VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

- ❑ **Subject -** Mathematics

- ❑ **Chapter –** Areas of Parallelograms & Triangles

Today's Targets

1. Planar Region and it's Area, Area Axioms
2. Figures on the same Base and between the same Parallels
3. Parallelograms on the same Base and Between the same Parallels
4. Triangles and Parallelogram on the same Base and between same Parallels
5. Triangles on the same Base and between the same Parallels
6. Median of a Triangle and regarding Property

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Planar Region and it's Area

The part of the plane enclosed by a simple closed figure is called a planar region corresponding to that figure. The magnitude or measure of this planar region is called its area. This magnitude or measure is always expressed with the help of a number (in some unit) such as 5 cm^2 , 8 m^2 , 3 hectares etc. So, we can say that area of a figure is a number (in some unit) associated with the part of the plane enclosed by the figure.

Area Axioms

Two figures are called congruent, if they have the same shape and the same size.

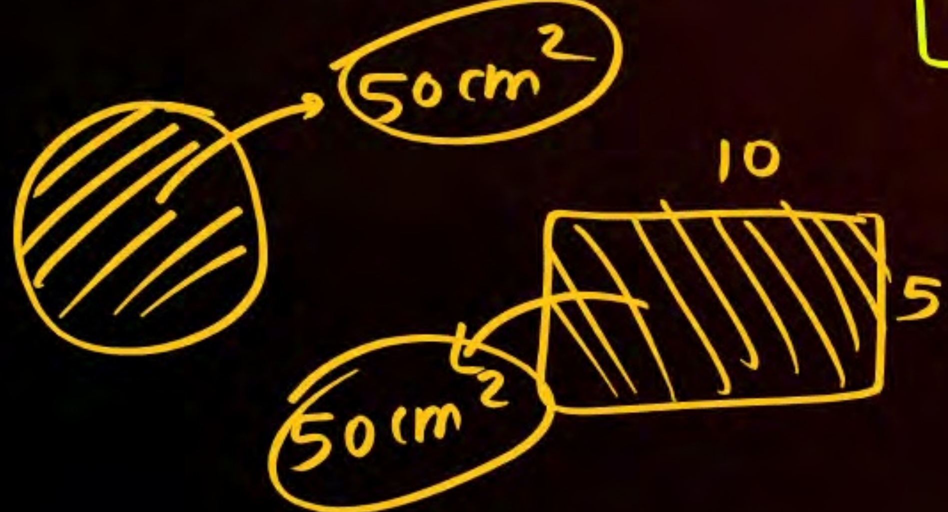
1. **Congruent area axiom:** If A and B are two congruent figures, then

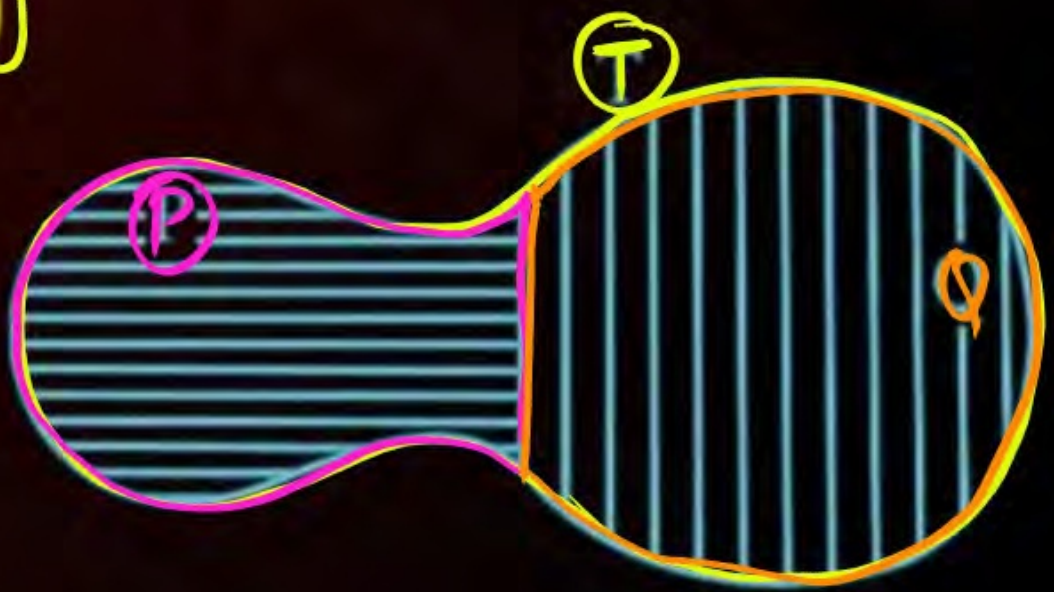
$$\boxed{\text{area}(A) = \text{area}(B)}$$

shape
size

2. **Area addition axiom:** If a planar region formed by a figure T is made up of two non-overlapping planar regions formed by figures P and Q, then

$$\boxed{\text{area}(T) = \text{area}(P) + \text{area}(Q)}$$

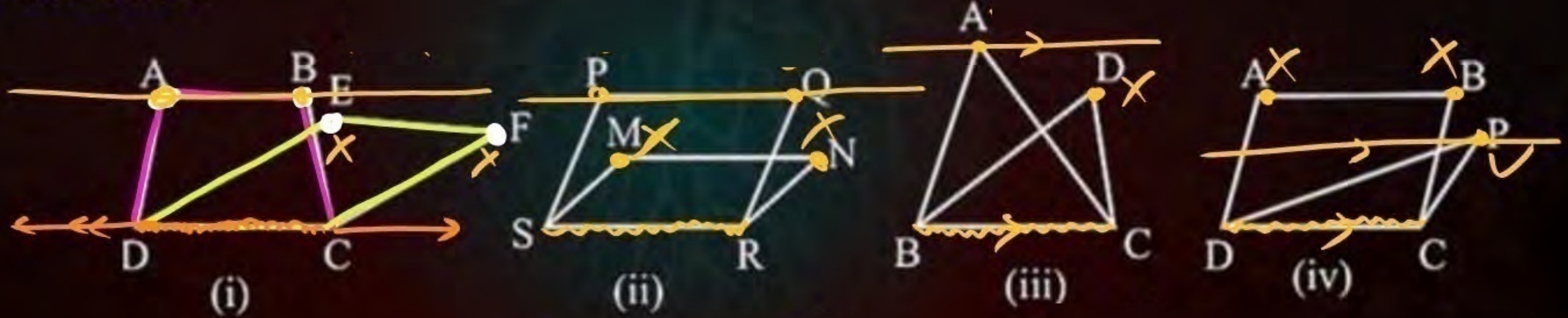


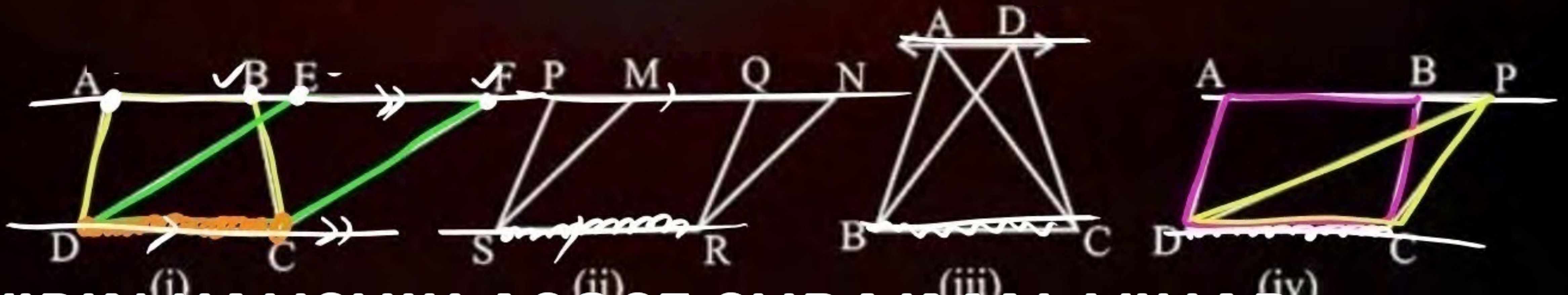


Figures on the Same Base and b/w the Same Parallels

Two figures are said to be on the same base and between the same parallels, if they have

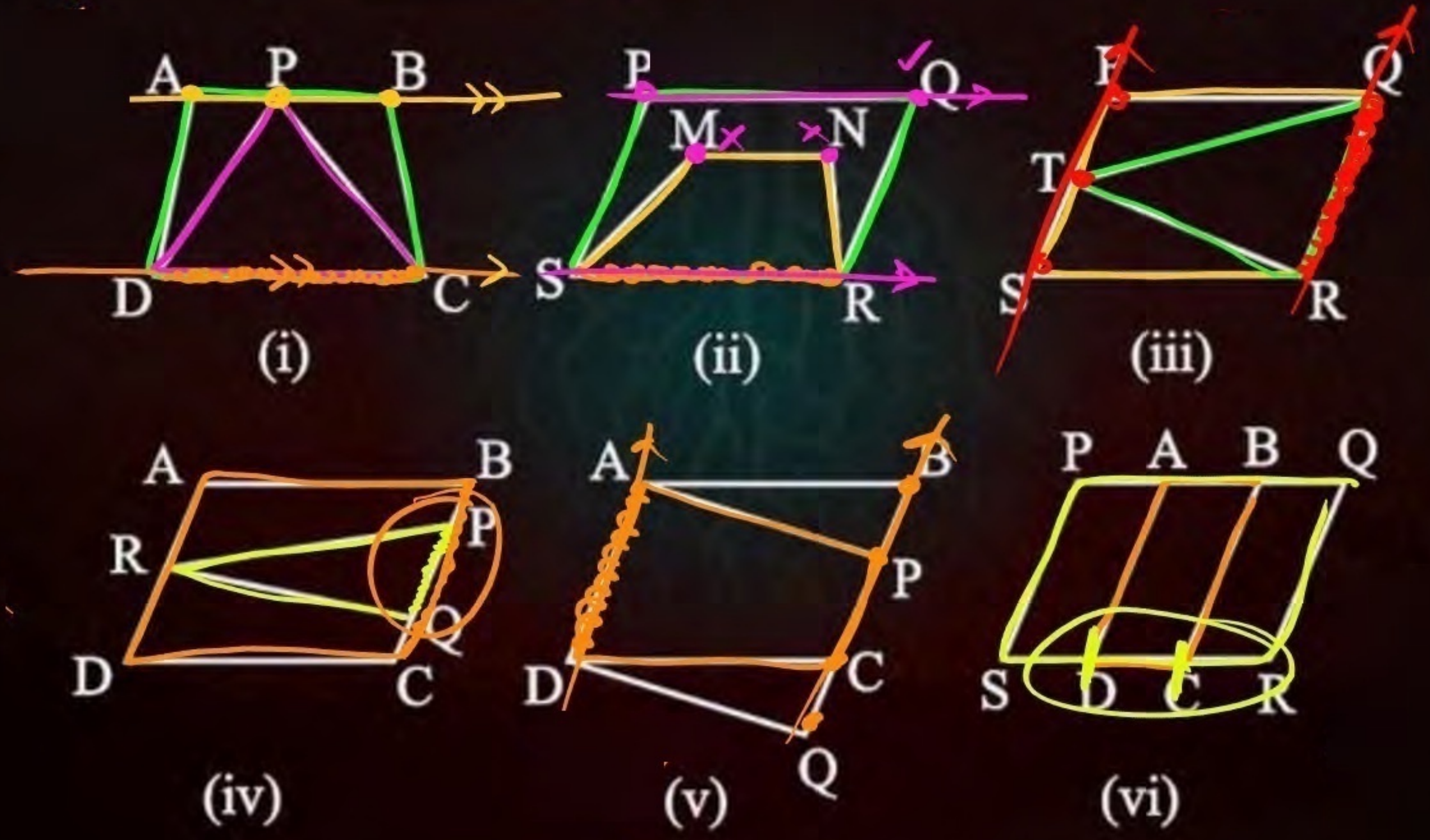
1. A common base (side) and
2. The vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.





Question

Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



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Question

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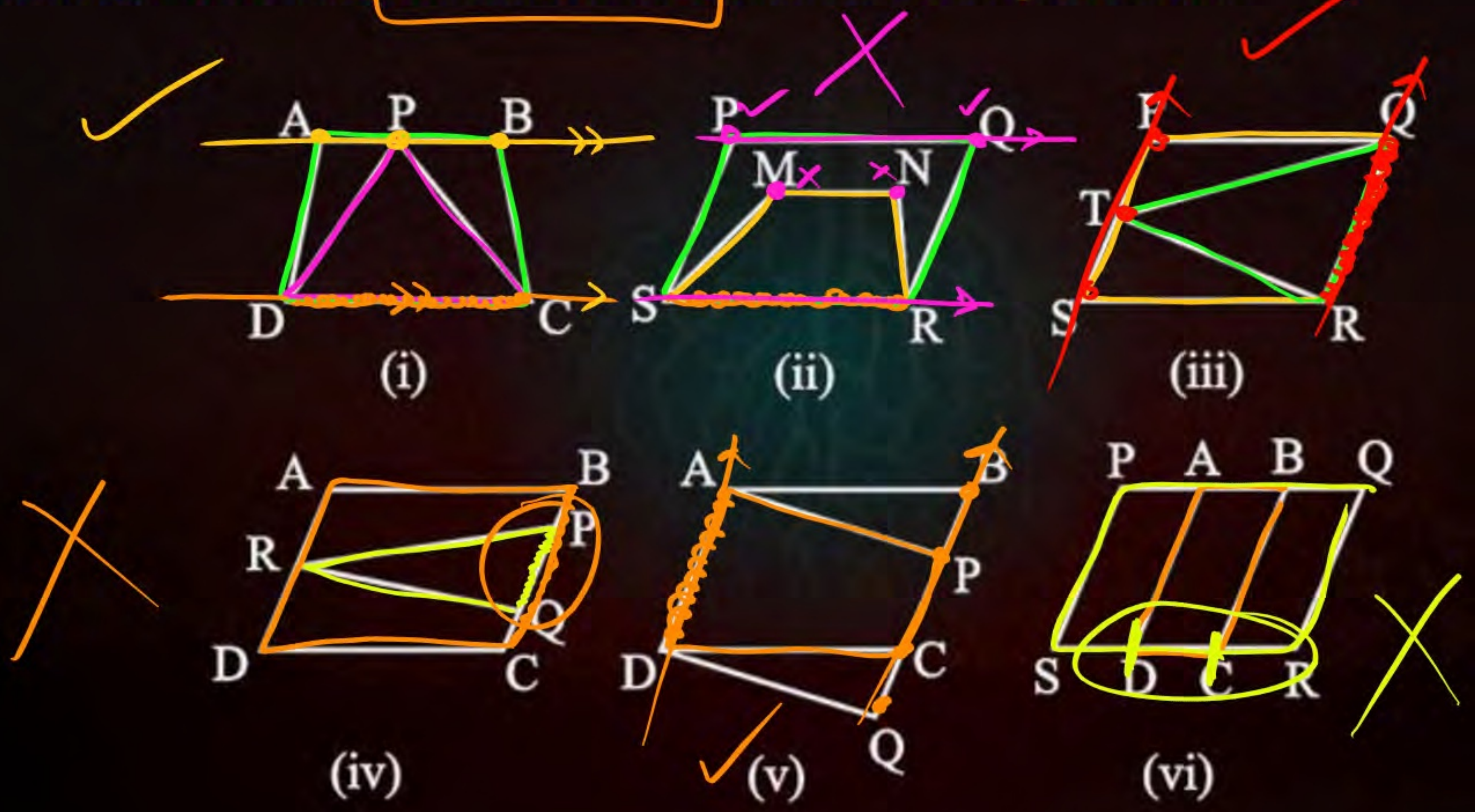
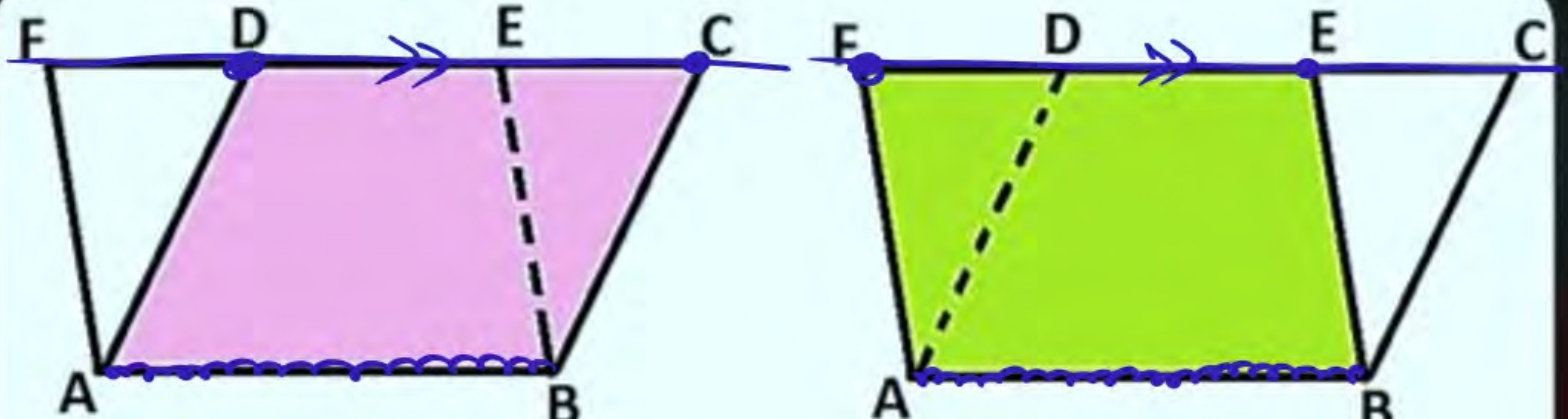


