VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

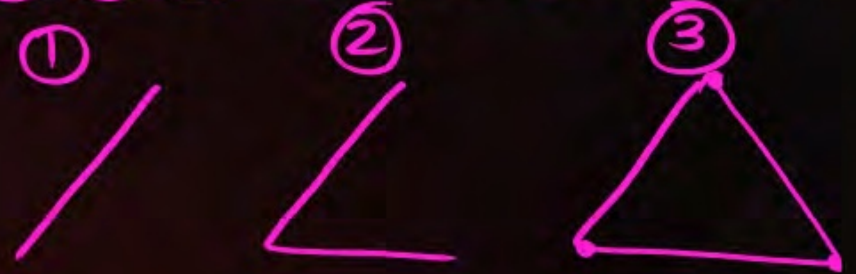
- **Subject –** Mathematics
- **Chapter –** Triangles

Today's Targets

1. Complete Chapter (one shot)
2. Congruence of Triangle
3. Rapid fire Question, Boggle Baba ke bhaukali sawal
4. One level up ke Brainstorming sawal

Polygon

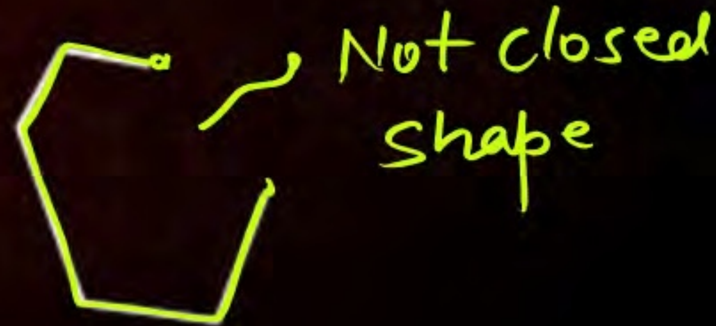
A polygon is a two-dimensional geometric figure that has a finite number of sides. The sides of a polygon are made of straight line segments connected to each other end to end.



- Polygons are 2-dimensional shapes. ✓
- They are made of straight lines only, and ✓
- The shape is "closed" (all the lines connect up). ✓

Polygon
(Straight Sides)

Not a polygon
(has a curve)

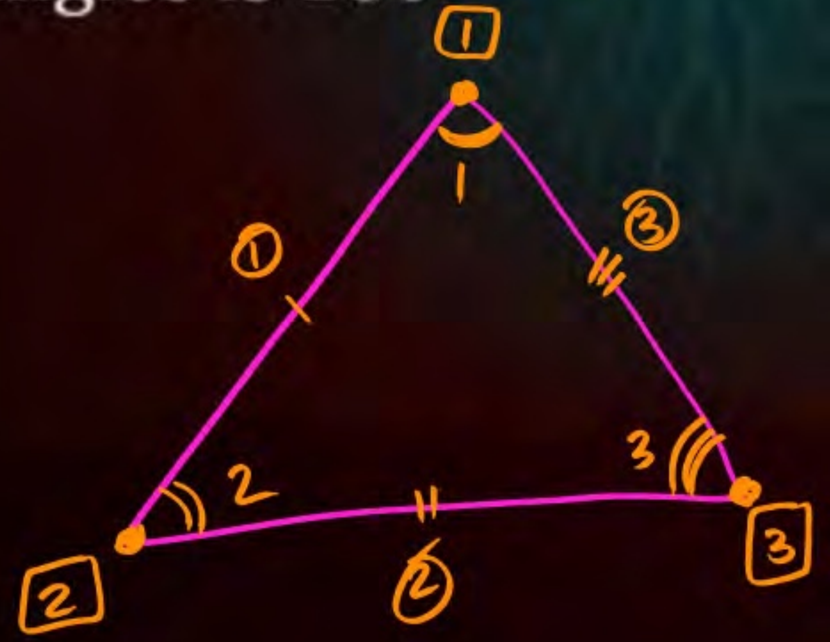


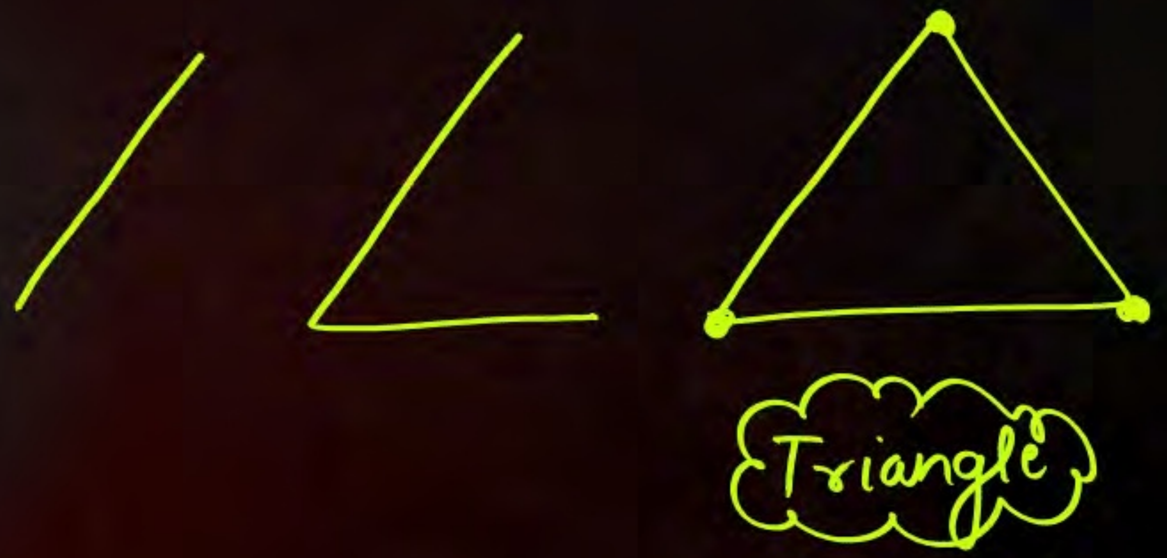
Not a polygon
(open not closed)

Triangle and Its Components

A triangle is a closed figure formed by three straight line segments

1. It is the simplest polygon.
2. Three sides, three angles, three vertices.
3. The sum of all the angles is 180°





Classification of Triangles

On the basis of sides

- Equilat. Δ
- Iso. Δ
- Scalene Δ

On the basis of angles

- Acute angled Δ — $0^\circ < \theta < 90^\circ$
- Right angled Δ — $\theta = 90^\circ$
- obtuse Angled Δ — $90^\circ < \theta < 180^\circ$

Congruent figures

The congruence of any two figures can be seen if they can be placed exactly over each other.

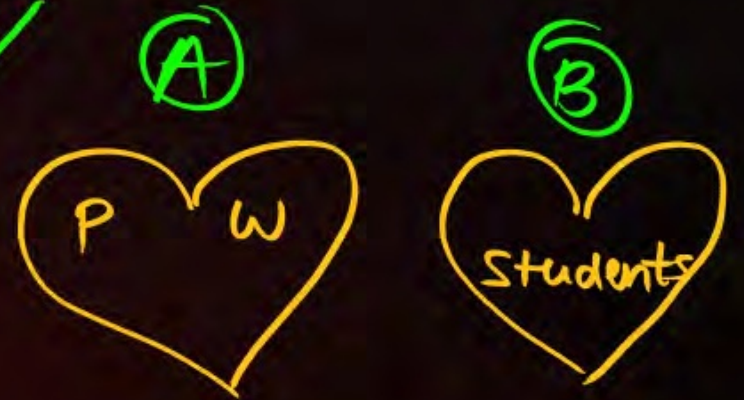
In other words, if any two geometrical figures can be superimposed on each other, they are termed as congruent figures

congruent → $\boxed{\text{shape and size}}$ → equal

Circle A ✓ Circle B ✓

Concept of congruence

- In geometry, congruent means identical in shape and size.
- It can be applied to line segments, angles and figures.
- We use the symbol $\boxed{\cong}$ for congruence.
- The word congruent means exactly equal.

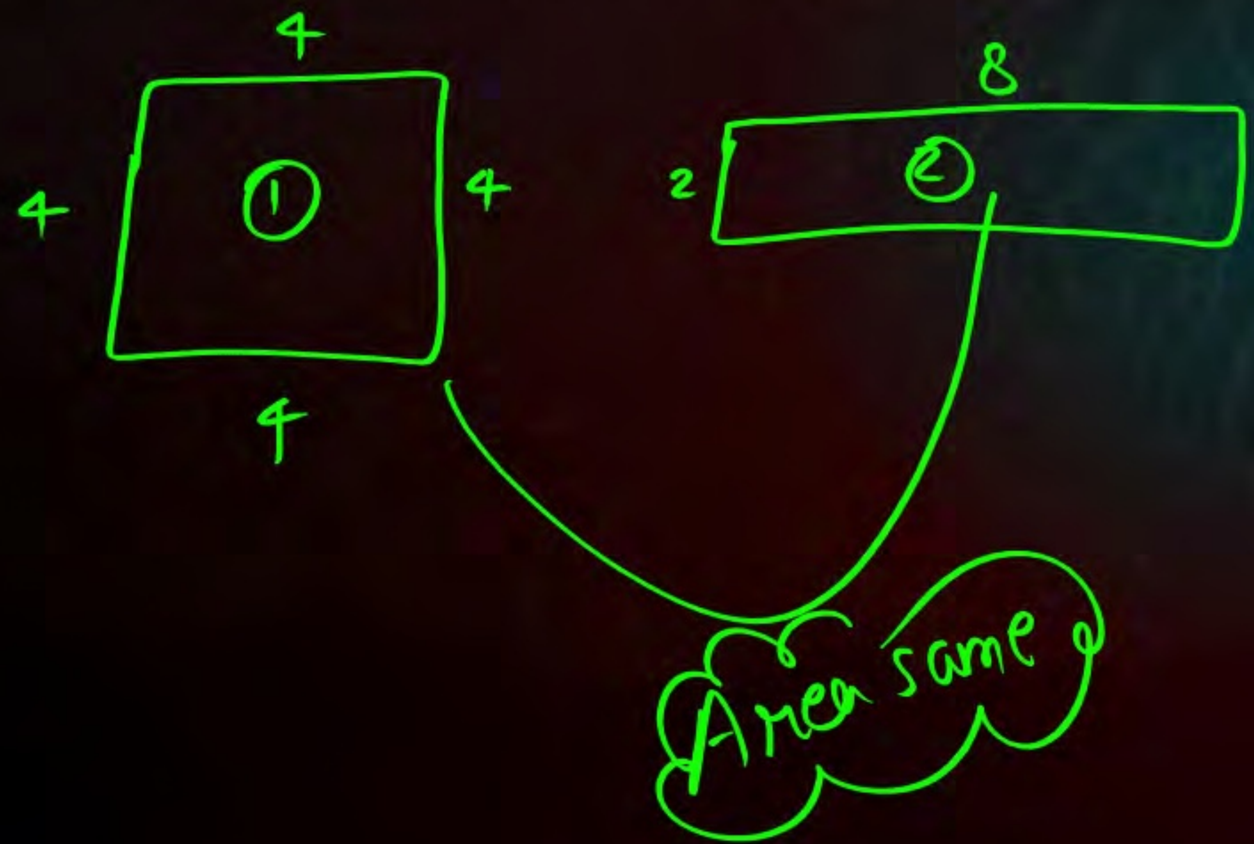


$\boxed{A \cong B}$

$\cong$ → sign of congruecy

Some Important points

- If two plane figures are congruent, then their areas are equal.
- If area of two plane figures are equal, they may or may not be congruent

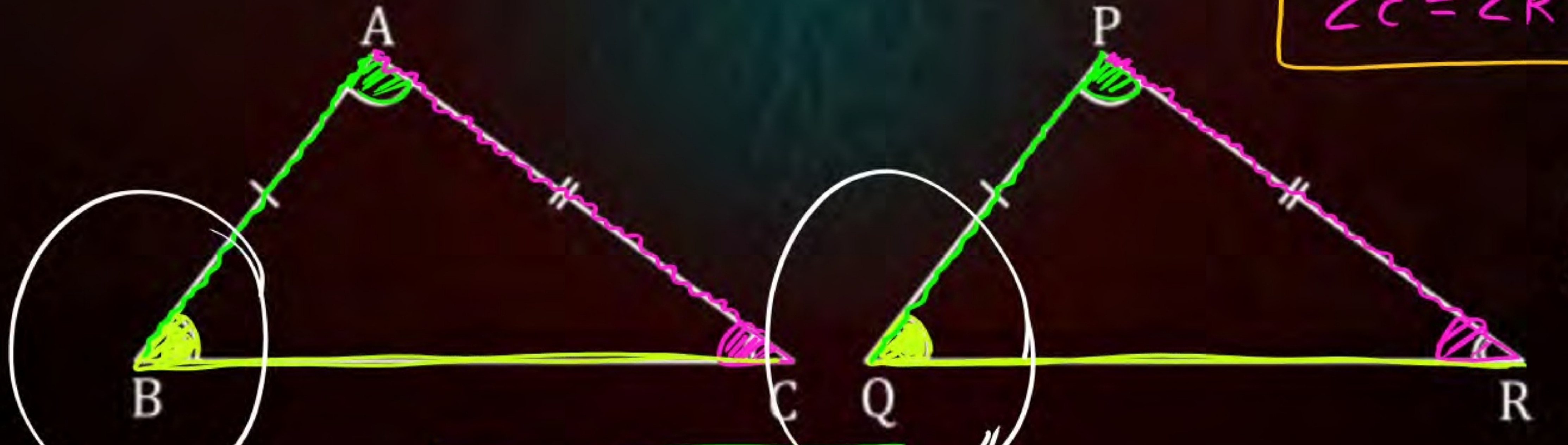


Congruence of Triangles

Two triangles are said to be congruent if,

- Their sides are equal in length ✓
- The angles are of equal measure. ✓
- And they can be superimposed on each other. ✓

Angles	side
$\angle A = \angle P$	$AB = PQ$
$\angle B = \angle Q$	$BC = QR$
$\angle C = \angle R$	$AC = PR$



$\cancel{\Delta ABC \cong \Delta PRQ}$

$\boxed{\Delta ABC \cong \Delta PQR}$ ✓

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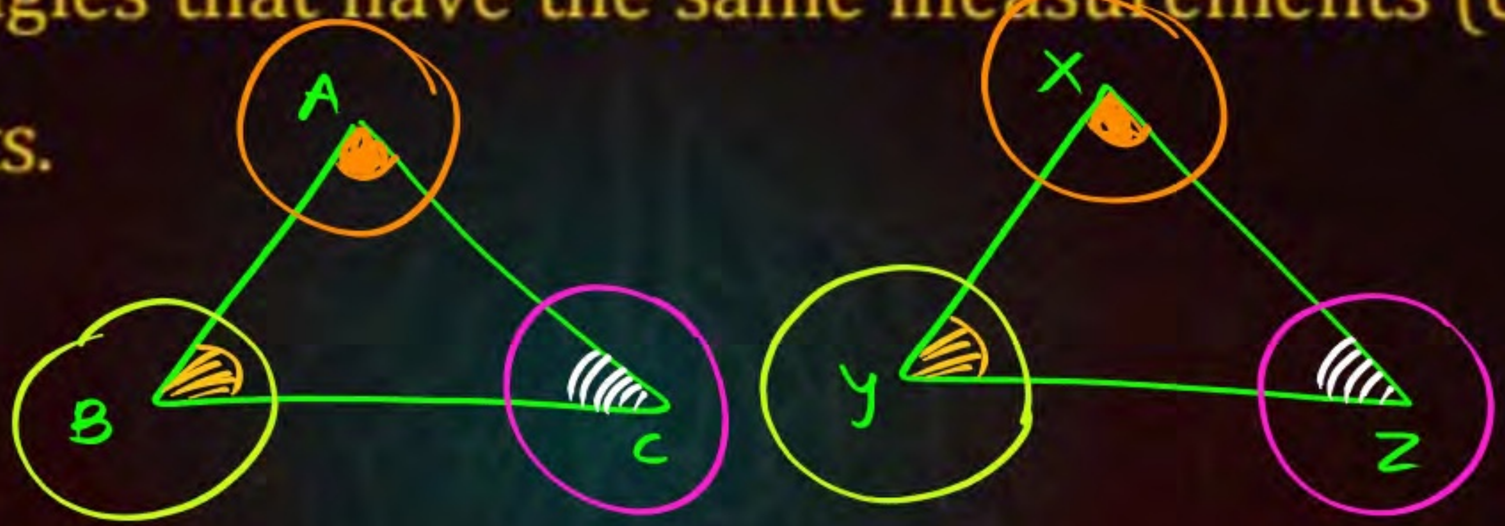
CPCT(Corresponding Parts of Congruent Triangles)

The parts of the two triangles that have the same measurements (congruent) are called corresponding parts.

$\angle A = \angle X$
$\angle B = \angle Y$
$\angle C = \angle Z$

$AB = XY$
$BC = YZ$
$AC = XZ$



$\boxed{\Delta ABC \cong \Delta XYZ}$

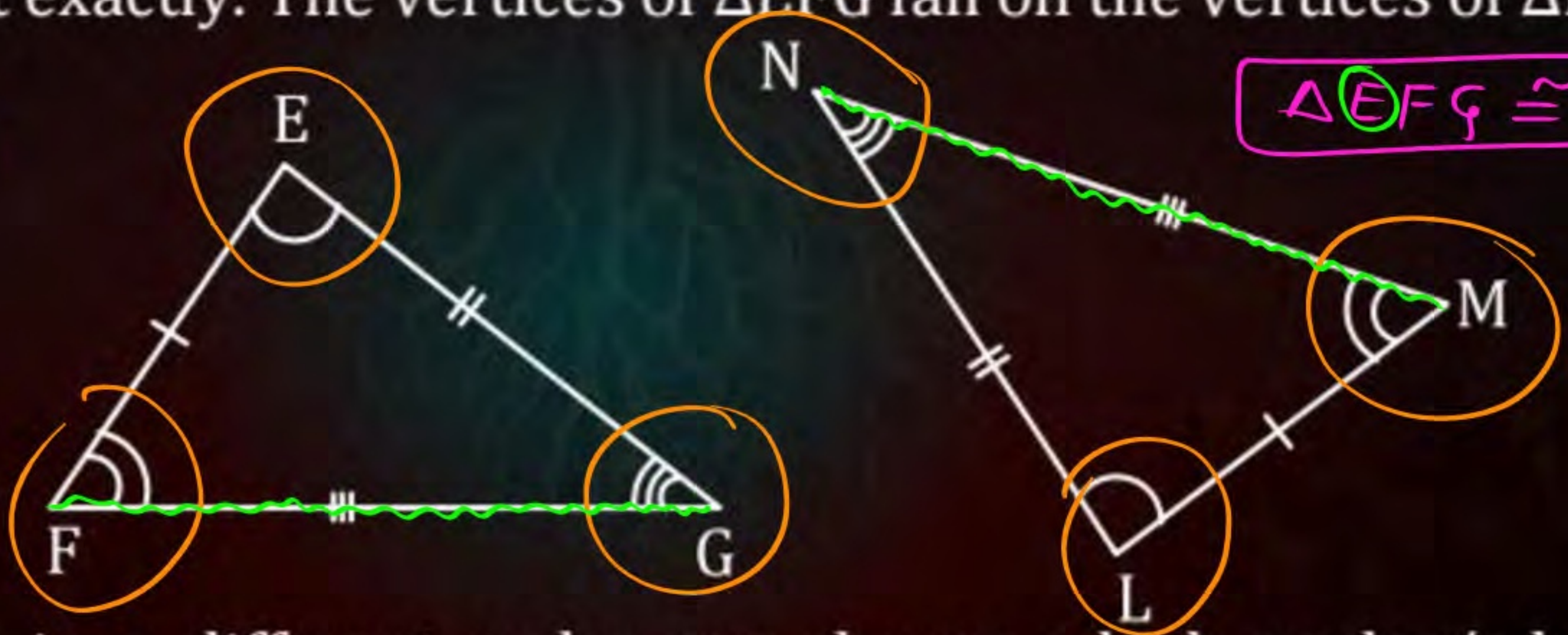
- Now, the question arises. What are the least possible conditions to ensure the congruence of two triangles?
- Do we need all the six condition to ensure the congruence of two triangles?
- Will lesser number of condition will do our work?

Order of Vertex in Congruency of Triangles

Let ΔEFG and ΔLMN be two congruent triangles, then we can superpose ΔEFG on ΔLMN so as to cover it exactly. The vertices of ΔEFG fall on the vertices of ΔLMN in the following order

$\boxed{\Delta EFG \cong \Delta LMN}$

- $E \leftrightarrow L$,
- $F \leftrightarrow M$,
- $G \leftrightarrow N$



$\boxed{\Delta EFG \cong \Delta NLM}$ X

Naming the triangles in a different order can change which angles/edges are implied to be congruent. This is why we must be careful to name the triangles in corresponding order. Otherwise, we may be saying two angles (or line segments) are congruent, when in fact, they are not.

Criteria's for congruence of triangles

1. $\boxed{\text{SAS}}$ congruence criterion ✓
2. ASA congruence criterion ✓
3. AAS congruence criterion ✓
4. SSS congruence criterion ✓
5. RHS congruence criterion ✓

S – A – S ⇒ $\boxed{\text{side - Included angle - side}}$

$\boxed{\text{SSA}}$ ✗

$\boxed{\text{ASS}}$ ✗

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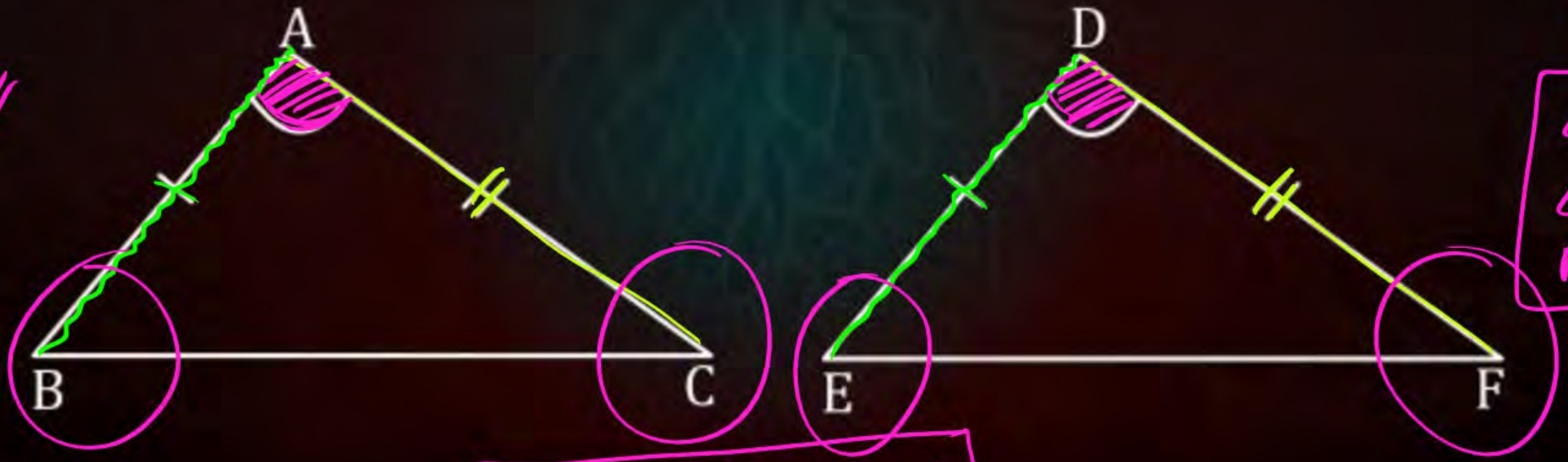
Side – Angle – Side (SAS) Congruence Criteria

Axiom 7.1 (SAS congruence rule) : Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

AB = DE
∠A = ∠D
AC = DF



By CPCT

∠B = ∠E
∠C = ∠F
BC = EF

$\Delta ABC \cong \Delta DEF$

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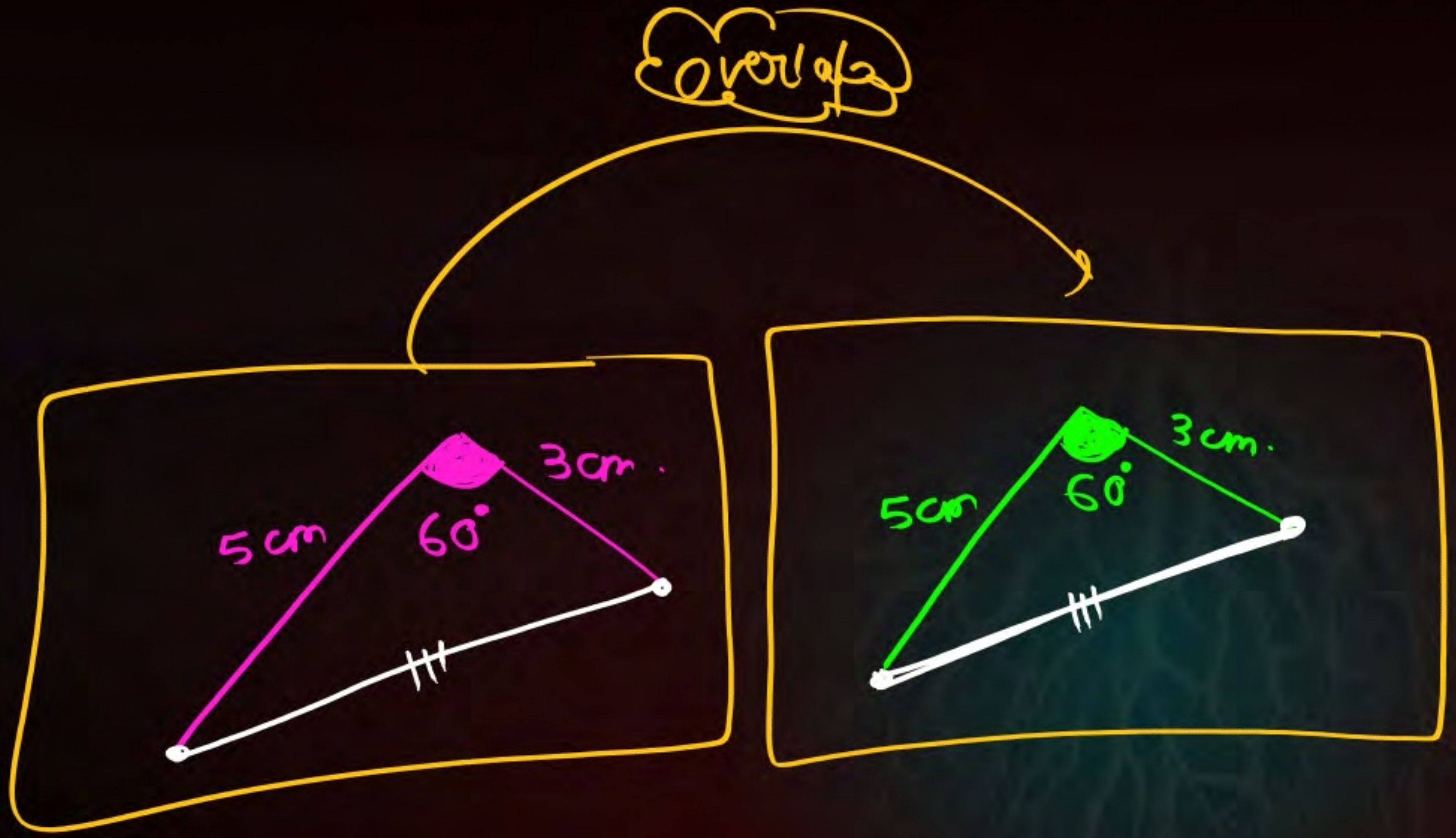

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Question

It is given that $\Delta ABC \cong \Delta FDE$ and $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$. Then which of the following is true?

