Department of Electrical Engineering

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT

DSP ASSIGNMENT 2

SUBMITTED BY

K CHARUKESH

EEE B BATCH

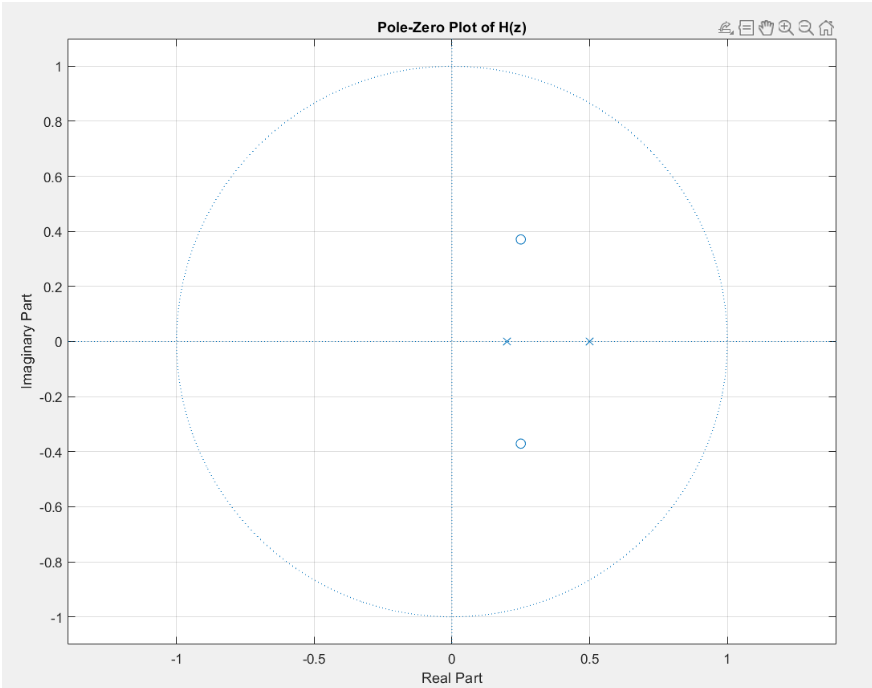
B220953EE

1Q:- A discrete-time system has the following transfer function:

$H(z) = (z^2 - 0.5z + 0.2) / (z^2 - 0.7z + 0.1)$

a) Determine the poles and zeros of the system and plot them on the z-plane.

```
>> Q1_a
Zeros of H(z):
   0.2500 + 0.3708i
   0.2500 - 0.3708i

Poles of H(z):
    0.5000
    0.2000
```



MATAB CODE:

```
% Define numerator and denominator coefficients
num = [1 -0.5 0.2]; % Coefficients of the numerator
den = [1 -0.7 0.1]; % Coefficients of the denominator

% Compute poles and zeros
zerosH = roots(num);
polesH = roots(den);

% Plot the z-plane
figure;
zplane(num, den);
title('Pole-Zero Plot of H(z)');
xlabel('Real Part');
ylabel('Imaginary Part');
grid on;

% Display results
disp('Zeros of H(z):');
disp(zerosH);
disp('Poles of H(z):');
disp(polesH);
```

b) Analyze the stability of the system using the unit circle criterion.

A discrete time system is stable if all the poles lie inside the unit circle, i.e, the magnitude should be less than 1. The unit circle criterion states that all poles must satisfy $|p_i| < 1$ for a system to be BIBO stable (Bounded Input Bounded Output).