Parallelograms on the same Base and Between the same Parallels

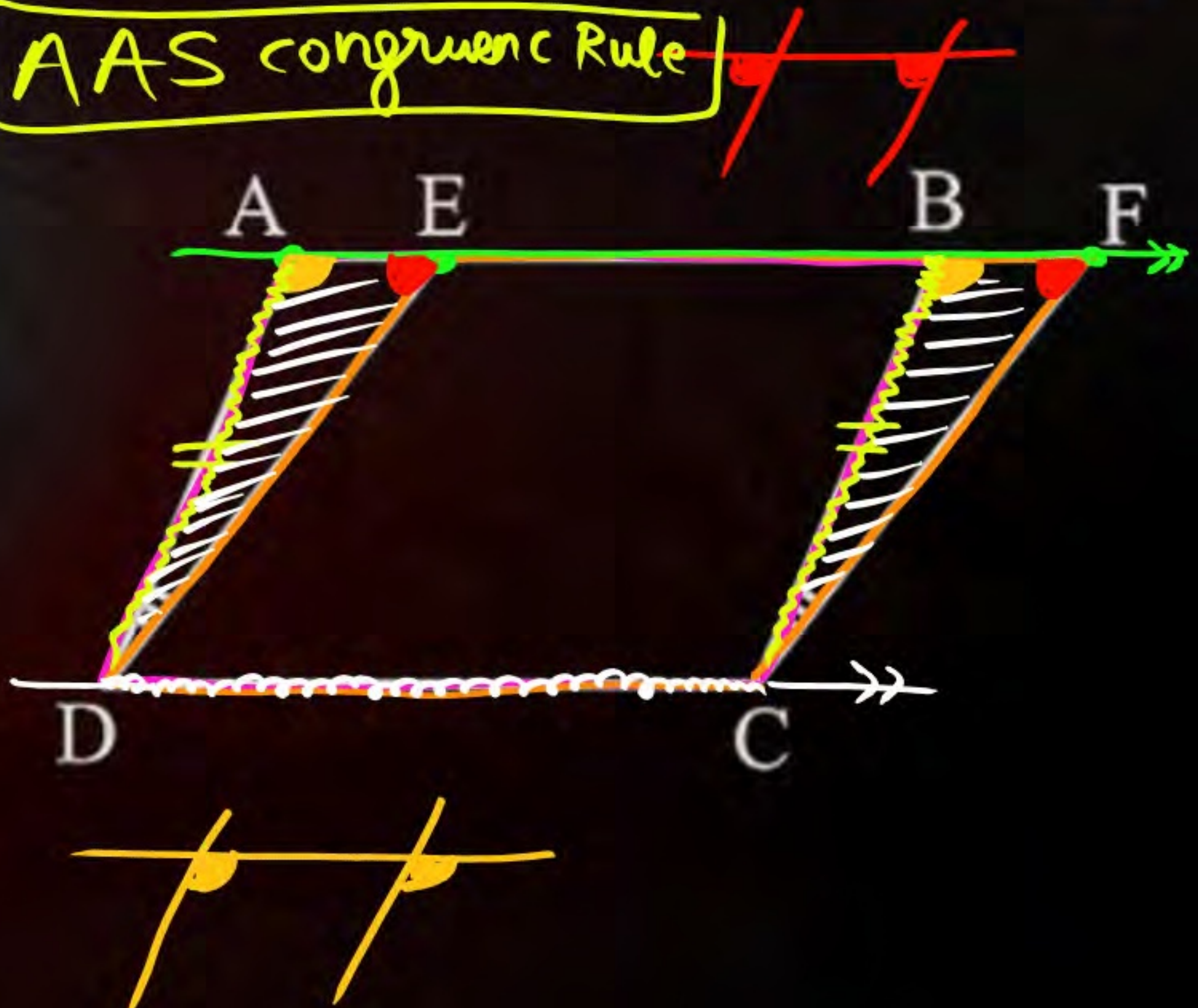
Theorem: Parallelograms on the same base and between the same parallels are equal in area.

AAS congruenc Rule



Parallelogram ABCD (highlighted in pink)

Paralleologram ABEF (highlighted in green)



$Ar(\Delta AED) = ar(\Delta BFC)$

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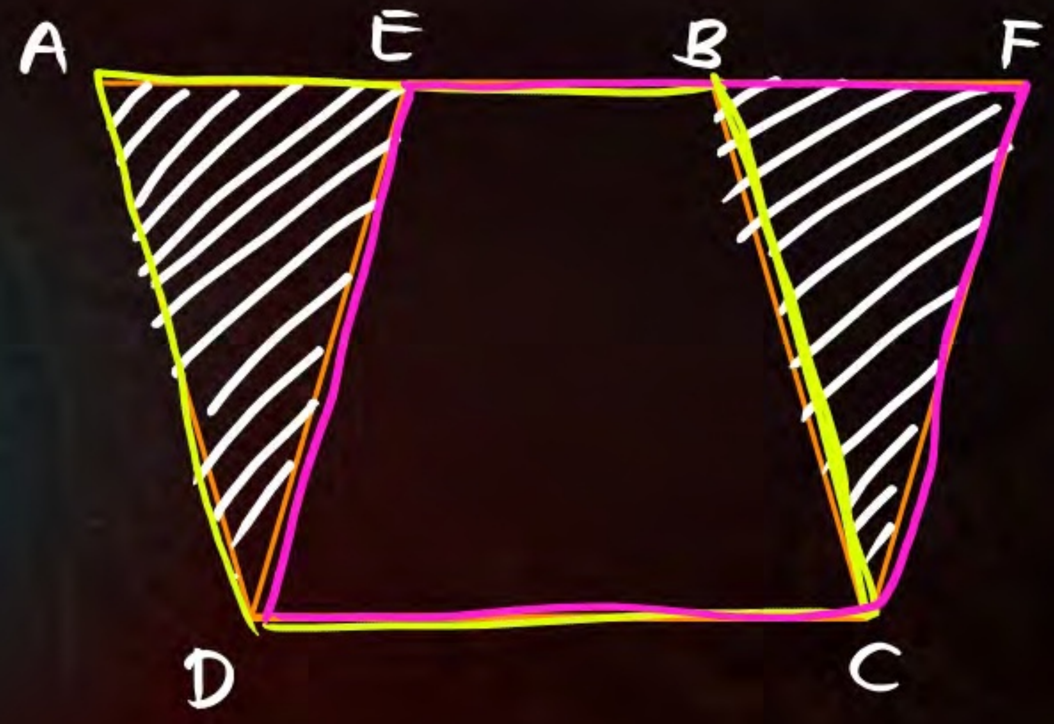
$$\left[ar(\triangle AED) + ar(\square EBCD) \right] = \left[ar(\triangle BFC) + ar(\square EBCD) \right]$$

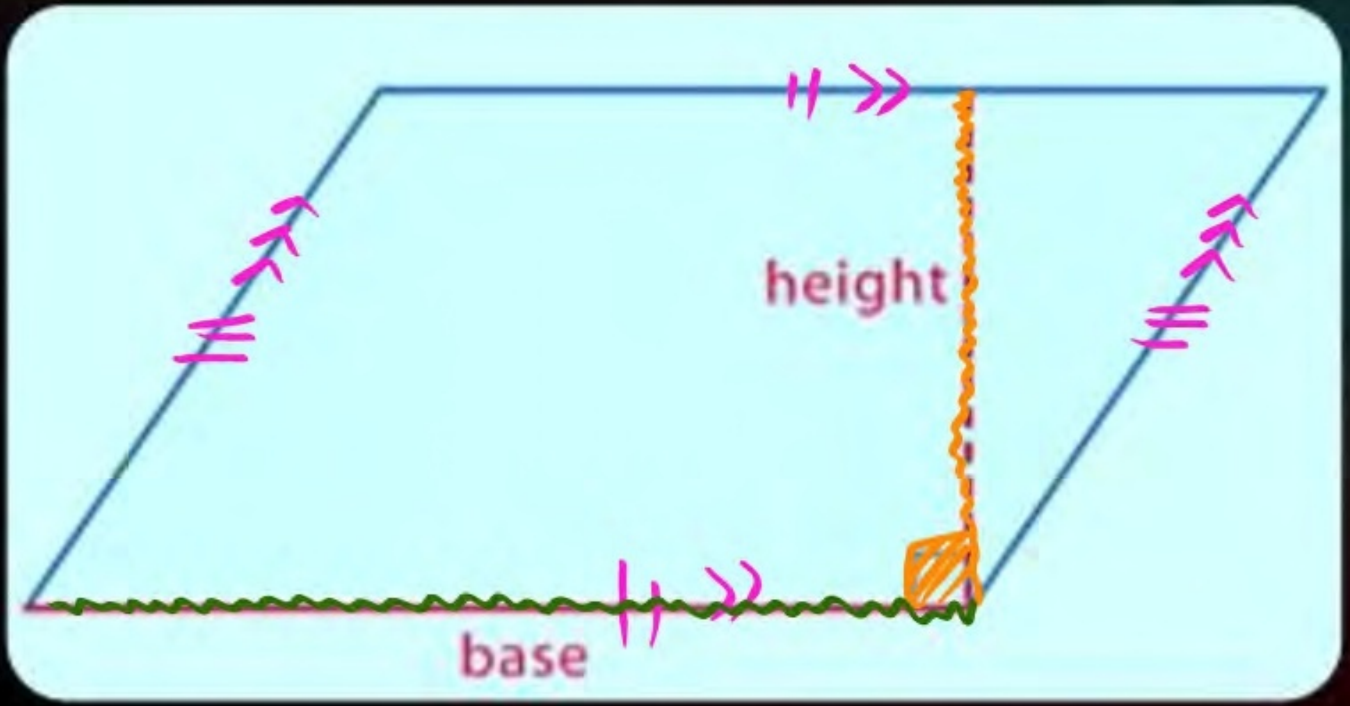
$$\boxed{ar(\parallel^{gram} ABCD) = area(\parallel^{gram} EFCD)}$$

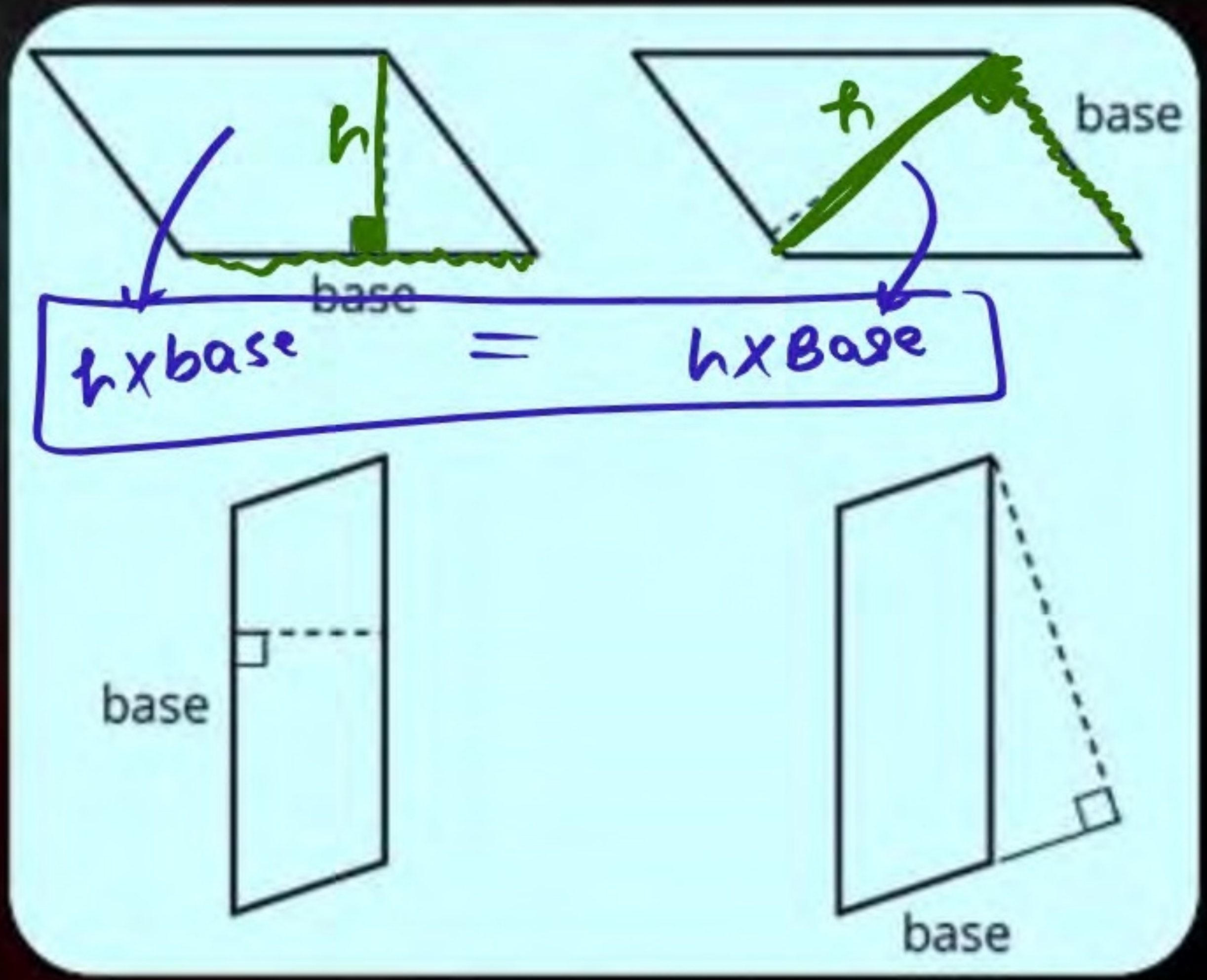
Hence, proved!!



