VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

- Subject - Mathematics
- Chapter – Heron's Formula

Today's Targets

1. Area of Different Types of Triangles
2. Why Heron's formula Needed??
3. Application of Heron's formula : Area of Quadrilaterals

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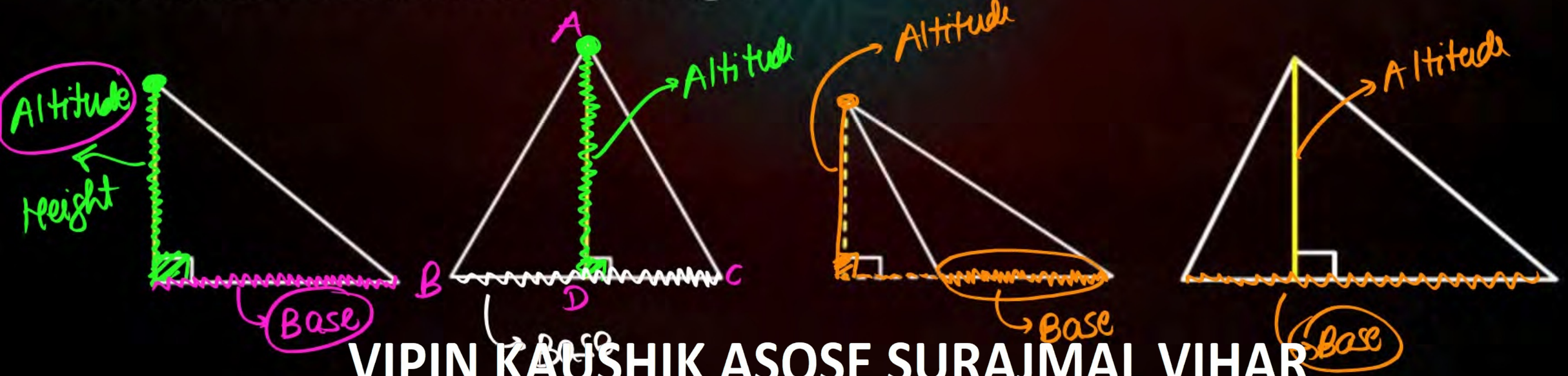
Triangle and Its Components

In a triangle, a line segment from a vertex and perpendicular to the opposite side (know as base) is called an altitude. It is also called the height of a triangle.

When a triangle is a right triangle, the altitude, or height, is the leg. If the triangle is obtuse, then the altitude will be outside of the triangle. If the triangle is acute, then the altitude will be inside the triangle.



Key point :-

Jisane perpendicular dala → Altitude / Height → (opposite vertex se nikla Hua line)

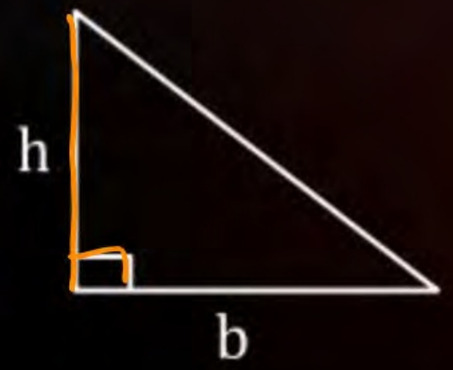
Jisape perpendicular data gya → Base

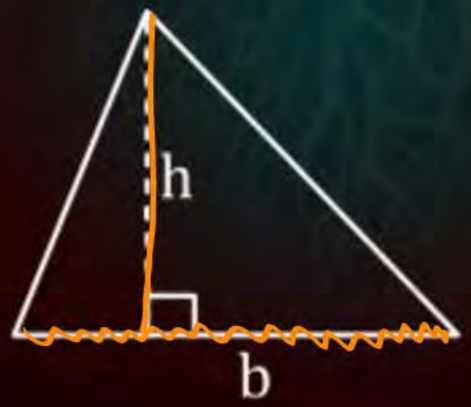
Area of a Triangle

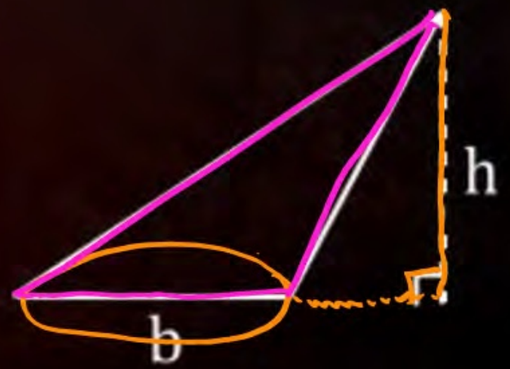
Area of a triangle is given by below formula;

$$\text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \text{base} \times \text{Altitude}$$







Question

The base of a triangular field is $2\frac{1}{2}$ times its height. If the cost of turfing it at Rs. 40 per 100 m^2 is Rs. 800, then find its base.

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Base $= (2\frac{1}{2}) \times$ Height

$b = \frac{5}{2} \times h \Rightarrow h = \frac{2b}{5}$

Area of $\triangle$

$= \frac{1}{2} \times b \times h$

$2000 = \frac{1}{2} \times b \times \frac{2b}{5}$

$2000 = \frac{2b^2}{10}$

$\Rightarrow \frac{2000 \times 10}{2} = b^2$

$\Rightarrow 10000 = b^2 \Rightarrow b^2 = (100)^2$

$\Rightarrow \boxed{b = 100 \text{ m}}$ ✓

₹40 → 100 m^2

₹1 → $\frac{100}{40}$ m^2

₹800 → $\left(800 \times \frac{100}{40}\right) m^2$

$= 200 \times 10 = \boxed{2000 m^2}$

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Question

Find the altitude of a triangle whose area is 540 m^2 and base is 30 m.

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$$\text{Area of } \Delta = \frac{1}{2} \times b \times h$$

$$540 = \frac{1}{2} \times 30 \times h$$

$$540 \times 2 = 30 \times h$$

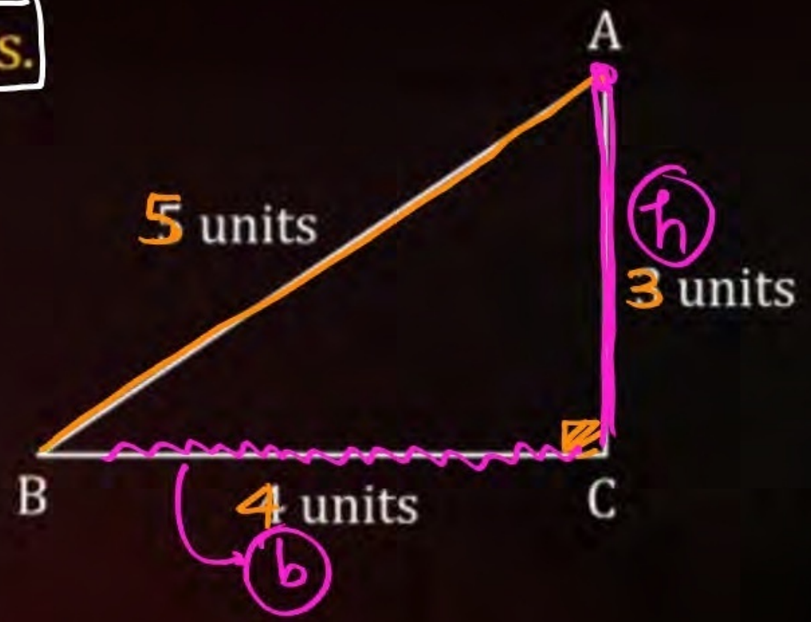
$$h = \frac{54\cancel{0} \times 2}{3\cancel{0}}$$

