- Subject – Mathematics
- Chapter – Polynomials

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Today's Targets

1. Polynomial, General Form, Terms and Degree, Types of Polynomials
2. Value of a Polynomial, Zeros of Polynomials and Number of Zeros (Imp.)
3. Division Algorithm/ Long Division, Remainder and Factor Theorem, (Excluded)
4. Factorisation, Factorisation using factor theorem, Algebraic Identities

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Variables

vary → Jisaki value fix Naa Hoo!!

Constants

fix

ⓐ 18, 15, 21, 24, 9, ... → constant

ⓑ a, b, c, p, q, r, x, y, z, α, β, ... → Represation of variables

ⓒ mathematical operation → $+, -, \times, \div$

operator sign

Algebraic Expression

An algebraic expression is a combination of constant and variables connected by the signs of fundamental operations.

variable constant

+, −, ×, ÷

Algebraic Expression $\longrightarrow$ $\left\{ \boxed{2x+5},\ \boxed{5},\ \boxed{a^2} \right.$

variable

+

×

−

constant

÷

$\boxed{\text{Ex:}}$ $\boxed{x^2y+\frac{5}{z}}$, $\boxed{x+\frac{1}{x}}$

Polynomials

Polynomials are mathematical expressions made up of variables and constants by using arithmetic operations like addition, subtraction, and multiplication. They represent the relationship between variables. In polynomials, the exponents of each of the variables should be a whole number. The exponents of the variables in any polynomial have to be a non-negative integer.

whole: $W = \{0, 1, 2, 3, 4, 5, \ldots\}$

Ex: $x^2 + 5x^1 + 6 \longrightarrow$ Polynomial

$x + \frac{1}{x} = \boxed{x^1 + x^{\boxed{-1}}} \longrightarrow$ Not a Polynomial

Algebraic Expression

Exponent of each variable of every term in always a whole number/ non-negative Integers

Polynomials

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Polynomial Kab Hoga!!

Polynomial Kab $\boxed{\text{nahi}}$ Hoga!!

ⓐ $\boxed{\text{Exponents}}$:- Whole Number

$\{0, 1, 2, 3, 4, \ldots\}$

Jaldi Vaho se Hato

$\boxed{\text{Exponents}}$:-

ⓐ Negative Integers $\Rightarrow \{-5, -1, -2, \ldots\}$

ⓑ Fractional $\Rightarrow \{-\frac{1}{2}, \frac{3}{5}, \frac{7}{2}, \ldots\}$

ⓒ Decimal $\Rightarrow \{0.5, -0.69, \ldots\}$

ⓓ Surds $\Rightarrow \{\sqrt{\ }, \sqrt[3]{\ }, \sqrt[4]{\ }, \ldots\}$

General form/ Standard form of a polynomial

Let x be a variable (literal), n be a positive integer and $a_0, a_1, a_2 \ldots\ldots, a_n$ be constants (real number). $\{n, n-1, n-2, \ldots, 1, 0\} \rightarrow$ whole Number

$a_n \neq 0$

Then, $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} \ldots\ldots a_1 x^1 + a_0$ is known as a polynomial in variable x.

In order to write a polynomial in Standard form, we need to arrange the terms in descending order of their ~~degree~~ Exponent.

$5 + 6x^9 + 8x^{1000} + 2x^1 \Rightarrow \boxed{8x^{1000} + 6x^9 + 2x^1 + 5}$

$\hookrightarrow$ Degree $= 1000$

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Question

Identify the polynomials in the following algebraic expression

(i) $\boxed{\sqrt{x}} + 2$

(ii) $2 - \frac{1}{x} + x^2$

(iii) $y + \sqrt{x}$

(iv) $x^{3} + 3x^{1} + 5$

(v) $x\left(x + \frac{1}{x}\right) =$

Question

Identify the polynomials in the following algebraic expression

(i) $\boxed{\sqrt{x}} + 2$ $= x^{\frac{1}{2}} + 2 \rightarrow \times$

(ii) $2 - \frac{1}{x} + x^2$ $= 2 - x^{-1} + x^2 \longrightarrow \times$

(iii) $y + \sqrt{x}$ $\longrightarrow y^{1} + x^{\frac{1}{2}} \longrightarrow \times$

(iv) $x^{3} + 3x^{1} + 5$ $\longrightarrow \checkmark$

(v) $x\left(x + \frac{1}{x}\right)$ $= x^2 + \cancel{x} \times \frac{1}{\cancel{x}} = x^{2} + 1 \longrightarrow \checkmark$

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Question

Identify polynomials in the following:

(i) $f(x) = 4x^{3} - x^{2} - 3x^{1} + 7$

(ii) $g(x) = 2x^{3} - 3x^{2} + \boxed{\sqrt{x}} - 1$

(iii) $p(x) = \frac{2}{3}x^{2} - \frac{7}{3}x^{1} + 9$

(iv) $g(x) = 2x^{2} - 3x^{1} + \boxed{\frac{4}{x}} + 2$

(v) $h(x) = x^{4} - x^{\boxed{\frac{3}{2}}} + x - 1$

(vi) $f(x) = 2 + \boxed{\frac{3}{x}} + 4x^{1}$

Question

Identify polynomials in the following:

(i) $f(x) = 4x^{3} - x^{2} - 3x^{1} + 7$ ✓

(ii) $g(x) = 2x^{3} - 3x^{2} + \boxed{\sqrt{x}} - 1$ ✗

(iii) $p(x) = \frac{2}{3}x^{2} - \frac{7}{3}x^{1} + 9$ ✓

(iv) $g(x) = 2x^{2} - 3x^{1} + \boxed{\frac{4}{x}} + 2$ ✗

(v) $h(x) = x^{4} - x^{\boxed{\frac{3}{2}}} + x - 1$ ✗

(vi) $f(x) = 2 + \boxed{\frac{3}{x}} + 4x^{1}$ ✗

Question

State whether the given algebraic expression is polynomial or not? (olympiad)

(a) $x^3 + \frac{3x^{3/2}}{x^{1/2}}$

(b) $\frac{x-1}{\boxed{x+1}}$

Question

State whether the given algebraic expression is polynomial or not? $\boxed{\text{(olympiad)}}$

(a) $x^3 + \frac{3x^{3/2}}{x^{1/2}}$

$= x^3 + 3(x)^{\frac{3}{2}-\frac{1}{2}} = x^3 + x^{\frac{3-1}{2}}$

$= x^3 + x^{2/2}$

$= \boxed{x^3 + x^1}$

$\frac{a^m}{a^n} = a^{m-n}$

$\boxed{\text{yes, a polynomial}}$

(b) $\frac{x-1}{\boxed{x+1}}$

$x+1 = a$

$x = \boxed{a-1}$

$= \frac{a-1-1}{a}$

$= \frac{a-2}{a}$

$= \frac{a}{a} - \frac{2}{a}$

$= \boxed{1 - \frac{2}{a}}$ ✗

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Terms, Coefficient, Leading Coefficient and Numerical Coefficient

↳ coefficient

- Terms are the parts of an algebraic expression separated by $\boxed{+ \text{ or } -}$ signs. It can be a single number, a variable, product of two or more variables, or product of a number and variable. An algebraic expression may contain single or more terms.

$5x^2\boxed{y}$

$\boxed{5}x^2 + 9x - 7 \rightsquigarrow$ No. of term = 3

↓ coefficient

- In mathematics, a coefficient is a multiplicative factor in some term of a polynomial or an expression; it is usually a number, but may be any expression (including variables such as a, b and c). The variable which does not contain any constant value has a coefficient of 1.

Coefficient:

$3\cancel{x}xy$

$3x^2\cancel{y} + 5\cancel{z}$

y ka coefficient $= 3x^2$

z ka coefficient $= 5$

x ka coefficient $= 3xy$

Numerical coefficent:

$3x^2y + 5z$

y ka numerical coefficent

$= \boxed{3}\cancel{x^2}\cancel{y}$

Question

Write the coefficient of x^2 in each of the following:

(i) $17 - 2x + 7x^2$

(ii) $9 - 12 + x^3$

(iii) $\frac{\pi}{6}x^2 - 3x + 4$

(iv) $\sqrt{3x} - 7$

Question

Write the coefficient of x^2 in each of the following:

(i) $17 - 2x + 7\boxed{x^2} \Rightarrow 7$

(ii) $9 - 12 + x^3 = -3 + x^3 + \boxed{0} \times x^2 \Rightarrow 0$

(iii) $\boxed{\frac{\pi}{6}} x^2 - 3x + 4 \Rightarrow \frac{\pi}{6}$

(iv) $\sqrt{3x} - 7 = \sqrt{3} \times \sqrt{x} - 7$

$= \sqrt{3} \times x^{\frac{1}{2}} - 7 + \boxed{(0)}(x^2) \Rightarrow 0$

Degree of a polynomial

The degree of a polynomial is the greatest power of a variable in the polynomial equation.

$\boxed{Ex:}$ $5x^2 + 7x - 9$ → Highest Exponent = Degree

$\boxed{\text{Degree} = 2}$

Agar aapne isko degree 2 vala polynomial bola hai, to toh Highest exponent vale variable ka coefficient $\boxed{\text{zero}}$ nahi Hona Hai !!

Question

Write the degrees of each of the following polynomials:

(i) $7x^{3} + 4x^{2} - 3x^{1} + 12$

(ii) $12 - x^{1} + 2x^{3}$

(iii) $5y^{1} - \sqrt{2}$

(iv) $7x^{0}$

(v) 0

Core-concept

Degree ⇝ Highest exponent vale variable ka coefficient $\boxed{\text{zero}}$ nahi hona chahiye

Question

Write the degrees of each of the following polynomials:

(i) $7x^3 + 4x^2 - 3x + 12$

Degree = 3

(ii) $12 - x + 2x^3$

Degree = 3

(iii) $5y - \sqrt{2}$

Degree = 1

(iv) $7x^0$

Degree = 0

(v) 0

$\boxed{a^0 = 1}$ +

Core-concept

Degree ⇝ Highest exponent wale variable ka coefficient $\boxed{\text{zero}}$ nahi hona chahiye

Question

Find the degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$.

Question

Find the degree of the polynomial $4x^4 + \boxed{\cancel{0x^3}} + \boxed{\cancel{0x^5}} + 5x + 7$.

Jaldi Vaha Se Hata !!

Degree $= \boxed{4}$ Ans

Some Important Facts

The degree of a polynomial with more than one variable can be computed by adding the exponents of each variable in terms.

