UNIT-4

TUTORIAL-1

Q1. $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix}$

$|A - \lambda I| \Rightarrow (5+\lambda)(2+\lambda) - 4 = 0$

$\Rightarrow 10 + 5\lambda + 2\lambda + \lambda^2 - 4 = 0$

$\Rightarrow \lambda^2 + 7\lambda + 6 = 0$

$\therefore \lambda = -1, 6$

When, $\lambda = -1$

$[A - \lambda I] X = 0$

$\Rightarrow \begin{bmatrix} -5+1 & 2 \\ 2 & -2+1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -4 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-2x + y = 0$

$\cancel{2x - 3y = 0}$

$2x = y$

$\frac{x}{1} = \frac{y}{2}$

$\boxed{X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$

$\cancel{2x - 3y = 0}$

$\cancel{y - 3y = 0}$

$-4y = 0$

When, $\lambda = -6$

$$[A-\lambda I]X=0$$

$$\begin{bmatrix} -5+6 & 2 \\ 2 & -2+6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=0$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=0$$

$$x+2y=0$$

$$x=-2y$$

$$\frac{x}{-2}=\frac{y}{1}$$

$$\boxed{X_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}}$$

Q2. $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$\because$ A is triangular matrix

$\therefore \lambda = 3, 2, 5$

~~$\lambda = 3$~~

~~$A - \lambda I$~~ ~~$\begin{bmatrix} 3- \end{bmatrix}$~~

When, $\lambda = 3$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 3-3 & 1 & 4 \\ 0 & 2-3 & 6 \\ 0 & 0 & 5-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$0x + y + 4z = 0$

$0x - y + 6z = 0$

$$\frac{x}{6+4} = \frac{-y}{0} = \frac{z}{0}$$

$$\boxed{\underline{\underline{X_1}} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$

When, $\lambda = 2$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 3-2 & 1 & 4 \\ 0 & 2-2 & 6 \\ 0 & 0 & 5-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + y + 4z = 0$

$0x + 0y + 6z = 0$

$$\frac{x}{6} = \frac{-y}{6} = \frac{z}{0}$$

$$\boxed{X_2 = \begin{bmatrix} 6 \\ -6 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}$$

When, $\lambda = 5$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x + y + 4z = 0$

$0x - 3y + 6z = 0$

$$\frac{x}{6+12} = \frac{-y}{-12} = \frac{z}{6}$$

$$\boxed{X_3 = \begin{bmatrix} 18 \\ 12 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}}$$

Q3. $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

$\lambda^3 - 5\lambda^2 + \lambda(4-2+2+2+2) - 4 = 0$

$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$

$\lambda = 1, 2, 2$

When, $\lambda = 1$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 0 & 2 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$0x + 2y + 2z = 0$

$\Rightarrow 0x + y + z = 0$

$-x + 2y + z = 0$

$$\frac{x}{1-2} = \frac{-y}{0+1} = \frac{z}{0+1}$$

$$\boxed{X_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}}$$

When, $\lambda = 2$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} -1 & 2 & 2 \\ 0 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-x + 2y + 2z = 0$

$-x + 2y + 0z = 0$

$$\frac{x}{0-4} = \frac{-y}{0+2} = \frac{z}{-2+2}$$

$$\frac{x}{-4} = \frac{-y}{2} = \frac{z}{0}$$

$$\boxed{X_2 = \begin{bmatrix} -4 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}}$$

Q4. $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$

$\lambda = 1, 1, 1$

When, $\lambda = 1$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-x + y + 0z = 0$
$0x - y + z = 0$

$$\frac{x}{1} = \frac{-y}{-1} = \frac{z}{1}$$

$$\boxed{X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

Q5.

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$\lambda^3 + \lambda^2 + \lambda(\cancel{7}12 - 3 - 2 - 4) - 45 = 0$$

$$\lambda^3 + \lambda^2 \cancel{\lambda} - 21\lambda - 45 = 0$$

$$\lambda = 5, -3, -3$$

When, $\lambda = 5$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} -2-5 & 2 & -3 \\ 2 & 1-5 & -6 \\ -1 & -2 & 0-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x - 2y - 3z = 0$$
$$-x + 2y + 5z = 0$$

$$\frac{x}{-10+6} = \frac{-y}{5-3} = \frac{z}{2-2}$$

$$\boxed{X_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}}$$

When, $\lambda = -3$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} -2+3 & 2 & -3 \\ 2 & 1+3 & -6 \\ -1 & -2 & 0+3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\cancel{5x + 2y - 3z = 0}$

$\cancel{x + 2y - 3z = 0}$

$\cancel{\dfrac{x}{-6+6} = \dfrac{-y}{-15+3} = \dfrac{z}{10-2}}$

$\cancel{X_2 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}}$

$\cancel{x + 2y - 3z = 0}$

$\cancel{x + 2y - 3z = 0}$

$\cancel{\dfrac{x}{-6+6} = \dfrac{-y}{-3+3} = \dfrac{z}{2-2}}$

$\cancel{\dfrac{x}{0} = \dfrac{y}{0} = \dfrac{z}{0}}$

$\cancel{x + 2y - 3z = 0}$

$x + 2y - 3z = 0$

$\cancel{\text{When, } z = 0}$

$\cancel{x = 3z}$

$y = 0$

$\cancel{\dfrac{x}{3} = \dfrac{z}{1}}$

$x_2 z$

$y = 0$

$x + 2y - 3z = 0$

When, $z = 0$

$\therefore x = -2y$

$\therefore \dfrac{x}{-2} = \dfrac{y}{1}$

$X_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

When, $y = 0$

$x = 3z$

$\dfrac{x}{3} = \dfrac{z}{1}$

$X_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

$\cancel{X_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$

Q6.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

$\lambda^3 - 7\lambda^2 + \lambda(5-1+1-9+5-1) + 36 = 0$

$\lambda^3 - 7\lambda^2 + 36 = 0$

$\boxed{\lambda = -2, 6, 3}$

When $\lambda = -2$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} 1+2 & 1 & 3 \\ 1 & 5+2 & 1 \\ 3 & 1 & 1+2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$3x + y + 3z = 0$

$x + 7y + z = 0$

$$\frac{x}{1-24} = \frac{-y}{3-3} = \frac{z}{24-1}$$

$$\frac{x}{-23} = \frac{-y}{0} = \frac{z}{23}$$

$$\boxed{X_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}$$

Scanned with CamScanner

When, $\lambda = 6$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 1-6 & 1 & 3 \\ 1 & 5-6 & 1 \\ 3 & 1 & 1-6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-5x + y + 3z = 0$$

$$x - y + z = 0$$

$$\frac{x}{1+3} = \frac{-y}{-5-3} = \frac{z}{5-1}$$

$$\frac{x}{4} = \frac{-y}{-8} = \frac{z}{4}$$

$$\boxed{X_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}$$

When, $\lambda = 3$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 1-3 & 1 & 3 \\ 1 & 5-3 & 1 \\ 1 & 1 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x + y + 3z = 0$