$$h = \frac{\overset{18}{\cancel{54}} \times 2}{\cancel{3}}$$

$$h = 18 \times 2 = \boxed{36\text{ m}}$$ Ans.

Question

Find the area of a triangle whose sides are 3, 4 and 5 units.



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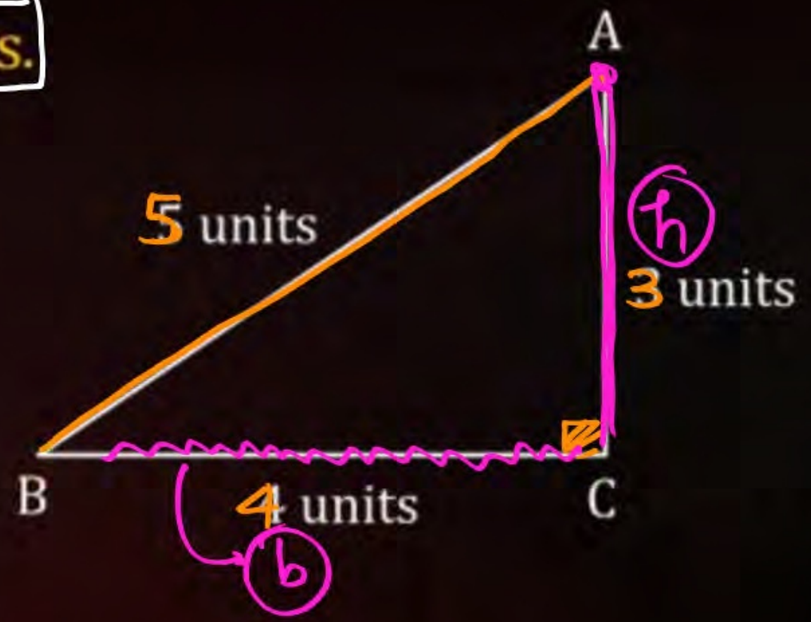


using pythagoras theorem.

$H^2 = P^2 + B^2$ → only applicable in Right angled Δ

Now, $(3)^2 + (4)^2 = 9 + 16$

$(3)^2 + (4)^2 = 25$

$(3)^2 + (4)^2 = (5)^2$ → It means 3, 4 & 5 will be sides of Right angled Δ.

Area of $\Delta = \frac{1}{2} \times 4 \times 3 =$ 6 unit^2

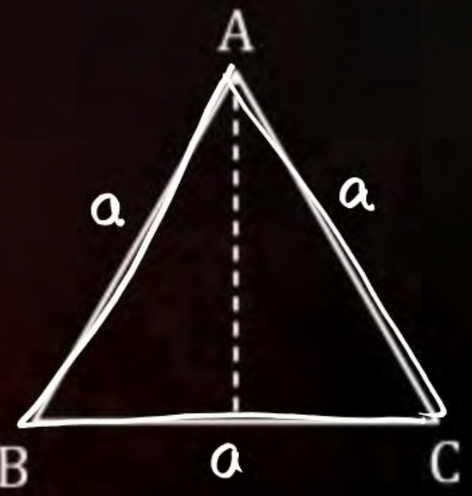
Question

Find the area of a triangle whose base is 7 cm and height is 10 cm.

Area of an Equilateral Triangle

If we have, side length of an equilateral triangle as 'a', then area is given by

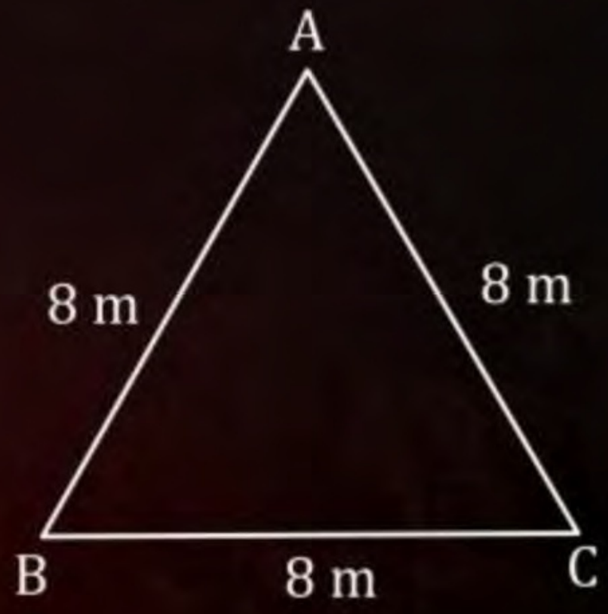
- Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times a^2$



- Height/ Altitude of equilateral triangle $= \frac{\sqrt{3}}{2} \times a$

Question

Find the area of an equilateral triangle of sides 8 m.



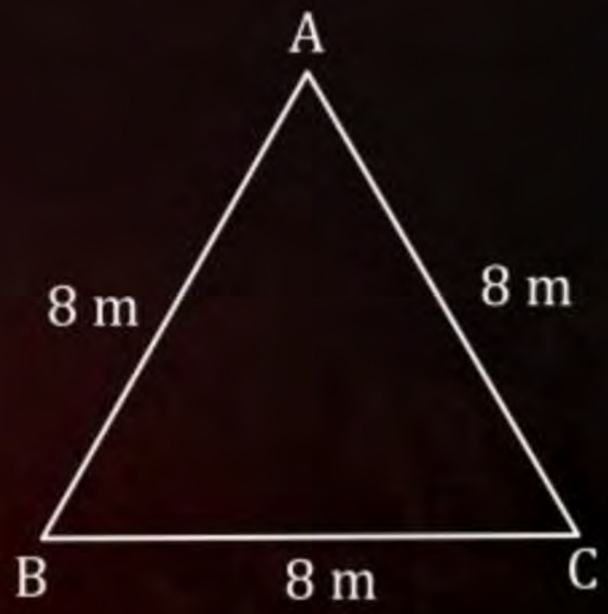
Question

Find the area of an equilateral triangle of sides 8 m.