For example: $\boxed{7a^4} - \boxed{3ab^4}$

Ek Hi Term ke Andar, Jo-Jo variable Hai, unake exponent ko collectively bolna Hai

- The exponent of variable in term $7a^4$ is 4.
- The exponent of variable in term $-3ab^4$ is 5. (a has exponent 1, b has 4, so 1 + 4 = 5)

$$\boxed{7a^{4} - 3a^{1}b^{4}} \quad \boxed{1\ 4} \approx 5 \Rightarrow \text{Degree} = 5$$

Types of Polynomial : In general, the polynomials are divided into three categories.

Types of polynomial

- Based on number of distinct variables
- Based on degree
- Based on number of terms

3 variable

Understood

$\boxed{3x^2y + 9xy + z}$ → Ek hi variable Kitni Bar Aaya Hai, USS se koi Matlab Nahi

On the basis of number of terms

Name

No. of terms

Examples

Monomial :

Binomials :

Two

Trinomial :

Three

If number of terms are more than three : Polynomial of that much term

$\Rightarrow 7x^5+9x^4+3x^3+2x-8 \rightarrow$ Polynomial with terms

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On the basis of number of terms

Name	No. of terms	Examples
Monomial :	1	x, $a/5$, $\frac{2a}{3}$, 7
Binomials : Two	2	$2x+5$, $\frac{2a}{5}+9$, ...
Trinomial : Three	3	$2x^2+9x+5$, $2x^{100000000}+5x^2+8x^3$, ...

If number of terms are more than three : Polynomial of that much term

$\Rightarrow 7x^5+9x^4+3x^3+2x-8 \longrightarrow$ Polynomial with 5 terms

Question

Which of the following expression are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 5$

(ii) $y^2 + 2\sqrt{3}$

(iii) $3\sqrt{x} + \sqrt{2}x$

(iv) $x - \frac{4}{x}$

(v) $x^{12} + y^3 + t^{50}$

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Question

Which of the following expression are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2 - 4x + 5$ → ✓

(ii) $y^2 + 2\sqrt{3}$ ✓

(iii) $3\sqrt{x} + \sqrt{2}x$ ✗

(iv) $x - \frac{4}{x}$ ✗

(v) $x^{12} + y^3 + t^{50}$ ✗

Polynomial in 3 variables

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On the basis of power of the variable/on the basis of degree of the polynomial

1. Zero polynomial : A polynomial of undefined degree is called as zero polynomial

$$\boxed{p(x) = 0} \Rightarrow p(x) = \begin{matrix} 0 \times x^{1} \\ 0 \times x^{1000} \\ 0 \times x^{99999} \\ 0 \times x \end{matrix} \Big\} \text{ Highest exponent is not fixed}$$

Note : *The constant polynomial 0 or f(x) = 0 is called a zero polynomial. The degree of zero polynomial is not defined.*

2. Constant polynomial : A polynomial of degree zero is called a constant polynomial.

$$\left.\begin{aligned} p(x) &= 7 = 7x^{\boxed{0}} \\ &= \frac{3}{9} = \frac{3}{9} \times p^{\boxed{0}} \end{aligned}\right\} \boxed{\text{Degree} = 0}$$

On the basis of power of the variable/on the basis of degree of the polynomial

General form | Example

Linear polynomials: Polynomial of degree 1. ax^1+b a & b → Real No. $2x^1+5$, $9x^1$

Quadratic polynomials: A polynomial of degree 2. ax^2+bx^1+c x^2-5x+6, $7x^2+2$

Cubic polynomial : A polynomial of degree 3.

If degree is more than 3 :

Question

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

1. $x + x^2 + x^3 + x^4$,
2. $7 + y + 5x$,
3. $2y - 3y^2 + 4y^3$,
4. $5x - 4y + 3xy$,
5. $4z - 15z^2$,
6. $ab + bc + cd + da$
7. pqr,
8. $p^2q + pq^2$,
9. $2p + 2q$.

Question

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

1. $x + x^2 + x^3 + x^4$, → Polynomial with 4 terms
2. $7 + y + 5x$, → Trinomial
3. $2y - 3y^2 + 4y^3$, → Trinomial
4. $5x - 4y + 3xy$, → Trinomial
5. $4z - 15z^2$, → Bionomial
6. $ab + bc + cd + da$, → Polynomial with 4 term
7. pqr, → Monomial
8. $p^2q + pq^2$, → Binomial
9. $2p + 2q$. → Binomial

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Question

$\sqrt{2}$ is a polynomial of degree

2

0

1

$\frac{1}{2}$

Question

$\sqrt{2}$ is a polynomial of degree

A 2

$\boxed{\text{B } 0}$

C 1

D $\frac{1}{2}$

constant Polynomial $\Rightarrow p(x) = k$, k is any constant number excluding zero

$p(x) = \sqrt{2}$

$= \sqrt{2} \times x^{\boxed{0}}$

Question

Degree of the zero polynomial is

0

1

2

Not defined

Question

Degree of the zero polynomial is

A 0

B 1

C 2

D Not defined

$$p(x) = 0 = 0 \times x^{1} = 0 \times x^{100000} = 0 \times x^{9} = 0 \times a$$

Highest exponent change Hote rahne ki vajah se defined nahi ho paa rha hai !!

Question

Classify the following polynomials are linear, quadratic, cubic and biquadratic polynomials:

(i) $x + x^2 + 4$

(ii) $3x - 2$ 1

(iii) $2x + x^2$

(iv) $3y$

(v) $t^2 + 1$

(vi) $7t^4 + 4t^3 + 3t - 2$ 1

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Question

Classify the following polynomials are linear, quadratic, cubic and biquadratic polynomials:

(i) $x + x^2 + 4$ ⇒ Degree = 2

Quadratic

(ii) $3x - 2$ ⇒ Degree = 1 ⇒ Linear

(iii) $2x + x^2$ ⇒ Degree = 2 → Quadratic

(iv) $3y$ → Linear

(v) $t^2 + 1$ → Quadratic

(vi) $7t^4 + 4t^3 + 3t - 2$ ⇒ Degree = 4

Polynomial of degree 4

#Machha Diya

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Question

Classify the following polynomials as polynomials in one-variable, two variables etc:

(i) $x^2 - xy + 7y^2$

(ii) $x^2 - 2tx + 7t^2 - x + 1$

(iii) $t^3 - 3t^2 + 4t - 5$

(iv) $xy + yz + zx$

Question

Classify the following polynomials as polynomials in one-variable, two variables etc:

(i) $x^2 - xy + 7y^2$

$x, y \rightarrow 2$

(ii) $x^2 - 2tx + 7t^2 - x + 1$

$x, t \rightarrow 2$

(iii) $t^3 - 3t^2 + 4t - 5$

$t \rightarrow 1$

(iv) $xy + yz + zx$

$x, y, z \rightarrow 3$

Question

Write whether the following statements are True or False. Justify your answer.

1. A binomial can have atmost two terms.

2. Every polynomial is a binomial.

3. A binomial may have degree 5.

4. Zero of a polynomial is always 0.

5. A polynomial cannot have more than one zero.

6. The degree of the sum of two polynomials each of degree 5 is always 5.

Question

Write whether the following statements are True or False. Justify your answer.

Exactly 2 term

1. A binomial can have $\boxed{\text{atmost}}$ two terms. → False

   ↳ Jyada se Jyada → Diye huye se kam vala bhi

2. Every polynomial is a binomial. → False

   $x^{900} + 7x^3 + 2x^2 + 5$

3. A binomial may have degree 5. → True

   Hame sirf terms se matlab hai - degree jitani bhi ho.

4. Zero of a polynomial is always 0. → False

5. A polynomial cannot have more than one zero. Later on → False

6. The degree of the sum of two polynomials each of degree 5 is always 5. → True

   $(x^5 + 100, + 2x^5 + 9x^3 + ..) \rightarrow$ Highest exponent $= 5$

Question

Give one example each of the a binomial of degree 35 and of a monomial of degree 100.

Question

Give one example each of the a binomial of degree 35 and of a monomial of degree 100.

$a^{35} + 2a^{30}$ ✓

$\frac{x^{35}}{9} - 2x$

Binomial of degree 35

x^{100}

$2t^{1000}$

$\frac{p^{100}}{500}$

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Value of a polynomial

The value of a polynomial $f(x)$ at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.

Example : $f(x) = 8x^2 - 3x + 7$, at $x = -1$ and $x = 2$

Value of a polynomial

Solution

The value of a polynomial f(x) at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.

Example : $\boxed{f(x) = 8x^2 - 3x + 7,}$ at $\boxed{x = -1}$ and $x = 2$

$p(x)$

$f(x)$

$q(s)$

$r(t)$

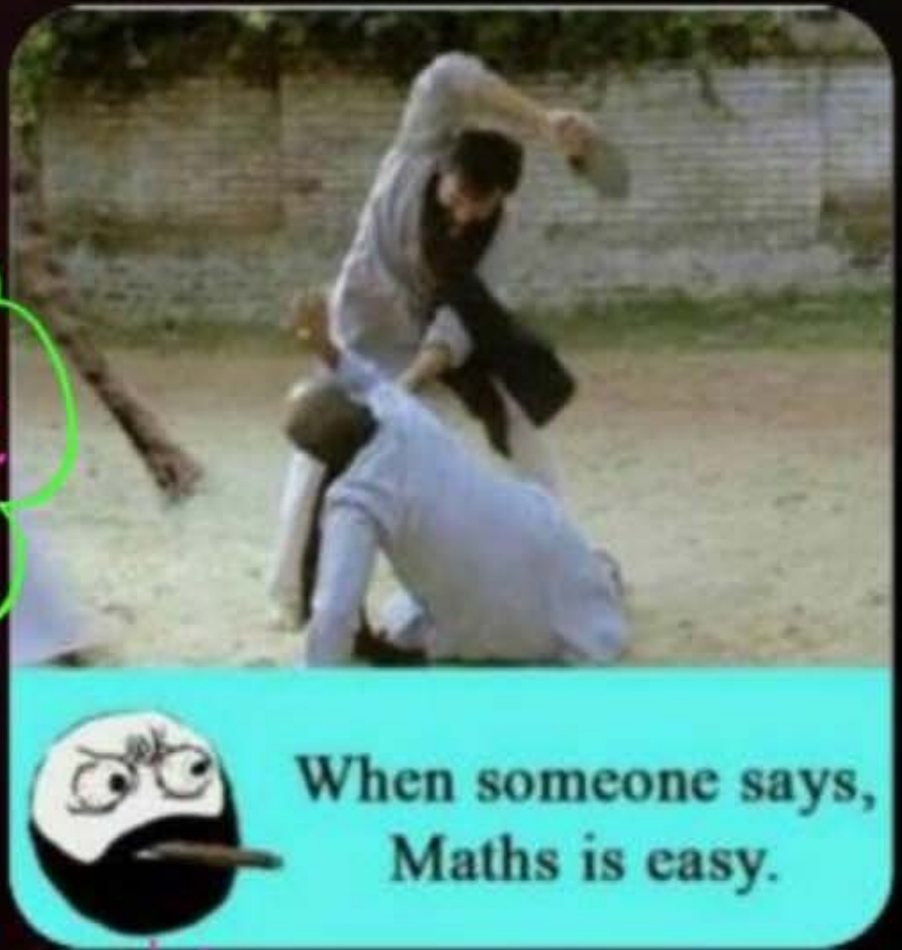
x ki kisi value pe, p(x) ki jo value aaye, use hi polynomial ki value uss x vali value ke liye bolenge!!

