VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

- Subject - Mathematics
- Chapter – Number System

Today's Targets

1. Recalling Numbers and Number System
2. Rational Number, Representation of Rational Number, Rational Number Between Two Rational Numbers
3. Decimal Part of Rational Number, Conversion of decimals into p/q form
4. Irrational Numbers, Representation over number line,
5. Power and Exponents
6. Rationalization and Rationalizing Factor

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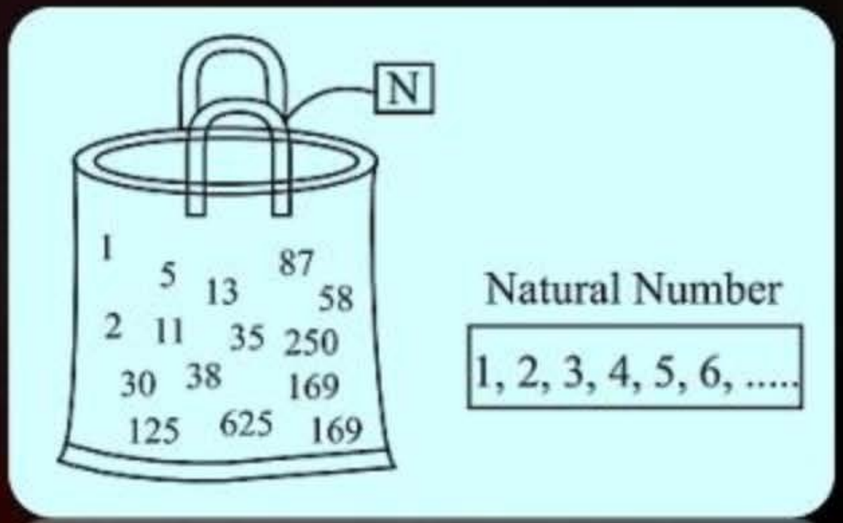
Let's recall what we have already studied i.e.,

Natural Numbers : The numbers used for counting objects are called Natural numbers. These are 1, 2, 3, 4, 5, 6..... and so on.
The set of natural numbers is denoted by N.

Note: 1 is the smallest natural number.

Counting Number

$N = \{1, 2, 3, 4, 5, \ldots\}$

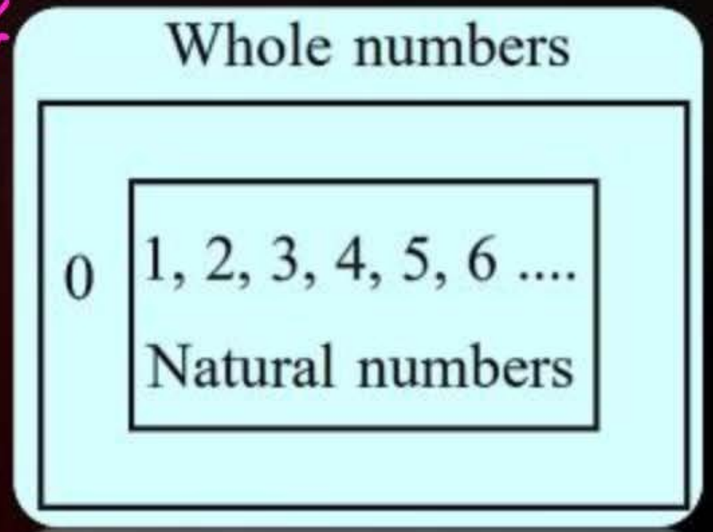


Let's recall what we have already studied i.e.,

Whole Numbers : The natural numbers together with zero are called the whole numbers.

Thus, W = {0, 1, 2, 3, 4, 5, 6 .....} = {0 + Natural No.}

Note: 0 is the smallest whole number



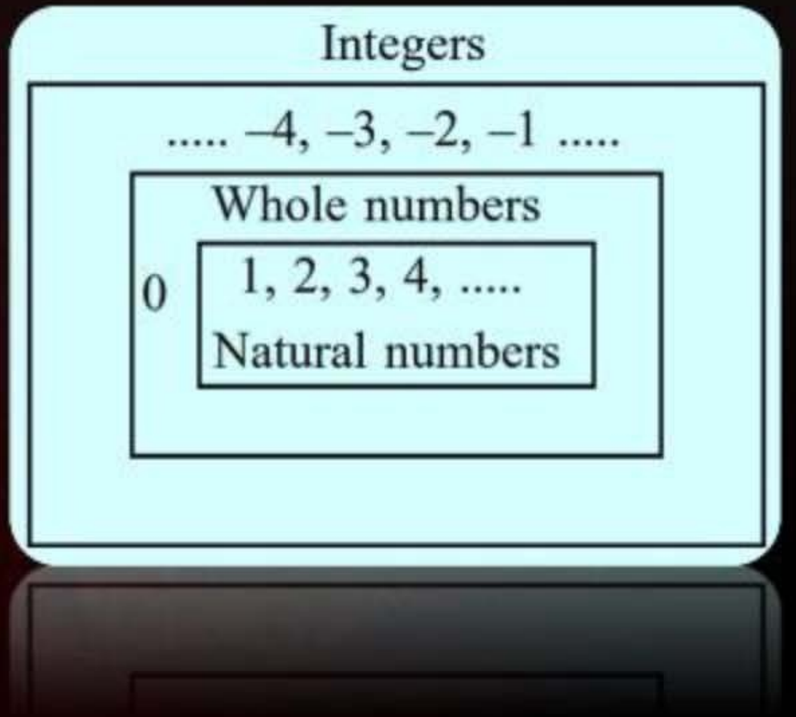
Let's recall what we have already studied i.e.,

Integers : All natural numbers, 0 and negatives of natural number together are known as integers.

The set of integers is Denoted by I or Z.

Thus, I or Z = {.... –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}

Natural No.s + whole No
with (–)ve sign
→ Integers



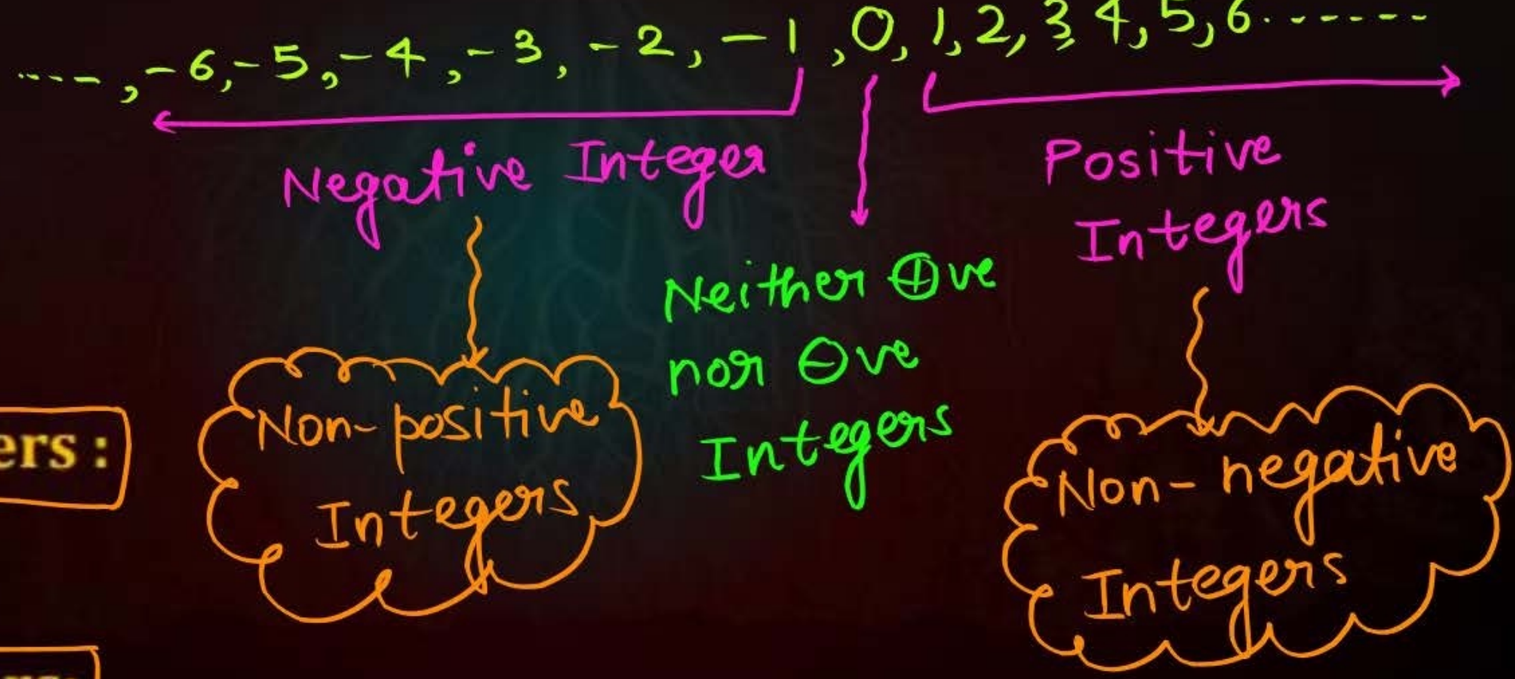
Ye important Hai

Negative integers:

Positive integers :

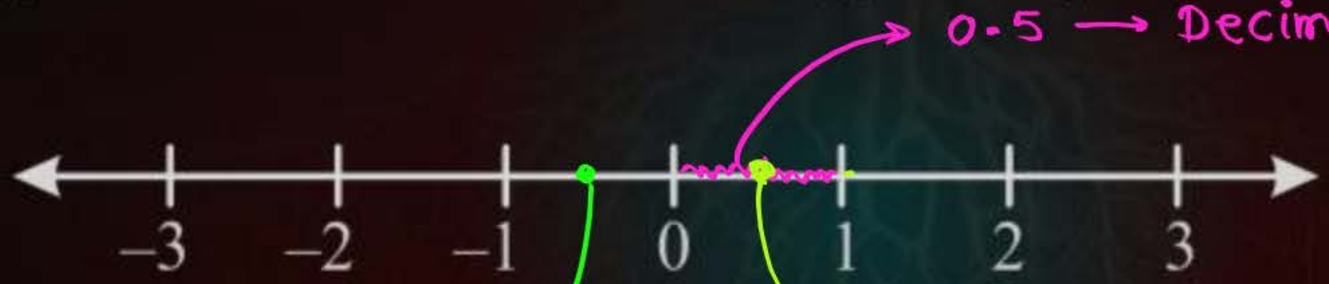
Non-negative integers :

Non-positive integers:



Number Line

A straight line on which points are identified with real numbers is called a number line. At regular intervals, successive integers are placed on the number line.



$\frac{1}{2}$ ⇝ fractional

(i) $q \neq 0$

$\frac{p}{q} \Leftarrow \frac{-1}{2}$

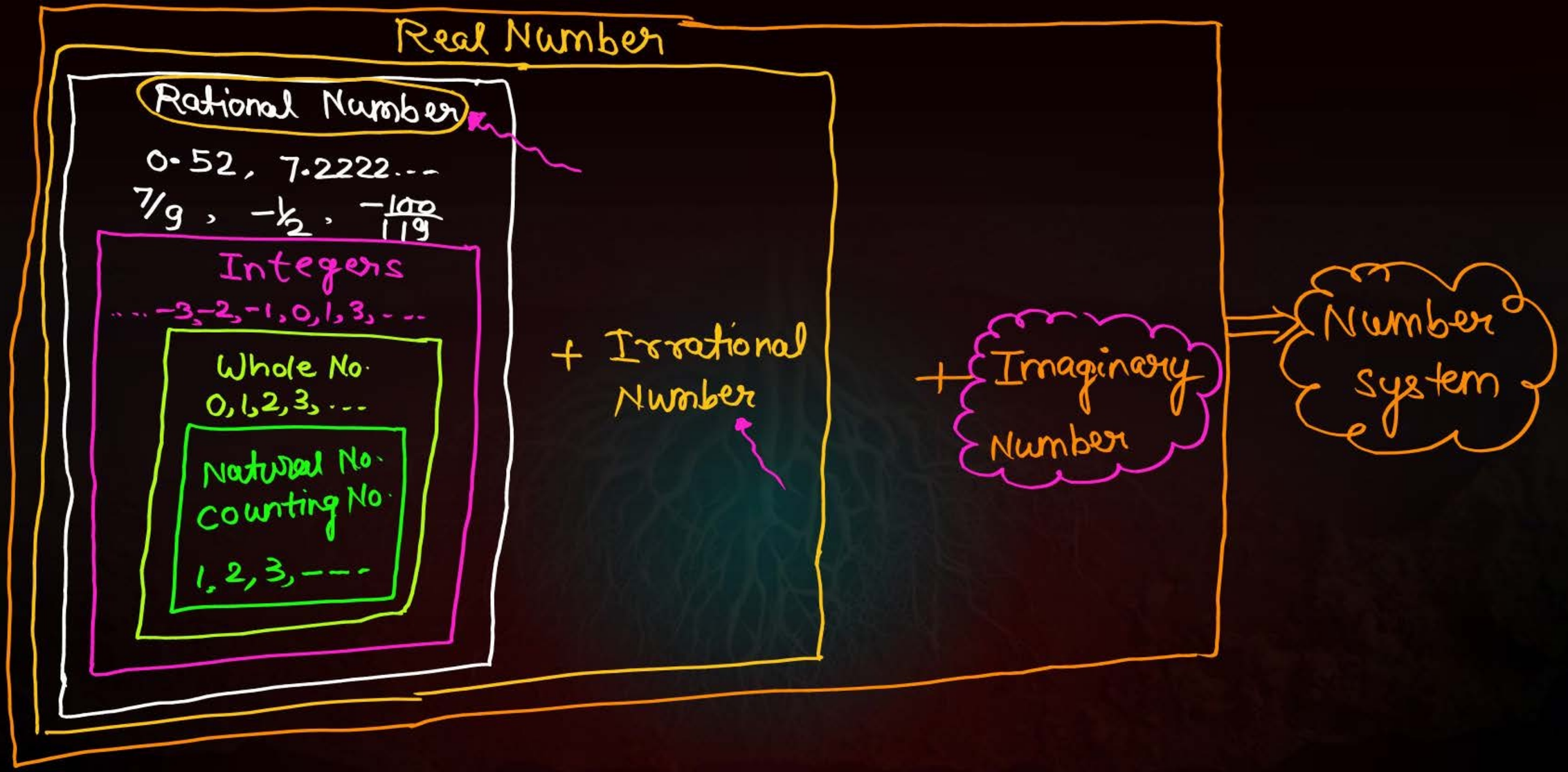
(ii) p & q

Integers → Rational Number

$\frac{1}{2} \Rightarrow \frac{p}{q}$

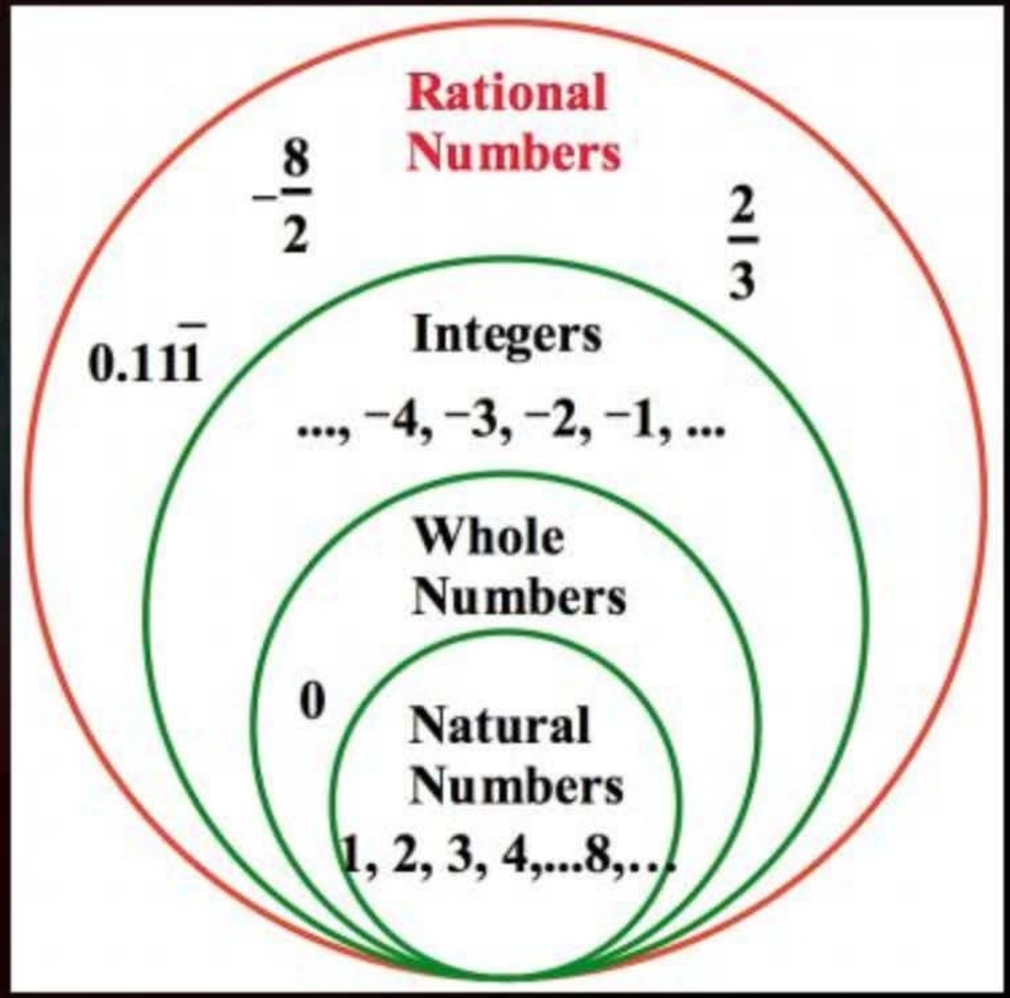
(i) $q \neq 0$ (ii) p, & q

Whole Number



Ab Ye Kya Hai ???

Rational Number



Rational Numbers

A number that can be written in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Example : $\frac{5}{8}, \frac{-5}{6}, \frac{7}{-5}, 3, -5, 0$

$3 = \frac{3}{1}$, $-5 = \frac{-5}{1}$

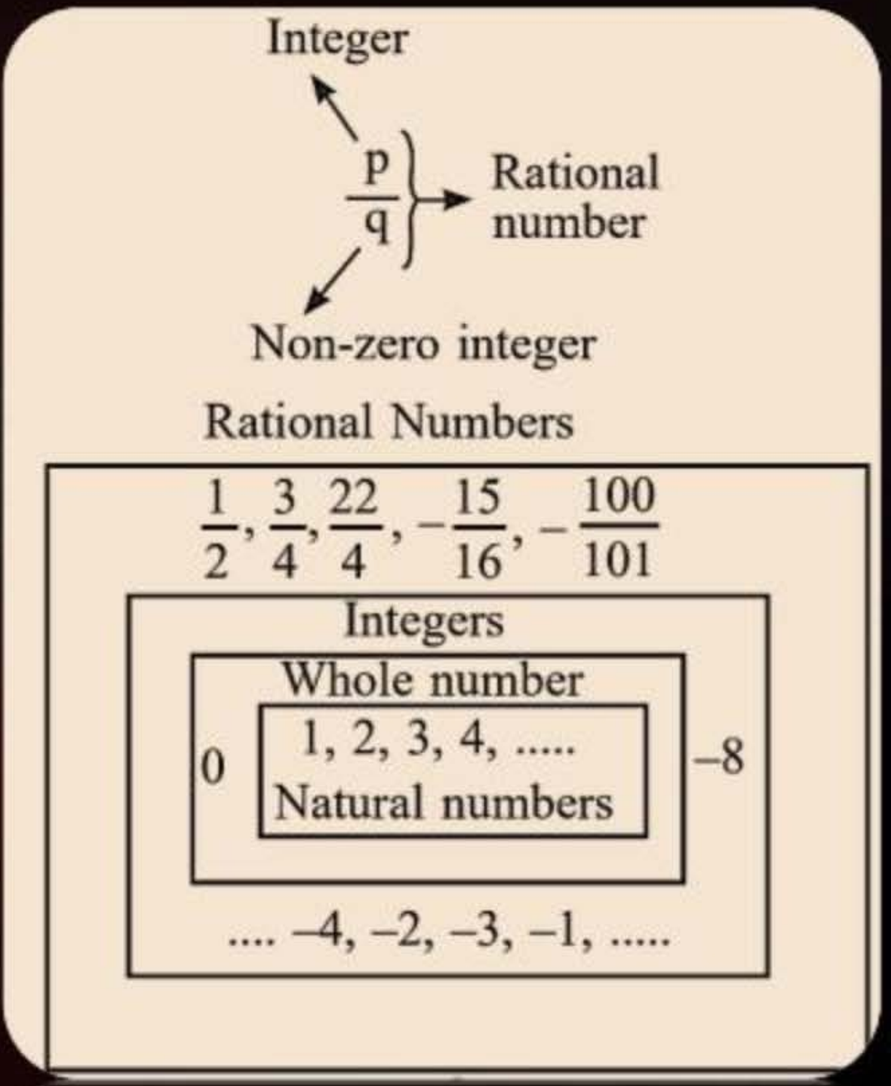
Rational Number

$\frac{p}{q}$ form

i $q \neq 0$

ii $p \neq q$

Integers



Are these Rational Numbers?

