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Battle of Brains Problem Submission July 3, 2024

Problem Name TBD. Given an array of n integers $A_0, A_1, \dots, A_{n-1}$ where $0 \le i, j < n$ and $\texttt{gcd}(A_i, A_j) = 1$. Your task is to partition this array into k disjoint sets $S_0, S_1, \dots S_{k-1}$ where $1 \le k \le n$.

The score of partition is defined to be as:

$$\sum_{\mathtt{i}=0}^{\mathtt{k}-1} \mathtt{f}\Big(\prod_{\mathtt{x}\in \mathtt{S_i}} \mathtt{x}\Big)$$

Here $f(n)$ is:

$$\mathtt{f(n)} = |\{(\mathtt{a},\mathtt{b}) \in \mathbb{N}\times\mathbb{N} : \mathtt{gcd(a,b)} = 1 \land \mathtt{ab} = \mathtt{n}\}|$$

where $|S|$ denotes the cardinality of the set S, and $\mathbb{N}$ is the set of natural numbers. A partition is deemed `Correct` iff all of $f(\prod_{x\in S_i} x)$ are distinct for all the sets S_i.

Find the maximum possible score over all `Correct Partitions` of the array A. Since the answer might be too big to print it modulo 998244353.

Constraints:

- $n \le 10^5$
- $A_i \le 10^{12}$

Solution: Some useful observations here are as follows:

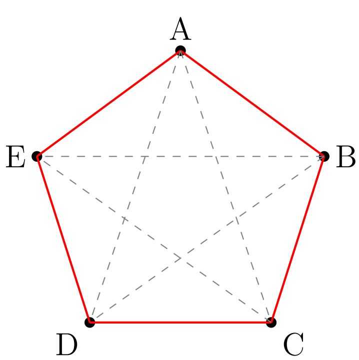
- $f(n) = 2^{|g(n)|}$ where $g(n)$ is the set of prime divisors of n.
- Thus the count of distinct primes in each subset determine the score.

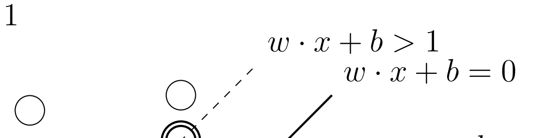
Now the crucial thing to realize here is that, since each partition corresponds to a power of 2 we are essentially summing up powers of 2 over all the partitions. Given the fact that they must all be distinct, the score can be thought of a binary number e.g. 10100101.

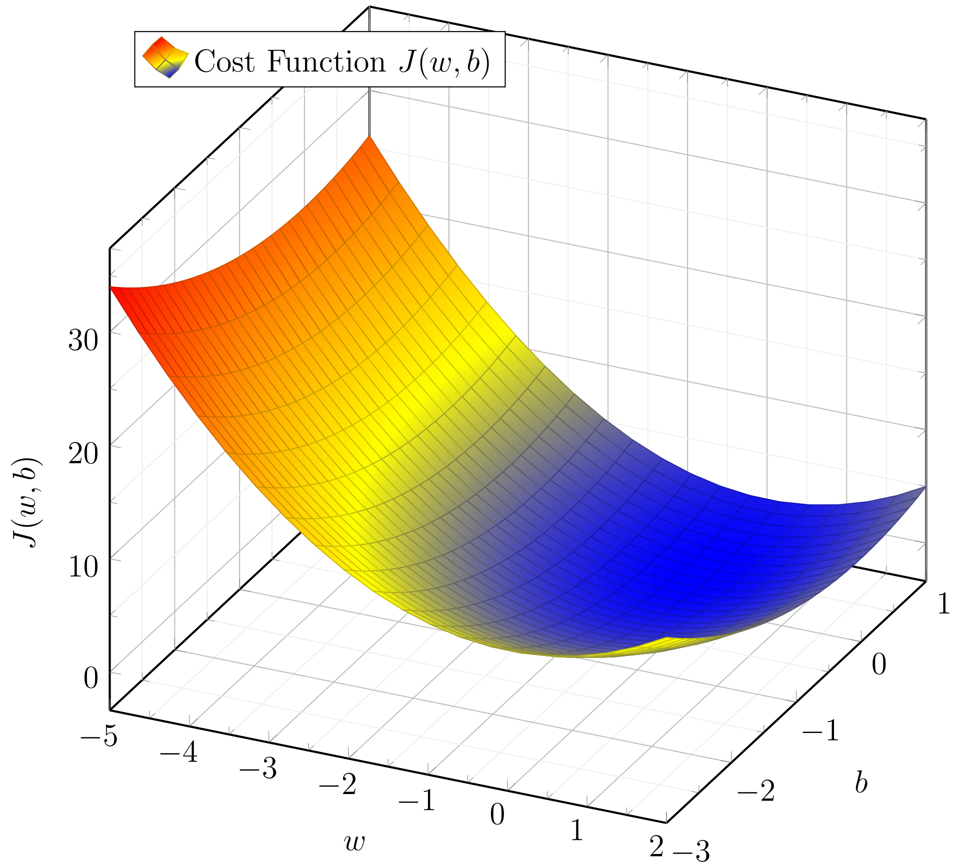
Now suppose we have > 1 subsets the score will of the form 1010010. So we have $2^6, 2^4, 2^1$ as the f scores. Since all pairs are coprime, the product of the elements in the subsets too will be coprime. Thus rather than summing up individual smaller powers of 2 combining them into a larger subset yields a larger power of 2 i.e. a larger binary number.