$f(-1) = (8)(-1)^2 - (3)(-1) + 7$

$f(-1) = 8 + 3 + 7 = 18$

$\boxed{(-1, 18)}$

Solution of that polynomial



$P(x) =$

#Ques: value the value of polynomial $\boxed{x^2 - 5x + 6}$ at

Solution

(i) $x = 0$

(ii) $x = 1$

(iii) $x = 2$

(iv) $x = 3$

zeros

$P(x) =$

#Ques: value the value of polynomial $\boxed{x^2 - 5x + 6}$ at

Solution

(i) $x = 0 \Rightarrow P(0) = (0)^2 - (5)(0) + 6 = 0 - 0 + 6 = \boxed{6} \Rightarrow (0, 6)$

(ii) $x = 1 \Rightarrow P(1) = (1)^2 - (5)(1) + 6 = 1 - 5 + 6 = \boxed{2} \Rightarrow (1, 2)$

(iii) $x = 2 \Rightarrow P(2) = (2)^2 - (5)(2) + 6 = 4 - 10 + 6 = 10 - 10 = \boxed{0} \Rightarrow (2, 0)$

(iv) $x = 3 \Rightarrow P(3) = (3)^2 - (5)(3) + 6 = 9 - 15 + 6 = 15 - 15 = \boxed{0} \Rightarrow (3, 0)$

zeros

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Question

Find the value of the polynomial $5x - 4x^2 + 3$ at

(a) $x = 0$

(b) $x = -1$

(c) $x = 2$

Question

Find the value of the polynomial $5x - 4x^2 + 3$ at

(a) $x = 0$ $\Rightarrow P(0) = 3$

$-5-4+3 = -9+3 = -6$

(b) $x = -1$ $\Rightarrow P(-1) = -6$

$10-16+3 = 13-16$

(c) $x = 2$ $\Rightarrow P(2) = -3$

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Question

If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

A 3

B $2x$

C 0

D 6

Question

If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to

A 3

B 2x

C 0

D 6

$$p(x) + p(-x)$$

$$= [x+3] + [(-x)+3]$$

$$= \cancel{x}+3 - \cancel{x}+3$$

$$= 6$$

Question

If $f(x) = 2x^3 - 13x^2 + 17x + 12$, find ;

(i) $f(2)$

(ii) $f(-3)$

Question

If $f(x) = 2x^3 - 13x^2 + 17x + 12$, find ;

(i) $f(2)$

$= (2)(2)^3 - (13)(2)^2 + (17)(2) + 12$

$= 16 - 52 + 34 + 12$

$= 62 - 52$

$= \boxed{10}$

(ii) $f(-3)$

$\boxed{\text{H.W.}}$

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Zeros of a polynomial

Zeroes of Polynomial are the real values of the variable for which the value of the polynomial becomes zero. A real number $x = a$ is zeros of a polynomial $f(x)$ if $f(a) = 0$.

For $f(x) = 3x + 1$, if we have $f\left(\frac{-1}{3}\right) = 0$ then $x = \frac{-1}{3}$ is the zeros of $f(x)$.

Zeros of a polynomial

Zeroes of Polynomial are the real values of the variable for which the value of the polynomial becomes zero. A real number $x = a$ is zeros of a polynomial $f(x)$ if $f(a) = 0$.

For $f(x) = 3x + 1$, if we have $f\left(\boxed{\frac{-1}{3}}\right) = 0$ then $\boxed{x = \frac{-1}{3}}$ is the zeros of f(x).

- Value ham uss polynomial ka nikalate hai,
- Lekin zeros, x ki vo value hoti hai, jha pe polynomial ki value zero aa jaye !!

Number of Zeros of a polynomial

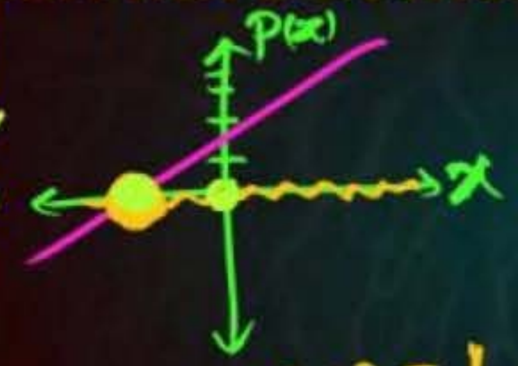
Note: Number of zeros will be always less than or equal to degree of that Polynomial.

An n^{th} degree polynomial can have at most 'n' real roots.

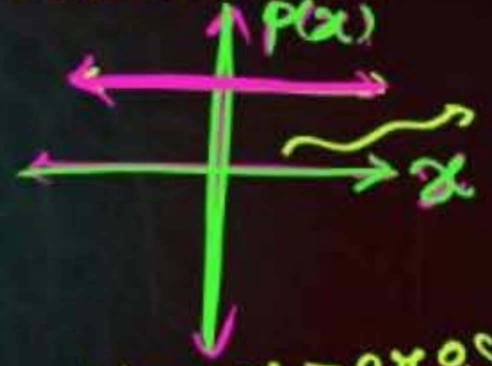
- Linear Polynomial

Degree = 1

$P(x) = 0$



No. of zeros = 1

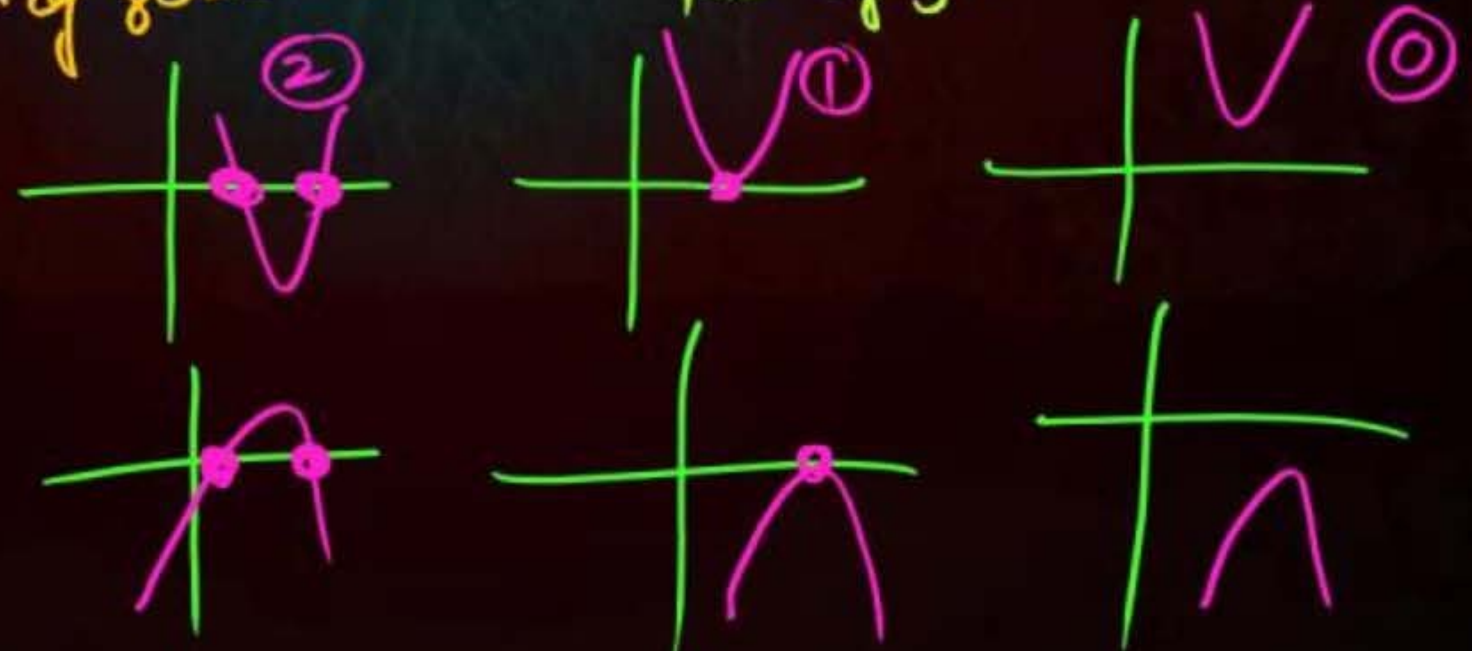


No cut or touch with x-axis

No. of zeros = 0

- Quadratic Polynomial

Degree = 2



Question

zeros

Show that $x = 1$ is a root of a polynomial $P(x)=2x^3 - 3x^2 + 7x - 6$.

Question

Zeros

Show that $x = 1$ is a ~~root~~ of a polynomial $P(x)=2x^3 - 3x^2 + 7x - 6$.

$$P(1) = (2)(1)^3 - (3)(1)^2 + (7)(1) - 6$$

$$P(1) = 2 - 3 + 7 - 6$$

$$P(1) = 9 - 9$$

$\boxed{P(1) = 0}$ → Jab $x=1$ ek zeros ho iss polynomial $P(x)$ ka.

Question

zeros

If $x = \frac{4}{3}$ is a $\boxed{\cancel{\text{root}}}$ of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$, find the value of k.

Question

If $x = \frac{4}{3}$ is a ~~root~~ zeros of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$, find the value of k.

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - (11)\left(\frac{4}{3}\right)^2 + (k)\left(\frac{4}{3}\right) - 20 = 0$$

$$\Rightarrow \overset{2}{\cancel{6}} \times \frac{64}{\underset{9}{\cancel{27}}} - \frac{11 \times 16}{9} + \frac{4k}{3} - 20 = 0$$

$$\frac{128 - 176 + 12k - 180}{9} = 0$$

$$128 - 176 + 12k - 180 = 0$$

$$-48 + 12k - 180 = 0$$

$$12k = 180 + 48$$

$$12k = 228$$

$$k = \frac{228}{12}$$

$$\boxed{k = 19}$$ Ans

Question

If $x = 2$ is a zeros of a polynomial $f(x) = 2x^2 - 3x + 7a$, find the value of a.

