1 \quad V_2 = -5 \quad V_3 = 0 \quad V_4 = -9$

$m+n-1 = 7$ ∴ ~~Non~~ Deg Soln. ∵ Allocated Cells are 6

$Cost = 9\times200 + 13\times200 + 12\times100 + 13\times50 + 4\times250 + 0\times200$
$= 1800 + 2600 + 1200 + 650 + 1000 + 0$
$= 7250$

∴ Allocate Art. var. d in C_{32}

Now, $C_{ij} = U_i + V_j$
Let, $V_3 = 0$. So,
$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 0 \Rightarrow U_1 = 9$
$C_{23} = U_2 + V_3 \Rightarrow 13 = U_2 + 0 \Rightarrow U_2 = 13$
$C_{33} = U_3 + V_3 \Rightarrow 13 = U_3 + 0 \Rightarrow U_3 = 13$
$C_{32} = U_3 + V_2 \Rightarrow 8 = 13 + V_2 \Rightarrow V_2 = -5$
$C_{21} = U_2 + V_1 \Rightarrow 12 = 13 + V_1 \Rightarrow V_1 = -1$
$C_{34} = U_3 + V_4 \Rightarrow 4 = 13 + V_4 \Rightarrow V_4 = -9$
$C_{42} = U_4 + V_2 \Rightarrow 0 = U_4 - 5 \Rightarrow U_4 = 5$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [for Unocc. Cells]
$\Delta_{11} = C_{11} - (U_1 + V_1) = 10 - (9-1) = 2$
$\Delta_{12} = 16 - (9-5) = 12$
$\Delta_{14} = 12 - (9-9) = 12$
$\Delta_{22} = 12 - (13-5) = 4$
$\Delta_{24} = 5 - (13-9) = 1$
$\Delta_{31} = 14 - (13-1) = 2$
$\Delta_{41} = 0 - (5-1) = -4$
$\Delta_{43} = 0 - (5+0) = -5$
$\Delta_{44} = 0 - (5-9) = 4$

Now, We Choose $\Delta_{43} = -5$ [As Most -ve]

$x = \text{Min}(200, 50)$

$= 50$

Modified Table

10	16	9 (200)	12	200	$U_1 = 9$
12 (100)	12	13 (200)	5	300	$U_2 = 13$
14	8 (50)	13	4 (250)	300	$U_3 = 8$
0	0 (150)	0 (50)	0	200	$U_4 = 0$
100	200	450	250		

$V_1 = -1$ $\quad V_2 = 0$ $\quad V_3 = 0$ $\quad V_4 = -4$

Now, $C_{ij} = U_i + V_j$

Let, $V_3 = 0$, so,

$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 0 \Rightarrow U_1 = 9$

$C_{23} = U_2 + V_3 \Rightarrow 13 = U_2 + 0 \Rightarrow U_2 = 13$

$C_{43} = U_4 + V_3 \Rightarrow 0 = U_4 + 0 \Rightarrow U_4 = 0$

$C_{21} = U_2 + V_1 \Rightarrow 12 = 13 + V_1 \Rightarrow V_1 = -1$

$C_{42} = U_4 + V_2 \Rightarrow 0 = 0 + V_2 \Rightarrow V_2 = 0$

$C_{32} = U_3 + V_2 \Rightarrow 8 = U_3 + 0 \Rightarrow U_3 = 8$

$C_{34} = U_3 + V_4 \Rightarrow 4 = 8 + V_4 \Rightarrow V_4 = -4$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [Un-occ]

$\Delta_{11} = C_{11} - (U_1 + V_1) = 10 - (9 - 1) = 2$

$\Delta_{12} = 16 - (9 + 0) = 7$

$\Delta_{14} = 12 - (9 + 0) = 3$

$\Delta_{22} = 12 - (13 + 0) = -1$

$\Delta_{24} = 5 - (13 - 4) = -4$

$\Delta_{31} = 14 - (8 - 1) = 7$

$\Delta_{33} = 13 - (8 + 0) = 5$

$\Delta_{41} = 0 - (0 - 1) = 1$

$\Delta_{44} = 0 - (0 - 4) = 4$

Now, We Choose $\Delta_{24} = -4$ [As most -ve]

$x = \text{Min}(200, 150, 250)$

$= 150$

Modified Table

10	16	9 (200)	12	200	$U_1 = 9$
12 (100)	12	13 (50)	5 (150)	300	$U_2 = 13$
14	8 (200)	13	4 (100)	300	$U_3 = 12$
0	0	0 (200)	0	200	$U_4 = 0$
100	200	450	250		
$V_1 = -1$	$V_2 = -4$	$V_3 = 0$	$V_4 = -8$		

