VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

- Subject - Mathematics
- Chapter - Euclid's Geometry

Today's Targets

1. Geometry and History behind Euclid's Geometry ✓
2. Statements, Euclid's Definitions, Axions, Postulates, Theorems and Corollary ✓
3. Euclid's Axioms and Postulates with Proper demonstrations ✓
4. Some Undefined Terms, Incident Axioms, Some terms related to Geometry ✓
5. Kuch Swag Vale Extra Bhaukali Sawal ✓

What Is Geometry?

The word ‘Geometry’ comes from the Greek words ‘Geo’ meaning the ‘earth’ and ‘Meterin’ meaning ‘to measure’. Geometry appears to have originated from the need for measuring lands. This branch of Mathematics was studied in various forms in ancient civilization.

Euclid's Geometry was introduced by the Greek mathematician Euclid, where Euclid defined a basic set of rules and theorems for a proper study of geometry.

History Behind Introduction to Euclid’s Geometry

The geometry developed by Egyptians mainly consisted of the statements of results. There were no general rules of the procedure. A Greek mathematician, Thales is credited with giving the first known proof. This proof was of the statement that a circle is bisected (i.e., cut into two equal parts) by its diameter. One of Thales’ most famous pupils was Pythagoras (572 BCE), whom you have heard about. Pythagoras and his group discovered many geometric properties and developed the theory of geometry to a great extent. This process continued till 300 BCE. At that time Euclid, a teacher of mathematics at Alexandria in Egypt, collected all the known work and arranged it in his famous treatise, called ‘Elements’. He divided the ‘Elements’ into thirteen chapters, each called a book.

Statements

A sentence which can be judged to be true or false is called a statements

Examples

(i) The sum of the angles of a triangle is $180°$, is a true statement.

(ii) The sum of the angles of a quadrilateral is $180°$, is a false statement.

(iii) $\boxed{x + 10 > 15}$ is a sentence but not a statement.

Euclide's Definitions

A solid has shape, size, position, and can be moved from one place to another. Its boundaries are called surfaces. They separate one part of the space from another, and are said to have no thickness. The boundaries of the surfaces are curves or straight lines. These lines end in points.

Euclide's Definitions

1) A point is that which has no part.

2) A line is breadthless length.

3) The ends of a line are points.

4) A straight line is a line which lies evenly with the points on itself.

5) A surface is that which has length and breadth only.

6) The edges of a surface are lines.

7) A plane surface is a surface which lies evenly with the straight lines on itself.

These definitions are not complete and required further assumptions due to which they were not accepted by the other Mathematicians. These geometric terms which were not completely defined were termed as undefined terms.

Axioms or postulates

Axioms or Postulates are assumptions that are obvious universal truths. They are not proven. Euclid axioms are the assumptions that are used throughout mathematics while Euclid Postulates are the assumptions that are specific to geometry.

Self-evident true statements used throughout mathematics and not specifically linked to geometry are called *axioms* while those specific to geometry are known as *postulates*. However, nowadays, we do not distinguish between axioms and postulates.

Euclid's axioms

(1) Things which are equal to the same thing are equal to one another.

(2) If equals are added to equals, the wholes are equal.

(3) If equals are subtracted from equals, the remainders are equal.

(4) Things which coincide with one another are equal to one another.

(5) The whole is greater than the part.

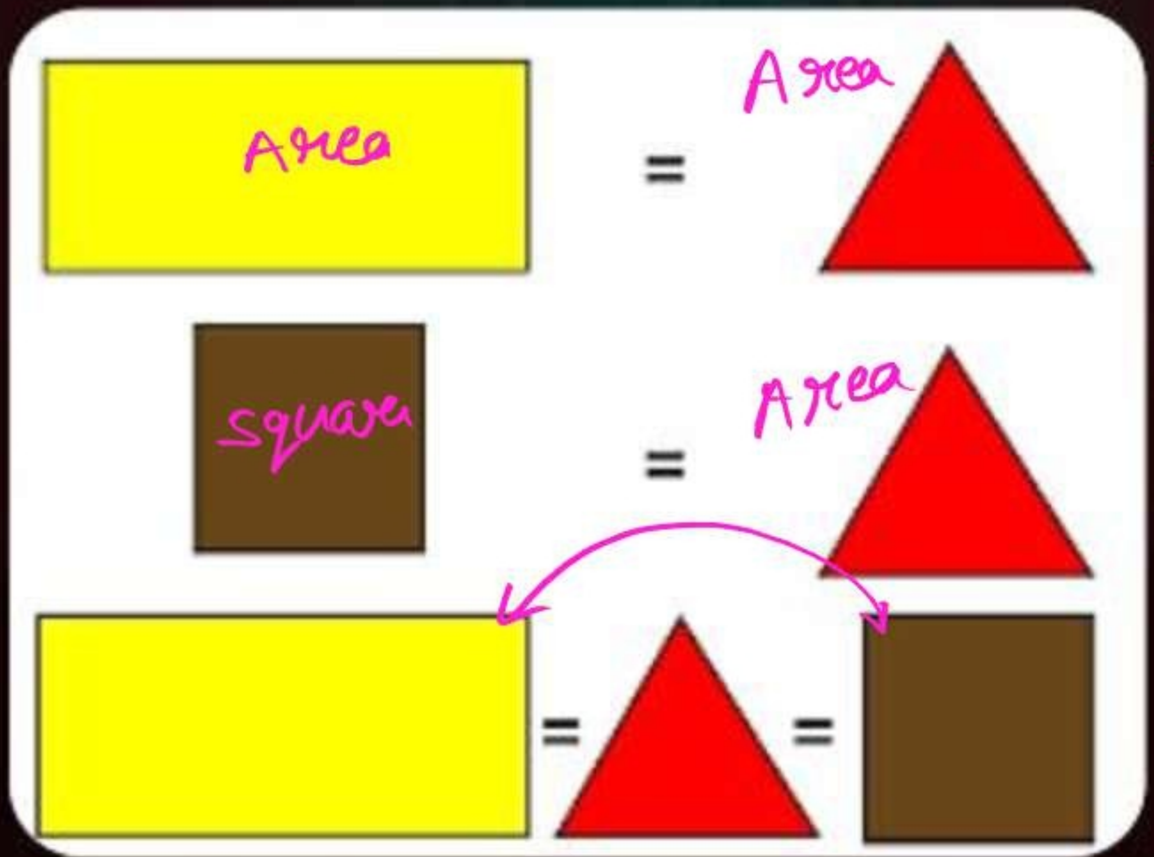
(6) Things which are double of the same things are equal to one another.

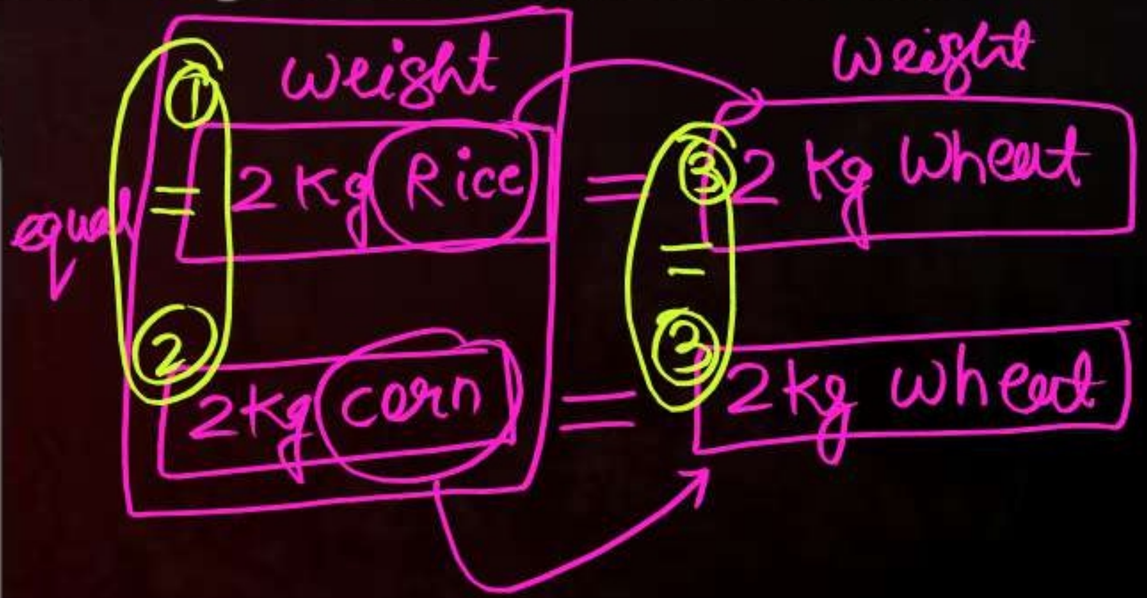
(7) Things which are halves of the same things are equal to one another

Euclid's axioms (1)

The things which are equal to the same thing are equal to one another.

If an area of a triangle equals the area of a rectangle and the area of the rectangle equals that of a square, then the area of the triangle also equals the area of the square.





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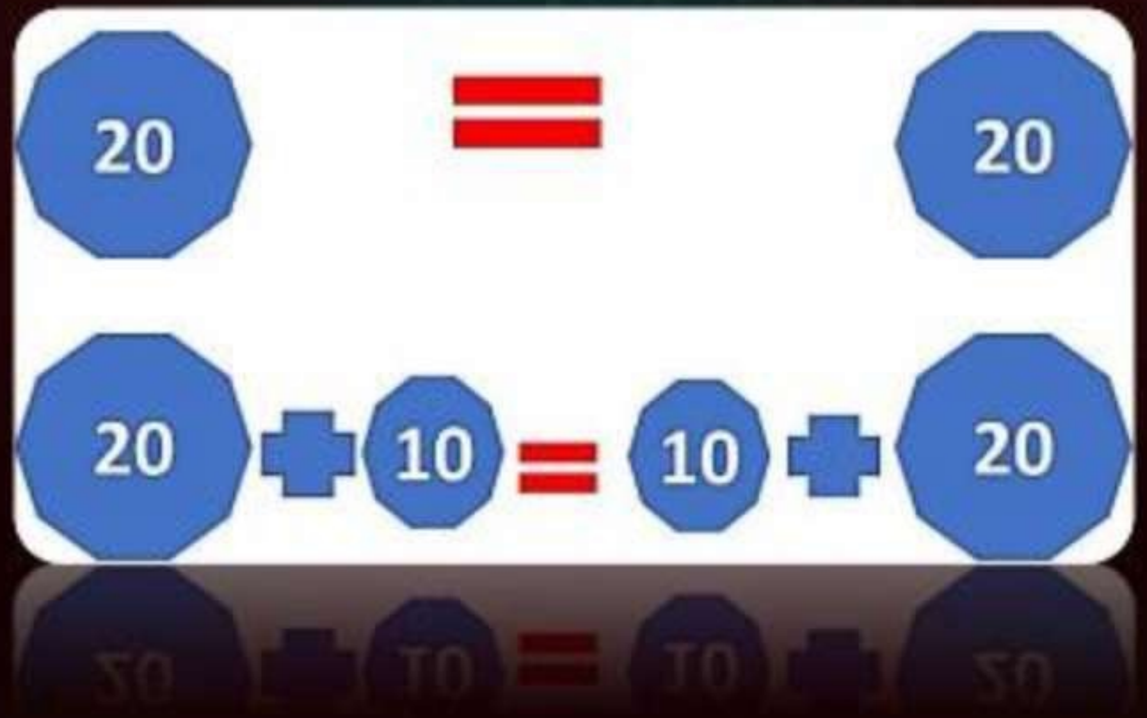
Euclid's axioms (2)

If equals are added to equals, the wholes are equal.