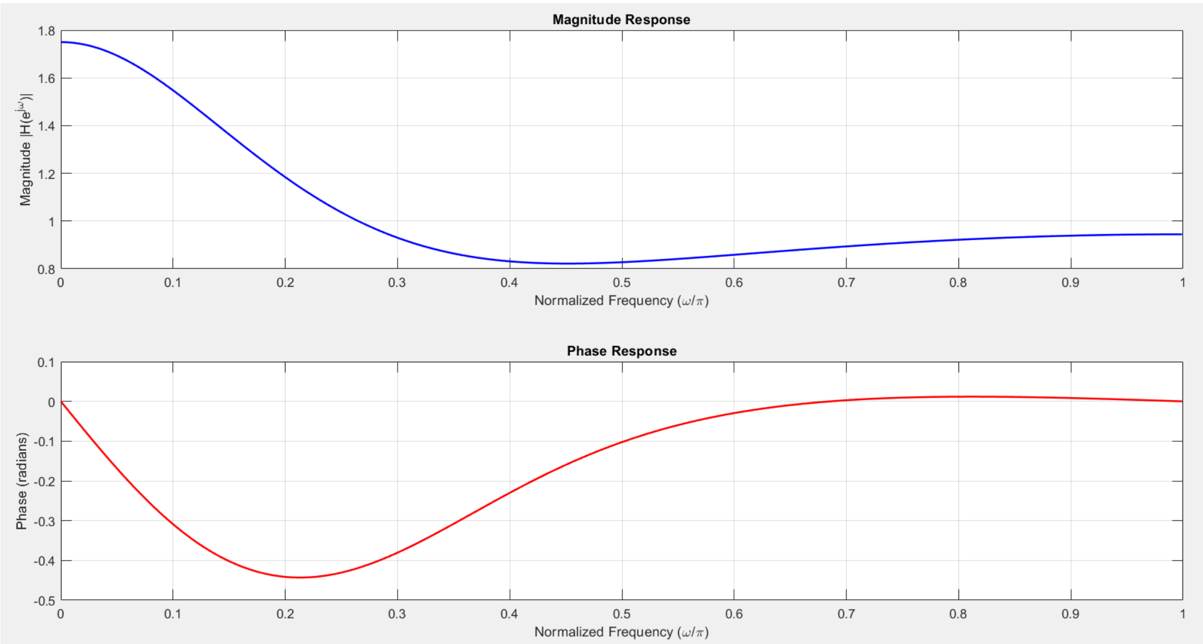
~~As~~ the poles of the system are $z_1 = 0.5$, $z_2 = 0.2$

As both the poles $|z_1| = 0.5 < 1$

$|z_2| = 0.2 < 1$

satisfy the criteria, and the poles are inside the unit circle, the system is stable.

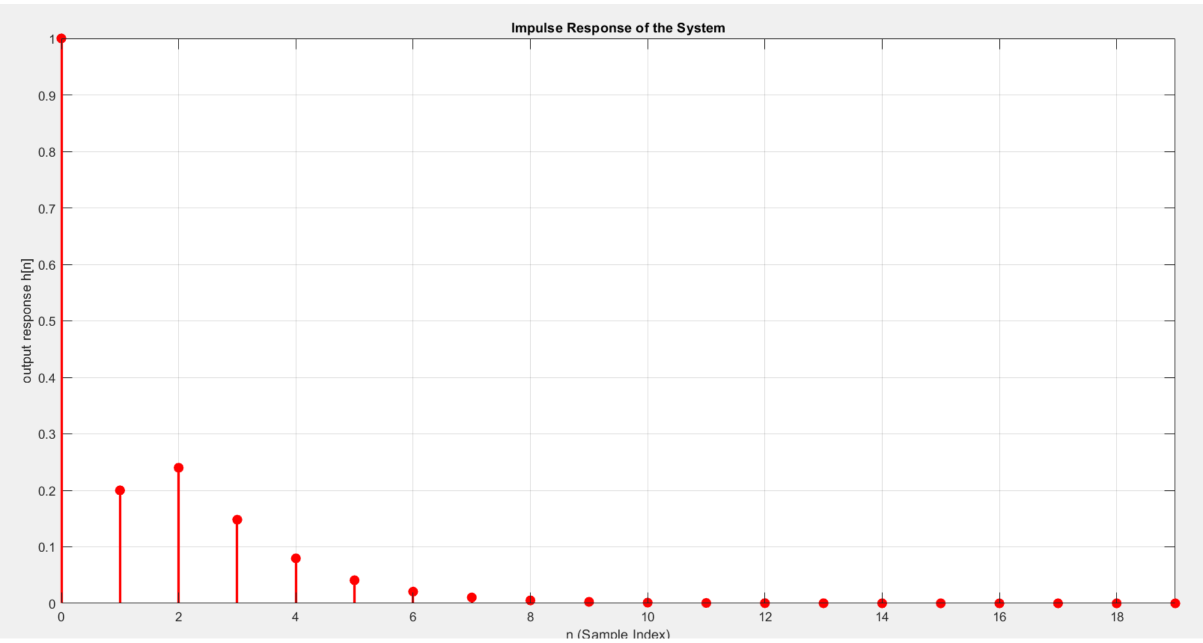
c) Sketch the frequency response function (FRF) directly from the pole-zero plot.