Natural Numbers

Whole Numbers

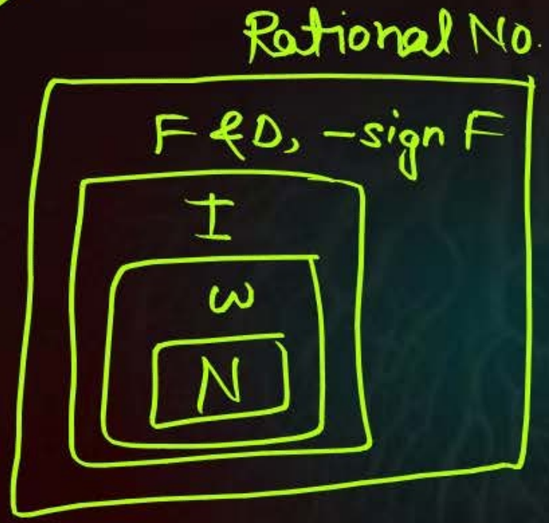
Integers :

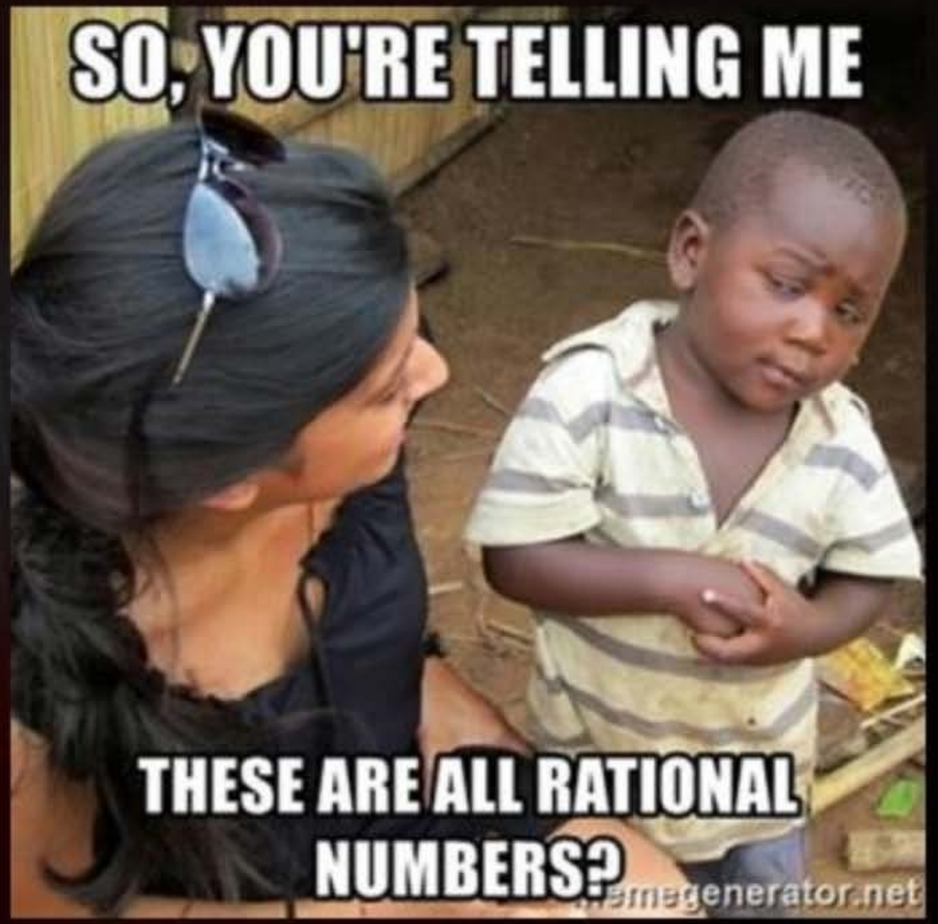
Are these Rational Numbers?

Natural Numbers : ✓

Whole Numbers : ✓

Integers : ✓





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State true or false : -

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

(iv) Every natural number is a whole number.

(v) Every integer is a whole numbers.

(vi) Every rational number is a whole number.

State true or false : -

(i) Every whole number is a natural number. (F)

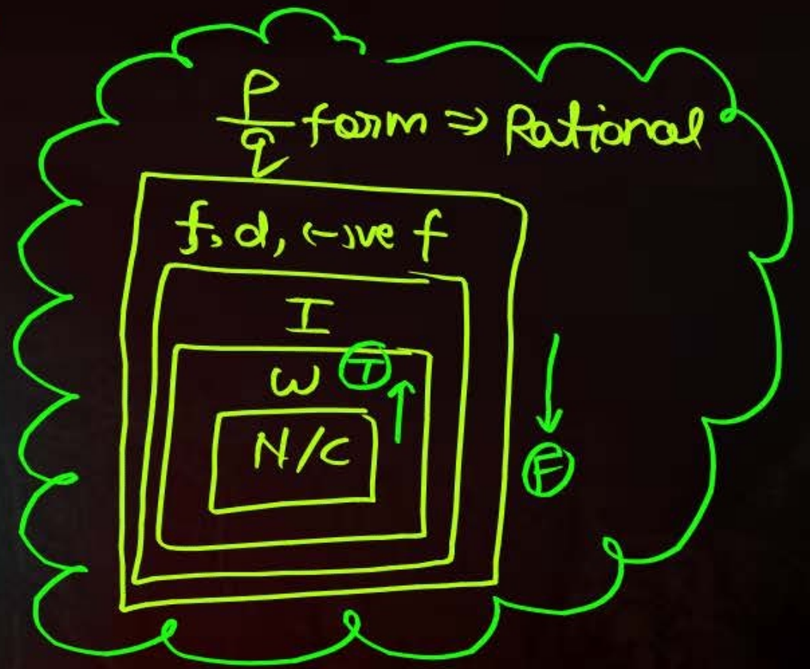
(ii) Every integer is a rational number. (T)

(iii) Every rational number is an integer. (F)

(iv) Every natural number is a whole number. (T)

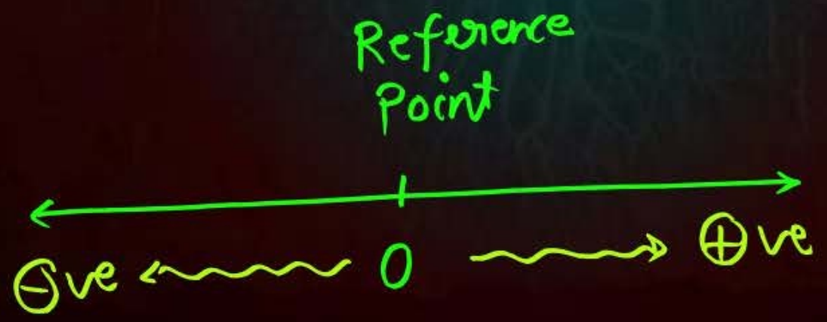
(v) Every integer is a whole numbers. (F)

(vi) Every rational number is a whole number. (F)



Representation of Rational Numbers

Representation of rational numbers on a number line is very similar to the representation of fractions on a number line. On a number line, keeping '0' as the reference, the left-hand side of '0' represents the negative region, and the right-hand side of '0' represents the positive region.



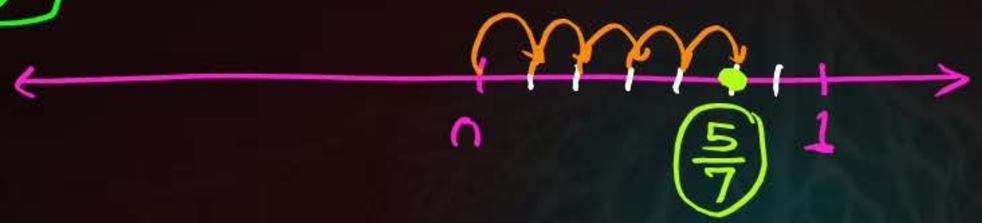
Question

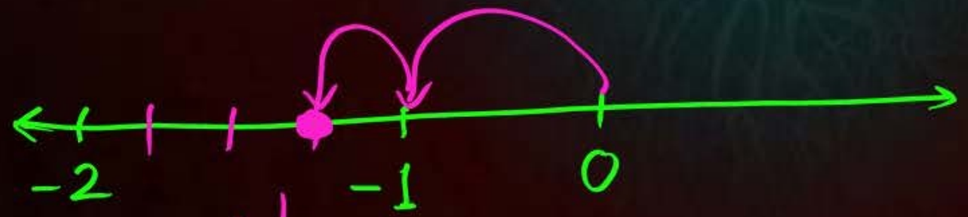
How to Represent Rational numbers $\frac{5}{7}$ and $\frac{-5}{4}$ on the number line?

Question

How to Represent Rational numbers $\frac{5}{7}$ and $\frac{-5}{4}$ on the number line?

$-\frac{5}{4} = -1\frac{1}{4}$





① Proper Rational No.

smaller $\frac{N}{D}$, Bigger

N < D $\Rightarrow$ 0 ..... 1 (+), -1 ..... 0 (−)

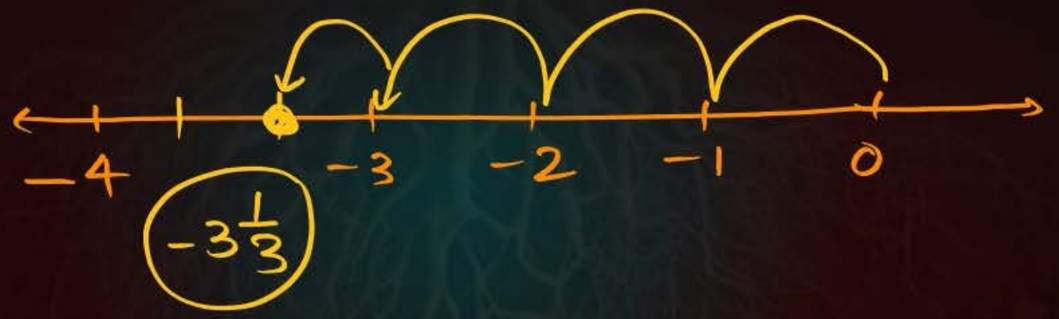
② Improper Rational

$\frac{N}{D}$, N > D → Process → Mixed fraction

③ Mixed → $I + \frac{N}{D}$ → Proper

Represent $-3\frac{1}{3}$ over the number the number line.

Represent $-3\frac{1}{3}$ over the number the number line.



Question

Is zero a rational number? Can you write it in the form $\frac{p}{q}$, when p and q are integers and $q \neq 0$?

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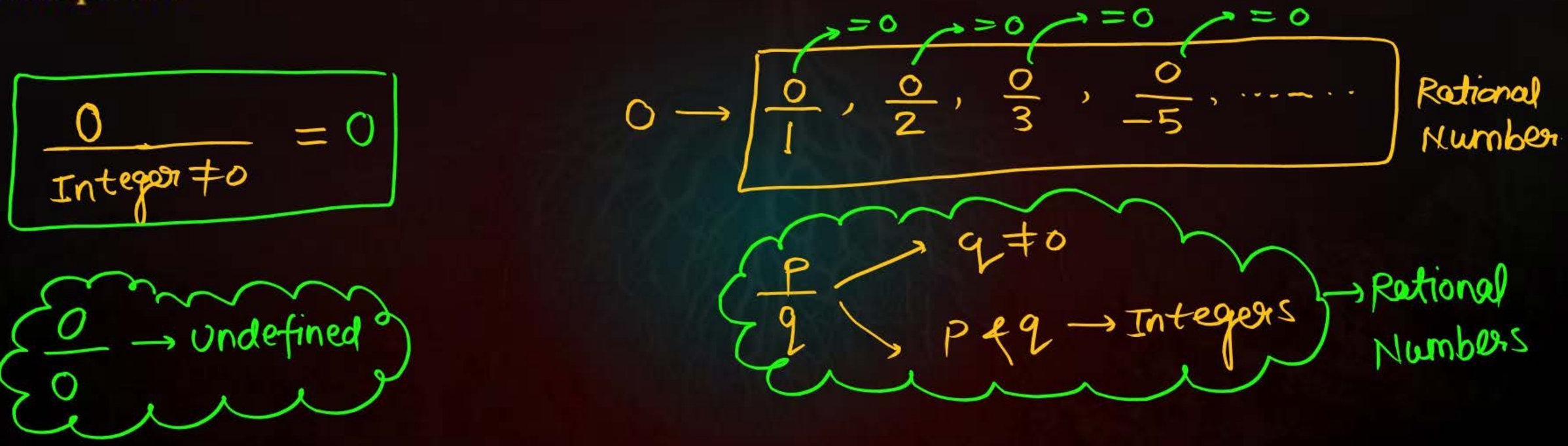
Is zero a rational number? Can you write it in the form $\frac{p}{q}$, when p and q are integers and $q \neq 0$?



Question

State true or false!

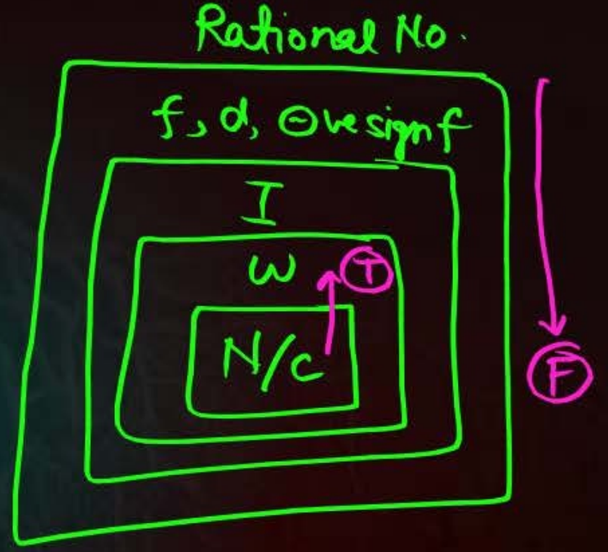
Every rational number is a whole number.

Question

State true or false!

Every rational number is a whole number.

False



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Inserting Rational numbers between two rational numbers

Mean or Average

Trick : $\frac{a+b}{2}$ for two Rational numbers a & b.

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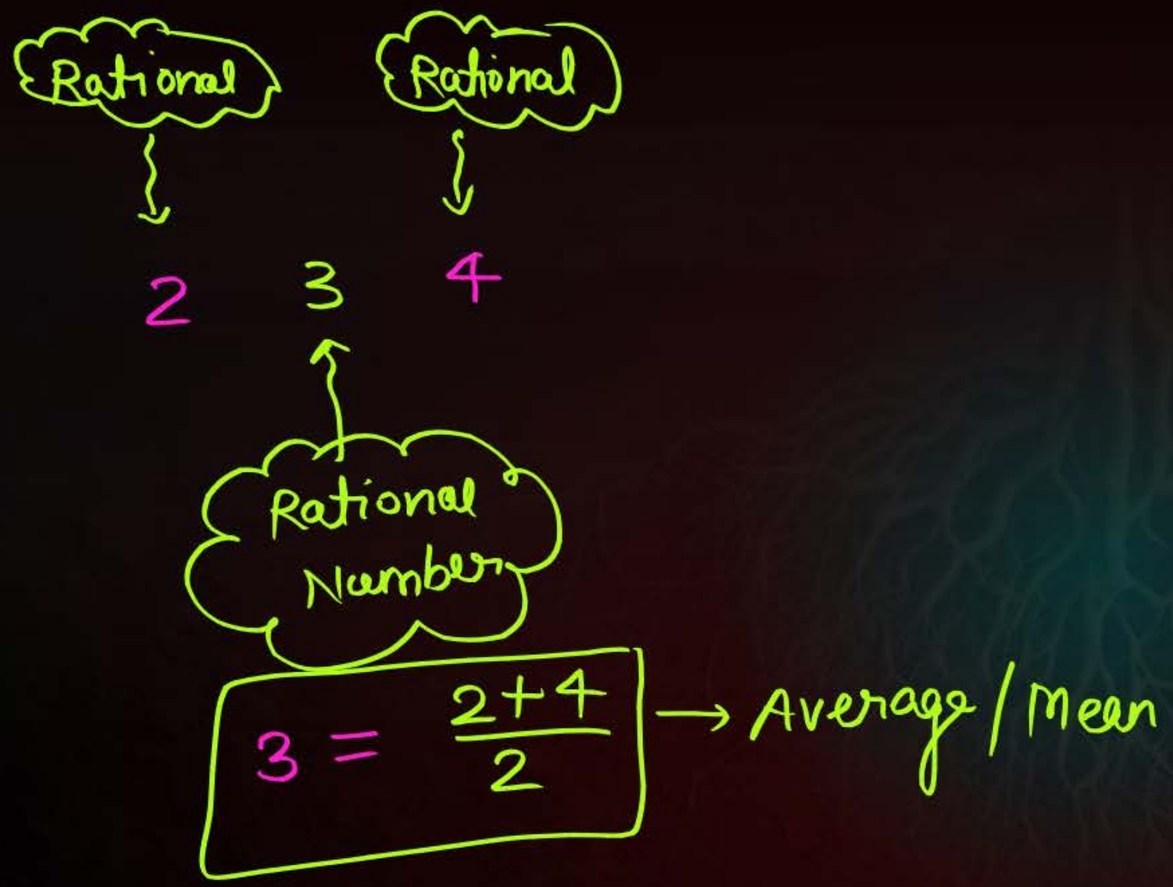


If we have two Rational No.

a & b

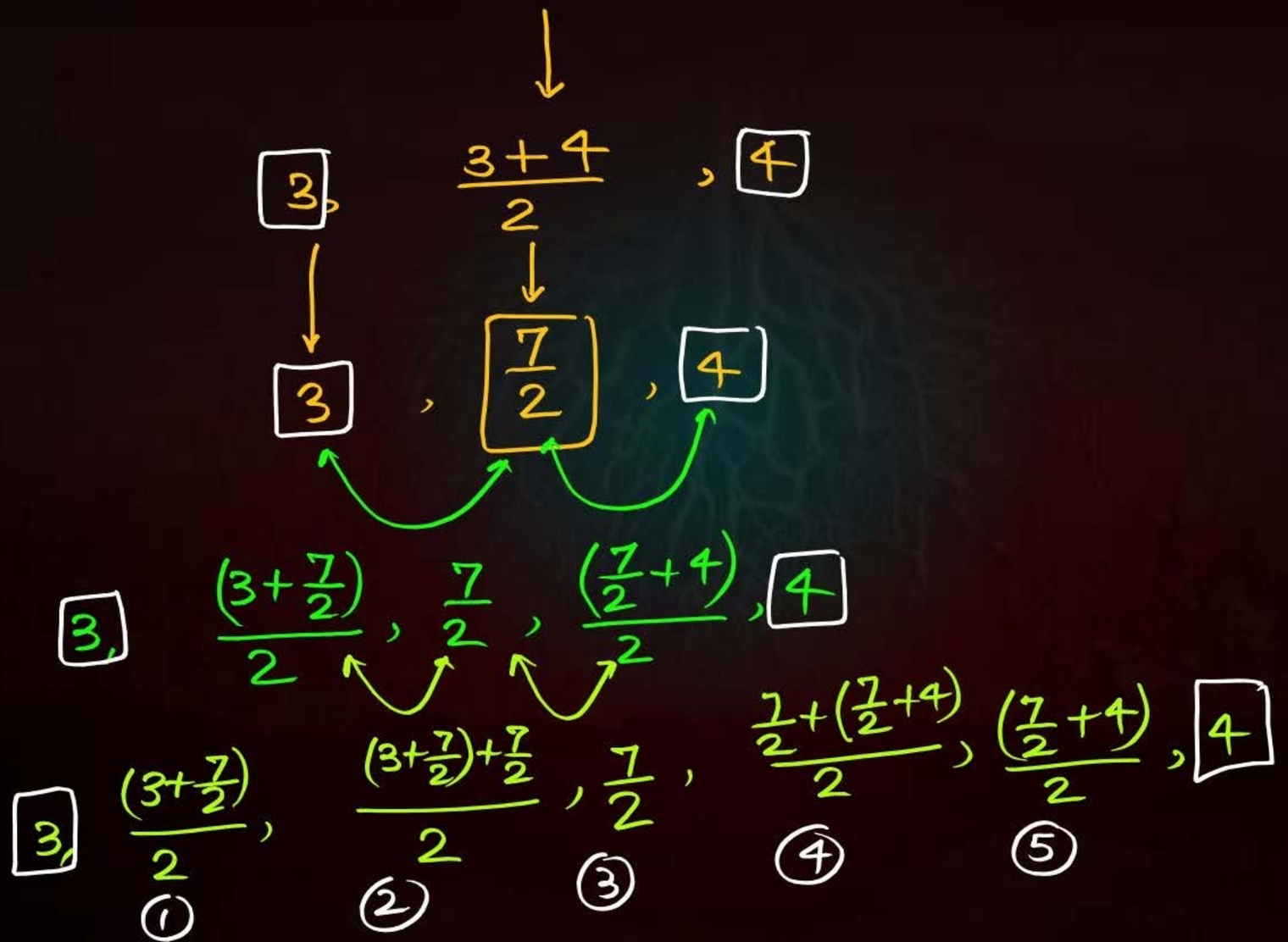
then $\frac{a+b}{2}$ will be a Rational number between them.