Now, $C_{ij} = U_i + V_j$

Let, $V_3 = 0$. So,

$C_{13} = U_1 + V_3 \Rightarrow 9 = U_1 + 0 \Rightarrow U_1 = 9$

$C_{23} = U_2 + V_3 \Rightarrow 13 = U_2 + 0 \Rightarrow U_2 = 13$

$C_{43} = U_4 + V_3 \Rightarrow 0 = U_4 + 0 \Rightarrow U_4 = 0$

$C_{24} = U_2 + V_4 \Rightarrow 5 = 13 + V_4 \Rightarrow V_4 = -8$

$C_{34} = U_3 + V_4 \Rightarrow 4 = U_3 - 8 \Rightarrow U_3 = 12$

$C_{32} = U_3 + V_2 \Rightarrow 8 = 12 + V_2 \Rightarrow V_2 = -4$

$C_{21} = U_2 + V_1 \Rightarrow 12 = 13 + V_1 \Rightarrow V_1 = -1$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{11} = C_{11} - (U_1 + V_1) = 10 - (9 - 1) = 2$

$\Delta_{12} = 16 - (9 - 4) = 11$

$\Delta_{14} = 12 - (9 - 8) = 11$

$\Delta_{22} = 12 - (13 - 4) = 3$

$\Delta_{31} = 14 - (12 - 1) = 3$

$\Delta_{33} = 13 - (12 + 0) = 1$

$\Delta_{41} = 0 - (0 - 1) = 1$

$\Delta_{42} = 0 - (0 - 4) = 4$

$\Delta_{44} = 0 - (0 - 8) = 8$

$\because$ All $\Delta_{ij} > 0$ $\therefore$ Optimal Sol^n is obtained

Cost $= 9 \times 200 + 13 \times 50 + 5 \times 150 + 12 \times 100 + 8 \times 200 + 4 \times 100 + 0 \times 200$

$= 1800 + 650 + 750 + 1200 + 1600 + 400$

$= 6400$

VOGEL'S

				Supply	P_1	P_2	P_3	P_4	
10	16	9 (250)	12	200	1	3	–	–	$U_1 = 9$
12	12 (50)	13	5 (250)	~~50~~ ~~300~~	7	7	7	–	$U_2 = 17$
14	8 (150)	13 (150)	4	150 ~~300~~	4	4	4	8	$U_3 = 13$
0 (100)	0	0 (100)	0	100 ~~200~~	0	0	–	–	$U_4 = 0$
100	150 ~~200~~	350 150 ~~450~~	250						
P_1: 10↑	8	9	4						
P_2: –	8	9↑	4						
P_3: –	4	–	1						
P_4: –	8↑	–	–						

$V_1 = 0 \quad V_2 = -5 \quad V_3 = 0 \quad V_4 = 0$

$m+n-1 = 7$ ∴ Non-Deg soln.

$\text{Cost} = 9\times250 + 12\times50 + 5\times250 + 8\times150 + 13\times150 + 0\times100 + 0\times100$

$= 2250 + 600 + 1250 + 1200 + 1950 + 0 + 0$

$= 7250$

$\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{11} = 1 \quad \Delta_{12} = 12 \quad \Delta_{14} = 3$

$\Delta_{21} = -5 \quad \Delta_{23} = -4 \quad \Delta_{31} = 1$

$\Delta_{34} = -9 \quad \Delta_{42} = 5 \quad \Delta_{44} = 0$

Now, We Choose $\Delta_{34} = -9$ [As Most -ve]

$x = \min(150, 50, 250)$
$= 50$

Modified Table

					P_1	P_2	P_3
10	16	9 (250)	12	200	$U_1 = -4$		
12	12	13	5 (200)	300	$U_2 = 1$		
14	8 (200)	13 (100)	4 (50)	300	$U_3 = 0$		
0 (100)	0	0 (100)	0	200	$U_4 = -13$		
100	200	450	250				

P_1
P_2 $V_1 = 13$ $V_2 = 8$ $V_3 = 13$ $V_4 = 4$
P_3

$C_{ij} = U_i + V_j$
Let, $U_3 = 0$. So,
~~$C_{32} = U_3 + V_2 \Rightarrow 8 = 0 + V_2 \Rightarrow V_2$~~

$\Delta_{ij} = (C_{ij} - (U_i + V_j))$ [For Un-Occ Cells]

