Analysis of Class E Power Amplifier With Shunt Filter under Different Duty Cycles

Hua Zaijun, Huang Fengchen, Li Jianni and Chen Zhao
School of Computer and Information
Hohai University, Nanjing, China
e-mail: huazaijun@hhu.edu.cn, huang@hhu.edu.cn, lijianni@hhu.edu.cn,enino@139.com

***Abstract*—Class E power amplifier is a class of power amplifier with high efficiency. It is widely used in wireless communications. In this paper, class E power amplifier with shunt filter under different duty cycles is analysis by ideal models, and the theoretical analysis is validated by simulations. Analysis reveals that the waveform and efficiency for the power amplifier varies with different duty cycles. The efficiency drops down much when the duty cycle is far way from 50%. A Class E power amplifier with shunt filter working at 1GHz with 0.5W output is designed. The power amplifier under duty cycles of 30%, 40%, 50%, 60% and 70% are simulated. Simulation results agree well with the theoretical analysis. The class E power amplifier with shunt filter can keep efficiency above 95% with duty cycles in 40% to 60%.**

Keywords-power amplifier; class E; efficiency; duty cycle;

I. INTRODUCTION

Switch mode power amplifier (SMPA) is power efficient, and is suitable for communication systems with constant envelope signals. SMPA is now used in communication systems with non-constant envelope signals by shaping the signal to maintain a constant envelope. One of these methods is delta sigma modulation [1]. The non-constant envelope signal is transformed in to two-level digital stream by delta sigma modulator. In a band-pass delta sigma modulator based transmitter, the duty cycle of the drive signal is not fixed at 50%, and the power amplifier efficiency varies with the duty cycle of the drive signal [2].

Class E power amplifier is one of the high efficient power amplifiers. Class E power amplifier with shunt filter [3] was invented in 2016, and was analyzed further in [4]-[7]. The efficiency under fixed duty cycle of 50% is 100% theoretically. However, the characters of the Class E power amplifier with shunt filter under different duty cycles have not been reported.

This paper discusses the behavior of the class E power amplifier with shunt filter under different duty cycles. In Section II a general class E power amplifier with shunt filter is described. The component values for the power amplifier are obtained under duty cycles of 50%. This class E power amplifier with shunt filter is driven with different duty cycles. Due to the violation of zero voltage switching (ZVS) and zeros voltage derivative switching (ZVDS) condition, the output capacitor consumes energy in every operational period. Thus, the theoretical efficiency of the power amplifier drops down. Section III gives the general waveforms for the class E power amplifier under different duty cycles. In Section IV circuit simulation is conducted to validate the theoretical analysis. The simulated waveforms and efficiency are compared with the theoretical ones. In the last section, the conclusion is draw.

II. CLASS E POWER AMPLIFIER WITH SHUNT FILTER UNDER DIFFERENT DUTY CYCLES

A. General Class E power Amplifier with Shunt Filter

The class E power amplifier with shunt filter is shown in Fig.1. It consists of a switch transistor S, a RF choke, a series inductor L, a dc blocking capacitor C_{DC}, a shunt capacitor C, a shunt L_P-C_P filter and load resistor R. The power amplifier is powered by dc voltage supply V_{DD}, and outputs RF signal.

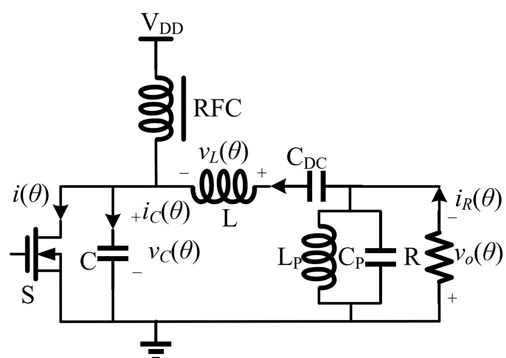


Figure 1. Class E power amplifier with shunt filter

In order to simplify the derivation processes, some assumptions of the class E power amplifier with shunt filter are made as follow.

1) All the passive elements in the power amplifier have no parasitic parameters.
2) The transistor is an ideal switch that has no parasitic parameters. It has a zero switch-on resistor and switches to on and off states instantaneously.
3) The RF choke is high enough to stop the ac current flowing to the dc supply while the capacitance of the dc blocking capacitor C_{dc} is high enough to prevent the dc current flowing to the load resistor R.
4) The shunt filter is resonant at frequency f_0, and the quality factor is high enough to make the output voltage sinusoid.

Let θ denote angular time $\theta=\omega_0 t=2\pi f_0 t$.