Question

If $x = 2$ is a zeros of a polynomial $f(x) = 2x^2 - 3x + 7a$, find the value of a.

$f(2) = 0$

$\boxed{H.W}$

Question

If the zeros of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and −3, then

- **A** $a = -7, b = -1$
- **B** $a = 5, b = -1$
- **C** $a = 2, b = -6$
- **D** $a = 0, b = -6$

Question

If the zeros of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

A $a = -7, b = -1$

B $a = 5, b = -1$

C $a = 2, b = -6$

D $a = 0, b = -6$

$p(x) = x^2 + (a+1)x + b$

$p(2) = 0$

$4 + (a+1)2 + b = 0$

$4 + 2a + 2 + b = 0$

$\boxed{2a + b + 6 = 0}$

$\boxed{b = -2a - 6}$

$p(-3) = 0$

$9 + (a+1)(-3) + b = 0$

$9 - 3a - 3 + b = 0$

$\boxed{-3a + b + 6 = 0}$

$(-3a) + (-2a - 6) + 6 = 0$

$-3a - 2a - \cancel{6} + \cancel{6} = 0$

$-5a = 0$

$a = \frac{0}{-5}$

$\boxed{a = 0}$

$b = (-2 \times 0) - 6$

$b = 0 - 6$

$\boxed{b = -6}$

Question

Divide $2x^3 + x^2 + x$ by x.

Question

Divide $2x^3 + x^2 + x$ by x.

$$= \frac{2x^3+x^2+x}{x}$$

$$= \frac{2x^3}{x} + \frac{x^2}{x} + \frac{x}{x}$$

$$= \boxed{2x^2+x+1}$$

Question

Divide $f(x) = 2x^3 + x^5 - 3x^4 + x^2 + 5$ by $g(x) = x + 2$

Question

Divide $f(x) = 2x^3 + x^5 - 3x^4 + x^2 + 5$ by $g(x) = x + 2$

$$\frac{f(x)}{g(x)} = \frac{2x^3 + x^5 - 3x^4 + x^2 + 5}{x+2}$$

Division of polynomials (Long division method)

Step (1): We need to write the dividend and divisor in standard form i.e., after arranging the terms in descending order of their degree.

Step (2): We divide first term of dividend by first term of divisor . This gives us the first term of quotient.

Step (3): We multiply the divisor by first term of quotient and subtract this product from dividend. This gives us reminder.

Step (4): Now, we need to treat this reminder as new dividend and repeat from step(2).

Step (5): This process continues till reminder is zero or degree of new dividend is less than degree of divisor.

Question

Find $f(x) \div g(x)$ where $f(x) = x^2 + 1 + 5x^3$ and $g(x) = x - 2$.

Question

Find f(x) ÷ g(x) where f(x) = $x^2 + 1 + 5x^3$ and g(x) = $x - 2$. $= 0 \rightsquigarrow x = 2$

$f(2) = 4 + 1 + 40$
$= 45$

① $x - 2 \overline{)\, 5x^3 + x^2 + 1\,}\ (\,5x^2 + 11x + 22$

$\cancel{5x^3} - 10x^2$
$\ominus \quad \oplus$

$\cancel{11x^2} + 1$
$\cancel{11x^2} - 22x$
$\ominus \quad \oplus$

$\cancel{22x} + 1$
$\cancel{22x} - 44$
$\ominus \quad \oplus$

$45\,x^0$

Rough

ⓐ $\frac{5x^3}{x} = 5x^2$

ⓑ $x \xrightarrow{5x^2} 5x^3$

$\frac{11x^2}{x} = 11x$

$x \xrightarrow{11x} 11x^2$

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Ye Toh Magic Hai

(1) $f(x) = 2x^3 + x^5 - 3x^4 + x^2 + 5 \div g(x) = x + 2$

(2) $f(x) = x^2 + 1 + 5x^3 \div g(x) = x - 2.$

(3) $x^4 - 3x^3 + 2x^2 + 1 \div 2x - 1$

Ye Toh Magic Hai

(1) $f(x) = 2x^3 + x^5 - 3x^4 + x^2 + 5 \div g(x) = x + 2 = 0 \Rightarrow \boxed{x = -2}$

Rem $= f(-2)$

(2) $f(x) = x^2 + 1 + 5x^3 \div g(x) = x - 2. = 0 \Rightarrow x = 2$

Rem$= f(2)$

(3) $x^4 - 3x^3 + 2x^2 + 1 \div 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

Rem$= f(\frac{1}{2})$

Remainder Theorem

Let $\boxed{p(x)}$ be any polynomial of degree greater, than or equal to one and 'a' be any real number. If p(x) is divided by $\boxed{x - a}$, then the remainder is equal to $p(a)$.

$= 0$

$x = a \rightarrow p(a) = \text{Remainder}$

q(x) = quotient

r(x) = remainder

p(x) = dividend

$x - a$ = divisor

Simple Bhasha main ...

The remainder theorem is used to find the remainder of a polynomial when a polynomial is divided by another polynomial.

Notice: When an integer is divided by another integer, we obtained R < divisor or R = 0. Similarly, when a polynomial is ÷ by another polynomial, we get

R = 0 or degree of 'R' < degree of 'Divisor'

Question

Find the reminder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.

Question

P(x)

Find the reminder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.

$x - 1 = 0 \Rightarrow x = 1$

Remainder of $\dfrac{x^4 + x^3 - 2x^2 + x + 1}{x - 1} = P(1)$

$= 1 + 1 - 2 + 1 + 1 = \boxed{2}$

$$x - 1 \overline{)\, \cancel{x^4} + x^3 - 2x^2 + x + 1 \,}\,(x^3 + 2x^2 + 1$$

$$\cancel{x^4} - x^3$$

$$\ominus \quad \oplus$$

$$\cancel{2x^3} - \cancel{2x^2} + x + 1$$

$$\cancel{2x^3} - \cancel{2x^2}$$

$$\ominus \quad \oplus$$

$$\cancel{x} + 1$$

$$\cancel{x} - 1$$

$$\ominus \quad \oplus$$

$$2$$

Question

Find the value of k if $p(x) = (3x - 2)(x - k) - 8$ is divided by $(x - 2)$ leaving the remainder 4.

Question

Find the value of k if $p(x) = (3x - 2)(x - k) - 8$ is divided by $(x - 2)$ leaving the remainder 4.

Rem $= 4$

$p(2) = 4$ (using Remainder theorem)

#chamak gya

$[(3 \times 2) - 2][2 - k] - 8 = 4$

$4 \times (2 - k) - 8 = 4$

$\cancel{8} - 4k - \cancel{8} = 4$

$-4k = 4$

$k = \frac{4}{-4} \Rightarrow \boxed{k = -1}$ Ans

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Question

For what value of m is $x^3 - 2mx^2 + 16$ completely divisible by $x + 2$?

Question

For what value of m is $x^3 - 2mx^2 + 16$ completely divisible by $x + 2$?

Remainder $= 0$

$x + 2 = 0$

$\boxed{x = -2}$

$$(-2)^3 - (2)(m)(-2)^2 + 16 = 0$$

$$-8 - 8m + 16 = 0$$

$$-8m + 8 = 0$$

$$8m = 8$$

$$m = \frac{8}{8} \Rightarrow \boxed{m = 1}$$

Question

If $x^{51} + 51$ is divided by $x + 1$, the remainder is

0

1

49

50

Question

If $x^{51} + 51$ is divided by $x + 1$, the remainder is

$=0$

$\boxed{x=-1}$

A 0

B 1

C 49

D 50

$Rem = (-1)^{51} + 51$

$= -1 + 51$

$= \boxed{50}$

$(-1)^{even} = 1$

$(-1)^{odd} = -1$

Question

If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leaves the same remainder when divided by $z - 3$, find the value of 'a'.

Question

If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leaves the $\boxed{\text{same remainder}}$ when divided by $z - 3$, find the value of 'a'.

$=0$

$z=3$

$(a)(3)^3 + (4)(3)^2 + (3)(3) - 4 = (3)^3 - (4)(3) + a$

$27a + 36 + 9 - 4 = 27 - 12 + a$

$27a - a = 27 - 12 - 36 - 9 + 4$

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Question

Let R_1 and R_2 are the remainder when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$ find the value of 'a'.

Question

Let R_1 and R_2 are the remainder when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$ find the value of 'a'.

$x+1=0 \qquad x-2=0$

$\boxed{x=-1} \qquad \boxed{x=2}$

$R_1 = (-1)^3 + (2)(-1)^2 - (5)(a)(-1) - 7 = -1+2+5a-7 = \boxed{5a-6} = R_1$

$R_2 = 8 + 4a - 24 + 6 \Rightarrow \quad R_2 = 4a - 10$

Given, $2R_1 + R_2 = 6$

$2(5a-6) + (4a-10) = 6$

$10a - 12 + 4a - 10 = 6$

$14a = 28 \Rightarrow \boxed{a=2}$

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Aaa Jao Thoda Level Up Karte Hai…

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Question

Find the remainder when $\boxed{x^{100}}$ is divided by $\boxed{x^2 - 3x + 2.}$ [Olympiad]

Question

Find the remainder when $\boxed{x^{100}}$ is divided by $\boxed{x^2 - 3x + 2.}$

[Olympiad]

linear ✗

$$x^{2} - 3x + 2 \overline{\big) \, x^{100} \, \big(} \; q(x)$$

98 steps ✗

$$r(x) = ax + b$$

$$\boxed{x^{100} = \left[(x^2 - 3x + 2) \times q(x)\right] + ax + b}$$

$$x^{100} = (x-1)(x-2) \times q(x) + ax + b$$

Concept Area

$$\boxed{5 = 3 \times 1 + 2}$$

$$g(x) \overline{\big) \, p(x) \, \big(} \; q(x)$$

$$r(x)$$

$$3 \overline{\big) \, 5 \, \big(} \; 1$$

$$\frac{3}{2}$$

Division Algorithm:

$$\boxed{p(x) = g(x) \times q(x) + r(x)}$$

$$x^{100} = \boxed{(x-2)(x-1)\times q(x)} + ax+b$$

at $\boxed{x=1}$

$$(1)^{100} = (1-2)\underbrace{(1-1)}_{0}\times q(1) + a+b$$

$$1 = 0 + a+b$$

$$\boxed{1 = a+b} \rightarrow b = 1-a$$

at $\boxed{x=2}$

$$2^{100} = \boxed{(\overset{=0}{2-2})(2-1)\times q(2)} + 2a+b$$

$$2^{100} = 0 + 2b + b \Rightarrow \boxed{2^{100} = 2a+b}$$

$$2^{100} = 2a + 1 - a$$

$$2^{100} = a+1 \Rightarrow \boxed{a = 2^{100} - 1}$$

$$b = 1 - (2^{100} - 1) = 1 - 2^{100} + 1$$

$$\boxed{b = 2 - 2^{100}}$$

$$\text{Remainder} = ax+b$$

$$= \boxed{(2^{100}-1)x + (2-2^{100})}$$

Factor theorem

If p(x) is a polynomial of degree $n \geq 1$ and 'a' is any real number, then

(i) $x - a$ is a factor of $p(x)$, if $p(a) = 0$, and

(ii) If $p(a) = 0$, then $x - a$ is a factor of $p(x)$.