MATLAB CODE:

```
% Define the numerator and denominator coefficients
num = [1 -0.5 0.2]; % Coefficients of the numerator
den = [1 -0.7 0.1]; % Coefficients of the denominator

% Compute the frequency response
[H, w] = freqz(num, den, 1024, 'half'); % 1024 points over [0, pi]

% Plot Magnitude Response
figure;
subplot(2,1,1);
plot(w/pi, abs(H), 'b', 'LineWidth', 1.5);
grid on;
xlabel('Normalized Frequency (\omega/\pi)');
ylabel('Magnitude |H(e^{j\omega})|');
title('Magnitude Response');

% Plot Phase Response
subplot(2,1,2);
plot(w/pi, angle(H), 'r', 'LineWidth', 1.5);
grid on;
xlabel('Normalized Frequency (\omega/\pi)');
ylabel('Phase (radians)');
title('Phase Response');
```

d) Calculate and plot the impulse response for the first 20 samples.

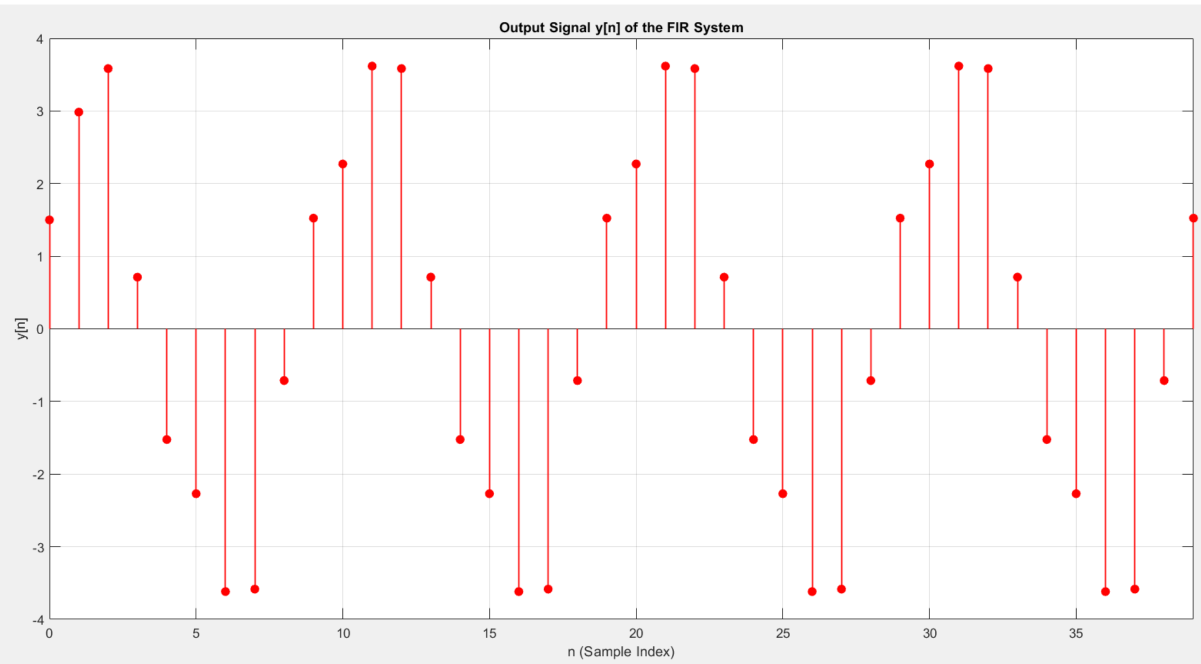


MATLAB CODE:

```
% Define the numerator and denominator coefficients
num = [1 -0.5 0.2]; % Coefficients of the numerator
den = [1 -0.7 0.1]; % Coefficients of the denominator

% Generate an impulse signal (first 20 samples)
N = 20; % Number of samples
impulse = [1; zeros(N-1, 1)]; % Unit impulse signal

% Compute the impulse response using filter()
h = filter(num, den, impulse);