The power amplifier is drive with signal duty cycle D. In one operational period [0, 2π], the switch is turned on during [0, θ_D], and turned off during [θ_D, 2π]. θ_D=2πD is the switching time.

According to Kirchhoff's Current Law (KCL), the current relation among the switch current $i(\theta)$, shunt capacitor current $i_C(\theta)$and inductor L current $i_L(\theta)$ is express as (1).

$$i(\theta)+i_C(\theta)=i_L(\theta) \tag{1}$$

According to Kirchhoff's Voltage Law (KVL), the voltage relation among the shunt capacitor voltage $v_C(\theta)$, inductor L voltage $v_L(\theta)$, load resistor voltage $v_R(\theta)$ is express as (2).

$$v_C(\theta)=V_{\mathrm{DD}}-v_L(\theta)-v_R(\theta) \tag{2}$$

The current and voltage relationship for the shunt capacitor C and the series inductor L can be written as (3) and (4).

$$i_C(\theta)=\omega C\frac{dv_C(\theta)}{d\theta} \tag{3}$$

$$v_L(\theta)=\omega L\frac{di_L(\theta)}{d\theta} \tag{4}$$

According to assumption 4), the output voltage can be expressed as (5).

$$v_R(\theta)=V_R\sin(\theta+\phi) \tag{5}$$

where φ is the initial phase shift, V_R is the output voltage amplitude which is

$$V_R=aV_{\mathrm{DD}} \tag{6}$$

Suppose the normalized slope of capacitor voltage v_C at the instance of switch S turning on is s, it is defined as

$$s=\frac{1}{V_{\mathrm{DD}}}\frac{dv_C(\theta)}{d\theta}\bigg|_{\theta=2\pi} \tag{7}$$

In interval [0, θ_D], the switch is on, so $v_C(\theta)$ = 0, thus $i_C(\theta)$=0.

According to (2), (4) and (7), the current in the series inductor L is

$$i_L(\theta)=\frac{1}{\omega L}\int_0^{\theta}\left(V_{\mathrm{DD}}-aV_{\mathrm{DD}}\sin(\theta+\phi)\right)d\theta+i_L(0_+) \tag{8}$$

where

$$i_L(0_+)=s\omega CV_{\mathrm{DD}} \tag{9}$$

The current in the switch is

$$i(\theta)=\frac{V_{\mathrm{DD}}}{\omega L}\theta+\frac{aV_{\mathrm{DD}}}{\omega L}\left(\cos(\theta+\phi)-\cos(\phi)\right)+s\omega CV_{\mathrm{DD}} \tag{10}$$

In interval [θ_D, 2π] , the switch is off, so $i(\theta)$ = 0, thus

$$i_L(\theta)=i_C(\theta) \tag{11}$$

According to (2), (3), (4) and (11), the equation (12) can be got.

$$\omega^2 CL\frac{d^2v_C(\theta)}{d\theta^2}+v_C(\theta)+aV_{\mathrm{DD}}\sin(\theta+\phi)-V_{\mathrm{DD}}=0 \tag{12}$$

The solution for (12) can be written as (13).

$$v_C(\theta)=V_{\mathrm{DD}}\left(C_1\cos(q\theta)+C_2\sin(q\theta)+1+\frac{q^2}{1-q^2}\cdot a\sin(\theta+\phi)\right) \tag{13}$$

where

$$q=\frac{1}{\omega\sqrt{LC}} \tag{14}$$

At the instance θ_D, voltage on capacitor C is zero as expressed in (15).

$$v_C(\theta_D)=0 \tag{15}$$

And voltage slope on capacitor C satisfies (16).

$$\omega C\frac{v_C(\theta)}{d\theta}\bigg|_{\theta_D}=i_L(\theta)\big|_{\theta_D} \tag{16}$$

B. Case for duty cycle D=0.5

The optimum condition is the switch is driven by signal with duty cycle D=0.5. The power amplifier works with zero voltage switching (ZVS) and zeros voltage derivative switching (ZVDS) conditions, which are

$$v(\theta)\big|_{\theta=2\pi}=0 \tag{17}$$

$$s=\frac{1}{V_{\mathrm{DD}}}\frac{dv(\theta)}{d\theta}\bigg|_{\theta=2\pi}=0 \tag{18}$$

The amplitude of the output voltage can be calculated by Fourier series as (19).

$$aV_{\mathrm{DD}}=-\frac{1}{\pi}\int_{\theta_D}^{2\pi}v_C(\theta)\sin(\theta+\varphi)d\theta \tag{19}$$

According to (15), (16), (17), (18) and (19), parameters C_1, C_2, q, φ, a can be calculated as shown in Table I.