Base and Altitude of a Parallelogram

A parallelogram is a four-sided figure, quadrilateral, with both pairs of opposite sides parallel. It doesn't matter what the angles are in a parallelogram if the opposite sides are parallel.





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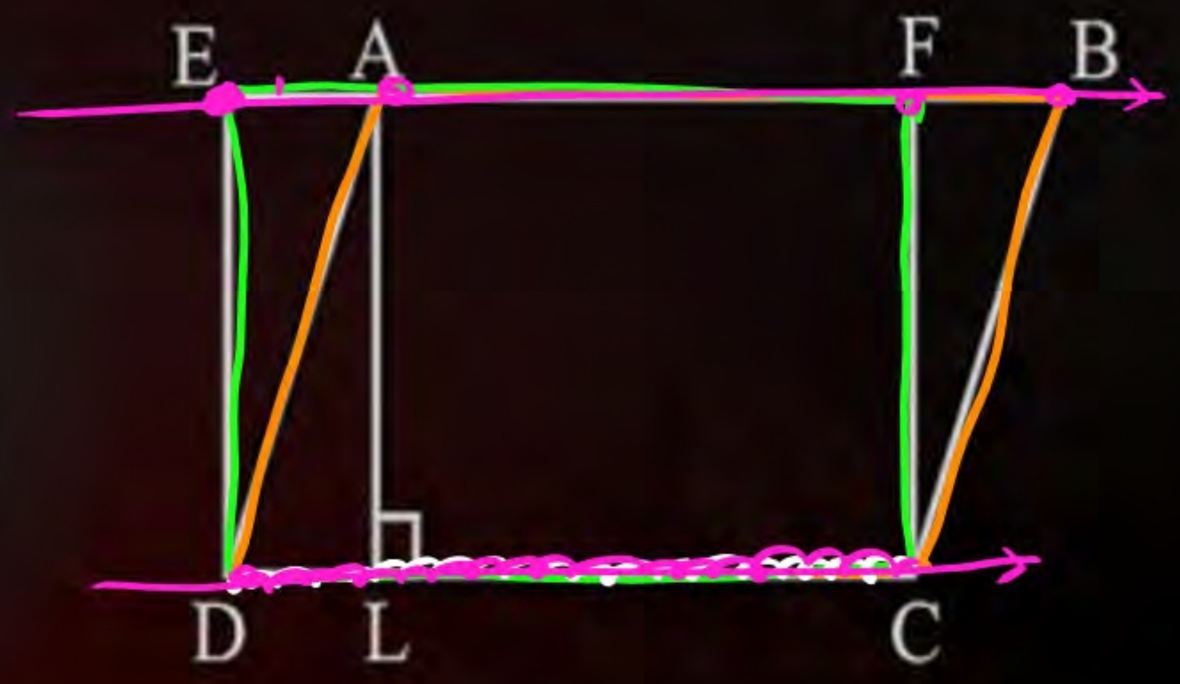
Area of Parallelogram

Area of a parallelogram is the product of its any side and the corresponding altitude.



$ar(\square EFCD) = area(\square ABCD)$

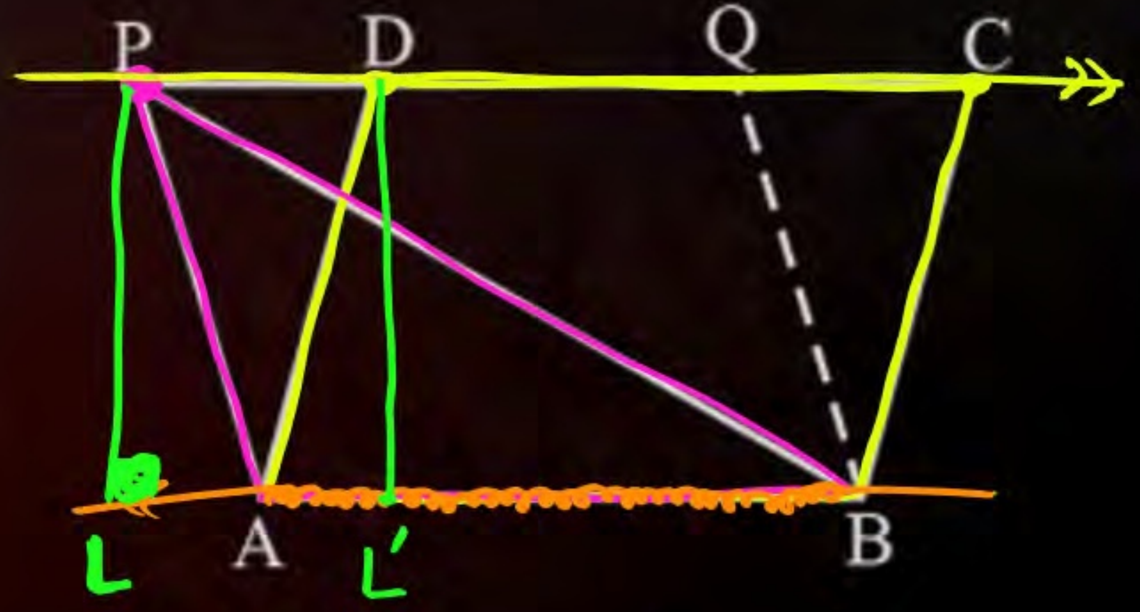
$DC \times ED = area(\parallel^{gm} ABCD)$

$DC \times AL = area(\parallel^{gm} ABCD)$

$\boxed{Base \times Height = area(\parallel^{gm} ABCD)}$

Triangles and Parallelogram on the same Base and between same Parallels

Theorems: If a triangles and a parallelogram are on the same base (or equal bases) and between the same parallels then area of triangle must be half of area of parallelogram.



We know that, gap between two parallel lines will be uniform

$$\boxed{PL = DL'}$$

$$\text{Area}(\triangle PBA) = \frac{1}{2} \times AB \times PL$$

$$\text{Ar}(\triangle PBA) = \frac{1}{2} \times AB \times DL'$$

$$\boxed{\text{Ar}(\triangle PBA) = \frac{1}{2} \text{ara}(\|^{gram} ABCD)}$$

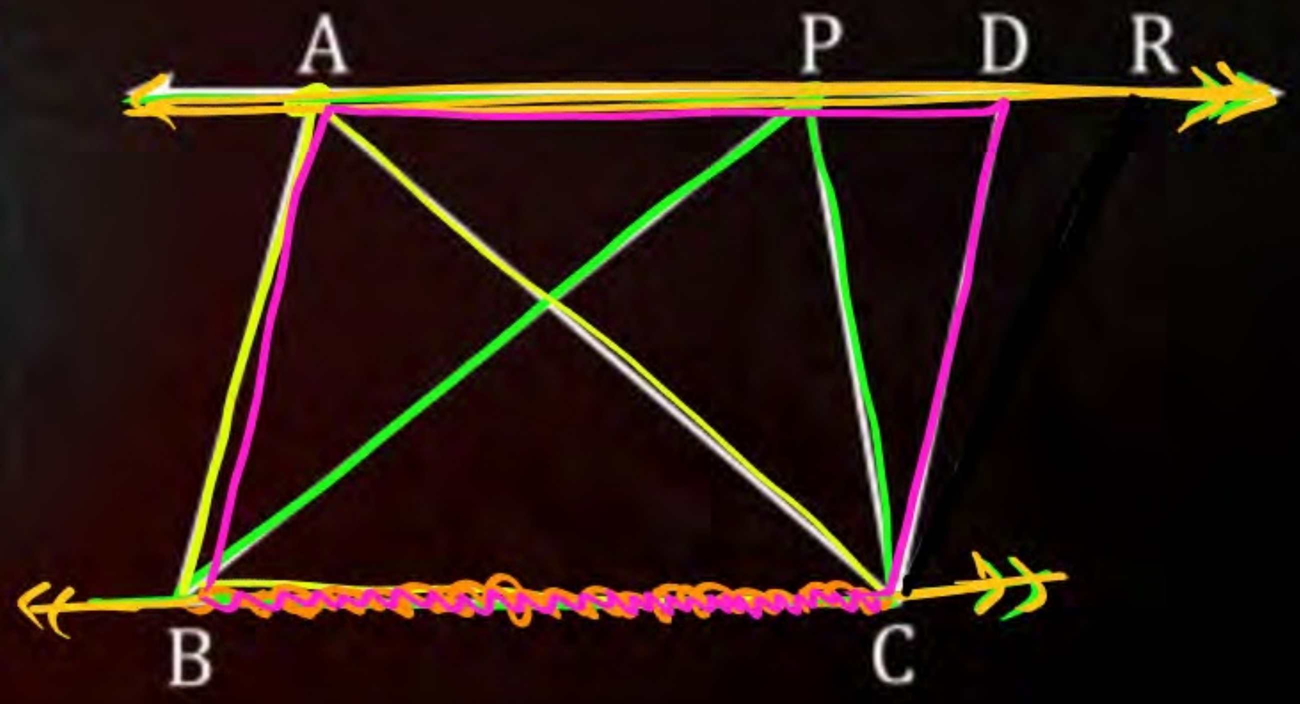
Triangles on the same Base and between the same Parallels

Theorems: Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

$$\text{Area}(\Delta ABC) = \frac{1}{2}\,\text{ar}(\|^{\text{gram}}\, ABCD) \quad \cdots\cdots (1)$$

$$\text{Area}(\Delta PBC) = \frac{1}{2}\,\text{area}(\|^{\text{gm}}\, ABCD) \quad \cdots\cdots (2)$$

$$\boxed{\text{Area}(\Delta ABC) = \text{area}(\Delta PBC)}$$



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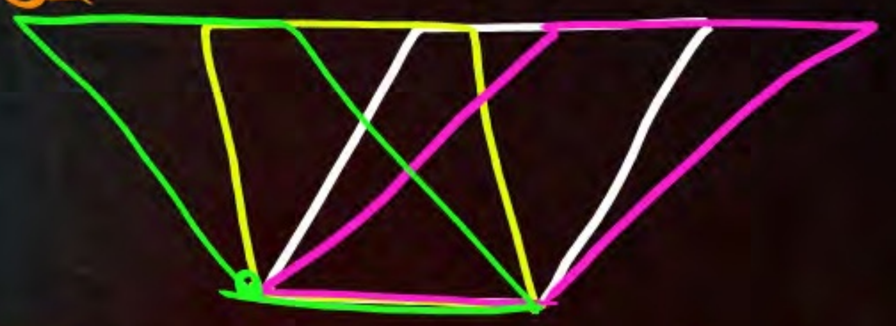
① on the same base and between same parallel : ✓

- Area of parallelogram following the same have same area



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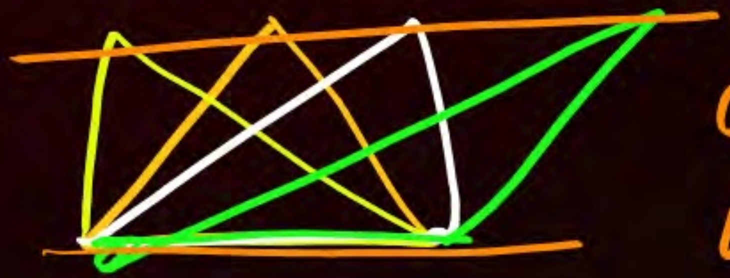
area of $\Delta = \frac{1}{2}$ area of $\|^{gram}$

- 

areas of all Δ following the same will be equal.

Question

If ABCD and EFGH are two parallelograms between same parallel lines and on the same base, then:

ar(ABCD) > ar(EFGH)

ar(ABCD) < ar(EFGH)

ar(ABCD) = ar(EFGH)

None of these

Question

If ABCD and EFGH are two parallelograms between same parallel lines and on the same base, then:

A. ar(ABCD) > ar(EFGH)

B. ar(ABCD) < ar(EFGH)

C. ar(ABCD) = ar(EFGH)

D. None of these

Question

In the given figure, ABCD is a parallelogram. If AB = 12 cm, AE = 7.5 cm, CF = 15 cm, then AD =

A 6 cm

B 8 cm

C 7 cm

D 3 cm

Question

In the given figure, ABCD is a parallelogram. If AB = 12 cm, AE = 7.5 cm, CF = 15 cm, then AD =

$\boxed{AB = DC} = 12$

A 6 cm

B 8 cm

C 7 cm

D 3 cm

$\boxed{\text{Area}_1 = \text{Area}_2}$

$AE \times DC = CF \times AD$

$\boxed{7.5 \times 12 = 15 \times AD}$

Assertion and Reason Type Problem

Direction: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true, but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true, but reason (R) is false.

(d) Assertion (A) is false, but reason (R) is true.

Question

Assertion: A triangle and a rhombus are on the same base and between the same parallels. The ratio of the areas of the triangle and the rhombus is 1:2.

Reason: The area of a triangle is half of the area of a parallelogram on the same base and between the same parallels.

Question

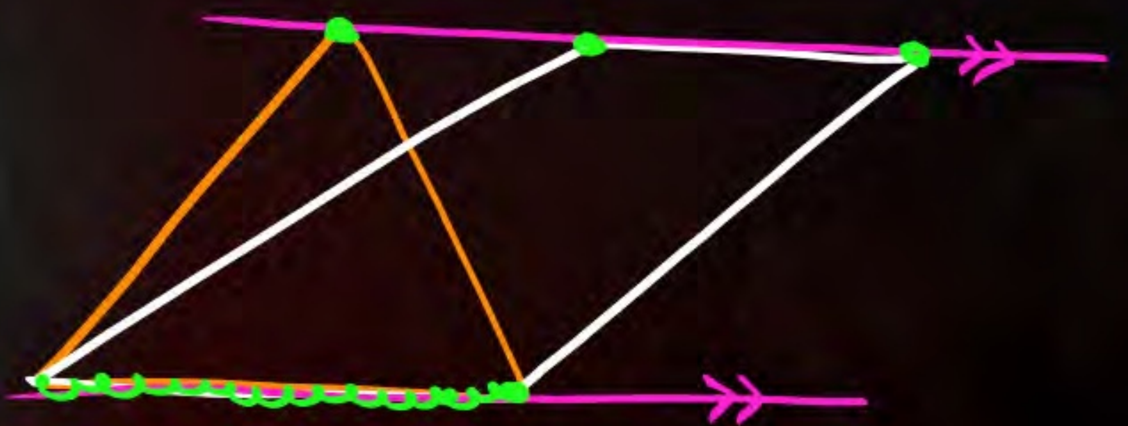
Assertion: A triangle and a rhombus are on the same base and between the same parallels. The ratio of the areas of the triangle and the rhombus is 1:2. True

Reason: The area of a triangle is half of the area of a parallelogram on the same base and between the same parallels. True

Explain

$$\text{area}(\Delta) = \frac{1}{2}\,\text{area}(\text{Rhombus})$$

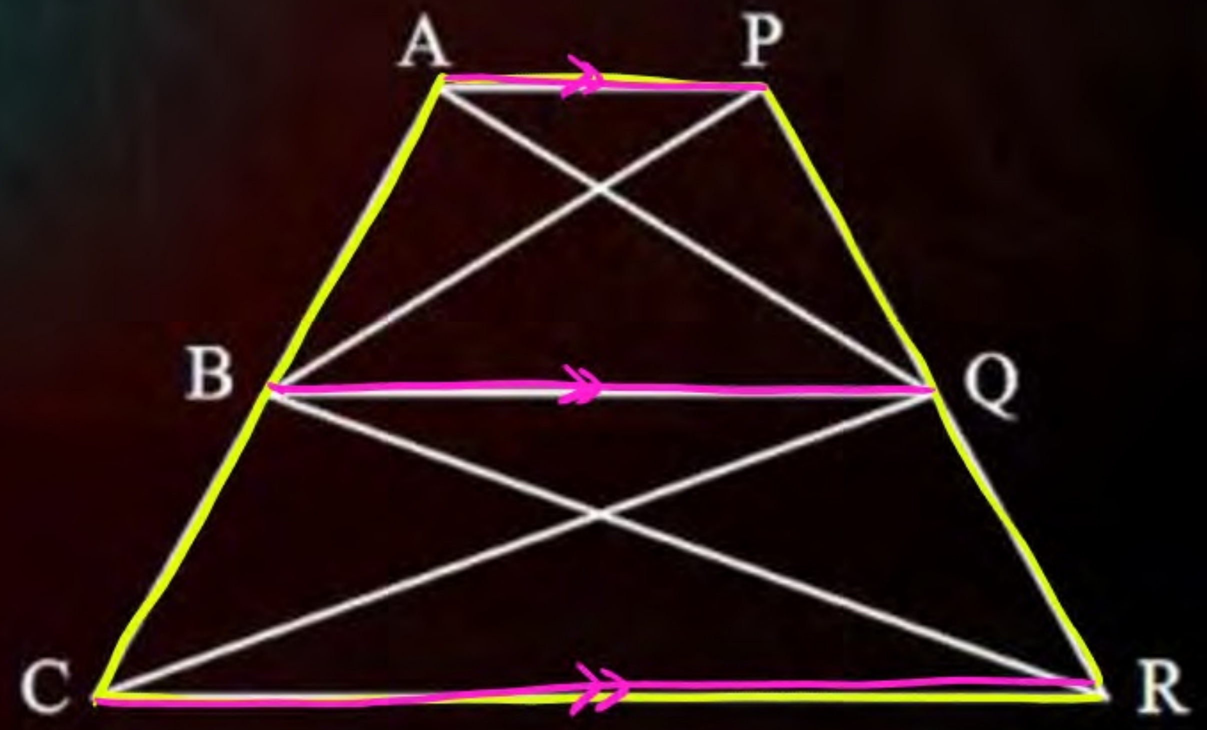
$$\frac{\text{area}(\Delta)}{\text{area of}(\text{Rhombus})} = \frac{1}{2} \Rightarrow 1:2$$



$\boxed{\text{option (A)}}$

Cased-Based Type Questions

Cased-Based: : Rangoli is a traditional form that brightens up an occasion and is believed to be harbinger of good luck. Beautiful patterns are created on the floor using coloured rice, flowers, coloured sand or paints. Keeping up with this thought, the talent and creativity of the students was well brought out in an Inter-class Rangoli making Competition. The students of class 9th made a rangoli in the form of a trapezium APRC as shown in the figure in which AP || BQ || CR.



Question

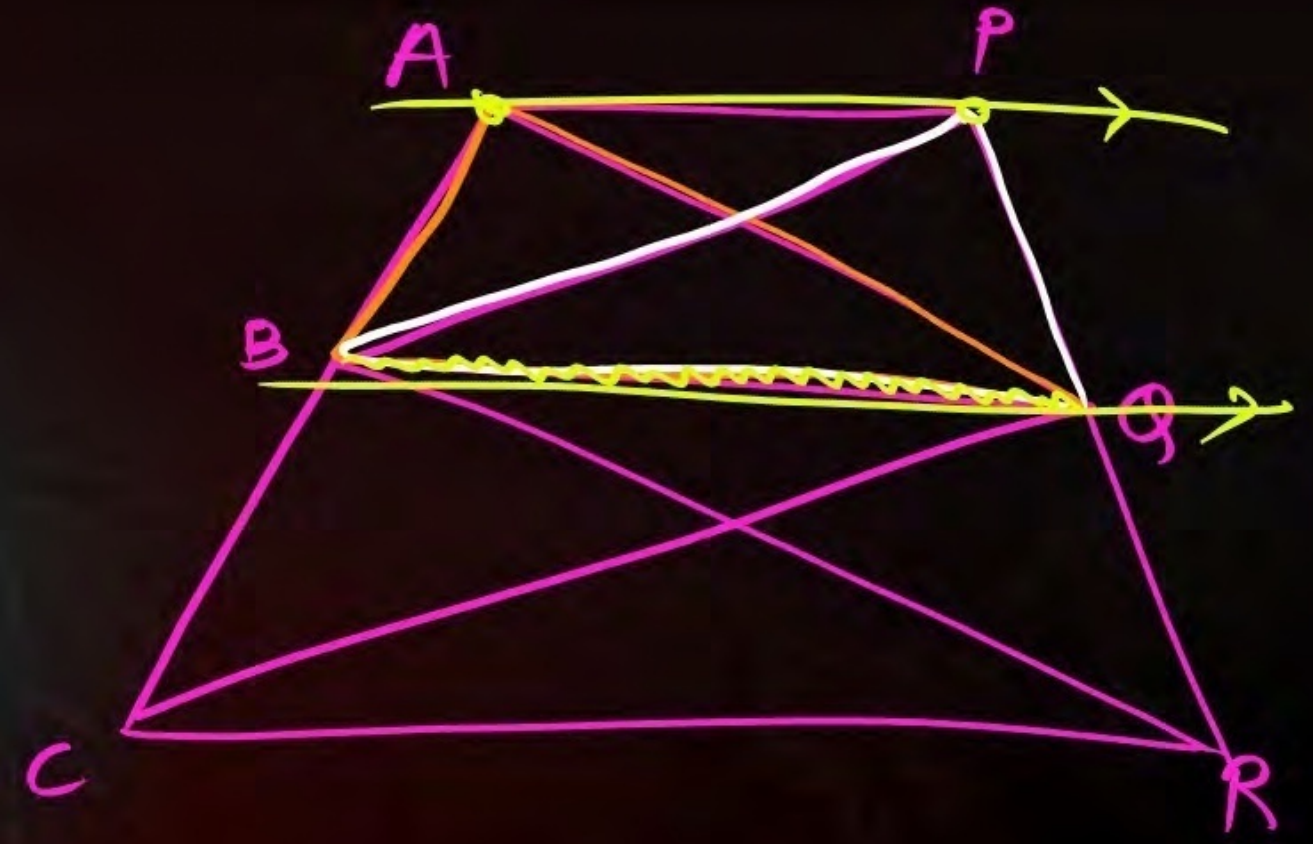
The area of ΔABQ is equal to

ar(ΔBCQ)

ar(ΔAPQ)

ar(ΔPBQ)

ar(ΔAPB)



Question

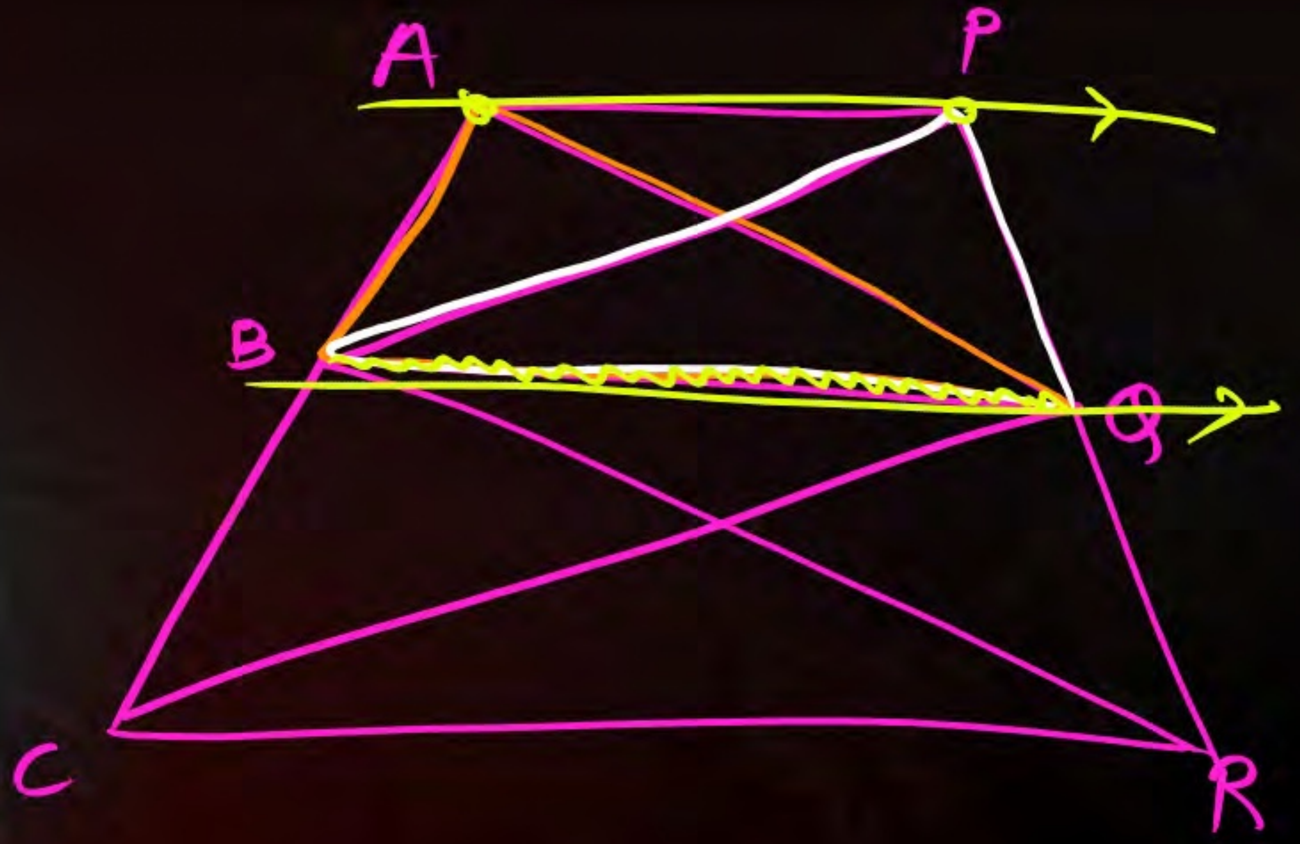
The area of ΔABQ is equal to

A ar(ΔBCQ)

B ar(ΔAPQ)

C ar(ΔPBQ)

D ar(ΔAPB)