$P(x) \rightsquigarrow (x-a) \rightarrow \boxed{Rem = P(a)}$

$\boxed{P(a) = 0} \rightarrow$ factor ka matlab hi hota Hai ki pura-pura divide kare!! $\downarrow$ Rem = 0

$4 \rightarrow 1, 2, 4$

$2 \overline{)4}(2$

4

Rem = $\boxed{0}$

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Question

Show that $(x - 3)$ is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$.

Question

Show that $(x - 3)$ is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$.

$= 0$

$\boxed{x = 3}$

$\text{Rem} = p(3) = 27 - 27 + 12 - 12$

$= 0$ → Thus $x - 3$ must be a factor of $p(x)$.

Question

Find the value of l, so that $y - 2p$ is a factor of $\frac{y^3}{4p^2} - 2y + lp$. [IMO Set A, 2015]

0

1

2

3

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Question

Find the value of l, so that $y - 2p$ is a factor of $\frac{y^3}{4p^2} - 2y + lp$. [IMO Set A, 2015]

A 0

B 1

C 2

D 3

$= 0$

$\Rightarrow \boxed{y = 2p}$

Rem $= 0$

$$\frac{(2p)^3}{4p^2} - 2(2p) + lp = 0$$

$$\frac{\cancel{8}\,2p^3}{4p^2} - 4p + lp = 0$$

$$2p - 4p + lp = 0$$

$$-2p + lp = 0 \Rightarrow l \times p = 2 \times p \Rightarrow l = \frac{2\cancel{p}}{\cancel{p}} \Rightarrow \boxed{l = 2}$$

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Question

Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 + 5k + 6$.

A $\frac{13}{5}$

B $\frac{5}{13}$

C $\frac{-13}{5}$

D $\frac{-5}{13}$

Question

Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 + 5k + 6$.

A $\frac{13}{5}$

B $\frac{5}{13}$

C $\frac{-13}{5}$

D $\frac{-5}{13}$

Question

If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

Question

If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

$= 0$

$\boxed{x = 2}$

$\text{Rem}(2) = 0$

$(p \times 4) + 10 + r = 0$

$\boxed{4p + r + 10 = 0}$

$r = -10 - 4p$

$= 0$

$x = \frac{1}{2}$

$\text{Rem}(\frac{1}{2}) = 0$

$\frac{p}{4} + \frac{5}{2} + r = 0$

$\frac{p + 10 + 4r}{4} = 0$

$\boxed{p + 4r + 10 = 0}$

$p + 4(-10 - 4p) + 10 = 0$

$p - 40 - 16p + 10 = 0$

$-15p - 30 = 0$

$-15p = 30 \Rightarrow \boxed{p = -2}$

$r = -10 - (4 \times -2)$

$r = -10 + 8$

$\boxed{r = -2}$

$\boxed{p = r}$

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Question

If $(x+2)$ and $(x-2)$ are factors of $ax^4 + 2x - 3x^2 + bx - 4$, then the value of $a + b$ is

A -7

B 7

C 14

D 8

Question

If $(x + 2)$ and $(x - 2)$ are factors of $ax^4 + 2x - 3x^2 + bx - 4$, then the value of $a + b$ is

A -7

B 7

C 14

D 8

Question

Without actual division prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

Question

$2\,\cancel{-6}+\cancel{3}\,\cancel{+3}\,\cancel{-2}=\boxed{0}$

Without actual division prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

Pura - Pura divide Karna

Rem = 0

Factor ban jana

$$x^2-3x+2\,\overline{)\,2x^4-6x^3+3x^2+3x-2\,}\,(\,q(x)$$

$$0$$

$$\boxed{2x^4-6x^3+3x^2+3x-2=(x^2-3x+2)\times q(x)+0}$$

$$(\quad\quad " \quad\quad)=(x-1)(x-2)\times q(x)$$

$$\boxed{x=1 \text{ or } x=2}\Rightarrow\boxed{P(1) \text{ or } P(2)=0}$$

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Question

Which of the following is the factor of the polynomial $p(x) = x^4 + 5x^3 + 9x^2 + 15x + 18$?

[IMO 2015 Set A]

A. $x^2 + 5x + 6$

B. $x^2 - 5x + 6$

C. $x^2 + 5x - 6$

D. $x^2 - 5x - 6$

Question

Which of the following is the factor of the polynomial $p(x) = \boxed{x^4 + 5x^3 + 9x^2 + 15x + 18}$?

[IMO 2015 Set A]

A $x^2 + 5x + 6 = (x+2)(x+3) \Rightarrow \boxed{x = -2 \text{ \& } -3}$

B $x^2 - 5x + 6 = (x-2)(x-3) \Rightarrow \boxed{x = 2, 3}$

C $x^2 + 5x - 6 = (x+6)(x-1) \Rightarrow \boxed{x = -6, 1}$

D $x^2 - 5x - 6 = (x-6)(x+1) \Rightarrow \boxed{x = 6, -1}$

Jaha pe $P(\underline{\cdot}), P(\_)$

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Question

If $(x - 3)$ and $(x - \frac{1}{3})$ are both factors of $ax^2 + 5x + b$, then

A $a = 5b$

B $2a = b$

C $a = 2b$

D $a = b$

Question

If $(x - 3)$ and $(x - \frac{1}{3})$ are both factors of $ax^2 + 5x + b$, then

A $a = 5b$

B $2a = b$

C $a = 2b$

D $a = b$

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Factorisation

- Method of common factors
- Grouping
- Identity ✓
- Middle term splitting (Quadratic) ✓

Middle Term Splitting

Factorise: $x^2 + 5x + 6$

Middle Term Splitting

Factorise: $x^2 + 5x + 6$

1 4 × 6

$\boxed{2 \quad 3}$ ✓ $2 \times 3 = 6$

$ax^2 + bx + c$

$b_1 \times b_2 = a \times c$

$x^2 + (2+3)x + 6$

$= x^2 + 2x + 3x + 6$

$= x(x+2) + 3(x+2)$

$= (x+2)(x+3)$

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Question

Factors of $y^2 + 10y + 24$ will be

$(y + 6)(y - 4)$

$(y - 6)(y + 4)$

$(y + 6)(y + 4)$

$(y - 6)(y - 4)$

Question

Factors of $y^2 + 10y + 24$ will be

A $(y+6)(y-4)$

B $(y-6)(y+4)$

C $\boxed{(y+6)(y+4)}$ ✓

D $(y-6)(y-4)$

10

6 4 $\Rightarrow 6 \times 4 = 24$

$= y^2 + 6y + 4y + 24$

$= y(y+6) + 4(y+6)$

$= (y+6)(y+4)$

Question

Factorise:

(i) $x^2 + 15x + 56$

(ii) $x^2 + 13x + 40$

(iii) $p^2 + 6x - 16$

(iv) $x^2 - 23x + 42$

Question

Factorise:

(i) $x^2 + 15x + 56$

$7 \quad 8$

$\boxed{(x+7)(x+8)}$

(ii) $x^2 + 13x + 40$

$8 \quad 5$

$\boxed{(x+8)(x+5)}$

(iii) $p^2 + 6x - 16$

$8 \quad -2$

$\boxed{(x+8)(x-2)}$

(iv) $x^2 - 23x + 42$

$-21 \quad -2$

$\boxed{(x-21)(x-2)}$

Question

Factorise $x^3 - 2x^2 - x + 2$.

Question

Factorise $x^3 - 2x^2 - x + 2$.

$= x^2(x-2) - 1(x-2)$

$= (x-2)(x^2-1)$ $a^2-b^2 = (a-b)(a+b)$

$= (x-2)(x^2-1^2)$

$= \boxed{(x-2)(x-1)(x+1)}$

Factorisation by Trial and Error Method

Factorise: $x^3 - 6x^2 + 11x - 6.$

Factorisation by Trial and Error Method

Factorise: $x^3 - 6x^2 + 11x - 6.$ $= (x-1)(x-2)(x-3)$

2

$x = 0,$ $0 - 0 + 0 - 6 = -6$ ✗

$x = 1,$ $1 - 6 + 11 - 6 = 12 - 12 = \boxed{0} \Rightarrow \boxed{(x-1)}$ ✓

$x = -1,$ $-1 - 6 - 11 - 6 = \ominus ve$ ✗

$x = 2,$ $8 - 24 + 22 - 6 = 30 - 30 = \boxed{0} \Rightarrow (x-2)$ ✓

$x = 3,$ - - - - - - - - $= \boxed{0} \Rightarrow (x-3)$ ✓

Drawback: It was really difficult to figure out those value of x which can satisfy the polynomial.

Factorising a polynomial by factor theorem

constant

Let us understand this process by taking an example. Given that, $f(x) = x^2 - 5x + \boxed{6}$

Highest exponent vale variable ka coefficient

If leading coefficient of given polynomial is one, then

(a) Find factor of constant term of f(x) i.e., 6
i.e., 1, –1, 2, –2, 3, –3, 6, –6

$\boxed{6 \rightarrow \pm 1, \pm 2, \pm 3, \pm 6}$ ✓

Possible value of x

(b) Take the above factors one by one and find out which of them satisfies the polynomial. Since, it has eight factors and degree of polynomial is two, so any two out of these eight will be the root of the given polynomial.

Now, $f(2) = (2)^2 - 5(2) + 6 = 0$

Question

Factorise $x^3 - 6x^2 + 11x - 6$ using factor theorem.

Question

Factorise $x^3 - 6x^2 + 11x - 6$ using factor theorem.