TABLE I. THE WAVEFORM PARAMETER FOR DUTY CYCLE D=0.5

C_1	C_2	q	φ	a
2.0047	0.6972	1.6069	-0.7263	0.9253

ZVS condition means that the dc power is totally converted into RF power.

$$P_o=V_{\mathrm{DD}}I_{\mathrm{DD}}=\frac{V_R^2}{2R} \tag{20}$$

The dc current is

$$I_{\mathrm{DD}}=\frac{1}{2\pi}\int_0^{2\pi}i(\theta)d\theta \tag{21}$$

Thus

$$I_{\mathrm{DD}}=\frac{V_{\mathrm{DD}}}{2\pi\omega L}\left(0.5\pi^2-a\pi\cos(\phi)-2a\sin(\phi)\right) \tag{22}$$

In practice application, the output power P_o and dc voltage V_{DD} are given parameters. Other parameters are expressed as the function of P_o and V_{DD}. According to (14), (20) and (22), equations (23)-(25) can be got.

$$\omega L=\frac{V_{\mathrm{DD}}^2}{2\pi P_o}\left(\frac{\pi^2}{2}-2a\sin(\varphi)-a\pi\cos(\varphi)\right) \tag{23}$$

$$R = \frac{1}{2}\frac{V_R^2}{P_o} \tag{24}$$

$$C = \frac{1}{q^2\omega^2 L} \tag{25}$$

For a shunt filter with load quality factor of Q_L, the inductor and capacitor are calculated as (26) and (27).

$$L_0 = \frac{R}{\omega Q_L} \tag{26}$$

$$C_0 = \frac{Q_L}{\omega R} \tag{27}$$

C. Case for different Duty Cycles

A class E power amplifier with shunt filter designed under D of 0.5 can be driven with signal of other duty cycles in delta sigma modulator based transmitters. When the duty cycle varies from 0.5, the switch voltage does not meet the ZVS and ZVDS condition at the switch turning on instance. Other conditions should be used to find the solution for the amplifier.

The amplitude of the voltage across inductor L for fundamental frequency is

$$V_{\mathrm{L},1} = -\frac{1}{\pi}\int_{\theta_D}^{2\pi} v_{\mathrm{C}}(\theta)\cos(\theta+\varphi)d\theta \tag{28}$$

The relationship between $V_{\mathrm{L},1}$ and V_{R} can be expressed in (29).

$$V_{\mathrm{L},1} = \frac{\omega L}{R}V_{\mathrm{R}} \tag{29}$$

So,

$$\frac{\omega L}{R} = -\frac{1}{\pi a V_{\mathrm{DD}}}\int_{\theta_D}^{2\pi} v_{\mathrm{C}}(\theta)\cos(\theta+\varphi)d\theta \tag{30}$$

The parameters L, C and q are fixed for a designed amplifier. Parameters C_1, C_2, s, φ, a under different duty cycles D are determined by (7), (15), (16), (19) and (30). They listed in table II for duty cycle from 0.3 to 0.7.

TABLE II. WAVEFORM PARAMETERS FOR DIFFERENT DUTY CYCLES

D	C_1	C_2	a	φ	s
0.7	-2.0169	1.7471	1.0084	-1.7692	-3.8905
0.6	0.4697	2.1179	0.9458	-1.3205	-2.5757
0.5	2.0047	0.6972	0.9253	-0.7263	0.0000
0.4	1.6366	-1.2865	0.8820	-0.1248	1.8269
0.3	-0.0029	-1.7891	0.6430	0.3868	1.2752

The dc current for the amplifier can be calculated as (31).

$$\begin{aligned} I_{\mathrm{DD}} &= \frac{1}{2\pi}\int_0^{2\pi} i(\theta)d\theta \\ &= \frac{1}{2\pi}\left(\int_0^{\theta_{\mathrm{D}}} i(\theta)d\theta + \int_{\theta_{\mathrm{D}}}^{2\pi} i_C(\theta)d\theta\right) \\ &= \frac{1}{2\pi}\left(\int_0^{\theta_{\mathrm{D}}} i(\theta)d\theta + \omega C v_C(2\pi)\right) \end{aligned} \tag{31}$$

It can be expressed as (32) after additional calculation.

$$\begin{aligned} I_{\mathrm{DD}} = \frac{V_{\mathrm{DD}}}{2\pi\omega L}\Bigg(&\frac{s\theta_{\mathrm{D}}}{q^2} + \frac{\theta_{\mathrm{D}}^2}{2} + a\sin(\theta_{\mathrm{D}}+\varphi) \\ &- a\sin(\varphi) - a\sin(\varphi)\theta_{\mathrm{D}} + \frac{v_{\mathrm{C}}(2\pi)}{q^2 V_{\mathrm{DD}}}\Bigg) \end{aligned} \tag{32}$$

The power efficiency can be calculated as (33).

$$\begin{aligned} \eta &= \frac{P_{\mathrm{o}}}{P_{\mathrm{DC}}} = \frac{V_{\mathrm{o}}^2/2R}{V_{\mathrm{DD}}I_{\mathrm{DD}}} \\ &= \pi a^2\frac{X}{R}\Bigg(\frac{s\theta_{\mathrm{D}}}{q^2} + \frac{\theta_{\mathrm{D}}^2}{2} + a\sin(\theta_{\mathrm{D}}+\varphi) \\ &\quad - a\sin(\varphi) - a\sin(\varphi)\theta_{\mathrm{D}} + \frac{v_{\mathrm{s}}(2\pi)}{q^2 V_{\mathrm{DD}}}\Bigg)^{-1} \end{aligned} \tag{33}$$

The calculated efficiency under different duty cycles are plotted in Fig. 2. It can be seen that the efficienty drops down much as duty cycle is far away from 0.5.

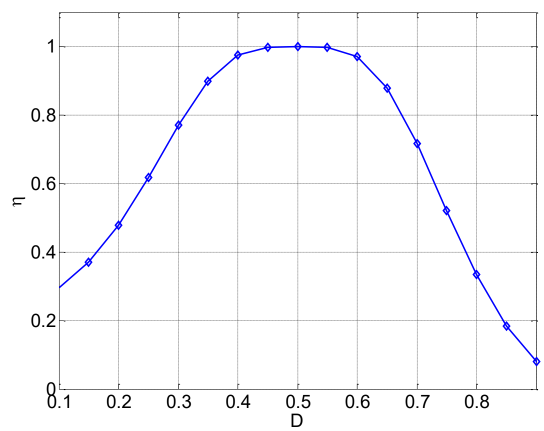


Figure 2. The efficiency under different duty cycles

III. WAVEFORMS

The normalized switch voltage waveform can be expressed as (34).

$$\begin{aligned} \bar{v}(\theta) = \frac{v(\theta)}{V_{\mathrm{DD}}} &= C_1\cos(q\theta) + C_2\sin(q\theta) + 1 \\ &+ \frac{q^2}{1-q^2}\cdot a\sin(\theta+\phi) \end{aligned} \tag{34}$$

The normalized switch current waveform can be expressed as (35).

$$\bar{i}(\theta) = \frac{i(\theta)}{I_{\mathrm{DD}}} = 2\pi\frac{\theta + a[\cos(\theta+\phi) - \cos(\phi)] + s/q^2}{M} \tag{35}$$

where

$$\begin{aligned} M = &\frac{s\theta_{\mathrm{D}}}{q^2} + \frac{\theta_{\mathrm{D}}^2}{2} + a\sin(\theta_{\mathrm{D}}+\varphi) \\ &- a\sin(\varphi) - a\sin(\varphi)\theta_{\mathrm{D}} + \frac{v_C(2\pi)}{q^2 V_{\mathrm{DD}}} \end{aligned} \tag{36}$$

TABLE IV. RESULTS OF THEORETICAL CALCULATION AND SIMULATION UNDER DIFFERENT DUTY CYCLES

D		$V_{\mathrm{sw,ON}}$/V	P_{Loss}/mW	V_o/V	P_o/mW	I_{DD}/mA	P_{DC}/mW	η/-
0.3	theoretical	8.6	72	3.21	241	62.6	313	77%
	simulated	9	79	3.18	236	63	315	75%
0.4	theoretical	3.5	12	4.41	454	93.2	466	97.5%
	simulated	3	8.7	4.42	457	95	475	96.2%
0.5	theoretical	0	0	4.63	500	100	500	100%
	simulated	0	0	4.65	506	102	510	99.2%
0.6	theoretical	4.05	16	4.73	522	107.6	538	97%
	simulated	4	15.5	4.74	525	109	545	96.3%
0.7	theoretical	15.5	233	5.04	594	166.5	827	71.6%
	simulated	15	218	5.03	591	165	825	71.6%

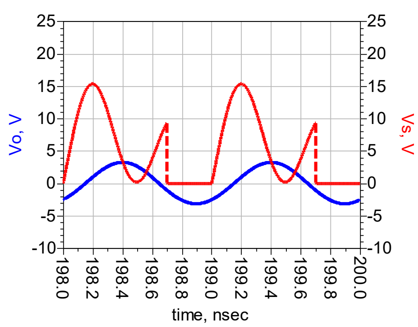
IV. SIMULATION

A class E power amplifier is designed in Advanced Design System software. DC supply V_{DD}=5V, output power P_o=0.5W, operation frequency f_0=1GHz, quality factor Q_L=20. All of the passive components are ideal with no parasitic. According to (23) to (27) the components values are calculated and show in Table III. This amplifier is drive with signal at 1 GHz in different duty cycles.

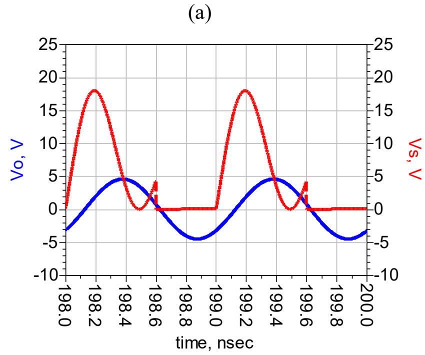
TABLE III. COMPONENT VALUES FOR THE DESIGNED POWER AMPLIFIER

components	values
R	21.4Ω
L	5.05nH
C	1.94pF
L_p	0.17nH
C_p	149pF

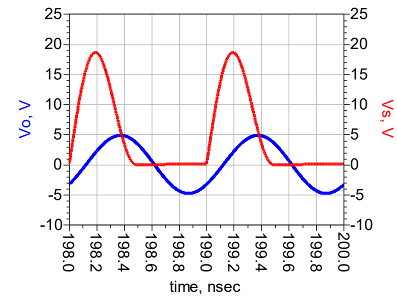
Waveform for switch voltage and output voltage are shown in Fig. 3. and the key parameters for these waveforms are measured.



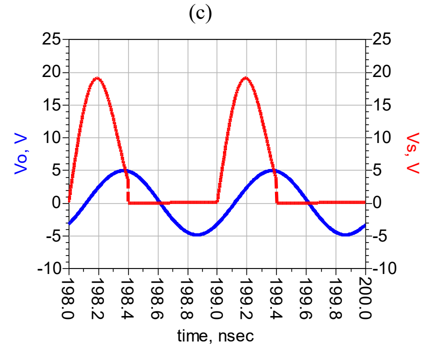
(a)



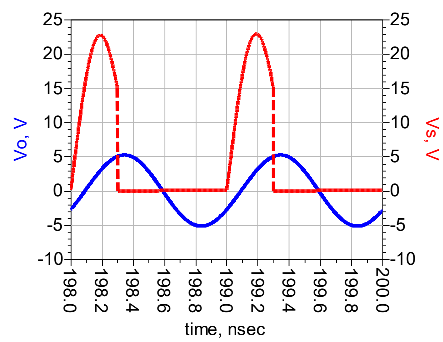
(b)



(c)



(d)



(e)

Figure 3. Simulated waveform under different duty cycles (a) D=0.3, (b) D=0.4, (c) D=0.5, (d) D=0.6, (e) D=0.7.

In Fig.3 the voltage on capacitor $V_{\mathrm{sw,on}}$ is not zero when the switch is turning on. The capacitor consumes energy, the power loss on capacitor C can be calculated as (37).

$$P_{\mathrm{Loss}} = \frac{1}{2} fCV_{\mathrm{sw,ON}}^2 \tag{37}$$

The efficiency can be recalculated as (38).

$$\eta = \frac{P_o}{P_{Loss} + P_o} \tag{38}$$

From (37) and (38), it can be seen that the larger the voltage $V_{sw,on}$ is , the lower the efficiency is.

Measurement results on the waveforms for duty cycle from 0.3 to 0.7 in Fig.3 are listed in Table IV.

The efficiency for theoretical calculation and simulation agrees well. Efficiency of the class E power amplifier with shunt filter can be above 95% with duty cycle kept from 0.4 to 0.6.

V. CONCLUSION

This paper studies the class E power amplifier with shunt filter under different duty cycles. By ideal assumptions, the waveform equations for the class E power amplifier with shunt filter are first setup. The zero voltage switching (ZVS) and zeros voltage derivative switching (ZVDS) conditions are used to solve the waveform equations in duty cycles of 50%. And the component values are calculated accordingly. The waveform and efficiency for the class E power amplifier with shunt filter in different duty cycles are analysis. To validate the theoretical analysis, simulations are conducted. The power amplifier under duty cycles of 30%, 40%, 50%, 60% and 70% are simulated. Simulations results agrees well with the theoretical analysis. Efficiency of the class E power amplifier with shunt filter can be high with duty cycle kept from 40% to 60%.

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