$x + 2y + z = 0$

$$\frac{x}{1-6} = \frac{-y}{-2-3} = \frac{z}{-4-1}$$

$$\frac{x}{-5} = \frac{-y}{-5} = \frac{z}{-5}$$

$$\boxed{X_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}}$$

Q7. $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

$$\lambda^3 - 3\lambda^2 + \lambda(8 - 6 - 6 + 5 - 12 + 14) - 1 = 0$$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$$

$$\lambda = 1, 1, 1$$

When, $\lambda = 1$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} -3-1 & -7 & -5 \\ 2 & 4-1 & 3 \\ 1 & 2 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$4x + 7y + 5z = 0$$
$$x + 2y + z = 0$$

$$\frac{x}{7-10} = \frac{-y}{4-5} = \frac{z}{8-7}$$

$$\boxed{X_1 = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}}$$

Q8. $A = \begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$

$\lambda^3 + \lambda^2 + \lambda(3-5+-9-10+-3+8) + 20 = 0$

$\lambda^3 + \lambda^2 - 16\lambda + 20 = 0$

$\lambda = -5, 2, 2$

When, $\lambda = -5$

$[A - \lambda I]X = 0$

$\begin{bmatrix} 3+5 & -2 & -5 \\ 4 & -1+5 & -5 \\ -2 & -1 & -3+5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\begin{bmatrix} 8 & -2 & -5 \\ 4 & 4 & -5 \\ -2 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$4x + 4y - 5z = 0$

$-2x + y - 2z = 0$

$\frac{x}{-8+5} = \frac{-y}{-8-10} = \frac{z}{4+8} \Rightarrow \frac{x}{-3} = \frac{y}{18} = \frac{z}{12}$

$\boxed{X_1 = \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}}$

When $\lambda = 2$

$$\begin{bmatrix} 3-2 & -2 & -5 \\ 4 & -1-2 & -5 \\ -2 & -1 & -3-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -2 & -5 \\ 4 & -3 & -5 \\ -2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x - 2y - 5z = 0$$
$$2x + y + 5z = 0$$

$$\frac{x}{-10+5} = \frac{-y}{5+10} = \frac{z}{1+4}$$

$$\frac{x}{-5} = \frac{-y}{15} = \frac{z}{5}$$

$$\boxed{x_2 = \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}}$$

Scanned with CamScanner

Q9. $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$. $4A^{-1} + 3A + 2I$

$\because$ In lower triangular form

$\lambda = 1, 4$

When, $\lambda = 1$

$4 \cdot \frac{1}{1} + 3 \cdot 1 + 2 \cdot 1$

$\Rightarrow 4 + 3 + 2$

$\Rightarrow \boxed{9}$

When, $\lambda = 4$

$4 \cdot \frac{1}{4} + 3 \cdot 4 + 2 \cdot 4$

$\Rightarrow 1 + 12 + 8$

$\Rightarrow \boxed{21}$

Q12.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\lambda^3 - 12\lambda^2 + \lambda(9 - 1 + 18 - 4 + 18 - 4) - 32 = 0$$

$$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$\therefore \lambda = 8, 2, 2$$

~~Now,~~ $\cancel{\lambda_1 \cdot \lambda_2 = 16}$

$\cancel{\therefore \lambda_1 = 8, \; \lambda_2 = 2}$

~~And,~~ $\cancel{\lambda_1 \lambda_2 \lambda_3 = |A|}$

OR

$$\lambda_1 \lambda_2 = 16$$

$$\lambda_1 \lambda_2 \lambda_3 = 32$$

$$16 \lambda_3 = 32$$

$$\boxed{\lambda_3 = 2}$$

Q14

a.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 5 & 4 \end{bmatrix}$$

$\lambda^3 - 9\lambda^2 + \lambda(16-25+4+4) + 9 = 0$

$\lambda^3 - 9\lambda^2 + 4 - \lambda + 9 = 0$

$\lambda = 9, 1, -1$

When, $\lambda = 9$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 1-9 & 0 & 0 \\ 0 & 4-9 & 5 \\ 0 & 5 & 4-9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -8 & 0 & 0 \\ 0 & -5 & 5 \\ 0 & 5 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-8x = 0$

$\boxed{x = 0}$

$-5y + 5z = 0$

$5y = 5z$

$\boxed{y = z}$

$$\boxed{X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}$$

When, $\lambda = -1$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} 1+1 & 0 & 0 \\ 0 & 4+1 & 5 \\ 0 & 5 & 4+1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\cancel{2x = 0} \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 5 \\ 0 & 5 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$2x = 0$

$\boxed{x = 0}$

$5y + 5z = 0$

$5y = -5z$

$\frac{y}{-5} = \frac{z}{5}$

$$\boxed{X_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}$$

When, $\lambda = 1$

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 0 & 4-1 & 5 \\ 0 & 5 & 4-1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 5 \\ 0 & 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$0x + 3y + 5z = 0$
$0x + 5y + 3z = 0$

$$\frac{x}{9-25} = \frac{-y}{0} = \frac{z}{0}$$

$$\frac{x}{-16} = \frac{-y}{0} = \frac{z}{0}$$

$$\boxed{X_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$

b. $A = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix}$

$\lambda^3 - 9\lambda^2 + \lambda(4-4+16-16+4-4) = 0$

$\lambda^3 - 9\lambda^2 = 0$

$\lambda - 9 = 0$

$\boxed{\therefore \lambda = 9, 0, 0}$

when, $\lambda = 0$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$2x + y + 2z = 0$

$\cancel{2x + y + 2z = 0}$

when, $x = 0$

$y = -2z$

$\frac{y}{-2} = \frac{z}{1}$

$X_1 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$

when, $z = 0$

$2x = -z$

$\frac{x}{-1} = \frac{z}{2}$

$X_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

When, $\lambda = 9$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} 4-9 & 2 & 4 \\ 2 & 1-9 & 2 \\ 4 & 2 & 4-9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 2 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-5x + 2y + 4z = 0$

$x - 4y + z = 0$

$$\frac{x}{2+16} = \frac{-y}{-5-4} = \frac{z}{20-2}$$

$$\frac{x}{18} = \frac{y}{9} = \frac{z}{18}$$

$$\boxed{X_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}}$$

c. $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

$\lambda^3 - 6\lambda^2 + \lambda[-4 + 9 - 16 + -4] - 8 = 0$

$\lambda^3 - 6\lambda^2 - 15\lambda - 8 = 0$

$\therefore \boxed{\lambda = 8, -1, -1}$

When, $\lambda = 8$

$[A - \lambda I]X = 0$

$\begin{bmatrix} 3-8 & 2 & 4 \\ 2 & 0-8 & 2 \\ 4 & 2 & 3-8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\begin{bmatrix} -5 & 2 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$-5x + 2y + 4z = 0$

$x - 4y + z = 0$

$\frac{x}{2+16} = \frac{-y}{-5-4} = \frac{z}{20-2}$

$\frac{x}{18} = \frac{y}{9} = \frac{z}{18}$

$\boxed{X_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}}$

Scanned with CamScanner

When, $\lambda = -1$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 3+1 & 2 & 4 \\ 2 & 0+1 & 2 \\ 4 & 2 & 3+1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$2x + y + 2z = 0$

When, $x = 0$

$y = -2z$

$\frac{y}{-2} = \frac{z}{1}$

$$\boxed{X_2 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}}$$

When, $y = 0$

$2x = -2z$

$x = -z$

$\frac{x}{-1} = \frac{z}{1}$

$$\boxed{X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}$$

Q15 $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A - \lambda I = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = 0$

$\lambda^2 + 1 = 0$

$\therefore \boxed{\lambda = \pm i}$

Hence, Proved.