$\Delta_{11} = 1$ $\Delta_{21} = -2$
$\Delta_{12} = 12$
$\Delta_{14} = 12$

VOGEL'S

					P_1	P_2	P_3	
10	16	9 (200)	12	200	1	3	4	$u_1 = 9$
12	12 (50)	13	5 (250)	~~300~~ 250	7	7	7	$u_2 = 17$
14	8 (150)	13 (150)	4	~~300~~ ~~150~~	4	4	4	$u_3 = 13$
0 (100)	0	0 (100)	0	~~200~~ ~~100~~	0	—		$u_4 = 0$
100	~~200~~ 50	~~450~~ ~~350~~ 150	250					
P_1: 10 ↑	8	9	4					
P_2: —	8	9 ↑	4					
P_3: —	4	—	1					

$v_1 = 0 \quad v_2 = -5 \quad v_3 = 0 \quad v_4 = -12$

$m+n-1 = 7$ ∴ Non-Deg Soln.

Cost $= 9\times200 + 5\times250 + 13\times150 + 8\times150 + 12\times50 + 0\times100 + 0\times100$

$= 1800 + 1250 + 1950 + 1200 + 600 + 0 + 0$

$= 6800$

$\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{11} = 1 \quad \Delta_{14} = 15 \quad \Delta_{23} = -4 \quad \Delta_{34} = 3 \quad \Delta_{42} = 5$

$\Delta_{12} = 12 \quad \Delta_{21} = -5 \quad \Delta_{31} = 1 \quad \Delta_{44} = 12$

Now, We Choose $\Delta_{21} = -5$ [As Most -ve]

$x = \min(50, 150, 100)$
$= 50$

Modified Table

10	16	9 (200)	12	200	$u_1 = 9$
12 (50)	12	13	5 (250)	300	$u_2 = 12$
14	8 (200)	13 (100)	4	300	$u_3 = 13$
0 (50)	0	0 (150)	0	200	$u_4 = 0$
100	200	450	250		
$v_1 = 0$	$v_2 = -5$	$v_3 = 0$	$v_4 = -7$		

$\Delta_{ij} = C_{ij} - (u_i + v_j)$ [For Un-Occ Cells]

$\Delta_{11} = 1$ $\Delta_{14} = 10$ $\Delta_{23} = 1$ $\Delta_{34} = -2$ $\Delta_{44} = 7$

$\Delta_{12} = 12$ $\Delta_{22} = 5$ $\Delta_{31} = 1$ $\Delta_{42} = 5$

We Choose $\Delta_{34} = -2$ [As Most -ve]

$x = \min(100, 50, 250)$

$= 50$

Modified Table

10	16	9 (200)	12	200	$u_1 = 9$
12 (100)	12	13	5 (200)	300	$u_2 = 14$
14	8 (200)	13 (50)	4 (50)	300	$u_3 = 13$
0	0	0 (200)	0	200	$u_4 = 0$
100	200	450	250		
$V_1 = -2$	$V_2 = -5$	$V_3 = 0$	$V_4 = -9$		

$\Delta_{ij} = C_{ij} - (U_i + V_j)$, [For Un-Occ Cells]

$\Delta_{11} = 3$ $\quad \Delta_{22} = 3$ $\quad \Delta_{41} = 2$

$\Delta_{12} = 12$ $\quad \Delta_{23} = -1$ $\quad \Delta_{42} = 5$

$\Delta_{14} = 12$ $\quad \Delta_{31} = 3$ $\quad \Delta_{44} = 9$

We Choose $\Delta_{23} = -1$ [As Most -ve]

$x = \min(250, 50)$

$= 50$

Modified Table

10	16	9 (200)	12	200	$U_1 = 9$
12 (100)	12	13 (50)	5 (150)	300	$U_2 = 13$
14	8 (200)	13	4 (100)	300	$U_3 = 12$
0	0	0 (200)	0	200	$U_4 = 0$
100	200	450	250		
$V_1 = -1$	$V_2 = -4$	$V_3 = 0$	$V_4 = -8$		

$\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{11} = 2$ $\quad \Delta_{22} = 3$ $\quad \Delta_{41} = 1$

$\Delta_{12} = 11$ $\quad \Delta_{31} = 3$ $\quad \Delta_{42} = 4$

$\Delta_{13} = 11$ $\quad \Delta_{33} = 1$ $\quad \Delta_{44} = 8$

$\because$ All $\Delta_{ij} > 0$ $\therefore$ Optimal Sol^n is obtained

Cost $= 9\times200 + 12\times100 + 13\times50 + 5\times150 + 8\times200 + 4\times100 + 0\times200$