Area of equilateral $\Delta = \frac{\sqrt{3}}{4} \times (8)^2$

$= \frac{\sqrt{3}}{4} \times 64^{16}$

$= \boxed{16\sqrt{3}\ m^2}$



Question

The area of [redacted] equilateral triangular children park is $6400\sqrt{3}$ m². Find each side of the park.

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Question

The area of [redacted] equilateral triangular children park is $6400\sqrt{3}$ m². Find each side of the park.

$$\text{Area} = \frac{\sqrt{3}}{4} \times (a)^2$$

$$6400\sqrt{3} = \frac{\sqrt{3}}{4} \times (a)^2$$

$$(a)^2 = \frac{6400 \times \sqrt{3} \times 4}{\sqrt{3}}$$

$$a^2 = 64 \times 100 \times 4$$

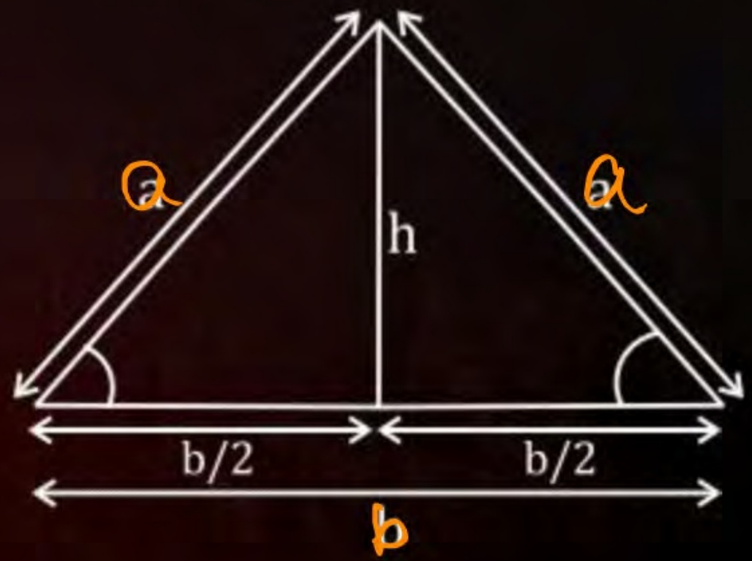
$$a^2 = 8 \times 8 \times 10 \times 10 \times 2 \times 2 \Rightarrow a^2 = (8 \times 10 \times 2)^2$$

$$a = 160 \text{ m}$$

Area of an Isosceles triangle

If we have, equal side length of an Isosceles triangle as 'a', and remaining unequal side as 'b' then area is given by

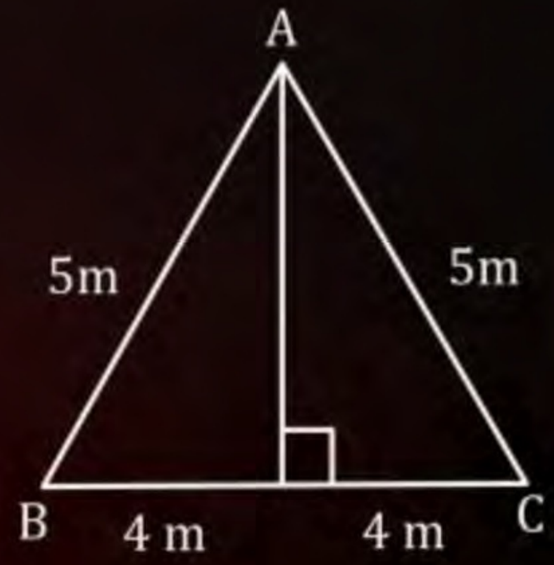
- Area of Isosceles triangle $= \frac{b}{2} \times \sqrt{a^2 - \frac{b^2}{4}}$



- Height/ Altitude of Isosceles triangle $= \sqrt{a^2 - \frac{b^2}{4}}$

Question

Find the area of an Isosceles triangle ABC in which AB = AC = 5 m and BC = 8 m.



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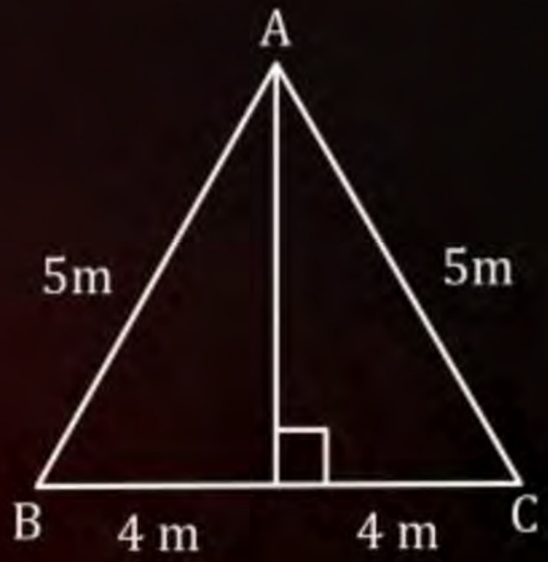