Thus the answer to this problem is:

$$2^{\sum_i \mathtt{g(a_i)}}$$







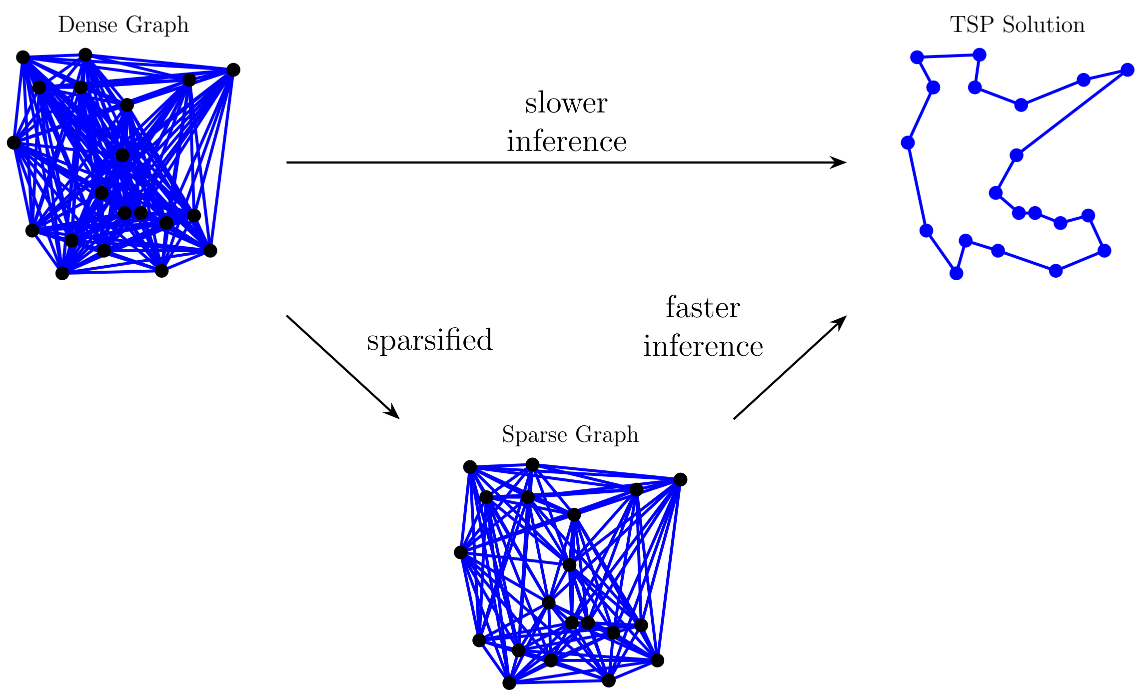