$6 \rightarrow \pm 1, \pm 2, \pm 3, \pm 6$

$x = 1 \longrightarrow = 0$ ✓

$x = -1$

$x = 2 \longrightarrow = 0$ ✓

$x = -2$

$x = 3 \longrightarrow = 0$ ✓

$x = -3$

$x = 6$

$x = -6$

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Factorizing a Cubic polynomial using factor theorem (Alternate Method)

This method can be used, if it is easier to obtain one factor by factor theorem and then the given polynomial is divided by the factor obtained to get quotient. It can be very helpful if the quotient is a quadratic polynomial, which can be factorized by splitting the middle term.

$x \to -3, -2, -1, \boxed{0}, 1, 2, 3$

check at only these 4 on priority

Question

Factorise the given polynomial $g(x) = x^4 - 5x^2 + 4$ given that $x - 2$ is a factor of $g(x)$.

Question

Factorise the given polynomial $g(x) = x^4 - 5x^2 + 4$ given that $x - 2$ is a factor of $g(x)$.

$$
\begin{array}{r|l}
x-2 & \cancel{x^4} - 5x^2 + 4 \quad (x^3 + 2x^2 - x - 2 \\
 & \underline{\cancel{x^4} - 2x^3} \\
 & 2x^3 - 5x^2 + 4 \\
 & \underline{2x^3 - 4x^2} \\
 & -\cancel{x^2} + 4 \\
 & \underline{-\cancel{x^2} + 2x} \\
 & -\cancel{2x} + \cancel{4} \\
 & \underline{-\cancel{2x} + \cancel{4}} \\
 & 0
\end{array}
$$

$\pm 1, \pm 2$

$\boxed{(x-1)} \Leftarrow x = 1, \quad 1+2-1-2=0$ ✓

$\boxed{x=2}, \quad 8+8-2-2 \neq 0$ ✗

$(x+1) \Leftarrow x=-1, \quad -1+2+1-2 = 0$ ✓

$= 0$ ✓

$(x+2) \Leftarrow x=-2$

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Question

Give possible expressions for the length and breadth of the rectangle whose area is given by $\boxed{4a^2 + 4a - 3}$

$(2a - 1)(2a - 3)$

$(2a - 1)(2a + 3)$

$(a - 2)\left(a + \frac{3}{2}\right)$

$(2a + 1)(2a - 3)$

Question

Give possible expressions for the length and breadth of the rectangle whose area is given by $\boxed{4a^2 + 4a - 3}$

-12

-6×2

A $(2a - 1)(2a - 3)$

B $(2a - 1)(2a + 3)$

C $(a - 2)\left(a + \frac{3}{2}\right)$

D $\boxed{(2a + 1)(2a - 3)}$ ✓

$4a^2 - 6a + 2a - 3$

$2a(2a-3) + 1(2a-3)$

$(2a+1)(2a-3) = \text{Area}$

Area of Rectangle $= l \times b$

Question

What must be added to $\boxed{x^3 - 3x^2 + 4x - 15}$ to obtain a polynomial which is exactly divisible by $(x - 3)$?

Question

What must be added to $\boxed{x^3 - 3x^2 + 4x - 15}$ to obtain a polynomial which is exactly divisible by $(x - 3)$?

$$x - 3 \,\overline{\big)\, x^3 - 3x^2 + 4x - 15 + (k) \,\big(}\; q(x)$$

$x - 3 = 0$

$\boxed{x = 3}$

rem $= 0$

$$(3)^3 - 3(3)^2 + (4)(3) - 15 + k = 0$$

Question

What must be subtracted from $x^4 + 2x^3 - 2x^2 + 4x + 6$ so that the final result is exactly divisible by $(x^2 + 2x - 3)$?

Question

What must be subtracted from $x^4 + 2x^3 - 2x^2 + 4x + 6$ so that the final result is exactly divisible by $(x^2 + 2x - 3)$?

$$P(x) = \underbrace{\left[x^4 + 2x^3 - 2x^2 + 4x + 6 - (ax+b)\right]} = (x^2 + 2x - 3) \times q(x) + 0$$

$$= \boxed{(x+3)}\boxed{(x-1)} \times \boxed{q(x)}$$

Quadratic

$P(-3) = 0 \Rightarrow \underline{a} \quad \underline{b} \rightarrow$ linear

$P(1) = 0 \rightarrow a \quad b \rightarrow$ linear

Algebraic Identities

Identity (I) : $(x+y)^2 = x^2 + 2xy + y^2$

Identity (II) : $(x-y)^2 = x^2 - 2xy + y^2$

Identity (III) : $x^2 - y^2 = (x+y)(x-y)$

Identity (IV) : $(x+a)(x+b) = x^2 + (a+b)x + ab$

Class 8

Identity (V) : $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Square Identities

Identity (VI) : $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Identity (VII) : $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Identity (VIII) : $x^3 + y^3 + z^3 - 3xyz = (x+y+z)\left(x^2 + y^2 + z^2 - xy - yz - zx\right)$

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$$

Identity (IX) : If $(x+y+z) = 0$ then $x^3 + y^3 + z^3 = 3xyz$ Imp.

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Square Identities

Identity (I) : $(x+y)^2 = x^2 + 2xy + y^2$

Identity (II) : $(x-y)^2 = x^2 - 2xy + y^2$

$x^2 \cancel{-xy} \cancel{-yx} - y^2$

Identity (III) : $x^2 - y^2 = \boxed{(x+y)(x-y)} = x^2 - y^2$

Identity (IV) : $(x+a)(x+b) = x^2 + (a+b)x + ab$

Class 8^{th}

Identity (V) : $\boxed{(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx}$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$$(x+y+z)^2 = [(x+y)+z]^2$$

$$= (x+y)^2 + (z)^2 + 2(x+y)(z)$$

$$= x^2+y^2+2xy + z^2 + 2xz + 2yz$$

$$= \boxed{x^2+y^2+z^2+2xy+2yz+2zx}$$

$$(x+y+z)^2 = [(x+y)+z]^2$$

$$= (x+y)^2 + (z)^2 + 2(x+y)(z)$$

$$= x^2+y^2+2xy + z^2 + 2xz + 2yz$$

$$= \boxed{x^2+y^2+z^2+2xy+2yz+2zx}$$

Question

The value of the term $249^2 - 248^2$

1

477

487

497

Question

$a^2 - b^2 = (a-b)(a+b)$

The value of the term $249^2 - 248^2$ $= (249-248)(249+248) = 1 \times 497$

$= \boxed{497}$

A 1

B 477

C 487

D 497

$$\left\{ \begin{array}{r} 249 \\ \times\ 249 \\ \hline \\ \hline \end{array} - \begin{array}{r} 248 \\ \times\ 248 \\ \hline \\ \hline \end{array} = (\quad) \right.$$

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Question

Expand each of the following:

(i) $(3x + 4y)^2$

(ii) $(5a - 2b)^2$

Question

Expand each of the following:

(i) $(\underbrace{3x}_{a} + \underbrace{4y}_{b})^2$

$(a+b)^2 = a^2 + b^2 + 2ab$

$= (3x)^2 + (4y)^2 + 2(3x)(4y)$

$= 9x^2 + 16y^2 + 24xy$

(ii) $(5a - 2b)^2$

$(x - y)^2 = (5a)^2 + (2b)^2 - 2(5a)(2b)$

$= [\underbrace{(5a)}_{x} + \underbrace{(-2b)}_{y}]^2$

$= (5a)^2 + (-2b)^2 + 2(5a)(-2b)$

$= 25a^2 + 4b^2 - 20ab$

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Question

Use suitable identities to find the products:

(i) $(x+4)(x+10)$

(ii) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

Question

Use suitable identities to find the products:

(i) $(x+4)(x+10)$

$x^2+(4+10)x+(4\times 10)$

$x^2+14x+40$

(ii) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$(a+b)(a-b)=a^2-b^2$

$=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2$

$=\boxed{y^4-\frac{9}{4}}$

Question

Evaluate the following using suitable identity:

(i) 103×107 (ii) 207×193 (iii) $(102)^2$

Question

Evaluate the following using suitable identity:

(i) 103×107

$(105-2)(105+2)$

$= (105)^2 - (2)^2$

$= 11025 - 4$

$= \boxed{11021}$ Ans

(ii) 207×193

$= (200+7)(200-7)$

$= (200)^2 - (7)^2$

$= 40000 - 49$

$= \boxed{39951}$

(iii) $(102)^2$

$\boxed{(a-b)(a+b) = a^2 - b^2}$

$(100+2)^2$

$= (100)^2 + (2)^2 + 2(100)(2)$

$= 10000 + 4 + 400$

$= \boxed{10404}$ Ans

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Question

The value of the term 104×96

9894

9984

9684

9884

Question

The value of the term 104×96

A 9894

B 9984

C 9684

D 9884

$= (100+4)(100-4)$

$= (100)^2 - (4)^2$

$= 10000 - 16$

$= \boxed{9984}$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

Expand each of the following:

(i) $(x + 2y + 4z)^2$

(ii) $(-2x + 3y + 2z)^2$

(iii) $\left(\frac{1}{4}a - \frac{1}{2}b - 1\right)^2$

Question

Expand each of the following:

(i) $(x + 2y + 4z)^2$ (ii) $(-2x + 3y + 2z)^2$ (iii) $\left(\frac{1}{4}a - \frac{1}{2}b - 1\right)^2$

$(x)^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(4z)(x)$ (i)

$= \boxed{x^2+4y^2+16z^2+4xy+16yz+8zx}$

$= [(-2x)+(3y)+(2z)]^2$

$= (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)$ (ii)

$\boxed{= 4x^2+9y^2+4z^2-12xy+12yz-8zx}$

$\boxed{(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca}$

$\left[\left(\frac{a}{4}\right)+\left(-\frac{b}{2}\right)+(-1)\right]^2$

$\left(\frac{a}{4}\right)^2+\left(-\frac{b}{2}\right)^2+(-1)^2+2\left(\frac{a}{4}\right)\left(-\frac{b}{2}\right)+2\left(-\frac{b}{2}\right)(-1)+2(-1)\left(\frac{a}{4}\right)$

$= \boxed{\frac{a^2}{16}+\frac{b^2}{4}+1+\frac{1}{4}(-ab)+b-\frac{a}{2}}$ (iii)

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Question

Factorise the following:

(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$ (iii) $x^2 - \frac{y^2}{100}$

Question

Factorise the following:

(i) $9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x+y)^2$

$\boxed{a^2 + 2ab + b^2 = (a+b)^2}$

(ii) $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y-1)^2$

$\boxed{a^2 - 2ab + b^2 = (a-b)^2}$

(iii) $x^2 - \frac{y^2}{100}$

$(x)^2 - \left(\frac{y}{10}\right)^2$

$= \left(x - \frac{y}{10}\right)\left(x + \frac{y}{10}\right)$

$\boxed{a^2 - b^2 = (a-b)(a+b)}$

Question

Factorise: $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Question

Factorise: $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)\boxed{(-4z)} + 2\boxed{(-4z)}(2x)$$

$$= [(2x) + (3y) + (-4z)]^2$$

$$= (2x + 3y - 4z)^2$$

Question

Factorise: $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

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Question

Factorise: $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$$(-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + \boxed{2(-\sqrt{2}x)(y)}$$

$$+ 2(y)(2\sqrt{2}z)$$

$$+ \boxed{(2\sqrt{2}z)(-\sqrt{2}x)}$$

$$= [(-\sqrt{2}x) + (y) + (2\sqrt{2}z)]^2$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

Question

(a) Factorise: $x^2 - 1 - 2a - a^2$

Question

(a) Factorise: $x^2 - 1 - 2a - a^2$

$= x^2 - (1 + 2a + a^2)$

$= x^2 - \left((1)^2 + 2(1)(a) + (a)^2\right)$

$= x^2 - (1+a)^2 = [(x) - (1+a)][(x) + (1+a)] = \boxed{(x-1-a)(x+1+a)}$

(b) Factorise: $1 + 2ab - (a^2 - b^2) \longrightarrow$ (Imp)

Question

Factors of $x^4 + 4$ will be

$(x^2 - 2x - 2)(x^2 + 2x - 2)$

$(x^2 - 2x + 2)(x^2 + 2x - 2)$

$(x^2 - 2x + 2)(x^2 + 2x + 2)$

Not Possible to factorize

Question

Factors of $x^4 + 4$ will be

A $(x^2 - 2x - 2)(x^2 + 2x - 2)$

B $(x^2 - 2x + 2)(x^2 + 2x - 2)$

C $(x^2 - 2x + 2)(x^2 + 2x + 2)$ ✓

D Not Possible to factorize

$$x^4 + 4 = (x^2)^2 + (2)^2 + 2(x^2)(2) - 2(x^2)(2)$$

$$= (x^2+2)^2 - 4x^2$$

$$= (x^2+2)^2 - (2x)^2$$

$$= [(x^2+2) - (2x)][(x^2+2) + (2x)]$$

$$= [x^2 - 2x + 2][x^2 + 2x + 2]$$

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Question

Factorise : $a(a-1)-b(b-1)$

$(a+b)(a+b-1)$

$(a-b)(a-b+1)$

$(a-b)(a+b-1)$

Not Possible to factorize

Question

Factorise : $a(a-1)-b(b-1)$ $= a^2 - a - b^2 + b$

$= \underbrace{a^2 - b^2} - a + b$

$= (a-b)(a+b) - 1(a-b)$

$= (a-b)[(a+b)-1]$

A $(a+b)(a+b-1)$

B $(a-b)(a-b+1)$

C $\boxed{(a-b)(a+b-1)}$ ✓

D Not Possible to factorize

Question

If $a + b + c = 9$ and $ab + bc + ca = 26$ then $a^2 + b^2 + c^2$

26

27

28

29

Question

If $a + b + c = 9$ and $ab + bc + ca = 26$ then $a^2 + b^2 + c^2$

A 26

B 27

C 28

D 29 ✓

$$(9)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$81 = a^2+b^2+c^2+(2\times 26)$$

$$\Rightarrow a^2+b^2+c^2 = 81-52 = \boxed{29}$$

$$(a+b+c)^2 = \boxed{a^2+b^2+c^2}+\boxed{2ab+2bc+2ca}$$

Question

If $\left(x+\frac{1}{x}\right)=3$ find the value of $x^2+\frac{1}{x^2}$ and $x^4+\frac{1}{x^4}$.

Question

If $\left(x+\frac{1}{x}\right)=3$ find the value of $x^2+\frac{1}{x^2}$ and $x^4+\frac{1}{x^4}$.

$$\left(x+\frac{1}{x}\right)^2=(x)^2+\left(\frac{1}{x}\right)^2+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)$$

$$(3)^2=x^2+\frac{1}{x^2}+2$$

$$9=x^2+\frac{1}{x^2}+2$$

$$\boxed{x^2+\frac{1}{x^2}=7}$$

$$\left(x^2+\frac{1}{x^2}\right)^2=(x^2)^2+\left(\frac{1}{x^2}\right)^2+2(\cancel{x^2})\left(\frac{1}{\cancel{x^2}}\right)$$

$$(7)^2=x^4+\frac{1}{x^4}+2$$

$$49=x^4+\frac{1}{x^4}+2$$

$$\boxed{x^4+\frac{1}{x^4}=47}$$

Cubic Identities

Identity : $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Identity : $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Cubic Identities

Identity : $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$$(x+y)(x+y)^2 = (x+y)(x^2+y^2+2xy)$$
$$= x[x^2+y^2+2xy] + y[x^2+y^2+2xy]$$
$$= x^3 + xy^2 + 2x^2y + x^2y + y^3 + 2xy^2] = x^3+y^3+3x^2y+3xy^2$$
$$= x^3+y^3+3xy(x+y)$$

Identity : $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$(a+b)^3 = \boxed{a^3+b^3+3a^2b+3ab^2}$$

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Question

Write the following cube in expanded form:

$(3a + 4b)^3$

Question

Write the following cube in expanded form:

$(3a + 4b)^3$

$= (3a)^3 + (4b)^3 + 3(3a)(4b)[3a + 4b]$

$= 27a^3 + 64b^3 + 36ab(3a + 4b)$

$= \boxed{27a^3 + 64b^3 + 108a^2b + 144ab^2}$

Question

Write the following cube in expanded form: $(5p - 3q)^3$

$125p^3 - 27q^3 - 225p^2q + 135pq^2$

$125p^3 + 27q^3 + 225p^2q - 135pq^2$

$125p^3 - 27q^3 - 135pq^2 + 225p^2q$

$125p^3 - 27q^3 - 135p^2q + 225pq^2$

Question

Write the following cube in expanded form: $(5p - 3q)^3$

A $125p^3 - 27q^3 - 225p^2q + 135pq^2$

B $125p^3 + 27q^3 + 225p^2q - 135pq^2$

C $125p^3 - 27q^3 - 135pq^2 + 225p^2q$

D $125p^3 - 27q^3 - 135p^2q + 225pq^2$

$$(5p)^3 - (3q)^3 - 3(5p)(3q)(5p-3q)$$

$$= 125p^3 - 27q^3 - 45pq(5p-3q)$$

$$= \boxed{125p^3 - 27q^3 - 225p^2q + 135pq^2}$$

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Question

Evaluate using identity: $(999)^3$

A 997002998

B 999000299

C 999777991

D 997002999

Question

Evaluate using identity: $(999)^3 =$ $(1000-1)^3$

A 997002998

B 999000299

C 999777991

D 997002999

Question

$$\left(x-\frac{2}{3}y\right)^3 =$$

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Question

Factorise each of the following:

$8a^3 + b^3 + 12a^2b + 6ab^2$

Question

Factorise each of the following:

$8a^3 + b^3 + 12a^2b + 6ab^2$

$(2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2$

$= (2a+b)^3$

Cubic Identities

Identity: $\boxed{x^3 + y^3 + z^3 - 3xyz = (x + y + z)\left(x^2 + y^2 + z^2 - xy - yz - zx\right)}$

OR

$$\boxed{x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]}$$

Cubic Identities

Prove that : $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Proof:

Cubic Identities

Prove that : $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Proof:

We have

$(x^3 + y^3 + z^3 - 3xyz)$

$= (x^3 + y^3) + z^3 - 3xyz$

$= [(x + y)^3 - 3xy(x + y)] + z^3 - 3xyz,$ Because $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$= u^3 - 3xyu + z^3 - 3xyz,$ Let us consider $(x + y) = u$

$= (u^3 + z^3) - 3xy(u + z)$

$= [(u + z)(u^2 - uz + z^2)] - 3xy(u + z),$ Because $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$= (u + z)[(u^2 + z^2 - uz - 3xy]$

$= (x + y + z)[(x + y)^2 + z^2 - (x + y)z - 3xy]$

$= (x + y + z)(x^2 + y^2 + 2xy + z^2 - xz - yz - 3xy)$

$\therefore (x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

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Question

Factorise each of the following:

$27 - 125a^3 - 135a + 225a^2$

Question

Factorise each of the following:

$27 - 125a^3 - 135a + 225a^2$

$= (3)^3 + (-5a)^3 + 3(3)^2(-5a) + 3(3)(-5a)^2$

$= (3-5a)^3$ Ans

Question

Factorise:

$8x^3 + y^3 + 27z^3 - 18xyz$

Question

Factorise:

$8x^3 + y^3 + 27z^3 - 18xyz$

$(2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$

$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6zx)$

$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Question

Factorise for $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p =$

$\left(3p + \frac{1}{6}\right)^3$

$\left(3p - \frac{1}{8}\right)^3$

$\left(3p - \frac{1}{6}\right)^3$

$\left(3 - \frac{1}{8}p\right)^3$

Question

Factorise for $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p =$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

A $\left(3p + \frac{1}{6}\right)^3$

$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$

B $\left(3p - \frac{1}{8}\right)^3$

$= \left(3p - \frac{1}{6}\right)^3$

C $\boxed{\left(3p - \frac{1}{6}\right)^3}$ ✓

D $\left(3 - \frac{1}{8}p\right)^3$

Question

Factorise the following:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Question

Factorise the following:

(i) $27y^3 + 125z^3$

$(3y)^3 + (5z)^3$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3$

$(4m)^3 - (7n)^3$

$= (4n - 7n)(16m^2 - 28mn + 49n^2)$

$$\boxed{(a^3+b^3) = (a+b)(a^2-ab+b^2) \quad \& \quad (a^3-b^3) = (a-b)(a^2+ab+b^2)}$$

One More Important Cubic Identity

Identity : If $(x + y + z) = 0$ then $x^3 + y^3 + z^3 = 3xyz$

One More Important Cubic Identity

Identity : $\boxed{\text{If } (x + y + z) = 0 \text{ then } x^3 + y^3 + z^3 = 3xyz}$

$$x^3+y^3+z^3-3xyz = \underbrace{\underbrace{\boxed{(x+y+z)}}_{=0}(x^2+y^2+z^2-xy-yz-zx)}_{=0}$$

$$x^3+y^3+z^3-3xyz=0$$

$$\boxed{x^3+y^3+z^3=3xyz} \longrightarrow \text{If any only if } \boxed{x+y+z=0}$$

Question

Without actual calculating the cubes, find the value of:

(i) $(28)^3 + (-15)^3 + (-13)^3$

(ii) $(-12)^3 + (7)^3 + (5)^3$

Question

Without actual calculating the cubes, find the value of:

(i) $\underbrace{(28)}_{a}^3 + \underbrace{(-15)}_{b}^3 + \underbrace{(-13)}_{c}^3$

$a+b+c = (28)+(-15)+(-13) = 28-15-13 = 0$

$a^3+b^3+c^3 = 3abc \Rightarrow (28)^3+(-15)^3+(-13)^3 = \boxed{3\times 28\times -15\times -13}$ Ans

(ii) $\underbrace{(-12)}_{a}^3 + \underbrace{(7)}_{b}^3 + \underbrace{(5)}_{c}^3$

$a+b+c = -12+7+5 = 0$

$a^3+b^3+c^3 = 3abc \Rightarrow (-12)^3+(7)^2+(5)^3 = \boxed{3\times(-12)\times(7)\times(5)}$

Miscellaneous Questions

If $x - \frac{1}{x} = 5$, find the value of $x^3 - \frac{1}{x^3}$.