Area of Isosceles $\Delta = \frac{b}{2} \times \sqrt{a^2 - \frac{b^2}{4}}$

$= \frac{8}{2} \times \sqrt{(5)^2 - \frac{(8)^2}{4}}$

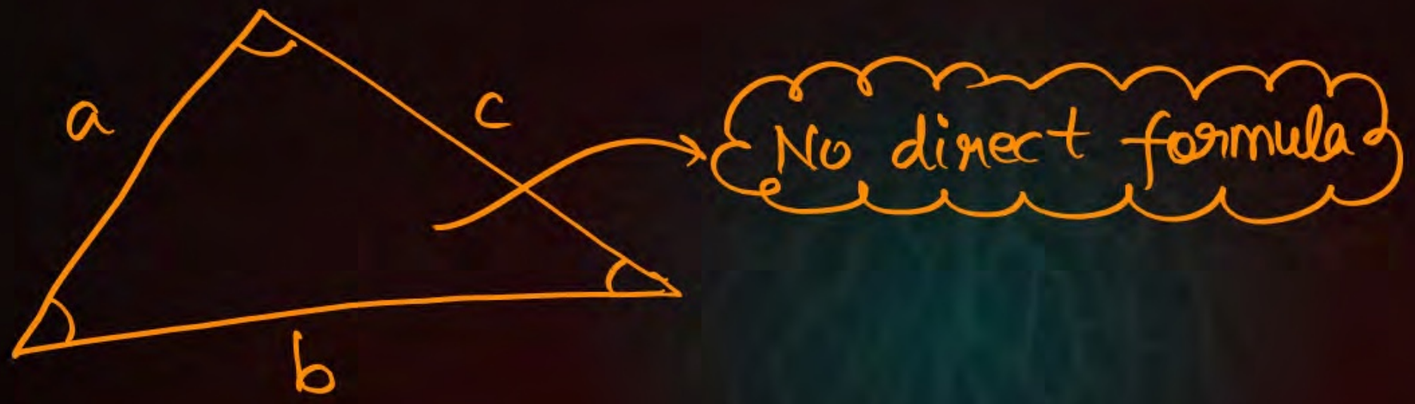
$= 4 \times \sqrt{25 - \frac{64}{4}}$

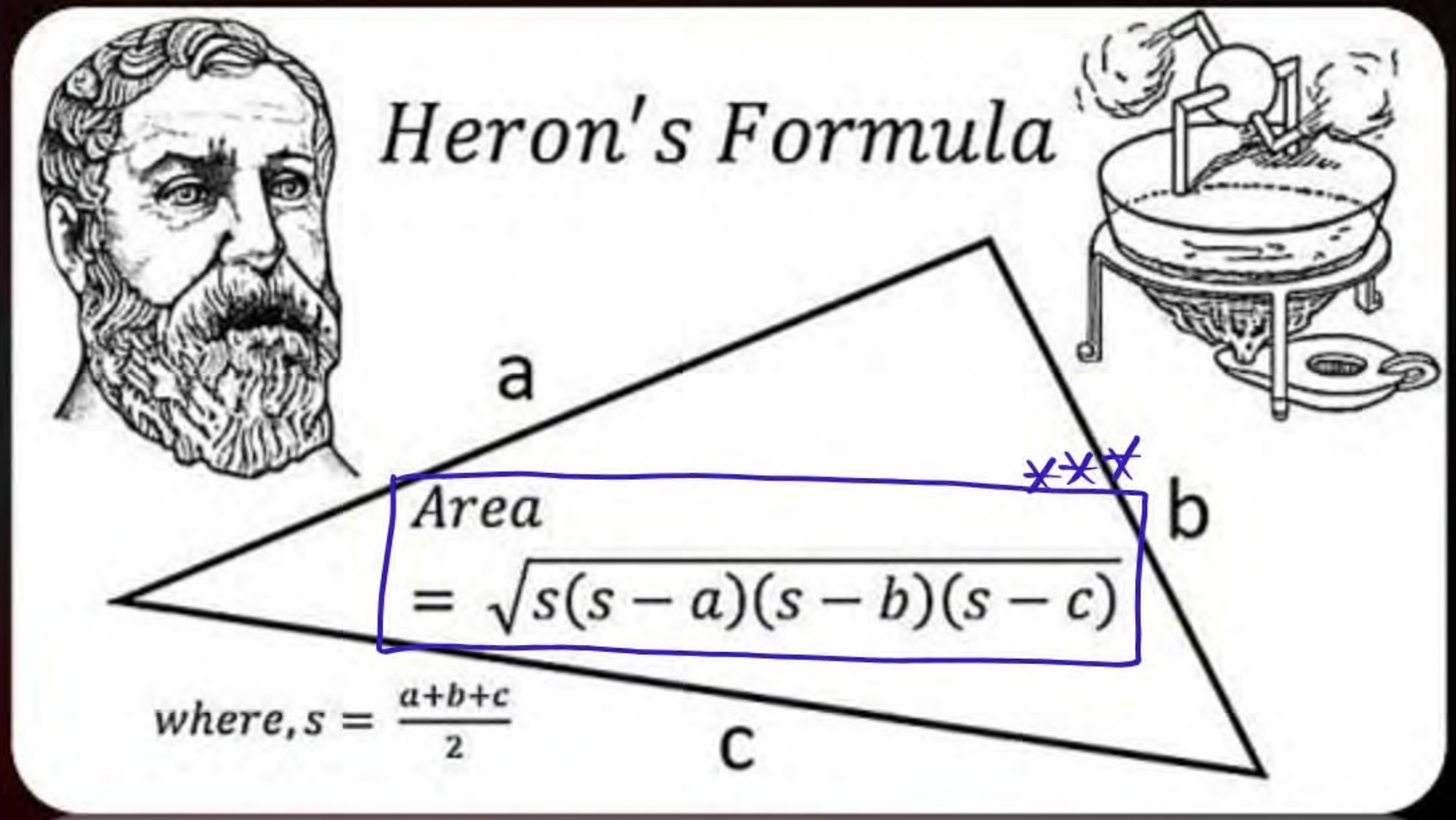
$= 4 \times \sqrt{25 - 16}$

$= 4 \times \sqrt{9} = 4 \times 3$

$= \boxed{12 \text{ m}^2}$ ✓

But, how we can calculate area of Scalene Triangle??





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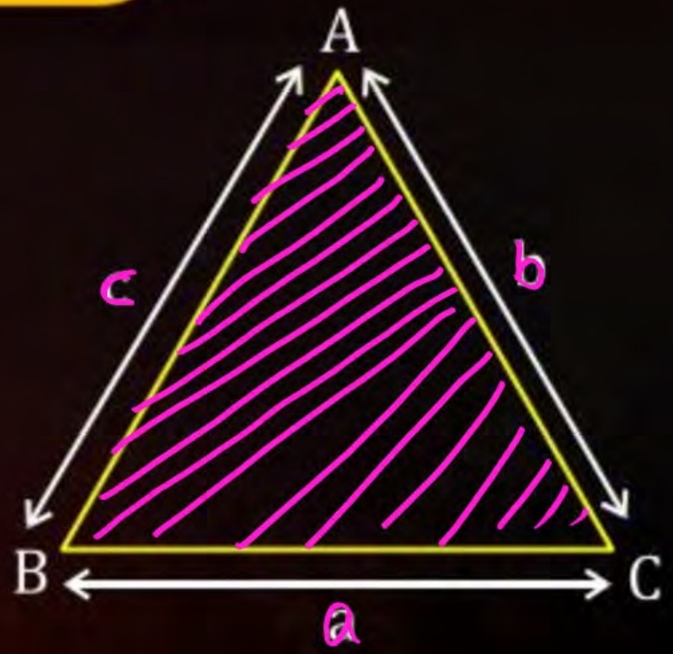
Area of Triangle Using Heron's Formula

In a ΔABC having its sides as a, b and c and perimeter as $a + b + c$, then $s = \frac{a+b+c}{2}$, where s is the semi-perimeter.



Now,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

This is known as the Heron's formula.

Thus, when we are given the lengths of three sides of a triangle, we can find its area by using formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ where, 's' stands for semi-perimeter of the triangle; a, b and c are the lengths of the sides of the triangle

Question

The side of a triangle are 4 cm. 5 cm and 7 cm. Find its perimeter, semi-perimeter and area.