Question

Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$.

Question

Find a rational number between 3 and 4.



Question

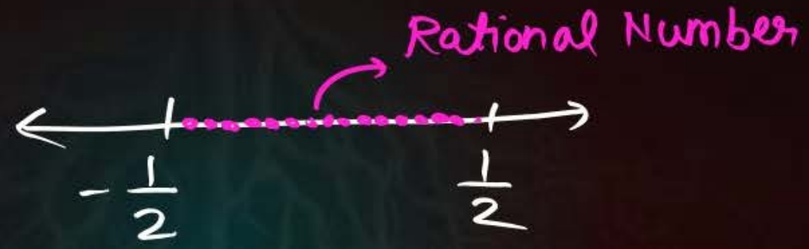
Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$.

$$\frac{\left(\frac{-2}{3}+\frac{1}{4}\right)}{2} = \frac{\left(\frac{-8+3}{12}\right)}{2} = \left(\frac{-5}{12}\right)\times\frac{1}{2}$$

$$= \frac{-5}{24}$$

Some important result

(1) There are infinite rational number between any two given rational number.



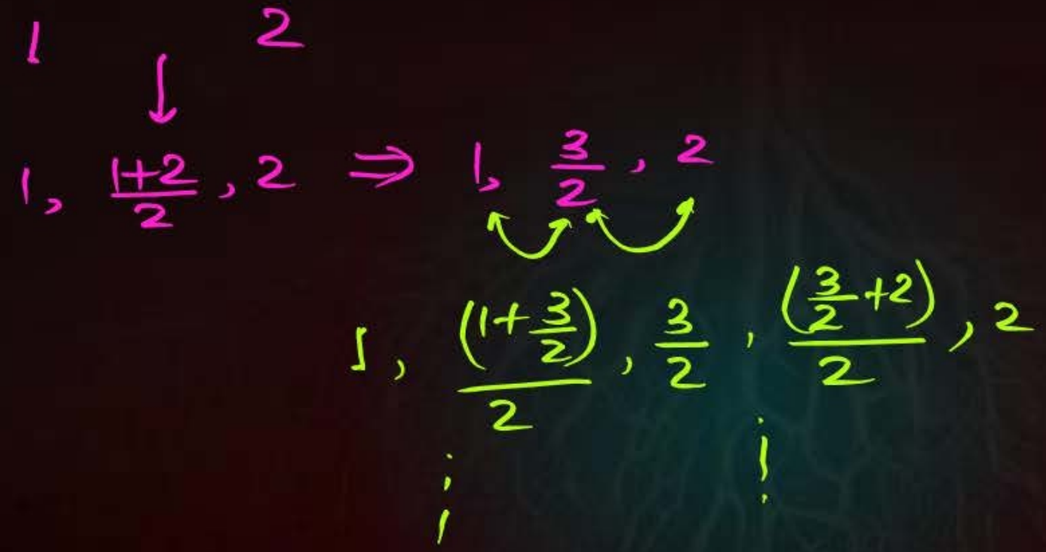
(2) Zero is rational number. ✓✓

Question

Find five rational numbers between 1 and 2.

Question

Find five rational numbers between 1 and 2.



Magic Karte Hai Ab

$\frac{1}{2}$ $\frac{1}{3}$

$\frac{3}{6}$ $\frac{2}{6}$

$\frac{p}{q}$ $\frac{x}{y}$

1. Make same denominator → LCM, multiply N & D by some non-zero number.
2. Now, if denominator look same, find gap b/w numerators
3. If asking for 'n' required Rational & you got 'n' gaps then write down those number keeping new deromination (after making equal denominator)
4. If asking for 'n' required rational then multiply N & D of each rationals by ('n+1').
5. You will see that is gap equal or more than required one, so repeat step ③.

Question

Tell me three rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Question

Tell me three rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

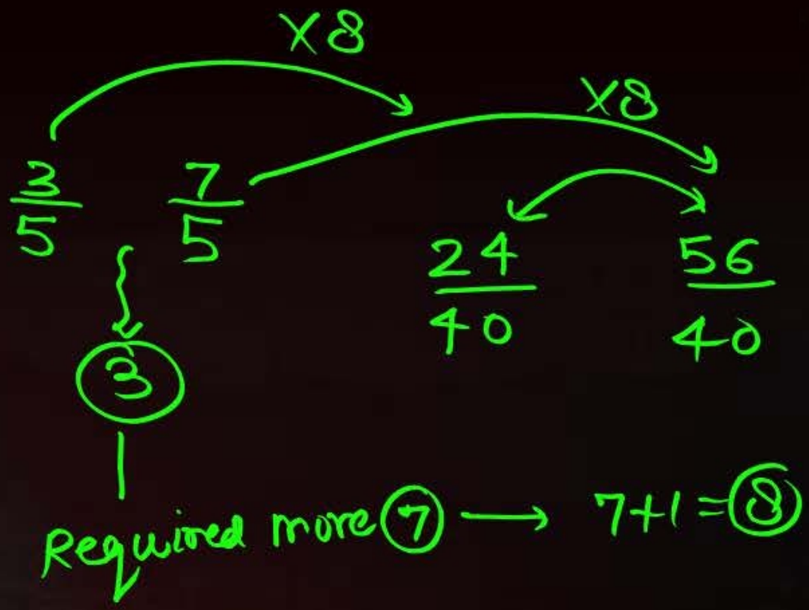
$3+1=4$

$\frac{3}{5}$ $\quad$ $\frac{4}{5}$

$\frac{3\times4}{5\times4}$ $\quad$ $\frac{4\times4}{5\times4}$

$= \frac{12}{20}$ $\quad$ $\frac{16}{20}$

$\frac{13}{20}, \frac{14}{20}, \frac{15}{20}$ Required Rational



Question

Find five rational numbers between $\frac{3}{13}$ and $\frac{9}{13}$.

Question

Find five rational numbers between $\frac{3}{13}$ and $\frac{9}{13}$.

$\frac{4}{13}, \frac{5}{13}, \frac{6}{13}, \frac{7}{13}, \frac{8}{13}$

Decimal Representation of Rational Numbers

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Important Points

- Rational number = $\frac{m}{n}$, m and n are integers and $n \neq 0$.
- A Rational number $\frac{m}{n}$ is said to be in lowest terms, if m and n have no common factor other than 1.
- Every integer m is also a rational number, as it can be written as $\frac{m}{1}$.

Question

Convert $\frac{35}{16}$ into decimal form by long division method.

Question

Convert $\frac{35}{16}$ into decimal form by long division method.

$$16 \overline{)35} \; 2.1875$$

32

30

16

140

128

120

112

80

80

0

$\frac{35}{16}$ → Terminating

→ Decimal Part → Terminates

$\frac{5 \times 7}{2 \times 2 \times 2 \times 2} = \frac{5 \times 7}{2^4}$

सिर्फ 2 हो, सिर्फ 5 हो या 2 & 5 का combo

↳ Denominator

$\frac{3}{10} = 0.3$

$\frac{3}{2 \times 5} \rightarrow 0.3$

$\frac{13}{5} = 2.6$

Question

Find the decimal expansion of $\frac{16}{45}$.

Question

Find the decimal expansion of $\frac{16}{45}$.

$\frac{2\times2\times2\times2}{3\times3\times5} = \frac{2\times2\times2\times2}{3^2\times5^1}$

3^2 → Extra, Not-allowed

5^1 → Allowed

$45 \overline{)160} (0.355555\ldots$

135

250

225

250

225

25

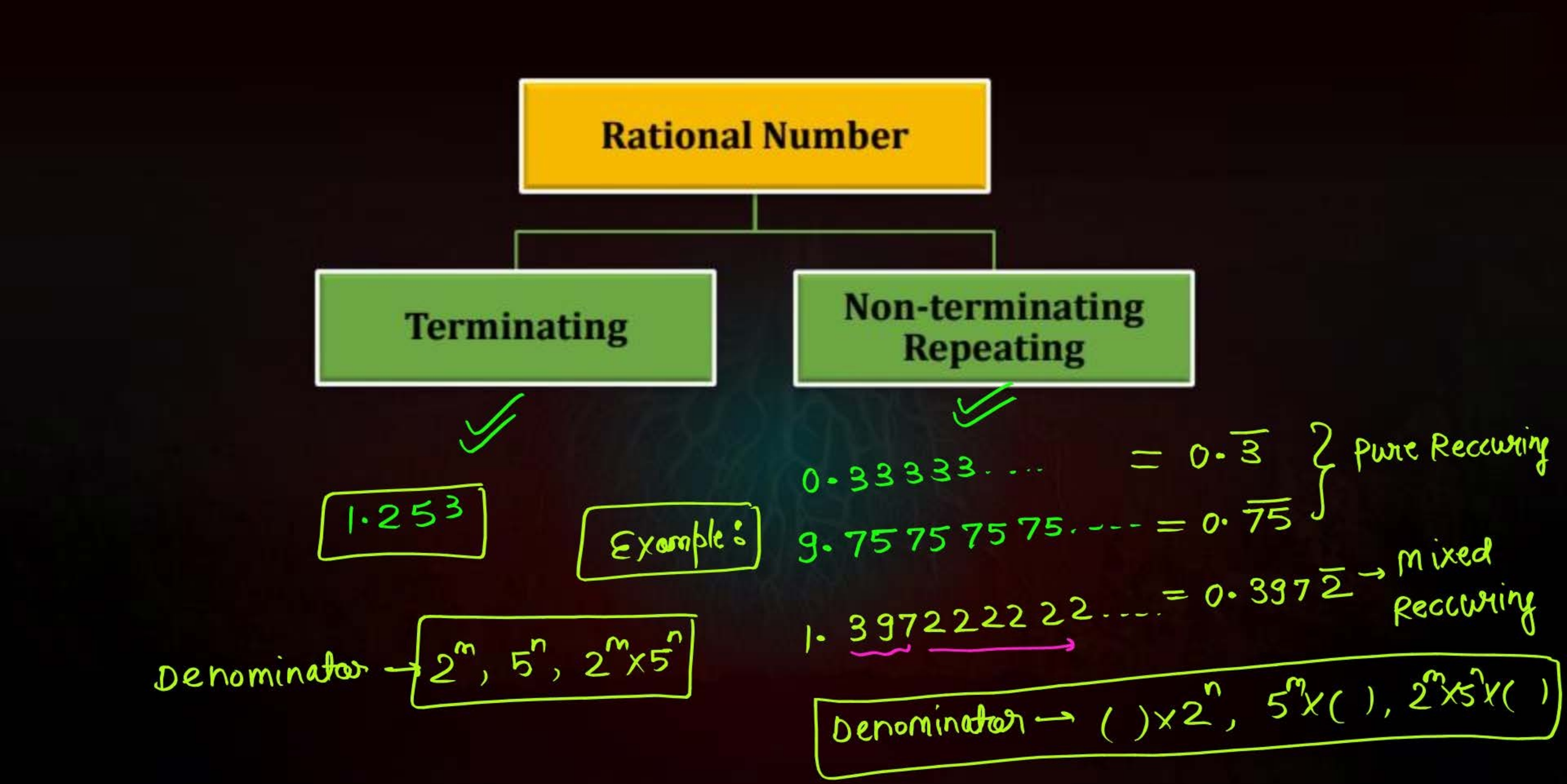
$\vdots$

Remainder $\neq 0$

Decimal Part → Non-terminating But reccuring / repeating

What have you noticed?

- **The remainder becomes zero.** → Decimal Expansion → Terminating
- **The remainder never becomes zero.** → Decimal Expansion → Non-terminating + Reccuring / Repeating



Condition for Rational Number

Termination: This indicates that the decimal representation terminates after a certain number of digits.

Condition for a rational number to be a terminated decimal: A rational number $\frac{p}{q}$ in its simplest form can be expressed as a terminating decimal only if its denominator has no other factors except 2 or 5 or both.

In other words, a rational number $\frac{p}{q}$ is a terminating decimal if and only if the denominator q can be expressed as $q = 2^m \times 5^n$ where m, n = 0, 1, 2, 3 ....

↳ कुछ भी extra नहीं।

Condition for Rational Number

Non- Terminating repeating: A decimal in which all the digit after the decimal point are repeated.

Condition for a rational number to be a recurring decimal: A rational number $\frac{p}{q}$ (in its lowest terms) is a recurring decimal if and only if its denominator q has a prime factor other than 2 or 5.

Question

You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}$ and $\frac{6}{7}$ are, without actually doing the long division ? If so, how?

Question

You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}$ and $\frac{6}{7}$ are, without actually doing the long division ? If so, how?

$\frac{1}{7} = 0.142857\ 142857\ 142857 \ldots \rightarrow$ Non-terminating

$\frac{6}{7}, \quad \frac{7}{7} = 1 = 1.0$

$\frac{6}{7}$ → Extra, No 2 & 5 or combo is there

Question

Decimal expansion of $\frac{329}{400}$ is terminating or non-terminating?

Question

Decimal expansion of $\frac{329}{400}$ is terminating or non-terminating?

$$\frac{329}{4 \times 100} = \frac{329}{2 \times 2 \times 10 \times 10} = \frac{329}{2 \times 2 \times 2 \times 5 \times 2 \times 5}$$

Terminating

Question

$\frac{5}{6}$ lies between?

1 and $\frac{5}{2}$

0 and $\frac{1}{2}$

-1 and $\frac{1}{3}$

$\frac{1}{3}$ and $\frac{12}{8}$

Question

$\frac{5}{6}$ lies between?

$\frac{5}{6} \rightarrow$ Proper $= 0.8333\ldots$

$6\overline{)50}(0.8333$
48
20
18
20

A 1 and $\frac{5}{2}$ ✗

B 0 and $\frac{1}{2}$ ⇒ 0 — 0.5 ✗

0 — 1

C −1 and $\frac{1}{3}$ −1, ..., 0.33 ✗

D $\frac{1}{3}$ and $\frac{12}{8}$ ✓

0.333... 1.5 ✓

Question

Between two rational numbers

there is no rational number

there is exactly one rational number

there are infinitely many rational numbers

None of these

Question

Between two rational numbers

A there is no rational number

B there is exactly one rational number

C there are infinitely many rational numbers

D None of these

Question

Decimal representation of a rational number cannot be

terminating

non-terminating

non-terminating repeating

non-terminating non-repeating

Question

Decimal representation of a rational number cannot be

A terminating

B non-terminating

C non-terminating repeating

D non-terminating non-repeating

Question

Find five rational numbers between $\frac{4}{5}$ and $\frac{7}{9}$.

H.W.

Question

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

(ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$

(iv) $\frac{3}{39}$

(v) $\frac{11}{22}$

(vi) $\frac{329}{400}$

Question

Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100} = \frac{4\times9}{4\times25} = \frac{3\times3}{5\times5}$ → T

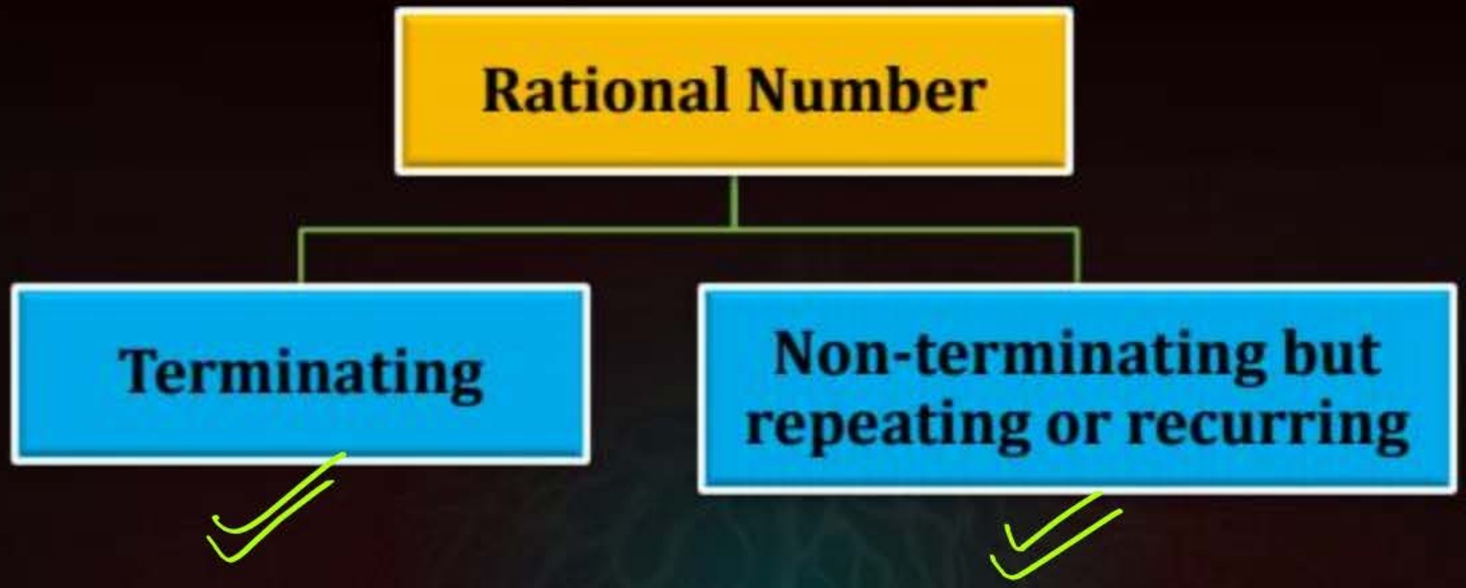
(ii) $\frac{1}{11} \Rightarrow$ N.T.+R

(iii) $4\frac{1}{8} = \frac{33}{2\times2\times2}$ → T

(iv) $\frac{3}{39} = \frac{3}{13\times3} = \frac{1}{13}$ → N.T +R

(v) $\frac{11}{22} \rightarrow \frac{11}{2\times11} = \frac{1}{2}$ → T

(vi) $\frac{329}{400}$ → T



Question

Classify the following as terminating, pure nonterminating repeating or mixed non-terminating repeating decimals.