Q16 $A = \begin{bmatrix} 0 & 9 & -12 \\ -9 & 0 & 20 \\ 12 & -20 & 0 \end{bmatrix}$

$A - \lambda I = 0 \Rightarrow \begin{bmatrix} -\lambda & 9 & -12 \\ -9 & -\lambda & 20 \\ 12 & -20 & -\lambda \end{bmatrix} = 0$

$\Rightarrow -\lambda^3 + 3\lambda^3 + \lambda[\lambda^2 + 400 + \lambda^2 + 144 + \lambda^2 + 81] = 0$

$\Rightarrow \lambda^3 + 3\lambda^3 + 3\lambda^3 + 625\lambda = 0 \quad \Rightarrow 4\lambda^3 - 3\lambda^3 + 625\lambda = 0$

$\therefore \lambda^2 \Rightarrow 12\lambda^2 + 625 = 0$

$\lambda =$

$$\begin{bmatrix} -\lambda & 9 & -12 \\ -9 & -\lambda & 20 \\ 12 & -20 & -\lambda \end{bmatrix}$$

$$\lambda^3 + 3\lambda^3 + \lambda[\lambda^2 + 400 + \lambda^2 + 144 + \lambda^2 + 81] = 0$$

$$4\lambda^3 + \lambda[3\lambda^2 + 625] = 0$$

$$7\lambda^3 + 625\lambda = 0$$

$$7\lambda^2 + 625 = 0$$

$$\lambda = 0, \frac{25\sqrt{7}}{7} i, \frac{-25\sqrt{7}}{7} i$$

Q 17. $A = \frac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$|A - \lambda I| = 0$

$$\frac{1}{3}\begin{bmatrix} 2-\lambda & 1 & 2 \\ -2 & 2-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} = 0$$

$$\frac{1}{3}[\lambda^3 - 6\lambda^2 + 3\lambda^3 + \lambda[4 - 2\lambda - 2\lambda + \lambda^2 - 2 + 4 - 2\lambda - 2\lambda + \lambda^2 - 2 + 4 - 2\lambda - 2\lambda + \lambda^2 + 2] + 1/3] = 0$$

$$4\lambda^3 - 6\lambda^2 + \lambda[10 + 3\lambda^2 - 12\lambda] + 1/3 = 0$$

$$7\lambda^3 - 18\lambda^2 + 10\lambda + 1/3 = 0$$

$\lambda^3 -$

Q 12

a. $A = \begin{bmatrix} 9 & -1 \\ 5 & 7 \end{bmatrix}$

$A - \lambda I = 0$

$$\begin{bmatrix} 9-\lambda & -1 \\ 5 & 7-\lambda \end{bmatrix} = 0$$

$$63 - 9\lambda - 7\lambda + \lambda^2 + 5 = 0$$

$$\lambda^2 - 16\lambda + 68 = 0$$

$$\lambda = 8+2i, 8-2i$$

When, $\lambda = 8+2i$

$$\begin{bmatrix} 9-8-2i & -1 \\ 5 & 7-8-2i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-2i & -1 \\ 5 & -1-2i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$\cancel{(1-2i)x \quad -y = 0}$

$5x \quad -(-1-2i)y = 0$

$5x = (-1-2i)y$

$$\frac{x}{-1-2i} = \frac{y}{5}$$

$$X_1 = \begin{bmatrix} -1-2i \\ 5 \end{bmatrix}$$

When, $\lambda = 8-2i$

$$\begin{bmatrix} 9-8+2i & -1 \\ 5 & 7-8+2i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} 1+2i & -1 \\ 5 & 1+2i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$5x + (1+2i)y = 0$

$5x = -(1+2i)y$

$\dfrac{x}{-(1+2i)} = \dfrac{y}{5}$

$$\boxed{X_2 = \begin{bmatrix} -(1+2i) \\ 5 \end{bmatrix}}$$

b. $A = \begin{bmatrix} -3 & -2 \\ 5 & -1 \end{bmatrix}$

$A - \lambda I = 0 \Rightarrow \begin{bmatrix} -3-\lambda & -2 \\ 5 & -1-\lambda \end{bmatrix} \Rightarrow$

$\Rightarrow (-3-\lambda)(-1-\lambda) + 10 = 0$

$\Rightarrow 3 + 3\lambda + \lambda + \lambda^2 + 10 = 0$

$\Rightarrow \lambda^2 + 4\lambda + 13 = 0$

$\lambda = -2+3i, -2-3i$

When, $\lambda = -2+3i$

$[A-\lambda I]X = 0 \Rightarrow \begin{bmatrix} -3+2-3i & -2 \\ 5 & -1+2-3i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -1-3i & -2 \\ 5 & 1-3i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$(-1-3i)x - 2y = 0$

$(-1-3i)x = 2y$

$\frac{x}{2} = \frac{y}{-1-3i}$

$\boxed{X_1 = \begin{bmatrix} 2 \\ -1-3i \end{bmatrix}}$

When, $\lambda = -2-3i$

$[A-\lambda I]X = 0 \Rightarrow \begin{bmatrix} -3+2+3i & -2 \\ 5 & -1+2+3i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -1+3i & -2 \\ 5 & 1+3i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$(-1+3i)x - 2y = 0$

$(-1+3i)x = 2y$

$\frac{x}{2} = \frac{y}{-1+3i}$

$\boxed{X_2 = \begin{bmatrix} 2 \\ -1+3i \end{bmatrix}}$

When $\lambda = 3-2i$

$[A-\lambda I]X = 0$

$$\begin{bmatrix} 6-3+2i & -13 \\ 1 & -3+2i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$x + (-3+2i)y = 0$$

$$x = -(3-2i)y$$

$$\frac{x}{3-2i} = \frac{-y}{1}$$

$$\boxed{X_2 = \begin{bmatrix} 3-2i \\ 1 \end{bmatrix}}$$

Scanned with CamScanner

d.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$\begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & -\lambda & 3 \\ 0 & -3 & -\lambda \end{bmatrix} = 0$$

$$\lambda^3 - \lambda^2(3 - 3\lambda) + \lambda(\lambda^2 + 9 \cancel{1} - 3\lambda + \lambda^2 \cancel{1} - 3\lambda + \lambda^2) - 27 =$$

$$\lambda^3 - 3\lambda^2 + 3\lambda^3 + \lambda(3\lambda^2 + 9 - 6\lambda) - 27 = 0$$

$$4\lambda\lambda^3 - 3\lambda^2 + 3\lambda^3 + 9\lambda - 6\lambda^2 - 27 = 0$$

$$7\lambda^3 - 9\lambda^2 + 9\lambda - 27 = 0$$

Q 19. $A = \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}$

$A - \lambda I = 0$

$$\begin{bmatrix} \frac{3}{5} - \lambda & -\frac{4}{5} \\ \frac{4}{5} & 3/5 - \lambda \end{bmatrix} = 0$$

$$\left(\frac{3}{5} - \lambda\right)^2 + \frac{16}{25} = 0 \Rightarrow \frac{9}{25} + \lambda^2 - \frac{6\lambda}{5} + \frac{16}{25} = 0$$

$$\Rightarrow \lambda^2 - \frac{6\lambda}{5} + 1 = 0$$

$$\lambda = \frac{3 + 4i}{5}, \frac{3 \quad 4i}{5}$$

When, $\lambda = \frac{3+4i}{5}$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} \frac{3-3-4i}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3-3-4i}{5} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} \frac{-4i}{5} & -\frac{4}{5} \\ \frac{4}{5} & -\frac{4i}{5} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$\frac{4}{5}$ $4x - 4i\, y = 0$

$x = iy$

$$\frac{x}{i} = \frac{y}{1}$$

$$\boxed{X_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}}$$

152. 09, 11, 13, 17

When $\lambda = \frac{3-4i}{5}$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} \frac{3-3+4i}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3-3+4i}{5} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} \frac{4i}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{4i}{5} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$\frac{4i}{5}x + \frac{4i}{5}y = 0$

$ix + iy = 0$

$ix = iy$

$\frac{x}{-i} = \frac{y}{1}$

$$\boxed{X_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}}$$

Q13

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\lambda^3 - 5\lambda^2 + \lambda[2 + 2 + 4 - 1] - 3 = 0$

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

$\lambda = 3, 1, 1$

Scanned with CamScanner

When, $\lambda = 1$

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + y + z = 0$

$x = 0$

$y = -z$

$\frac{y}{-1} = \frac{z}{1}$

$y = 0$

$x = -z$

$\frac{x}{-1} = \frac{z}{1}$

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \qquad X_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore AM = 2 \; ; GM = 2$

When, $\lambda = 3$

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x - y + z = 0$

$0x + 0y - 2z = 0$

$\frac{x}{2} = \frac{-y}{-2} = \frac{z}{0}$

$$X_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore A.M = 1, G.M = 1$

Q17. $A = \frac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\lambda^3 - 6\lambda^2 + \lambda(4-2+4-2+4+2) - \frac{2}{3} + 3 = 0$

$-1 - 6 - 10$

$\lambda^3 - 6\lambda^2 + 10\lambda + 3 = 0$

Q11. $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$

$\lambda^3 - 5\lambda^2 + \lambda(-2+2+6-2) - 4 = 0$

$\lambda = 1, 2, 2$

~~When, $\lambda = 1$~~
~~$[A - \lambda I]$~~
$A^2 = \begin{bmatrix} 9 & 3 & -4 \\ 8 & 4 & -4 \\ 10 & 6 & -4 \end{bmatrix}$

When, $\lambda = 1$

$[A^2 - \lambda I]X = 0 \Rightarrow \begin{bmatrix} 9-1 & 3 & -4 \\ 8 & 4-1 & -4 \\ 10 & 6 & -4-1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 8 & 3 & -4 \\ 8 & 3 & -4 \\ 10 & -6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

Page No
Date

$8x+3y-4z=0$
$10x-6y-5z=0$

$$\frac{x}{-15-24} = \frac{-y}{-40+40} = \frac{z}{-48-30}$$

$$\frac{x}{-39} = \frac{-y}{0} = \frac{z}{-78}$$

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

when, $\lambda = 2$

$[A^2 - \lambda I] X = 0$ $\Rightarrow \begin{bmatrix} 9-2 & 3 & -4 \\ 8 & 4-2 & -4 \\ 10 & 6 & -4-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$$\begin{bmatrix} 7 & 3 & -4 \\ 8 & 2 & -4 \\ 10 & 6 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$7x+3y-4z=0$
$4x+y-2z=0$

$$\frac{x}{-6+4} = \frac{-y}{-14+16} = \frac{z}{7-12}$$

$$\frac{x}{-2} = \frac{y}{-2} = \frac{z}{-5} \qquad X_2 = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$$

Tuto - 2

Q1. $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

$\lambda^3 - 6\lambda^2 + \lambda(-3 - 8 + 8 + 6 + -24 + 32) - 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda = 1, 3, 2$

When, $\lambda = 1$

$[A - \lambda I]X = 0 \Rightarrow \begin{bmatrix} 8-1 & -8 & -2 \\ 4 & -3-1 & -2 \\ 3 & -4 & 1-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$7x - 8y - 2z = 0$

$2x - 2y - z = 0$

$\dfrac{x}{8-4} = \dfrac{-y}{-7+4} = \dfrac{z}{-14+16}$

$\dfrac{x}{4} = \dfrac{y}{3} = \dfrac{z}{2}$

$X_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$

When, $\lambda = 3$

$$[A-\lambda I]X = 0 \quad \begin{bmatrix} 8-3 & -8 & -2 \\ 4 & -3-3 & -2 \\ 3 & -4 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$5x - 8y - 2z = 0$$
$$2x - 3y \mp z = 0$$

$$\frac{x}{8-6} = \frac{-y}{-5+4} = \frac{z}{-15+16}$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$$

$$X_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

When, $\lambda = 2$

$[A - \lambda I]\,X = 0$

$$\Rightarrow \begin{bmatrix} 8-2 & -8 & -2 \\ 4 & -3-2 & -2 \\ 3 & -4 & 1-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$4x - 5y - 2z = 0$

$3x - 4y - z = 0$

$$\frac{x}{5-8} = \frac{-y}{-4+6} = \frac{z}{-16+15}$$

$$\frac{x}{-3} = \frac{-y}{2} = \frac{z}{-1}$$

$$X_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 4 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Q2. $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

$\cancel{A-\lambda I}$ $\lambda^3 - 8\lambda^2 + \lambda[3-8+4-4+12+10] - 10 =$

$\lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$

$\lambda = 5, 2, 1$

When, $\lambda = 5$

$[A-\lambda I]X = 0$ $\Rightarrow \begin{bmatrix} 4-5 & 2 & -2 \\ -5 & 3-5 & 2 \\ -2 & 4 & 1-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -1 & 2 & -2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$-x + 2y - 2z = 0$

$-5x - 2y + 2z = 0$

$\frac{x}{4-4} = \frac{-y}{-2+10} = \frac{z}{2+10}$

$X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

When, $\lambda = 2$

$[A - \lambda I]\,X = 0$

$$\Rightarrow \begin{bmatrix} 4-2 & 2 & -2 \\ -5 & 3-2 & 2 \\ -2 & 4 & 1-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + y - z = 0$

$-5x + y + 2z = 0$

$$\frac{x}{2+1} = \frac{-y}{2-5} = \frac{z}{1+5}$$

$$\frac{x}{3} = \frac{-y}{-3} = \frac{z}{6}$$

$$X_2 = \begin{bmatrix} 1 \\ \cancel{2}1 \\ 2 \end{bmatrix}$$

When, $\lambda = 1$

$[A - \lambda I] X = 0$

$$\Rightarrow \begin{bmatrix} 4-1 & 2 & -2 \\ -5 & 3-1 & 2-1 \\ -2 & 4 & 1-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$3x + 2y - 2z = 0$

$-5x + 2y + 2z = 0$

$$\frac{x}{4+4} = \frac{-y}{6-10} = \frac{z}{6+10}$$

$$\frac{x}{8} = \frac{-y}{4} = \frac{z}{16}$$

$$X_3 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q3. $A = \begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$\lambda^3 + \lambda^2 + \lambda[-12+3-2-4] - 27 = 0$

$\lambda^3 + \lambda^2 - 15\lambda - 27 = 0$

$\therefore \lambda = 5, -3, -3$

When, $\lambda = 5$

$[A - \lambda I]X = 0$ $\Rightarrow \begin{bmatrix} -2-5 & 2 & 3 \\ 2 & 1-5 & -6 \\ -1 & -2 & 0-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\begin{bmatrix} -7 & 2 & 3 \\ 2 & -4 & -6 \\ 1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$x - 2y - 3z = 0$

$x - 2y - 5z = 0$

$\Rightarrow \frac{x}{10-6} = \frac{-y}{-5+3} = \frac{z}{-2+2}$

$\Rightarrow \frac{x}{4} = \frac{y}{2} = \frac{z}{0}$

$\boxed{X_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}}$

When, $\lambda = -3$

$[A - \lambda I] X = 0$

$$\Rightarrow \begin{bmatrix} -2+3 & 2 & 3 \\ 2 & 1+3 & -6 \\ -1 & -2 & 0+3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + 2y + 3z = 0$

$x + 2y - 3z = 0$

$$\cancel{\frac{x}{-6-6} = \frac{-y}{-3-3} = \frac{z}{2-2}}$$

$$\frac{x}{-6-6} = \frac{-y}{-3-3} = \frac{z}{2-2}$$

$$X_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$x + 2y - 3z = 0$

$y = 0$

$x = 3z$

$\frac{x}{3} = \frac{z}{1}$

$$X_3 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$D = M^{-1}$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Scanned with CamScanner

Page No

Date

Q5. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $\lambda = 1, 2, 2$

When, $\lambda = 2$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-x + 0y + 3z = 0$

When, $x = 0$

$0y = 3z$

$\frac{y}{3} = \frac{z}{0}$

$$X = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$\therefore am = 2, gm = 1$

$\because am \neq gm$

$\therefore$ Not diagonalisable

Page No
Date

Q6. $A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ $\lambda = 1, 1, 1$

When, $\lambda = 1$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} 0 & 3 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$0x + 3y - 2z = 0$

~~When, $x = 0$~~

~~$3y = 2z$~~

~~$\frac{y}{2} = \frac{z}{3}$~~

~~$X = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$~~

$0x + 0y + 4z = 0$

$\frac{x}{12} = \frac{y}{0} = \frac{z}{0}$

$X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$am = 3$; $gm = 1$

$\because am \neq gm$

$\therefore$ Non-diagonalisable.

Q7. $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

Q9. $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\Rightarrow \lambda^3 - 6\lambda^2 + \lambda(4-1+4-1+4-1) - 4 = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

Applying Cley. Ham. Theo.

$A^3 - 6A^2 + 9A - 4I$

$$\Rightarrow \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} + \begin{bmatrix} -36 & 30 & -30 \\ 30 & -36 & 30 \\ -30 & 30 & -36 \end{bmatrix}$$

$$+ \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & \cancel{-18}-9 \\ 9 & -9 & 18 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For A^{-1}

$A^{-1}[A^3-6A^2+9A-4I]=0$

$\Rightarrow A^2-6A+9I-4A^{-1}=0$

$\Rightarrow A^{-1}=\frac{1}{4}[A^2-6A+9I]$

$$\therefore A^{-1}=\frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Q 10. $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ $\Rightarrow \lambda^3-6\lambda^2+\lambda[6+3-4+2]+2=0$

$\Rightarrow \lambda^3-6\lambda^2+7\lambda+2=0$

Applying CHT

$A^3-6A^2+7A+2I$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}+\begin{bmatrix} -30 & 0 & -48 \\ -12 & -24 & -30 \\ -48 & 0 & -78 \end{bmatrix}+\begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix}$$

$$+\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For A^{-1}

$A^{-1}[A^3 - 6A^2 + 7A + 2I] = 0$

$\Rightarrow A^2 - 6A + 7I + 2A^{-1} = 0$

$\cancel{\Rightarrow A^{-1} = \frac{1}{2}[-A^2 + 6A - 7I]}$

$$\Rightarrow \frac{1}{2}\left[\begin{bmatrix} -5 & 0 & -8 \\ -2 & -4 & -5 \\ -8 & 0 & -13 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 12 \\ 0 & 12 & 6 \\ 12 & 0 & 18 \end{bmatrix} + \begin{bmatrix} -7 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -7 \end{bmatrix}\right]$$

$$A^{-1} \Rightarrow \begin{bmatrix} -3 & 0 & 2 \\ -1 & 1/2 & 1/2 \\ 2 & 0 & -1 \end{bmatrix}$$

Q11.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \Rightarrow \lambda^3 + \lambda[-12 - 12 - 4 + 6 + 3 - 1] - 8 = 0$$

$\Rightarrow \lambda^3 - 20\lambda - 8 = 0$

Applying CHT $\Rightarrow A^3 - 20A - 8I = 0$

$$\Rightarrow \begin{bmatrix} 12 & 20 & 60 \\ 20 & 52 & -60 \\ -40 & -80 & -88 \end{bmatrix} + \begin{bmatrix} -20 & -20 & -60 \\ -20 & -60 & 60 \\ 40 & 80 & 80 \end{bmatrix} + \begin{bmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For A^{-1}

$A^{-1}[A^3-20A-8I]=0$

$\Rightarrow A^2-20I-8A^{-1}=0$

$$A^{-1}=\frac{1}{8}[A^2-20I]$$

$$=\frac{1}{8}\left[\begin{bmatrix}-4 & -8 & -12\\ 10 & 22 & 6\\ 2 & 2 & 22\end{bmatrix}+\begin{bmatrix}-20 & 0 & 0\\ 0 & -20 & 0\\ 0 & 0 & -20\end{bmatrix}\right]$$

$$A^{-1}\Rightarrow\frac{1}{8}\begin{bmatrix}-24 & -8 & -12\\ 10 & 2 & 6\\ 2 & 2 & 2\end{bmatrix}$$

Q12. $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 1 & 1\\ 0 & 1 & 2\end{bmatrix}$ $\Rightarrow \lambda^3+\lambda[2-1+2+1-6]+2=0$

$\Rightarrow \lambda^3-2\lambda+2=0$

Applying CHT

$A^3-2A+2I=0$

$$\Rightarrow\begin{bmatrix}\cancel{29} & 32 & 50\\ 30 & \cancel{32} & 50\\ 12 & 14 & 22\end{bmatrix}+\begin{bmatrix}-2 & -4 & -6\\ -6 & -2 & -2\\ 0 & -2 & -4\end{bmatrix}+\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}28 & 28 & 44\\ 24 & 32 & 48\\ 12 & 12 & 20\end{bmatrix}$$