$= 1800 + 1200 + 650 + 750 + 1600 + 400$

$= 6400$

b. **NORTHWEST CORNER**

(10) 2	3	d 1	~~10~~	$u_1 = 10$
(10) 5	(25) 4	8	~~10~~ ~~35~~	$u_2 = 10$
5	6	(25) 8	25	$u_3 = 17$
~~20~~ ~~10~~	~~25~~	25		
$v_1 = 0$	$v_2 = -6$	$v_3 = -9$		

$m+n-1 = 7$ ∴ Deg Soln. ∴ Add Art. Var d. at C_{13}

$\text{Cost} = 2\times10 + 10\times5 + 4\times25 + 8\times25$

$= 20+50+100+200$

$= 370$

$C_{ij} = U_i + V_j$

Let $V_1 = 0$ So,

$C_{11} = U_1+V_1 \Rightarrow 2 = U_1+0 \Rightarrow U_1 = 2$

$C_{21} = U_2+V_1 \Rightarrow 5 = U_2+0 \Rightarrow U_2 = 5$

$C_{13} = U_1+V_3 \Rightarrow 1 = 10+V_3 \Rightarrow V_3 = -9$

$C_{22} = U_2+V_2 \Rightarrow 4 = 10+V_2 \Rightarrow V_2 = -6$

$C_{33} = U_3+V_3 \Rightarrow 8 = U_3 - 9 \Rightarrow U_3 = 17$

Now, $\Delta_{ij} = C_{ij} - (U_i+V_j)$ [Un-Occ]

$\Delta_{12} = C_{12} - (U_1+V_2) = 3-(10-6) = -1$

$\Delta_{23} = 8-(10-9) = 7$

$\Delta_{31} = 5-(17+0) = -12$

$\Delta_{32} = 6-(17-6) = -5$

We Choose, $\Delta_{31} = -5$ [Most -ve]
$x = \min(25, 10)$
$= 10$

Modified Table

2	3	1 (10)	10	$U_1 = -2$
5 (10)	4 (25)	8	35	$U_2 = 5$
5 (10)	6	8 (15)	25	$U_3 = 5$
20	25	25		
$V_1 = 0$	$V_2 = -1$	$V_3 = 3$		

$C_{ij} = U_i + V_j$
Let, $V_1 = 0$ so,
$C_{21} = U_2 + V_1 \Rightarrow 5 = U_2 + 0 \Rightarrow U_2 = 5$
$C_{31} = U_3 + V_1 \Rightarrow 5 = U_3 + 0 \Rightarrow U_3 = 5$
$C_{22} = U_2 + V_2 \Rightarrow 4 = 5 + V_2 \Rightarrow V_2 = -1$
$C_{33} = U_3 + V_3 \Rightarrow 8 = 5 + V_3 \Rightarrow V_3 = 3$
$C_{13} = U_1 + V_3 \Rightarrow 1 = U_1 + 3 \Rightarrow U_1 = -2$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [Un-Occ]
$\Delta_{11} = C_{11} - (U_1 + V_1) = 2 - (-2+0) = 4$
$\Delta_{12} = 3 - (-2-1) = 6$
$\Delta_{23} = 8 - (5+3) = 0$
$\Delta_{32} = 6 - (5-1) = 2$

$\because$ All $\Delta_{ij} > 0$ $\therefore$ Optimal Sol^n is obtained

Cost $= 1\times10 + 5\times10 + 4\times25 + 5\times10 + 8\times15$
$= 10 + 50 + 100 + 50 + 120$
$= 330$

LEAST COST

2	3	1 (10)	~~10~~	$U_1 = -7$
5 (10)	4 (25)	8	10 ~~35~~	$U_2 = 0$
5 (10)	6	8 (15)	15 ~~25~~	$U_3 = 0$
10 ~~20~~	~~25~~	15 ~~25~~		
$V_1 = 5$	$V_2 = 4$	$V_3 = 8$		

$m+n-1 = 5$ $\therefore$ Non-Deg Sol^n

$$\text{Cost} = 1\times10 + 5\times10 + 4\times25 + 5\times10 + 8\times15$$
$$= 10+50+100+50+120$$
$$= 330$$

$C_{ij} = U_i + V_j$

Let, $U_2 = 0$. so,

$C_{21} = U_2 + V_1 \Rightarrow 5 = 0 + V_1 \Rightarrow V_1 = 5$

$C_{22} = U_2 + V_2 \Rightarrow 4 = 0 + V_2 \Rightarrow V_2 = 4$

$C_{31} = U_3 + V_1 \Rightarrow 5 = U_3 + 5 \Rightarrow U_3 = 0$

$C_{33} = U_3 + V_3 \Rightarrow 8 = 0 + V_3 \Rightarrow V_3 = 8$

$C_{13} = U_1 + V_3 \Rightarrow 1 = U_1 + 8 \Rightarrow U_1 = -7$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [Un-Occ Cells]

$\Delta_{11} = C_{11} - (U_1 + V_1) = 2 - (-7+5) = 4$

$\Delta_{12} = 3 - (-7+4) = 6$

$\Delta_{23} = 8 - (0+8) = 0$

$\Delta_{32} = 6 - (0+4) = 2$

$\because$ All $\Delta_{ij} \geq 0$ $\therefore$ Optimal Sol^n has obtained.

Cost = 330.

VOGEL'S

				P_1	P_2	P_3	
2	3	1 (10)	10	1	–	–	$U_1 = -7$
5 (10)	4 (25)	8	~~35~~ 10	1	1	1	$U_2 = 0$
5 (10)	6	8 (15)	~~25~~ 10	1	1	1	$U_3 = 0$
~~20~~ 10	~~25~~	~~25~~ 15					
P_1 3	1	7 ↑					
P_2 0	2 ↑	–					
P_3 0	–	–					

$V_1 = 5 \quad V_2 = 4 \quad V_3 = 8$

$m+n-1 = 5$ ∴ Non Deg Sol^n

$C_{ij} = U_i + V_j$

Let, $U_2 = 0$ So,

$C_{21} = U_2 + V_1 \Rightarrow 5 = 0 + V_1 \Rightarrow V_1 = 5$

$C_{22} = U_2 + V_2 \Rightarrow 4 = 0 + V_2 \Rightarrow V_2 = 4$

$C_{31} = U_3 + V_1 \Rightarrow 5 = U_3 + 5 \Rightarrow U_3 = 0$

$C_{33} = U_3 + V_3 \Rightarrow 8 = 0 + V_3 \Rightarrow V_3 = 8$

$C_{13} = U_1 + V_3 \Rightarrow 1 = U_1 + 8 \Rightarrow U_1 = -7$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [Un-Occ cells]

$\Delta_{11} = 2 - (-7+5) = 4$

$\Delta_{12} = 3 - (-7+4) = 6$

$\Delta_{23} = 8 - (0+8) = 0$

$\Delta_{32} = 6 - (0+4) = 2$

∵ All $\Delta_{ij} \geq 0$ ∴ Optimal Sol^n is obtained

Cost $= 1\times10 + 5\times10 + 4\times25 + 5\times10 + 8\times15$

$= 10 + 50 + 100 + 50 + 120$

$= 330$

d. NORTHWEST CORNER

25 (200)	45	10 d	0	~~200~~	$U_1 = 10$
30	65 (100)	15 d	0	~~100~~	$U_2 = 15$
15	40	55 (300)	0 (100)	~~400~~ 100	$U_3 = 55$
~~200~~	~~100~~	~~300~~	100		
$V_1 = 15$	$V_2 = 50$	$V_3 = 0$	$V_4 = -55$		

$m+n-1 = 3+4-1 = 6$ $\therefore$ Deg Sol^n $\therefore$ Add art. var. @ C_{13} & C_{23}

Cost $= 25\times200 + 65\times100 + 55\times300 + 0\times100$
$= 5000 + 6500 + 16500 + 0$
$= 28000$

Now, $C_{ij} = U_i + V_j$
Let, $V_3 = 0$. So,
$C_{13} = U_1 + V_3 \Rightarrow 10 = U_1 + 0 \Rightarrow U_1 = 10$
$C_{23} = U_2 + V_3 \Rightarrow 15 = U_2 + 0 \Rightarrow U_2 = 15$
$C_{33} = U_3 + V_3 \Rightarrow 55 = U_3 + 0 \Rightarrow U_3 = 55$
$C_{34} = U_3 + V_4 \Rightarrow 0 = 55 + V_4 \Rightarrow V_4 = -55$
$C_{22} = U_2 + V_2 \Rightarrow 65 = 15 + V_2 \Rightarrow V_2 = 50$
$C_{11} = U_1 + V_1 \Rightarrow 25 = 10 + V_1 \Rightarrow V_1 = 15$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$
$\Delta_{12} = C_{12} - (U_1 + V_2) = 45 - (10+50) = -15$
$\Delta_{14} = 0 - (10 - 55) = 45$
$\Delta_{21} = 30 - (15 + 15) = 0$
$\Delta_{24} = 0 - (15 - 55) = 40$
$\Delta_{31} = 15 - (55 + 15) = -55$
$\Delta_{32} = 40 - (55 + 50) = -65$

We Choose, $\Delta_{32} = -65$ [As Most -ve]

$x = \min(100, 300)$

$= 100$

Modified Table

25 (200)	45	10	0	200	$u_1 = 10$
30	65	15 (100)	0	100	$u_2 = 15$
15	40 (100)	55 (200)	0 (100)	400	$u_3 = 55$
200	100	300	100		
$V_1 = 15$	$V_2 = -15$	$V_3 = 0$	$V_4 = -55$		

$C_{ij} = U_i + V_j$

Let, $V_3 = 0$. So,

$C_{13} = U_1 + V_3 \Rightarrow 10 = U_1 + 0 \Rightarrow U_1 = 10$

$C_{23} = U_2 + V_3 \Rightarrow 15 = U_2 + 0 \Rightarrow U_2 = 15$

$C_{33} = U_3 + V_3 \Rightarrow 55 = U_3 + 0 \Rightarrow U_3 = 55$

$C_{11} = U_1 + V_1 \Rightarrow 25 = 10 + V_1 \Rightarrow V_1 = 15$

$C_{32} = U_3 + V_2 \Rightarrow 40 = 55 + V_2 \Rightarrow V_2 = -15$

$C_{34} = U_3 + V_4 \Rightarrow 0 = 55 + V_4 \Rightarrow V_4 = -55$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [Un-Occ Cells]

$\Delta_{12} = C_{12} - (U_1 + V_2) = 45 - (10 - 15) = 40$

$\Delta_{14} = 0 - (10 - 55) = 45$

$\Delta_{21} = 30 - (15 - 15) = 30$

$\Delta_{22} = 65 - (15 - 15) = 65$

$\Delta_{24} = 0 - (15 - 55) = 40$

$\Delta_{31} = 15 - (55 + 15) = -55$

We Choose, $\Delta_{31} = -55$ [As Most -ve]

$x = \min(200, 200)$

$= 200$

Modified Table

25 d	45	10 (200)	0	200	$U_1 = 10$
30	65	15 (100)	0	100	$U_2 = 15$
15 (200)	40 (100)	55	0 (100)	400	$U_3 = 0$
200	100	300	100		
$V_1 = 15$	$V_2 = 40$	$V_3 = 0$	$V_4 = 0$		

$m+n-1 = 6$ Add Art. Var. @ C_{11}.

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$

$\Delta_{12} = 45 - (10+40) = -5$ $\Delta_{22} = 65 - (15+40) = 10$

$\Delta_{14} = 0 - (10) = -10$ $\Delta_{24} = 0 - (15) = -15$

$\Delta_{21} = 30 - (15-15) = 30$ $\Delta_{33} = 55 - (0-0) = 55$

Now, When we choose $\Delta_{24} = -15$ [As most -ve] the process will stop since art. var is being selected.

LEAST COST

25	45	10 (100)	0 (100)	200	$u_1 = 10$
30	65	15 (100)	0	100	$u_2 = 15$
15 (200)	40 (100)	55 (100)	0	~~400~~ ~~200~~ 100	$u_3 = 55$
~~200~~	~~100~~	~~300~~ 100	~~100~~		

$V_1 = -40$ $V_2 = -15$ $V_3 = 0$ $V_4 = -10$

$m + n - 1 = 6$ $\therefore$ Non-Deg soln.

$\text{Cost} = 10 \times 100 + 0 \times 100 + 15 \times 100 + 15 \times 200 + 40 \times 100 + 55 \times 100$

$= 1000 + 1500 + 3000 + 4000 + 5500$

$= 15000$

$\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{11} = 55$ $\Delta_{21} = 55$ $\Delta_{24} = -5$

$\Delta_{12} = 50$ $\Delta_{22} = 65$ $\Delta_{34} = -45$

We Choose $\Delta_{34} = -45$ [As Most -ve]

$x = \min(100, 100)$

$= 100$

Modified Table

25 d	45	10 (200)	0	200	$U_1 = 10$
30	65	15 (100)	0	100	$U_2 = 15$
15 (200)	40 (100)	55	0 (100)	400	$U_3 = 0$
200	100	300	100		
$V_1 = 15$	$V_2 = 40$	$V_3 = 0$	$V_4 = 0$		

$m+n-1 \neq 6$ ∴ Deg Soln. ∴ Add Art. Var. @ C_{11}

$C_{ij} = U_i + V_j$

Let, $V_3 = 0$. So

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{12} = 45 - (10+40) = -5$

$\Delta_{14} = 0 - (10+0) = -10$

$\Delta_{21} = 30 - (15+15) = 0$

$\Delta_{22} = 65 - (15+40) = 10$

$\Delta_{24} = 0 - (15+0) = -15$

$\Delta_{33} = 55 - (0+0) = 55$

Now, when we choose $\Delta_{24} = -15$ [As Most -Ve] the process will stop since an art. var. is being selected.