Question

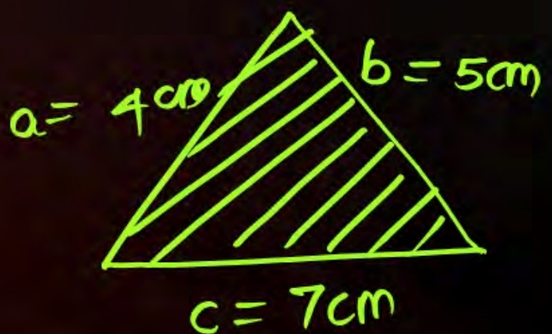
The side of a triangle are 4 cm, 5 cm and 7 cm. Find its perimeter, semi-perimeter and area.

(a = 4 cm, b = 5 cm, c = 7 cm)

We have, the given sides of triangle as $a = 4\text{cm}$, $b = 5\text{cm}$ & $c = 7\text{cm}$

Now, semi-perimeter

$$s = \frac{a+b+c}{2} = \frac{4+5+7}{2} = \frac{16}{2} = \boxed{8\text{ cm}}$$



Therefore, required area of Δ

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{8 \times 4 \times 3 \times 1}$$

$$= \sqrt{2\times2\times2\times2\times3\times1} = 2\times2\times\sqrt{6} = \boxed{4\sqrt{6}\text{ cm}^2}$$

Question

The sides of triangular plot are in the ratio 3 : 5 : 7 and its perimeter is 300m. Find the area of field. Also find the length of the shortest altitude.

Question

The sides of triangular plot are in the ratio 3 : 5 : 7 and its perimeter is 300m. Find the area of field. Also find the length of the shortest altitude.

Let sides of Δ is $3x$, $5x$ & $7x$

Now, perimeter $= 300$

$3x + 5x + 7x = 300$

$15x = 300$

$x = 20$

$a = 3 \times 20 = 60\,m$

$b = 5 \times 20 = 100\,m$

$c = 7 \times 20 = 140\,m$

$s = \frac{300}{2} = 150\,m$

Area of $\Delta = \sqrt{150 \times 90 \times 50 \times 10}$

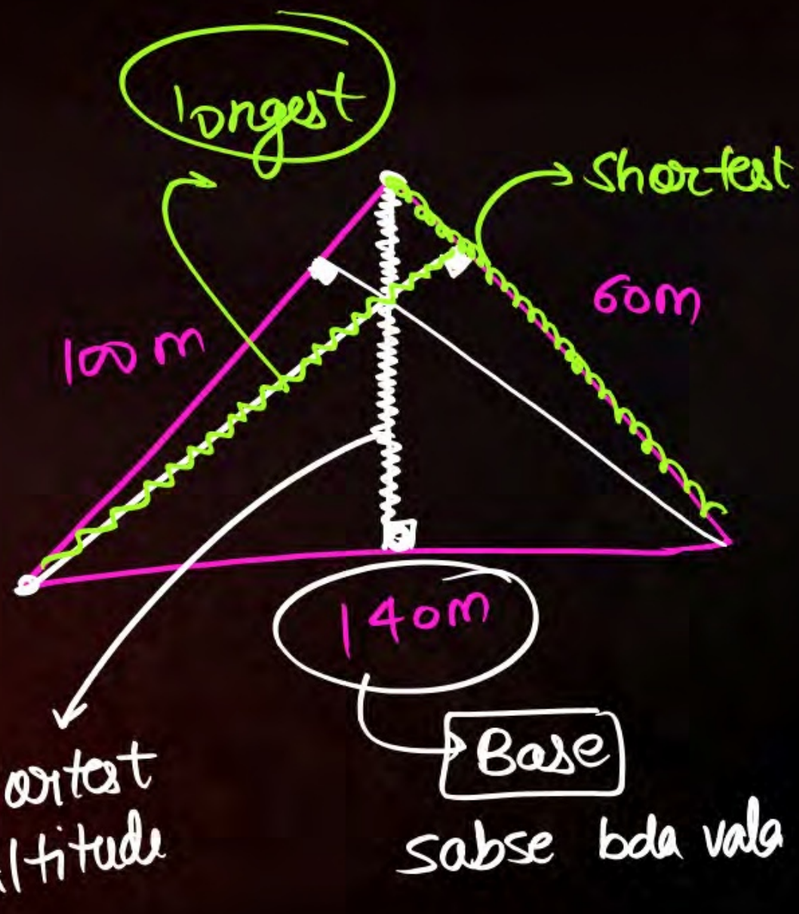
$= \sqrt{15 \times 10 \times 9 \times 10 \times 5 \times 10 \times 10} = 10 \times 10 \sqrt{3 \times 5 \times 3 \times 3 \times 5}$

$= 100 \times 5 \times 3\sqrt{3}$

$= 1500\sqrt{3}\ m^2$

$A = \frac{1}{2} \times b \times h$

- Jo side sabse bda → Uska altitude sabse chhota
- Jo side sabse chhota → Uska altitude sabse bda



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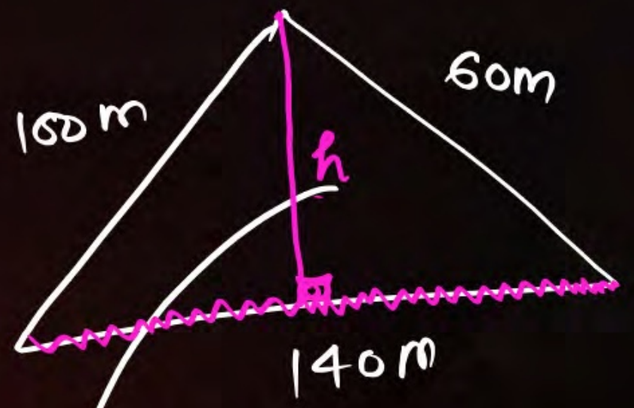
Area of $\Delta = \frac{1}{2} \times 140 \times$ smallest Altitude

$1500\sqrt{3} = \frac{1}{2} \times 140 \times h$

$$h = \frac{1500\sqrt{3} \times 2}{140}$$

XXX



$A = 1500\sqrt{3}\ m^2$

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Question

The base of an isosceles triangle is 16 cm and its area is 48 cm^2. The perimeter of the triangle is

A 41 cm

B 36 cm

C 45 cm

D 324 cm

Question

The base of an isosceles triangle is 16 cm and its area is 48 cm^2. The perimeter of the triangle is