Let us look at the line segment AB, where AP = QB. When PQ is added to both sides, then according to axiom 2, AP + PQ = QB + PQ i.e AQ = PB.



You | = | Your's Sister
₹10 | | ₹10
₹5 | | ₹5

₹15 = ₹15

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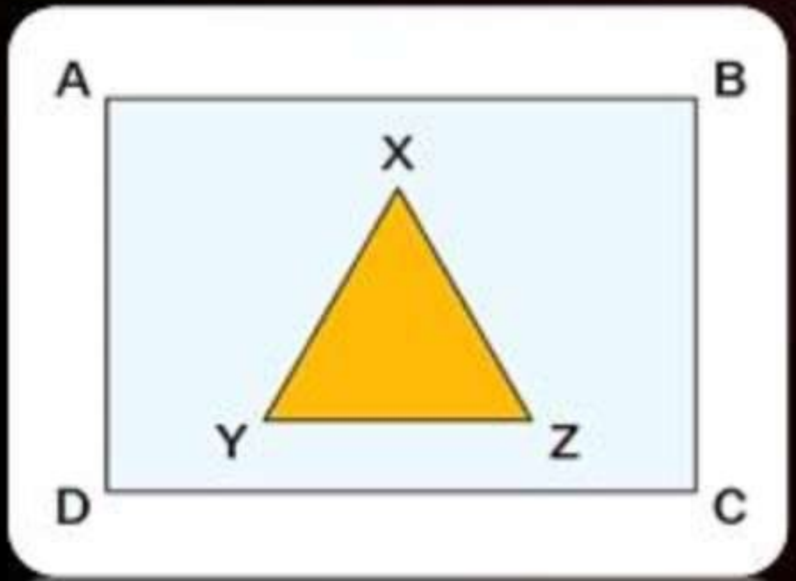
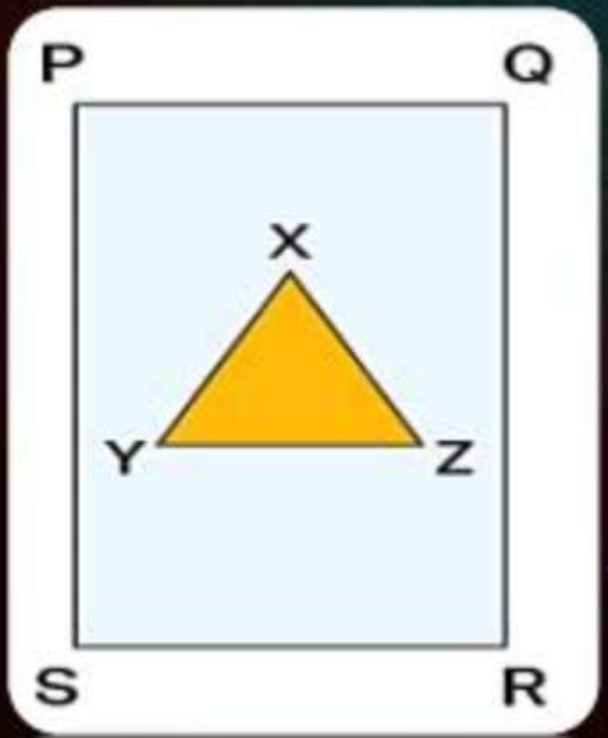
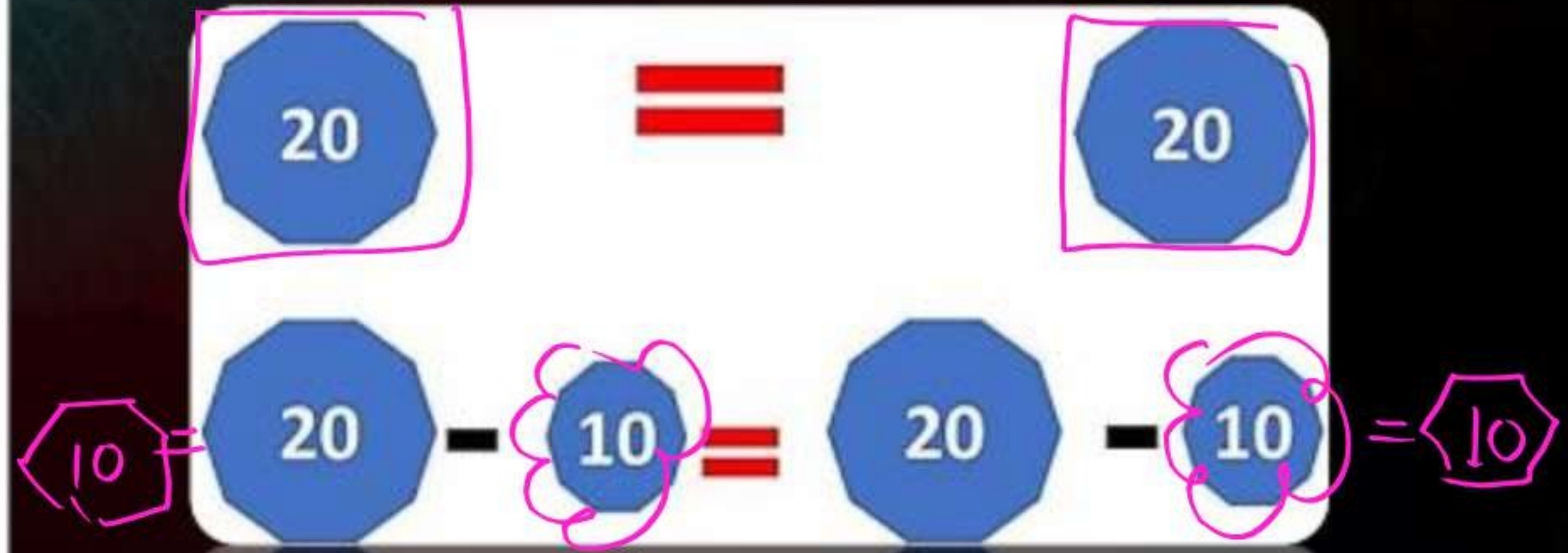
Euclid's axioms (3)

If equals are subtracted from equals, the remainders are equal.

Consider rectangles ABCD and PQRS, where the areas are equal. If the triangle XYZ is removed from both the rectangles then according to axiom 3, the areas of the remaining portions of the two triangles are equal.



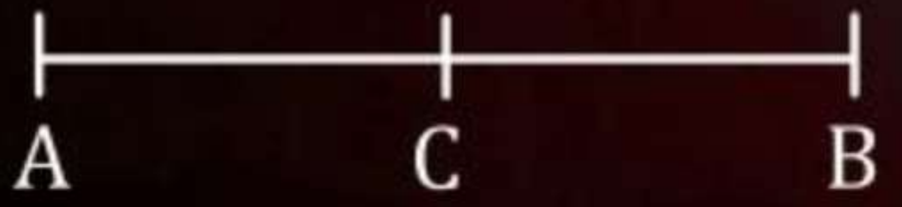




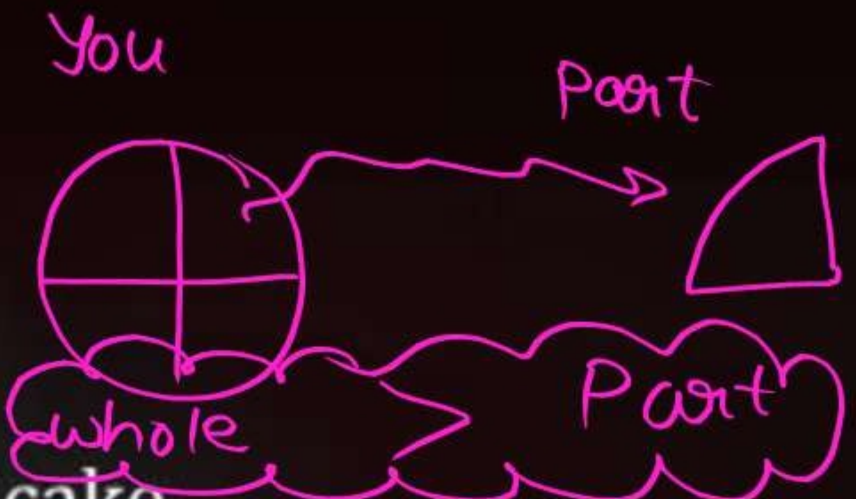
Euclid's axioms (4)

Things which coincide with one another are equal to one another.

Consider line segment AB with C in the center. AC + CB coincides with the line segment AB. Thus by axiom 4, we can say that AC + CB = AB.