(i) 0.5984

(ii) $0.76\overline{8}$

(iii) $9.\overline{34975}$

(iv) 9.20

(v) $1178.\overline{93}$

(vi) $73.390\,\overline{714}$

(vii) $3.93479347\ldots\ldots$

(viii) $89.76287287\ldots$

Question

Classify the following as terminating, pure nonterminating repeating or mixed non-terminating repeating decimals.

(i) 0.5984 → Terminating

(ii) $0.76\overline{8} = 0.768888\ldots$ → Mixed Repeating

(iii) $9.\overline{34975} = 9.34975\ 34975\ 34975\ldots$ → Pure Repeating

(iv) 9.20 → Terminating

(v) $1178.\overline{93}$ → Pure Repeating

(vi) $73.390\ \overline{714}$ → Mixed Repeating

(vii) $3.93479347\ldots\ldots$ Pure Repeating

(viii) $89.76287287\ldots$ Mixed Repeating

Question

Decimal expansion of $\frac{75}{300}$ is terminating or non-terminating?

Question

Decimal expansion of $\frac{75}{300}$ is terminating or non-terminating?

$$\frac{75}{300} = \frac{3 \times 5 \times 5}{3 \times 10 \times 10} = \frac{1}{2 \times 2}$$ → Terminating

Conversion of decimal numbers into Rational numbers of the form $\frac{p}{q}$

[Deleted Part]

Two cases !

- When the decimal number is of terminating nature.

- When the decimal representation is of non-terminating **nature.**

+ Reccwring / Repeating

Case-I: When the decimal is terminating :

(i) 0.15

(ii) 0.00026

Case-I: When the decimal is terminating :

(i) $0.15 = \frac{15}{100}$

(ii) $0.00026 = \frac{26}{100000}$

Question

Express each of the following in the form $\frac{p}{q}$.

(i) 15.75 (ii) –8.0025

Question

Express each of the following in the form $\frac{p}{q}$.

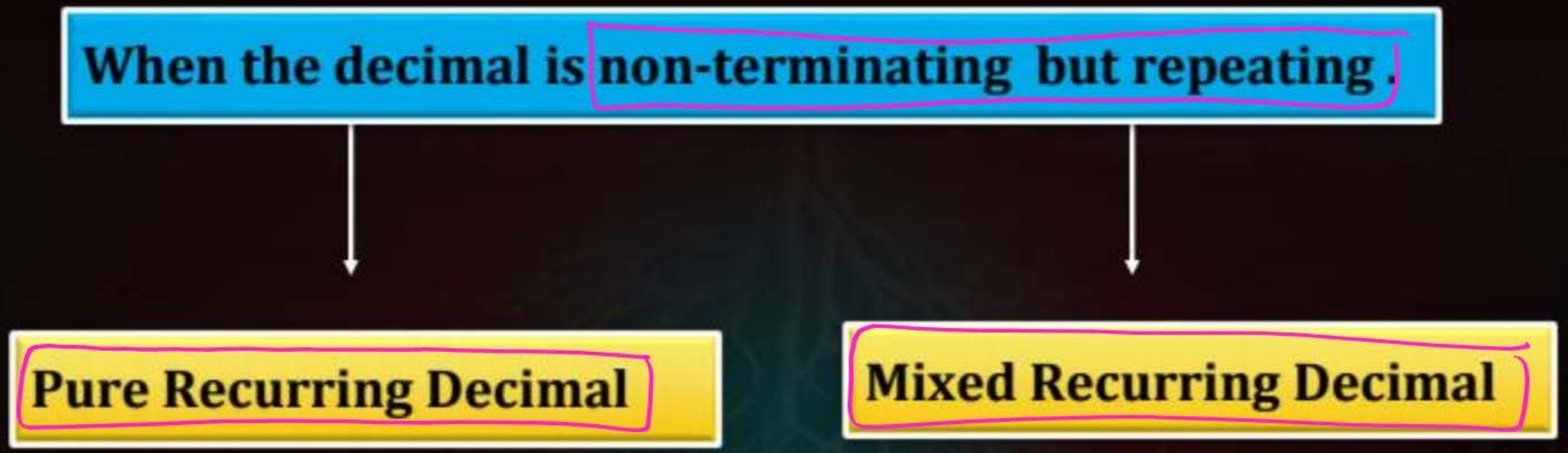
(i) 15.75

$\frac{1575}{100}$

(ii) −8.0025

$\frac{-80025}{10000}$

Case-II:



Question

Express $0.\overline{2}$ in the form $\frac{p}{q}$.

Question

Express $0.\overline{2}$ in the form $\frac{p}{q}$.

$\overline{\quad}$ → Bar

( ) (same set of digit is repeating)

Let $x = 0.\overline{2}$

$x = 0.22222\ldots$ → eqn ①

$10x = 2.22222\ldots$ → eqn ②

eqn ② − eqn ①

$10x - x = 2 - 0$

$9x = 2$

$x = \frac{2}{9}$ ⇒ $\frac{p}{q}$ form

Question

Express each of the following in the form $\frac{p}{q}$.

(i) $0.\overline{3}$

(ii) $0.\overline{35}$

Question

Express each of the following in the form $\frac{p}{q}$.

(i) $0.\overline{3}$

Let,

$a = 0.\overline{3}$

$a = 0.3333\ldots$ — eqn ①

$10a = 3.3333\ldots$ — eqn ②

eqn ② − eqn ①

$10a - a = 3 - 0$

$9a = 3$

$a = \frac{3}{9} \Rightarrow \boxed{a = \frac{1}{3}}$

(ii) $0.\overline{35}$

Let,

$y = 0.\overline{35}$

$y = 0.3535\ldots$ → eqn ①

$100y = 35.3535\ldots$ → eqn ②

eqn ② − eqn ①

$100y - y = 35 - 0$

$99y = 35$

$\boxed{y = \frac{35}{99}}$

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Question

Express $0.\overline{001}$ as a fraction in the simplest form.

Question

Express $0.\overline{001}$ as a fraction in the simplest form.

Let,

$x = 0.\overline{001}$

$x = 0.001\,001\,001\,001\ldots$ ---eqn ①

$1000x = 1.001\,001\,001\,001\ldots$ ---eqn ②

eqn ② − eqn ①

$1000x - x = 1 - 0$

$999x = 1$

$$x = \frac{1}{999}$$

Question

Convert the following decimal number in the form $\frac{p}{q}$.

(i) $5.\overline{2}$

(ii) $23.\overline{43}$

H.W.

Mixed recurring decimal

$5.12533333\ldots$

$0.59232323\ldots$

$1.92\overline{4876}\,\overline{4876}\,\overline{4876}\ldots$

} Mixed Reccuring Decimal

Question

Express $0.3\overline{2}$ in the form $\frac{p}{q}$.

Question

Express $0.3\overline{2}$ in the form $\frac{p}{q}$.

Let, $x = 0.3\overline{2}$

$x = 0.32222\ldots$

Mixed Recurring

Pure Recurring

$10x = 3.2222222\ldots$ → Eq^n ①

$100x = 32.2222\ldots$ → Eq^n ②

Eq^n ② − Eq^n ①

$100x - 10x = 32 - 3$

$90x = 29 \Rightarrow \boxed{x = \frac{29}{90}}$

Question

Express each of the following decimal in the form $\frac{p}{q}$.

(i) $0.12\overline{3}$

(ii) $4.3\overline{2}$

Question

Express each of the following decimal in the form $\frac{p}{q}$.

(i) $0.12\overline{3}$

(ii) $4.3\overline{2}$

$a = 0.12333\ldots$

$100a = 12.3333\ldots$ — Pure → eqn ①

$1000a = 123.333\ldots$ — eqn ②

$1000a - 100a = 123 - 12$

$900a = 111$

$a = \frac{111}{900}$

H.W.

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Question

Show that $0.2353535..... = 0.2\overline{35}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Question

Show that $0.2353535\ldots\ldots = 0.2\overline{35}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

$a = 0.2\,353535\ldots$ —

$10a = 2.\,\overline{35}\,3535\ldots$ —

$1000a = 235.\,\overline{353535}\ldots$ —

$990a = 233$

$$a = \frac{233}{990}$$

Question

Express $0.\bar{3} + 0.\bar{4}$ in form of $\frac{p}{q}$.

Question

Express $0.\bar{3} + 0.\bar{4}$ in form of $\frac{p}{q}$.

$x = 0.3333\ldots$

$10x = 3.3333\ldots$

$9x = 3$

$x = \frac{3}{9}$

$y = 0.4444\ldots$

$10y = 4.4444\ldots$

$9y = 4$

$y = \frac{4}{9}$

$0.\bar{3} + 0.\bar{4}$

$= \frac{3}{9} + \frac{4}{9}$

$= \frac{7}{9}$

Question

Express $0.6 + 0.\bar{3} + 2.2\bar{5}$ in form of $\frac{p}{q}$.

Question

Express $0.6 + 0.\bar{3} + 2.2\bar{5}$ in form of $\frac{p}{q}$.

$\frac{6}{10}$ $\frac{3}{9}$

$a = 2.25555\ldots$

$10a = 22.555555\ldots$

$100a = 225.5555\ldots$

$90a = 203$

$a = \frac{203}{90}$

Final Ans: $\frac{6}{10} + \frac{3}{9} + \frac{203}{90}$

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Question

Express 0.99999..... in the form of $\frac{p}{q}$. Are you surprised with your Answer?

Question

Express 0.99999..... in the form of $\frac{p}{q}$. Are you surprised with your Answer?

$x = 0.\overline{9999\text{-}} \rightarrow$

$10x = 9.\overline{9999\text{-}} \rightarrow$

$9x = 9$

$x = \frac{9}{9}$

$\boxed{x = 1} \approx 0.99999$

Question

$0.12\bar{3}$ can be expressed in rational form of $\frac{p}{q}$ as

A $\frac{900}{111}$

B $\frac{111}{900}$

C $\frac{123}{10}$

D $\frac{121}{900}$

H.W

Question

Show that $0.2353535..... = 0.2\overline{35}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

H.W.

From all the discussion that we how had, we can conclude that the decimal representation of a number can be: → Kisi bhi type ke Real Numbers

(i) Terminating
(ii) Non-terminating but repeating } Rational Numbers
(iii) Non-terminating and non-repeating → Irrational Numbers

Irrational Numbers

Rational No → T, N·T·R

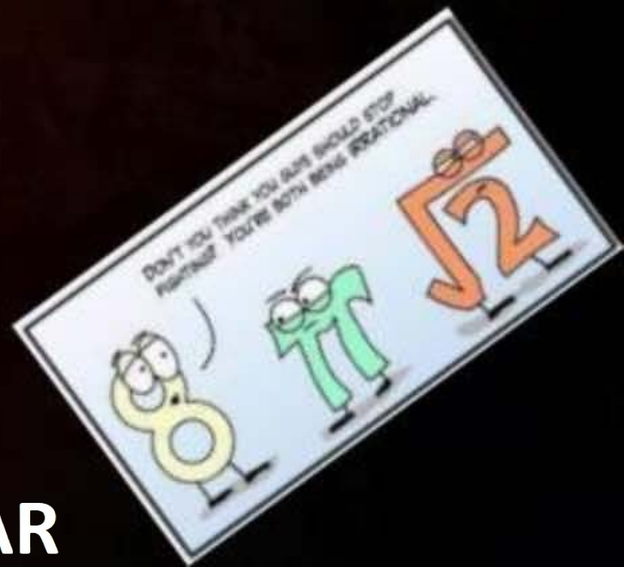
- A number is an irrational number, if it has a non-terminating and non-repeating decimal representation.

  → Irrational → N·T·N·R

- Irrational number cannot be written in the form p/q, where p and q are integers and $q \neq 0$

  For Example: $\sqrt{2}$, $\sqrt{5}$, $\sqrt[4]{5}$, $\sqrt[7]{35}$ etc...

$\sqrt{\quad}$ $\sqrt[3]{\quad}$ swords



FUNFACT : Discovered by Pythagoras around 400 BC.

Remember $\sqrt{n}$ is an irrational number, if n is not a perfect square.

$\sqrt{\text{'n'}}$ ⟹ Irrational Number

↘ Non-perfect square

Some useful results on irrational numbers

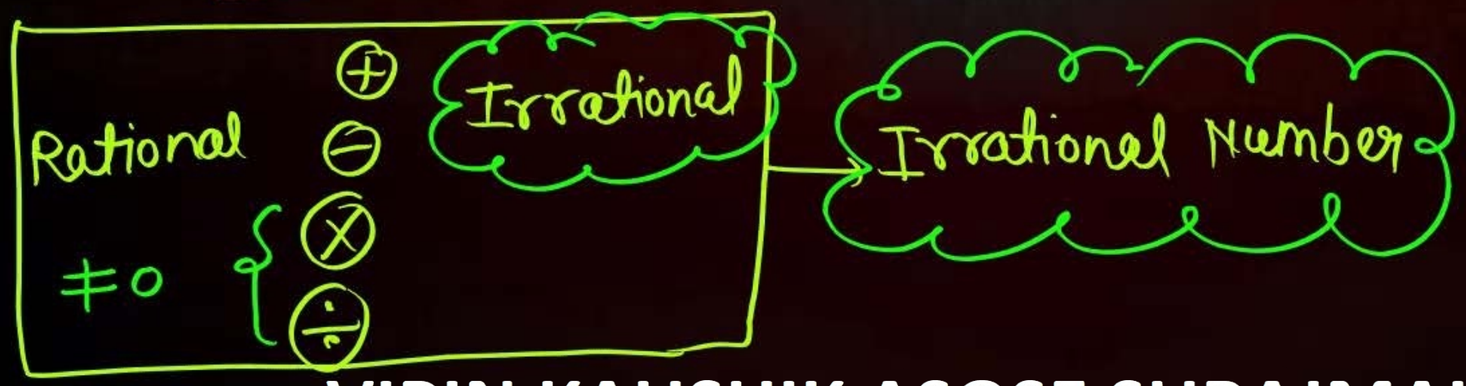
- **Negative of an irrational number is an irrational number.**

$\sqrt{2}$, $-\sqrt{2}$

Irrational Irrational

- **The sum of a rational and an irrational number is an irrational number.**

$2 + \sqrt{3}$ → Irrational



Some useful results on irrational numbers

- The product of a non-zero rational number and an irrational numbers is an irrational numbers.

$$\otimes \begin{cases} (\sqrt{2}) + (-\sqrt{2}) = \sqrt{2} - \sqrt{2} = 0 \\ (\sqrt{2}) \times (-\sqrt{2}) = -(\sqrt{2})^2 = -2 \end{cases}$$

- The sum, difference, product and quotient of two irrational numbers need not be an irrational number.

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$

Irrational $\oplus \ominus \otimes \oslash$ Irrational → Jaruri Nahin Hai Ki Result Irrational Aaaye!!

Properties of Irrational Numbers

Property (i):

There are infinitely many irrational numbers between any two rational numbers.

Property (ii):

An irrational number can be approximated by irrational numbers. For example, if x be an irrational number such that $0.2 < x < 0.3$ where 0.2 and 0.3 are two rational numbers, then $0.201 < x < 0.2012$ or $0.201001 < x < 0.201002$ and so on.

$\pi \approx \frac{22}{7}$

Property (iii):

The set of irrational numbers is dense everywhere, i.e., "Between two irrational numbers a and b with $a < b$, there lies at least one irrational number and hence an infinite number of irrational numbers."

Question

Give examples of two irrational numbers, the product of which is:

(i) A rational number

(ii) An irrational number

Question

Give examples of two irrational numbers, the product of which is:

(i) A rational number ⟶ $(\sqrt{2}) \times (-\sqrt{2})$ or $(\sqrt{3}) \times \left(\frac{1}{\sqrt{3}}\right)$

(ii) An irrational number

↳ $\sqrt{2} + \sqrt{3}$

Question

State whether given statement is true of false:

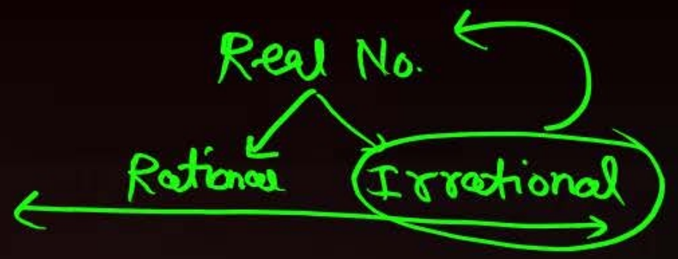
(a) Every irrational number is a real number.

(b) Every point on the number line is of form $\sqrt{m}$, where m is a natural numbe

(c) Every real number is an irrational number.

Question

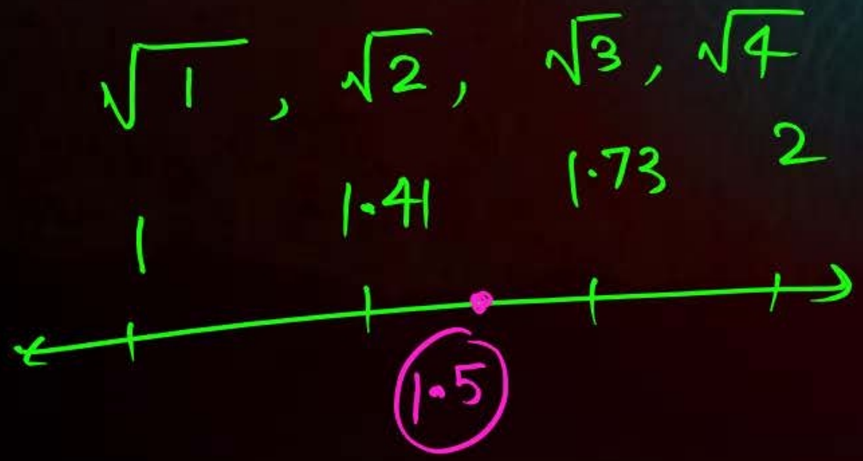
State whether given statement is true of false:



(a) Every irrational number is a real number. True

(b) Every point on the number line is of form $\sqrt{m}$, where m is a natural number. False

(c) Every real number is an irrational number. False



Brainstorming Time

Examine whether $\sqrt{2}$ is rational or irrational.

Brainstorming Time

Examine whether $\sqrt{2}$ is rational or irrational.

When we will continue this process, we will get the decimal expansion of $\sqrt{2}$ as 1.4142135 623730950488016887..... which is clearly nonterminating and non-repeating. Therefore, $\sqrt{2}$ is irrational.

	1.4142135.....
1	2.00000000
	1
24	100
	96
281	400
	281
2824	11900
	11296
28282	60400
	56564
282841	383600
	282841
2828423	10075900
	8485269
28284265	159063100
	141421325
	17641775

Question

Is $\sqrt{45}$ a rational or an irrational number?

Question

Is $\sqrt{45}$ a rational or an irrational number?

$\sqrt{45} = \sqrt{3 \times 3 \times 5}$

$= 3 \times \sqrt{5}$

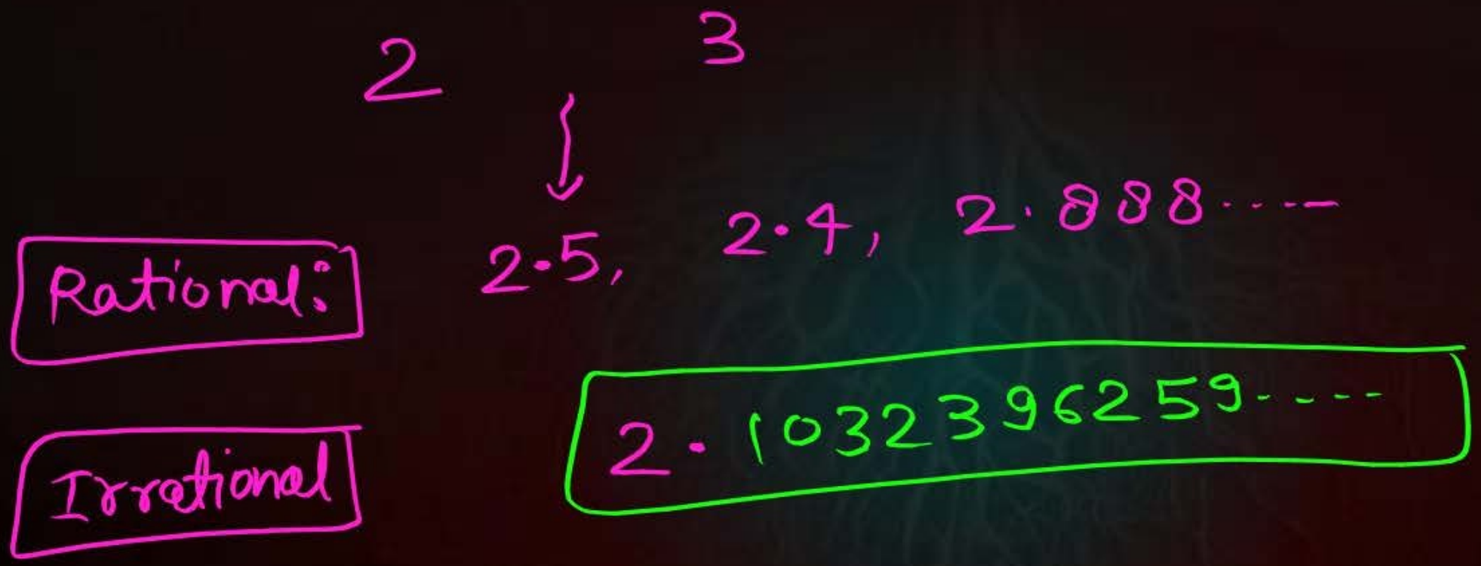
$= R \times I \rightarrow$ Irrational

Question

Insert a rational and an irrational number between 2 and 3.

Question

Insert a rational and an irrational number between 2 and 3.



Question

Find two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.

Question

Find two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.

$\sqrt{2} \approx 1.41$

$\sqrt{3} \approx 1.71$

$\rightsquigarrow$ 1.523796588 3289...

Question

Find two irrational numbers between 0.12 and 0.13.

Question

Find two irrational numbers between 0.12 and 0.13.

0.12397582....

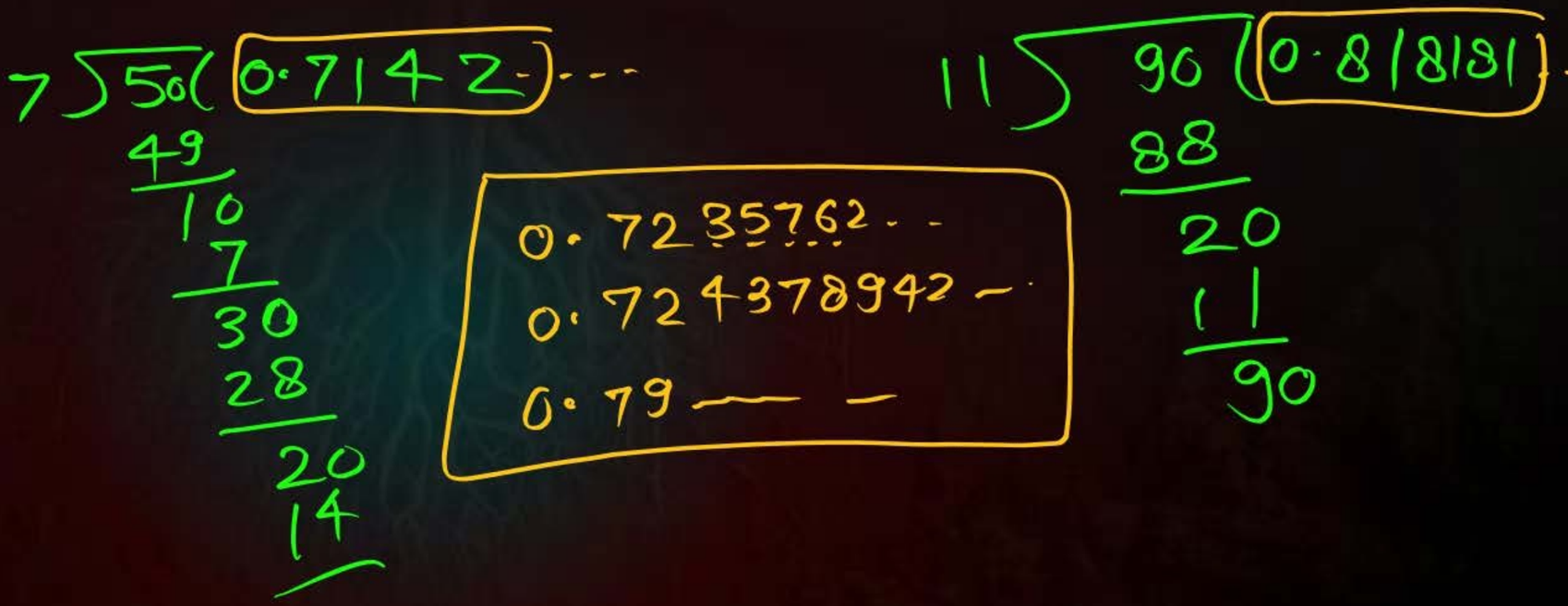
H.W

Question

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Question

Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.



Question

Write three numbers whose decimal expansions are non-terminating non-recurring.

Question

Write three numbers whose decimal expansions are non-terminating non-recurring.

Irrational

Question

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

(iii) 0.3796

(iv) $7.478478....$

(v) $1.101001000100001....$

Question

Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ → (I)

(ii) $\sqrt{225} = \sqrt{15 \times 15} = 15$ → (R)

(iii) 0.3796 → (R)

(iv) $7.478478....$

N.T + Reccuring → (R)

(v) $1.101001000100001....$

N.T & N.R → (I)

Revision

Identify the following as rational or irrational numbers.

(i) $\sqrt{4}$ (ii) $3\sqrt{18}$ (iii) $\sqrt{1.44}$

(iv) $\sqrt{3}+\sqrt{2}$ (v) $\sqrt{5}-2$ (vi) $\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$

(vii) $7.478478\ldots..$ (viii) $1.101001000100000\ldots\ldots$

Revision

Identify the following as rational or irrational numbers.

(i) $\sqrt{4}$ → R

(ii) $3\sqrt{18}$ → I

(iii) $\sqrt{1.44} = 1.2$ → R

(iv) $\sqrt{3}+\sqrt{2}$ → I

(v) $\sqrt{5}-2$

(vi) $(2-\sqrt{2})(2+\sqrt{2})$

$(2)^2-(\sqrt{2})^2 = 4-2$

$= 2$

R

(vii) $7.478478\ldots$

R

(viii) $1.101001000100000\ldots\ldots$

I

Types of Irrational Numbers

Like Irrational Numbers: Irrational numbers having the same irrational factor are called similar or like irrational number.

For example: $\sqrt{3}$, $3\sqrt{3}$, $\frac{8}{2}\sqrt{3}$, $-7\sqrt{3}$ are like irrational numbers since all of these have same irrational factor i,e. $\sqrt{3}$.

Unlike Irrational number: Irrational numbers having no common irrational factor are known as unlike irrational numbers

Question

Check whether $3\sqrt{5}$, $\frac{7}{\sqrt{3}}$, $\pi - 2, \sqrt{11} - 4$ and $\frac{3}{\sqrt{9}}$ are Irrational or Rational Number?

Question

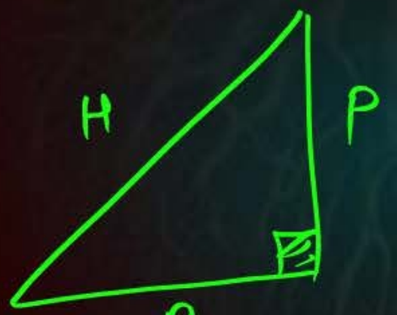
Check whether $3\sqrt{5}$, $\frac{7}{\sqrt{3}}$, $\pi - 2$, $\sqrt{11} - 4$ and $\frac{3}{\sqrt{9}}$ are Irrational or Rational Number?

(I) (I) (I) (I)

$\frac{3}{3} \rightarrow$ (R)

Pythagoras Theorem

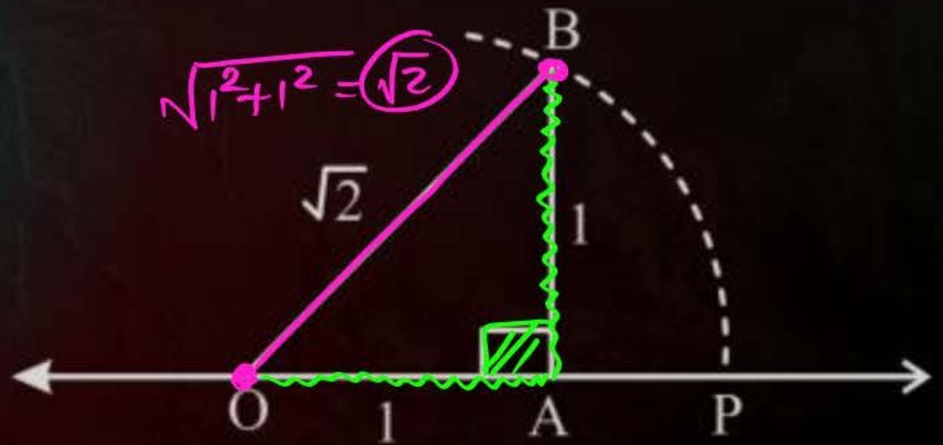
We use the Pythagoras property of a right angled triangle, according to which, in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



$$H^2 = P^2 + B^2$$

$$H = \sqrt{P^2 + B^2}$$

***



Representation of irrational Number of the number line

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Example

Represent $\sqrt{2}$ on the number line.

Example

$\sqrt{n} = \sqrt{(n-1)+1}$

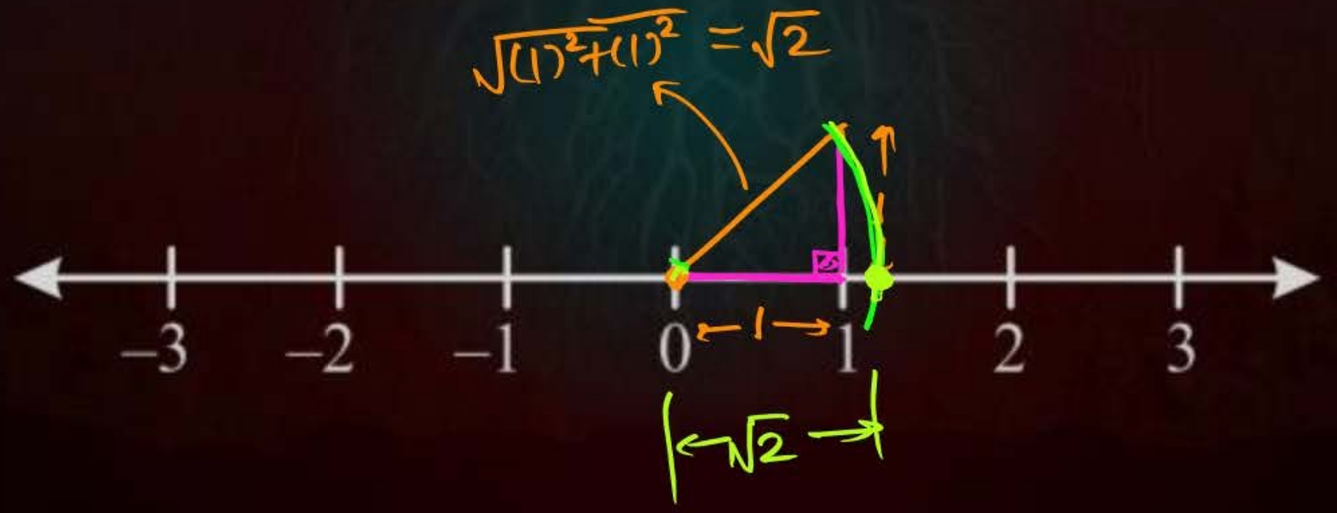
$\sqrt{2} = \sqrt{1+1}$

$H = \sqrt{(1)^2+(1)^2}$

Represent $\sqrt{2}$ on the number line.

$\sqrt{3} = \sqrt{2+1}$

$= \sqrt{(\sqrt{2})^2+(1)^2}$



① compass ✓✓
② Protactor
③ scale
④ Pencil
⑤ Eraser
⑥ set square
⑦ pivider

Example

Represent $\sqrt{3}$ on the number line.

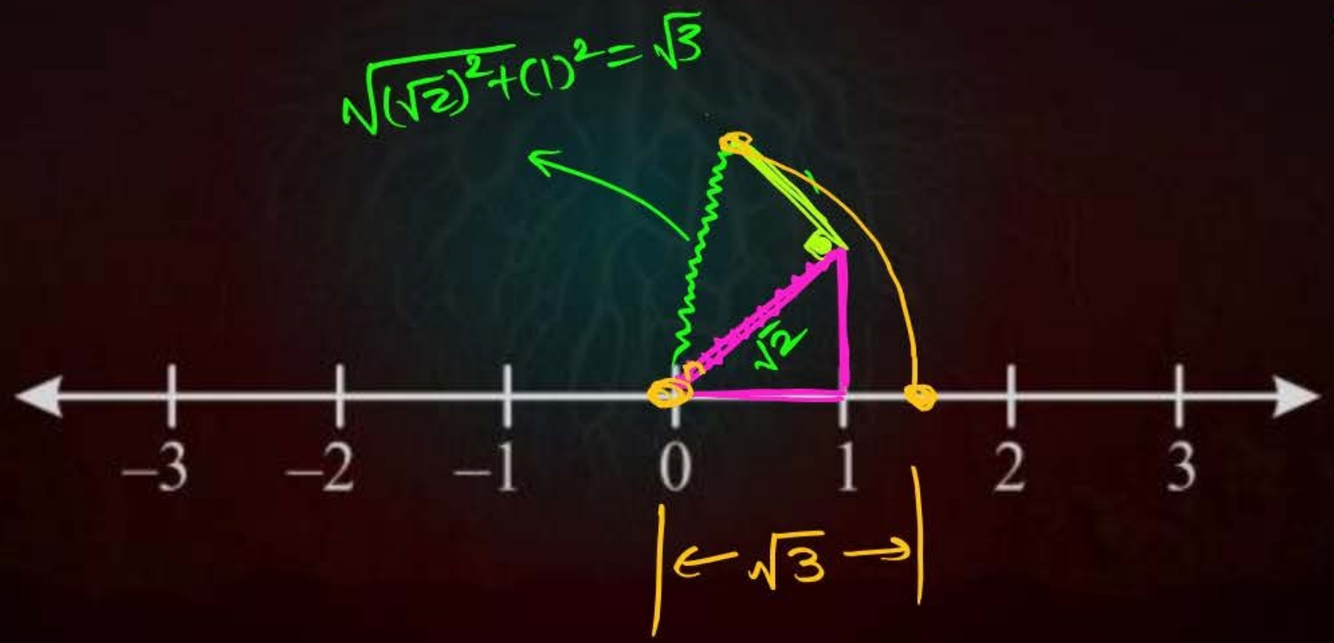
Example

Represent $\sqrt{3}$ on the number line.

$$\sqrt{3} = \sqrt{2+1}$$

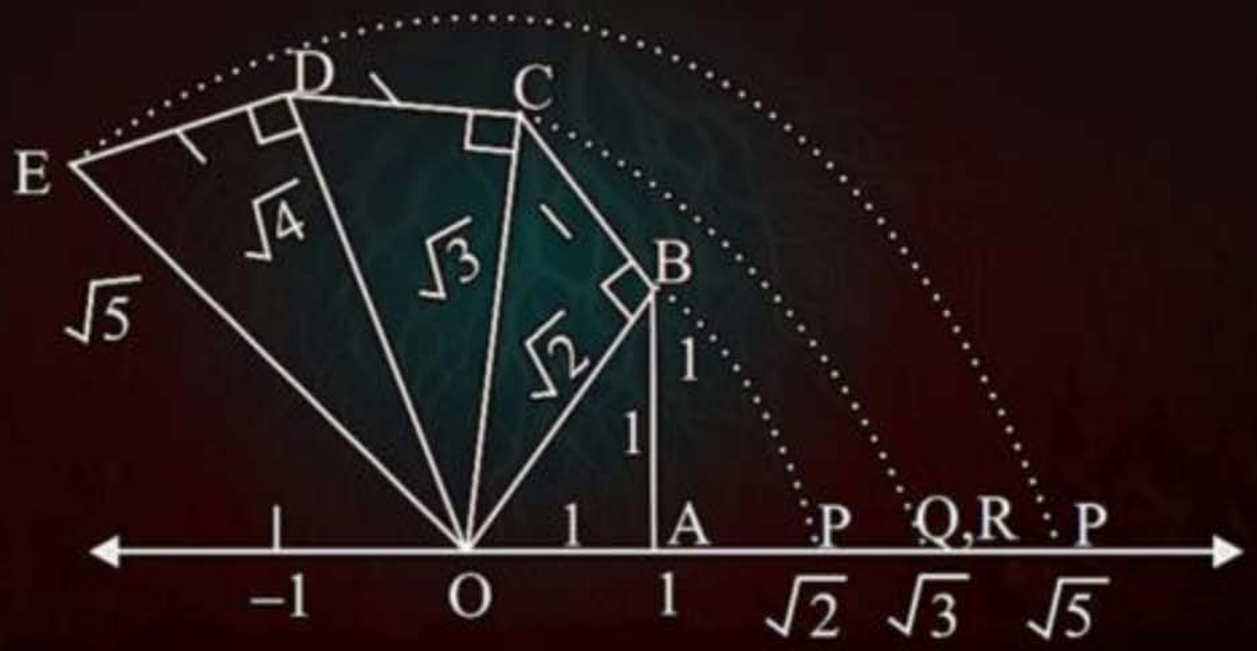
$$= \sqrt{(\sqrt{2})^2 + (1)^2}$$

B P



Remark: We can locate $\sqrt{n}$ for any positive integer n after $\sqrt{n-1}$ has located.

Spiral Method



Question

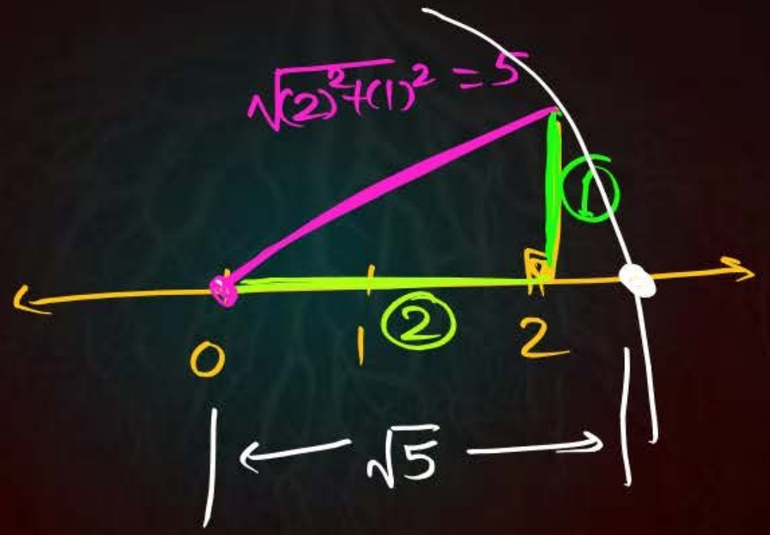
Represent $\sqrt{5}$ on the number line.