A 41 cm

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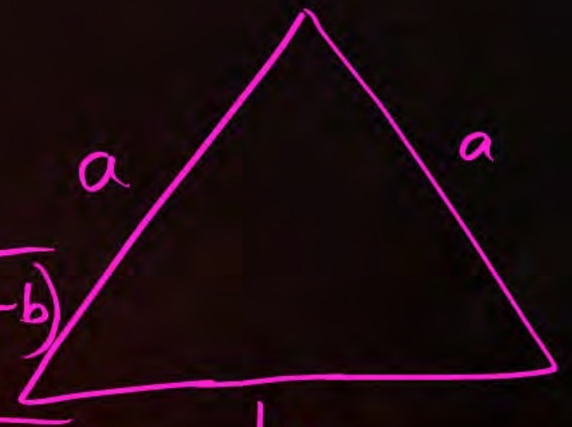
D 324 cm

$$s = \frac{a+a+b}{2} = \frac{2a+b}{2} = \left(a+\frac{b}{2}\right)$$



$$\text{Area} = \sqrt{s(s-a)(s-a)(s-b)}$$

$$= \sqrt{\left(a+\frac{b}{2}\right)\left(a+\frac{b}{2}-a\right)\left(a+\frac{b}{2}-a\right)\left(a+\frac{b}{2}-b\right)}$$

$$= \sqrt{\left(a+\frac{b}{2}\right)\times\left(\frac{b}{2}\right)\times\left(\frac{b}{2}\right)\times\left(a+\left(\frac{b-2b}{2}\right)\right)}$$

$$= \frac{b}{2}\times\sqrt{\left(a+\frac{b}{2}\right)\times\left(a-\frac{b}{2}\right)}$$

$$= \frac{b}{2}\times\sqrt{a^2-\left(\frac{b}{2}\right)^2} = \boxed{\frac{b}{2}\times\sqrt{a^2-\frac{b^2}{4}}}$$

$$(x+y)(x-y) = x^2-y^2$$

$$48 = \frac{16}{2} \times \sqrt{a^2 - \frac{(16)^2}{4}}$$

$$48 = 8 \times \sqrt{a^2 - \frac{256}{4}}$$

$$\frac{48}{8} = \sqrt{a^2 - 64}$$

$$6 = \sqrt{a^2 - 64}$$

squaring on both of the sides

$$36 = a^2 - 64 \Rightarrow a^2 = 100 \Rightarrow \boxed{a = 10}$$

$$P = a + a + b$$

$$= 10 + 10 + 16$$

$$= \boxed{36 \text{ cm}}$$ Ans

Question

The area of an equilateral triangle is $36\sqrt{3}$ cm^2. Its perimeter is

36 cm

$12\sqrt{2}$

24 cm

30 cm

Question

The area of an equilateral triangle is $36\sqrt{3}$ cm^2. Its perimeter is

A 36 cm

B $12\sqrt{2}$

C 24 cm

D 30 cm

$$\frac{\sqrt{3}}{4}a^2 = 36\sqrt{3}$$

$$\frac{a^2}{4} = 36$$

$$a^2 = 36 \times 4$$

$$a^2 = 6 \times 6 \times 2 \times 2$$

$$a^2 = (6 \times 2)^2 \Rightarrow \boxed{a = 12 \text{ cm}}$$

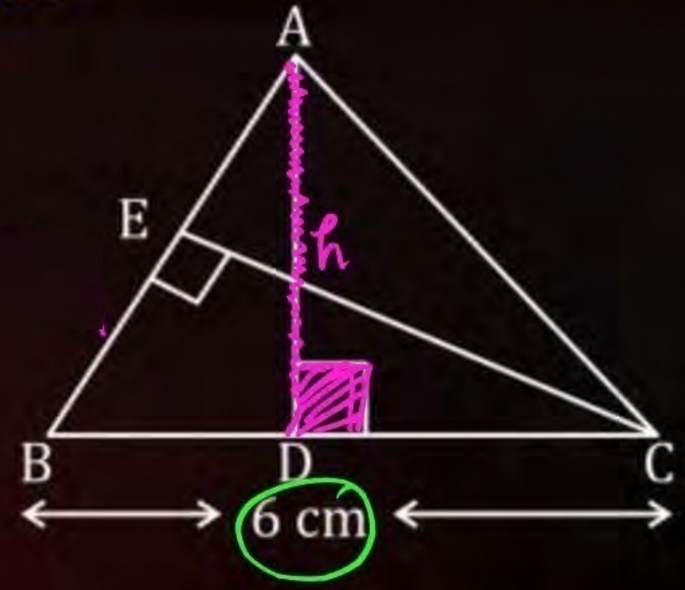
$$P = a + a + a$$

$$= 12 + 12 + 12$$

$$= \boxed{36 \text{ cm}}$$

Question

Find the length of AD in given figure, if EC = 6 cm and AB = 5 cm.



6 cm

5 cm

15 cm

30 cm

Question

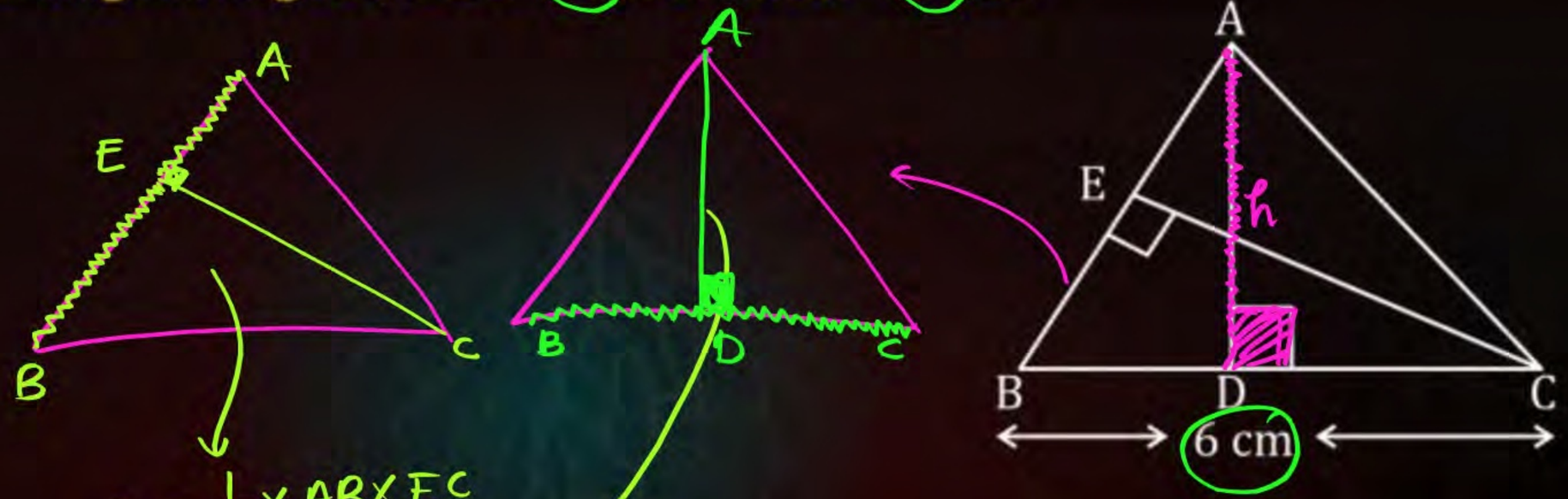
Find the length of AD in given figure, if EC = 6 cm and AB = 5 cm.