% Plot the impulse response
figure;
stem(0:N-1, h, 'r','filled', 'LineWidth', 2);
grid on;
xlabel('n (Sample Index)');
ylabel('output response h[n]');
title('Impulse Response of the System');
xlim([0 N-1]);
```

2Q:-A signal $x[n] = \cos(0.2\pi n) + 0.5\sin(0.6\pi n)$ is processed through an FIR system with transfer function $H(z) = 1 + 2z^{-1} + z^{-2}$.

(a) Determine the output signal y[n] using the unilateral Z-transform to solve the difference equation with initial conditions $y[-1] = 0.5$ and $y[-2] = -0.3$



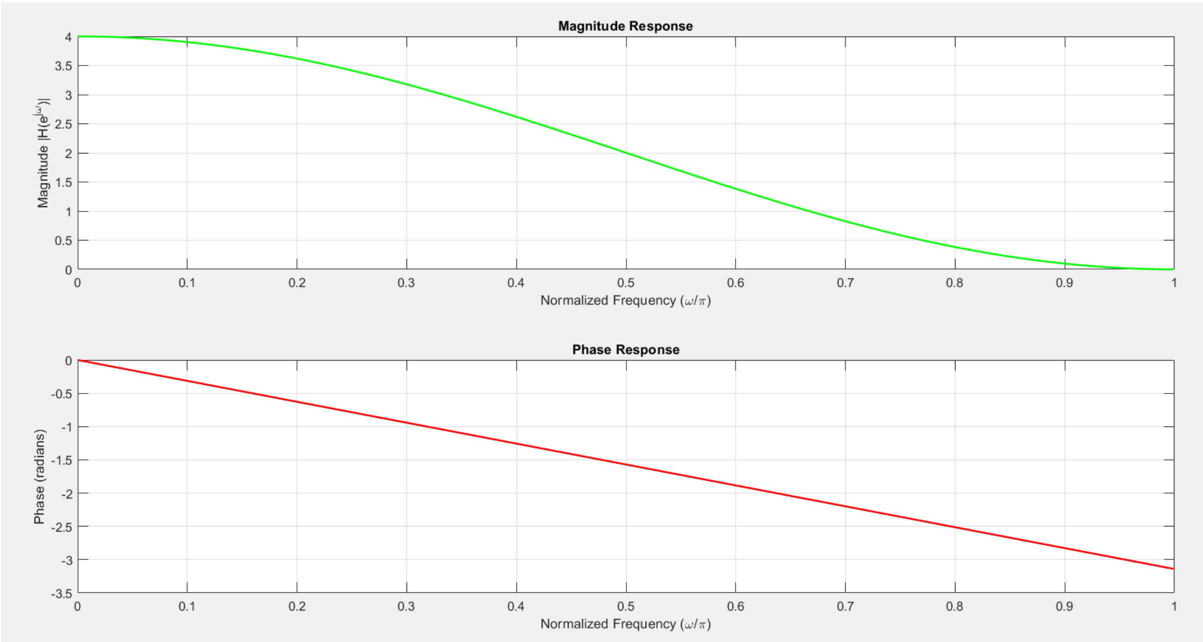
```
>> q2_a
Computed Output y[n]:
  Columns 1 through 6

    1.5000    2.9845    3.5842    0.7119   -1.5242   -2.2699

  Columns 7 through 12

   -3.6180   -3.5842   -0.7119    1.5242    2.2699    3.6180

  Columns 13 through 18

    3.5842    0.7119   -1.5242   -2.2699   -3.6180   -3.5842

  Columns 19 through 24

   -0.7119    1.5242    2.2699    3.6180    3.5842    0.7119

  Columns 25 through 30

   -1.5242   -2.2699   -3.6180   -3.5842   -0.7119    1.5242

  Columns 31 through 36

    2.2699    3.6180    3.5842    0.7119   -1.5242   -2.2699

  Columns 37 through 40

   -3.6180   -3.5842   -0.7119    1.5242
```

MATLAB CODE:

```
% FIR system coefficients (H(z) = 1 + 2z^-1 + z^-2)
b = [1 2 1]; % FIR numerator coefficients
a = 1;      % Denominator is 1 for FIR filters

% Compute the frequency response
[H, w] = freqz(b, a, 1024, 'half'); % 1024 points over [0, pi]

% Plot Magnitude Response
figure;
subplot(2,1,1);
plot(w/pi, abs(H), 'g', 'LineWidth', 1.5);
grid on;
```

```
xlabel('Normalized Frequency (\omega/\pi)');
ylabel('Magnitude |H(e^{j\omega})|');
title('Magnitude Response');

% Plot Phase Response
subplot(2,1,2);
plot(w/pi, angle(H), 'r', 'LineWidth', 1.5);
grid on;
xlabel('Normalized Frequency (\omega/\pi)');
ylabel('Phase (radians)');
title('Phase Response');
```

(b) Calculate and plot the system's Frequency Response Function (FRF).



MATLAB CODE:

```
% FIR system coefficients (H(z) = 1 + 2z^-1 + z^-2)
b = [1 2 1]; % FIR numerator coefficients
a = 1;      % Denominator is 1 for FIR filters

% Compute the frequency response
[H, w] = freqz(b, a, 1024, 'half'); % 1024 points over [0, pi]

% Plot Magnitude Response
figure;
subplot(2,1,1);
plot(w/pi, abs(H), 'g', 'LineWidth', 1.5);
```

```
grid on;
xlabel('Normalized Frequency (\omega/\pi)');
ylabel('Magnitude |H(e^{j\omega})|');
title('Magnitude Response');

% Plot Phase Response
subplot(2,1,2);
plot(w/pi, angle(H), 'r', 'LineWidth', 1.5);
grid on;
xlabel('Normalized Frequency (\omega/\pi)');
ylabel('Phase (radians)');
title('Phase Response');
```

3Q:- Consider the discrete-time system described by the difference equation:
y[n]−0.5y[n−1]=x[n]+2x[n−1]

(a) Determine the system transfer function H(z).

$$y[n]-0.5y[n-1]=x[n]+2x[n-1]$$

Applying z transform on both sides (Assume zero initial conditions)