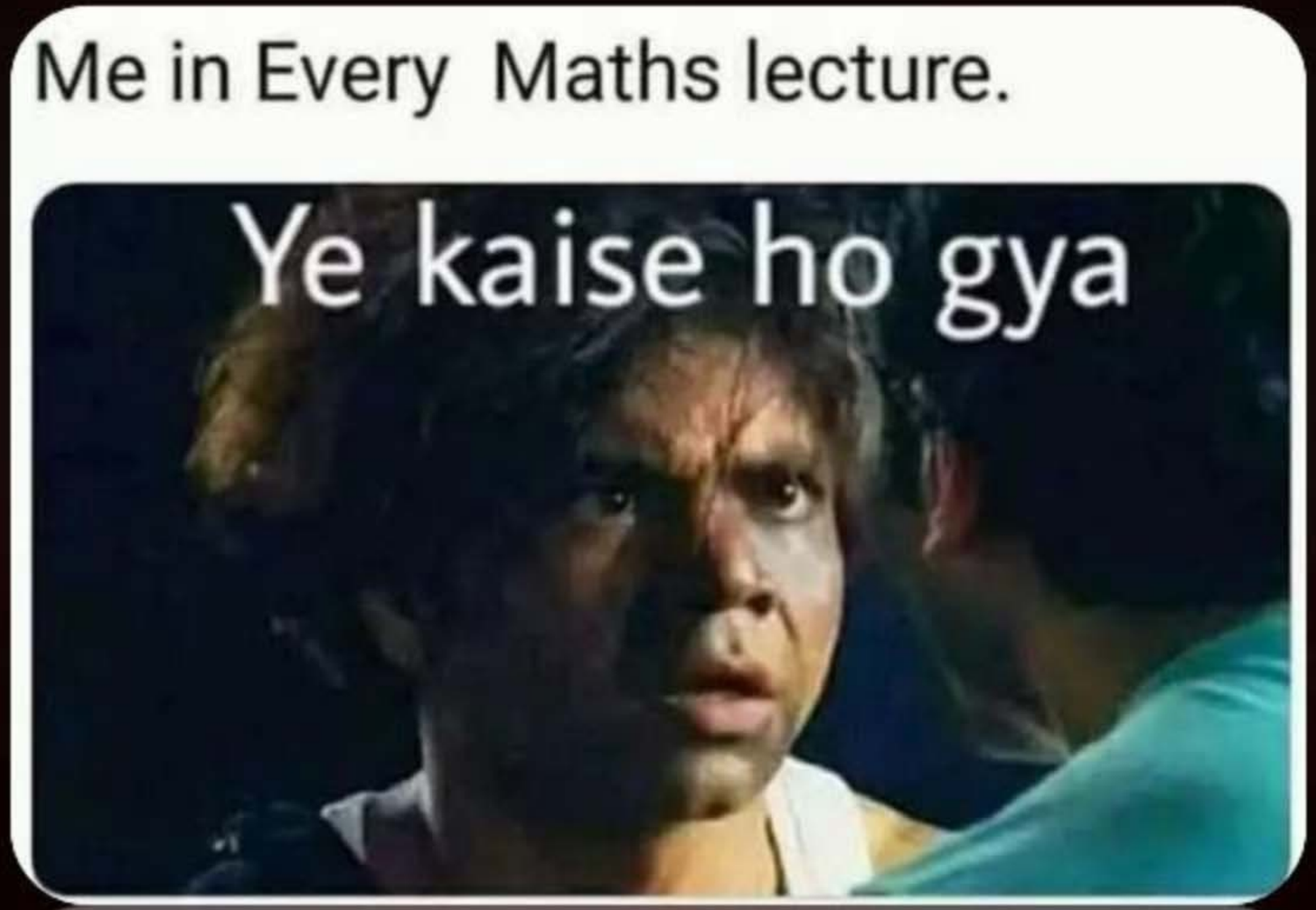
Question

Represent $\sqrt{5}$ on the number line.

$$\sqrt{5} = \sqrt{4+1}$$

$$= \sqrt{(2)^2+(1)^2}$$





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Question

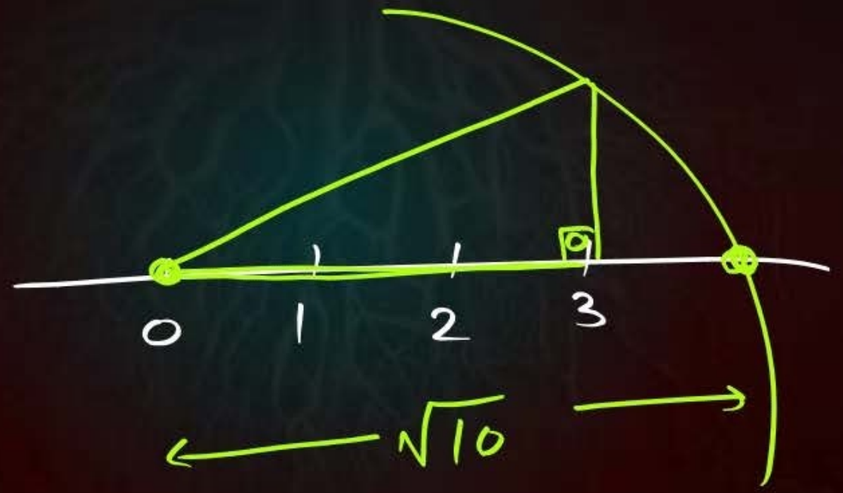
Represent $\sqrt{10}$ on the number line.

Question

Represent $\sqrt{10}$ on the number line.

$$\sqrt{10} = \sqrt{9+1}$$

$$= \sqrt{(3)^2+(1)^2}$$



Real numbers and the Real number line

- There are infinitely many irrational numbers.
- Square roots of all the +ve integers which are not perfect squares are irrational
- Cube roots of all the integers which are not perfect cubes are irrational numbers.
- In fact there are many more numbers which are irrational numbers.
- Rational and irrational numbers milkar bante hai Real numbers.
- The set of real numbers is denoted by R.
- All the numbers (R) can be represented on the number line, so we call the number line as real number line.

Representation of real numbers on number line

Represent $\sqrt{x}$ on the number line, where x is a real number.

For any two real number x, we have

$$\sqrt{\left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2} = \sqrt{\frac{x^2+2x+1}{4} - \frac{x^2-2x+1}{4}} = \sqrt{\frac{4x}{4}} = \sqrt{x}$$

$\sqrt{(\text{Natural})}$

$\sqrt{2}$

$\sqrt{3.7}$

$\sqrt{(\text{Positive Real No})}$

Positive Real numbers on number Line

To draw the square root of a positive real number, we follow the following algorithm -

Algorithm:

Step-I :- Obtain the positive real number x

Step-II :- Draw a line and mark a point A on it.

Step-III :- Mark a point B on the line such that AB = x units.

Step-IV :- From a point B mark a distance of 1 unit and mark the new point as C.

Step-V :- Find the mid-point of AC and mark the point as O.

Step-VI :- Draw a circle with centre O and radius OC.

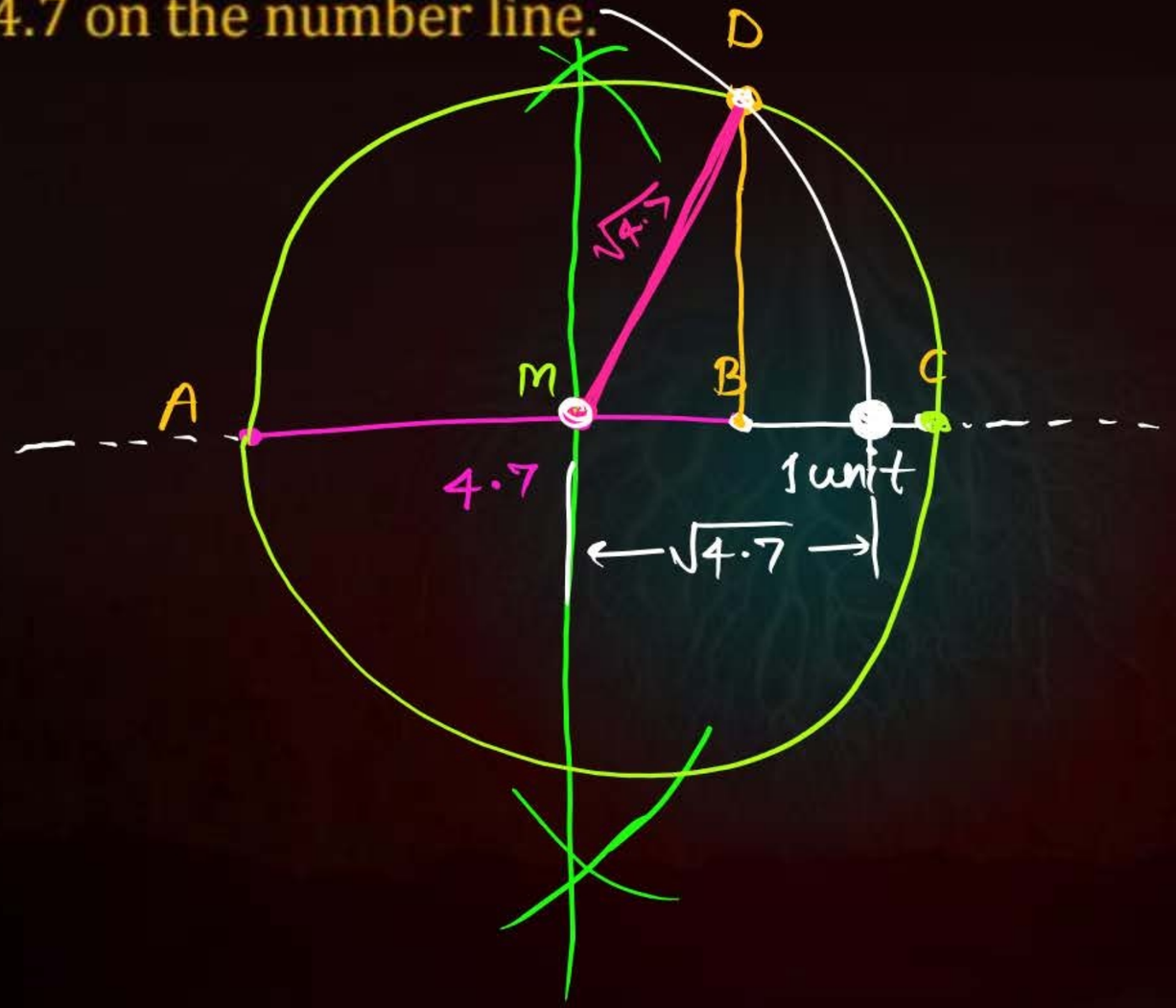
Step-VII :- Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Length BD is equal to $\sqrt{x}$.

Question

Represent $\sqrt{4.7}$ on the number line.

Question

Represent $\sqrt{4.7}$ on the number line.



$\frac{2.85}{5.7}$
$\frac{}{2}$

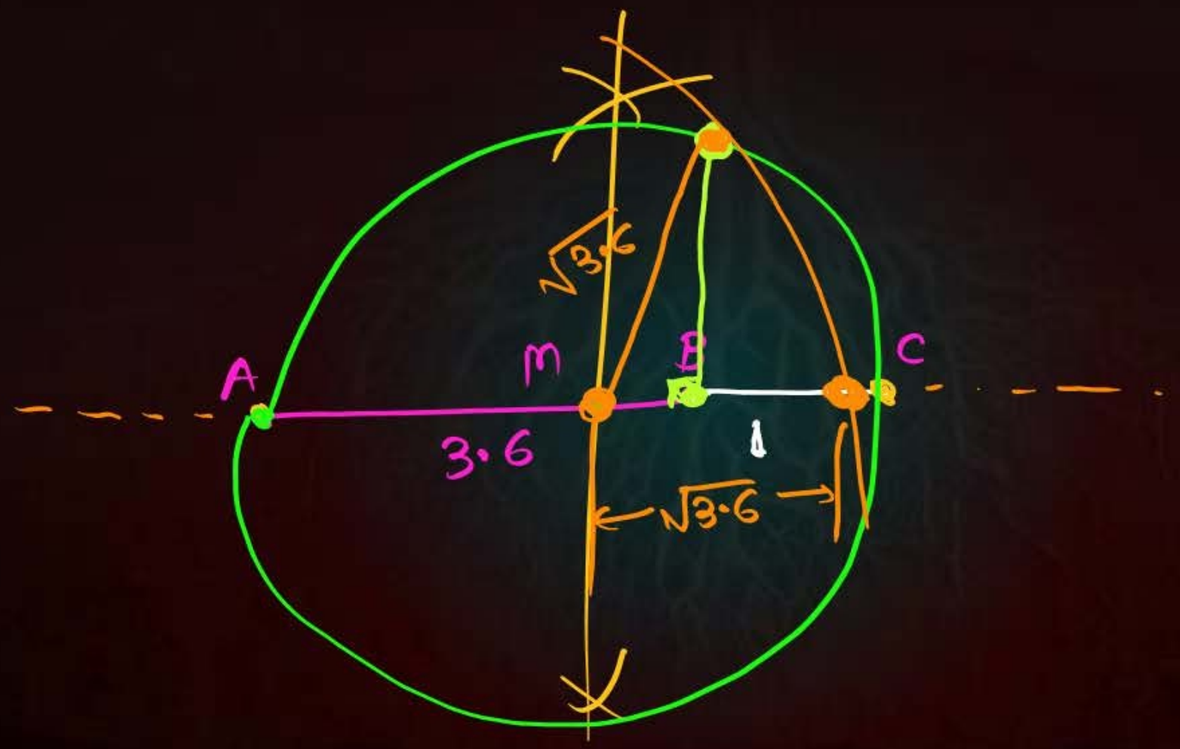
AM = Radius

Question

Represent $\sqrt{3.6}$ on the number line.

Question

Represent $\sqrt{3.6}$ on the number line.

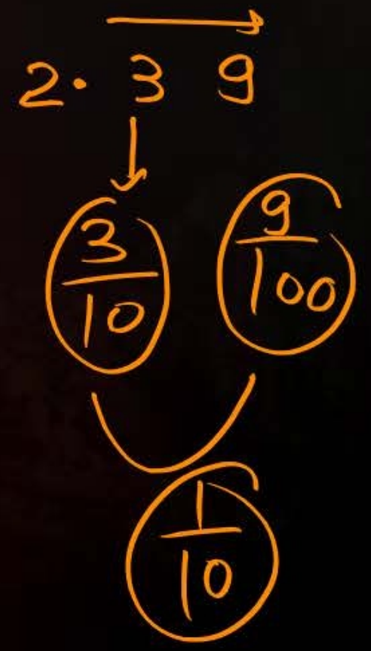


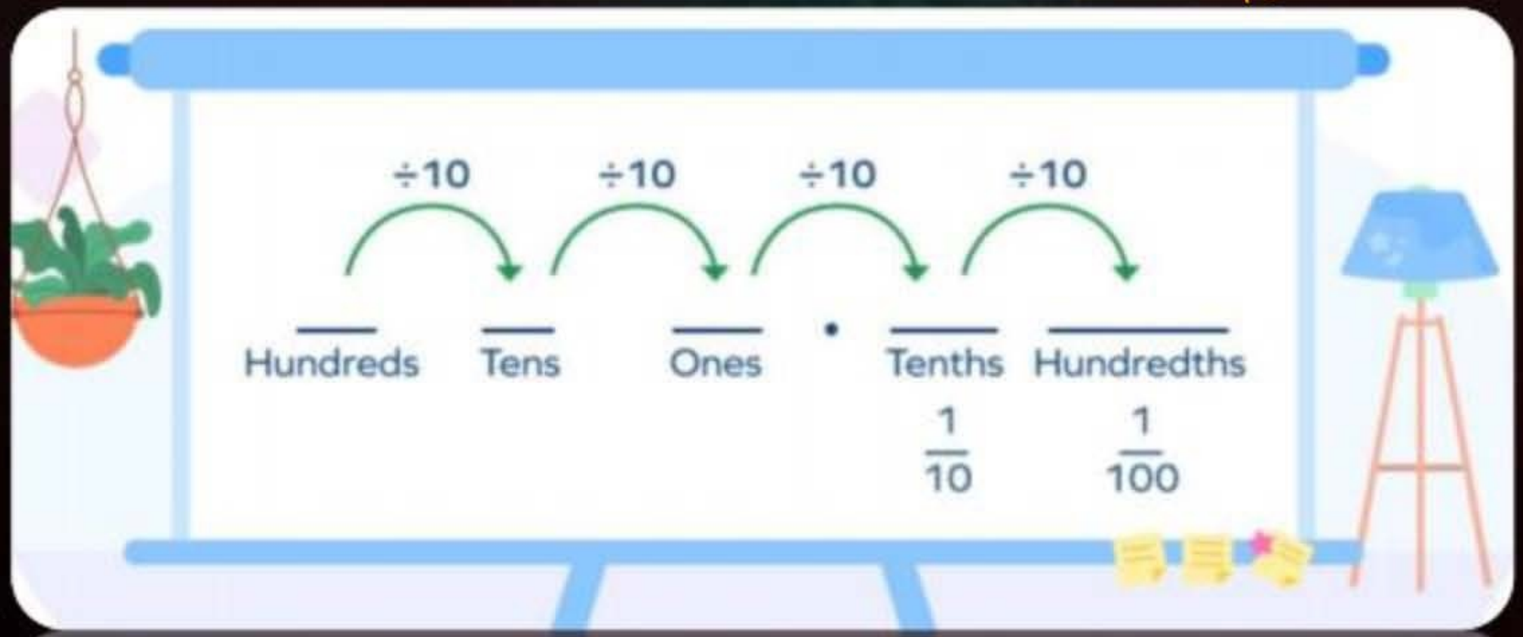
Decimal Numbers

When we go from left to right in the decimal place value system, each values is $\frac{1}{10}$ times smaller than the value to its left.

$2 \cdot 39$

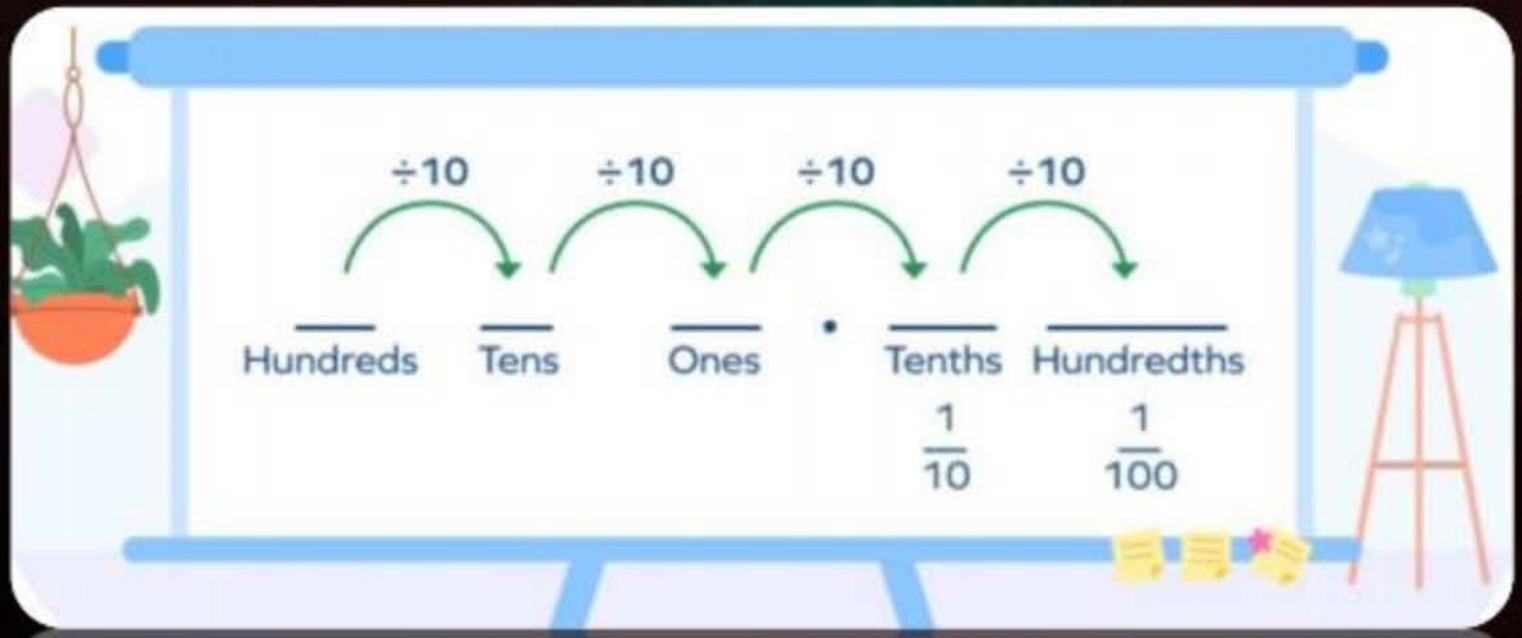
$2 \times 1 + \frac{3}{10} + \frac{9}{100}$





Visualization of real number by using the process of successive magnification

When we go from left to right in the decimal place value system, each values is $\frac{1}{10}$ times smaller than the value to its left.