$DF = 5$ cm $\angle F = 60°$

$DF = 5$ cm, $\angle E = 60°$

$DE = 5$ cm $\angle E = 60°$

$DE = 5$ cm, $\angle D = 40°$

Question

It is given that $\Delta ABC \cong \Delta FDE$ and $AB = 5$ cm, $\angle B = 40°$ and $\angle A = 80°$. Then which of the following is true?

A $DF = 5$ cm, $\angle F = 60°$

B $DF = 5$ cm, $\angle E = 60°$

C $DE = 5$ cm, $\angle E = 60°$

D $DE = 5$ cm, $\angle D = 40°$

Angle sum property

$40° + 80° + x = 180°$

$\boxed{x = 60°}$

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Question

In triangles ABC and DEF, $AB = FD$ and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if

$BC = EF$

$AC = DE$

$AC = EF$

$BC = DE$

Question

In triangles ABC and DEF, AB = FD and $\angle A = \angle D$. The two triangles will be congruent by SAS axiom if

A BC = EF

B AC = DE

C AC = EF

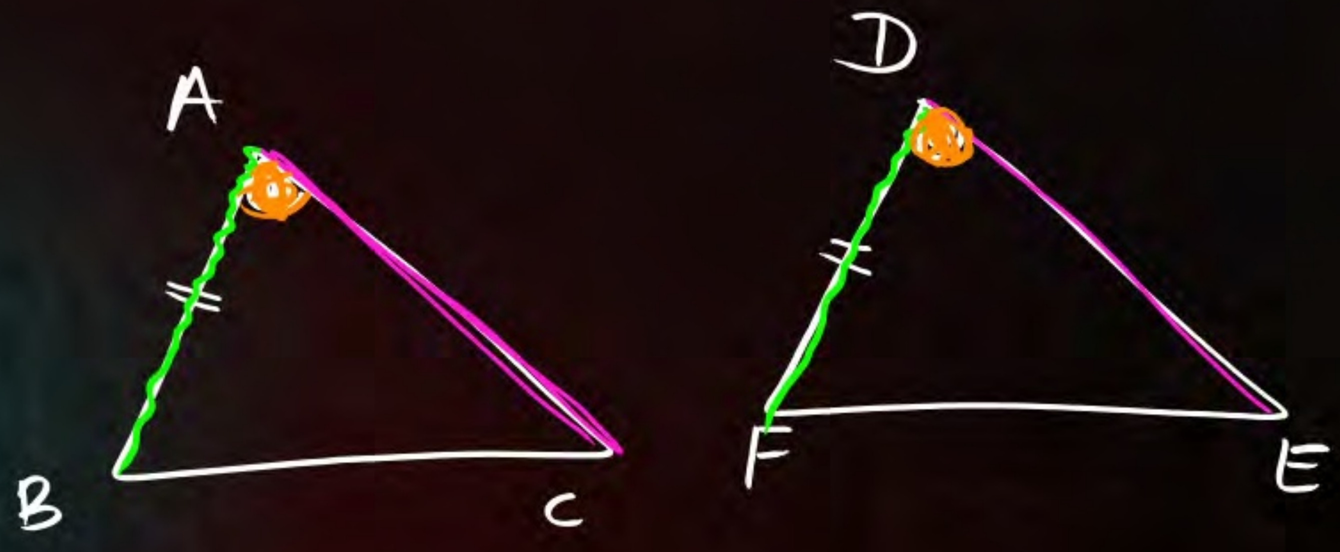
D BC = DE

In $\triangle ABC$ & $\triangle DFE$

S $AB = DF$

A $\angle A = \angle D$

S $\boxed{AC = DE}$

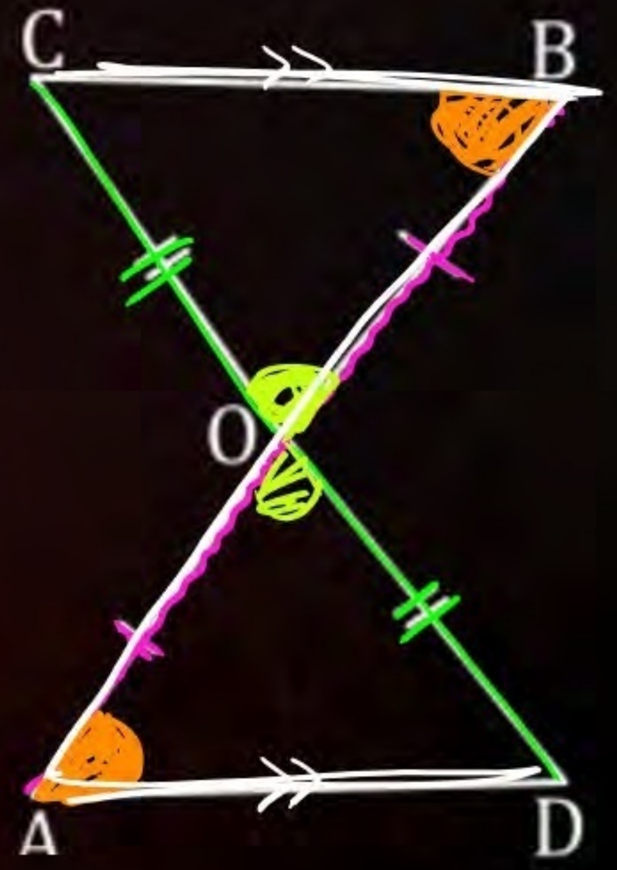


Question

In figure, two lines AB and CD are such that $OA = OB$ and $OD = OC$. Show that

(i) $\Delta AOD \cong \Delta BOC$ and

(ii) $AD \parallel BC$.

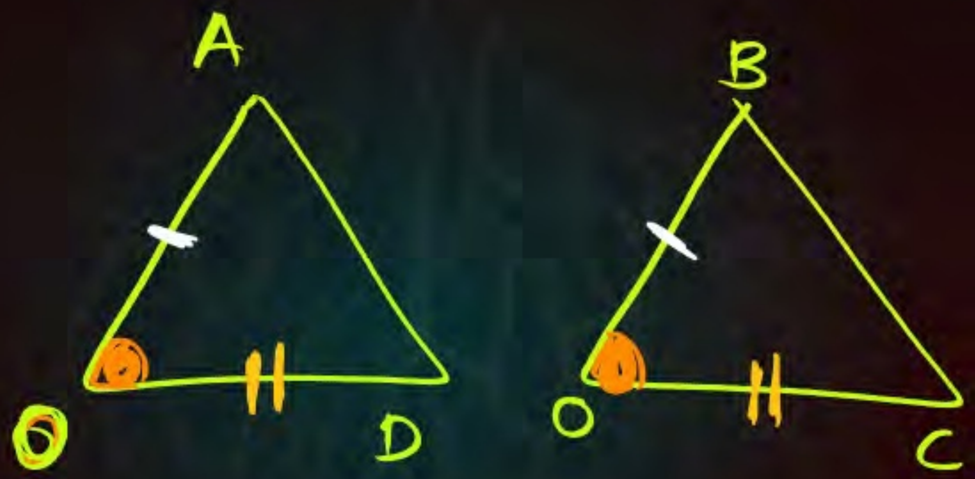


Question

In figure, two lines AB and CD are such that OA = OB and OD = OC. Show that

(i) $\Delta AOD \cong \Delta BOC$ and

(ii) AD || BC.



(i) By using SAS criteria,

$\boxed{\Delta AOD \cong \Delta BOC}$

By CPCT,

AD = BC

(ii) $\boxed{\angle OAD = \angle OBC}$

↳ If alternate interior angle are same, then

$\boxed{AD \parallel BC}$

In ΔAOD & ΔBOC

S: OA = OB (given)

A: $\angle AOD = \angle BOC$ (vertically opposite)

S: OD = OC

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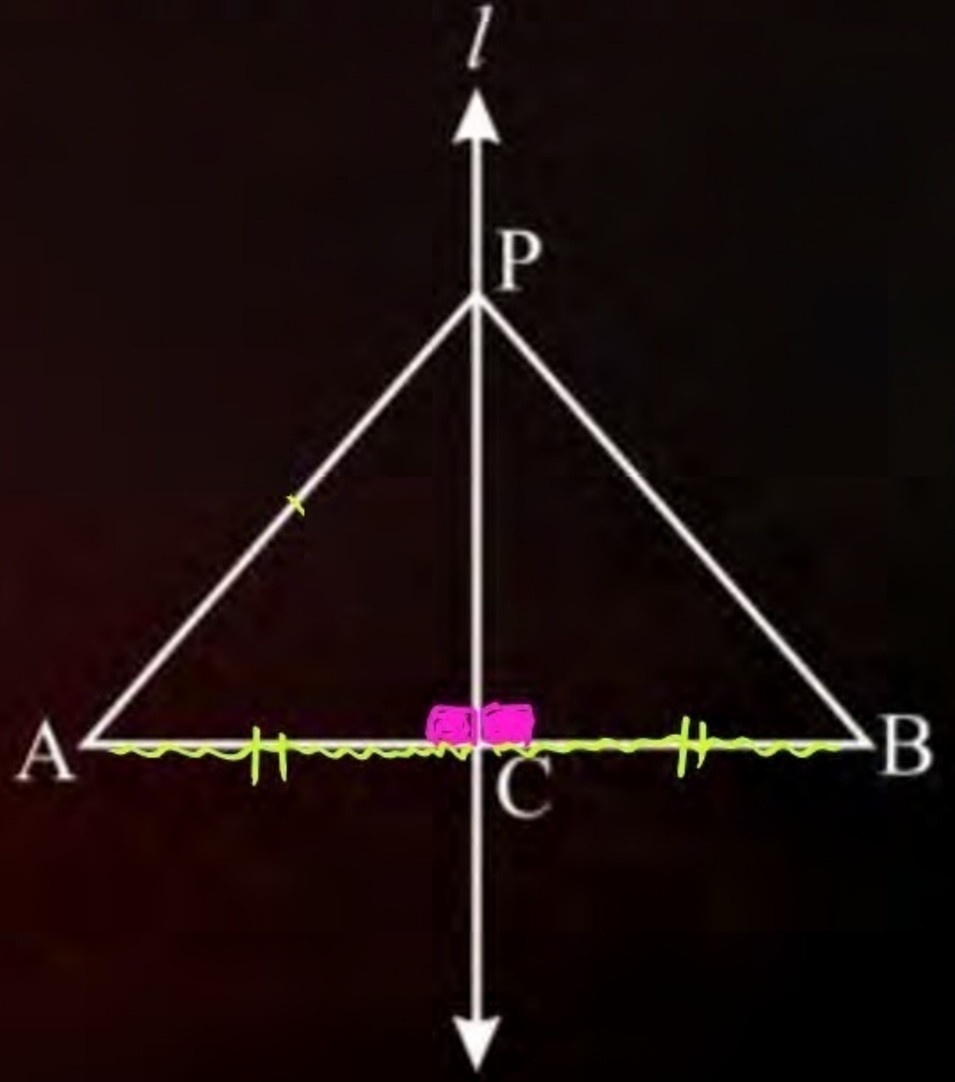
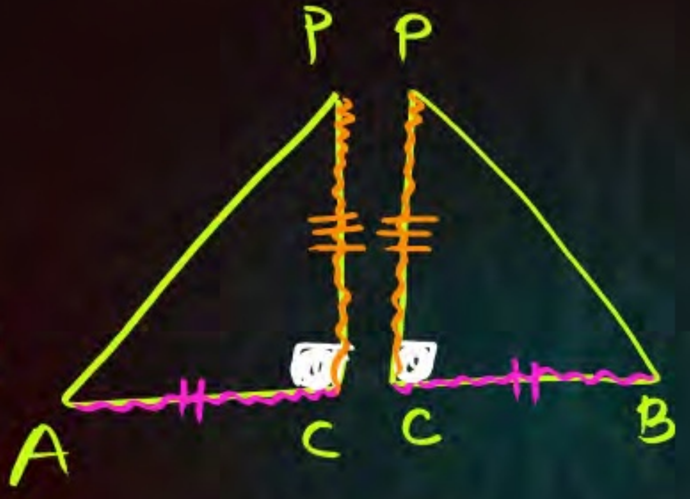
Question

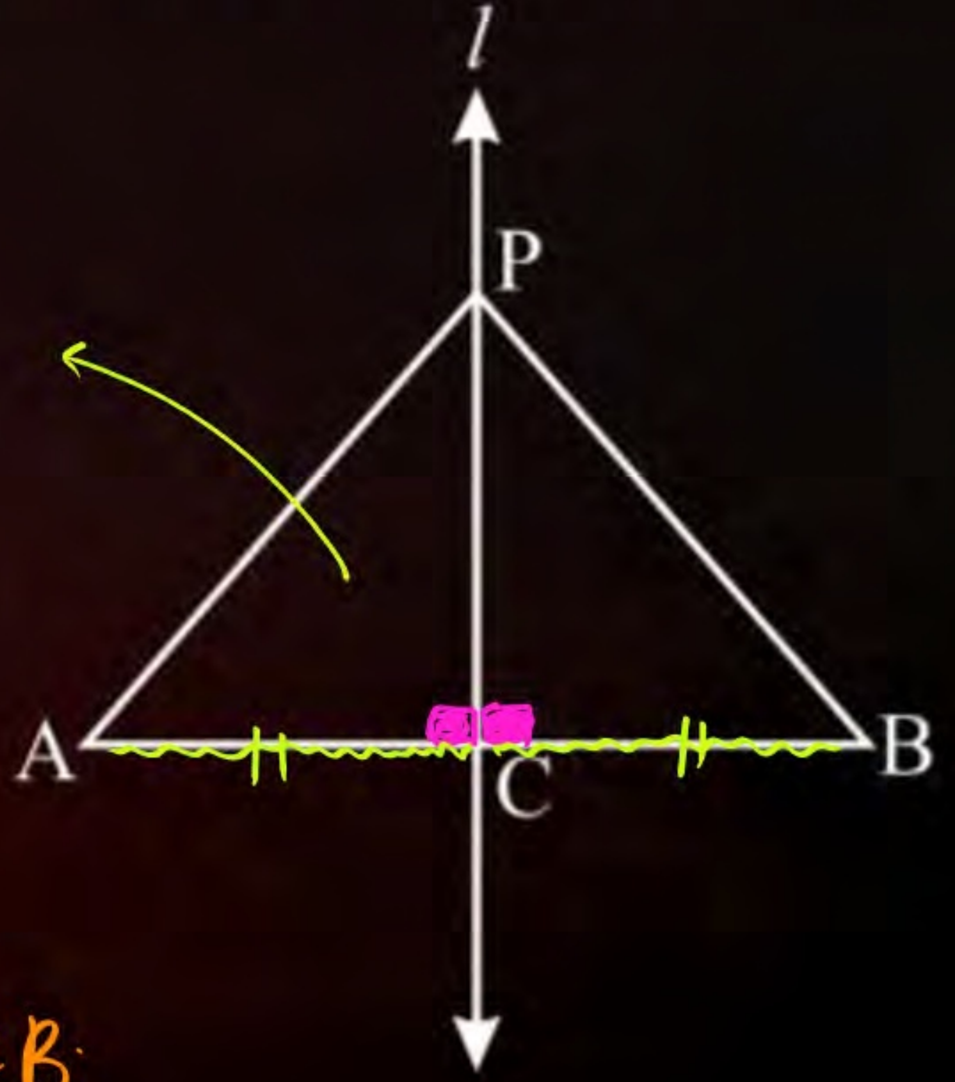
AB is a line segment and line l is its perpendicular bisector. If a point P lies on l, show that P is equidistant from A and B.



Question

AB is a line segment and line l is its perpendicular bisector. If a point P lies on l, show that P is equidistant from A and B. $\Rightarrow \boxed{PA = PB}$





In $\triangle PCA$ & $\triangle PCB$

S $AC = BC$ (Bisected)

A $\angle PCA = \angle PCB$ (Both 90°)

S $PC = PC$ (common)

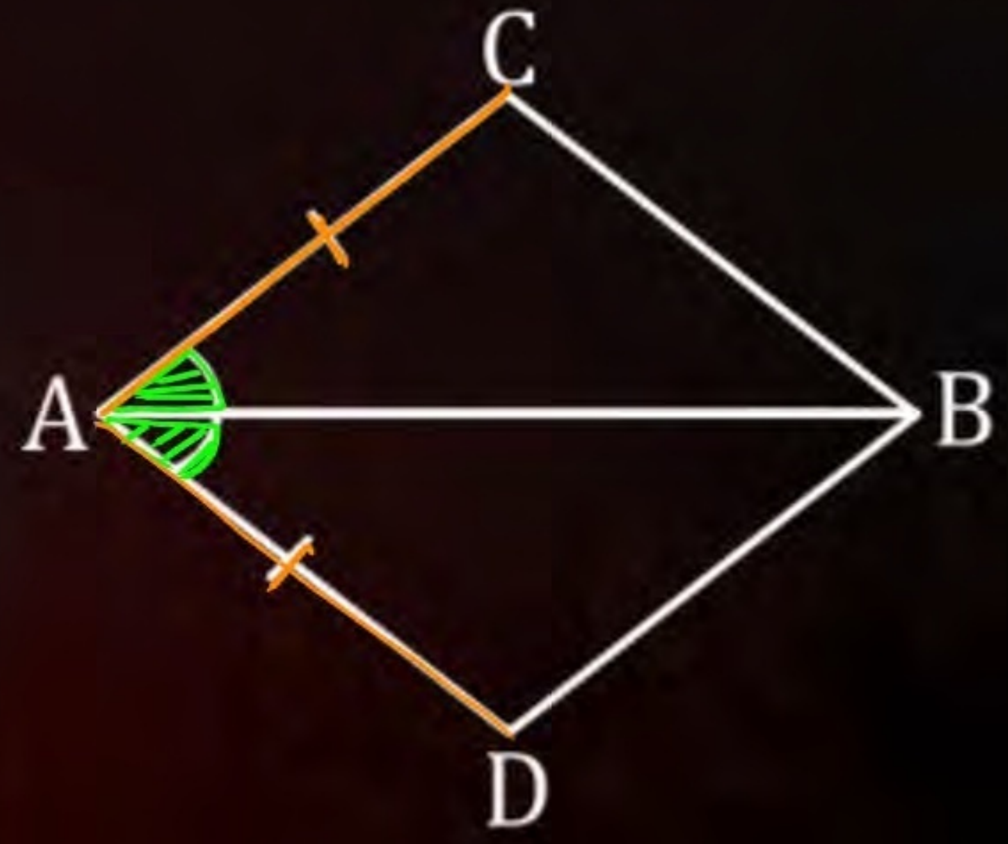
$\boxed{\triangle PCA \cong \triangle PCB}$

By CPCT,

$\boxed{PA = PB} \rightarrow$ P is equidistant from point A & B.

Question

In quadrilateral ACBD, $\boxed{AC = AD}$ and AB bisects $\angle A$. Show that $\boxed{\Delta ABC \cong \Delta ABD.}$ What can you say about BC and BD?

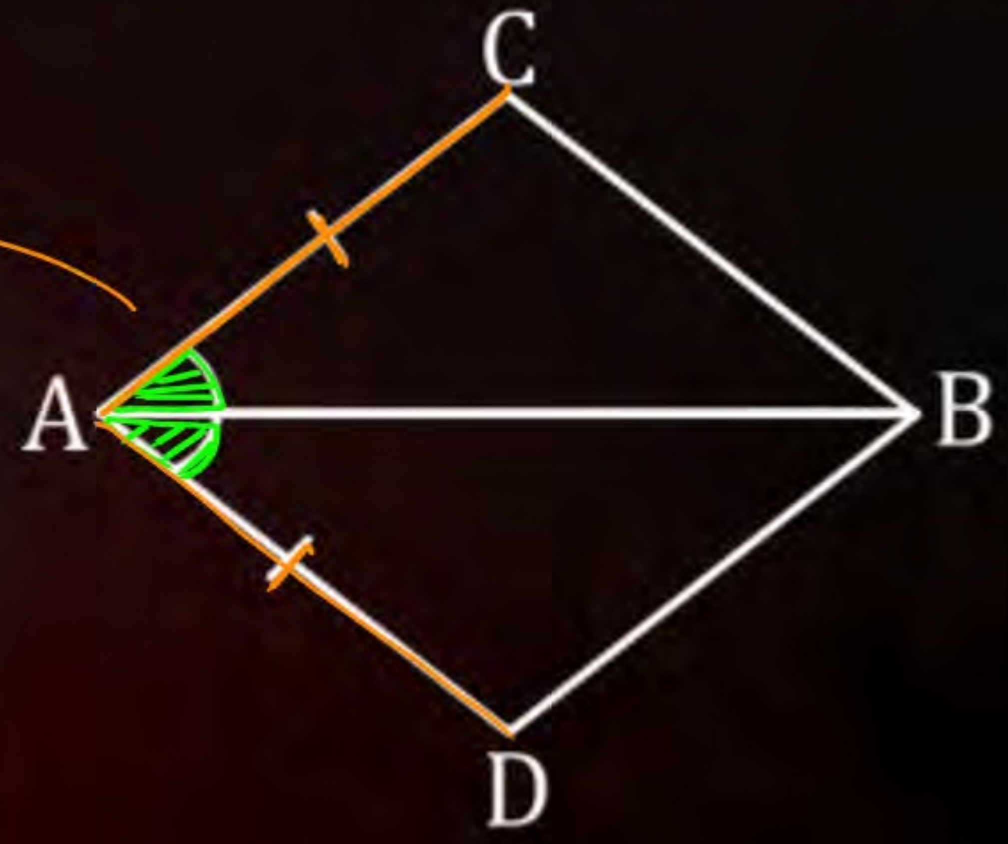


Question

In quadrilateral ACBD, $\boxed{AC = AD}$ and AB bisects $\angle A$. Show that $\boxed{\Delta ABC \cong \Delta ABD}$. What can you say about BC and BD?

(i) since $\angle A$ is bisected by AB,

$\boxed{\angle CAB = \angle DAB}$



In ΔABC & ΔABD

S — $AC = AD$ (given)

A — $\angle CAB = \angle DAB$ ($\angle A$ bisected)

S — $AB = AB$ (common)

$\boxed{\Delta ABC \cong \Delta ABD}$

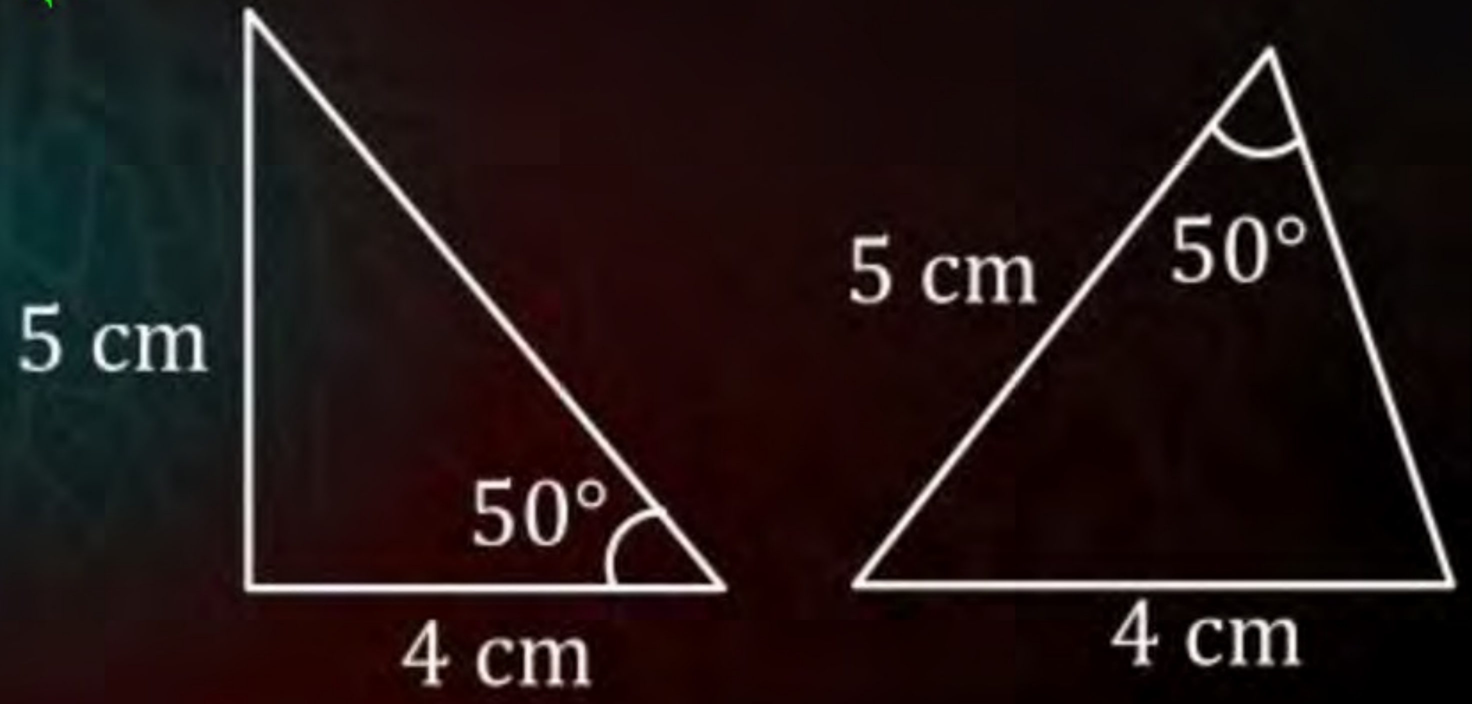
By CPCT, $\boxed{BC = BD}$ ✓

Misconception Points

SAS congruence rule holds but not ASS or SSA rule.