$$Y(z)-0.5z^{-1}Y(z)=X(z)+2z^{-1}X(z)$$

$$\Rightarrow Y(z)(1-0.5z^{-1})=X(z)(1+2z^{-1})$$

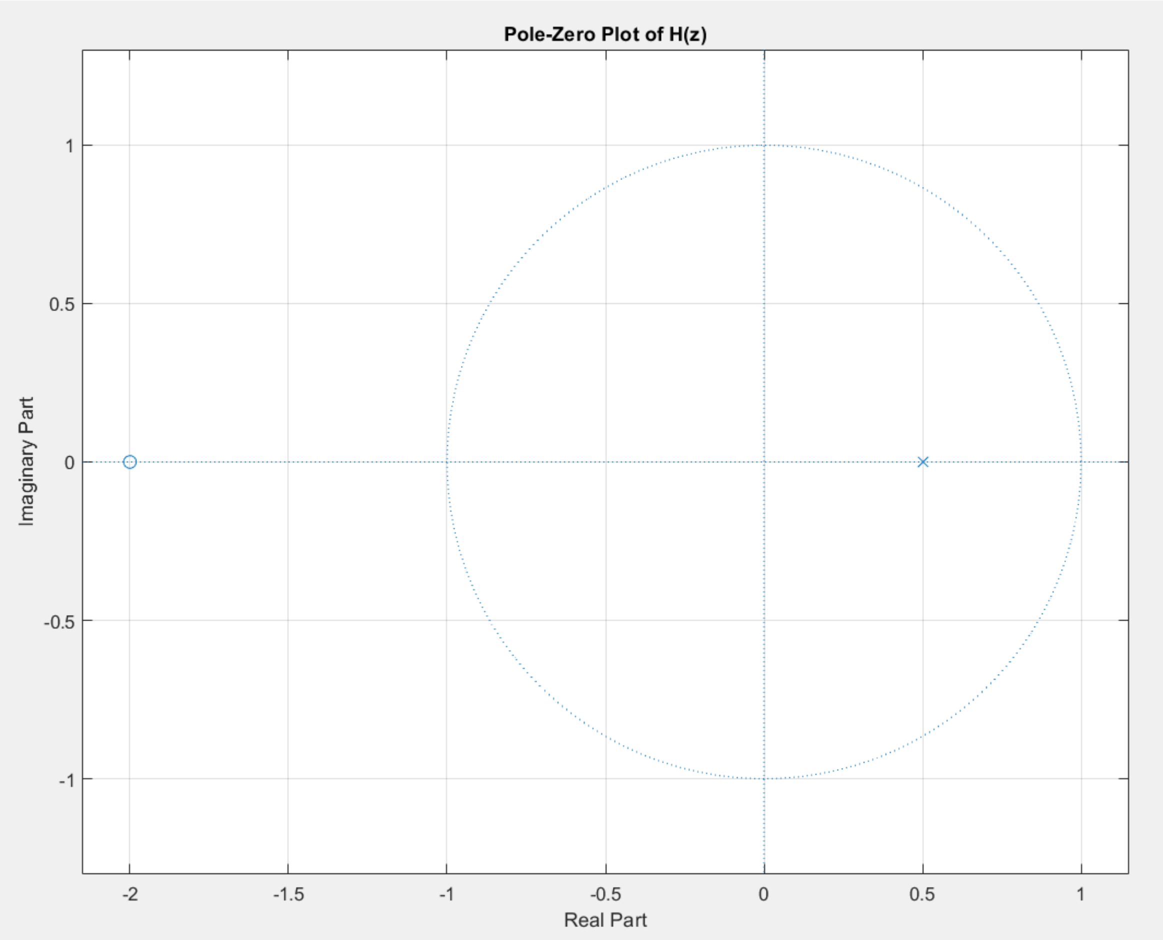
$$\Rightarrow \frac{Y(z)}{X(z)}=\frac{1+2z^{-1}}{1-0.5z^{-1}}$$

$$\Rightarrow \frac{Y(z)}{X(z)}=\frac{z+2}{z-0.5}$$

$\therefore$ The transfer function $H(z)=\frac{z+2}{z-0.5}$

(b) Find the poles and zeros and plot them in the z-plane.

```
>> q3_b
Zeros of H(z):
    -2

Poles of H(z):
    0.5000
```



MATLAB CODE:

```
% Define the transfer function coefficients
num = [1 2];    % Numerator (z + 2)
den = [1 -0.5]; % Denominator (z - 0.5)

% Compute poles and zeros
zeros_H = roots(num);
poles_H = roots(den);

% Plot poles and zeros in the z-plane
figure;
zplane(num, den);
grid on;
title('Pole-Zero Plot of H(z)');
xlabel('Real Part');
ylabel('Imaginary Part');

% Display the computed values
disp('Zeros of H(z):');
disp(zeros_H);
disp('Poles of H(z):');
disp(poles_H);
```

(c) Analyze the system stability using the unit circle criterion.

A discrete time system is stable if all the poles lie inside the unit circle, i.e, the magnitude should be less than 1. The unit circle criterion states that all poles must satisfy $|p_i| < 1$ for a system to be BIBO stable (Bounded Input Bounded Output).

As the poles of the system are $z_1 = 0.5$

$|z_1| = 0.5 < 1$

As this satisfies the criteria, and the pole is inside the unit circle, so the system is stable.