Question

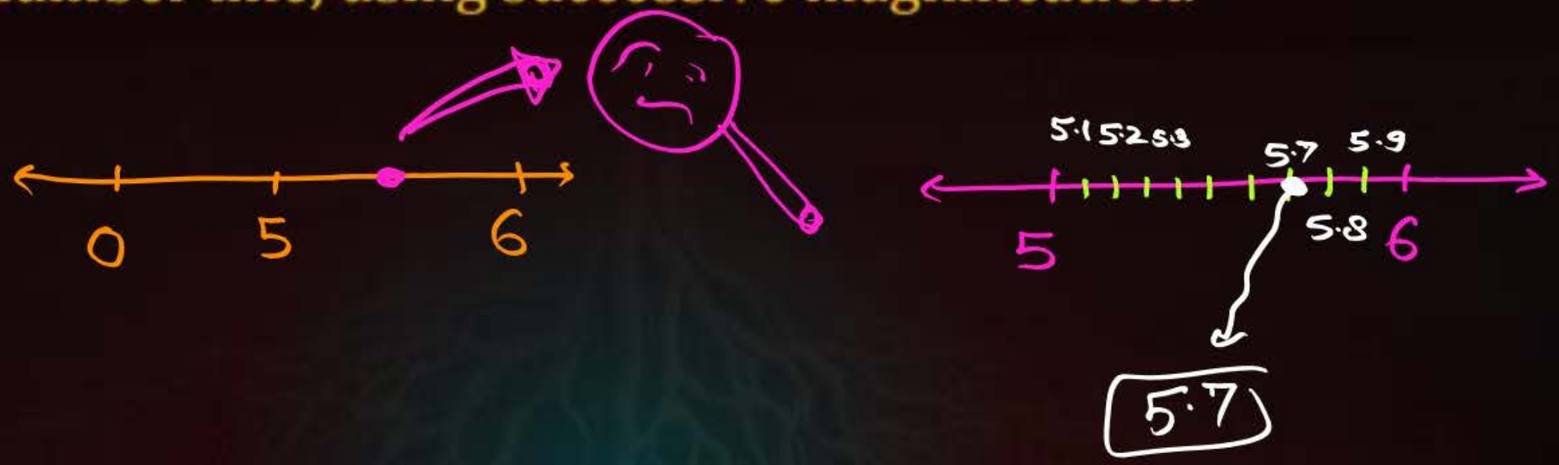
Visualization of representation of real number by using the process of successive magnification.

Question

Visualize 5.7 on the number line, using successive magnification.

Question

Visualize 5.7 on the number line, using successive magnification.

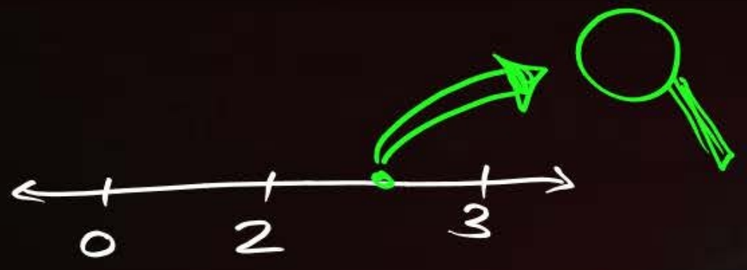


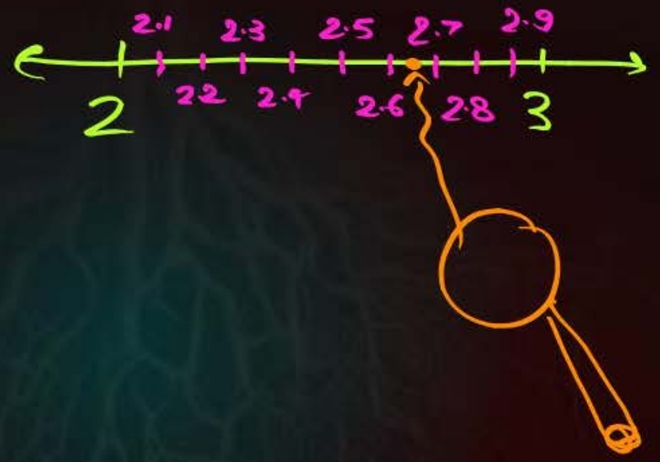
Question

Visualize 2.65 on the number line, using successive magnification.

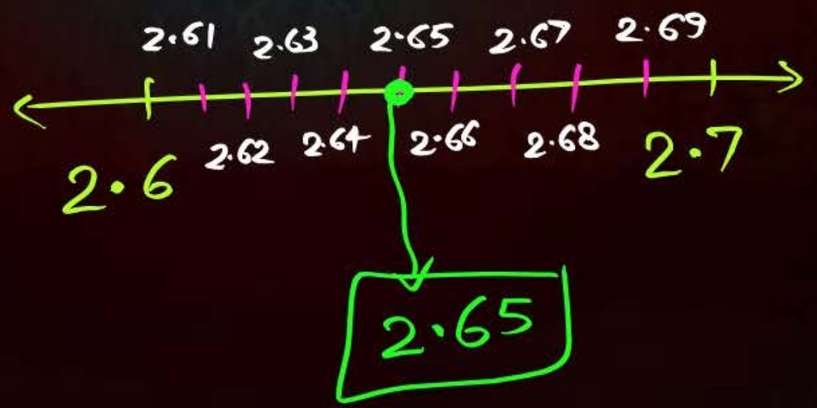
Question

Visualize 2.65 on the number line, using successive magnification.





Successive magnification



Question

Visualize 1.675 on the number line, using successive magnification.

H.W

Question

Visualize 3.24 on the number line, using successive magnification.

H.W.

Power and Exponents

Ten — Hundred

10, 100, 1000, 100000

⇒ 1000000000000000000000000000
000000000000000000000000000

$= (10)^{54}$ → To read & write → Huge Bigger Number & Huge Smaller Number

Laws of exponents for Real Number

(Base) (Exponent)

$(3)^{(3)}$ → 3 to the power 3

→ multiplication

⇒ Base vala number → Exponent times multiply Ho Rha hai

$3^3 = 3 \times 3 \times 3$

Laws of Exponents and Powers

Power of a Power Rule:

(i) $(a^m)^n =$

Laws of Exponents and Powers

Power of a Power Rule:

(i) $(a^m)^n = a^{m \cdot n}$

$(2^2)^3 = 2^6$

$2^2 \times 2^2 \times 2^2$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

Laws of Exponents and Powers

Negative Exponent:

(ii) $a^{-n} =$

Laws of Exponents and Powers

Negative Exponent:

(ii) $a^{-n} = \frac{1}{a^n}$

$2^{-3} = \frac{1}{2^3}$ & $\frac{1}{3^9} = 3^{-9}$

Important Fact:

A negative exponent in power for any non-zero integer is basically a reciprocal of the power

Laws of Exponents and Powers

(a) If base is same but exponent different:

(1) $a^m \times a^n =$

(2) $\frac{a^m}{a^n} =$

Laws of Exponents and Powers

(a) If base is same but exponent different:

(1) $a^m \times a^n = a^{m+n}$

$2^3 \times 2^2 = 2^5$

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

(2) $\frac{a^m}{a^n} = a^{m-n}$; $m > n$

$\frac{2^3}{2^2} = 2^1$

$\frac{2 \times 2 \times 2}{2 \times 2} = 2^1$

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Laws of Exponents and Powers

Exponent as Zero: (iii) $a^0 =$

Exponent as 1: (iv) $a^1 =$

Laws of Exponents and Powers

Exponent as Zero: (iii) $a^0 = 1$

Exponent as 1: (iv) $a^1 = a$

$(\text{Non-zero})^0 = 1$

Laws of Exponents and Powers

(b) If base is different but exponent is same:

(1) $a^m \cdot b^m =$

(2) $\frac{a^m}{b^m} =$

Laws of Exponents and Powers

(b) If base is different but exponent is same:

(1) $a^m \cdot b^m = (ab)^m$

(2) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

$3^2 \times 2^2 = (3 \times 2)^2$

$3 \times 3 \times 2 \times 2 = (3 \times 2) \times (3 \times 2)$

$= (3 \times 2)^2$

Laws of Exponents and Powers

Miscellaneous:

(1) $\sqrt[n]{a} =$

(2) $(a^m \cdot b^n)^x =$

(3) $a^{m/n} =$

Laws of Exponents and Powers

Miscellaneous:

(1) $\sqrt[n]{a} = a^{\frac{1}{n}}$

(2) $(a^m \cdot b^n)^x = a^{mx} \cdot b^{nx}$

(3) $a^{m/n} = (a^m)^{1/n}$

$\sqrt[n]{a} = a^{\frac{1}{n}}$

Ab bari hai dhuadhar khelne ki

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Question

Evaluate :-

(i) $(-3)^{-3}$

(ii) $\left(\frac{2}{7}\right)^{-4}$

Question

Evaluate :-

(i) $(-3)^{-3}$

$= \frac{1}{(-3)^3}$

$= \frac{1}{-3 \times -3 \times -3}$

$= \frac{1}{-27}$

(ii) $\left(\frac{2}{7}\right)^{-4}$

$= \frac{1}{\left(\frac{2}{7}\right)^4} = \frac{1}{\frac{(2)^4}{(7)^4}} = \frac{(7)^4}{(2)^4}$

$\left(\frac{2}{7}\right)^{-4} = \left(\frac{7}{2}\right)^4$

Question

Simplify:

(i) $\left(2^{-1}+4^{-1}+3^{-1}\right)^{-1}$

(ii) $\left[5^{2}-\left(\frac{1}{4}\right)^{-2}\right]\times\left(\frac{3}{4}\right)^{-2}$

Question

Simplify:

(i) $(2^{-1}+4^{-1}+3^{-1})^{-1}$

$= \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{3}\right)^{-1}$

$= \left(\frac{6+3+4}{12}\right)^{-1}$

$= \left(\frac{13}{12}\right)^{-1} = \left(\frac{12}{13}\right)^{1} = \frac{12}{13}$

(ii) $\left[5^2-\left(\frac{1}{4}\right)^{-2}\right]\times\left(\frac{3}{4}\right)^{-2}$

$\left[25-\left(\frac{4}{1}\right)^{2}\right]\times\left(\frac{4}{3}\right)^{2}$

$= [25-16]\times\frac{16}{9}$

$= 9\times\frac{16}{9}$

$= 16$

Question

Simplify and writs in exponential form with positive exponent:

(i) $\left[\left\{\left(\frac{5}{7}\right)^{2}\right\}^{-1}\right]^{-3}$

(ii) $\left(\frac{2}{7}\right)^{2} \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4}$

(iii) $\frac{[8^{-1} \times 5^{3}]}{2^{-4}}$

Question

Simplify and writs in exponential form with positive exponent:

(i) $\left[\left\{\left(\frac{5}{7}\right)^2\right\}^{-1}\right]^{-3} = \left(\frac{5}{7}\right)^{2\times-1\times-3} = \left(\frac{5}{7}\right)^6$

(ii) $\left(\frac{2}{7}\right)^2 \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4} = \left[\left(\frac{2}{7}\right)^2 \times \left(\frac{2}{7}\right)^3\right] \div \left(\frac{7}{5}\right)^8$

$= \left(\frac{2}{7}\right)^5 \div \left(\frac{7}{5}\right)^8$

$= \frac{2^5}{7^5} \times \left(\frac{5}{7}\right)^8 = \frac{2^5}{7^5} \times \frac{5^8}{7^8} = \boxed{\frac{2^5 \times 5^8}{7^{13}}}$

(iii) $\frac{[8^{-1}\times 5^3]}{2^{-4}}$

$= \frac{\left(\frac{1}{8}\times 125\right)}{\left(\frac{1}{16}\right)} = \frac{125}{8} \times \frac{16}{1}^{2}$

$= 125 \times 2$

$= 250$

Question

Evaluate :

(i) $(64)^{1/3}$

(ii) $(125)^{-1/3}$

(iii) $(27)^{-2/3}$

(iv) $\left(\frac{64}{25}\right)^{-3/2}$

Question

Evaluate :

(i) $(64)^{1/3}$ $= (4\times4\times4)^{\frac{1}{3}}$
$= 4^{3\times\frac{1}{3}} = 4^1 = 4$

(ii) $(125)^{-1/3}$

(iii) $(27)^{-2/3}$

(iv) $\left(\frac{64}{25}\right)^{-3/2}$

H.W.

Question

Simplify each of the following, removing radical signs and negative indices wherever they occur:

(i) $\left(\sqrt{4}\right)^{-3}$

(ii) $\left(\sqrt{5}\right)^{-3}\left(\sqrt{2}\right)^{-3}$

(iii) $\left(\sqrt[3]{8}\right)^{-1/2}$

(iv) $\left(\frac{256}{81}\right)^{5/4}$

Question

Simplify each of the following, removing radical signs and negative indices wherever they occur:

(i) $\left(\sqrt{4}\right)^{-3} = \left(4^{1/2}\right)^{-3} = 4^{-3/2}$

(ii) $\left(\sqrt{5}\right)^{-3}\left(\sqrt{2}\right)^{-3} = 5^{-3/2} \times 2^{-3/2} = (10)^{-3/2}$

(iii) $\left(\sqrt[3]{8}\right)^{-1/2} = \left(8^{1/3}\right)^{-1/2} = (8)^{-1/6}$

(iv) $\left(\frac{256}{81}\right)^{5/4} = \left[\frac{(4)^4}{(3)^4}\right]^{5/4} = \frac{4^{4 \times 5/4}}{3^{4 \times 5/4}} = \frac{4^5}{3^5} = \left(\frac{4}{3}\right)^5$

Question

Find the value of $\left[\left((256)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right]^2$

Question

Find the value of $\left[\left((256)^{\frac{-1}{2}}\right)^{\frac{-1}{4}}\right]^2$

$(4\times4\times4\times4)^{-\frac{1}{2}\times-\frac{1}{4}\times2}$

$(2\times2\times2\times2\times2\times2\times2\times2)^{\frac{1}{4}}$

$= 2^{\frac{2}{8}\times\frac{1}{4}} = 2^2 = \boxed{4}$

Question

Simplify: $\frac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$

Question

Simplify: $\dfrac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$

$$\frac{5^{2\times\frac{3}{2}}\times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times 2^{3\times\frac{4}{3}}} = \frac{5^3\times 3^3}{2^5\times 2^4} = \frac{(15)^3}{2^9} = \frac{(15)^3}{(2^3)^3} = \frac{(15)^3}{(8)^3}$$

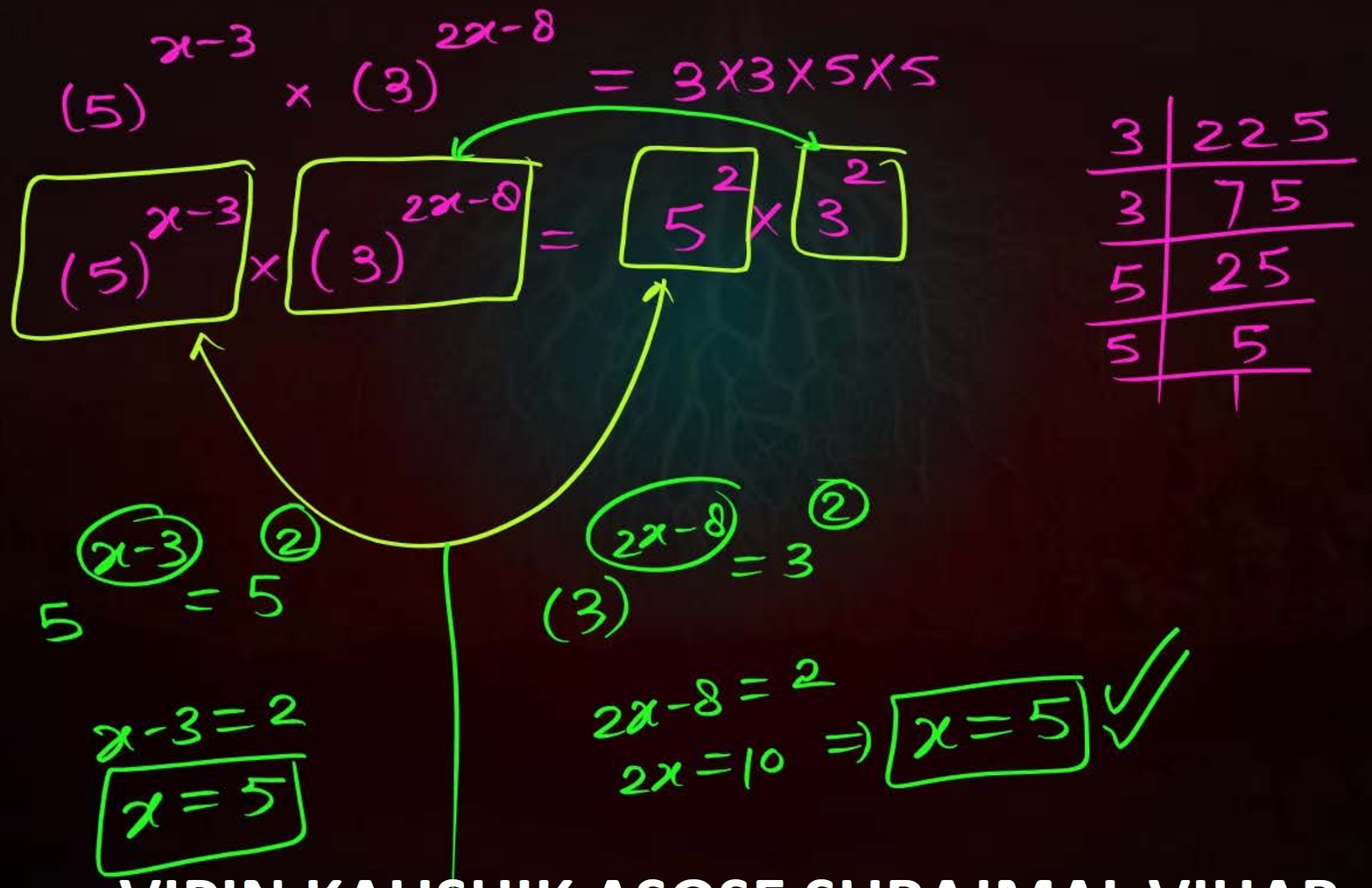
$$= \left(\frac{15}{8}\right)^3$$

Question

Find the value of x, if $5^{x-3}.3^{2x-8} = 225$

Question

Find the value of x, if $5^{x-3}.3^{2x-8} = 225$



3	225
3	75
5	25
5	5
	1

Question

Find the value of the term $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a}$

Question

Find the value of the term $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a}$

$$= \sqrt{\frac{1}{a} \times b} \times \sqrt{\frac{1}{b} \times c} \times \sqrt{\frac{1}{c} \times a}$$

$$= \sqrt{\frac{b}{a}} \times \sqrt{\frac{c}{b}} \times \sqrt{\frac{a}{c}}$$

$$= \left(\frac{b}{a}\right)^{1/2} \times \left(\frac{c}{b}\right)^{1/2} \times \left(\frac{a}{c}\right)^{1/2}$$

$$= \left[\frac{b}{a} \times \frac{c}{b} \times \frac{a}{c}\right]^{1/2} = (1)^{1/2} = \sqrt{1} = \boxed{1}$$