A 6 cm

B 5 cm ✓

C 15 cm

D 30 cm



$A = \frac{1}{2} \times AB \times EC$

$A = \frac{1}{2} \times 5 \times 6$

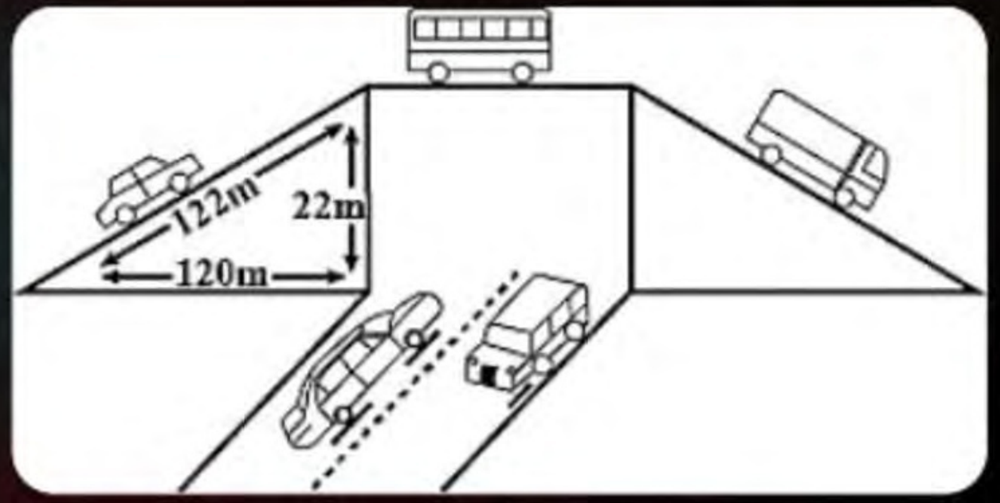
$A = 15\ cm^2$

$= \frac{1}{2} \times BC \times AD$

$15 = \frac{1}{2} \times 6 \times AD \Rightarrow AD = \frac{30}{6} = 5\ cm$ ✓

Question

The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig.). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?



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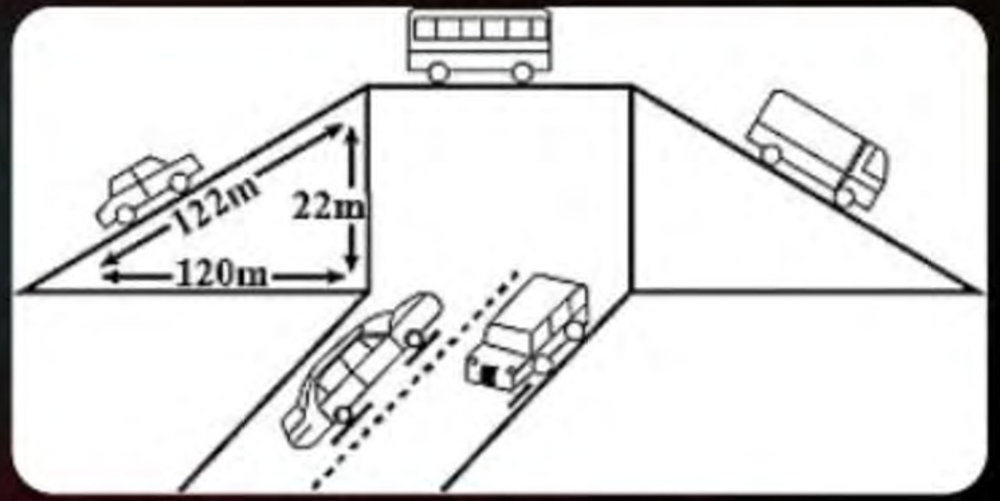


1 saal $\longrightarrow$ 1 m^2 $\longrightarrow$ ₹5000

12 month $\longrightarrow$ ₹5000 / m^2

1 month $\longrightarrow$ $\frac{₹5000}{12}$ / m^2

3 months $\longrightarrow$ $\frac{3 \times ₹5000}{12}$ / m^2

$= ₹\frac{15000}{12}$ / m^2

$= ₹1250$ / m^2 $\longrightarrow$ for 3 months

$a = 122,\ b = 22,\ c = 120 \Rightarrow s = \frac{264}{2} = \boxed{132\ m}$

Area of Δ wall $= \sqrt{132 \times 10 \times 110 \times 12}$

$= \sqrt{2 \times 2 \times 3 \times 11 \times 10 \times 11 \times 10 \times 12}$

$= 12 \times 11 \times 10$

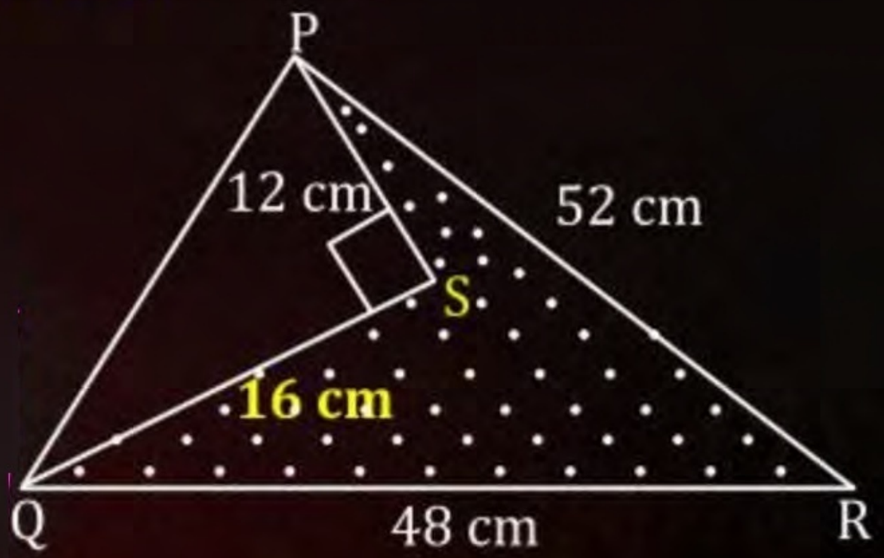
$= 12 \times 110$

$= 1320\ m^2$

Total Rent $= 1320 \times 1250 = \boxed{₹\ 1650000}$ ***

Question

Find the area of the shaded region in the figure given below.