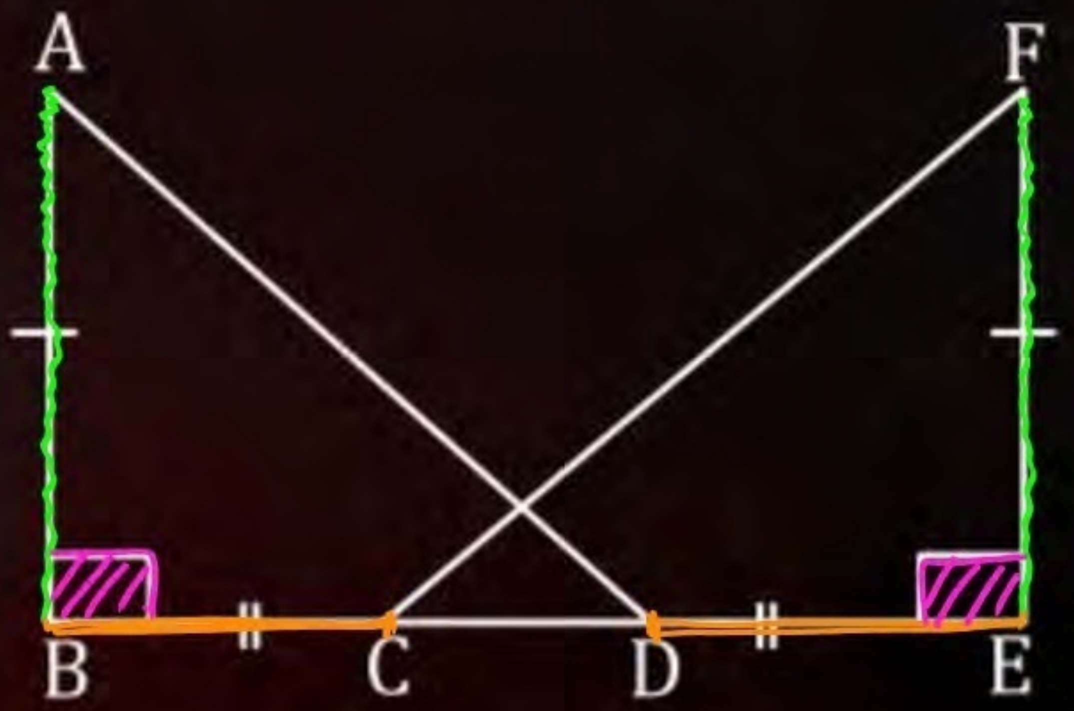
S — $\boxed{A}$ — S

Included Angle



Question

In the given figure, it is given that $AB = EF$, $\boxed{BC = DE}$, $AB \perp BD$ and $EF \perp CE$. Prove that $\boxed{\Delta ABD \cong \Delta FEC.}$



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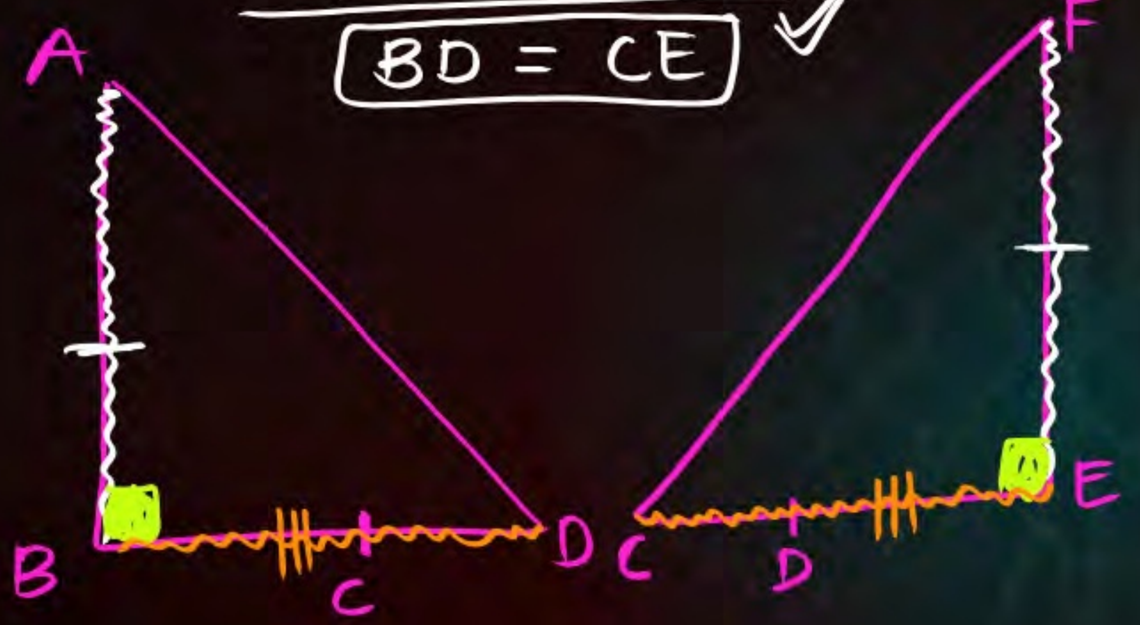
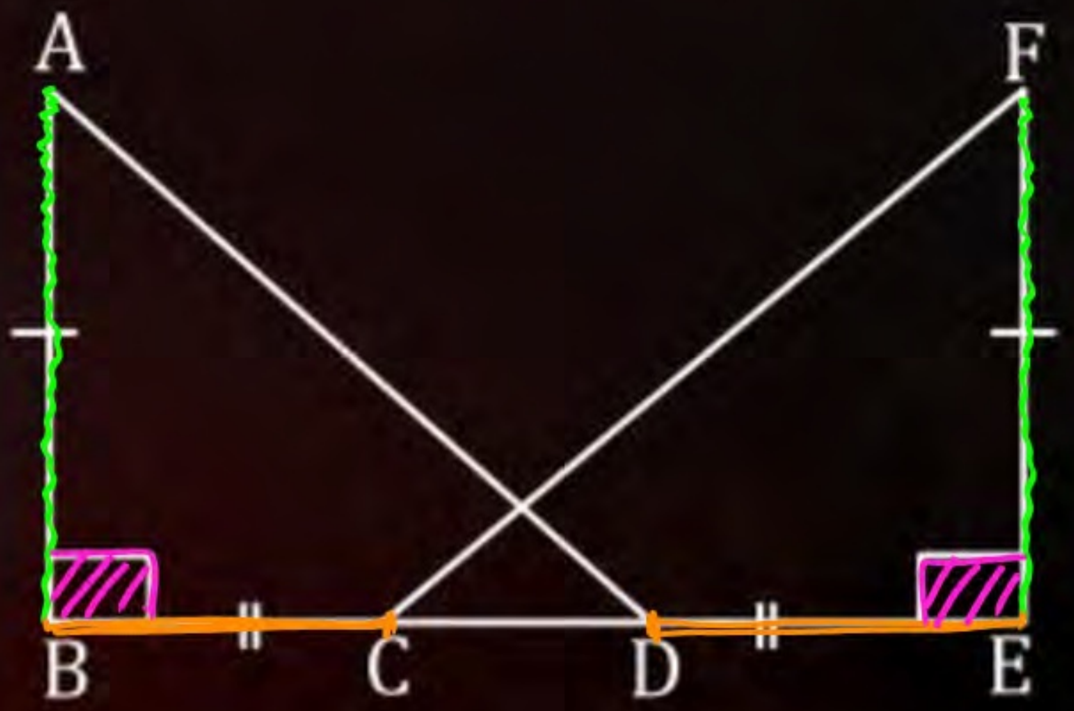
Question

In the given figure, it is given that AB = EF, $\boxed{BC = DE}$, $AB \perp BD$ and $EF \perp CE$. Prove that $\boxed{\Delta ABD \cong \Delta FEC}$.

$$\begin{array}{r} BC = DE \\ +CD \quad +CD \\ \hline \boxed{BD = CE} \end{array}$$





In ΔABD & ΔFEC

S $AB = FE$

A $\angle ABD = \angle FEC$ ($AB \perp BD$ & $EF \perp CE$)

S $BD = CE$

By SAS criteria,

$\boxed{\Delta ABD \cong \Delta FEC}$

Question

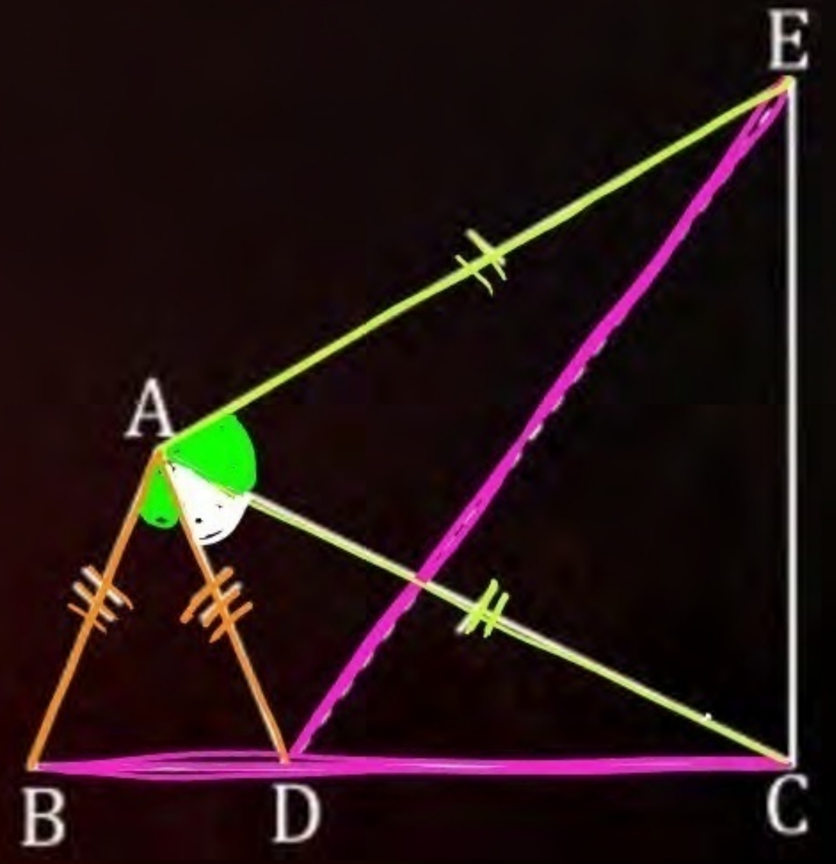
In the given figure, AC = AE, AB = AD and $\boxed{\angle BAD = \angle EAC}$.

$BC = DE$

$BC = CE$

$BC = AC$

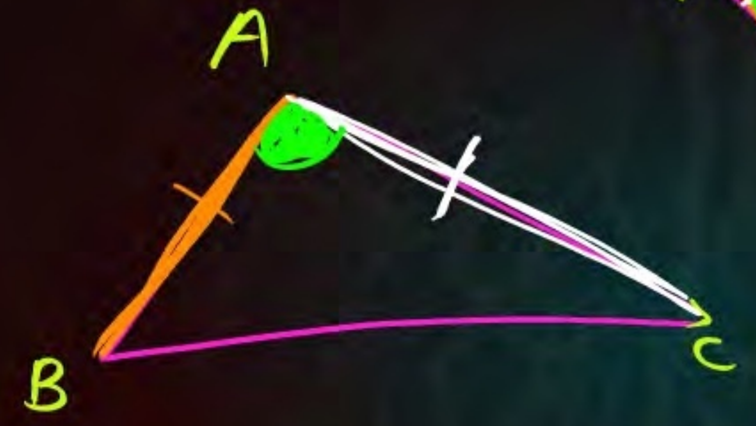
None of these

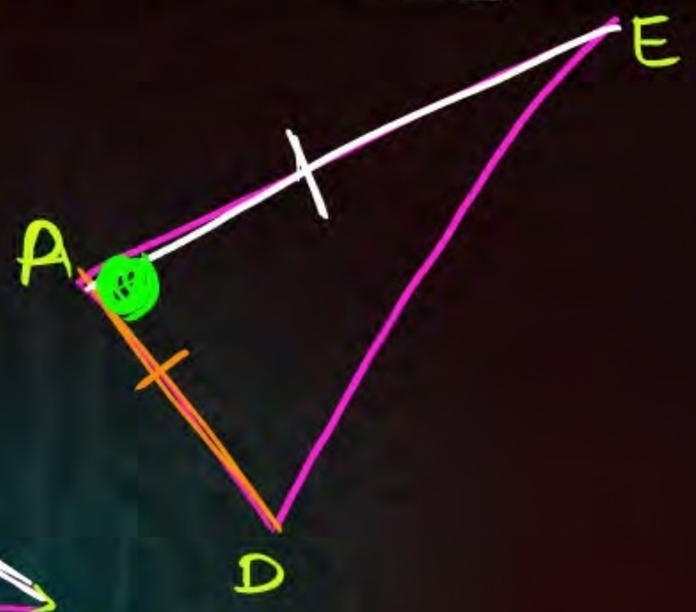


Question

In the given figure, AC = AE, AB = AD and $\boxed{\angle BAD = \angle EAC}$.

- **A** $BC = DE$ ✓
- **B** $BC = CE$
- **C** $BC = AC$
- **D** None of these

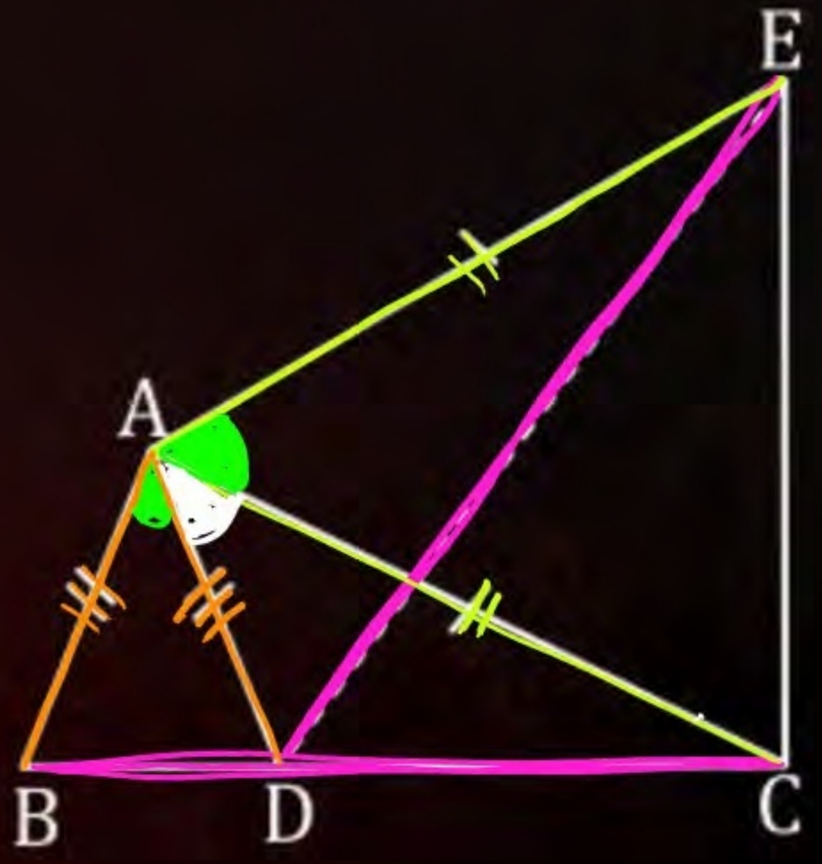




We have, $\angle BAD = \angle EAC$

$+\angle DAC \quad +\angle DAC$

$\underline{\angle BAC = \angle EAD}$



Question

In figures, it is given that AB = CF, EF = BD and $\angle AFE = \angle CBD$, then

$\Delta AFE \cong \Delta CBD$

$\Delta AFE \cong \Delta CDB$

$\Delta AEF \cong \Delta BDC$

None of these

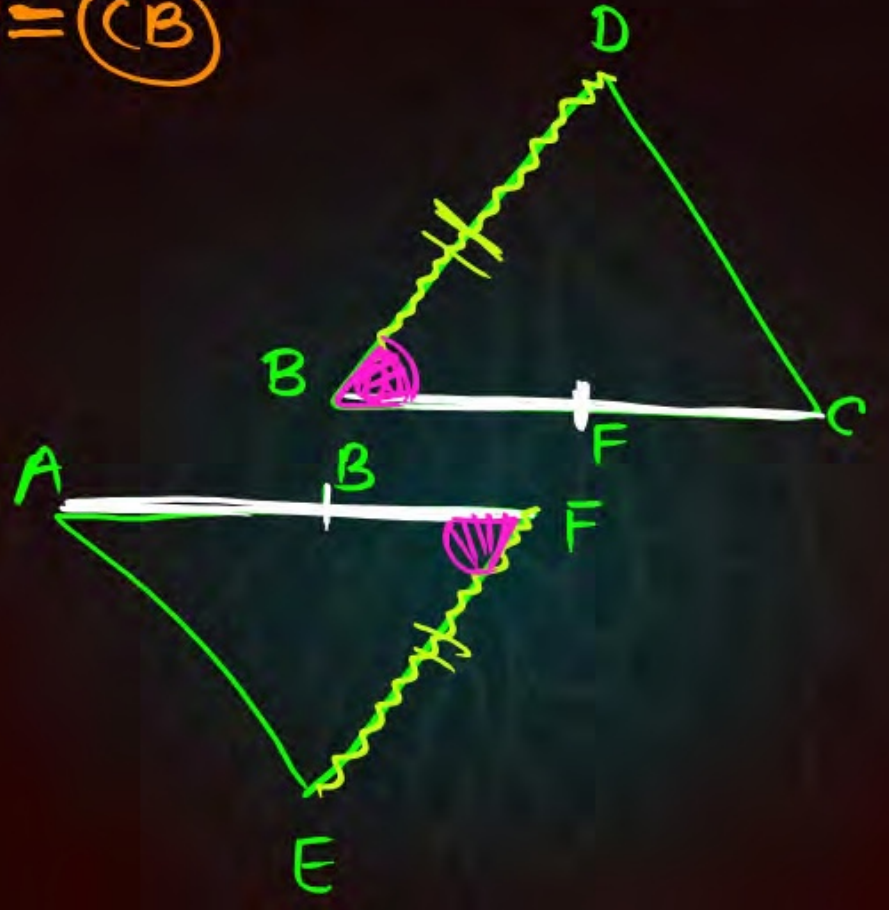
Question

In figures, it is given that AB = CF, EF = BD and ∠AFE = ∠CBD, then

+FB +FB

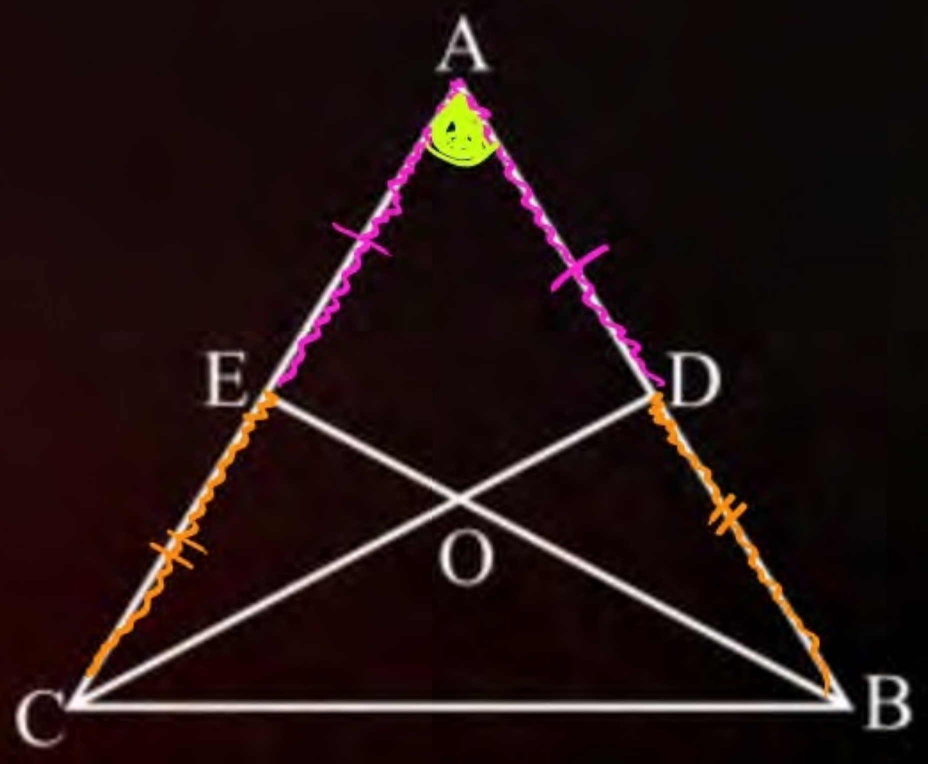
AF = CB

- **A** $\Delta AFE \cong \Delta CBD$ ✓
- **B** $\Delta AFE \cong \Delta CDB$ ✗
- **C** $\Delta AEF \cong \Delta BDC$ ✗
- **D** None of these



Question

In figure, it is given that AE = AD and BD = CE. Prove that $\boxed{\Delta AEB \cong \Delta ADC}$

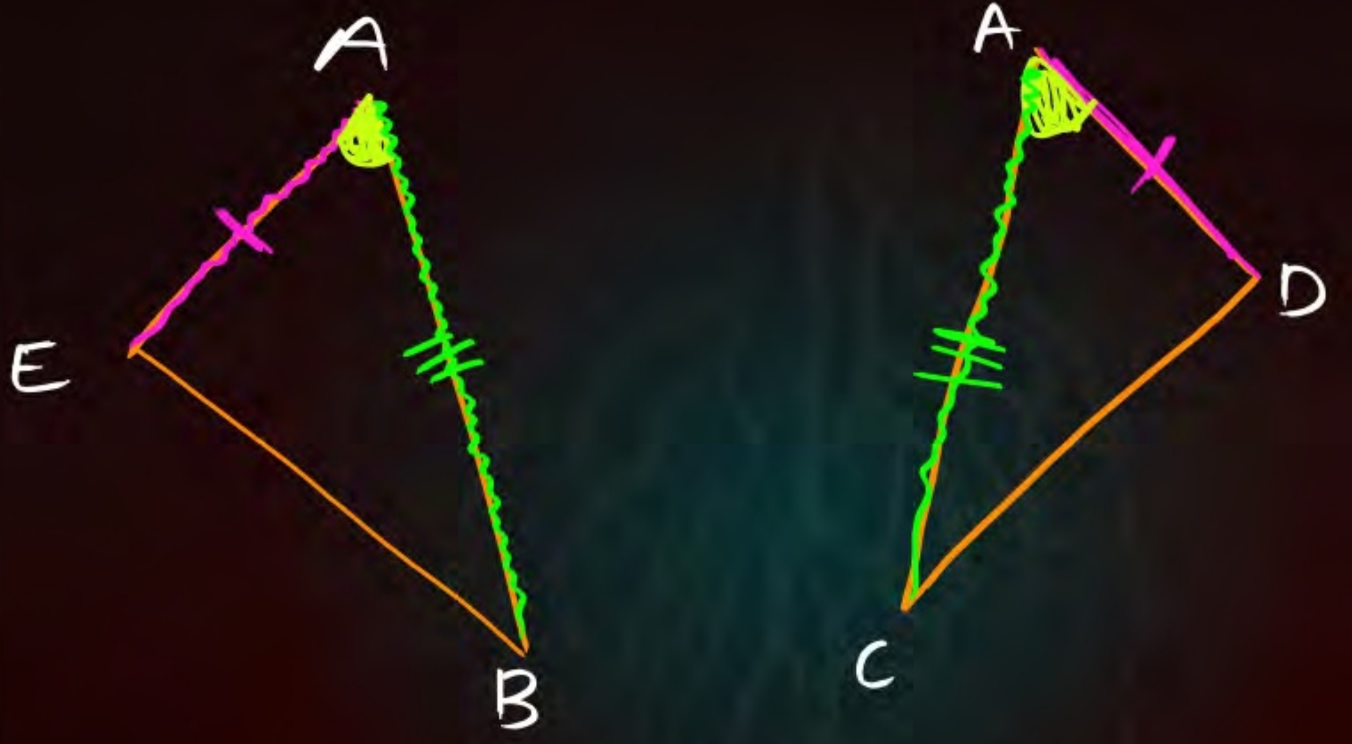


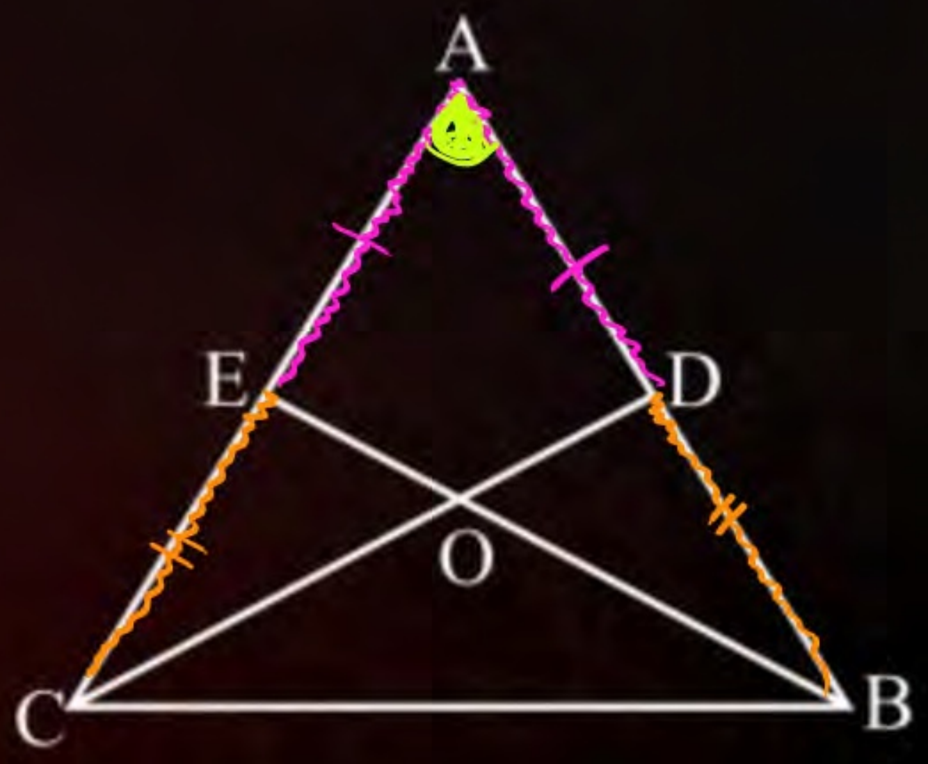
Question

In figure, it is given that AE = AD and BD = CE. Prove that $\boxed{\Delta AEB \cong \Delta ADC}$

given,

$$\begin{array}{r} AE = AD \\ +\ CE \quad +BD \\ \hline AC = AB \end{array}$$





Angle-Side-Angle (ASA)

Theorem :- Two triangles are congruent if two angle and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

$\boxed{\boxed{S} - \boxed{A} - \boxed{S}}$

cases
- ① AB = PQ
- ② AB > PQ
- ③ AB < PQ

Case (I)

$\boxed{AB = PQ}$

In $\triangle ABC$ & $\triangle PQR$

S — $BC = QR$ (given)

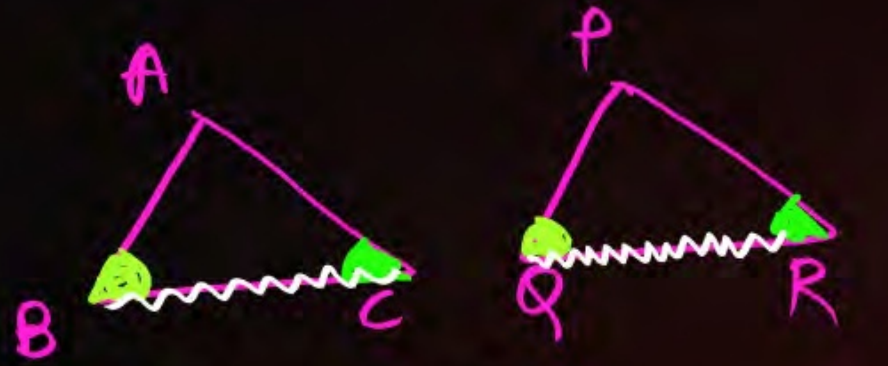
A — $\angle B = \angle Q$ (given)

S — $AB = PQ$ (Assume)

$\boxed{\triangle ABC \cong \triangle PQR}$

By using SAS

$\boxed{\text{Case-II}}$ $\boxed{AB > PQ}$ → marked a point G on extended part of PQ such that $\boxed{AB = GQ}$



$\angle B = \angle Q$
$BC = QR$
$\angle C = \angle R$

such that $\boxed{GQ = AB}$

$\Delta ABC \cong \Delta GQR$ (By using SAS)

By CPCT,

$\angle ACB = \angle GRQ$

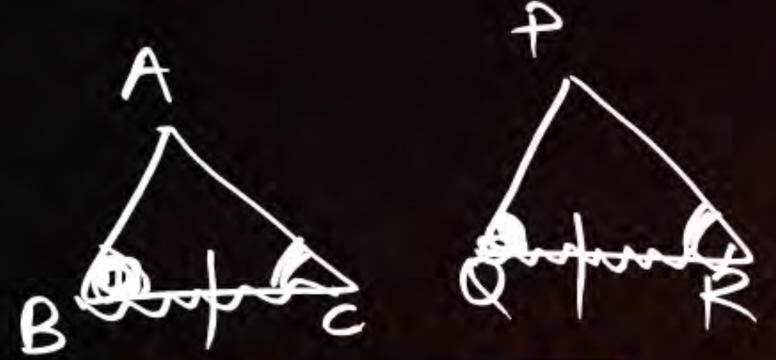
$\boxed{\angle PRQ = \angle GRQ}$

$\boxed{\angle GRP}$ ye kha gya?

Point G coincide with Point P

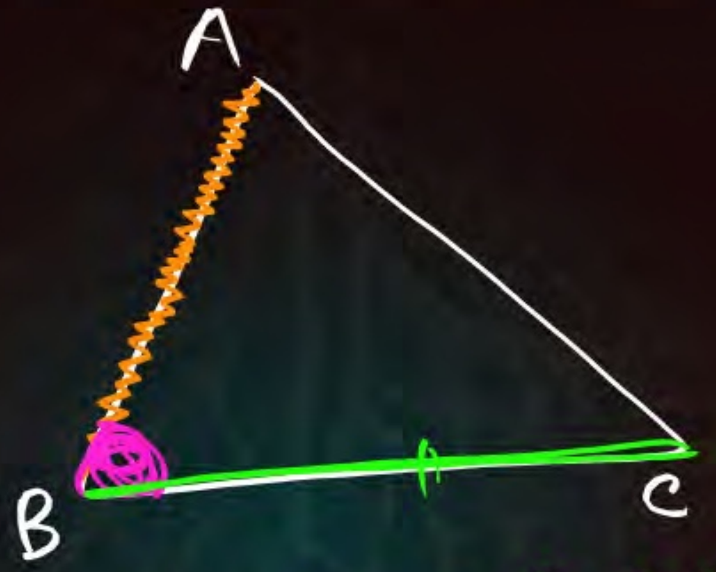
$\boxed{\Delta ABC \cong \Delta PQR}$

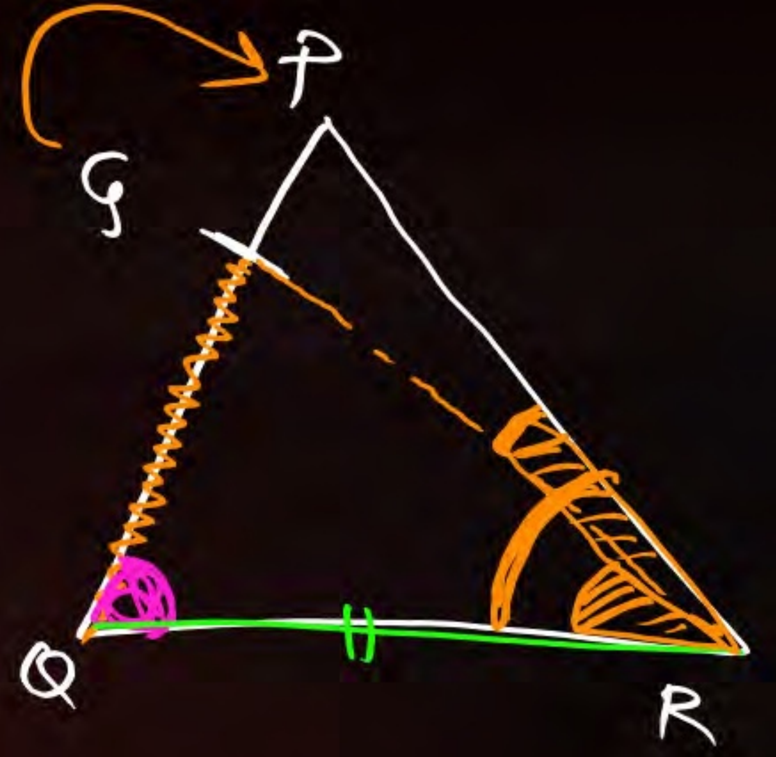
Case:→ AB < PQ → Marked a point S on PQ such that $\boxed{AB = SQ}$



$\boxed{\angle ACB = \angle PRQ}$

$\boxed{\Delta ABC \cong \Delta PQR}$





In ΔABC & ΔSQR

(S) $AB = SQ$

(A) $\angle B = \angle Q$

(S) $BC = QR$

→ $\boxed{\Delta ABC \cong \Delta SQR}$

↓ By CPCT,

$\angle ACB = \angle SRQ$

$\angle PRS = ?$ ye khagya

$\boxed{\angle PRQ = \angle SRQ}$ → Point S coincide with point P → $\Delta ABC \cong \Delta PQR$

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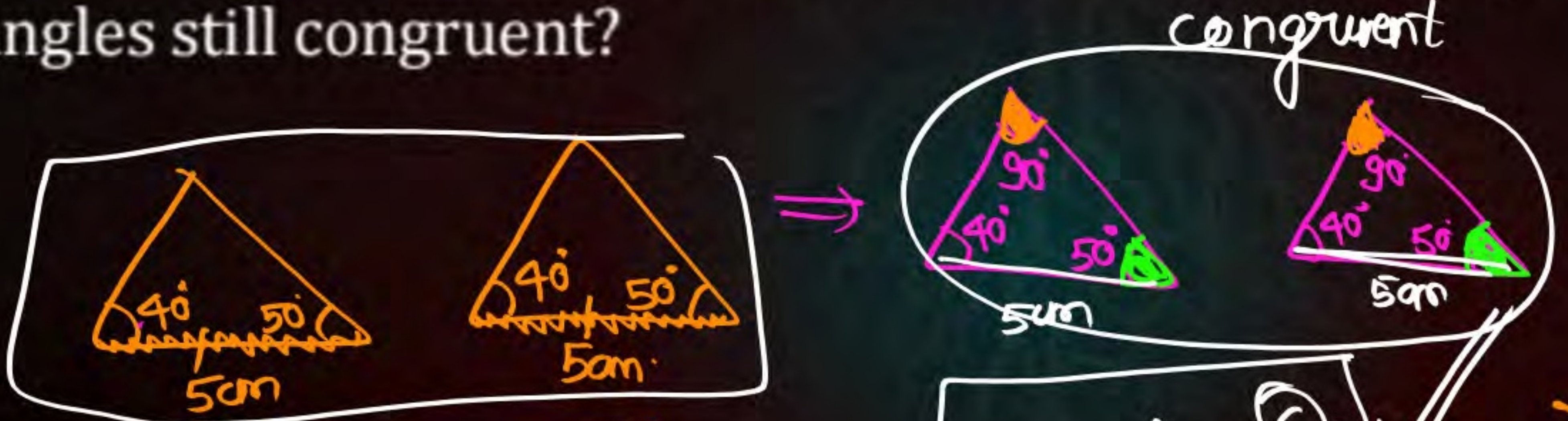
Misconception Points

In two triangles two pairs of angles and one pair of corresponding sides are equal but the side is not included between the corresponding equal pairs of angles. Are the triangles still congruent?



A-S-A

A-A-S ✓

Included

S-A-S

SSA ✗

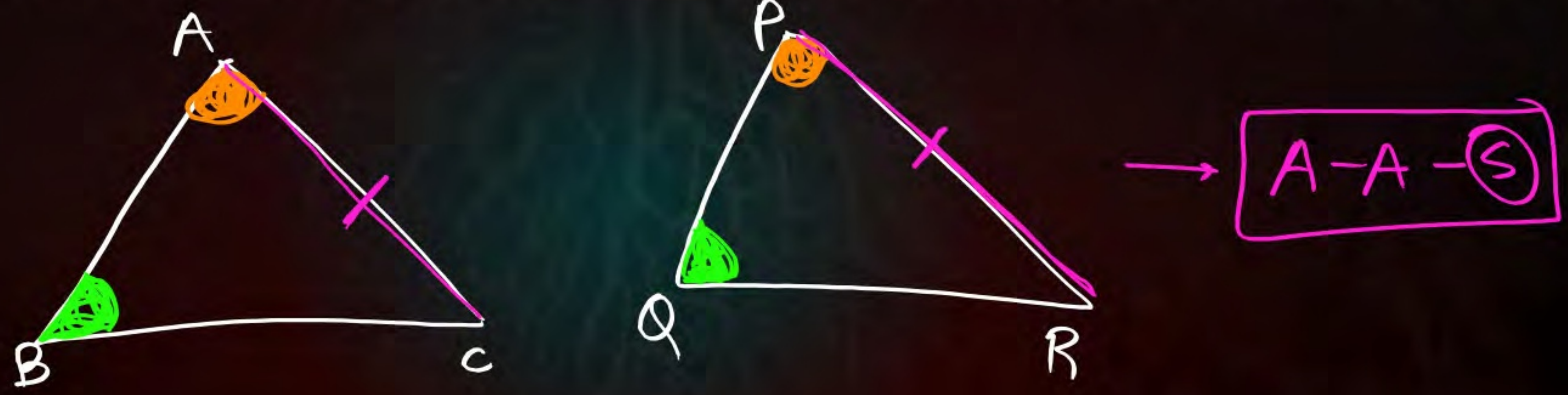
ASS ✗

A-S-A ✓

A-A-S ✓

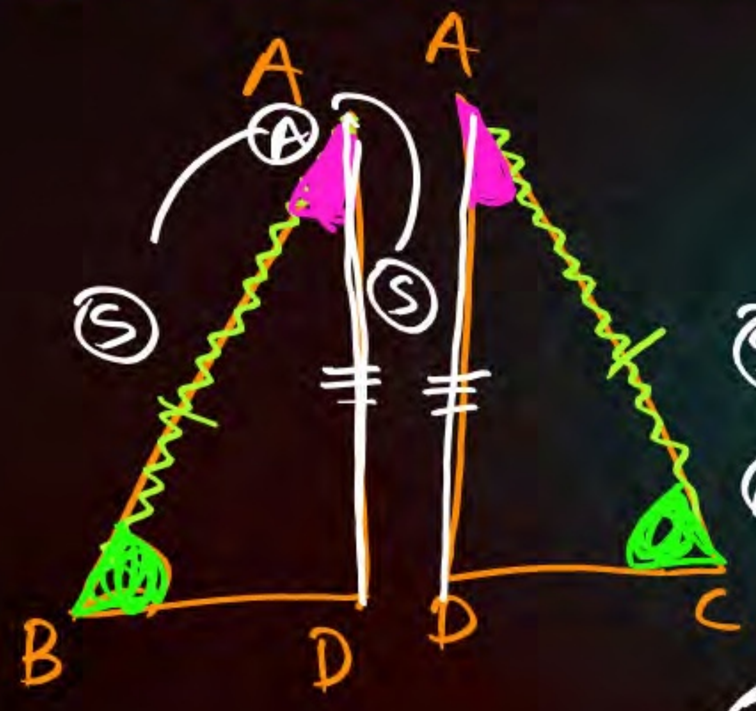
Angle-Angle-Side (AAS) congruence criterion

Theorem :- Two triangles are congruent if two angles and a non-included side of one triangle are equal to the corresponding angle and side of the another triangle.



Some Properties of Triangle

Theorem 7.2 : Angles opposite to equal sides of an isosceles triangle are equal.

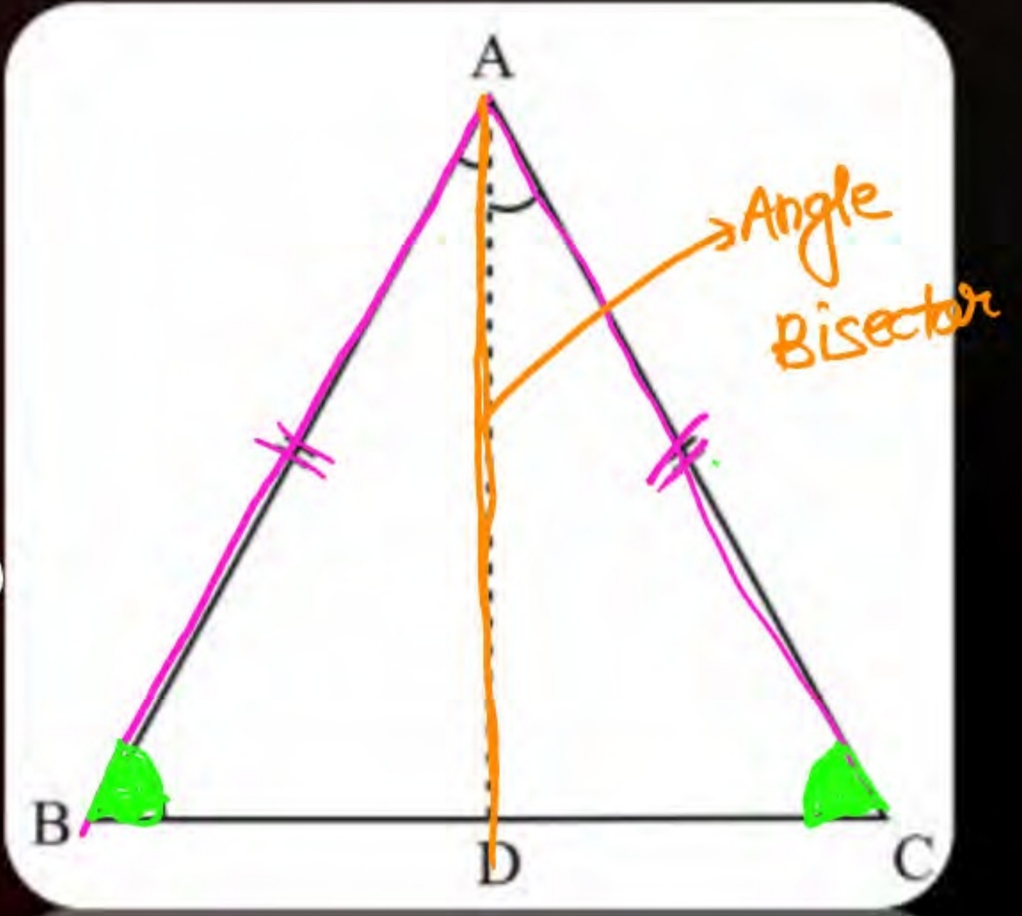


$\triangle ADB$ & $\triangle ADC$

S $AB = AC$ (given)

A $\angle BAD = \angle CAD$
(AD is angle bisector of $\angle A$)

S $AD = AD$ (common)



$\boxed{\angle B = \angle C}$ ⇐ By CPCT ← $\boxed{\triangle ADB \cong \triangle ADC}$

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Some Properties of Triangle

Theorem 7.3 : The sides opposite to equal angles of a triangle are equal.

$\boxed{\text{Draw } AD \perp BC}$

In $\triangle ADB$ & $\triangle ADC$

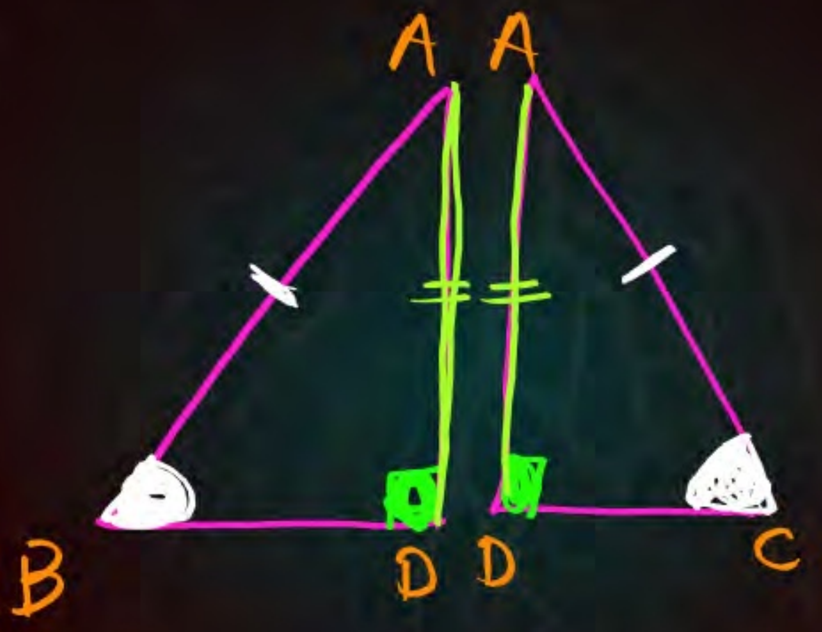
A $\angle B = \angle C$ (given)

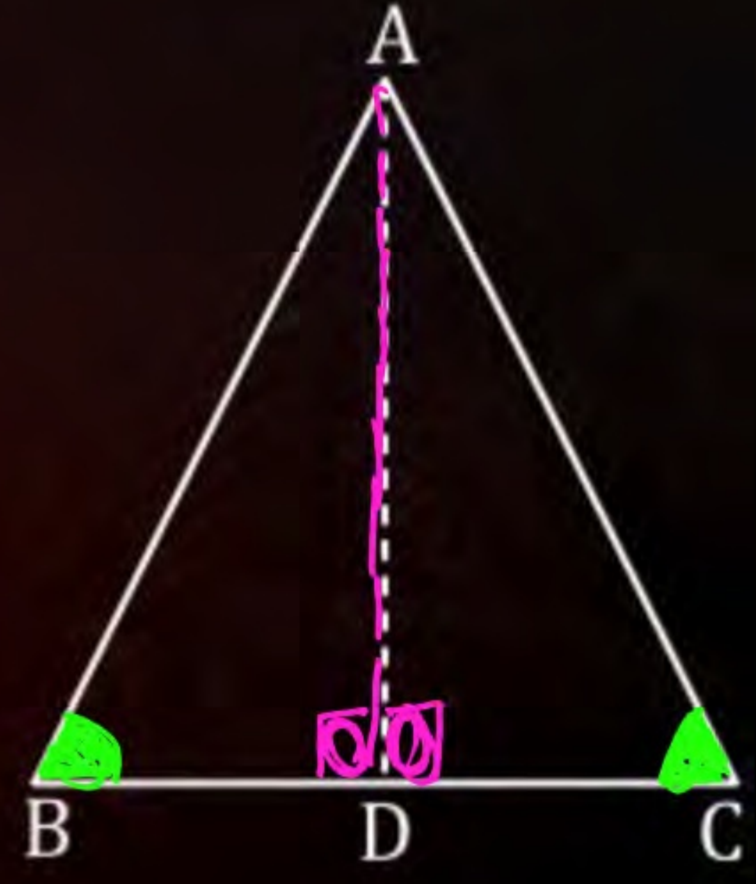
A $\angle ADB = \angle ADC$ ($AD \perp BC$)

S $AD = AD$

$\boxed{\triangle ADB \cong \triangle ADC}$

↳ By CPCT, $\boxed{AB = AC}$ Hence, prooved!!





Question

Which of the following is not a criterion of congruency ?

S-A-S

A-A-S

S-S-A

A-S-A

Question

Which of the following is not a criterion of congruency ?

A S-A-S

B A-A-S

C $\boxed{\text{S-S-A}}$ ✓

D A-S-A

S – A – A ⟶ SSA , ASS ✗

A – S – A

A – A – S

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Question

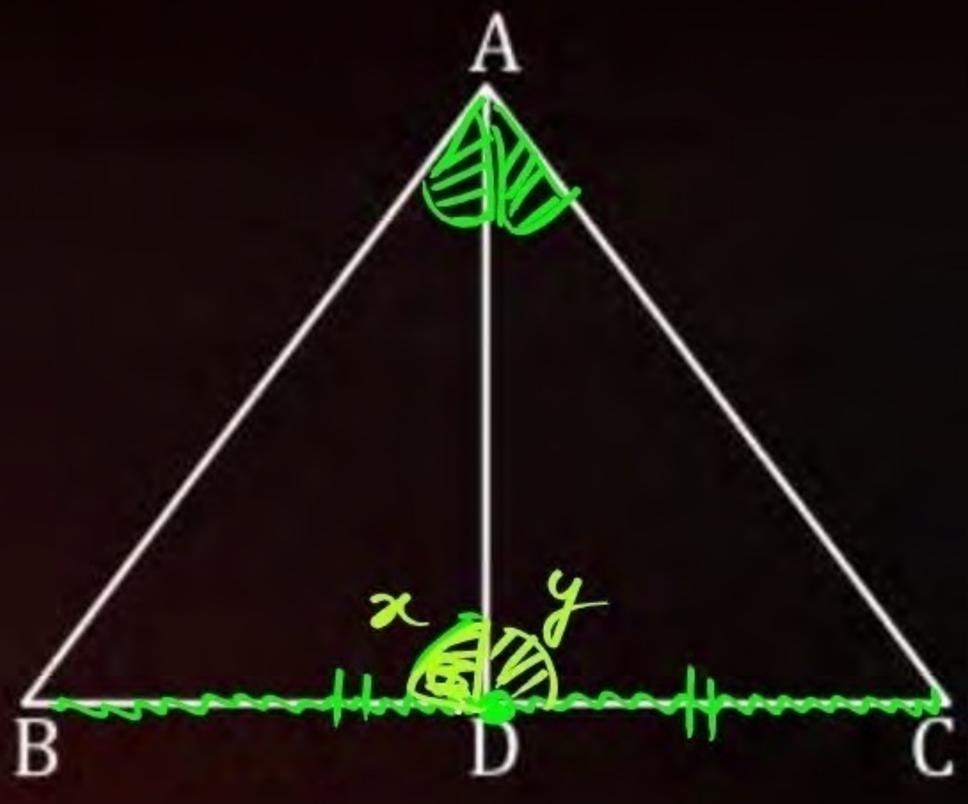
In the given figure, if $\Delta ABD \cong \Delta ACD$, then

D is the mid-point of BC

AD is the angle bisector of $\angle BAC$

AD is the perpendicular bisector of BC

All of these



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

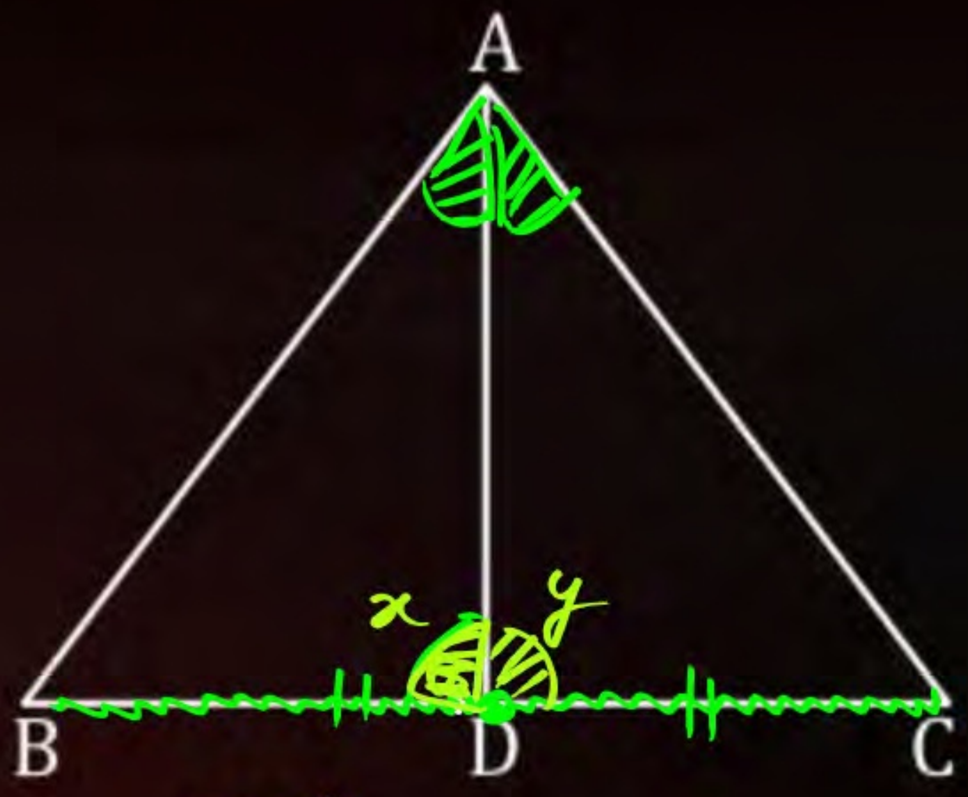
In the given figure, if $\Delta ABD \cong \Delta ACD$, then

A D is the mid-point of BC

B AD is the angle bisector of $\angle BAC$

C AD is the perpendicular bisector of BC

D All of these ✓



$x = y$

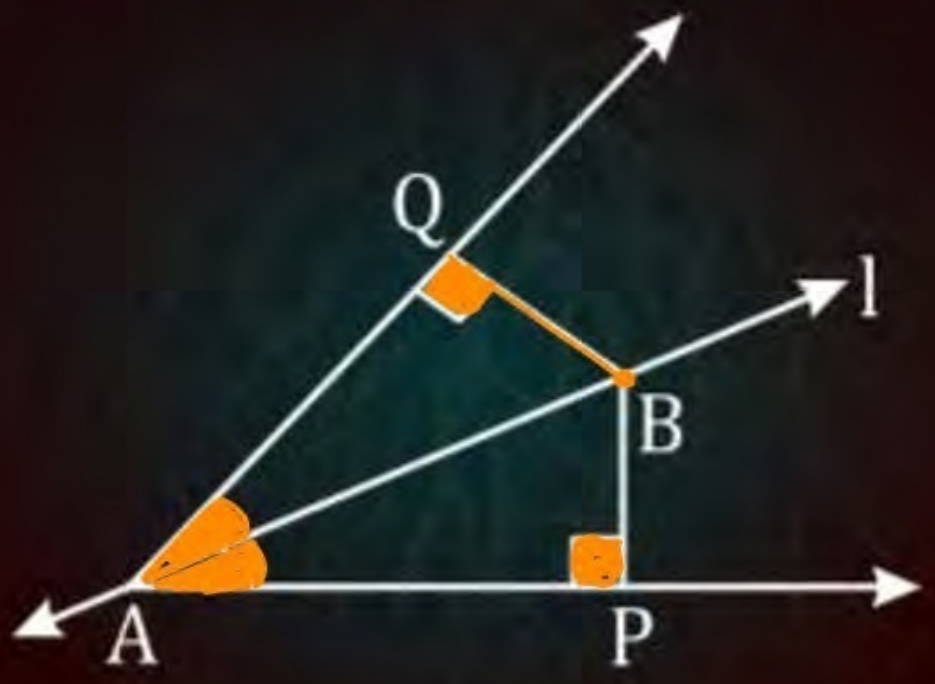
$x + y = 180^\circ$

$y + y = 180^\circ$

$2y = 180^\circ \Rightarrow \boxed{y = 90^\circ = x}$

Question

Line l is the bisector of an angle $\angle$ A and $\angle$ B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle$ A (see Fig.). Show that: (i) $\boxed{\Delta APB \cong \Delta AQB}$ (ii) BP = BQ or B is equidistant from the arms of $\angle$ A.



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Question

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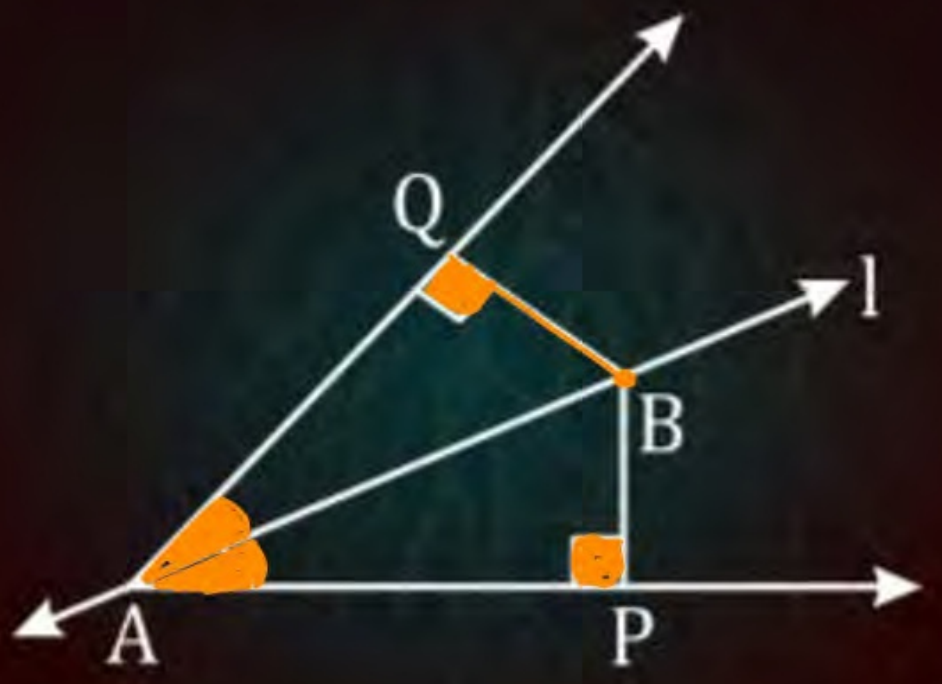
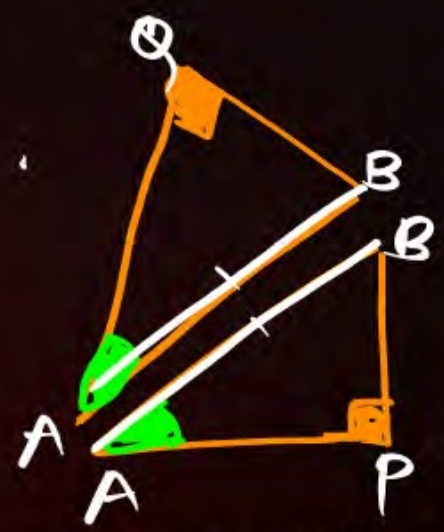


(ii) By CPCT,

$\boxed{BP = BQ}$

→ point B is equidistant from arm of $\angle A$



$\boxed{A - A - S}$

In ΔAPB & ΔAQB

(i) ($\angle A$ bisected) ← $\angle BAP = \angle BAQ$

(90°) ← $\angle BPA = \angle BQA$

(common) ← $AB = AB$

By (A)-(A)-(S) → $\boxed{\Delta APB \cong \Delta AQB}$

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Question

In the given figure, C is the mid-point of AB such that $\angle CAD = \angle CBE$ and $\angle ECA = \angle DCB$. Prove that:

(i) $\Delta DAC \cong \Delta EBC$

(ii) $DA = EB$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In the given figure, C is the mid-point of AB such that $\angle CAD = \angle CBE$ and $\angle ECA = \angle DCB$. Prove that:

(i) $\boxed{\Delta DAC \cong \Delta EBC}$

(ii) $DA = EB$

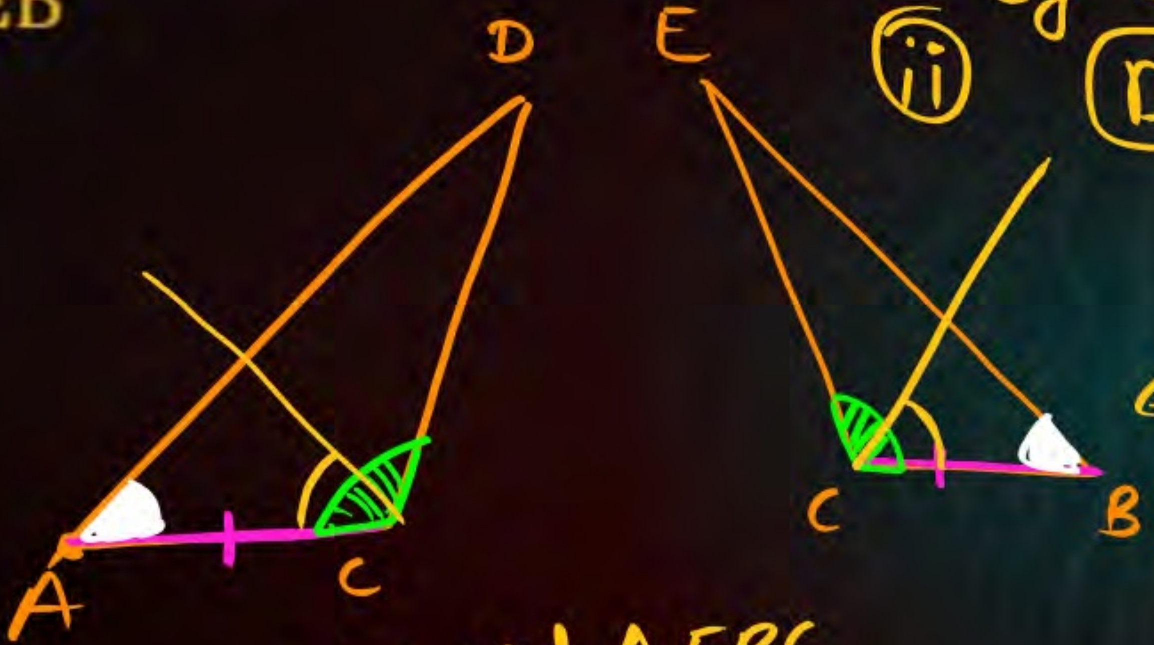
(i) $\Rightarrow \boxed{\Delta DAC \cong \Delta EBC}$

By CPCT,

(ii) $\boxed{DA = EB}$

SAS ~~X~~

A-S-A



In ΔDAC and ΔEBC,

(A) $\angle DAC = \angle EBC$ (given)

(S) $AC = CB$ (C is mid pt of AB)

(A) $\angle ACD = \angle ECB$ (concluded)

$\angle ECA = \angle DCB$

$+\angle ECD \quad +\angle ECD$

$\boxed{\angle ACD = \angle ECB}$

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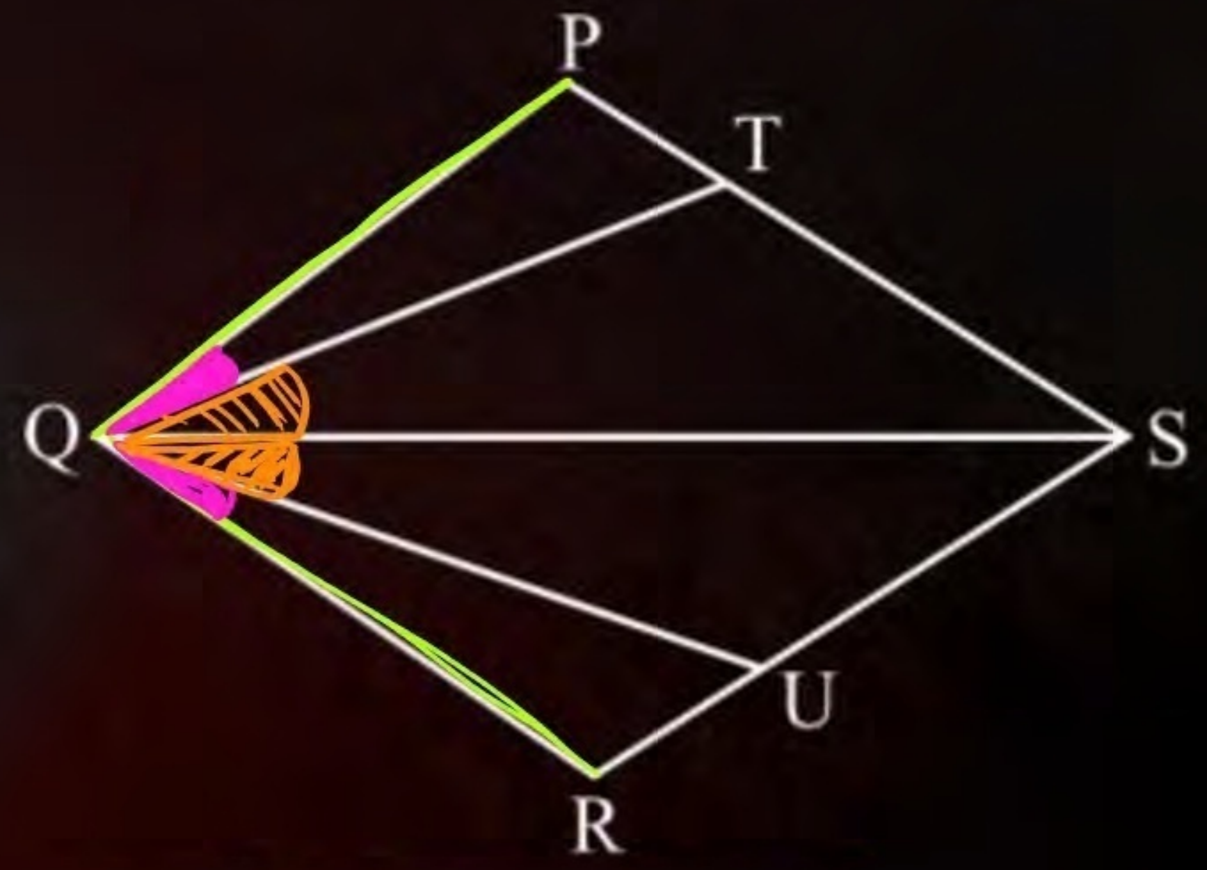
Question

In the given figure, PQRS is a quadrilateral in which PQ = RQ and T and U are points on PS and RS respectively such that $\angle PQT = \angle RQU$ and $\angle TQS = \angle UQS$. Prove that QT = QU



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

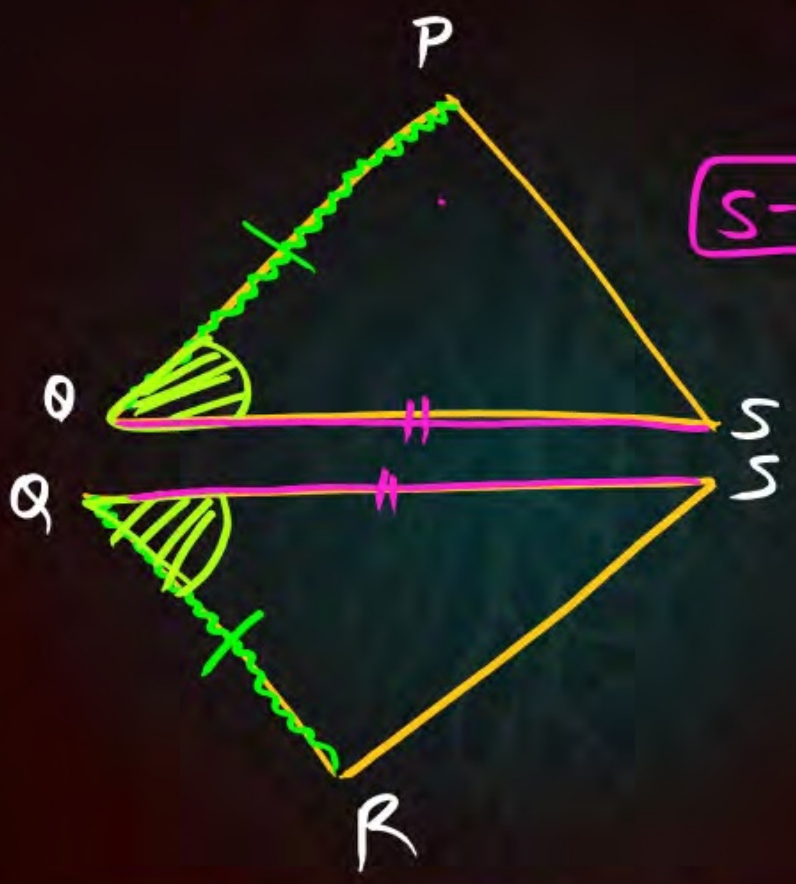
Question

In the given figure, PQRS is a quadrilateral in which PQ = RQ and T and U are points on PS and RS respectively such that ∠PQT = ∠RQU and ∠TQS = ∠UQS. Prove that QT = QU

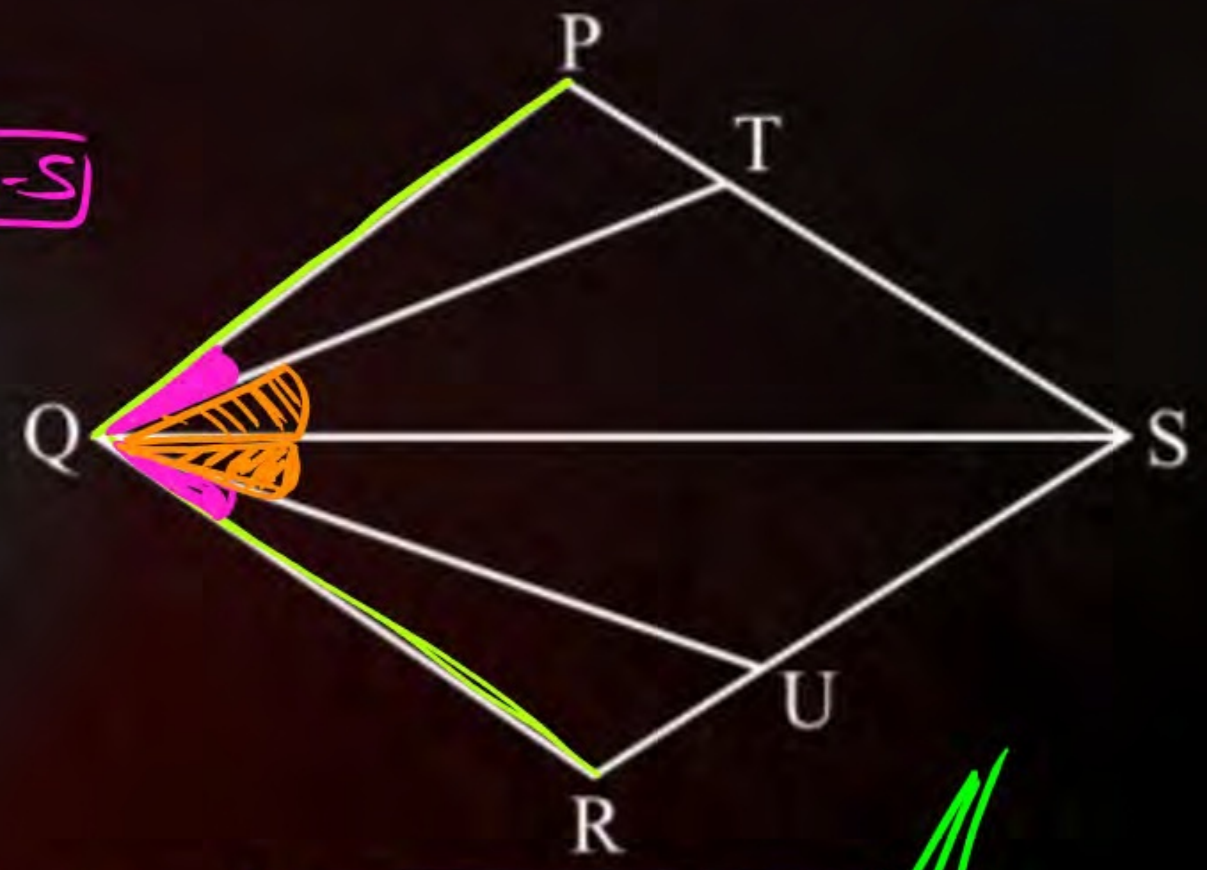
$\angle PQT = \angle RQU$

$+\ \angle TQS = \angle UQS$

$\angle PQS = \angle RQS$



$\boxed{S\text{-}A\text{-}S}$



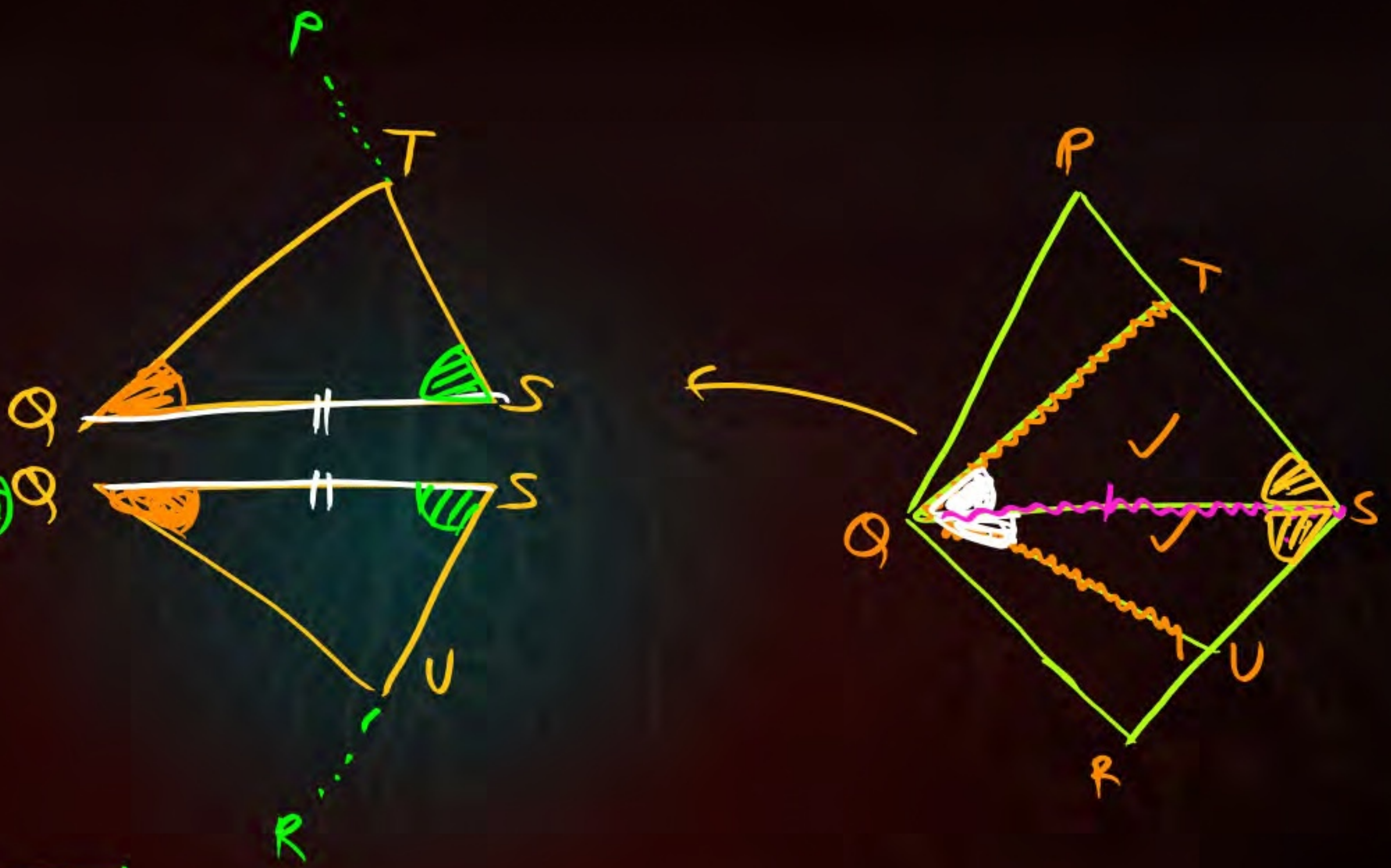
$\boxed{\Delta PQS \cong \Delta RQS} \rightarrow$ By CPCT, $\boxed{\angle PSQ = \angle RSQ}$ ✓

A-S-A criteria,

$$\left\{\begin{array}{l} \angle TQS = \angle UQS \text{ (given)} \\ \angle TSQ = \angle USQ \text{ (calculated)} \\ QS = QS \end{array}\right.$$

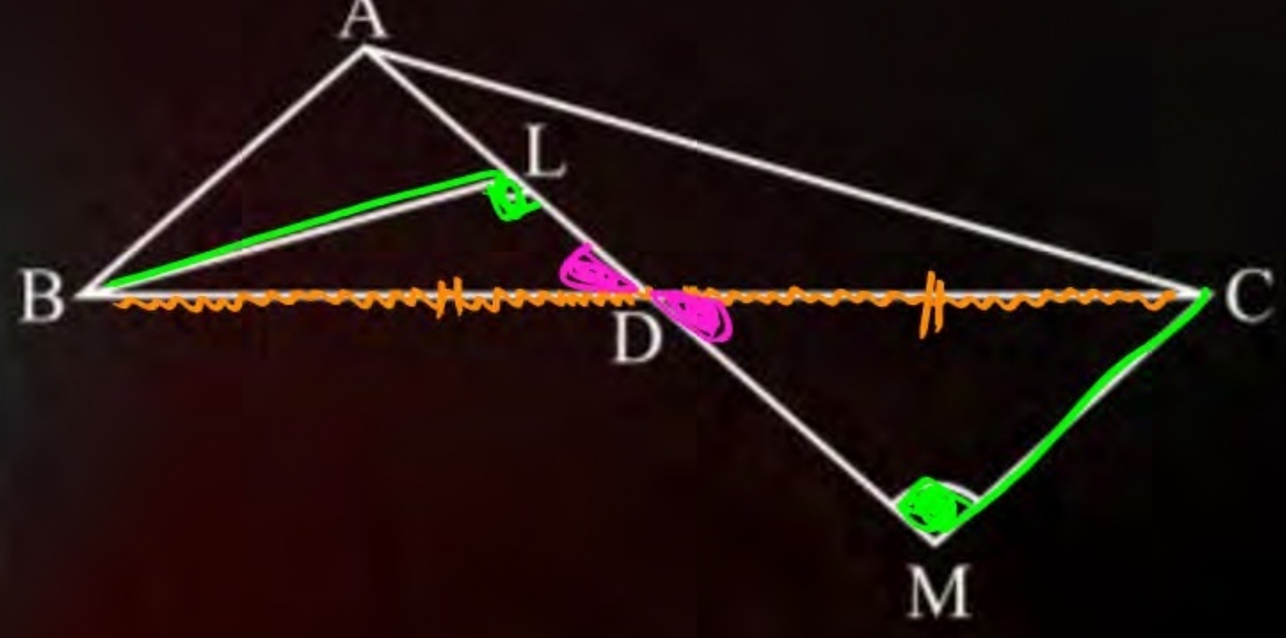
$$\boxed{\triangle QST \cong \triangle QSU}$$

By CPCT, $\boxed{QT = QU}$



Question

In the given figure, AD is the median and AD is produced to M. If BL and CM are perpendiculars draw, from B and C respectively on AM. Then prove that BL = CM.



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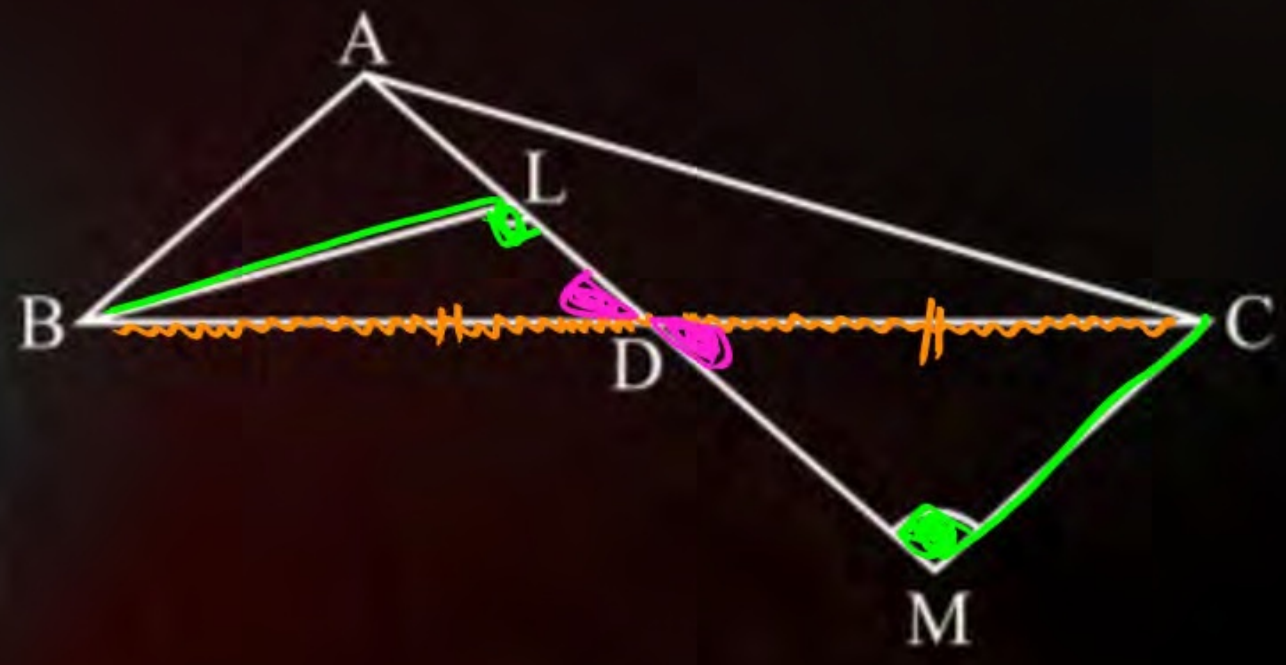
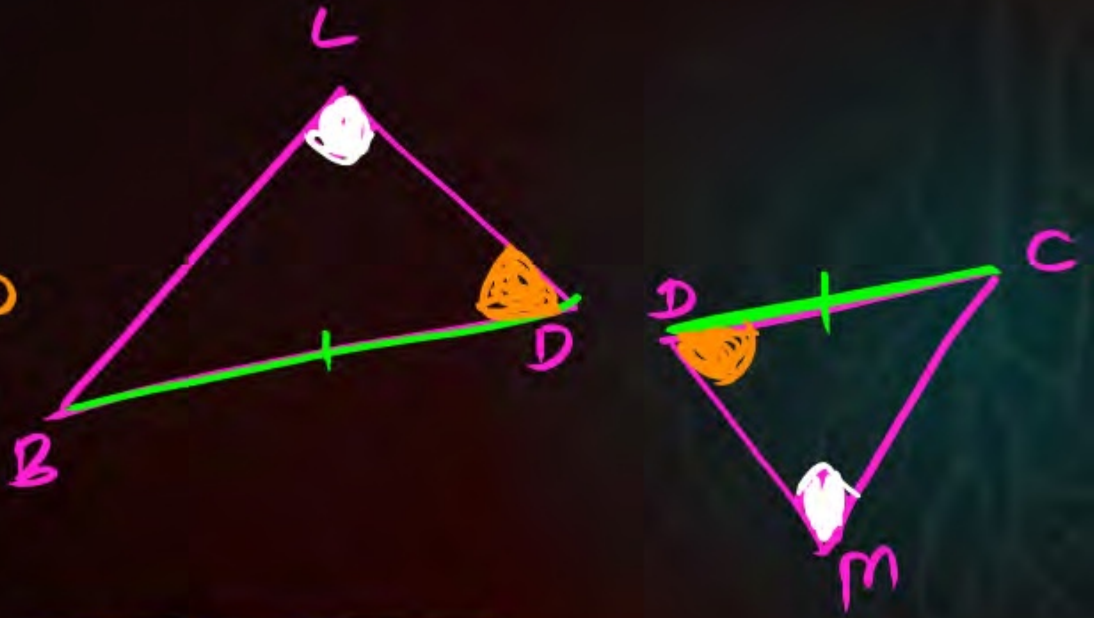


$\boxed{A-A-S}$

$\begin{cases} \angle BLD = \angle CMD \\ \angle BDL = \angle CDM \\ BD = CD \end{cases}$

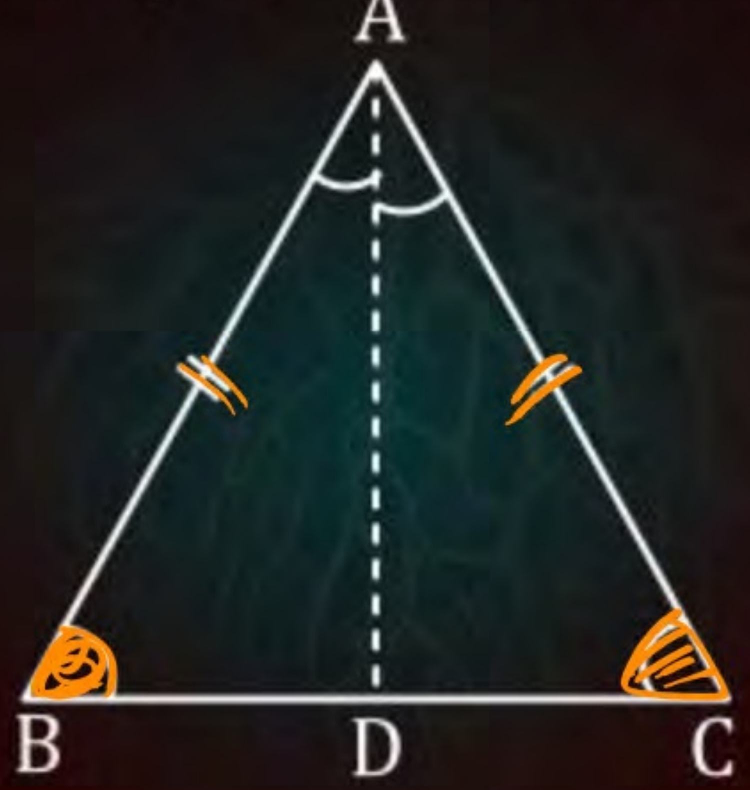


$\boxed{\triangle BDL \cong \triangle CDM}$

By CPCT, $\boxed{BL = CM}$

Some Properties of Triangle

Theorem 7.2 : Angles opposite to equal sides of an isosceles triangle are equal.



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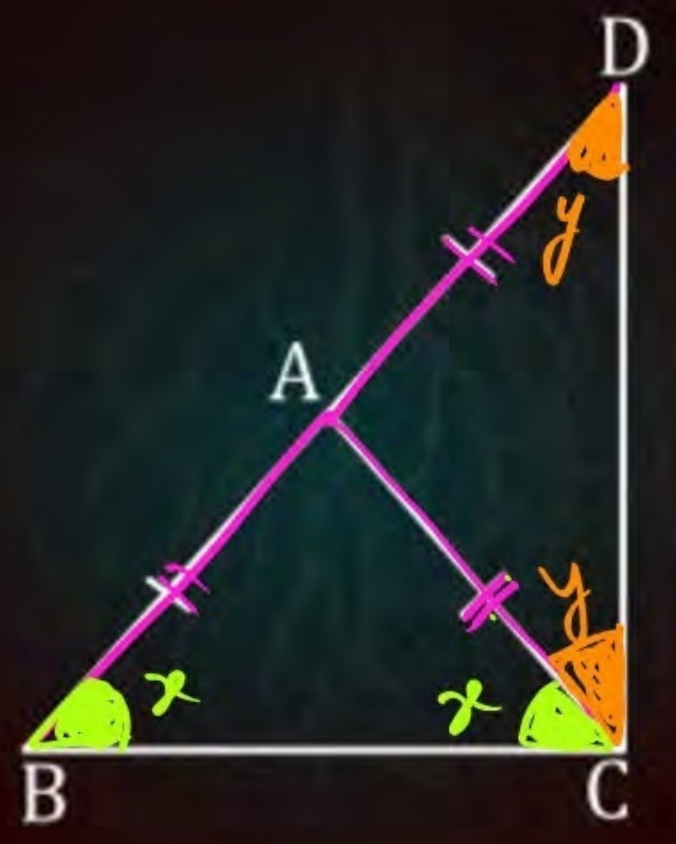
Question

ΔABC is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AC$ (see figure below). Show that $\angle BCD$ is a right angle.



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Question

ΔABC is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AC$ (see figure below). Show that $\angle BCD$ is a right angle.

In an isosceles Δ, angle opposite to equal side are equal.

In ΔBCD, angle sum property

$$x+y+(x+y)=180^\circ$$
$$2x+2y=180^\circ$$
$$2(x+y)=180^\circ$$
$$x+y=90^\circ$$
$$\boxed{\angle BCD=90^\circ}$$

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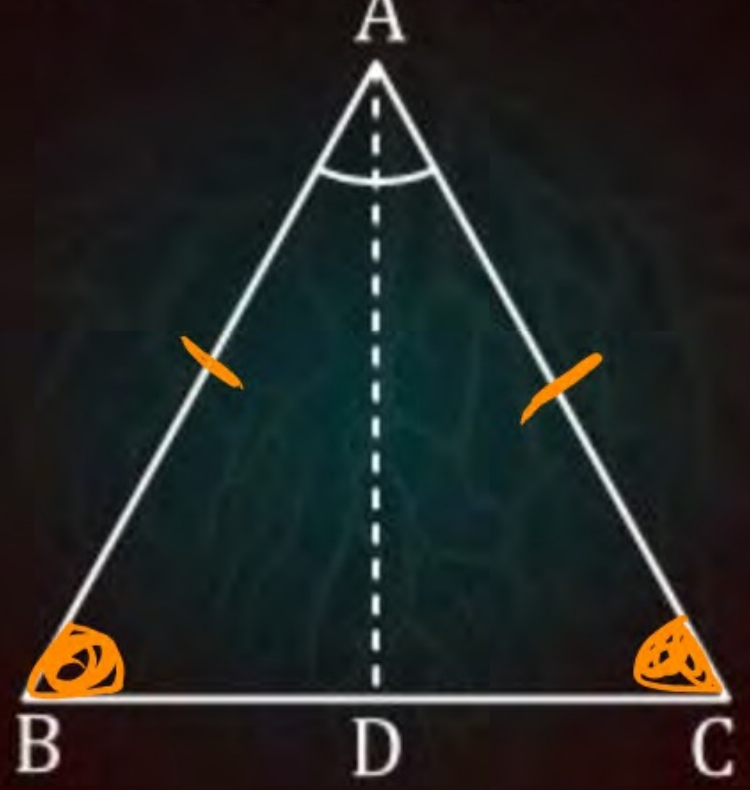
Some Properties of Triangle

Theorem 7.3 : The sides opposite to equal angles of a triangle are equal.



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Property-3

If the altitude from one vertex of a triangle bisects the opposite side then the triangle is isosceles.

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Property-4

In an isosceles triangle, altitude from the vertex bisects the base.

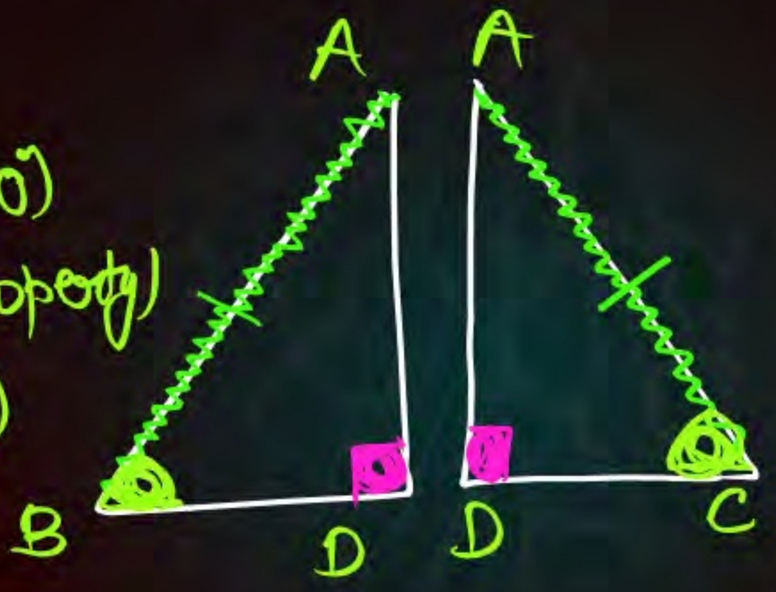
A
A
S

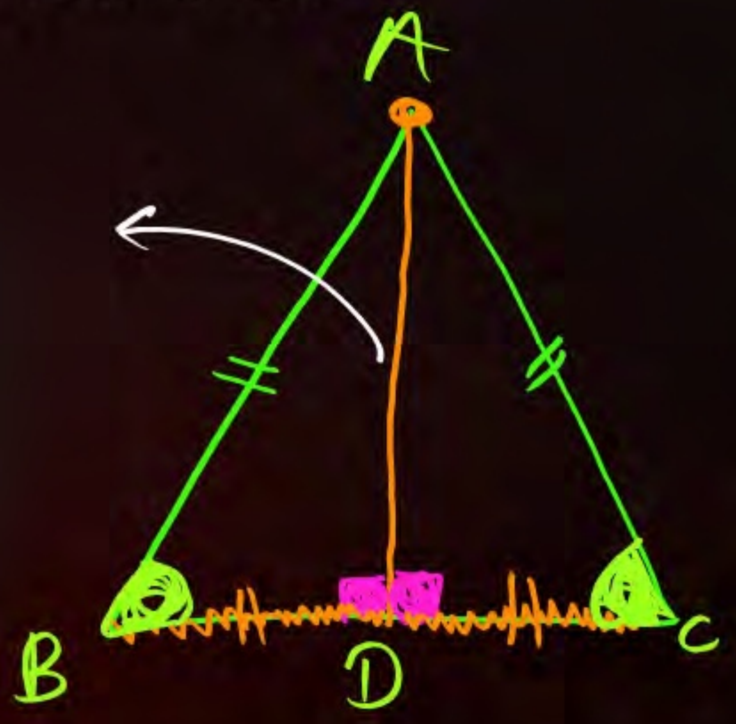
$\angle ADB = \angle ADC$ (90°)
$\angle ABD = \angle ACD$ (Property)
$AB = AC$ (Property)

By using A-A-S

$\boxed{\Delta ADB \cong \Delta ADC}$

By CPCT, $\boxed{DB = DC}$ ✓





Property-5

If the bisector of the vertical angle of a triangle bisects the base of the triangle, then the triangle is isosceles.

construction:- $\boxed{AC = CE \ \& \ \angle ACB = \angle ECB}$

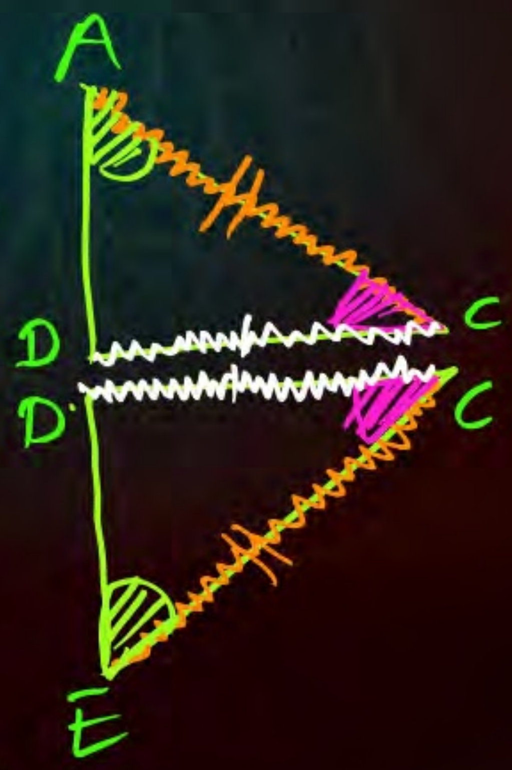
$$\left.\begin{array}{l} S \\ A \\ S \end{array}\right\{ \begin{array}{l} AC = EC \\ \angle ACD = \angle ECD \\ CD = CD \end{array}$$

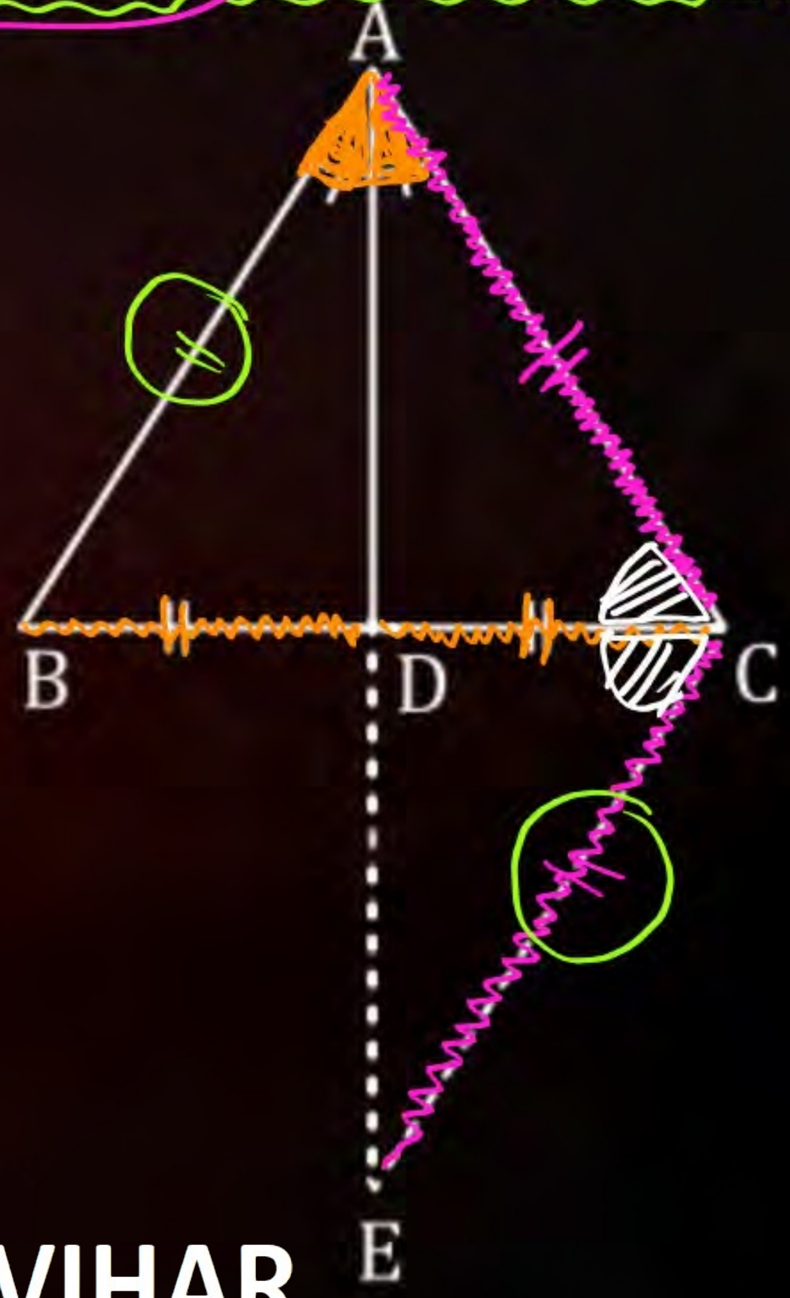
using SAS criteria

$\boxed{\triangle ADC \cong \triangle EDC}$

By CPCT,

$\angle CAD = \angle CED$

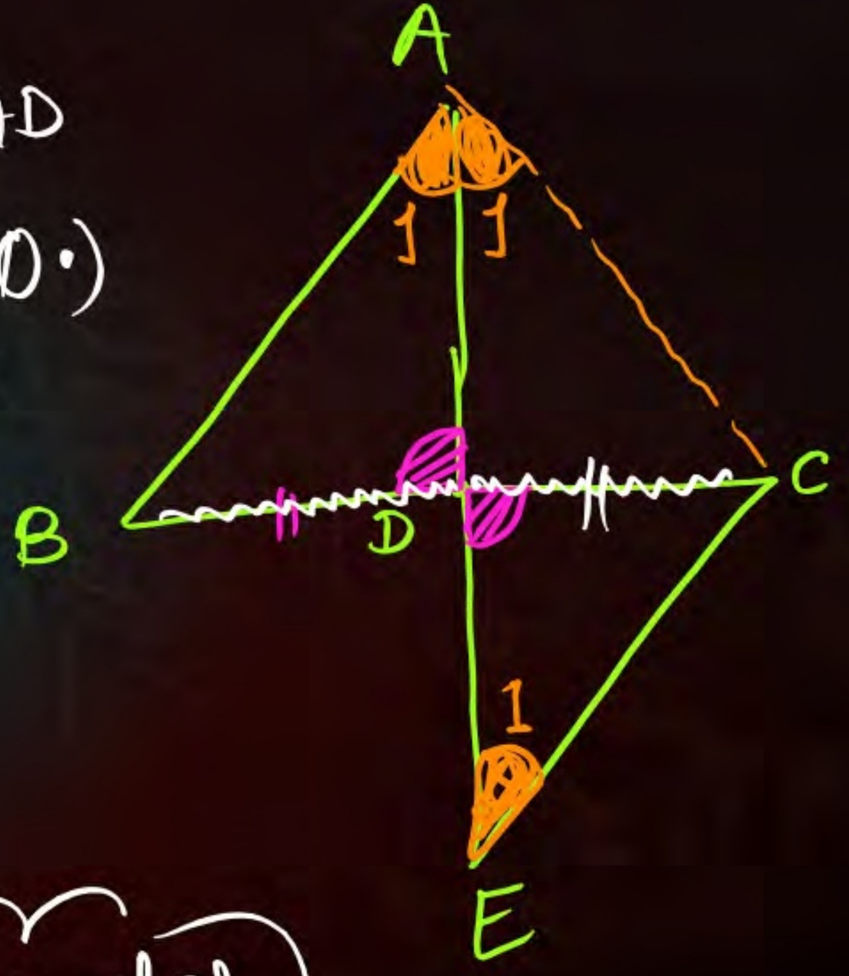




A $\left\{ \angle BAD = \angle CED = \angle CAD \right.$

A $\quad \angle ADB = \angle EDC \quad (V.O.)$

S $\quad BD = CD$ (Base bisect)



Using A-A-S, $\boxed{\Delta ADB \cong \Delta EDC}$

By CPCT, $AB = CE = AC$

$\Rightarrow \boxed{AB = AC}$ → Isosceles

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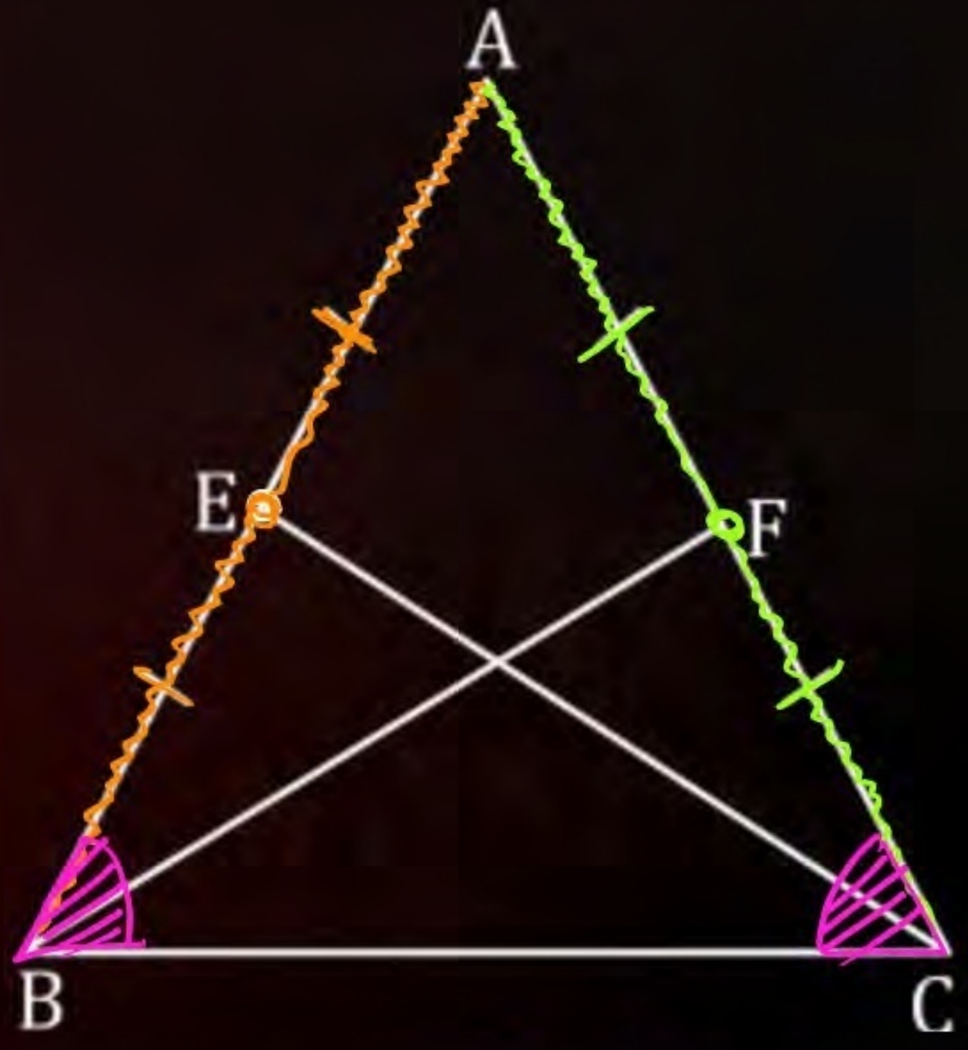
Question

E and F are respectively the mid-points of equal sides AB and AC of ΔABC. Show that BF = CE.



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Question

E and F are respectively the mid-points of equal sides AB and AC of ΔABC. Show that $BF = CE$.

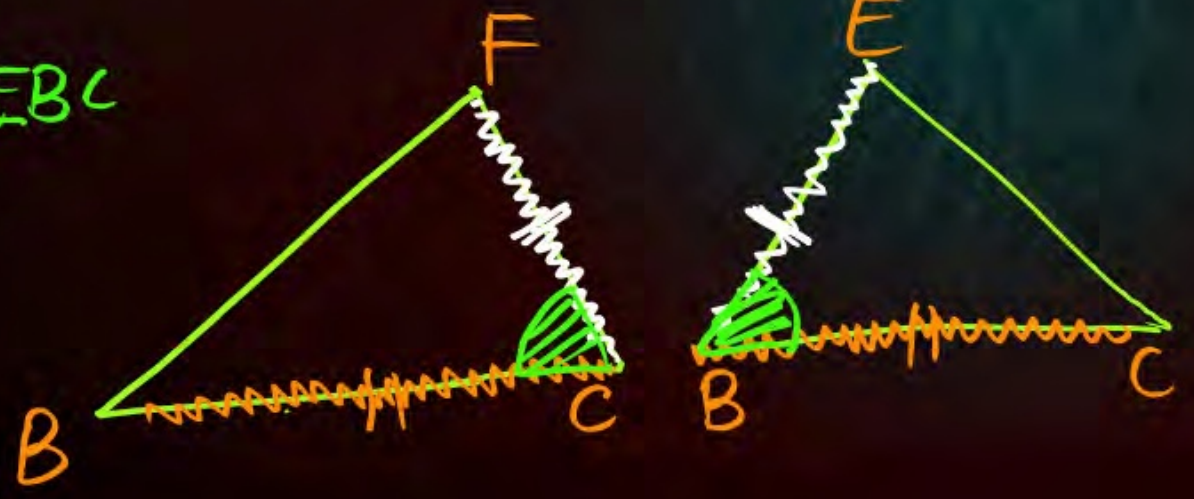
$AB = AC \Rightarrow \frac{AB}{2} = \frac{AC}{2} \Rightarrow \boxed{EB = FC}$ ✓

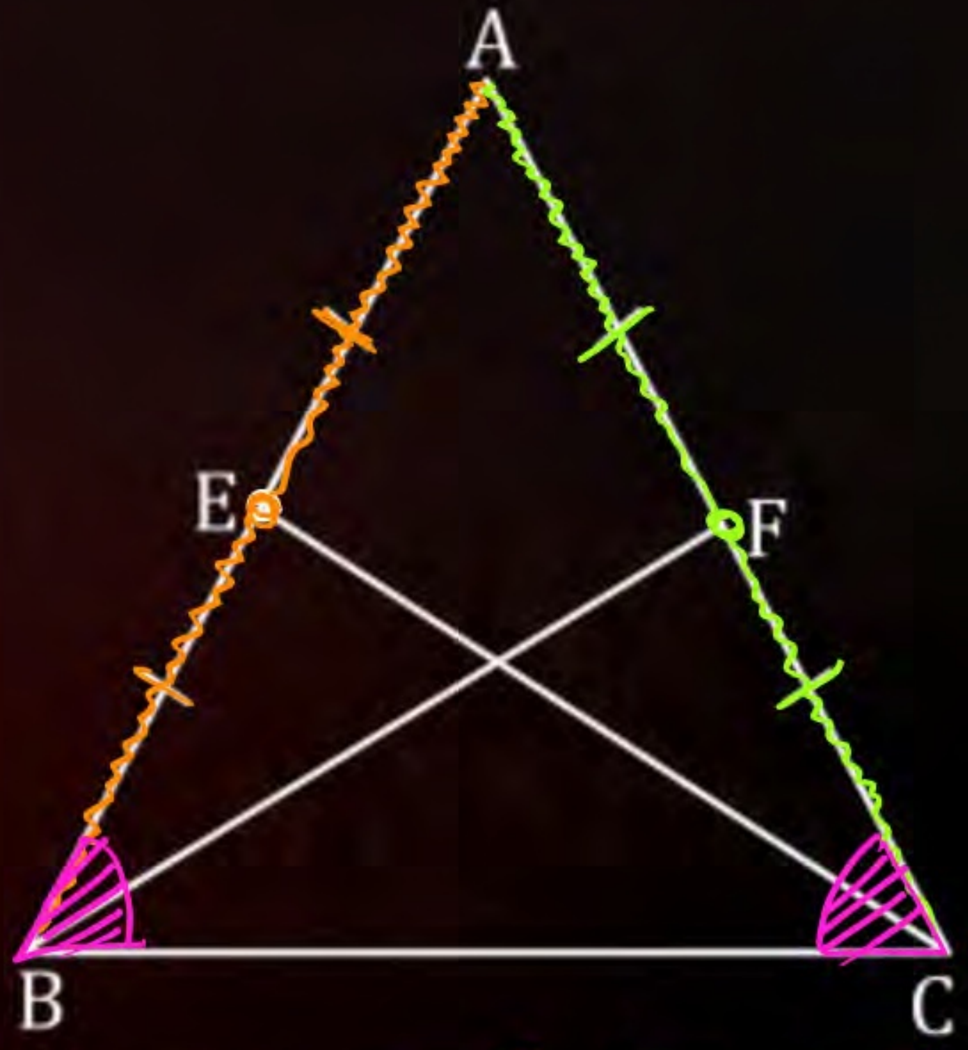
S: $FC = EB$
A: $\angle FCB = \angle EBC$
S: $BC = CD$

using SAS rule,

$\boxed{\Delta FCB \cong \Delta EBC}$

By CPCT, $\boxed{BF = CE}$ ***





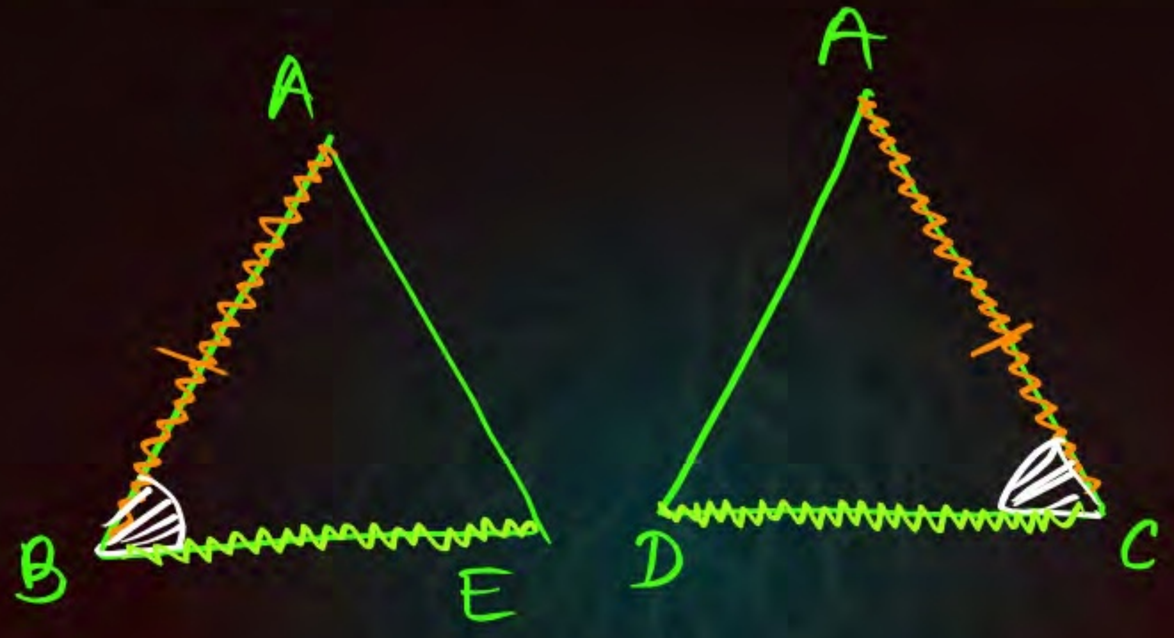
$\boxed{\angle EBC = \angle FCB}$ (Isosceles)

Question

In a isosceles triangle ABC with AB = AC, D and E are points on BC such that BE = CD. Show that AD = AE

Question

In a isosceles triangle ABC with AB = AC, D and E are points on BC such that BE = CD. Show that AD = AE



Using SAS rule,

$\boxed{\Delta ABE \cong \Delta ACD}$

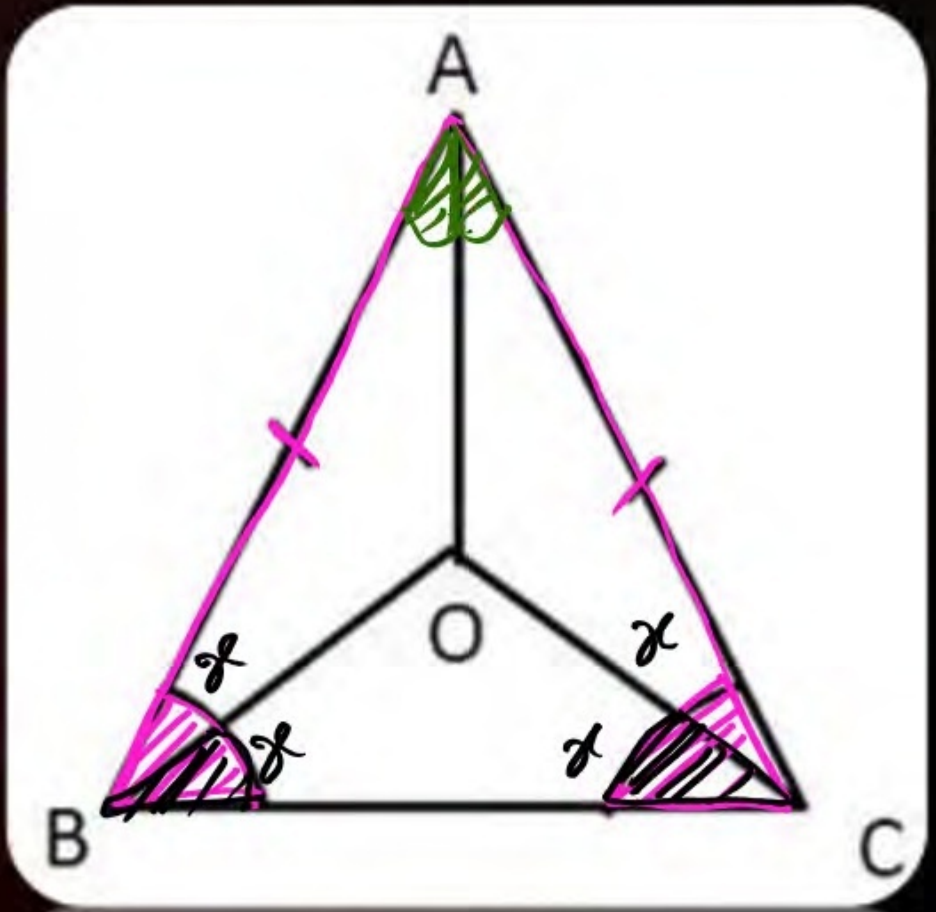
By CPCT, $\boxed{AD = AE}$ ***

Question

In an isosceles ΔABC, with $AB = AC$, the bisector or $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that.

(i) $OB = OC$

(ii) AO bisects $\angle A$



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Question

In an isosceles ΔABC, with $AB = AC$, the bisector or $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that.

(i) $OB = OC$

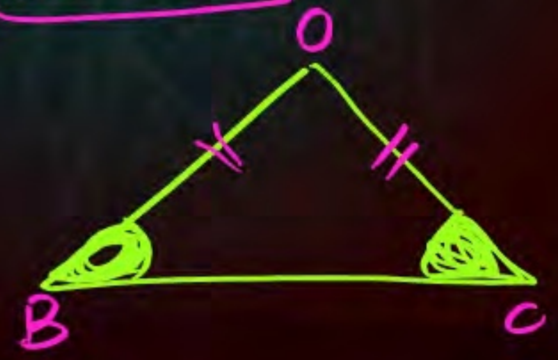
(ii) AO bisects $\angle A$

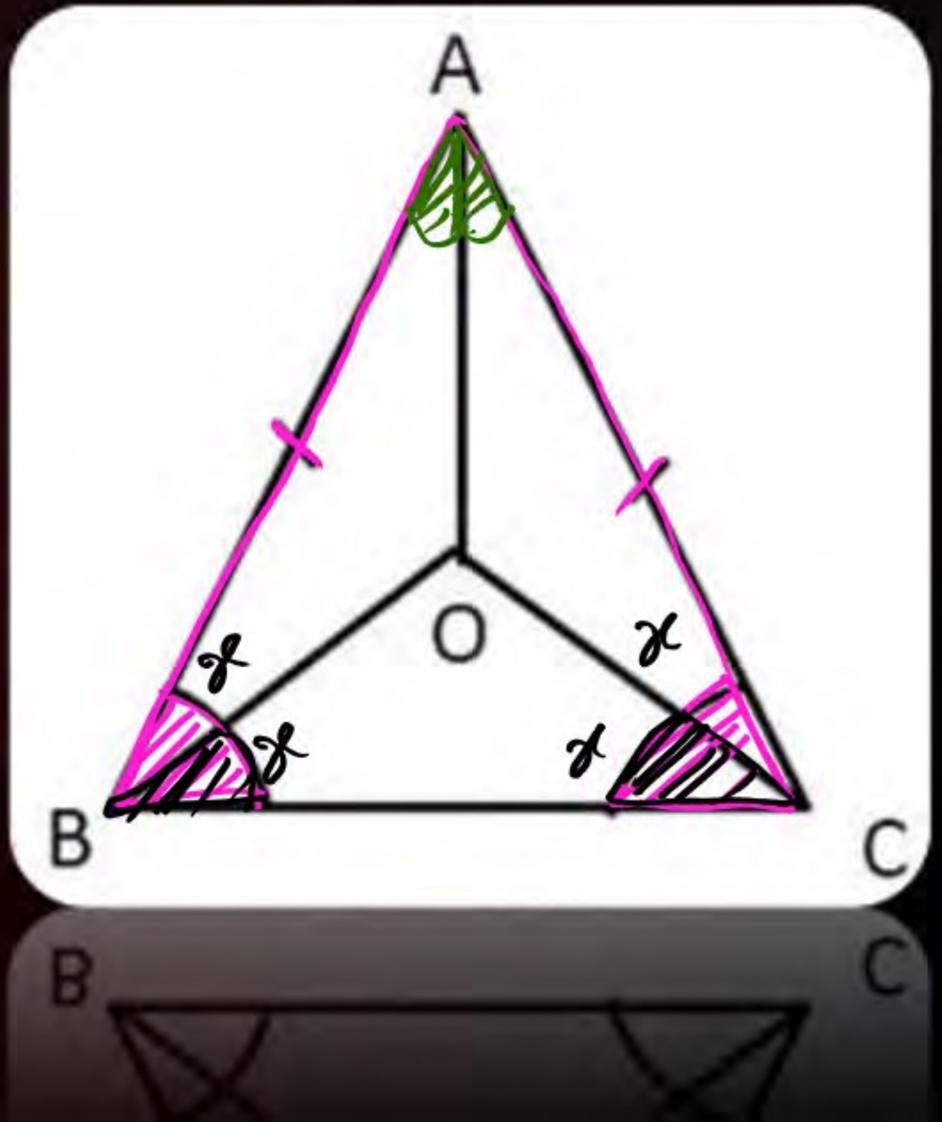
(i) since, ΔABC is isosceles Δ.

$AB = AC \Rightarrow \angle C = \angle B \Rightarrow \frac{\angle C}{2} = \frac{\angle B}{2}$

$\Rightarrow \boxed{\angle OCB = \angle OBC}$

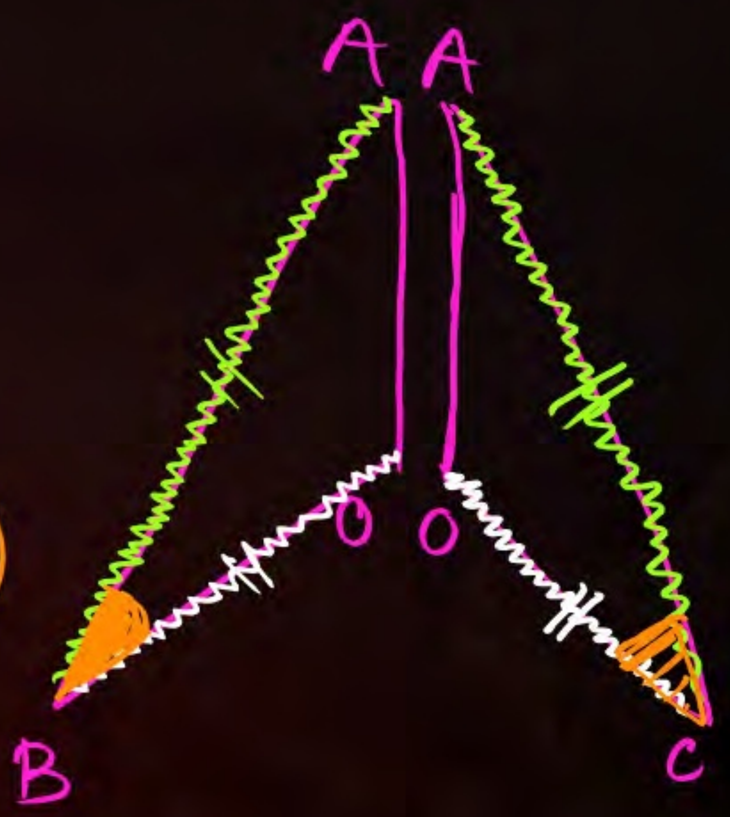
$\Rightarrow \boxed{OB = OC}$





(ii) since, $AB = AC \Rightarrow \angle B = \angle C \Rightarrow \frac{\angle B}{2} = \frac{\angle C}{2}$

$\Rightarrow \boxed{\angle ABO = \angle ACO}$



$$\left.\begin{array}{l} S \\ A \\ S \end{array}\right\{ \begin{array}{ll} AB = AC & \text{(given)} \\ \angle ABO = \angle ACO & \left(\frac{\angle B}{2} = \frac{\angle C}{2}\right) \\ OB = OC & \text{(proven)} \end{array}$$

using SAS rule,

$\boxed{\triangle ABO \cong \triangle ACO}$

By CPCT, $\boxed{\angle BAO = \angle CAO} \longrightarrow \boxed{OA \text{ bisect } \angle A}$

Question

In the given figure, if $\Delta ABC \cong \Delta DEF$, then

A $\angle A = \angle F$

B $\angle A = \angle B$

C $\angle B = \angle E$

D $\angle D = \angle E$

Question

In the given figure, if $\Delta ABC \cong \Delta DEF$, then

A $\angle A = \angle F$

B $\angle A = \angle B$

C $\angle B = \angle E$

D $\angle D = \angle E$

order of vertex

Question

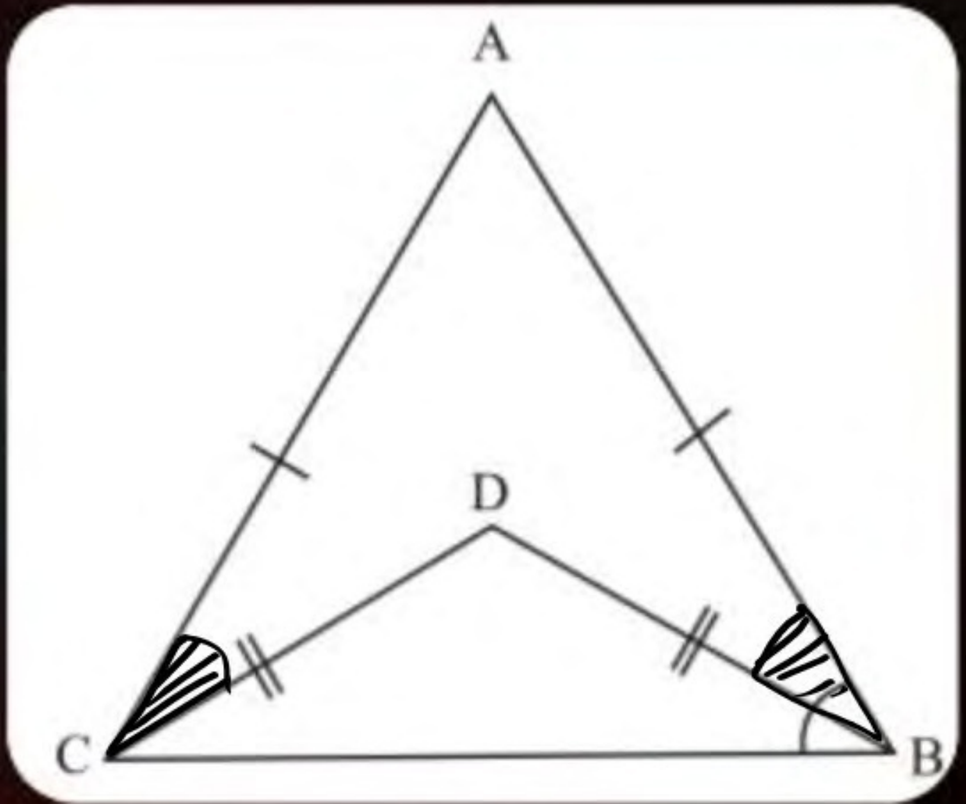
In the given figure, if AB = AC and CD = BD, then the ratio ∠ABD : ∠ACD is

2 : 1

1 : 2

1 : 1

3 : 4



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Question

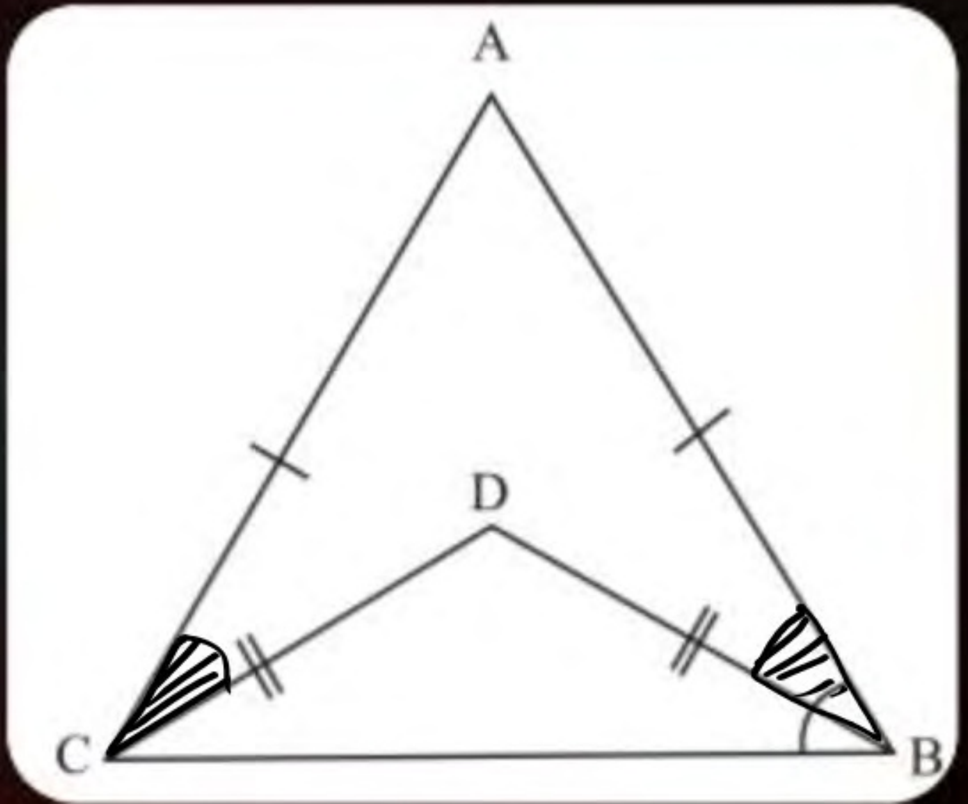
In the given figure, if AB = AC and CD = BD, then the ratio ∠ABD : ∠ACD is

A 2 : 1

B 1 : 2

C 1 : 1

D 3 : 4



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Side-Side-Side (SSS) Congruence Criterion

Theorem : Two triangles are congruent if there sides of one triangle are respectively equal to three sides of another triangle.

construction: $\boxed{AB = EG}$

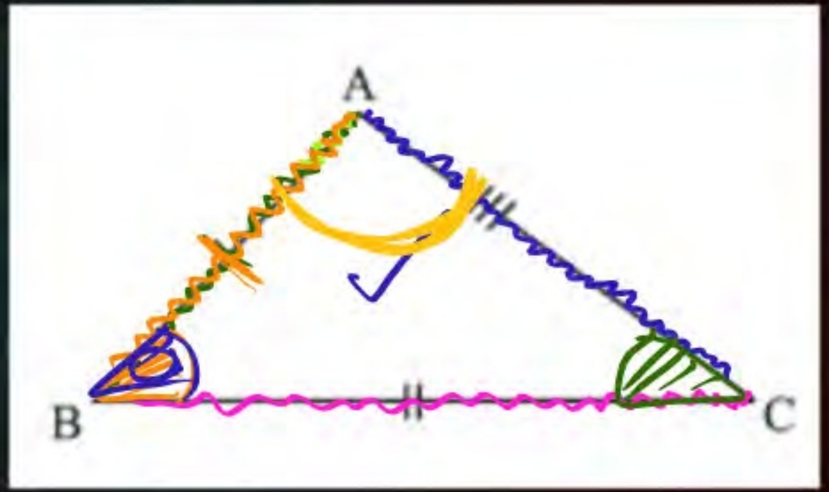
such that, $\boxed{\angle ABC = \angle GEF}$

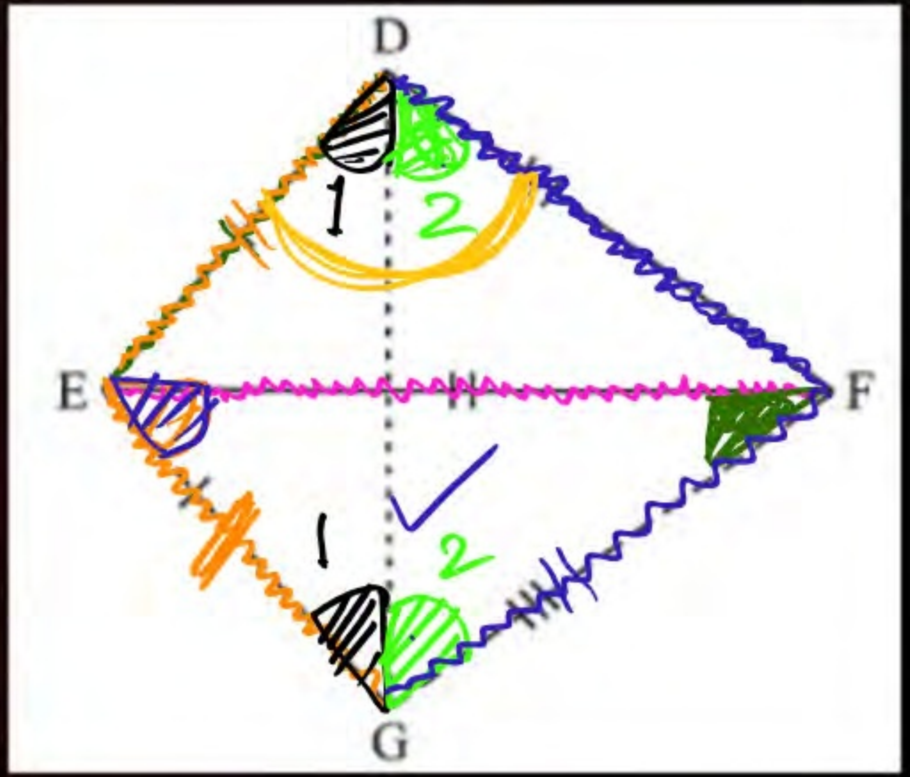
A/Q, $AB = \boxed{DE = EG}$

$\boxed{\angle EGD = \angle EDG}$

similarity,

$\Rightarrow \boxed{\angle G = \boxed{\angle D}} = \angle A$





$\boxed{\Delta ABC \cong GEF}$ SAS

$AC = \boxed{FG = DF}$

$\boxed{\Delta ABC \cong \Delta DEF}$

Question

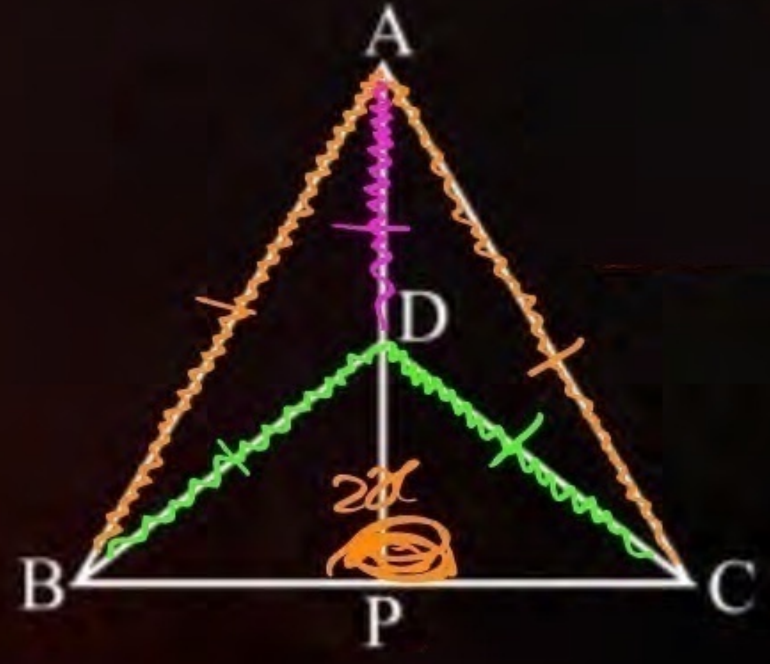
ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure given below). If AD is extended to intersect BC at P, show that

(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.



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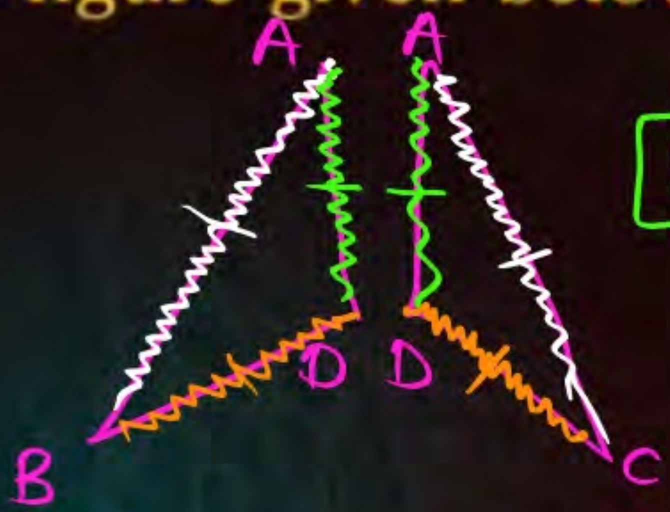
(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC.



$\boxed{S-S-S}$

$2x = 180°$

$\boxed{x = 90°}$

By CPCT,

$\boxed{\angle BAD = \angle CAD}$

$\boxed{\Delta ABP \cong \Delta ACP}$ $\boxed{SAS}$ $\Leftarrow$

iii

Right Angle-Hypotenuse-Side (RHS) Congruence Criterion

Theorem : Two right triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other triangle.

construction: $\boxed{AB = EG}$

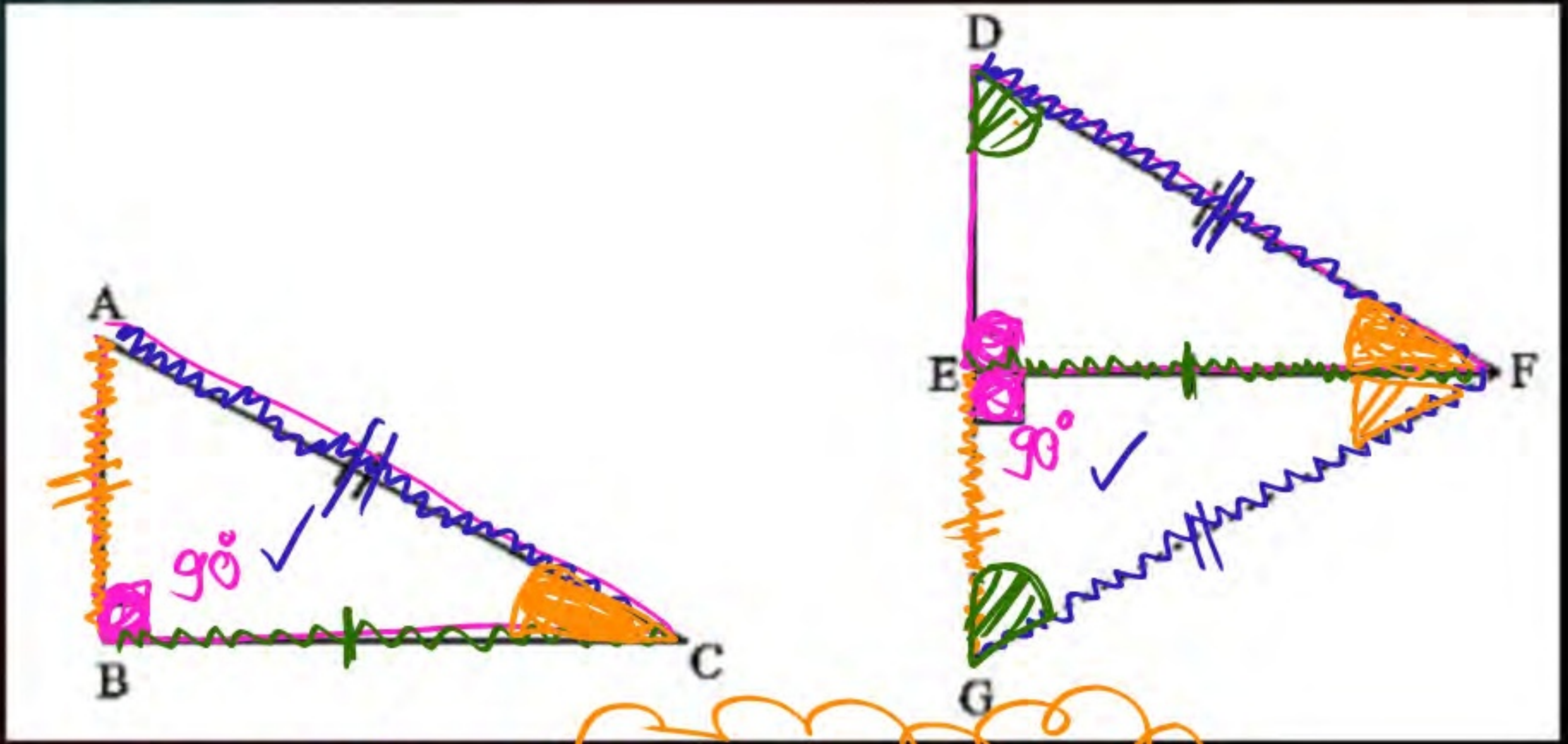
SAS rule

$\angle ABC = \angle GEF\ (90°)$

$BC = EF$

$\boxed{\triangle ABC \cong \triangle GEF}$

$\boxed{AC = GF}$



$\Rightarrow BC = EF, \angle BCA = \angle EFD, AC = DF \Rightarrow$ SAS rule $\Rightarrow \triangle ABC \cong \triangle DEF$

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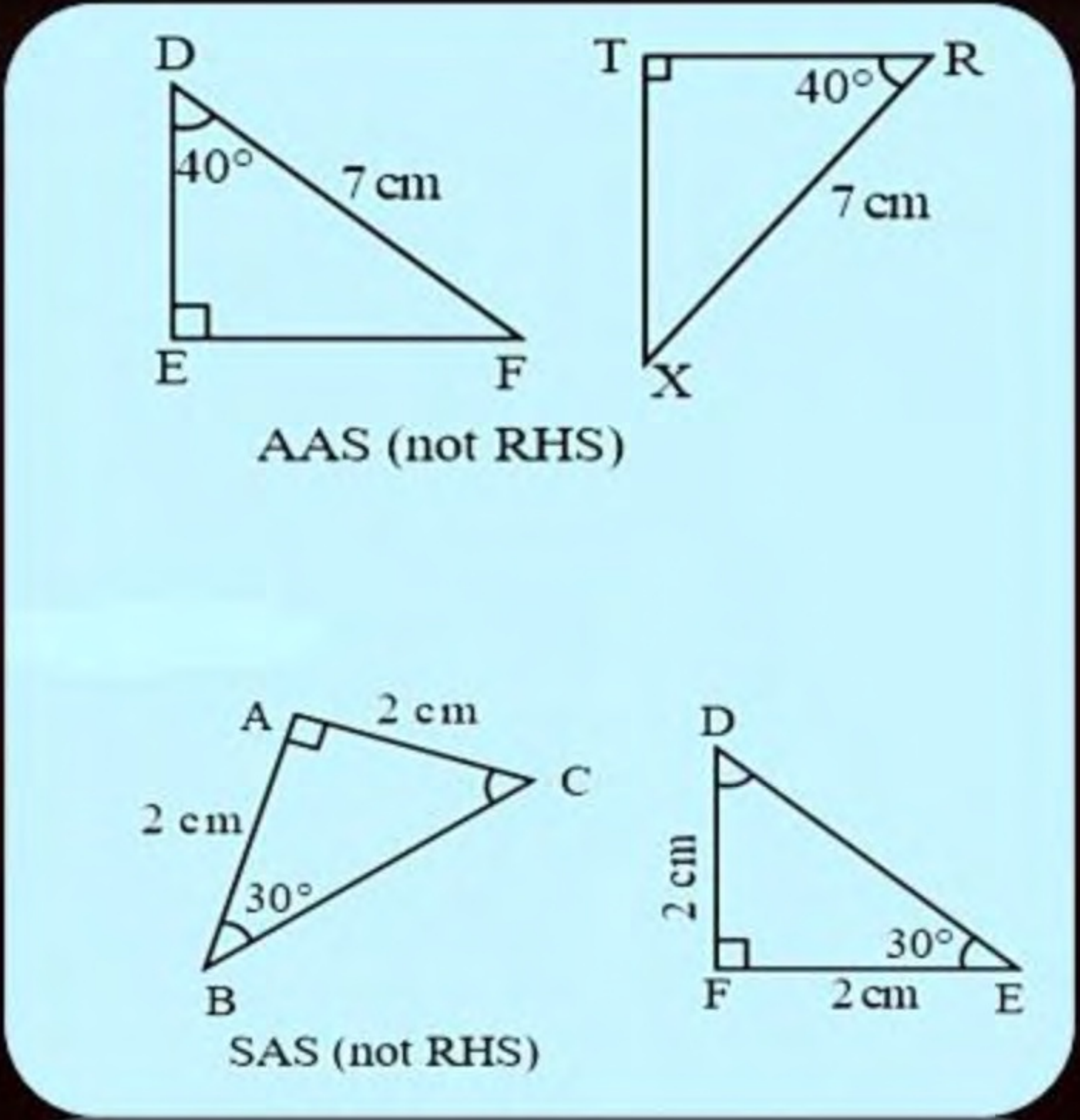
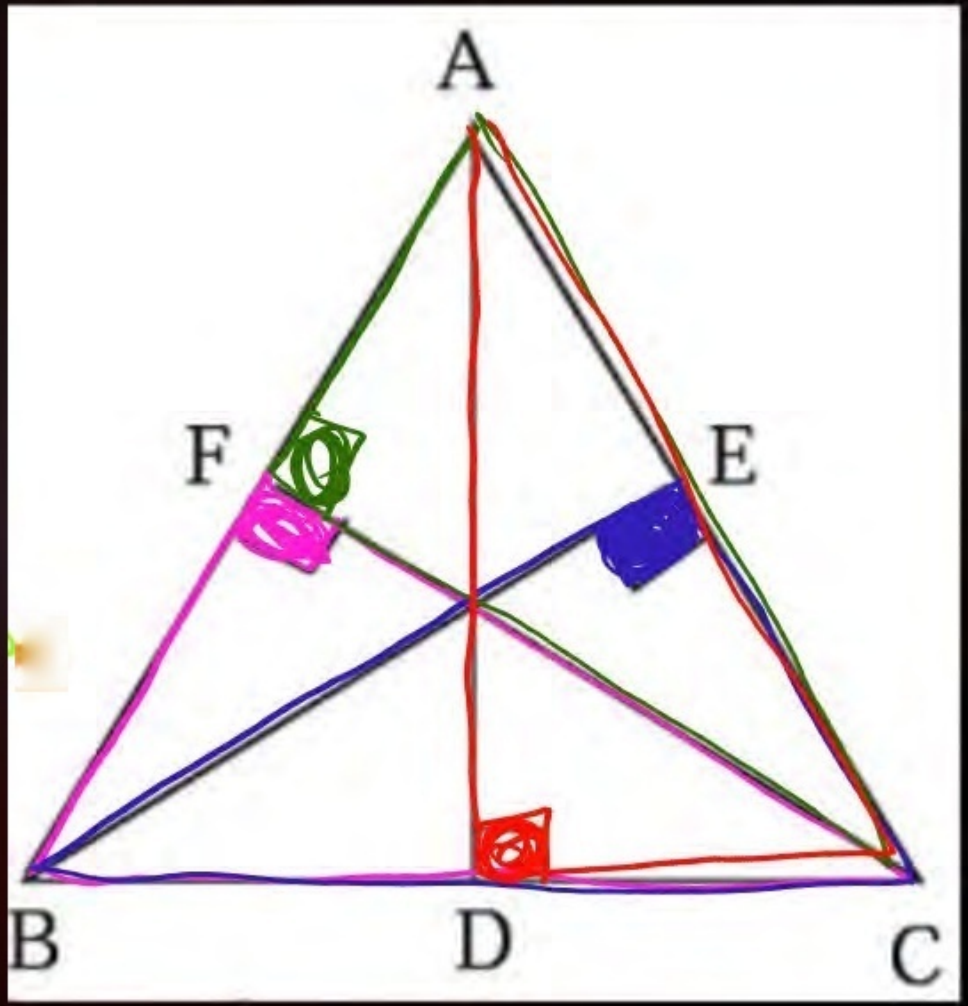
Some Area of Misconception



Question

AD, BE and CF, the altitudes of ΔABC are equal. Prove that ΔABC is an equilateral triangle.



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Question

AD, BE and CF, the altitudes of ΔABC are equal. Prove that ΔABC is an equilateral triangle.

$\boxed{AD = BE = CF}$

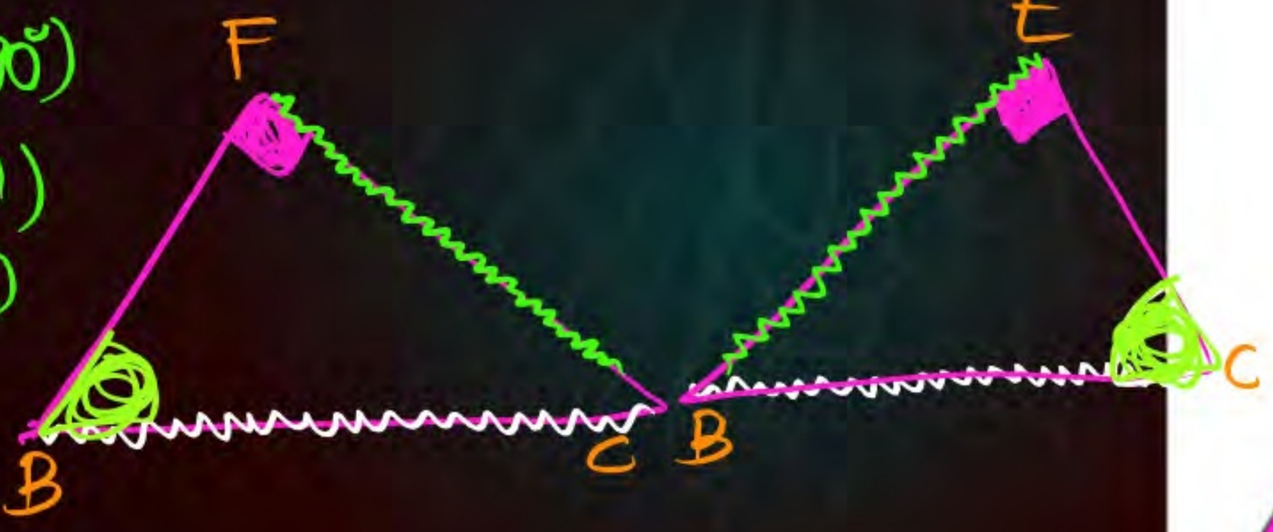
R $\angle BFC = \angle CEB (90^\circ)$

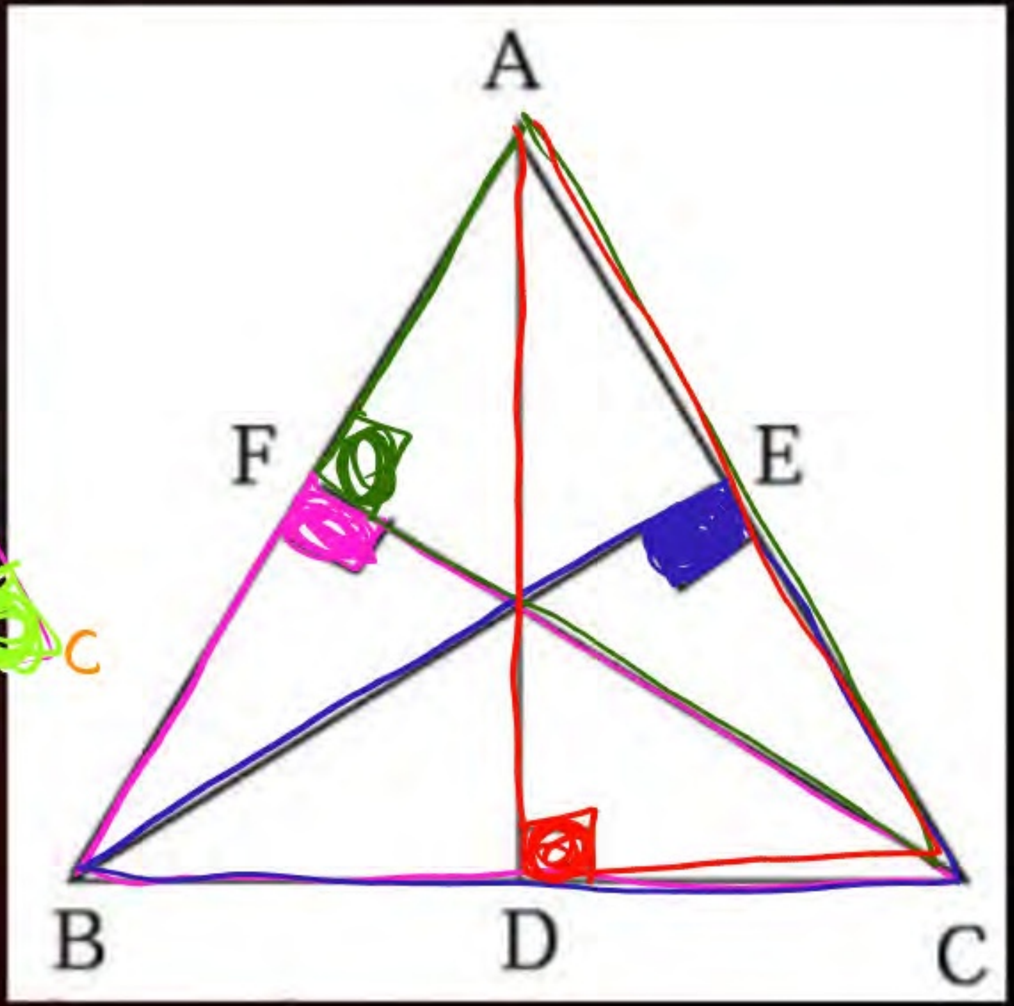
H $BC = CB$ (common)

S $CF = BE$ (Altitudes)

$\boxed{\Delta FBC \cong \Delta ECB}$

By, $\boxed{\angle B = \angle C}$





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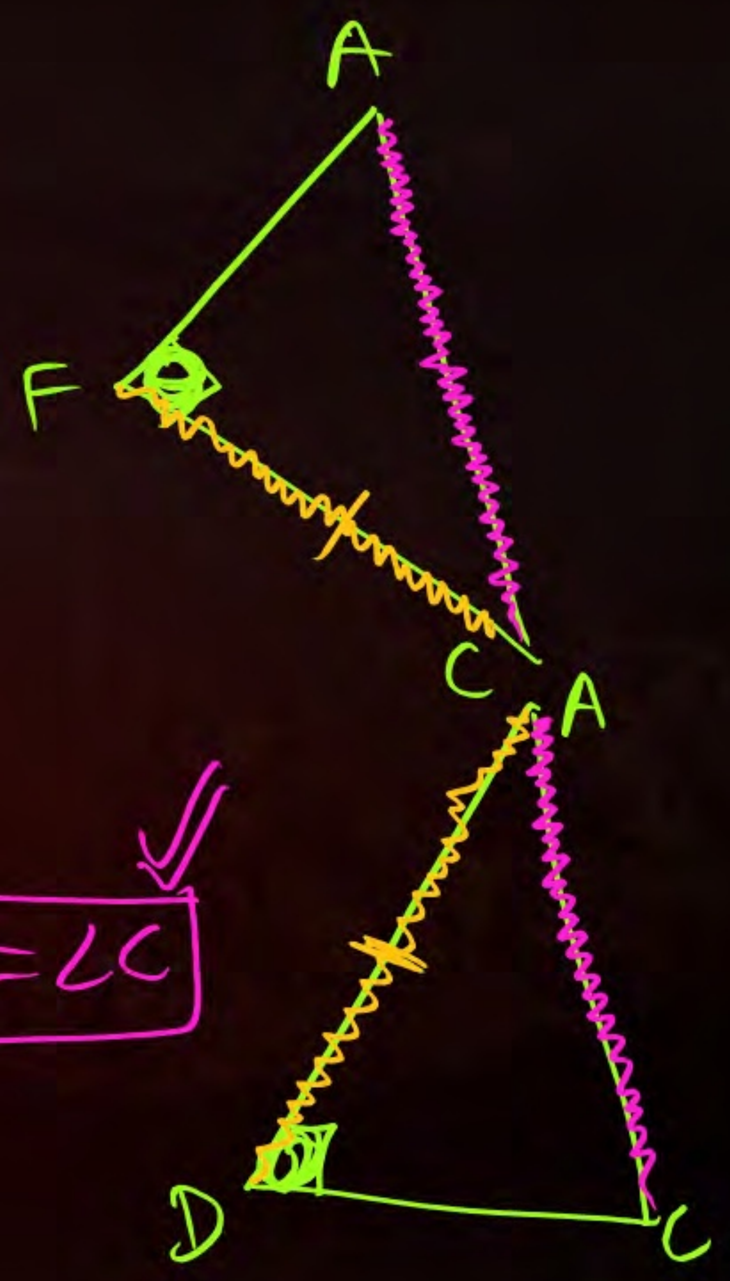
$$\left.\begin{array}{l} R \\ H \\ S \end{array}\right\{ \begin{array}{l} \angle AFC = \angle CDA \\ AC = CA \\ CF = AD \end{array}$$

$$\boxed{\Delta FCA \cong \Delta DAC}$$

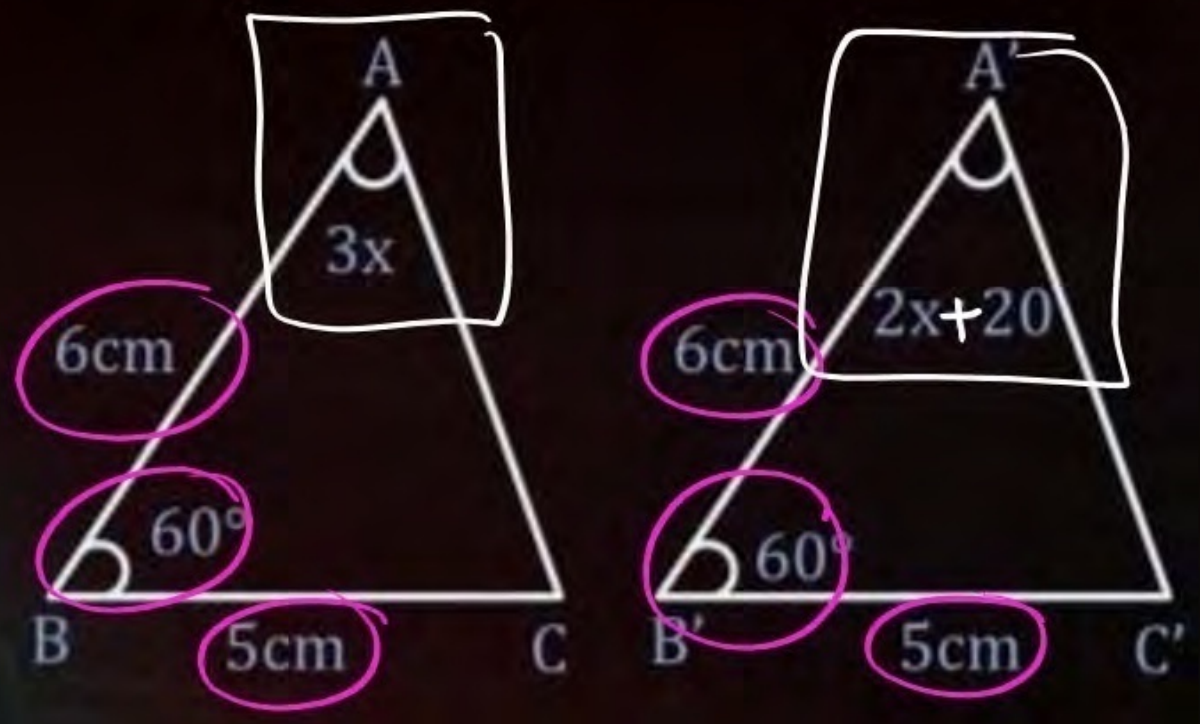
$\hookrightarrow$ By CPCT, $\boxed{\angle A = \angle C}$ ✓

$\Rightarrow \boxed{\angle A = \angle B = \angle C} \rightarrow$ Equilateral Δ



Question

In fig. the measure of $\angle B'A'C'$ is



50°

60°

70°

80°

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Question

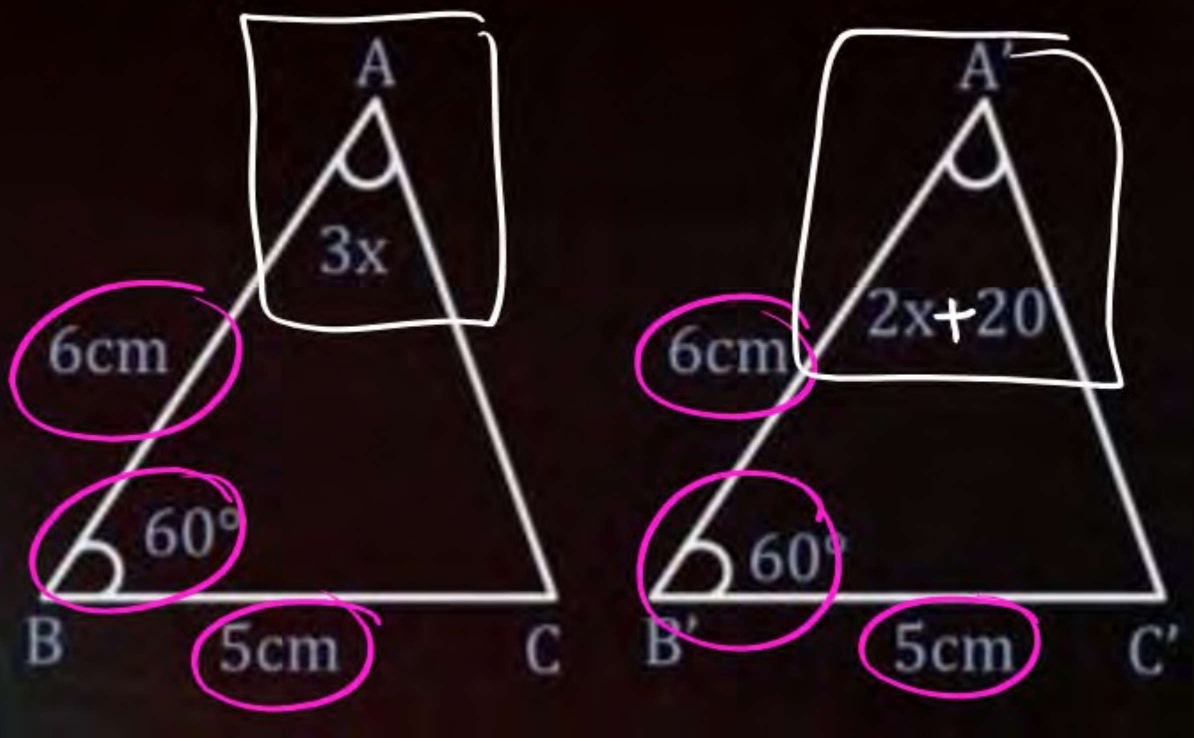
In fig. the measure of $\angle B'A'C'$ is

A $50°$

B $60°$

C $70°$

D $80°$



SAS rule

$3x = 2x + 20$

$\boxed{x = 20°}$

$\angle B'A'C' = 2x + 20°$

$= (2 \times 20°) + 20° = \boxed{60°}$

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THANK YOU
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