Question

Find the value of $\frac{2^n+2^{n-1}}{2^{n+1}-2^n}$

Question

Find the value of $\frac{2^n+2^{n-1}}{2^{n+1}-2^n}$ (most Imp)

$$\Rightarrow \frac{2^n+2^{n-1}}{2^{n+1}-2^n} = \frac{2^n+2^n\times 2^{-1}}{2^n\times 2^1-2^n} = \frac{2^n\left[1+2^{-1}\right]}{2^n\left[2^1-1\right]} = \frac{\left(1+\frac{1}{2}\right)}{(2-1)}$$

$$= \frac{\left(\frac{3}{2}\right)}{1}$$

$$= \boxed{\frac{3}{2}}$$

$$2^{n-1} = 2^{n+(-1)} = 2^n \times 2^{(-1)}$$

$$2^{n+1} = 2^n \times 2^1$$

key concepts

Question

If $(10)^x = 64$, then find the value of $10^{\left(\frac{x}{2}+1\right)}$

Question

If $(10)^x = 64$, then find the value of $10^{\left(\frac{x}{2}+1\right)}$

$$(10)^{\left(\frac{x}{2}+1\right)}$$

$$= (10)^{x/2} \times (10)^1$$

$$= (10^x)^{1/2} \times 10$$

$$= (64)^{1/2} \times 10$$

$$= (8)^{2 \times 1/2} \times 10$$

$$= 8^1 \times 10 = 8 \times 10 = \boxed{80}$$ Ans

Question

Simplify :-

(i) $\sqrt[5]{16} \times \sqrt[5]{2}$

(ii) $\dfrac{\sqrt[4]{243}}{\sqrt[4]{3}}$

Question

Simplify :-

(i) $\sqrt[5]{16} \times \sqrt[5]{2}$

$(16)^{\frac{1}{5}} \times 2^{\frac{1}{5}} = (16 \times 2)^{\frac{1}{5}}$

$= (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}}$

$= 2^{5 \times \frac{1}{5}}$

$= 2^1$

$= 2$

$= \boxed{2}$

(ii) $\dfrac{\sqrt[4]{243}}{\sqrt[4]{3}}$

$\dfrac{(243)^{\frac{1}{4}}}{(3)^{\frac{1}{4}}} = \left(\dfrac{243}{3}\right)^{\frac{1}{4}}$

$= (81)^{\frac{1}{4}}$

$= (3)^{4 \times \frac{1}{4}}$

$= 3^1$

$= 3$

$= \boxed{3}$

Question

If $a = 2$ and $b = 5$ then find value of following

(1) $(a^b + b^a)^{-1}$

(2) $(a^a + b^b)^{-1}$

Question

If $a = 2$ and $b = 5$ then find value of following

(1) $(a^b + b^a)^{-1} = \frac{1}{a^b + b^a} = \frac{1}{2^5 + 5^2} = \frac{1}{32 + 25} = \frac{1}{57}$

(2) $(a^a + b^b)^{-1} = \frac{1}{a^a + b^b} = \frac{1}{2^2 + 5^5} = \frac{1}{4 + 3125} = \frac{1}{3129}$

Adding or Subtracting two Irrational Number

Step 1: Figure out like and unlike terms

Step 2: Make group of like terms

Step 3: Apply algebraic Summation with their coefficients (for like terms)

Step 4: Check that in final step all the terms are unlike, unless repeat the step 1,2,3

Question

Add $2\sqrt{2} + 5\sqrt{3}$ and $\sqrt{2} - 3\sqrt{3}$

Question

Add $2\sqrt{2} + 5\sqrt{3}$ and $\sqrt{2} - 3\sqrt{3}$

$$= (2\sqrt{2} + 5\sqrt{3}) + (\sqrt{2} - 3\sqrt{3})$$

$$= (2\sqrt{2} + \sqrt{2}) + (5\sqrt{3} - 3\sqrt{3})$$

$$= \boxed{3\sqrt{2} + 2\sqrt{3}}$$

Multiplying or Dividing two Irrational Number

TRICK: (1) $\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a^{\frac{1}{2} \times 2} = a$

(2) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, where a and b are real number

Multiply/Divide Rational and Irrational part of given two terms separately and then find product/ division of these two.

Question

Find product of

(i) $6\sqrt{5}$ and $2\sqrt{5}$

(ii) $4\sqrt{3}$ and $3\sqrt{7}$

Question

Find product of

(i) $6\sqrt{5}$ and $2\sqrt{5}$

$6\sqrt{5} \times 2\sqrt{6}$

$= (6 \times 2) \times \sqrt{5 \times 6}$

$= \boxed{12\sqrt{30}}$

(ii) $4\sqrt{3}$ and $3\sqrt{7}$

$4\sqrt{3} \times 3\sqrt{7}$

$= (4 \times 3) \times \sqrt{3 \times 7}$

$= \boxed{12\sqrt{21}}$

Question

Divide

(i) $8\sqrt{5}$ by $2\sqrt{5}$

(ii) $12\sqrt{15}$ and $3\sqrt{5}$

Question

Divide

(i) $8\sqrt{5}$ by $2\sqrt{5}$

$$\frac{8\sqrt{5}}{2\sqrt{5}} = \frac{8}{2} \times \sqrt{\frac{5}{5}}$$

$$= 4 \times \sqrt{1}$$

$$= 4 \times 1$$

$$= \boxed{4}$$

(ii) $12\sqrt{15}$ and $3\sqrt{5}$

$$\frac{12\sqrt{15}}{3\sqrt{5}} = \frac{4}{\frac{12}{3}} \times \sqrt{\frac{15}{5}}$$

$$= 4 \times \sqrt{3}$$

$$= \boxed{4\sqrt{3}}$$

Some important result

- The sum, difference, product and quotient of a non-zero rational number with irrational number is always irrational.
- If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.

Some identities

(i) $(\sqrt{a})^2 =$

(ii) $\sqrt{a} \cdot \sqrt{b} =$

(iii) $\frac{\sqrt{a}}{\sqrt{b}} =$

(iv) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) =$

(v) $(a + \sqrt{b})(a - \sqrt{b}) =$

(vi) $(\sqrt{a} \pm \sqrt{b})^2 =$

(vii) $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) =$

Some identities

(i) $(\sqrt{a})^2 = a$

(ii) $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

(iii) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

(iv) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

(v) $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

(vi) $(\sqrt{a} \pm \sqrt{b})^2 = a \pm 2\sqrt{ab} + b$

(vii) $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

Question

Simplify:

(i) $\left(4+\sqrt{7}\right)\left(3+\sqrt{2}\right)$

(ii) $\left(\sqrt{5}-2\right)\left(\sqrt{3}-\sqrt{5}\right)$

Question

Simplify:

(i) $(4+\sqrt{7})(3+\sqrt{2})$

$= 4(3+\sqrt{2}) + \sqrt{7}(3+\sqrt{2})$

$= 12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}$

(ii) $(\sqrt{5}-2)(\sqrt{3}-\sqrt{5})$

$= \sqrt{15} - 5 - 2\sqrt{3} + 2\sqrt{5}$

Question

Simplify:

(i) $\left(5+\sqrt{7}\right)\left(5-\sqrt{7}\right)$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

Question

Simplify:

(i) $(5+\sqrt{7})(5-\sqrt{7})$

$= (5)^2 - (\sqrt{7})^2$

$= 25 - 7$

$= \boxed{18}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

$= (3)^2 - (\sqrt{3})^2$

$= 9 - 3$

$= \boxed{6}$

Question

Simplify:

(iii) $\left(\sqrt{3}+\sqrt{7}\right)^2$

(iv) $\left(2\sqrt{5}+3\sqrt{2}\right)^2$

Question

Simplify:

(iii) $\left(\sqrt{3}+\sqrt{7}\right)^2$

$(\sqrt{3})^2+(\sqrt{7})^2+2\sqrt{21}$

$= 3+7+2\sqrt{21}$

$= 10+2\sqrt{21}$

(iv) $\left(2\sqrt{5}+3\sqrt{2}\right)^2$

$(2\sqrt{5})^2+(3\sqrt{2})^2+2(6\sqrt{10})$

$= (4\times5)+(9\times2)+12\sqrt{10}$

$= \boxed{38+12\sqrt{10}}$

Rationalisation

When the denominator of an expression contains an irrational term, then the process of converting its denominator into a rational number is called rationalization.

→ $\frac{3+\sqrt{5}}{\sqrt{2}}$

→ $\frac{1}{2+\sqrt{3}}$

→ $\frac{1}{\sqrt{2}+\sqrt{3}}$

✓ **Rationalizing Factor**

Rationalisation

Denominator ko Irrational se Rational bnana!!

When the denominator of an expression contains an irrational term, then the process of converting its denominator into a rational number is called rationalization.

→ $\frac{3+\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(3+\sqrt{5})}{2}$

✓ **Rationalizing Factor**

→ $\frac{1}{2+\sqrt{3}}$

→ $\frac{1}{\sqrt{2}+\sqrt{3}}$

Rationalizing Factor

(a) Rationalizing factor of $\frac{1}{\sqrt{a}}$ is

(b) Rationalizing factor of $\frac{1}{a+\sqrt{b}}$ is

(c) Rationalizing factor of $\frac{1}{\sqrt{a}+\sqrt{b}}$ is

(d) Rationalizing factor of $\frac{1}{a+b\sqrt{c}}$ is

Rationalizing Factor

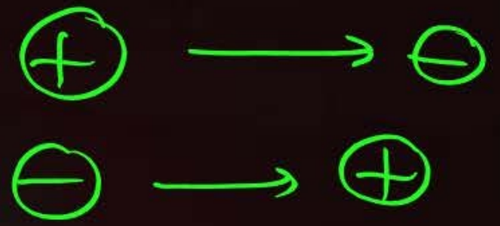
Rationalizing Factor

(a) Rationalizing factor of $\frac{1}{\sqrt{a}}$ is $\sqrt{a}$

(b) Rationalizing factor of $\frac{1}{a+\sqrt{b}}$ is $a-\sqrt{b}$

(c) Rationalizing factor of $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\sqrt{a}-\sqrt{b}$

(d) Rationalizing factor of $\frac{1}{a+b\sqrt{c}}$ is $a-b\sqrt{c}$



Rationalizing Factor

Question

Rationalizing factor of the denominator $\frac{1}{7+3\sqrt{2}}$ will be

$7 + 3\sqrt{2}$

$\frac{1}{7+3\sqrt{2}}$

$7 - 3\sqrt{2}$

$\frac{1}{7-3\sqrt{2}}$

Question

Rationalizing factor of the denominator $\frac{1}{7+3\sqrt{2}}$ will be

A $7 + 3\sqrt{2}$

B $\frac{1}{7+3\sqrt{2}}$

C $7 - 3\sqrt{2}$

D $\frac{1}{7-3\sqrt{2}}$

$7 - 3\sqrt{2}$

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Question

$\frac{1}{\sqrt{9}-\sqrt{8}}$ will be

$\frac{1}{2}(3-2\sqrt{2})$

$\frac{1}{3+2\sqrt{2}}$

$3-2\sqrt{2}$

$3+2\sqrt{2}$

Question

$\frac{1}{\sqrt{9}-\sqrt{8}}$ will be

A $\frac{1}{2}(3-2\sqrt{2})$

B $\frac{1}{3+2\sqrt{2}}$ ✗

C $3-2\sqrt{2}$ ✓

D $3+2\sqrt{2}$

$$\frac{1}{(\sqrt{9}-\sqrt{8})} \times \frac{(\sqrt{9}+\sqrt{8})}{(\sqrt{9}+\sqrt{8})} = \frac{(\sqrt{9}+\sqrt{8})}{(\sqrt{9})^2-(\sqrt{8})^2}$$

$(a-b)(a+b) = a^2-b^2$

$$= \frac{\sqrt{3\times3}-\sqrt{2\times2\times2}}{9-8}$$

$$= \frac{3-2\sqrt{2}}{1} = 3-2\sqrt{2}$$

Question

Given $\sqrt{2} = 1.414$, find the values to three decimal place of $\frac{1}{\sqrt{2}}$.

Question

Given $\sqrt{2} = 1.414$, find the values to three decimal place of $\frac{1}{\sqrt{2}}$.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{(\sqrt{2})^2} = \frac{1.414}{2} = 0.707$$

Question

Simplify each of the following by rationalizing the denominator.

(i) $\frac{5+\sqrt{6}}{5-\sqrt{6}}$

(ii) $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$

Question

Simplify each of the following by rationalizing the denominator.

(i) $\frac{5+\sqrt{6}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{(5+\sqrt{6})^2}{25-6}$

$= \frac{25+6+10\sqrt{6}}{19}$

$= \boxed{\frac{31+10\sqrt{6}}{19}}$

(ii) $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$ → H.W.

Question

If a and b are rational numbers, find the value of a and b in each. (Most Imp)

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

Question

If a and b are rational numbers, find the value of a and b in each. (Most Imp)

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = a + b\sqrt{3}$$

$$\frac{(\sqrt{3}-1)^2}{3-1} = a + b\sqrt{3}$$

$$\frac{3+1-2\sqrt{3}}{2} = a + b\sqrt{3}$$

$$\frac{4-2\sqrt{3}}{2} = a + b\sqrt{3}$$

$$\frac{4}{2} + \left(\frac{-2}{2} \times \sqrt{3}\right) = a + b\sqrt{3}$$

$$2 + (-2)\sqrt{3} = a + b\sqrt{3}$$

$a = 2$

$b = -2$

Question

If a and b are rational numbers, find the value of a and b in each.

(ii) $\frac{5+2\sqrt{3}}{7-4\sqrt{3}} = a + b\sqrt{3}$

Question

If a and b are rational numbers, find the value of a and b in each.

(ii) $\frac{5+2\sqrt{3}}{7-4\sqrt{3}} = a + b\sqrt{3}$

$$\frac{(5+2\sqrt{3})}{(7-4\sqrt{3})} \times \frac{(7+4\sqrt{3})}{(7+4\sqrt{3})} \Rightarrow \frac{35+20\sqrt{3}+14\sqrt{3}+24}{49-48} = a+b\sqrt{3}$$

$$= a + b\sqrt{3} \qquad \Rightarrow \qquad 59 + 34\sqrt{3} = a + b\sqrt{3}$$

$$\Rightarrow \boxed{\begin{array}{l} a = 59 \\ b = 34 \end{array}}$$

Question

Find value of $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$

Question

Find value of $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$

$$\frac{(\sqrt{2}-\sqrt{1})}{(\sqrt{2}+\sqrt{1})(\sqrt{2}-\sqrt{1})}+\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}+\cdots\cdots+\frac{\sqrt{9}-\sqrt{8}}{\sqrt{9}+\sqrt{8}}$$

$$=\frac{\sqrt{2}-\sqrt{1}}{2-1}+\frac{\sqrt{3}-\sqrt{2}}{3-2}+\frac{\sqrt{4}-\sqrt{3}}{4-3}+\cdots\cdots+\frac{\sqrt{8}-\sqrt{7}}{8-7}+\frac{\sqrt{9}-\sqrt{8}}{9-8}$$

$$=\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+\cdots+\sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8}$$

$$=-\sqrt{1}+\sqrt{9}=-1+3=\boxed{2}$$

Question

If $\sqrt{2} = 1.4142$ then value of $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

2.4142

5.8282

0.4142

0.1718

Question

If $\sqrt{2} = 1.4142$ then value of $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

A 2.4142

B 5.8282

C 0.4142

D 0.1718

$$\sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}} = \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{\frac{(\sqrt{2}-1)^2}{1}}$$

$$= \sqrt{2}-1$$

$$= 1.4142-1$$

$$= 0.4142$$

$$\sqrt{\frac{1.4142-1}{1.4142+1}} = \sqrt{\frac{0.4142}{2.4142}} = \sqrt{\frac{4142}{24142}}$$

$$= \sqrt{(\text{-ve})}$$

Question

If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then value of $x^{\frac{1}{2}} + y^{\frac{1}{2}}$.

Question

If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then value of $x^{\frac{1}{2}} + y^{\frac{1}{2}}$.

$$x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} = (\sqrt{3}+\sqrt{2})^2$$

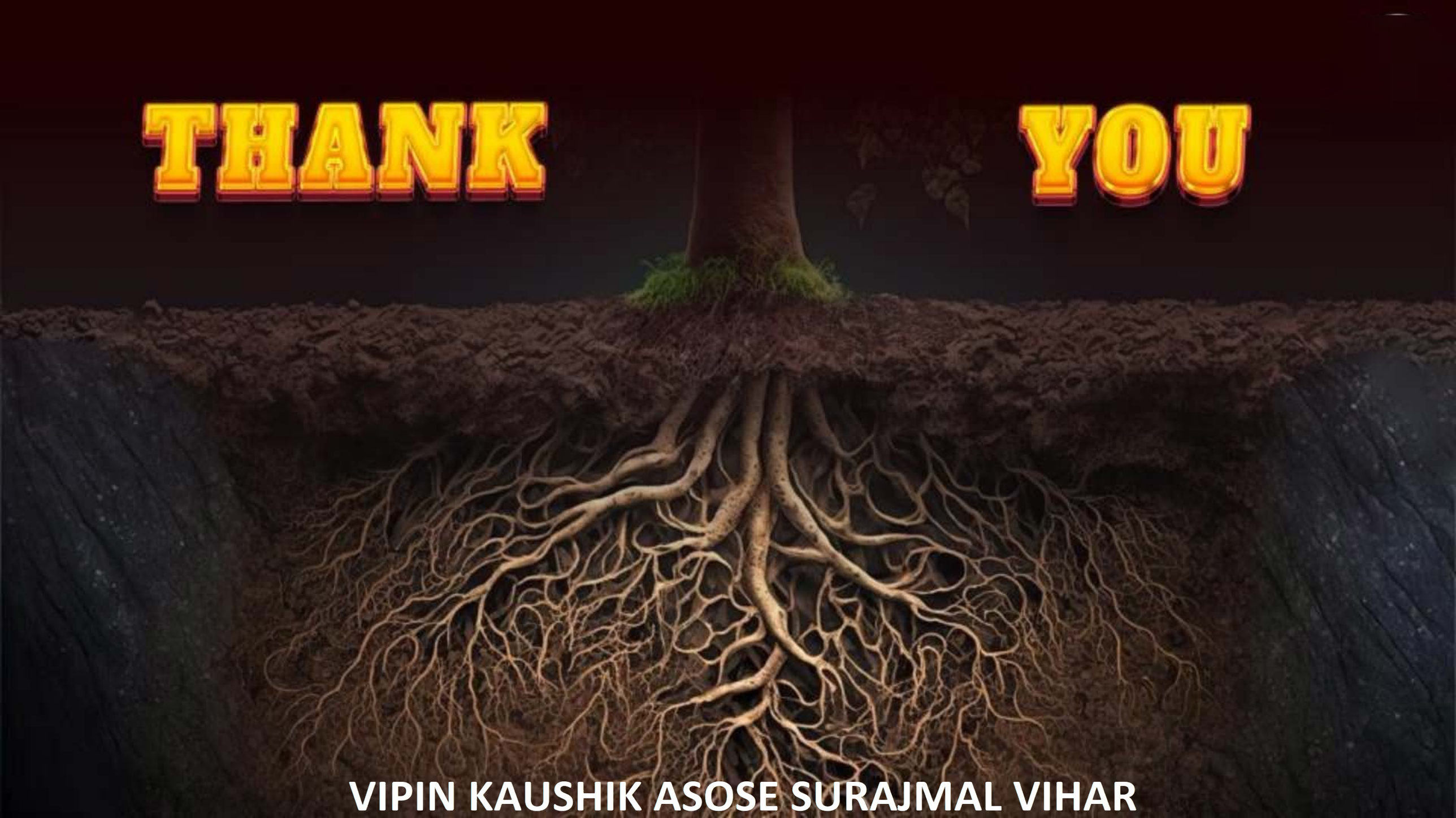
$$y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} = (\sqrt{3}-\sqrt{2})^2$$

$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = \left[(\sqrt{3}+\sqrt{2})^2\right]^{\frac{1}{2}} + \left[(\sqrt{3}-\sqrt{2})^2\right]^{\frac{1}{2}}$$

$$= \sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2} = \boxed{2\sqrt{3}}$$

Ab Dete Hai 21 Topo Ki Salami Ke Sath Aakhiri Bidayi

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR



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