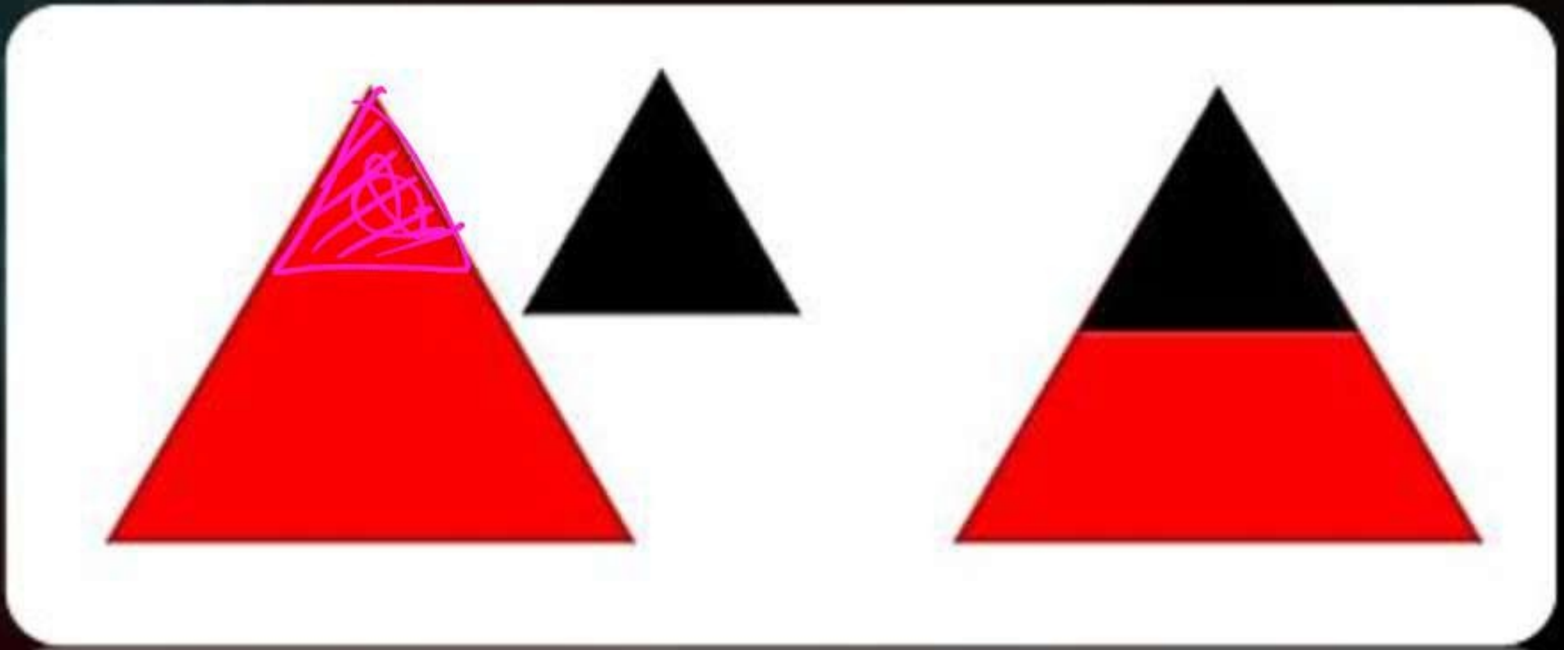


Euclid's axioms (5)



The whole is greater than the part.

A whole cake is greater than slice (a part) of cake.



Euclid's axioms (6) & (7)

same

20cm 20cm

Things which are double of the same thing are equal to one another.

Things which are halves of the same thing are equal to one another.

Consider two identical circles with radii r_1 and r_2 with diameters as d_1 and d_2 respectively. Since the circles are identical, using both axioms 6 and 7, we can say that $r_1 = r_2$ and $d_1 = d_2$.

Question

Euclid's second axiom (as per order given in the Textbook for Class IX) is

The things which are equal to the same thing are equal to one another.

If equals be added to equals, the wholes are equal.

If equals be subtracted from equals, the remainders are equals.

Things which coincide with one another are equal to one another.

Question

Euclid's second axiom (as per order given in the Textbook for Class IX) is

$\rightarrow$ I[st]

A The things which are equal to the same thing are equal to one another.

second

B $\boxed{\text{If equals be added to equals, the wholes are equal.}}$ ✓

₹10 = ₹10
+ ₹5 = ₹5 +
$\boxed{₹15 = ₹15}$

$\rightarrow$ III

C If equals be subtracted from equals, the remainders are equals.

D Things which coincide with one another are equal to one another.

Question

Axioms are assumed

- **A** Definition
- **B** Theorems
- **C** Universal truths specific to geometry
- **D** Universal truths in all branch of mathematics.

Question

Axioms are assumed

A. Definition

B. Theorems

C. Universal truths specific to geometry

D. $\boxed{\text{Universal truths in all branch of mathematics.}}$ ✓

Question

Boundaries of solids are

Surfaces

Curves

Lines

Points

Question

Boundaries of solids are

A Surfaces

B Curves

C Lines

D Points

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Question

A point C is called the midpoint of a line segment $\overline{AB}$ if

C is an interior point of AB

$\overline{AC} = \overline{CB}$

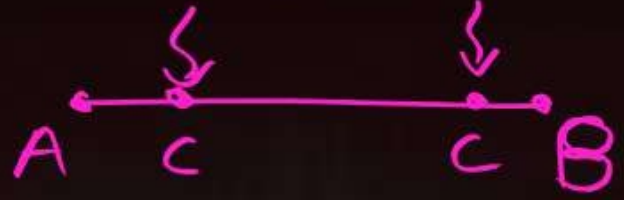
C is an interior point of AB such that $\overline{AC} = \overline{CB}$

AC + CB = AB

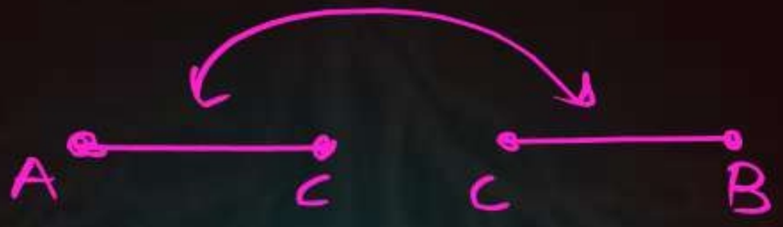
Question

A point C is called the midpoint of a line segment $\overline{AB}$ if

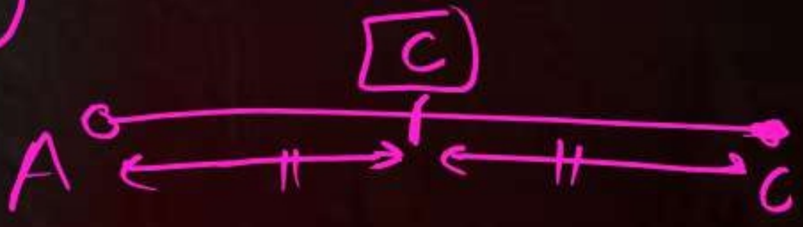
A C is an interior point of AB



B $\overline{AC} = \overline{CB}$



C C is an interior point of AB such that $\overline{AC} = \overline{CB}$



D AC + CB = AB

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Question

A is of the same age as B and C is of the same age as B. Euclid's which axiom illustrates the relative ages of A and C?

First Axioms

Second Axioms

Third Axioms

Fourth Axioms

Question

A is of the same age as B and C is of the same age as B. Euclid's which axiom illustrates the relative ages of A and C?

(Imp.)

A First Axioms ✓

B Second Axioms

C Third Axioms

D Fourth Axioms

Age of person A = Age of person B

equal

Age of person C = Age of person B

Question

It is known that if $x + y = 10$ then $\boxed{x + y + z = 10 + z.}$ The Euclid's axiom that illustrates this statement is :

First Axioms

Second Axioms

Third Axioms

Fourth Axioms

Question

It is known that if $x + y = 10$ then $\boxed{x + y + z = 10 + z}$. The Euclid's axiom that illustrates this statement is :

A First Axioms

B Second Axioms

C Third Axioms

D Fourth Axioms

We have,

$$x + y = 10$$
$$+z = +z$$
$$(x+y)+z = 10+z$$
$$\Rightarrow \boxed{x+y+z = 10+z}$$

If equals are added to equals the whole are equals

Hence, proved!!

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Question

A point C is said to lie between the points A and B if

$AC = CB$

$AC + CB = AB$

points A, C and B are collinear

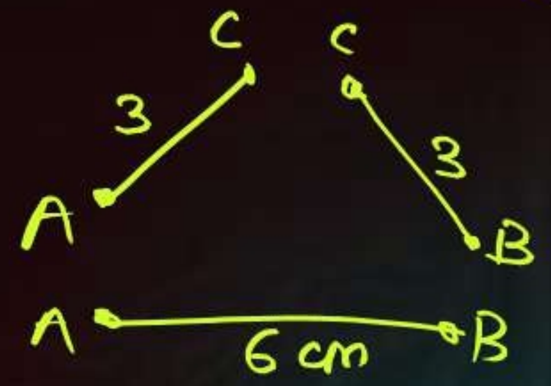
None of these

Question

A point C is said to lie between the points A and B if

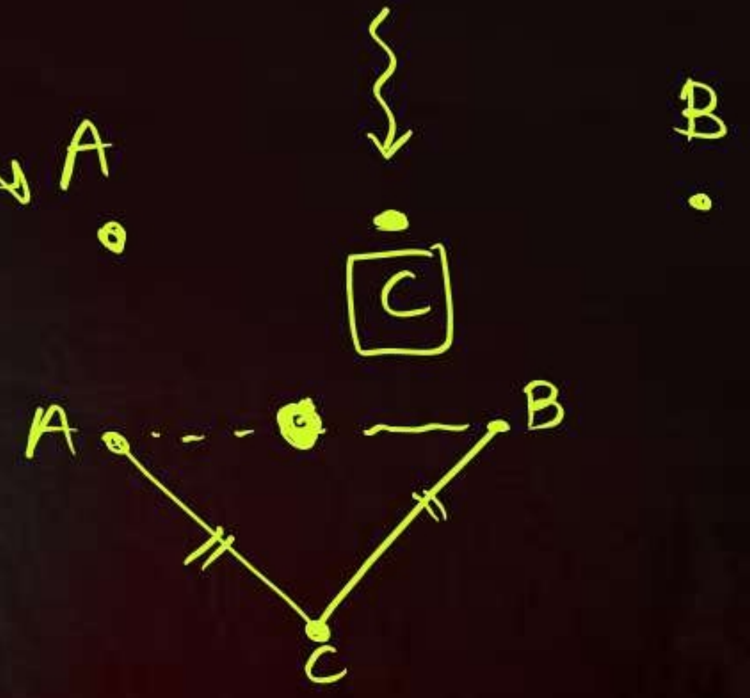
A) AC = CB

B) AC + CB = AB



C) points A, C and B are collinear

D) None of these



Question

Euclid's which axiom illustrates the statement that when $x + y = 15$, then $x + y + z = 15 + z$?

First

Second

Third

Fourth

Question

Euclid's which axiom illustrates the statement that when $x + y = 15$, then $x + y + z = 15 + z$?