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Question

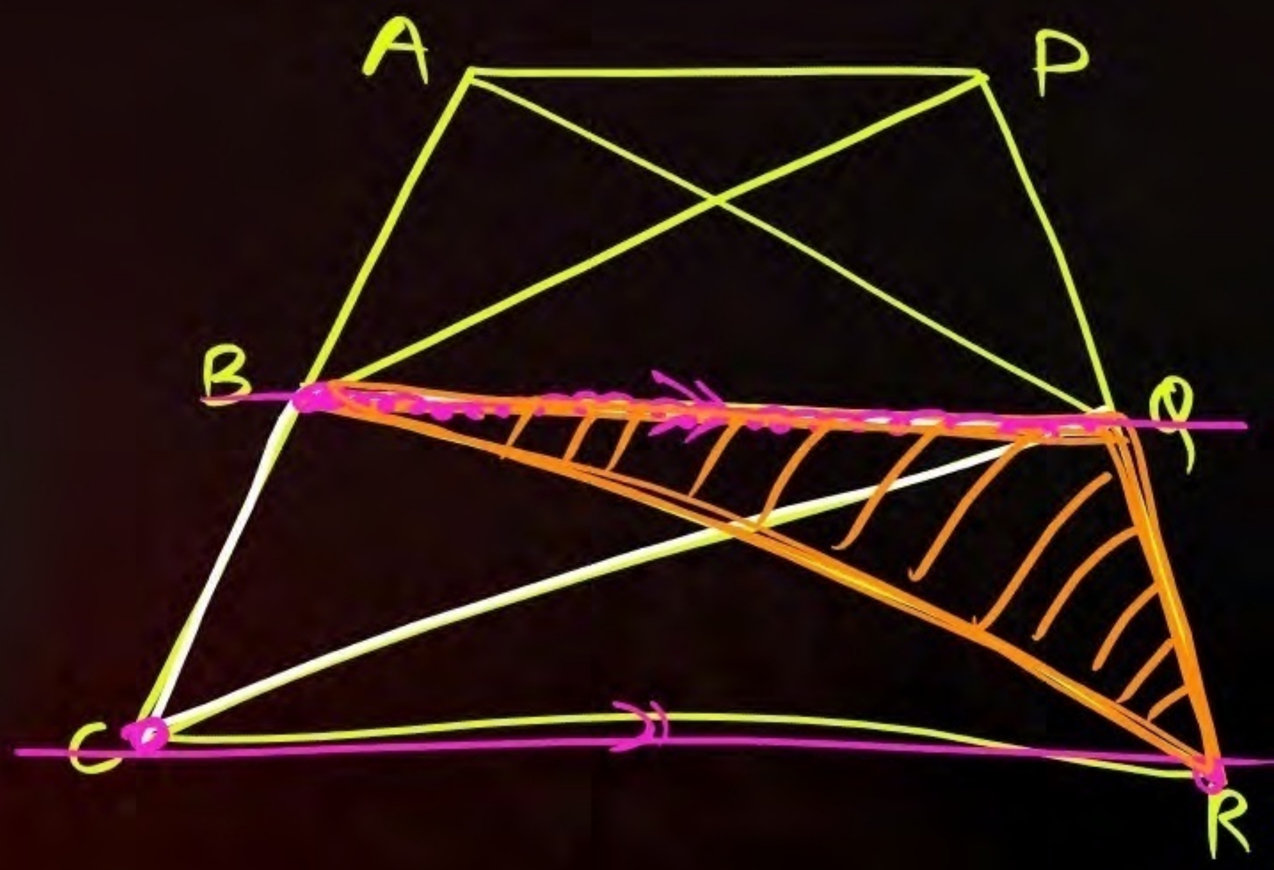
The area of ΔBCQ is equal to

$ar(\Delta BCR)$

$ar(\Delta CRQ)$

$ar(\Delta PBQ)$

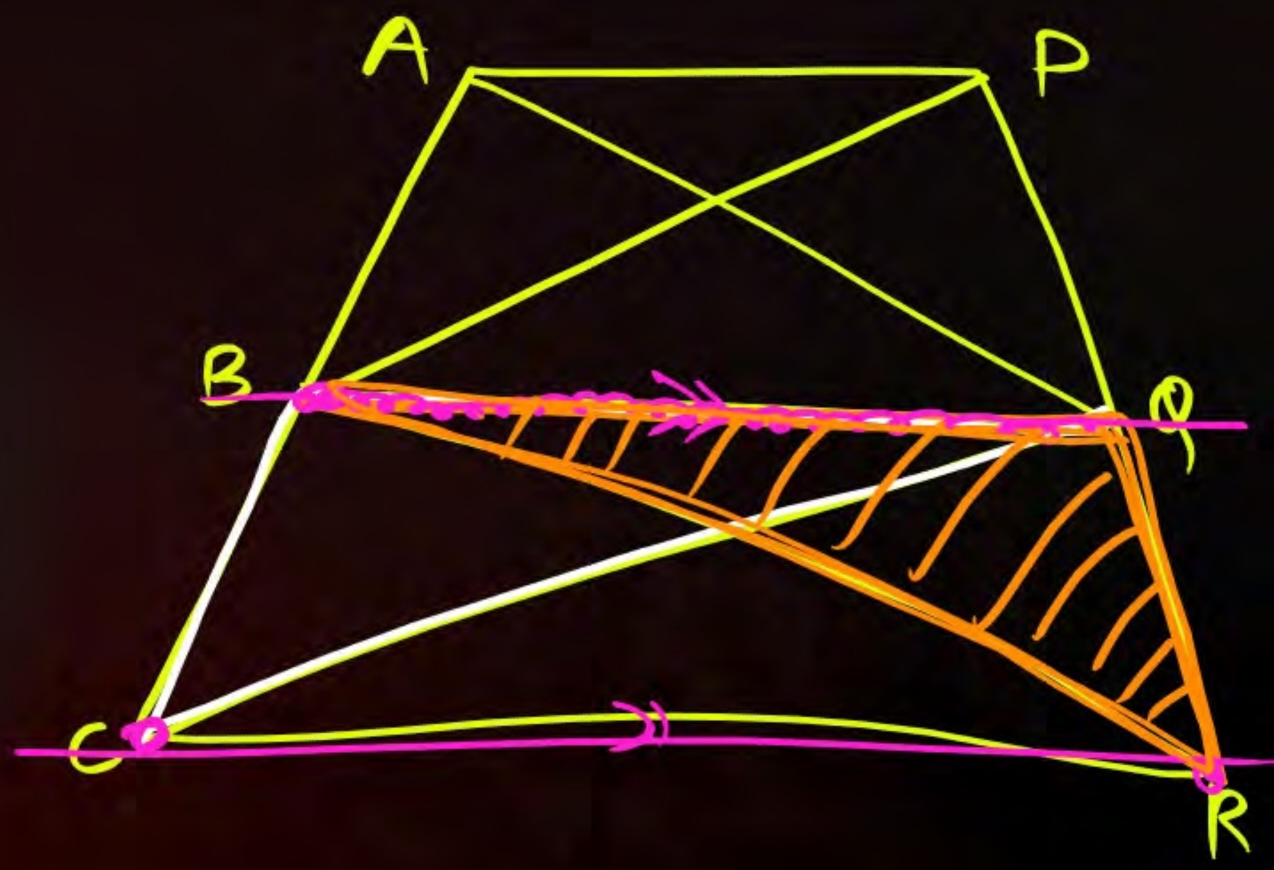
$ar(\Delta BQR)$



Question

The area of ΔBCQ is equal to

- **A** ar(ΔBCR)
- **B** ar(ΔCRQ)
- **C** ar(ΔPBQ)
- **D** ar(ΔBQR)



Question

If two parallelograms are on the same base and between the same parallels, then the ratio of their areas is

- **A** $1:1$
- **B** $1:2$
- **C** $1:3$
- **D** $1:4$

Question

If two parallelograms are on the same base and between the same parallels, then the ratio of their areas is

A $1:1$

B $1:2$

C $1:3$

D $1:4$

Question

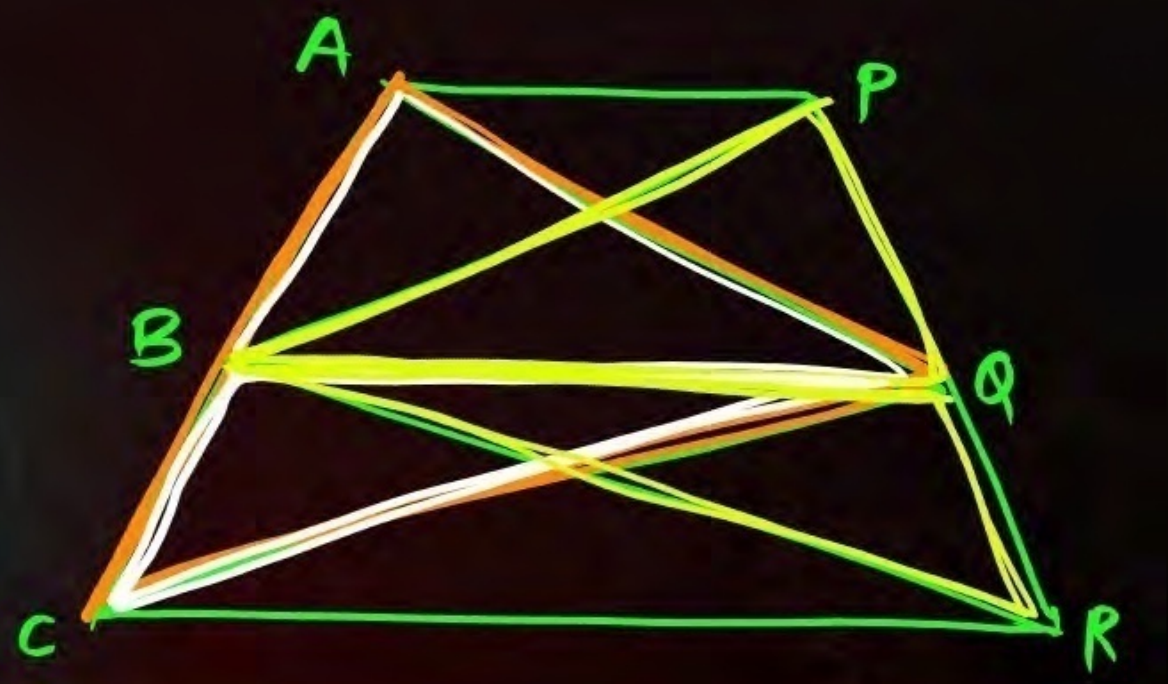
The area of ΔAQC is equal to

ar(ΔBCQ)

ar(ΔPBR)

ar(quadrilateral BCRQ)

ar(ΔAPB)



Question

The area of ΔAQC is equal to

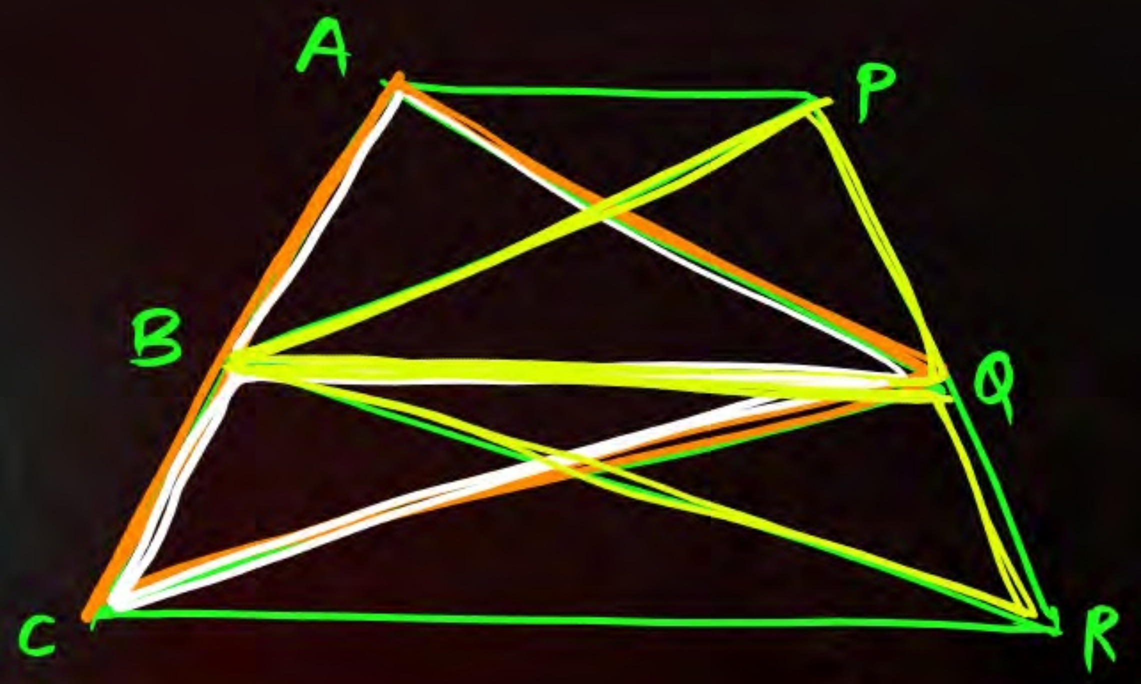
A ar(ΔBCQ)

B $\boxed{\text{ar}(\Delta PBR)}$ ✓

C ar(quadrilateral BCRQ)

D ar(ΔAPB)

$$ar(\Delta ABQ) = ar(\Delta PBQ)$$
$$ar(\Delta BQC) = ar(\Delta BQR)$$
$$(+)$$
$$\underline{ar(\Delta AQC) = ar(\Delta PBR)}$$



Question

In the given figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

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Area of same || gram

$AE \times DC = AD \times FC$

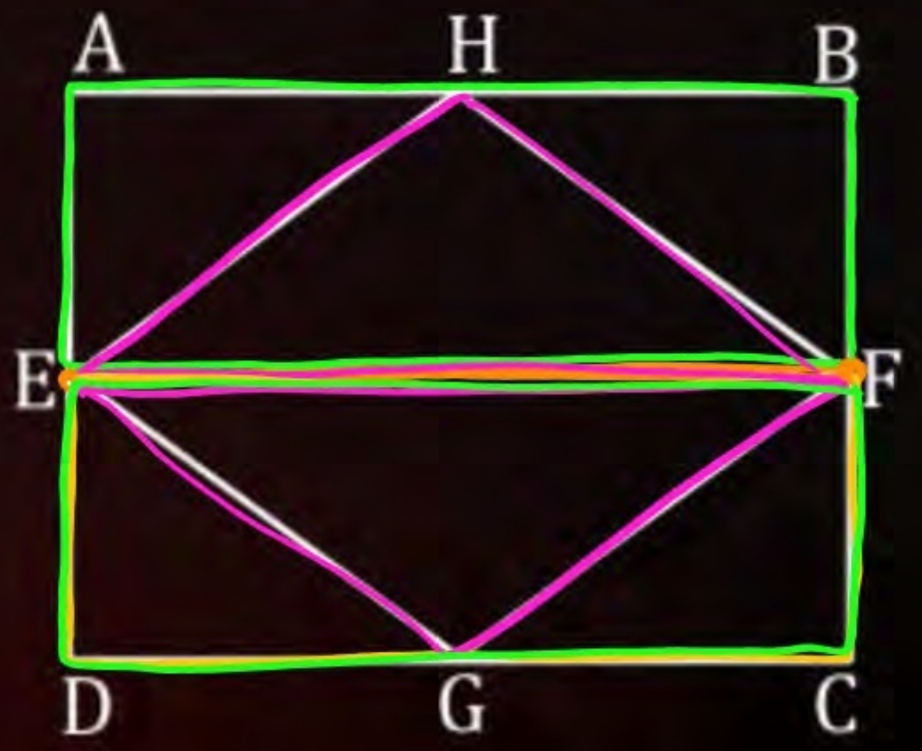
$8 \times AB = AD \times 10$

$8 \times 16 = AD \times 10$

$$\boxed{AD = \frac{8 \times 16}{10}}$$

Question

If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that area(EHFG) = 1/2 area(ABCD).



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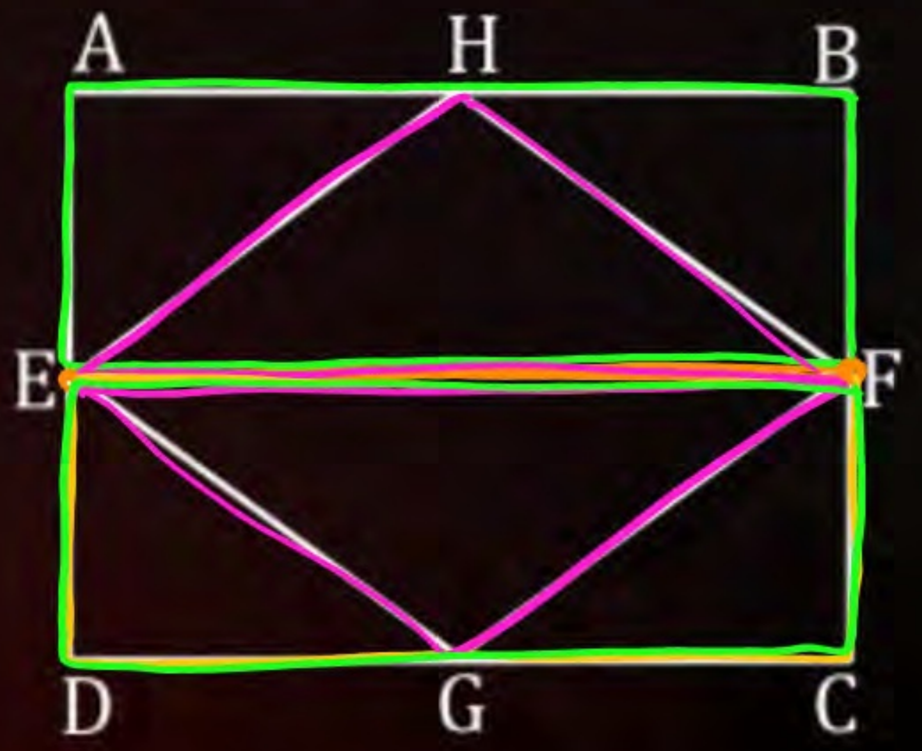


over the base EF,

AB||EF → $\text{Area}(\triangle EHF) = \frac{1}{2}\,\text{ar}(\square EABF)$ --- (1)

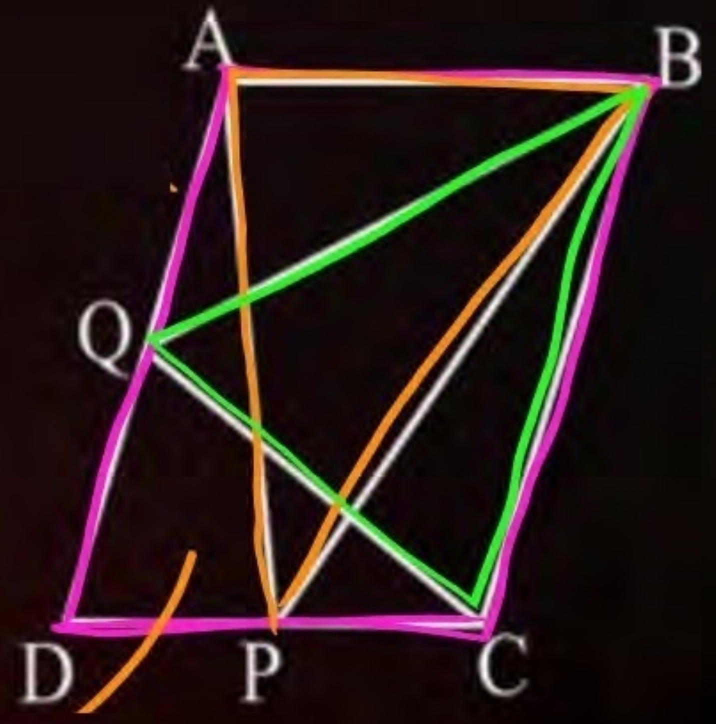
DC||EF → $\text{area}(\triangle EGF) = \frac{1}{2}\,\text{area}(\square EFCD)$ --- (2)

$$\text{ar}(\square EFHG) = \frac{1}{2}\left[\text{ar}(\square EABF) + \text{ar}(\square EFCD)\right]$$

$$\boxed{\text{ar}(\square EFHG) = \frac{1}{2}\,\text{area}(\square ABCD)}$$

Question

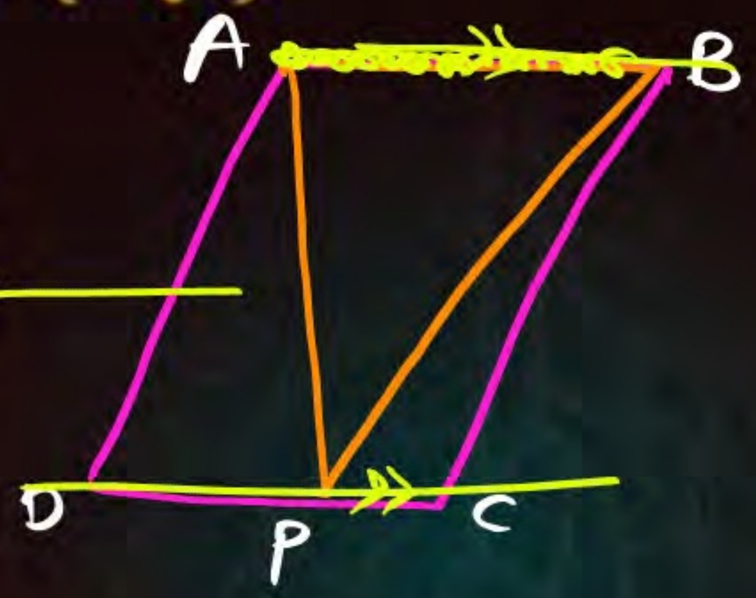
P & Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

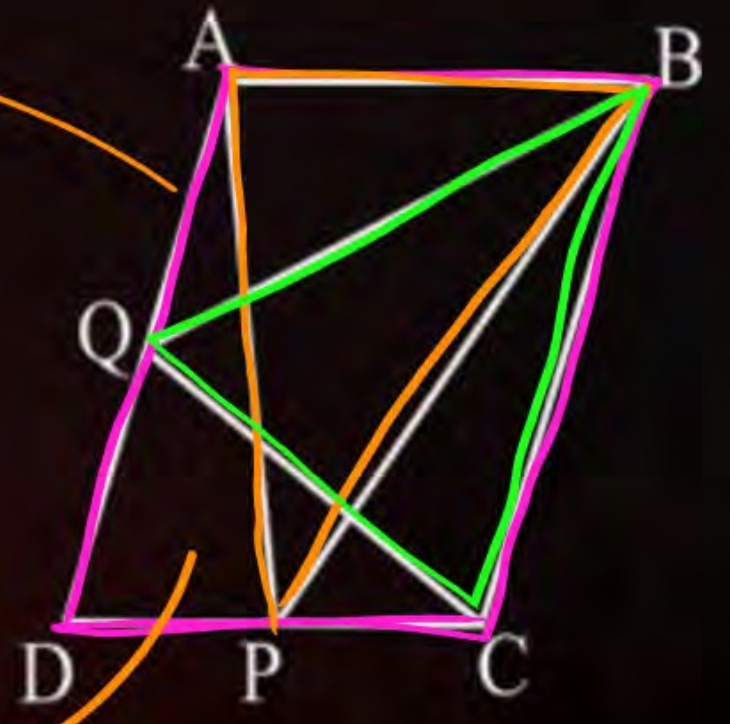


Question

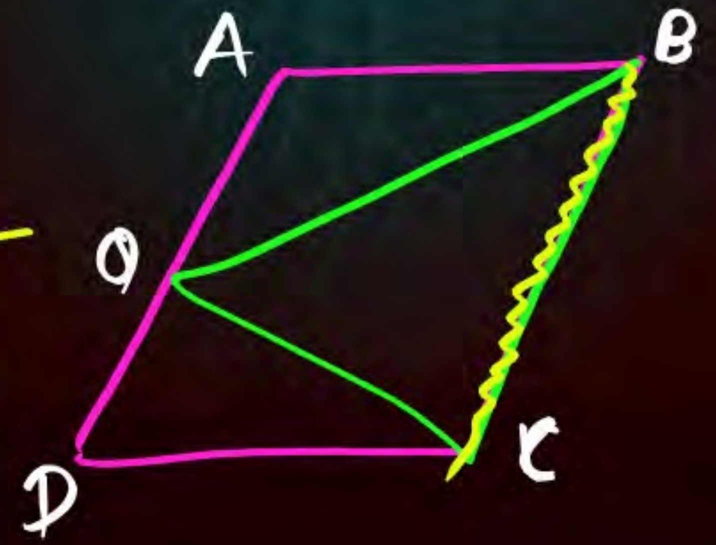
P & Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

$ar(\Delta APB) = \frac{1}{2} ar(\|^{gm} ABCD)$





$ar(\Delta BQC) = \frac{1}{2} ar(\|^{gm} ABCD)$



Thus, $ar(\Delta APB) = ar(\Delta BQC)$

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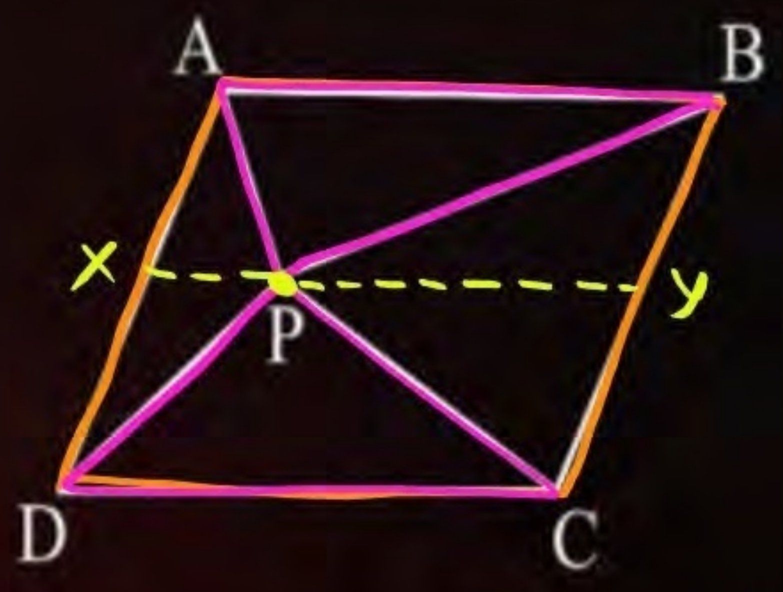
Question

In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

(i) $\boxed{ar (APB) + ar (PCD)} = 1/2 \, ar (ABCD)$

(ii) $ar (APD) + ar (PBC) = \boxed{ar (APB) + ar (PCD)}$

[Hint : Through P, draw a line parallel to AB.]



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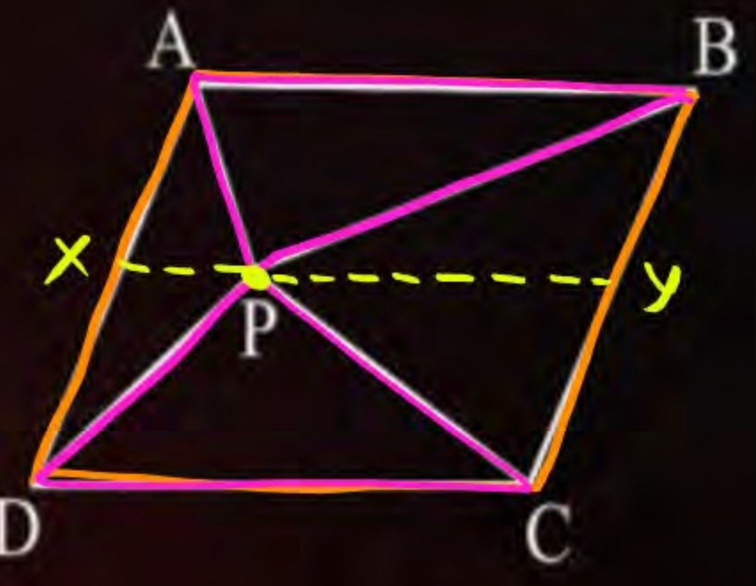
(i) ar (APB) + ar (PCD) = 1/2 ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

[Hint : Through P, draw a line parallel to AB.]



(i) In $\square ABCD$, $AB \parallel CD$

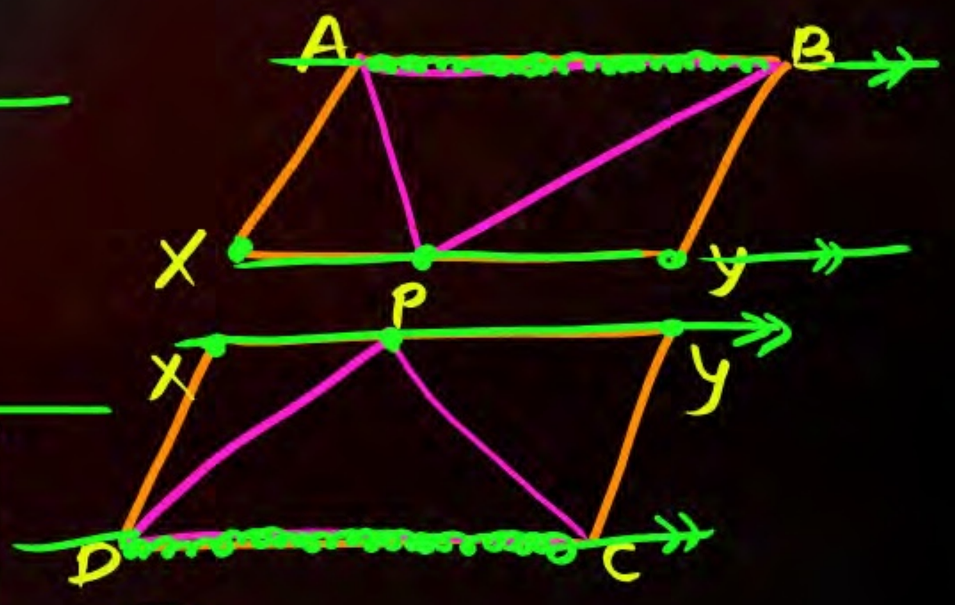
Now, draw a line XY passing through P and $\parallel^{el}$ to AB

Therefore, $\boxed{XY \parallel AB} \parallel CD$

$$ar(\Delta APB) = \frac{1}{2} ar(\parallel^{gram} ABYX)$$

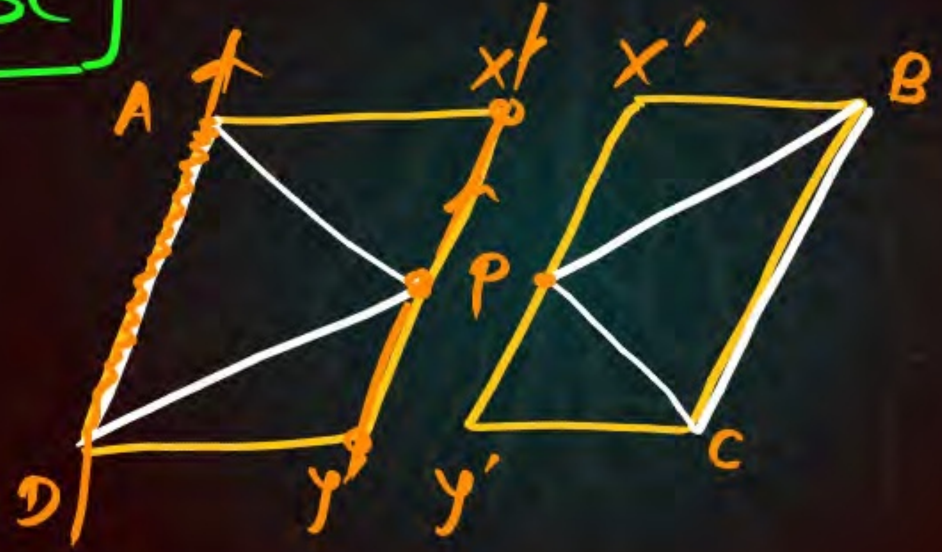
$$ar(\Delta PCD) = \frac{1}{2} ar(\parallel^{gram} DCYX)$$

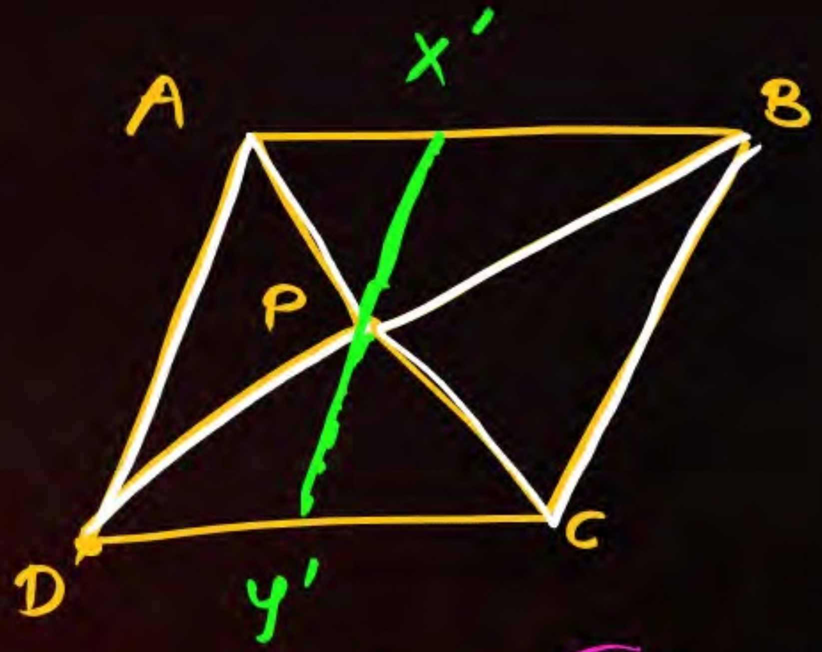
$$\boxed{ar(\Delta APB) + ar(\Delta PCD) = \frac{1}{2}\left[ar(\parallel^{gram} ABCD)\right]} \rightarrow (1)$$



Draw a line $X'Y'$ passing through P and $\|^{el}$ to AB.

Therefore $\boxed{X'Y' \parallel AD \parallel BC}$





$ar(\Delta APD) = \frac{1}{2} ar(\|^{gram} AX'Y'D)$

$ar(\Delta BPC) = \frac{1}{2} ar(\|^{gram} BX'Y'C)$

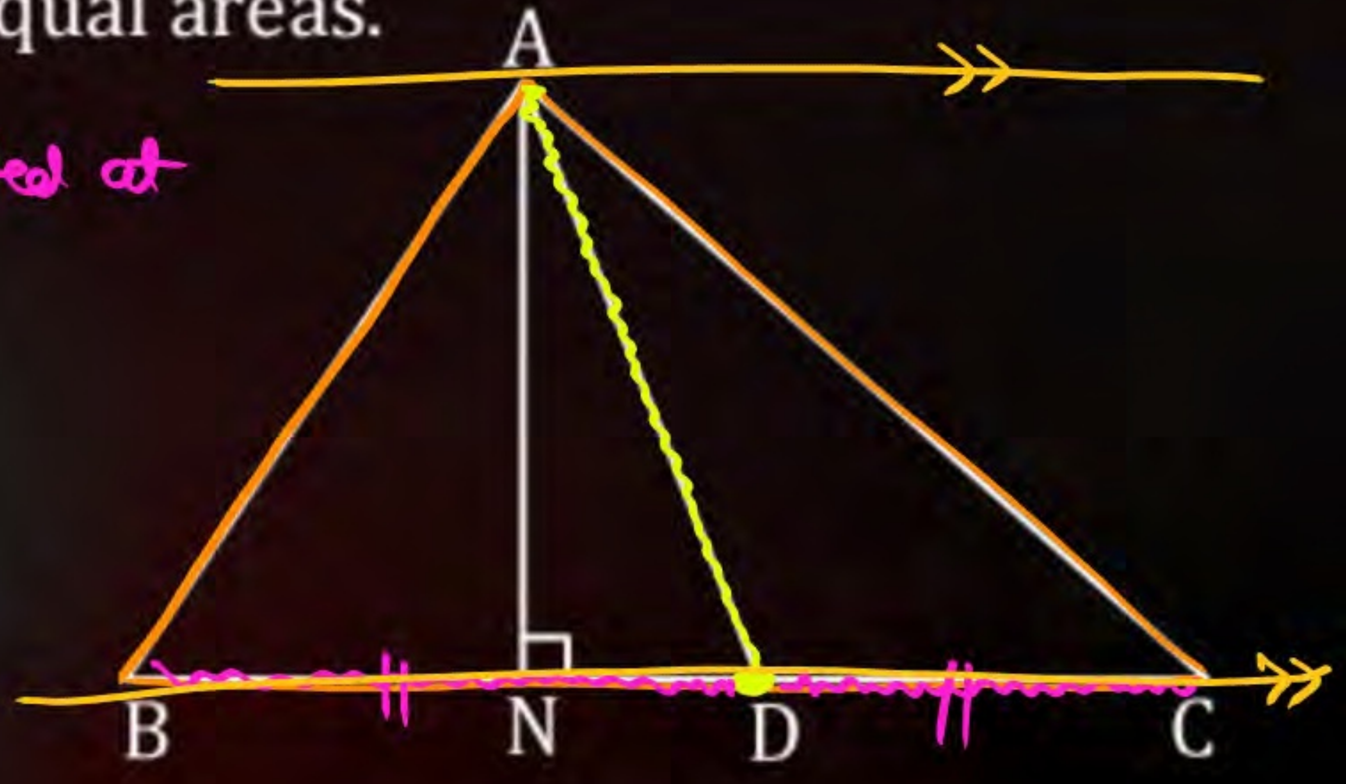
$\boxed{ar(\Delta APD) + ar(\Delta BPC) = \frac{1}{2}(ar(\|^{gram} ABCD))}$ (ii)

From (i) & (ii)

(iii) $\boxed{ar(\Delta APD) + ar(\Delta BPC) = ar(\Delta APB) + ar(\Delta PCD)}$

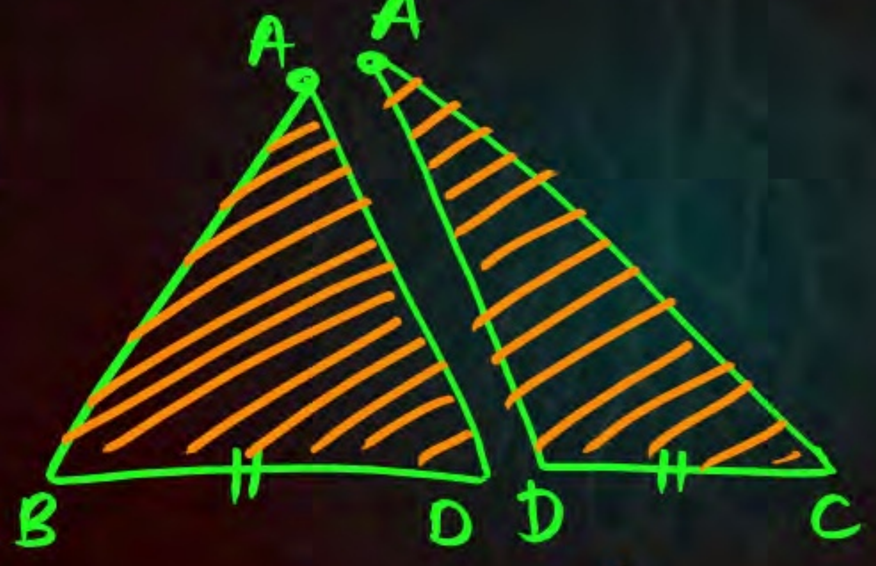
Median of a Triangle

Median of a triangle divides it into two triangles of equal areas.



since, AD is median, therefore BC will be bisected at point D.

Thus, $\boxed{BD = DC}$

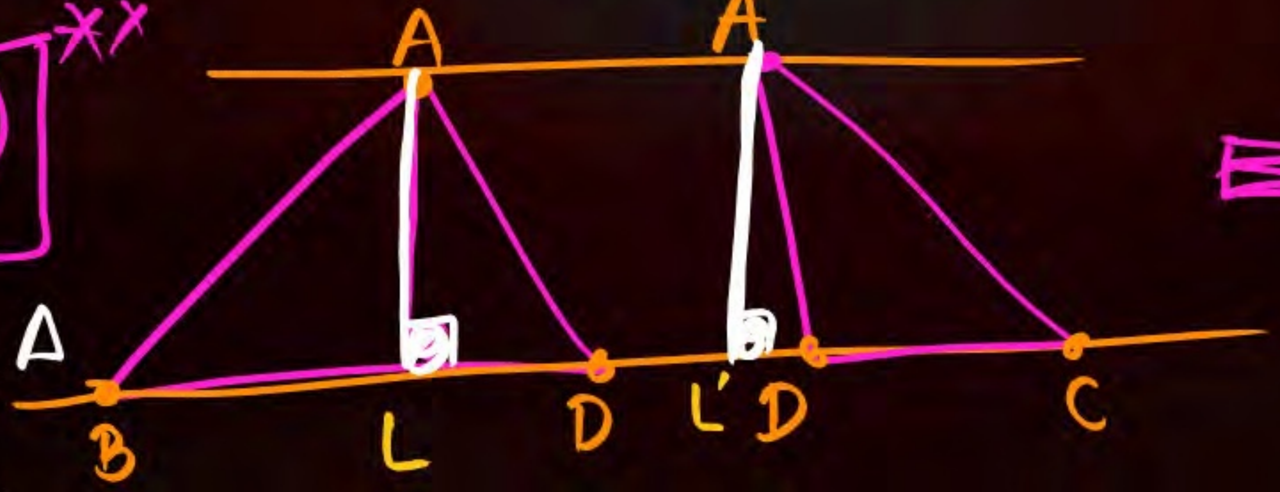


$$\boxed{Ar(\Delta ABD) = \frac{1}{2} \times BD \times AL}$$

$$Ar(\Delta ABD) = \frac{1}{2} \times DC \times AL'$$

$$\boxed{Ar(\Delta ABD) = ar(\Delta ADC)}$$

median AD bisect whole Δ in two Δ with equal area



$\Rightarrow$ $BD = DC$ & $\boxed{AL = AL'}$

Uniform Gap

Question

ABCD is a parallelogram where P is any point on one of its side i.e., CD. If $ar(\Delta DPA) = 15\ cm^2$ and $ar(\Delta APC) = 20\ cm^2$, then $ar(\Delta APB) =$

$15\ cm^2$

$20\ cm^2$

$35\ cm^2$

$30\ cm^2$

Question

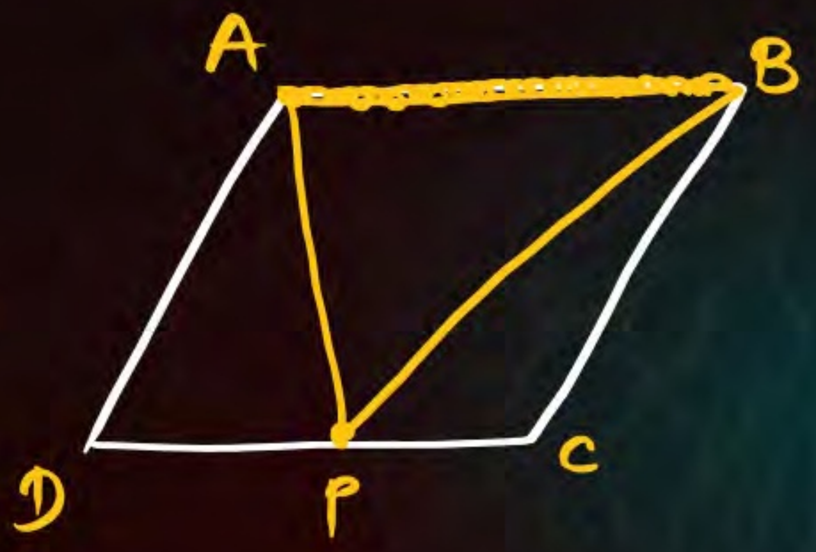
ABCD is a parallelogram where P is any point on one of its side i.e., CD. If ar(ΔDPA) = 15 cm^2 and ar(ΔAPC) = 20 cm^2, then ar(ΔAPB) =

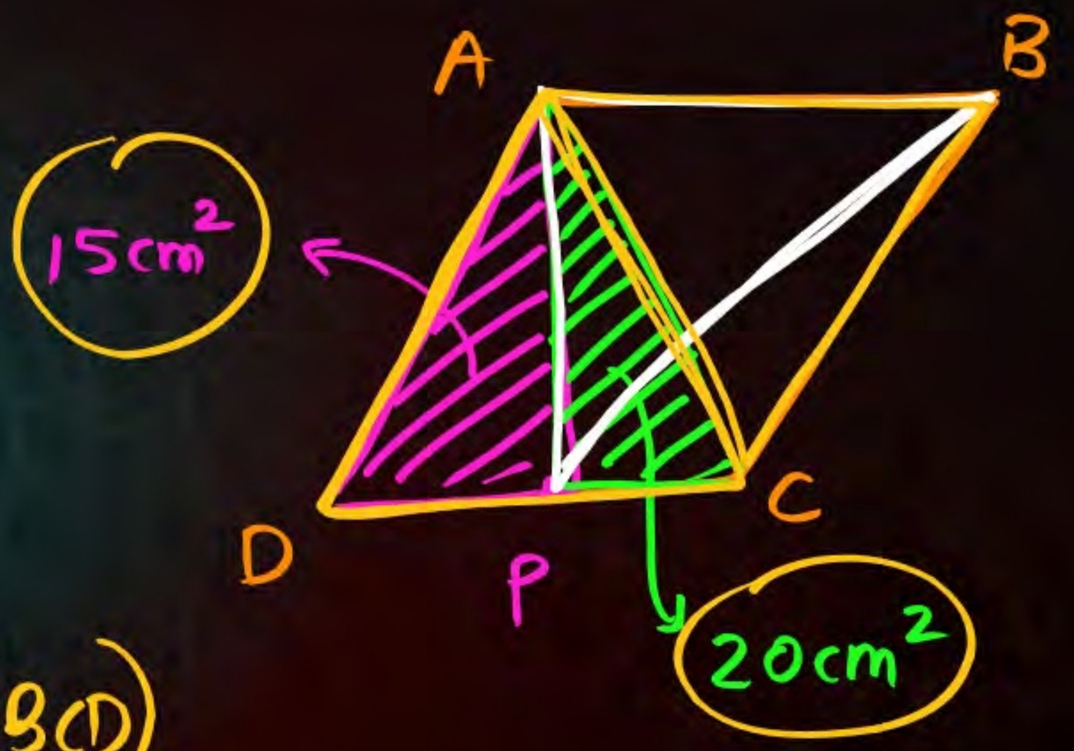
A 15 cm^2

B 20 cm^2

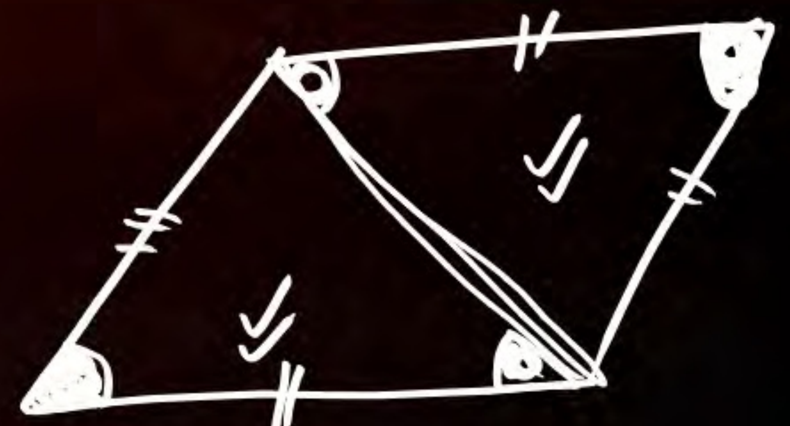
C 35 cm^2

D 30 cm^2



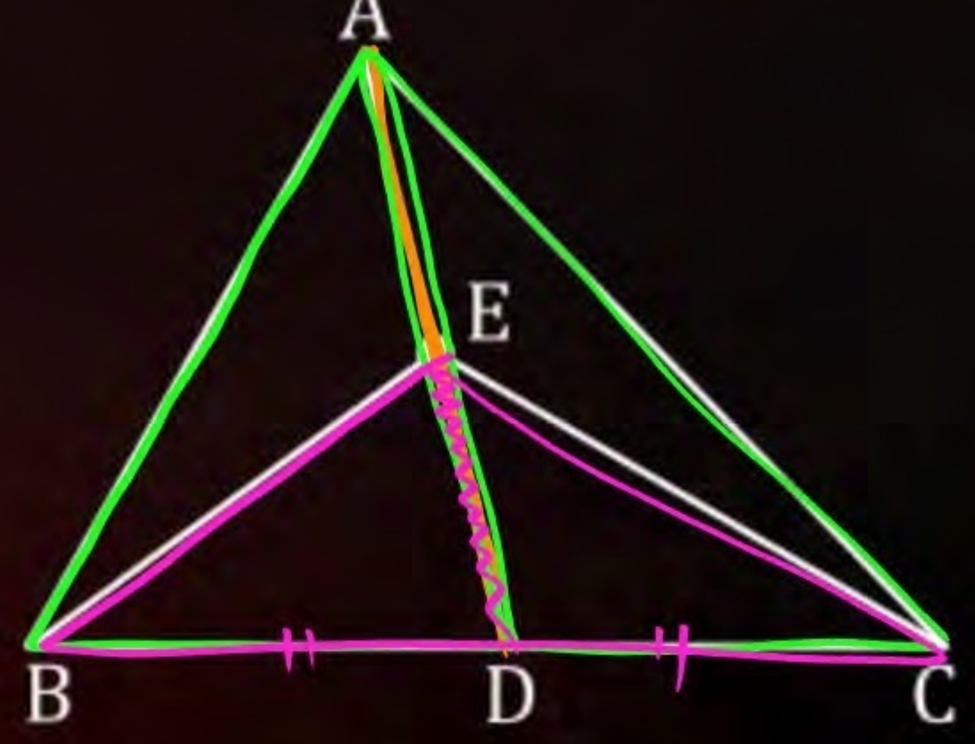


$$Ar(\Delta APB) = \frac{1}{2} ar(\parallel^{gm} ABCD)$$
$$= \frac{1}{2} \times (35 + 35)$$
$$= \frac{1}{2} \times 70 = 35 cm^2$$