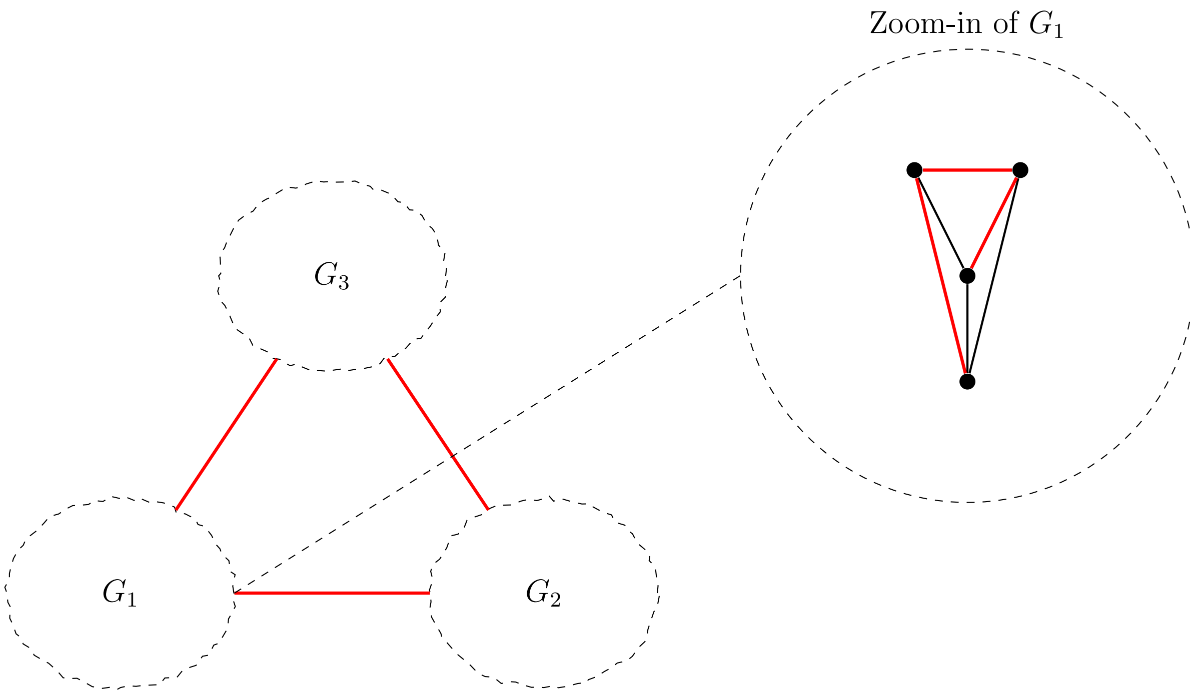
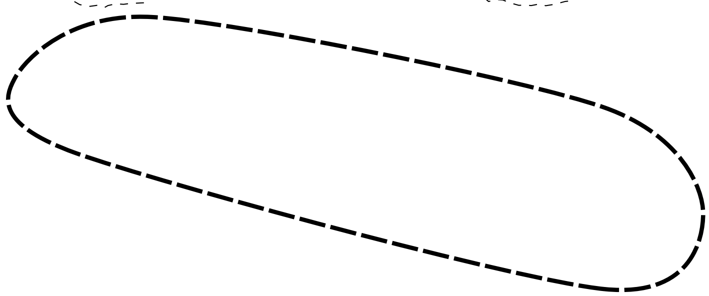
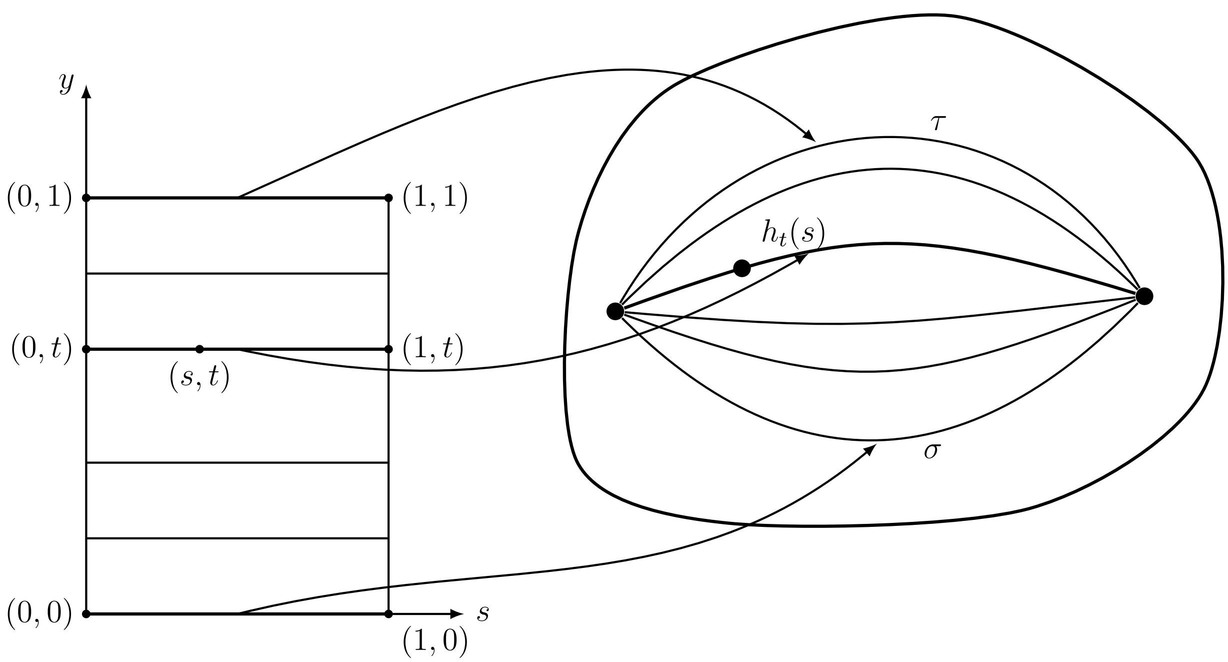