A. First

B. Second

C. Third

D. Fourth

equal

$\boxed{x+y = 15}$

$+$

$\boxed{z = z}$

equal

Add $\Rightarrow$

whole $\quad\quad$ whole

$\{x+y+z\} = \{15+z\}$

Question

If a point C lies between two points A and B such that $\boxed{AC = BC,}$ then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

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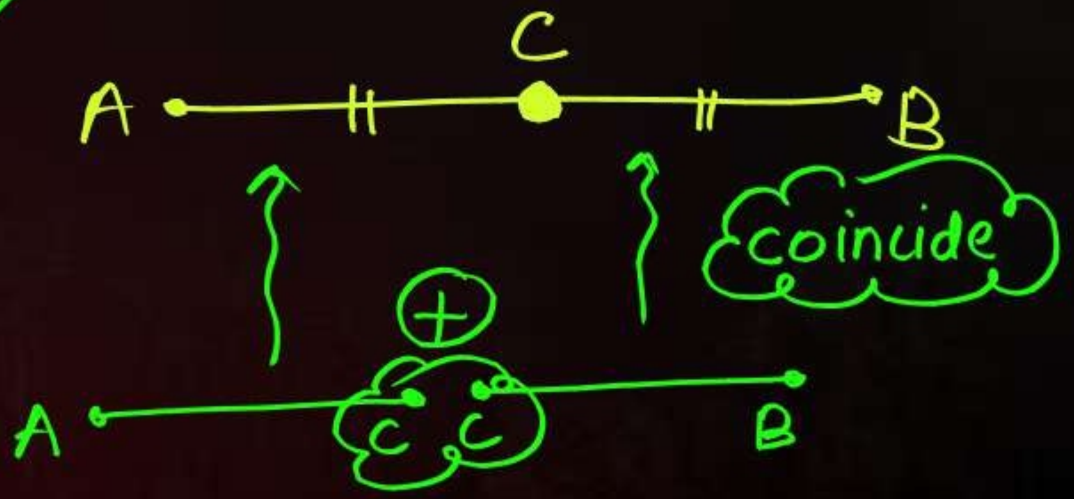


Since, we have C is between A & B such that AC = CB, then C must be mid point of AB.

Therefore,

using Axiom 4

$$AC + CB = AB$$
$$AC + AC = AB$$
$$2AC = AB$$
$$\boxed{AC = \frac{1}{2}AB}$$

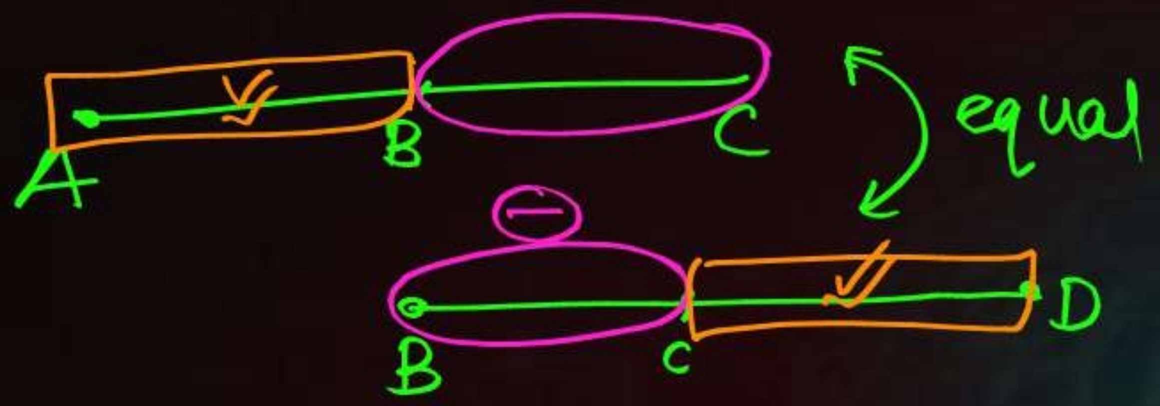
Hence, proved!!

Question

In fig. if AC = BD, then prove that AB = CD.

Question

In fig. if AC = BD, then prove that AB = CD.



Using Axiom 3
If equals are subtracted from equals then remainders are equals

$AC - BC = BD - BC$

$\boxed{AB = CD}$

Hence, proved!!

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Question

Ram and Ravi have the same weight. If they each gain weight by 2 kg, how will their new weights be compared ?

Question

Ram and Ravi have the same weight. If they each gain weight by 2 kg, how will their new weights be compared ?

weight of Ram = weight of Ravi

\+ \+

gain of weight 2kg = gain of weight 2 kg

$(W_{Ram} + 2)$ kg = $(W_{Ravi} + 2)$ kg

Using 2^{nd} axiom

If equals are added to equals then whole are equals

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Question

John is of the same age as Mohan. Ram is also of the same age as Mohan. State the Euclid's axiom that illustrates the relative ages of John and Ram

Question

John is of the same age as Mohan. Ram is also of the same age as Mohan. State the Euclid's axiom that illustrates the relative ages of John and Ram

equal

Age of John = Age of Mohan

Age of Ram = Age of Mohan

same

using $\boxed{\text{Axiom 1}}$

Things which are equal to same things are equal to one-another.

$\boxed{\text{Age of John} = \text{Age of Ram}}$ ✓

Question

In the given figure, we have $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Show that $\angle A = \angle C$.

Question

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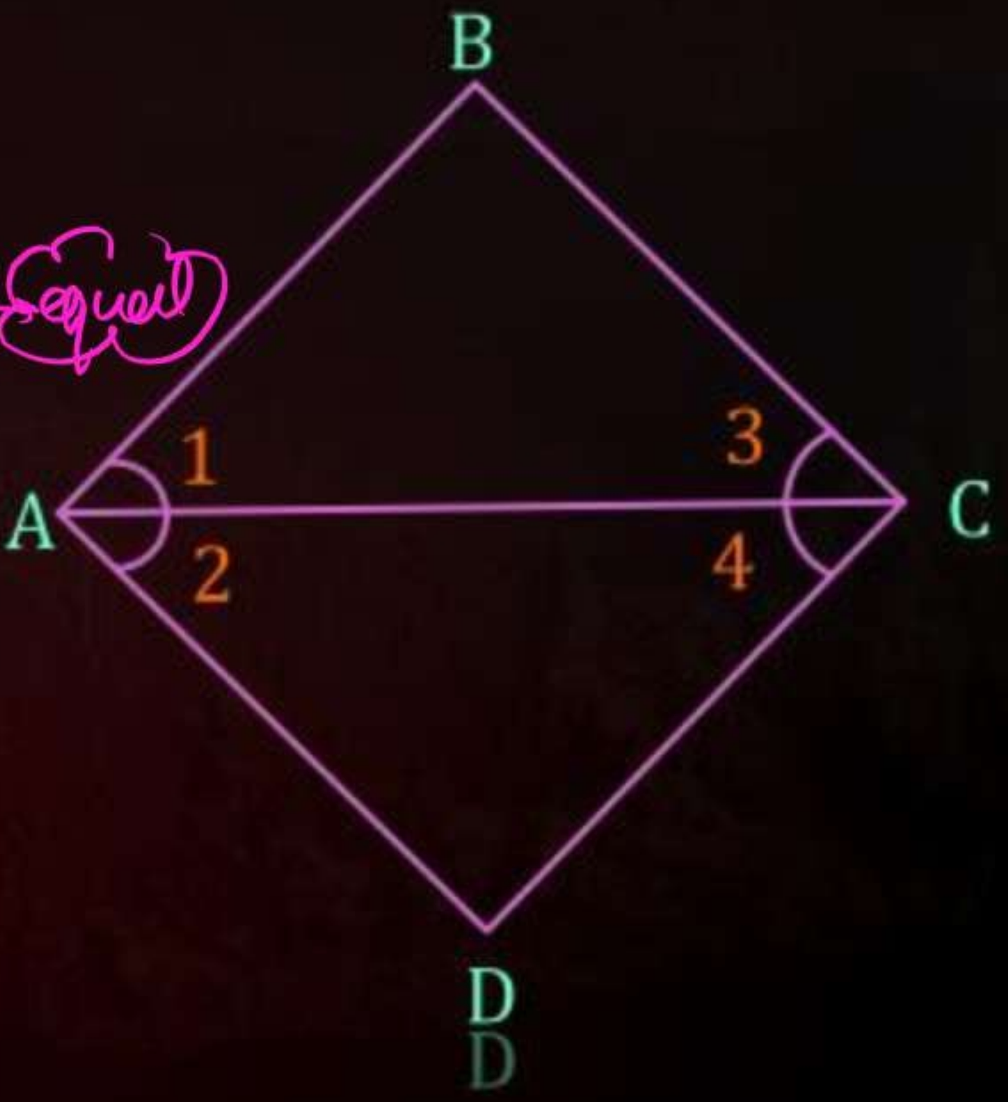


We have,

Axiom ②

$\boxed{\angle 1 = \angle 3}$ equals

Add $\Rightarrow$ whole equal

and $\boxed{\angle 2 = \angle 4}$ equals

$\boxed{\angle A = \angle C}$

Hence, proved !!

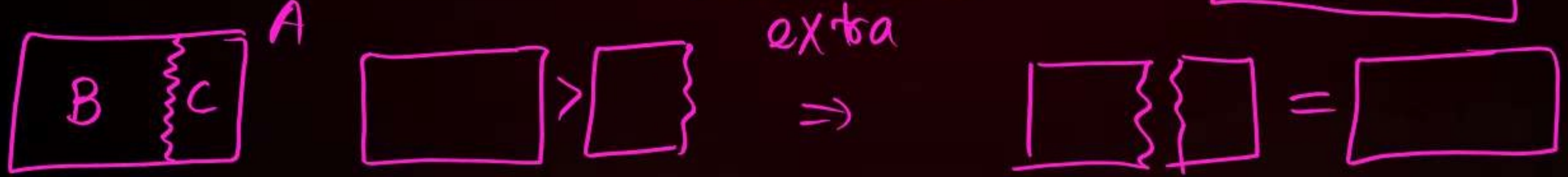
ILLUSTRATIONS

1. If the area of a triangle equals the area of a rectangle and the area of the rectangle equals that of a square, then the area of the triangle also equals the area of the square. → Axiom ①

2. Magnitudes of the same kind can be compared and added (or subtracted). But, magnitudes of different kind cannot be compared. → Axiom ② & ③

3. Everything equals itself. Axiom ④

4. If A > B, then there exists a quantity C such that A = B + C. → Axiom ⑤



Euclid's Postulate

Postulates in geometry is very similar to axioms, self-evident truths, and beliefs in logic, political philosophy, and personal decision-making. The five postulates of Euclidean Geometry define the basic rules governing the creation and extension of geometric figures with ruler and compass.

Euclid's Postulate

Postulates: The basic facts which are taken, for granted, without proof and which are specific to geometry are called postulates.

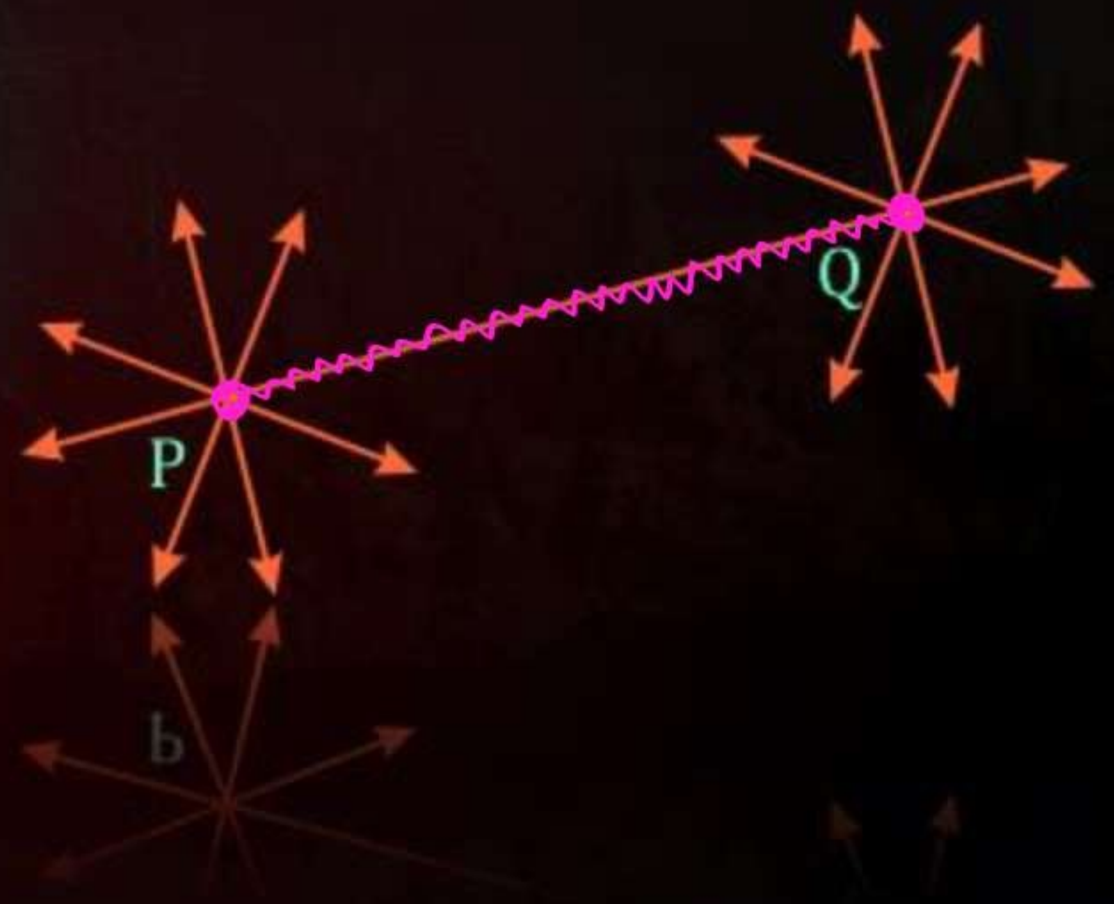
Axioms: The basic facts which are taken for granted, without proof and which are used throughout in the mathematics are called axioms.

Theorem: The conclusions obtained through logical reasoning based on previously proved results and some axioms constitute a statement known as a theorem or a proposition.

Euclid's Postulate (1)

POSTULATE 1: A straight line may be drawn from any one point to any other point.

Note that, this postulate tells us that at least one straight line passes through two distinct points, but it does not say that there cannot be more than one such line(Euclid assumed that there is a unique line joining two distinct points). However, in his work, Euclid has frequently assumed, without mentioning, that there is a unique line joining two distinct points. We state this result in the form of an axiom onwards.



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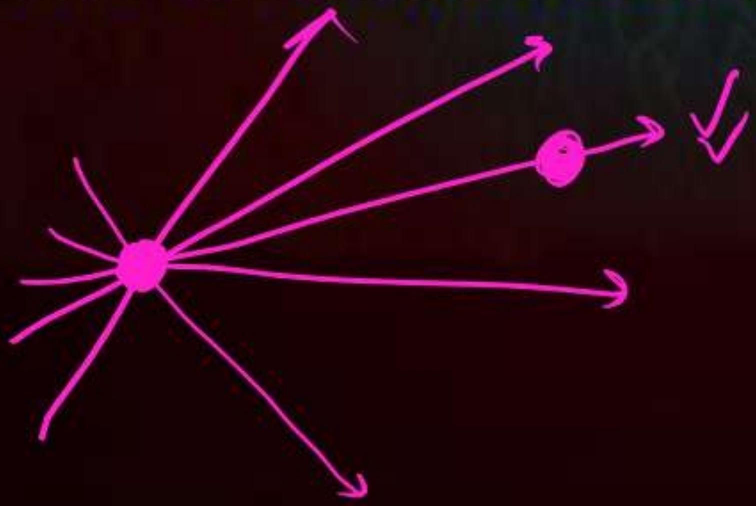
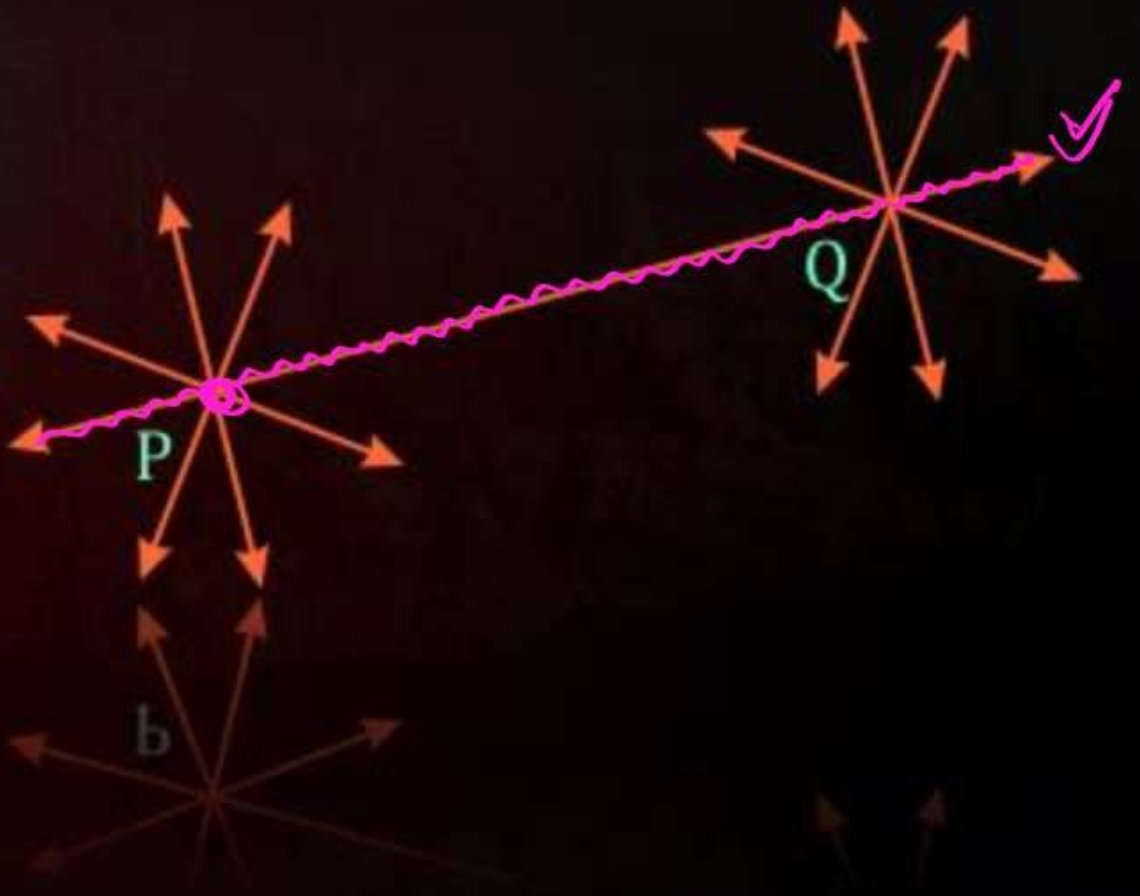
Euclid's Axioms (5.1)

Axiom 5.1 : Given two distinct points, there is a unique line that passes through them.

How many lines passing through P also pass through Q ? Only one, that is, the line PQ. How many lines passing through Q also pass through P? Only one, that is, the line PQ. Thus, the statement above is self-evident, and so is taken as an axiom.





Euclid's Postulate (2)

POSTULATE 2 : A terminated line can be produced indefinitely.

Note that what we call a line segment now-a-days is what Euclid called a terminated line. So, according to the present day terms, the second postulate says that a line segment can be extended on either side to form a line.

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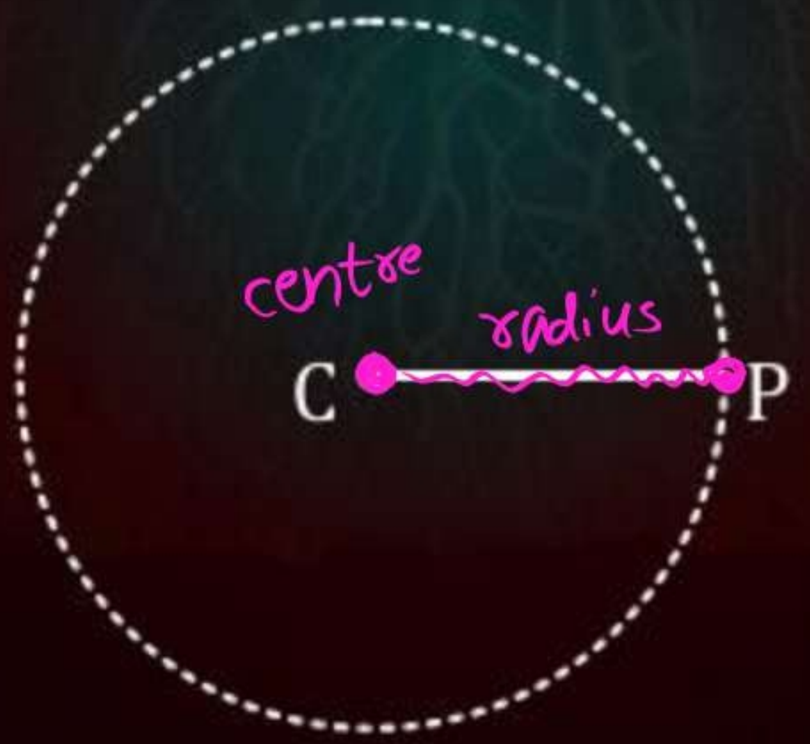
Euclid's Postulate (3)

POSTULATE 3 : A circle can be drawn with any centre and any radius.