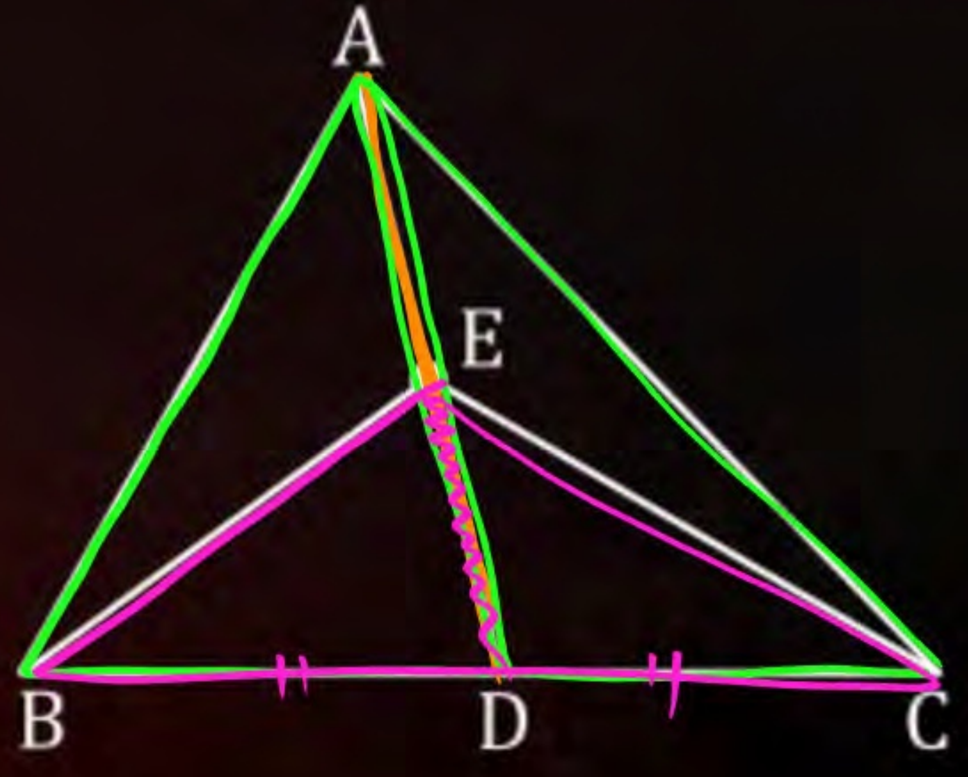
Question

In the given figure, E is any point on median AD of a Δ ABC. Show that : ar (ABE) = ar (ACE).



Question

In the given figure, E is any point on median AD of a Δ ABC. Show that : ar (ABE) = ar (ACE).



since AD is median of ΔABC,

Therefore,

$Ar(\Delta ABD) = ar(\Delta ACD)$ --- (1)

$Ar(\Delta EBD) = ar(\Delta ECD)$ --- (2)

(1) − (2)

$$\boxed{Ar(\Delta ABE) = Ar(\Delta ACE)}$$

Hence, proved!!

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Assertion and Reason Type Problem

Direction: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true, but reason (R) is not the correct explanation of assertion (A).

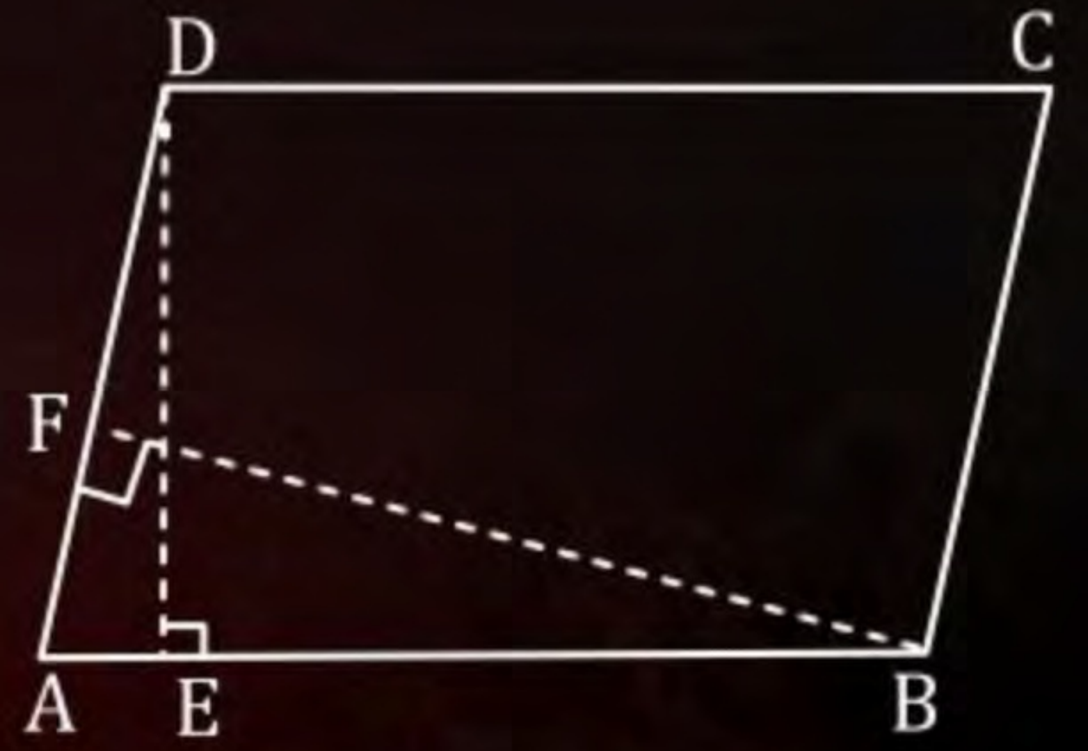
(c) Assertion (A) is true, but reason (R) is false.

(d) Assertion (A) is false, but reason (R) is true.

Question

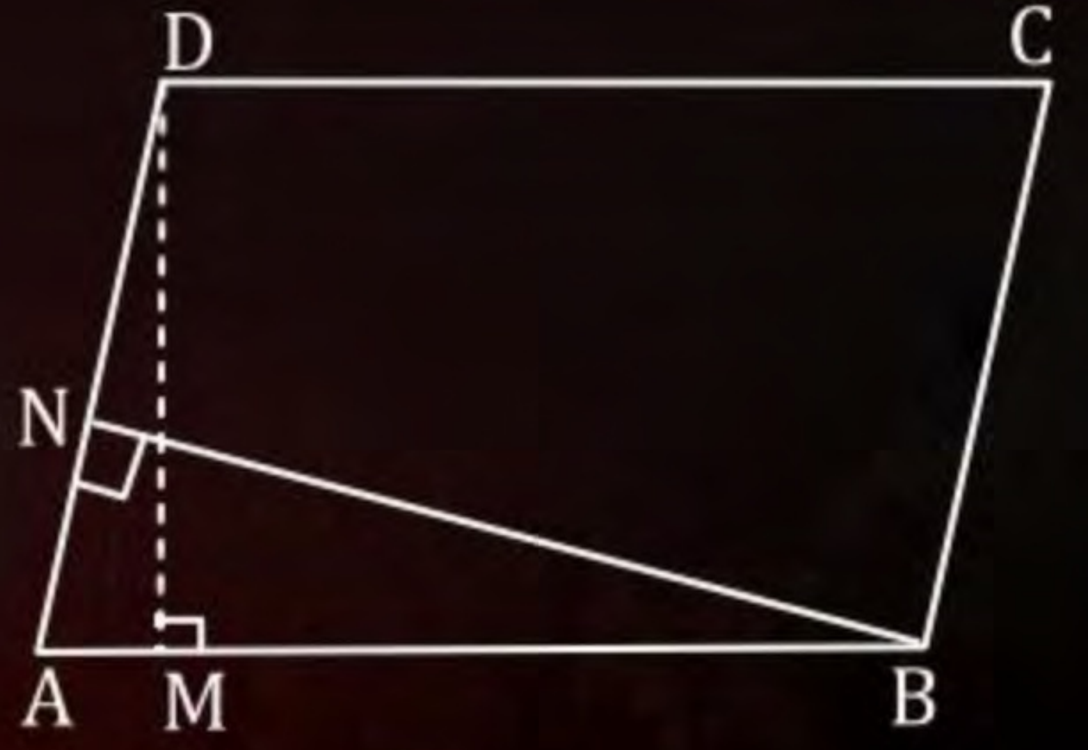
Assertion: In the given figure, ABCD is a ||gm in which DE ⊥ AB and BE ⊥ AD. If AB = 16 cm, DE = 8 cm and BF = 10 cm, then AD is 12 cm.

Reason: Area of a ||gm = base × height.



Question

In parallelogram ABCD, AB = 10 cm. The altitudes corresponding to side AB and AD are 7 cm and 8 cm respectively. Then AD is equal to



A $\frac{35}{4}$ cm

B $\frac{35}{8}$ cm

C 8.75 cm

D 4.375 cm

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Question

In ΔABC, it is given that D is the midpoint of BC, E is the midpoint of BD and O is the midpoint of AE. Then $\boxed{ar(\Delta BOE)} = ?$

$\frac{1}{3} ar(\Delta ABC)$

$\frac{1}{4} ar(\Delta ABC)$

$\frac{1}{6} ar(\Delta ABC)$

$\frac{1}{8} ar(\Delta ABC)$

Question

In ΔABC, it is given that D is the midpoint of BC, E is the midpoint of BD and O is the midpoint of AE. Then, $\boxed{ar(\Delta BOE)}$ = ?

A $\frac{1}{3}$ ar(ΔABC)

B $\frac{1}{4}$ ar(ΔABC)

C $\frac{1}{6}$ ar(ΔABC)

D $\frac{1}{8}$ ar(ΔABC) ✓

$Ar(\Delta ABD) = Ar(ACD)$

$\hookrightarrow Ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$

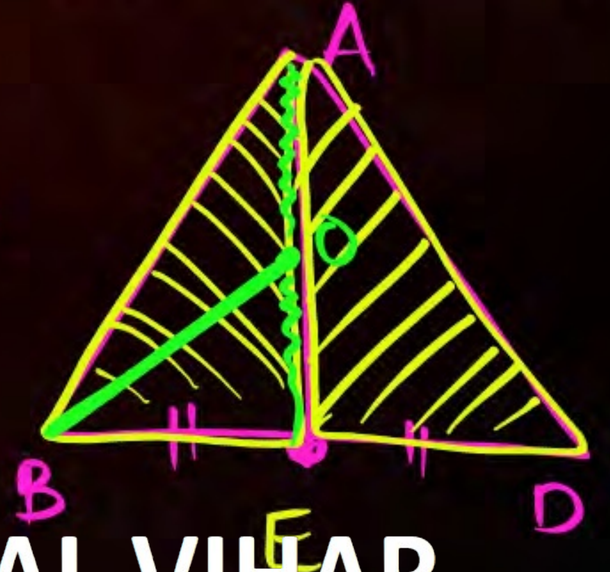
$Ar(\Delta ABE) = \frac{1}{2} \times \boxed{ar(\Delta ABD)}$

$Ar(\Delta ABE) = \frac{1}{2} \times \frac{1}{2} ar(\Delta ABC)$

$Ar(\Delta BOE) = \frac{1}{2} \times \boxed{ar(\Delta ABE)}$

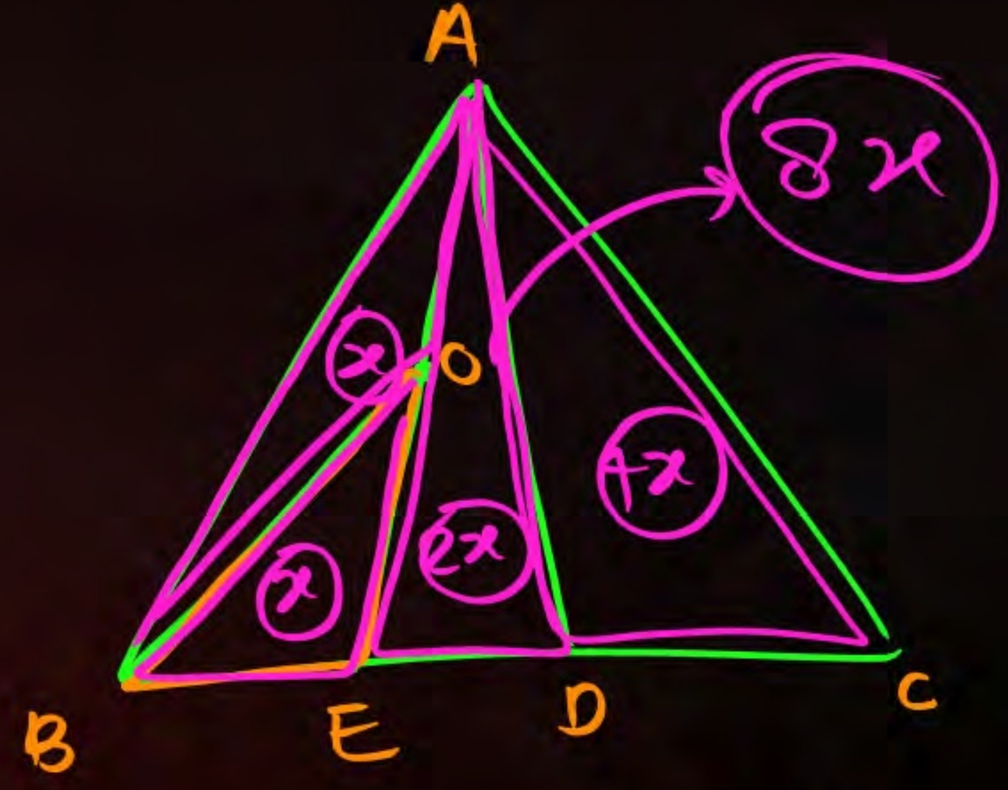
$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times ar(\Delta ABC)$

$= \boxed{\frac{1}{8} \times ar(\Delta ABC)}$



$$8 \times ar(\Delta BOE) = Ar(\Delta ABC)$$

$$\boxed{ar(\Delta BOE) = \frac{1}{8}\, ar(\Delta ABC)}$$



$8x = ar\,\Delta ABC$

$8 \times ar\,\Delta BOE = ar\,\Delta ABC$

$\boxed{ar\,\Delta BOE = \frac{1}{8}\, ar\,\Delta ABC}$

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THANK YOU