Miscellaneous Questions

If $x - \frac{1}{x} = 5$, find the value of $x^3 - \frac{1}{x^3}$.

$$\left(x - \frac{1}{x}\right) = 5 \quad \text{By doing cube}$$

$$(x)^3 - \left(\frac{1}{x}\right)^3 - 3 \times x \times \frac{1}{x}\left(x - \frac{1}{x}\right) = 125$$

$$x^3 - \frac{1}{x^3} - 3 \times 5 = 125$$

$$x^3 - \frac{1}{x^3} = 125 + 15 = \boxed{140}$$

Question

$x^6 - y^6$ is equal to:

$(x + y)(x - y)(x^4 + y^4 + x^2y^2)$

$(x^2 + y^2)(x^4 + y^4 + x^2y^2)$

$(x^2 - y^2)(x^4 + y^4 - x^2y^2)$

$(x + y)(x - y)(x^4 + y^4 - x^2y^2)$

Question

$x^6 - y^6$ is equal to:

A $(x+y)(x-y)(x^4+y^4+x^2y^2)$

B $(x^2+y^2)(x^4+y^4+x^2y^2)$

C $(x^2-y^2)(x^4+y^4-x^2y^2)$

D $(x+y)(x-y)(x^4+y^4-x^2y^2)$

$(x^3)^2-(y^3)^2 = (x^3-y^3)(x^3+y^3)$

$= (x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$

$= (x^2-y^2)[(x^2+y^2)-xy][(x^2+y^2)+xy]$

$= (x^2-y^2)[(x^2+y^2)^2-(xy)^2]$

$= (x^2-y^2)[x^4+y^4+2x^2y^2-x^2y^2]$

$= (x^2-y^2)(x^4+y^4+x^2y^2)$

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Question

The value of $\dfrac{(361)^3 + (139)^3}{(361)^2 - (361 \times 139) + (139)^2}$ is

300

500

400

600

Question

The value of $\dfrac{(361)^3 + (139)^3}{(361)^2 - (361 \times 139) + (139)^2}$ is

a b

A 300

B 500

C 400

D 600

$$\frac{a^3+b^3}{a^2-ab+b^2} = \frac{(a+b)\cancel{(a^2-ab+b^2)}}{\cancel{(a^2-ab+b^2)}} = \boxed{a+b}$$

$$= 361+139$$

$$= \boxed{500}$$

Question

The value of $\frac{0.85\times0.85\times0.85+0.15\times0.15\times0.15}{0.85\times0.85-0.85\times0.15+0.15\times0.15}$ is

0

2

1

0.7

Question

The value of $\frac{0.85\times0.85\times0.85+0.15\times0.15\times0.15}{0.85\times0.85-0.85\times0.15+0.15\times0.15}$ is

A 0

B 2

C 1 ✓

D 0.7

$$\frac{a^3+b^3}{a^2-ab+b^2} = \frac{(a+b)\cancel{(a^2-ab+b^2)}}{\cancel{(a^2-ab+b^2)}} = a+b$$

$$= 0.85+0.15$$

$$= \boxed{1.00}$$

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Question

Without finding the cubes, factorize $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$.

Question

Without finding the cubes, factorize $\underbrace{\boxed{(x-2y)}}_{a}{}^3 + \underbrace{\boxed{(2y-3z)}}_{b}{}^3 + \underbrace{\boxed{(3z-x)}}_{c}{}^3$.

$$a+b+c = (x-2y)+(2y-3z)+(3z-x)$$
$$= \cancel{x}-\cancel{2y}+\cancel{2y}-\cancel{3z}+\cancel{3z}-\cancel{x}$$
$$= 0$$

$$a^3+b^3+c^3 = 3abc$$

$$(x-2y)^3+(2y-3z)^3+(3z-x)^3 = \boxed{3(x-2y)(2y-3z)(3z-x)} \text{ Ans}$$

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Question

If $\frac{x}{y}+\frac{y}{x}=-1$ $(x, y \neq 0)$, then the value of x^3-y^3

$x+y$

$x-y$

3

0

Question

If $\frac{x}{y}+\frac{y}{x}=-1$ $(x, y \neq 0)$, then the value of x^3-y^3 $= (x-y)(x^2+xy+y^2)$

$= (x-y)\times 0$

$= \boxed{0}$

A) $x+y$

B) $x-y$

C) 3

D) 0 ✓

$$\frac{x}{y}+\frac{y}{x}=-1$$

$$\frac{x^2+y^2}{xy}=-1 \Rightarrow x^2+y^2=-xy \Rightarrow \boxed{x^2+xy+y^2=0}$$

Question

If $p = 2 - a$, prove that $a^3 + 6ap + p^3 - 8 = 0$

Question

If $p = 2 - a$, prove that $a^3 + 6ap + p^3 - 8 = 0$

$a+p=2$

$(a+p)^3=(2)^3$

$a^3+p^3+3ap(a+p)=8$

$a^3+p^3+3ab\times 2=8$

$\boxed{a^3+p^3+6ap-8=0}$

Hence, proved !!

Question

If $a > b > 0$ and $\boxed{a^3 + b^3 + 27ab = 729}$, then find the value of $(a + b)$. [olympiad]

Question

If $a > b > 0$ and $\boxed{a^3 + b^3 + 27ab = 729}$, then find the value of $(a + b)$. [olympiad]

$$a^3+b^3+27ab=729$$

$$a^3+b^3-729=-27ab$$

$$\boxed{(a)^3+(b)^3+(-9)^3=3(a)(b)(-9)} \longrightarrow (a)+(b)+(-9)=0$$

$$a+b-9=0$$

$$\boxed{a+b=9}$$

Question

If a, b, c are non-zero and $a + b + c = 0$, then find the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$.

1

2

3

0

Question

If a, b, c are non-zero and $a + b + c = 0$, then find the value of $\boxed{\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}}$.

A 1

B 2

C 3

D 0

$a^3 + b^3 + c^3 = 3abc$

$$\frac{a^3 + b^3 + c^3}{abc} = \frac{3\cancel{abc}}{\cancel{abc}} = \boxed{3}$$

Question

Khatarnaak dikhne vala question: $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

Question

Khatarnaak dikhne vala question: $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

$\rightsquigarrow \dfrac{\cancel{3}(a^2-b^2)(b^2-c^2)(c^2-a^2)}{\cancel{3}(a-b)(b-c)(c-a)}$

$= \dfrac{\cancel{(a-b)}(a+b)(b+c)\cancel{(b-c)}\cancel{(c-a)}(c+a)}{\cancel{(a-b)}\cancel{(b-c)}\cancel{(c-a)}}$

$= \boxed{(a+b)(b+c)(c+a)}$ ✓

$\boxed{(a^2-b^2)+(b^2-c^2)+(c^2-a^2)=0}$

$\hookrightarrow (a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3 = 3(a^2-b^2)(b^2-c^2)(c^2-a^2)$

$\boxed{(a-b)+(b-c)+(c-a)=0}$

$\hookrightarrow (a-b)^3+(b-c)^3+(c-a)^3 = 3(a-b)(b-c)(c-a)$

Question

If $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$, then

a b c

A $x + y + z = 27xyz$

B $x^3 + y^3 + z^3 = 0$

C $x^3 + y^3 + z^3 = 27xyz$

D $(x + y + z)^3 = 27xyz$

Question

If $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}} = 0$, then

A $x + y + z = 27xyz$

B $x^3 + y^3 + z^3 = 0$

C $x^3 + y^3 + z^3 = 27xyz$

D $\boxed{(x + y + z)^3 = 27xyz}$ ✓

$\boxed{a+b+c=0} \longrightarrow a^3+b^3+c^3 = 3abc$

$$\left(x^{\frac{1}{3}}\right)^3 + \left(y^{\frac{1}{3}}\right)^3 + \left(z^{\frac{1}{3}}\right)^3 = 3\left(x^{\frac{1}{3}}\right)\left(y^{\frac{1}{3}}\right)\left(z^{\frac{1}{3}}\right)$$

$$x^{3/3} + y^{3/3} + z^{3/3} = 3(xyz)^{\frac{1}{3}}$$

$$x^1 + y^1 + z^1 = 3(xyz)^{\frac{1}{3}}$$

cube on both sides

$$(x+y+z)^3 = \left[3(xyz)^{\frac{1}{3}}\right]^3$$

$$(x+y+z)^3 = 27(xyz)^{\cancel{\frac{1}{3}} \times \cancel{3}} = 27xyz$$

Question

Factorize : $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$ [IMO 2015 Set A]

$3pqr$

$3pqr(p-q)(q-r)(r-p)$

$p^3+q^3+r^3-3pqr$

None of these

Question

Factorize : $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$ [IMO 2015 Set A]

$[p(q-r)]^3+[q(r-p)]^3+[r(p-q)]^3$

$[\underbrace{pq-pr}_{x}]^3+[\underbrace{qr-qp}_{y}]^3+[\underbrace{pr-qr}_{z}]^3 = 3[(pq-pr)(qr-qp)(pr-qr)]$

$x+y+z=0$

$\cancel{pq}-\cancel{pr}+\cancel{qr}-\cancel{pq}+\cancel{pr}-\cancel{qr}=0$

$3[p(q-r)\times q(r-p)\times r(p-q)]$

$= 3pqr[(p-q)(q-r)(r-p)]$

A $3pqr$

B $\boxed{3pqr(p-q)(q-r)(r-p)}$ ✓

C $p^3+q^3+r^3-3pqr$

D None of these


THANK YOU
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR