

**Problem Name TBD.** Given an array of  $n$  integers  $A_0, A_1, \dots, A_{n-1}$  where  $0 \leq i, j < n$  and  $\gcd(A_i, A_j) = 1$ . Your task is to partition this array into  $k$  disjoint sets  $S_0, S_1, \dots, S_{k-1}$  where  $1 \leq k \leq n$ .

The score of partition is defined to be as:

$$\sum_{i=0}^{k-1} f\left(\prod_{x \in S_i} x\right)$$

Here  $f(n)$  is:

$$f(n) = |\{(a, b) \in \mathbb{N} \times \mathbb{N} : \gcd(a, b) = 1 \wedge ab = n\}|$$

where  $|S|$  denotes the cardinality of the set  $S$ , and  $\mathbb{N}$  is the set of natural numbers. A partition is deemed **Correct** iff all of  $f(\prod_{x \in S_i} x)$  are distinct for all the sets  $S_i$ .

Find the maximum possible score over all **Correct Partitions** of the array  $A$ . Since the answer might be too big to print it modulo 998244353.

**Constraints:**

- $n \leq 10^5$
- $A_i \leq 10^{12}$

**Solution:** Some useful observations here are as follows:

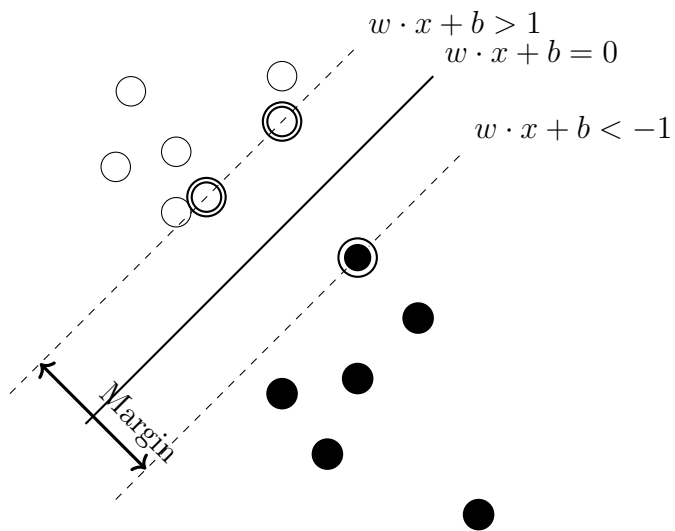
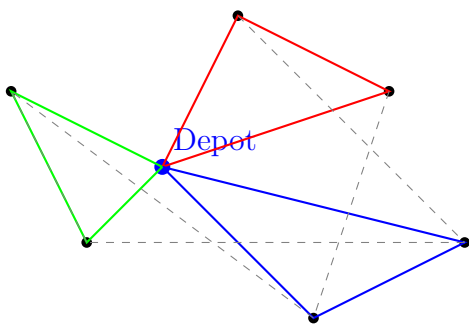
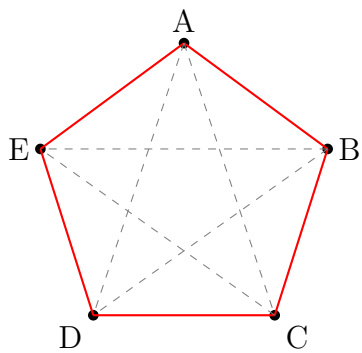
- $f(n) = 2^{|g(n)|}$  where  $g(n)$  is the set of prime divisors of  $n$ .
- Thus the count of distinct primes in each subset determine the score.

Now the crucial thing to realize here is that, since each partition corresponds to a power of 2 we are essentially summing up powers of 2 over all the partitions. Given the fact that they must all be distinct, the score can be thought of a binary number e.g. 10100101.

Now suppose we have  $> 1$  subsets the score will of the form 1010010. So we have  $2^6, 2^4, 2^1$  as the  $f$  scores. Since all pairs are coprime, the product of the elements in the subsets too will be coprime. Thus rather than summing up individual smaller powers of 2 combining them into a larger subset yields a larger power of 2 i.e. a larger binary number.

Thus the answer to this problem is:

$$2^{\sum_i g(a_i)}$$



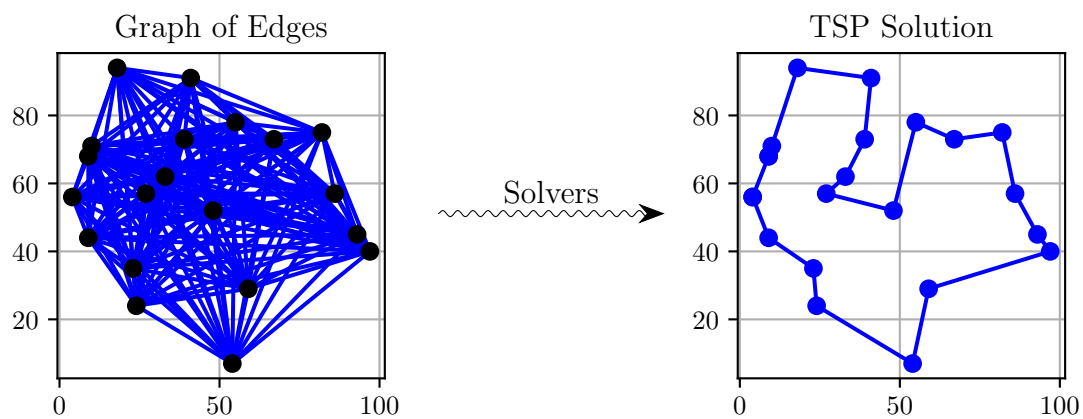


Figure 1: 2D-Euclidean TSP solution