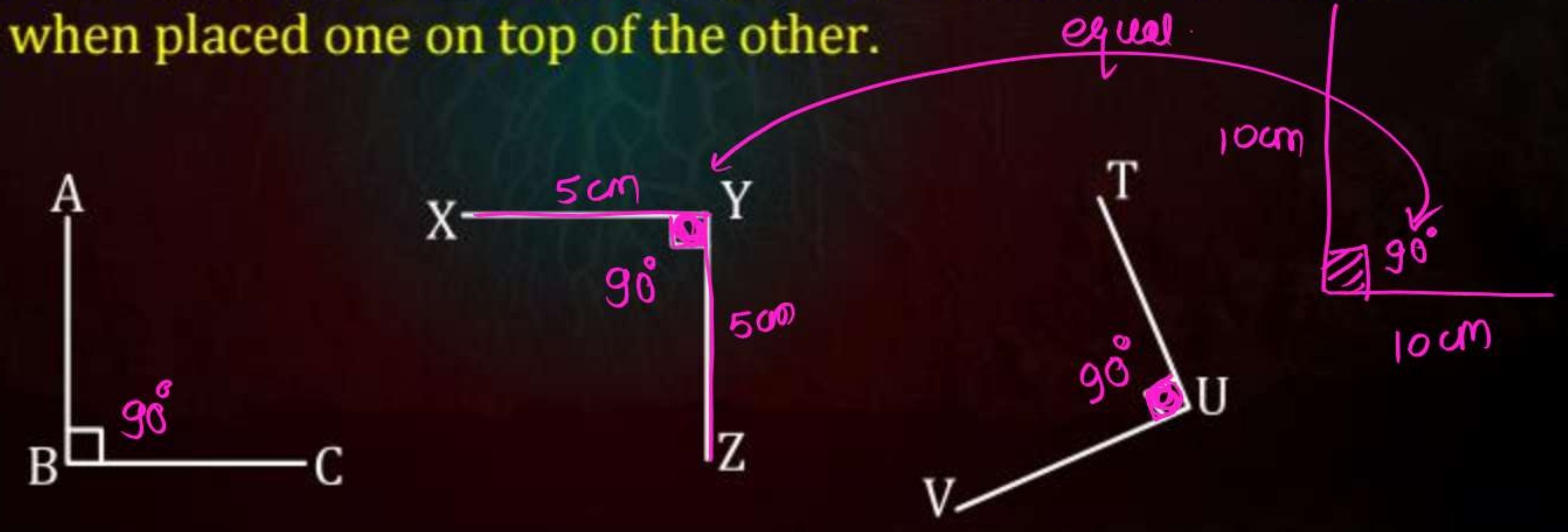
A circle can be drawn having the line segment as radius and one endpoint of the segment as center.



Euclid's Postulate (4)

POSTULATE 4 : All right angles are equal to one another.

A right angle is an angle measuring 90 degrees. So, irrespective of the length of a right angle or its orientation all right angles are identical in form and coincide exactly when placed one on top of the other.

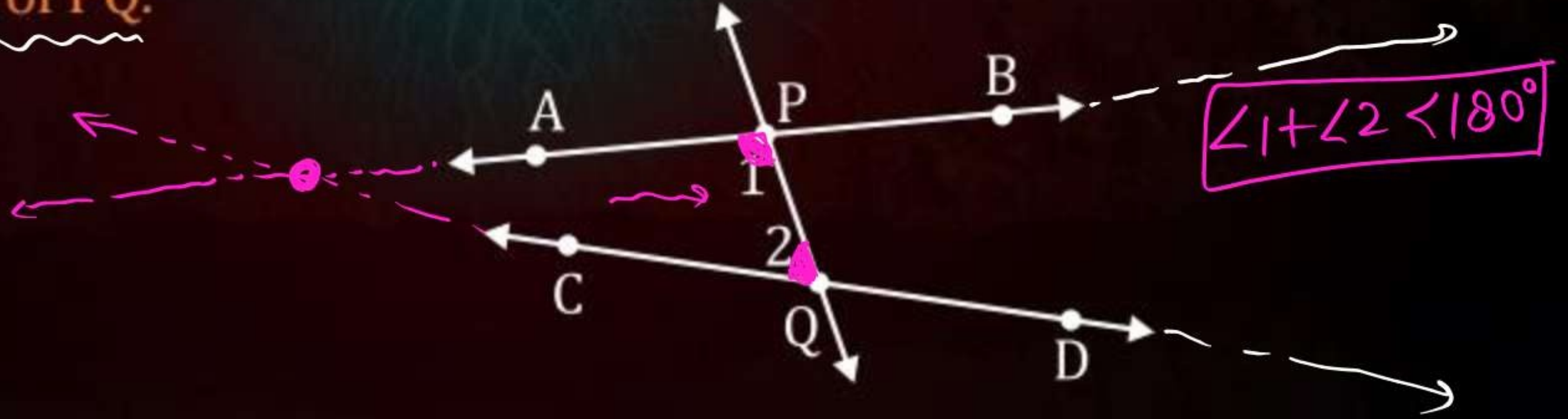


Euclid's Postulate (5)

(Most Imp)

POSTULATE 5 (Playfair's Axiom) : If a straight line falling on two straight lines makes the interior angles on the same side of tit taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of the angles is less than two right angles.

The line PQ falls on lines AB and CD such that the sum of the interior angles 1 and 2 is less than 180° on the left side of PQ. Therefore, the lines AB and CD will eventually intersect on the left side of PQ.



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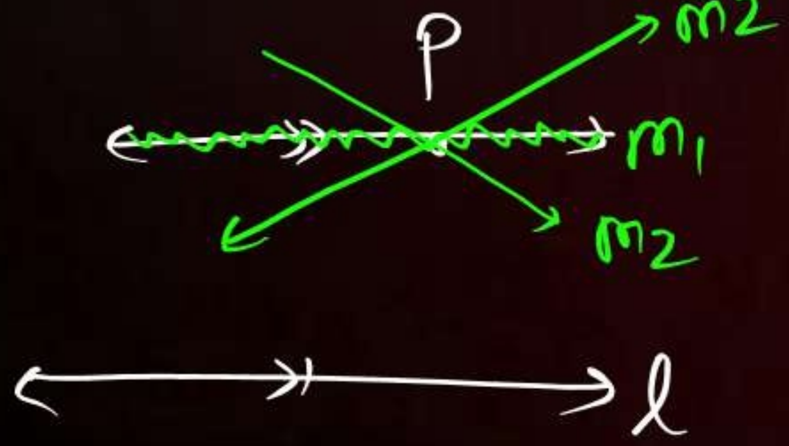
Modified Part of Euclid's 5th Postulate

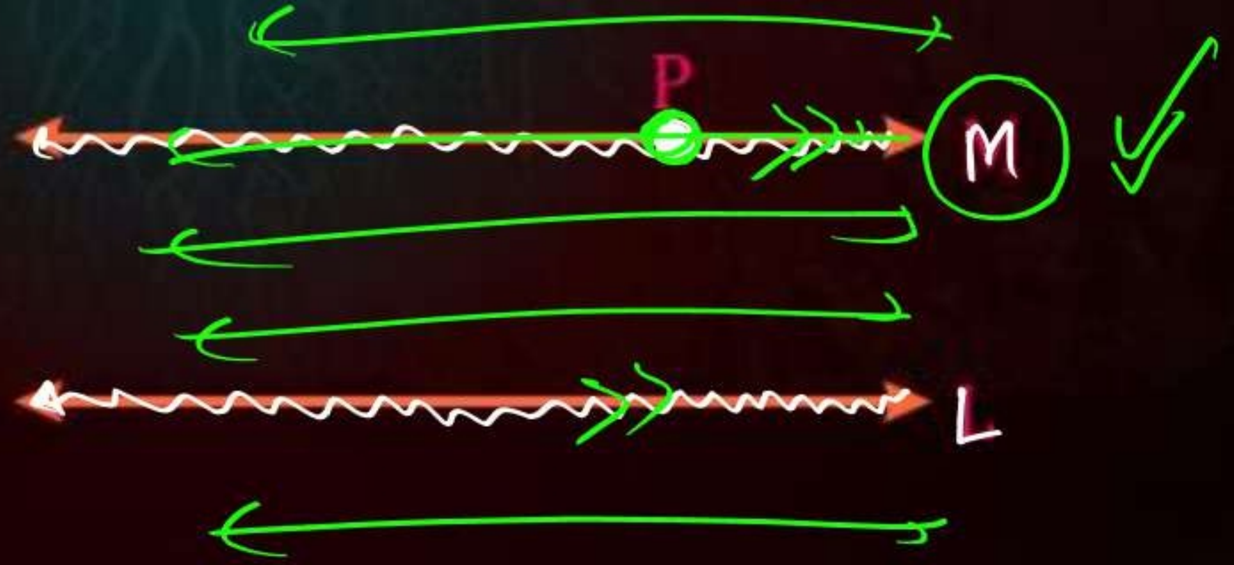
Later on, the fifth postulate was modified as under: 'For every line L and for every point P not lying on L, there exists a unique line M passing through P and parallel to L.

$\boxed{l \parallel n}$ $\boxed{m \nparallel n}$

An important result : Two distinct intersecting lines cannot be parallel to the same line.



$\boxed{m_1 \parallel l}$



After Euclid stated his postulates and axioms, he used them to prove other results. Then using these results, he proved some more results by applying deductive reasoning. The statements that were proved are called propositions or theorems. Euclid deduced 465 propositions in a logical chain using his axioms, postulates, definitions and theorems proved earlier in the chain.

Theorems and Corollary

THEOREMS :

A statement that requires a proof is called a theorem.
Establishing the truth of a theorem is known as proving the theorem.

Examples:

(i) The sum of all the angles around a point is 360°

(ii) The sum of the angles of a tringle is 180°

COROLLARY :

A statement whose truth can be deduced from a theorem called its corollary.

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Euclid's Theorem (5.1)

Prove that two distinct lines cannot have more than one point in common.

GIVEN : Two distinct lines l and m.

TO PROVE : l and m cannot have more than one point in common.

PROOF : If $l \parallel m$ then clearly there is no point common to both l and m. So, let us consider the case when l is not parallel to m. If possible, let P and Q be two points common to both l and m. Then, l contains both the points P and Q. And, m contains both the points P and Q. But, we know that there is only one line passing through two distinct points P and Q.

$\therefore l \parallel m$.

This contradicts the hypothesis that l and m are distinct.

Thus, our supposition is wrong.

Hence, two distinct lines cannot have more than one point in common.

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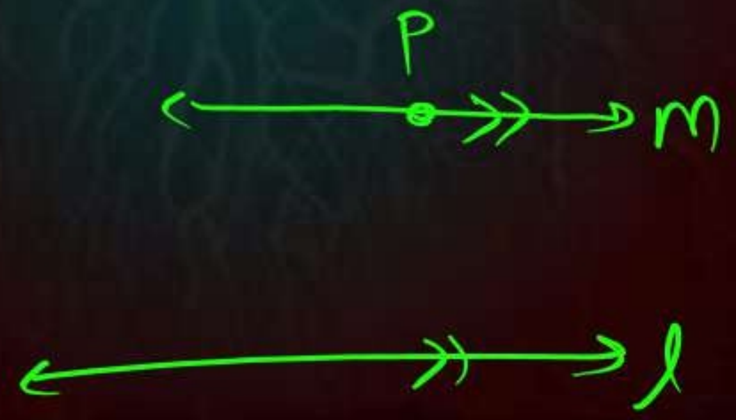
Two equivalent versions of Euclid's fifth postulate

Theorem 5.1 : Two distinct lines cannot have more than one point in common.

(1) For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.

(2) Two distinct intersecting lines cannot be parallel to the same line.



SOME UNDEFINED TERMS

There are three basic concepts in geometry, namely, "point", "line" and "plane". It is not possible to define these three concepts precisely. We can, however, have a good idea of these concepts by considering examples as given below.

POINT : A point is represented by a fine dot made by a sharp pencil on a sheet of paper. In our subsequent discussion, we shall denote a point by a capital letter A, B, P, Q, R etc.

SOME TREMS RELATED TO GEOMETRY

Point : A point is an exact location.

Line segment : The straight path between two points A and B is called the line segment $\overline{AB}$.

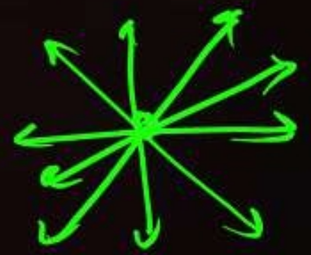
Ray : A line segment $\overline{AB}$ when extende indefinitely in one direction is a ray $\overrightarrow{AB}$.

Line : A line segment $\overline{AB}$ when extende indefinitely in both direction is a ray $\overleftrightarrow{AB}$.

INCIDENCE AXIOMS ON LINES

(i) A line contains infinitely many points.

(ii) Through a given point, infinitely many lines can be drawn.

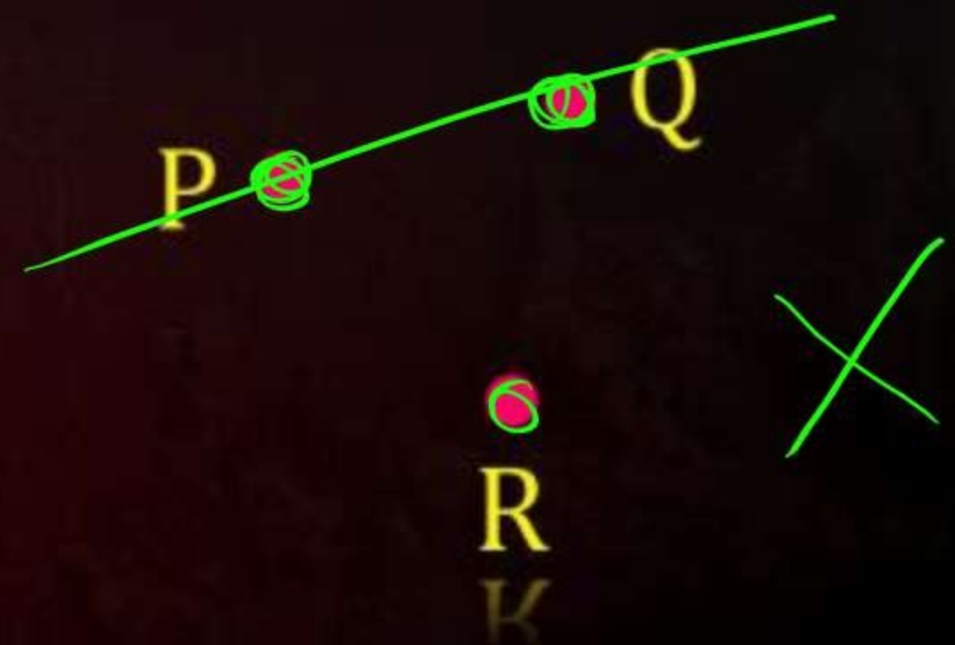


(iii) One and only one line can be drawn to pass through two given points A and B.

Unique line

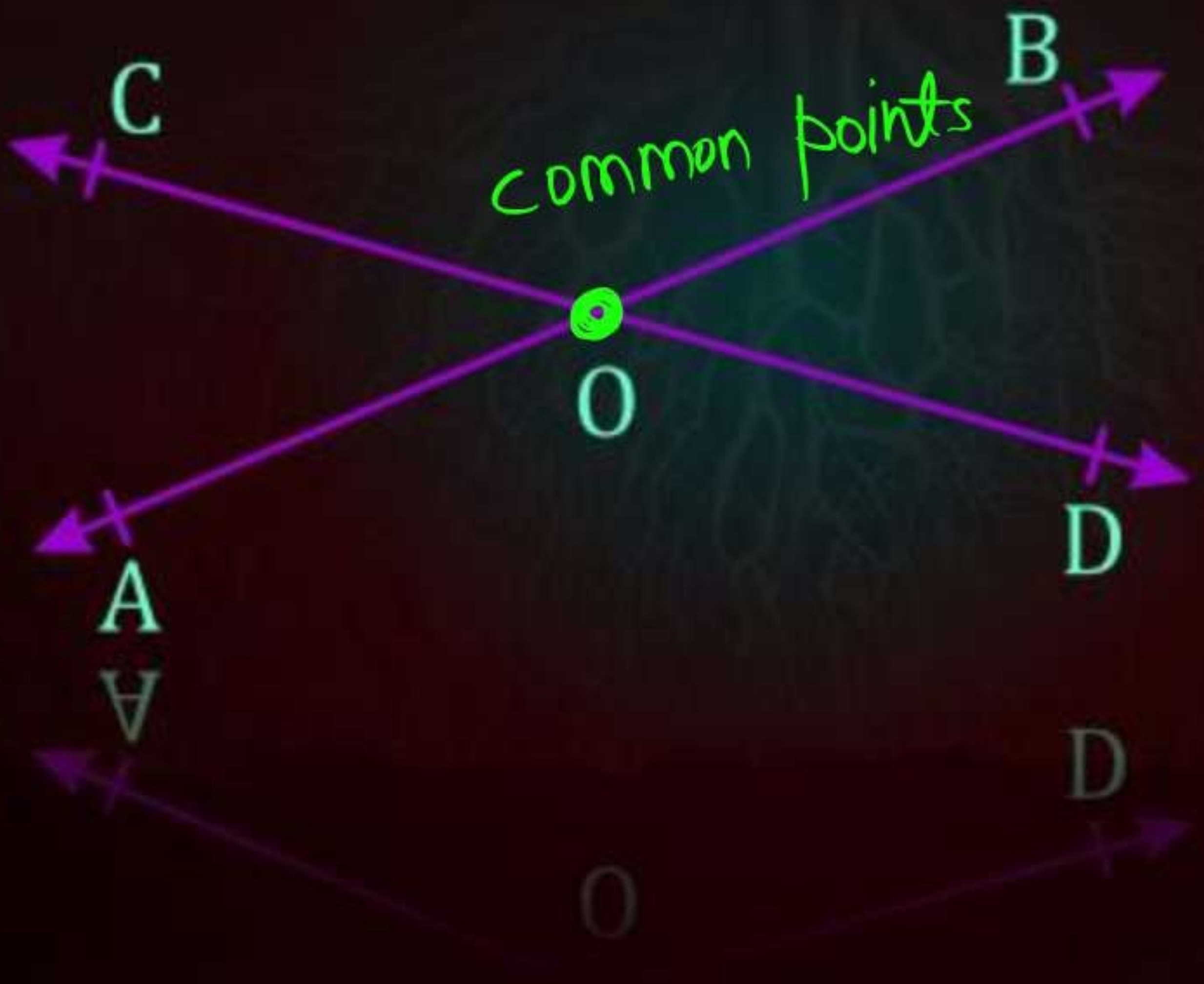
COLLINEAR POINTS

Three or more than three points are said to be collinear, if there is a line which contains them all.



INTERSECTING LINES

Two lines having a common points are called intersecting lines.



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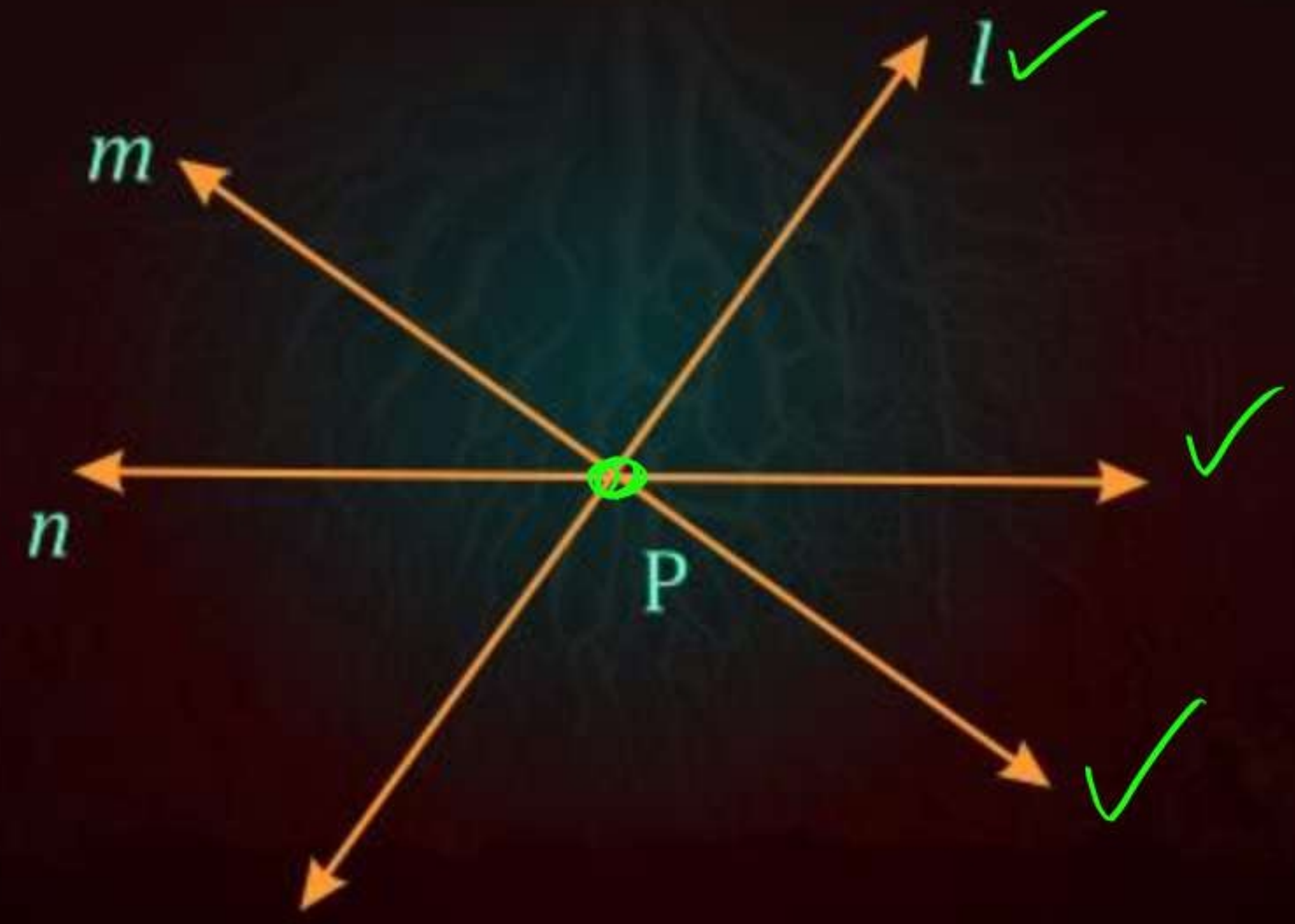
CONCURRENT LINES

Three or more lines intersecting at the same point are said to be concurrent.



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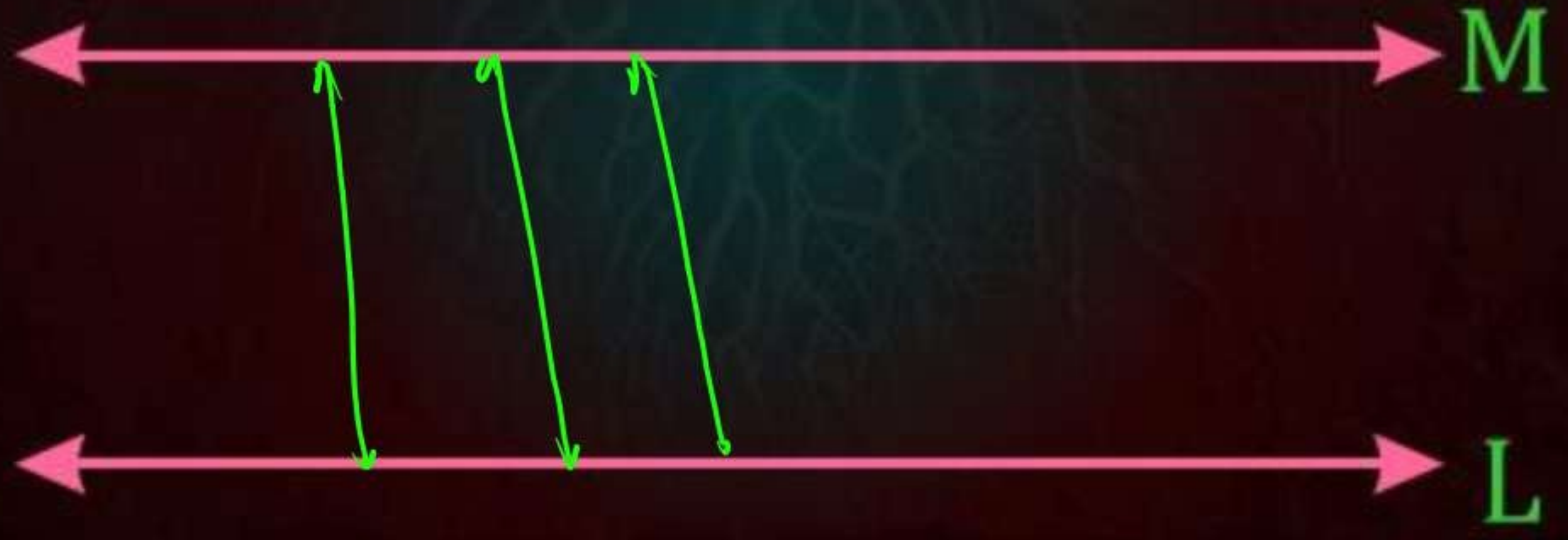
PARALLEL LINES

Two lines l and m in a plane are said to be parallel, if they have no point in common and we write, l ∥ m.

The distance between two parallel lines always remains the same.



Euclid's Definitions

A solid has shape, size, position, and can be moved from one place to another. Its boundaries are called surfaces. They separate one part of the space from another, and are said to have no thickness. The boundaries of the surfaces are curves or straight lines. These lines end in points.

Euclid's Definitions

1. A point is that which has no part.
2. A line is breadthless length.
3. The ends of a line are points.
4. A straight line is a line which lies evenly with the points on itself.
5. A surface is that which has length and breadth only.
6. The edges of a surface are lines.
7. A plane surface is a surface which lies evenly with the straight lines on itself.

Euclid's Definitions

If we carefully study these definitions, we will find that some of the terms like part, length, breadth, evenly etc. need to be further explained clearly. For example, in the definition of 'point', the word 'part' needs to be defined. If you define 'part' as something that occupies area, then again 'area' has to be defined.

$\therefore$ We can see that there was a chain of words which remained undefined. Thus, mathematicians agreed to leave some geometric terms UNDEFINED.

UNDEFINED TERMS OF GEOMETERY

Set of elements in geometry are point, line and plane. These terms are undefined but can be visualized by certain practical situations.

(i) Point : It has neither length nor breadth nor thickness. It is represented by a fine dot.

(ii) Line : If we fold a piece of paper, the crease of paper represents a line. A line cannot be drawn fully on the paper because it extends indefinitely in both the directions.

These terms are called undefined because the terms used in definitions of point, line and plane further needed an explanation or needed to be defined. So, to define one thing, you need to define may other things and you may get a long chain of definitions without an end

Euclid assumed certain properties, which were not to be proved. These assumptions are actually 'obvious Universal truths'. He divided them into two types: Axioms and postulates.

Question

Euclid stated that all right angles are equal to each other in the form of

An axiom

A definitions

A postulate

A theorem

Question

Euclid stated that all right angles are equal to each other in the form of

A An axiom

B A definitions

C A postulate ✓

D A theorem

Question

Two distinct intersecting lines l and m cannot have

Any point in common

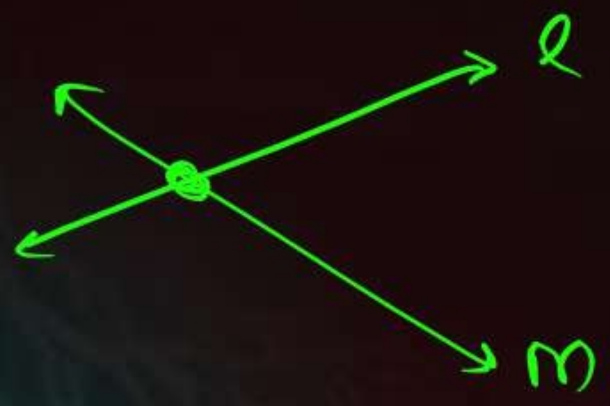
One point in common

Two point in common

None of these

Question

Two distinct intersecting lines l and m cannot have



- **A** Any point in common
- **B** One point in common
- **C** Two point in common
- **D** None of these

Question

Which of the following needs a proof?

An axiom

A definitions

A postulate

A theorem

Question

Which of the following needs a proof?

A An axiom

B A definitions

C A postulate

D $\boxed{\text{A theorem}}$

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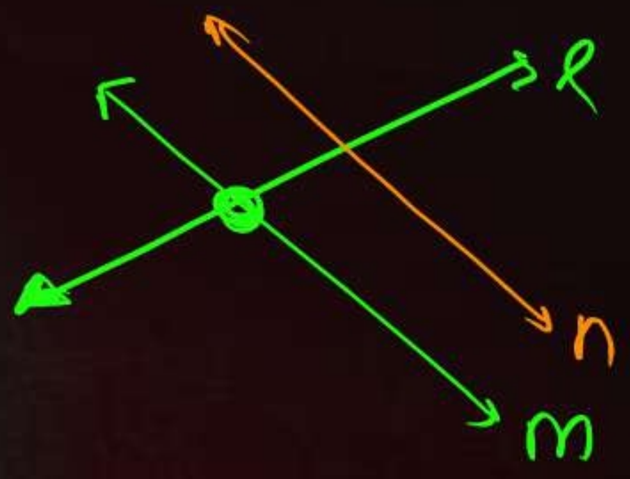
Question

If l, m, n are lines in the same plane such that l intersects m and $n || m$, then show that l intersect n also.

Question

If l, m, n are lines in the same plane such that l intersects m and $n||m$, then show that l intersect n also.



l & m are intersecting

$\boxed{m \parallel n}$ $\boxed{l \nparallel n}$

Intersecting

If l is not parallel to n. There l will be intersecting line of n.

Question

If l and m are intersecting lines, $l \parallel p$ and $m \parallel q$, then show that p and q also intersect.

Question

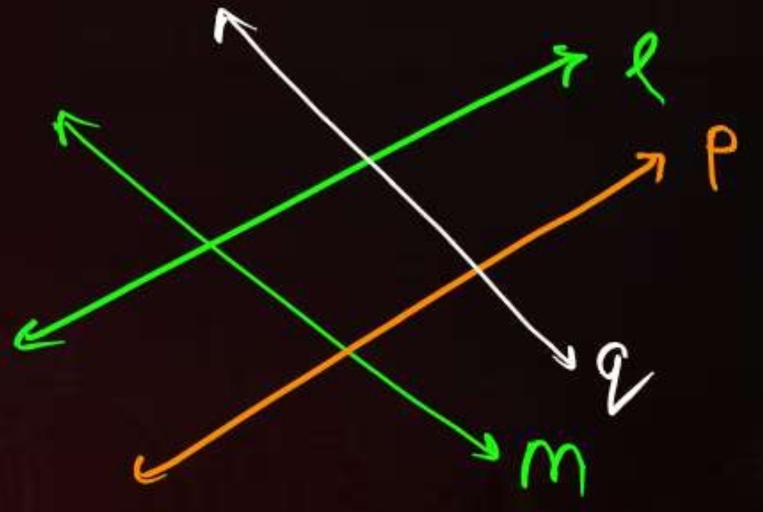
If l and m are intersecting lines, $l \parallel p$ and $m \parallel q$, then show that p and q also intersect.

$l \parallel p$ intersecting $m \nparallel p$ ✓

$\Rightarrow \boxed{m \parallel q}$

$\boxed{l \nparallel q}$

$q \nparallel l \parallel p \Rightarrow \boxed{q \nparallel p} \rightarrow$ Intersecting




THANK YOU
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