$A^3 - 2A + 2I = 0$

$A^2 - 2I + 2A^{-1} = 0$

$A^{-1} = \frac{1}{2}[2I - A^2]$

$$A^{-1} = \frac{1}{2}\left[\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -7 & -2 & -4 \\ -6 & -8 & -12 \\ -3 & -3 & -5 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -5 & -7 & -11 \\ -6 & -6 & -12 \\ -3 & -3 & -3 \end{bmatrix}$$

Q13

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

$\Rightarrow \lambda^3 - \lambda^2 + \lambda[-1-2] - 1 = 0$

$\Rightarrow \lambda^3 - \lambda^2 - 3\lambda - 1 = 0$

~~A~~ Applying CHT

$A^3 - A^2 - 3A - I = 0$

$A^2 - A - 3I - A^{-1} = 0$

$\therefore A^{-1} = A^2 - A - 3I$

$$\therefore A^{-1} = \begin{bmatrix} -3 & 0 & 1 \\ 2 & -2 & -3 \\ -4 & -1 & 4 \end{bmatrix}$$

Now

$A^4 - A^3 - 3A^2 - A = 0$

$\therefore A^4 = A^3 + 3A^2 + A$

$$\therefore A^4 = \begin{bmatrix} 7 & 6 & 2 \\ 4 & 3 & 0 \\ 4 & 4 & 3 \end{bmatrix}$$

Q14 $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

$= \lambda^3 - 6\lambda^2 + \lambda[4+4-1+4] - 6$

$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6.$

Applying CHT

$A^3 - 6A^2 + 11A - 6I = 0$

$\cancel{A^3 - 6A^2 + 11A - 6I = 0}$

$\cancel{\Rightarrow A}$ For A^{-1}

$A^2 - 6A + 11\cancel{A} - 6A^{-1} = 0$

$A^{-1} = \frac{1}{6}[A^2 - 6A + 11\cancel{I}]$

$$A^{-1} = \frac{1}{6}\left[\begin{bmatrix} 5 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 5 \end{bmatrix} + \begin{bmatrix} -12 & 0 & 6\cancel{12} \\ 0 & -12 & 0 \\ 6 & 0 & -12 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right]$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 0 & 2 \\ 0 & 3 & 6 \\ 2 & 0 & 4 \end{bmatrix}$$

For A^4

$A^4 - 6A^3 + 11A^2 - 6A = 0$

$A^4 = 6A^3 - 11A^2 + 6A$

$$\therefore A^4 = \begin{bmatrix} 41 & 0 & -40 \\ 0 & 16 & 0 \\ -40 & 0 & 41 \end{bmatrix}$$

Page No. Date

Q15.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

$\lambda^3 + \lambda[-12-12-4+6+3-1]$
$+8$
$\lambda^3 - 20\lambda + 8$.

Applying CHT

$A^3 - 20A + 8I = 0$

For A^{-1}

$A^2 - 20I + 8A^{-1} = 0$

$\Rightarrow A^{-1} = \frac{1}{8}[20I - A^2]$

$$\Rightarrow A^{-1} = \frac{1}{8}\begin{bmatrix} 24 & 8 & 12 \\ -10 & -2 & -6 \\ -2 & -2 & -2 \end{bmatrix}$$

For A^4

$A^4 - 20A^2 + 8A = 0$

$\Rightarrow A^4 = 20A^2 - 8A$

$$\Rightarrow A^4 = \begin{bmatrix} -88 & -168 & -264 \\ 192 & 416 & 144 \\ 56 & 72 & 472 \end{bmatrix}$$

Q16 $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

$\lambda^3 - 7\lambda^2 + \lambda[-21+20+21-15-9+20] - 12 = 0$

$\Rightarrow \lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

Applying CHT

$A^3 - 7A^2 + 16A - 12I = 0$

~~No~~

Now

$A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$

$\Rightarrow A^3(A^3 - 7A^2 + 16A - 12I) + A^5 - 7A^4 + 8A^3 - 12A^2 + 2A - I$

$\Rightarrow A^5 - 7A^4 + 8A^3 - 12A^2 + 2A - I$

$\Rightarrow A^2(A^3 - 7A^2 + 16A - 12I) - 8A^3 + 2A - I$

$\Rightarrow -8A^3 + 2A - I$

$$\Rightarrow \begin{bmatrix} 64 & -120 & 80 \\ 416 & 1256 & 944 \\ -736 & -2160 & -1664 \end{bmatrix} + \begin{bmatrix} 6 & 20 & 10 \\ -4 & -6 & -8 \\ 6 & 10 & 14 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 69 & -100 & 90 \\ 412 & 1249 & 936 \\ -730 & -2150 & -1651 \end{bmatrix}$$

$-8(A^3 - 7A^2 + 16A - 12I) + 56A^2 - $ [illegible]

Q17. $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$\lambda^3 - 6\lambda^2 + \lambda[6+2-3+3+2] - 3$

$\Rightarrow \lambda^3 - 6\lambda^2 + 10\lambda - 3$

Applying CHT

$A^3 - 6A^2 + 10A - 3I = 0$

Now

$A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$

$\Rightarrow A^6[A^3 - 6A^2 + 10A - 3I] + A + I$

$\Rightarrow A + I$

$\Rightarrow \begin{bmatrix} 2 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

Tuto - I

Q10. $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\lambda^3 - 7\lambda^2 + \lambda[6-2+4-1+6-2] - 5 = 0$

$\Rightarrow \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\therefore \lambda = \cancel{5,2,2}\ 5, 1, 1$

For $A^3 + I$ $\lambda = 5$, $A = 126$, $\cancel{\lambda = 2}\ \lambda = 1$, $A = 2$

For $A = 5$

$(A - \lambda I]X = 0$

$\Rightarrow \begin{bmatrix} 2-5 & 2 & 1 \\ 1 & 3-5 & 1 \\ 1 & 2 & 2-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$3x - 2y - z = 0$

$x - 2y + z = 0$

$\frac{x}{-2-2} = \frac{-y}{3+1} = \frac{z}{-6+2}$

$\Rightarrow \frac{x}{-4} = \frac{-y}{4} = \frac{z}{-4}$

$X_1 = \cancel{\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

For $A = 1$

$[A - \lambda I]X = 0$

$$\rightarrow \begin{bmatrix} 2-1 & 2 & 1 \\ 1 & 3-1 & 1 \\ 1 & 2 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + y + z = 0$

$0x + 2y + z = 0$

$$\frac{x}{1-2} = \frac{-y}{1-0} = \frac{z}{2}$$

$$X = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

$x + 2y + z = 0$

When, $x = 0$

$2y = -z$

$$\frac{y}{-1} = \frac{z}{2}$$

$$X_2 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

When, $y = 0$

$x = -z$

$$\frac{x}{-1} = \frac{z}{1}$$

$$X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Q17. $A = \frac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$|A - \lambda I| = 0$

$\Rightarrow \frac{1}{3}\left[2-\ldots\right.\ \frac{1}{3}\left[\lambda^3 - 6\lambda^2 + \lambda[4-2+4-2+4+2] + 1\right] =$

$\Rightarrow \lambda^3 - 6\lambda^2 + 10\lambda + 1 = 0$

$\therefore \lambda =$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 2-\lambda & 1 & 2 \\ -2 & 2-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} = 0$$

$\Rightarrow \lambda^3 - \lambda^2[6-3\lambda] + \lambda[4-2\lambda-2\lambda+\lambda^2 + 4-2\lambda-2\lambda+\lambda^2 \mp 2 + 4-2\lambda$
$-2\lambda+\lambda^2+2] + 1 = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + 3\lambda^3 + \lambda[12 - 12\lambda + 3\lambda^2] + 1 = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + 3\lambda^3 + 12\lambda - 12\lambda^2 + 3\lambda^3 + 1 = 0$

$\Rightarrow 7\lambda^3 - 18\lambda^2 + 12\lambda + 1 = 0$

d. $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & -3 & 0 \end{bmatrix}$

$\begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$\lambda^3 - 3\lambda^2 + 9\lambda - 27 = 0$

$\lambda = 3, -3i, +3i$

$\cancel{\lambda^3 - \lambda^2[9 - 3\lambda] + \lambda[+27} = 0$

$\lambda^3 - 3\lambda^2 + 9\lambda + 27 = 0$

~~When, $\lambda = 3$~~

When, $\lambda = 3$

$|A - \lambda I| X = 0$

$\begin{bmatrix} 3-3 & 0 & 0 \\ 0 & 0-3 & 3 \\ 0 & -3 & 0-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -3 & 3 \\ 0 & -3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$0x - 3y + 3z = 0$

$0x - 3y - 3z = 0$

$\frac{x}{9+9} = \frac{-y}{0} = \frac{z}{0}$

$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

When, $\lambda = 3i$

$|A - \lambda I| X = 0$

$\begin{bmatrix} 3-3i & 0 & 0 \\ 0 & 0-3i & 3 \\ 0 & -3 & 0-3i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\begin{bmatrix} 3-3i & 0 & 0 \\ 0 & -3i & 3 \\ 0 & -3 & -3i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$0x - 3iy + 3z = 0$

$0x - 3y - 3iz = 0$

$\frac{x}{3i^2 + 9} = \frac{-y}{0} = \frac{z}{0}$

$X_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

NOTES

$\lambda = -3i$

$$\begin{bmatrix} 3+3i & 0 & 0 \\ 0 & 3i & 3 \\ 0 & -3 & 3i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$(3+3i)x + 0y + 0z = 0$

$0x + 3iy + 3z = 0$

$$\frac{x}{0} = \frac{-y}{9+9i} = \frac{z}{9i-9}$$

$$= \begin{bmatrix} 0 \\ 9+9i \\ 9-9i \end{bmatrix}$$

$= 0$

Scanned with CamScanner

UNIT-IV

TUTORIAL-III

Q1.

(1) $x_1^2+x_2^2-3x_3^2+2x_1x_2-4x_1x_3+6x_2x_3$

$$A=\begin{bmatrix}1 & 1 & -2\\ 1 & 1 & 3\\ -2 & 3 & -3\end{bmatrix}$$

(2) $2x_1^2-3x_2^2+4x_3^2-x_4^2-x_1x_2+2x_1x_3-3x_1x_4+2x_2x_3-4x_2x_4+6x_3x_4$

$$A=\begin{bmatrix}2 & -1/2 & 1 & -3/2\\ -1/2 & -3 & 1 & -2\\ 1 & 1 & 4 & 3\\ -3/2 & -2 & 3 & -1\end{bmatrix}$$

Q2

$$Y=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 5\\ 1 & 3 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad (2,0,5) \text{ in } Y$$

$$\begin{bmatrix}2\\ 0\\ 5\end{bmatrix}=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 5\\ 1 & 3 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$R_2 \to R_2-R_1$; $R_3 \to R_3-R_1$

$$\begin{bmatrix}2\\ -2\\ 3\end{bmatrix}=\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 3\\ 0 & 2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$R_3 \to R_3-2R_2$

$$\begin{bmatrix}2\\ -2\\ 7\end{bmatrix}=\begin{bmatrix}1 & 1 & 2\\ 0 & 1 & 3\\ 0 & 0 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Eqn are

$x_1 + x_2 + 2x_3 = 2$ — (1)

$x_2 + 3x_3 = -2$ — (2)

$-5x_3 = 7$

$\Rightarrow \boxed{x_3 = \frac{-7}{5}}$

Put value of x_3 in (2)

$\Rightarrow x_2 - \frac{21}{5} = -2$

$\Rightarrow \boxed{x_2 = \frac{11}{5}}$

Put x_2 and x_3 values in (1)

$x_1 + \frac{11}{5} - \frac{14}{5} = 2$

$\Rightarrow x_1 - \frac{3}{5} = 2$

$\Rightarrow \boxed{x_1 = \frac{13}{5}}$

Q3

$$Y = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Y is (9, 6, 2)

$$\begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$; $R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 9 \\ 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$R_3 \to 3R_3 + R_2$

$$\begin{bmatrix} 9 \\ 3 \\ -12 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & -1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Eqn are

$2x_1 - x_2 + 3x_3 = 9$ -(1)

$\Rightarrow 2x_1 - 2 + 9 = 9$

$\Rightarrow 2x_1 + 7 = 9$

$\Rightarrow 2x_2 = 2$

$\Rightarrow \boxed{x_2 = 1}$

$3x_2 - x_3 = 3$ -(2)

$3x_2 - 3 = 3$

$\Rightarrow 3x_2 = 6$

$\Rightarrow \boxed{x_2 = 2}$

$-4x_3 = -D$

$\Rightarrow \boxed{x_3 = 3}$

Q4. $x_1 = 3y_1 + 2y_2$; $x_2 = -y_1 + 4y_2$

$y_1 = z_1 + 2z_2$; $y_2 = 3z_2$

x_1, x_2 in terms of z_1, z_2

$x_1 = 3[z_1 + 2z_2] + 2[3z_2]$

$\Rightarrow x_1 \Rightarrow 3z_1 + 6z_2 + 6z_2$

$\Rightarrow \boxed{x_1 = 3z_1 + 12z_2}$

$x_2 = -[z_1 + 2z_2] + 4[\cancel{-}3z_2]$

$\Rightarrow x_2 = -z_1 - 2z_2 + 12z_2$

$\Rightarrow \boxed{x_2 = 10z_2 - z_1}$

Q5. $x_1 = y_1 - 2y_2 + 3y_3$; $x_2 = 2y_1 + 3y_2 - y_3$; $x_3 = -3y_1 + y_2 + 2y_3$

$y_1 = z_1 + 2z_3$; $y_2 = z_2 + 2z_3$; $y_3 = z_1 + 2z_2$

$x_1 = z_1 + 2z_3 - 2[z_2 + 2z_3] + 3[z_1 + 2z_2]$

$\Rightarrow x_1 = z_1 + 2z_3 - 2z_2 - 4z_3 + 3z_1 + 6z_2$

$\Rightarrow \boxed{x_1 = 4z_1 + 4z_2 - 2z_3}$

$x_2 = 2[z_1 + 2z_3] + 3[z_2 + 2z_3] - [z_1 + 2z_2]$

$\Rightarrow x_2 = 2z_1 + 4z_3 + 3z_2 + 6z_3 - z_1 - 2z_2$

$\Rightarrow \boxed{x_2 = z_1 + z_2 + 10z_3}$

$x_3 = -3[z_1 + 2z_3] + [z_2 + 2z_3] + 2[z_1 + 2z_2]$

$\Rightarrow x_3 = -3z_1 - 6z_3 + z_2 + 2z_3 + 2z_1 + 4z_2$

$\Rightarrow \boxed{x_3 = -z_1 + 5z_2 - 4z_3}$

Q6. $6x_1^2+3x_2^2+3x_3^2-4x_1x_2+4x_1x_3-2x_2x_3$

$x_1 = y_1 + \frac{1}{3}y_2 - \frac{2}{7}y_3$

$x_2 = y_2 + \frac{1}{7}y_3$

$x_3 = y_3$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 1/3 & -2/7 \\ 0 & 1 & 1/7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ -2/7 & 1/7 & 1 \end{bmatrix}$$

$B = P'AP$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ -2/7 & 1/7 & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1/3 & -2/7 \\ 0 & 1 & 1/7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 7/3 & 0 \\ 0 & 0 & 16/7 \end{bmatrix}$$

$$\therefore Y'BY = 6y_1^2 + \frac{7}{3}y_2^2 + \frac{16}{7}y_3^2$$

Q9. $x^2+2y^2+2z^2-2xy-2yz+2zx$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_1 ; R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + C_1 ; C_3 \to C_3 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$YBY = u^2 + v^2 + w^2$

RANK = 3 INDEX = 3 SIGNATURE = 3

VALUE CLASS: Positive Definite

Q7.

1. $xy + yz + zx$

$$\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 + 2R_2$

$$\begin{bmatrix} 1 & 1/2 & 3/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_1 \to C_1 + 2C_2$

$$\begin{bmatrix} 2 & 1/2 & 3/2 \\ 1/2 & 0 & 1/2 \\ 3/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - \frac{1}{4} R_1 \,;\, R_3 \to R_3 - \frac{3}{4} R_1$

$$\begin{bmatrix} 2 & 1/2 & 3/2 \\ 0 & -1/8 & 1/8 \\ 0 & 1/8 & -9/8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ -1/4 & 1/2 & 0 \\ -3/4 & -3/2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - \frac{1}{4} C_1 \,;\, C_3 \to C_3 - \frac{3}{4} C_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1/8 & 1/8 \\ 0 & 1/8 & -9/8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ -1/4 & 1/2 & 0 \\ -3/4 & -3/2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1/4 & -3/4 \\ 2 & 1/2 & -3/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1/8 & -9/8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ -3/4 & -3/2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1/4 & -3/4 \\ 2 & 1/2 & -3/2 \\ 0 & 0 & 1 \end{bmatrix}$$

Scanned with CamScanner

$C_2 \to C_2 + C_3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -9/8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ -3/4 & -3/2 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -3/4 \\ 2 & -1 & -3/2 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1/8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 1/4 & -1/2 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -3/4 \\ 2 & -1 & -3/2 \\ 0 & 1 & 1 \end{bmatrix}$$

$C_3 \to C_3 - C_2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1/8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 1/4 & -1/2 & 0 \end{bmatrix} A \cancel{\begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1/340 & -1/2 \end{bmatrix}} \begin{bmatrix} 1 & 1 & 1/4 \\ 2 & -1 & -1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\therefore \boxed{2u^2 - v^2 - \frac{1}{8} w^2}$$

(2).

② $x^2+2y^2+2z^2-2xy-2yz+2zx$

$$\begin{bmatrix}1 & -1 & 1/2\\ -1 & 2 & -1\\ 1/2 & -1 & 2\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} A \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$R_2 \to R_2 + R_1 ;\ R_3 \to 2R_3 - R_1$

$$\begin{bmatrix}1 & -1 & 1/2\\ 0 & 1 & -1/2\\ \cancel{1/2}\,0 & -1 & 7/2\end{bmatrix} = \begin{bmatrix}1&0&0\\1&1&0\\-1&0&2\end{bmatrix} A \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$C_2 \to C_2 + C_1 ;\ C_3 \to 2C_3 - C_1$

$$\begin{bmatrix}1 & \cancel{-1/2}\,0 & 0\\ 0 & \cancel{2\,1/2}\,1 & -1\\ 0 & -1 & 7\end{bmatrix} = \begin{bmatrix}1&0&0\\1&1&0\\-1&0&2\end{bmatrix} A \begin{bmatrix}1&1&-1\\0&1&0\\0&0&2\end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ 0 & 0 & 6\end{bmatrix} = \begin{bmatrix}1&0&0\\1&1&0\\0&1&2\end{bmatrix} A \begin{bmatrix}1&1&-1\\0&1&0\\0&0&2\end{bmatrix}$$

$C_3 \to C_3 + C_2$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 6\end{bmatrix} = \begin{bmatrix}1&0&0\\1&1&0\\0&1&2\end{bmatrix} A \begin{bmatrix}1&1&0\\1&1&1\\0&0&2\end{bmatrix}$$

$\therefore\ u^2 + v^2 + 6w^2$

Q8. $2x^2 + y^2 - 3z^2 + 12xy - 8yz - 4zx$

$$\begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$; $R_3 \to R_3 + R_1$

$$\begin{bmatrix} 2 & 6 & -2 \\ 0 & -17 & 2 \\ 0 & 2 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - 3C_1$; $C_3 \to C_3 + C_1$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 2 \\ 0 & 2 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$R_3 \to 17R_3 + 2R_2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 2 \\ 0 & 0 & 81 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 11 & 2 & 17 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$C_3 \to 17C_3 + 2C_2$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 0 \\ 0 & 0 & 1377 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 11 & 2 & 17 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 11 \\ 0 & 1 & 2 \\ 0 & 0 & 17 \end{bmatrix}$$

$2u^2 - 17v^2 + 1377w^2$

$\therefore$ Rank $= 3$; Index $= 2$; Signature $= 1$

Value Class: Positive Indefinite

Q7. $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

$|A-\lambda I| = 0 \Rightarrow (1-\lambda)(-1-\lambda) - 4 = 0$

$\Rightarrow -1-\lambda+\lambda+\lambda^2-4=0$

$\Rightarrow \lambda^2 - 5 = 0$

$\Rightarrow \lambda^2 = 5$

Applying CHT

$A^2 = 5I$

Now,

$A^4 = A^2 \cdot A^2$

$= 5I \cdot 5I$

$\therefore A^4 = 25I$

And,

$A^8 = A^4 \cdot A^4$

$= 25I \cdot 25I$

$\therefore \boxed{A^8 = 625I}$