VOGEL'S

					P1	P2	P3	
25	45	10 (200)	0	~~200~~	10	15	—	$u_1 = 10$
30	65	15 δ	0 (100)	~~100~~	15	—	—	$u_2 = 15$
15 (200)	40 (100)	55 (100)	0	200 ~~300~~ ~~400~~	15	25	25	$u_3 = 55$
200	100	100 ~~300~~	~~100~~					
P1 10	5	5	0					
P2 10	5	~~45~~ 5↑	—					
P3 15	40↑	—	—					

$V_1 = -40$ $V_2 = -15$ $V_3 = 0$ $V_4 = -15$

Cost $= 10 \times 200 + 0 \times 100 + 55 \times 100 + 40 \times 100 + 15 \times 200$

$= 2000 + 0 + 5500 + 4000 + 3000$

$= 14500$

$n + m - 1 = 6$ $\therefore$ Deg soln. Add Art. var @ C_{23}

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-Occ Cells]

$\Delta_{11} = 55$ $\Delta_{12} = 50$ $\Delta_{14} = 5$

$\Delta_{21} = 55$ $\Delta_{22} = 65$ $\Delta_{34} = -40$

Now, We choose $\Delta_{34} = -40$ [As most -ve]

$x = \min(100, 100)$

$= 100$

Modified Table

25 (ε)	45	10 (200)	0	200	$U_1 = 10$
30	65	15 (100)	0	100	$U_2 = 15$
15 (200)	40 (100)	55	0 (100)	400	$U_3 = 0$
200	100	300	100		
$V_1 = 15$	$V_2 = 40$	$V_3 = 0$	$V_4 = 0$		

Add art. var at C_{11}

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$

$\Delta_{12} = -5$ $\quad$ $\Delta_{22} = 10$

$\Delta_{14} = -10$ $\quad$ $\Delta_{24} = -15$

$\Delta_{21} = 0$ $\quad$ $\Delta_{33} = 55$

Now, when we choose $\Delta_{24} = -15$ as most -ve a dummy (artificial variable is being selected

∴ Process will stop here.

Q4.

a.

8	4	2	6	1
0	9	5	5	4
3	8	9	2	6
4	3	1	0	3
9	5	8	9	5

Row = Col = 5

∴ Balanced.

n = Order of Mat. = 5

Row Reduction Matrix

7	3	1	5	0
0	9	5	5	4
1	6	7	0	4
4	3	1	0	2
4	0	3	4	0

Column Reduction Matrix

7	3	0	5	0
0	9	4	5	4
1	6	6	0	4
4	3	0	0	2
4	0	2	4	0

Draw min. no. of hor. & ver. lines to cover all 0's.

7	3	0	5	$\boxed{0}$
$\boxed{0}$	9	4	5	4
1	6	6	$\boxed{0}$	4
4	3	$\boxed{0}$	0	2
4	$\boxed{0}$	2	4	0

∵ 5=5 ∴ Go to Next step To find Cost.

Job	Machine
1	B
2	E
3	D
4	C
5	A

Cost = 0+5+1+2+1
= 9

b

5	15	20	25	10
10	12	5	15	19
5	17	18	9	11
8	9	10	5	12
9	10	5	11	7

Row = Col = 5 ∴ Balanced

$n = 5$

Row Reduction Matrix

0	10	15	20	5
5	7	0	10	14
0	12	13	4	6
3	4	5	0	7
4	5	0	6	2

Column Reduction Matrix

0	6	15	20	3
5	3	0	10	12
0	8	13	4	4
3	0	5	0	5
4	1	0	6	0

Draw min. no. of hor. & ver. lines to cover all 0's

$\boxed{0}$	6	15	20	3
5	3	$\boxed{0}$	10	12
0	8	13	4	4
9	$\boxed{0}$	5	0	5
4	1	0	6	$\boxed{0}$

∴ 4<5 ∴ ~~Go to Next Step n Repeat & Add 1 to unallocated~~ cells

&

∴ Add 1 to intersection cells & substract 1 for non-covered cells.

Modified Table

0	2	12	16	0
8	2	0	9	12
0	4	10	0	1
7	0	6	0	6
7	0	0	5	0

Draw min. no. of hor. & vertical lines to cover all 0's

$\boxed{0}$	2	12	16	0
8	2	$\boxed{0}$	9	12
0	4	10	$\boxed{0}$	1
7	$\boxed{0}$	6	0	6
7	0	0	5	$\boxed{0}$

Optimal Solⁿ is

Warehouse	Factory
1	1
2	3
3	4
4	2
5	5

Cost = 5+5+9+9+7

= 35

Q3.

10	(20) 2	(15) 3	15	9	35 15	$U_1 = -6$
(d) 5	10	15	(10) 2	(30) 4	~~40~~ 30 ~~10~~	$U_2 = 0$
(20) 15	5	14	7	15	~~20~~	$U_3 = 10$
20	15	(25) 13	25	(5) 8	~~30~~ 25	$U_4 = 4$
~~20~~	~~20~~	~~40~~ 25	10	~~35~~ 5		
$V_1 = 5$	$V_2 = 8$	$V_3 = 9$	$V_4 = 2$	$V_5 = 4$		

$Cost = 2\times20 + 3\times15 + 2\times10 + 4\times30 + 15\times20 + 13\times25 + 8\times5$

$= 40 + 45 + 20 + 120 + 300 + 325 + 40$

$= 850$

$m+n-1 = 8$ ∴ Deg - Soln. ∴ Add Art. Var. @ C_{21}

Now, $C_{ij} = U_i + V_j$

Let, $U_2 = 0$. So,

$C_{21} = U_2 + V_1 \Rightarrow 5 = 0 + V_1 \Rightarrow V_1 = 5$

$C_{24} = U_2 + V_4 \Rightarrow 2 = 0 + V_4 \Rightarrow V_4 = 2$

$C_{25} = U_2 + V_5 \Rightarrow 4 = 0 + V_5 \Rightarrow V_5 = 4$

$C_{31} = U_3 + V_1 \Rightarrow 15 = U_3 + 5 \Rightarrow U_3 = 10$

$C_{45} = U_4 + V_5 \Rightarrow 8 = U_4 + 4 \Rightarrow U_4 = 4$

$C_{43} = U_4 + V_3 \Rightarrow 13 = 4 + V_3 \Rightarrow V_3 = 9$

$C_{13} = U_1 + V_3 \Rightarrow 3 = U_1 + 9 \Rightarrow U_1 = -6$

$C_{12} = U_1 + V_2 \Rightarrow 2 = -6 + V_2 \Rightarrow V_2 = 8$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ } [For Un-Occ Cells]

$\Delta_{11} = C_{11} - (U_1 + V_1) = 10 - (-6+5) = 10 - (-1) = 11$

$\Delta_{14} = 15 - (-6+2) = 19$

$\Delta_{15} = 9 - (-6+4) = 11$

$\Delta_{22} = 10 - (0+8) = 2$

$\Delta_{23} = 15 - (0+9) = 6$

$\Delta_{32} = \cancel{15 - (10+9)} = 5 - (10+8) = -13$

$\Delta_{33} = 14 - (10+9) = -5$

$\Delta_{34} = 7 - (10+2) = -5$

$\Delta_{35} = 15 - (10+4) = 1$

$\Delta_{41} = 20 - (4+5) = 11$

$\Delta_{42} = 15 - (4+8) = -4$

$\Delta_{44} = 25 - (4+2) = 19$

Δ =

Now, We Choose $\Delta_{32} = -15$ [As Most -ve]

$x = \min(20, 30, 20, 25)$

$= 20$

Modified Table

10	2 d	3 (35)	18	9	35	$u_1 = -6$
5 (20)	10	15	2 (10)	4 (10)	40	$u_2 = 0$
15	5 (20)	14	7	15	20	$u_3 = -3$
20	15	13 (5)	25	8 (25)	30	$u_4 = 4$
20	20	40	10	35		
$v_1 = 5$	$v_2 = 8$	$v_3 = 9$	$v_4 = 2$	$v_5 = 4$		

$m+n-1 = 7$ ∴ Deg soln. ∴ Add dummy var. @ C_{22}

Now, $C_{ij} = U_i + V_j$

Let, $U_2 = 0$

Now, $\Delta_{ij} = C_{ij} - (U_i + V_j)$ [For Un-occ Cells]

$\Delta_{11} = 11$ $\Delta_{22} = 2$ $\Delta_{33} = 8$ $\Delta_{41} = 11$

$\Delta_{14} = 19$ $\Delta_{23} = 6$ $\Delta_{34} = 8$ $\Delta_{42} = 3$

$\Delta_{15} = 11$ $\Delta_{31} = 13$ $\Delta_{35} = 14$ $\Delta_{44} = 2$

∵ All $\Delta_{ij} \geq 0$ ∴ Optimal Soln has obtained.

Cost $= 3\times35 + 5\times20 + 2\times10 + 4\times10 + 5\times20 + 13\times5 + 8\times25$

$= 105 + 100 + 20 + 40 + 100 + 65 + 200$

$= 630$