Figure 1: Sparsification speeding up inference


Zoom-in of G_1
G_3
G_1
G_2




y
$(0,1)$
$(1,1)$
$(0,t)$
$(1,t)$
(s,t)
$(0,0)$
$(1,0)$
s
τ
$h_t(s)$
σ

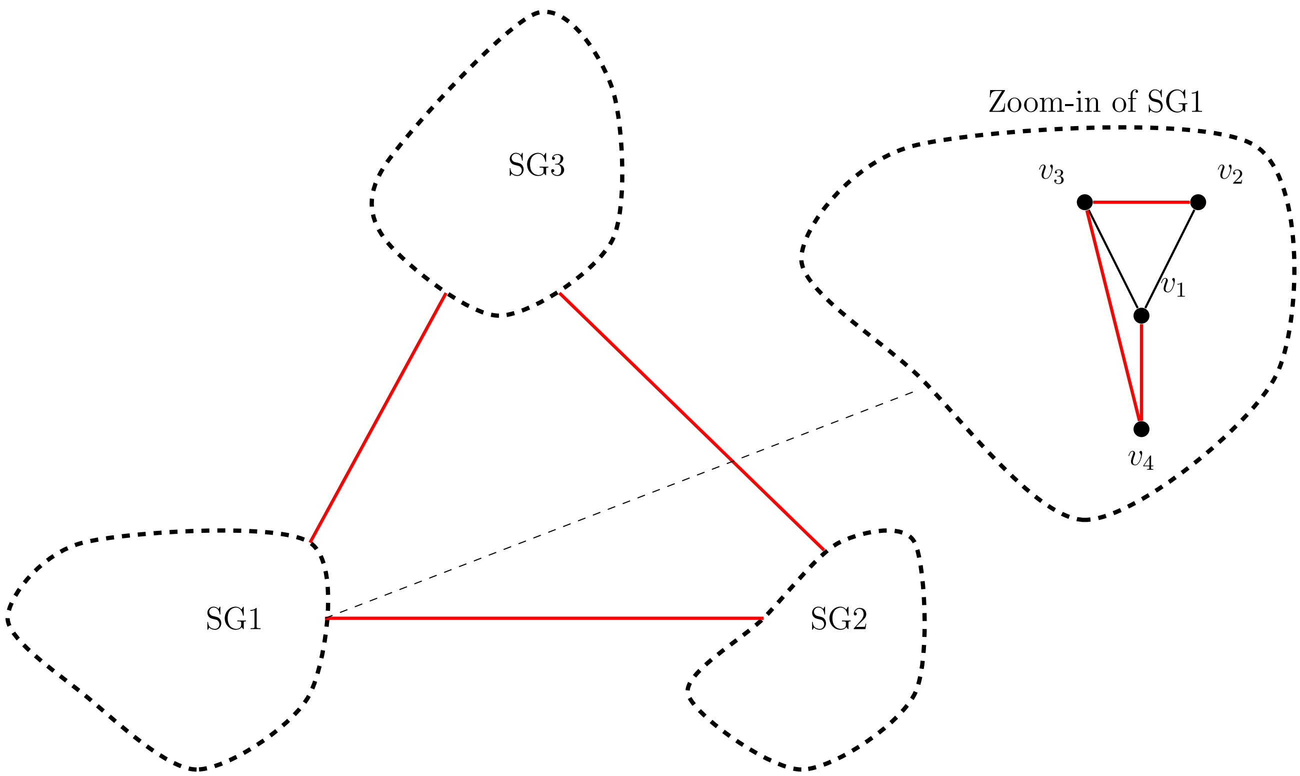


Figure 2: Partioned Graph Solution