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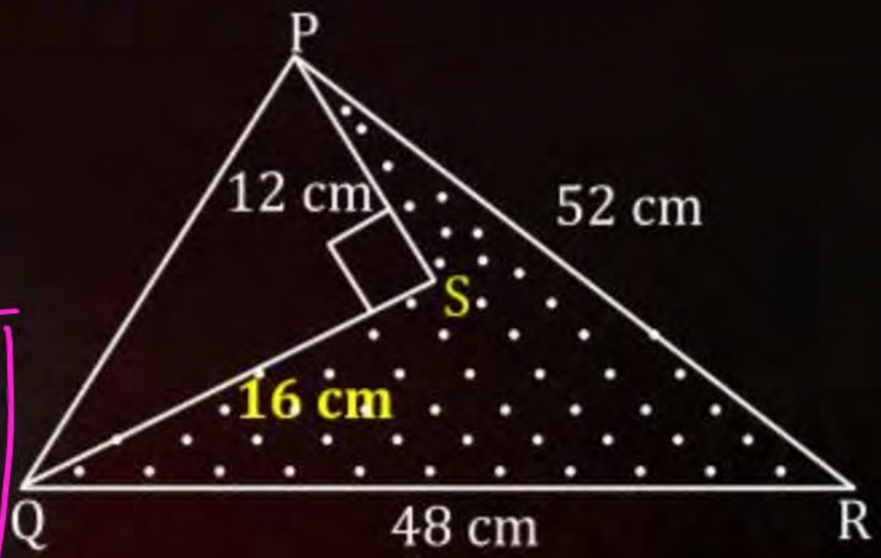
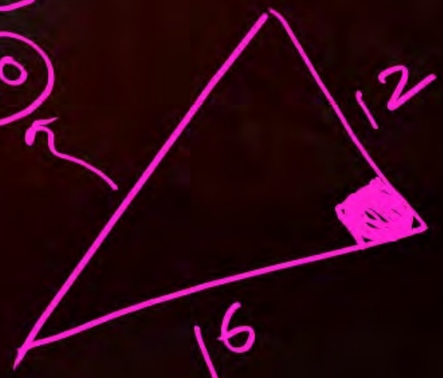


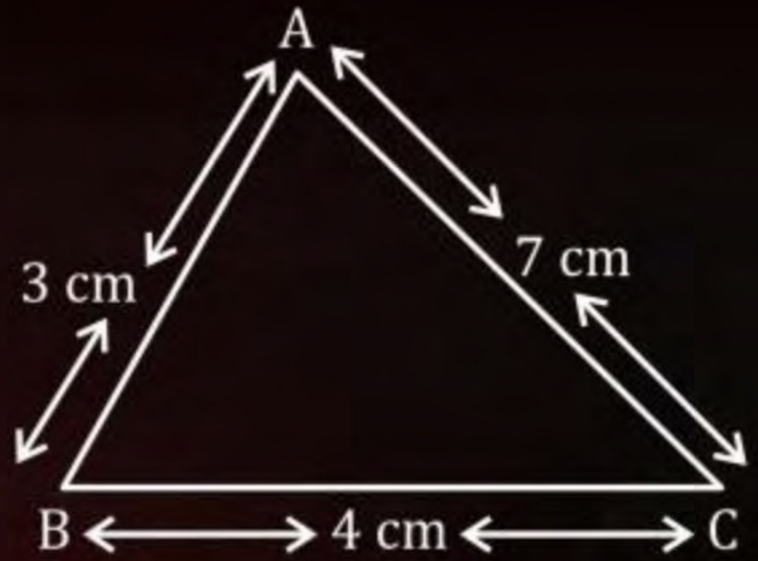
Shaded Region = ar(ΔPQR) − ar(ΔPSQ)

$= \left(\text{A with sides } 20, 52, 48\right) - \frac{1}{2} \times 12 \times 16$



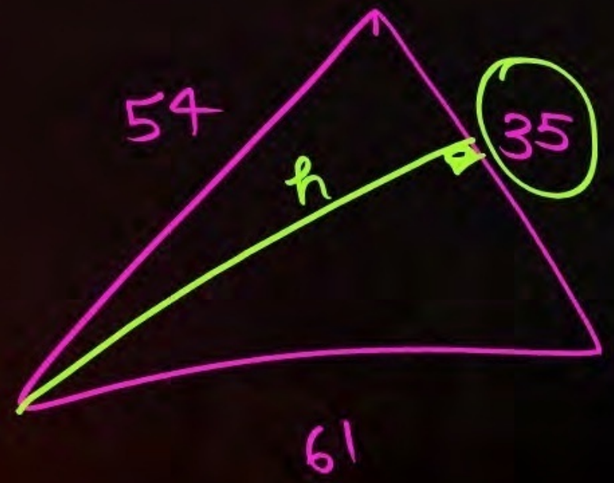
Question

For the given figure, find the s(s – a).



Question

The sides of a triangle are 35 cm, 54 cm and 61 cm respectively. Find the length of its longest altitude.



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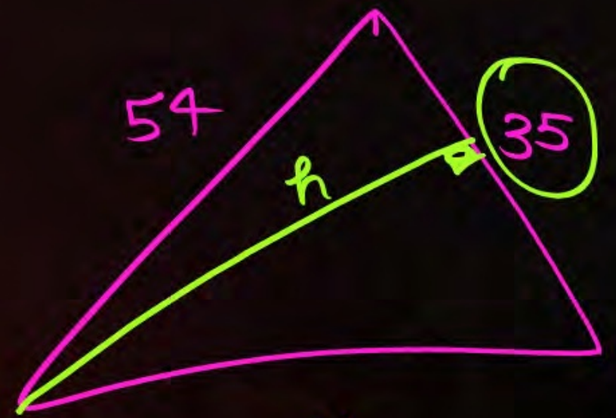
chutku vali side ke corresponding

Heron's formula

#machha Diya



$A = \frac{1}{2} \times 35 \times h$

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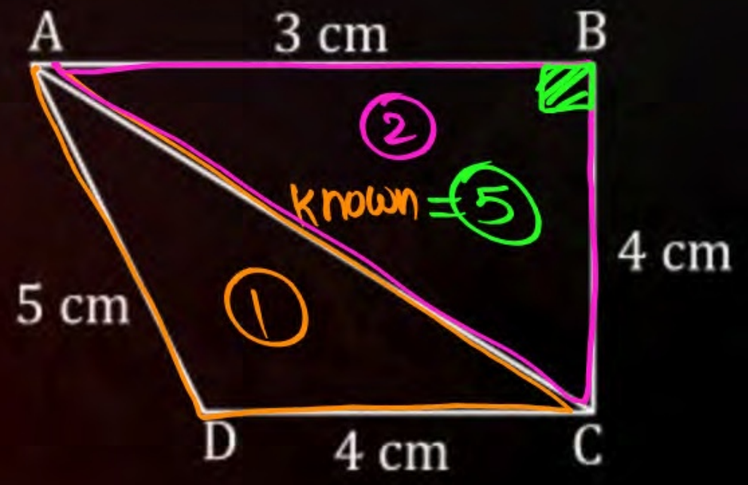
Application of Heron's Formula in Finding Area of Quadrilaterals

Heron's formula can be applied to find the area of a quadrilateral by dividing the quadrilateral into two triangular parts. If we join any of the two diagonals of the quadrilateral, then we get two triangles. Area of each triangle is calculated and the sum of two areas is the area of the quadrilateral.

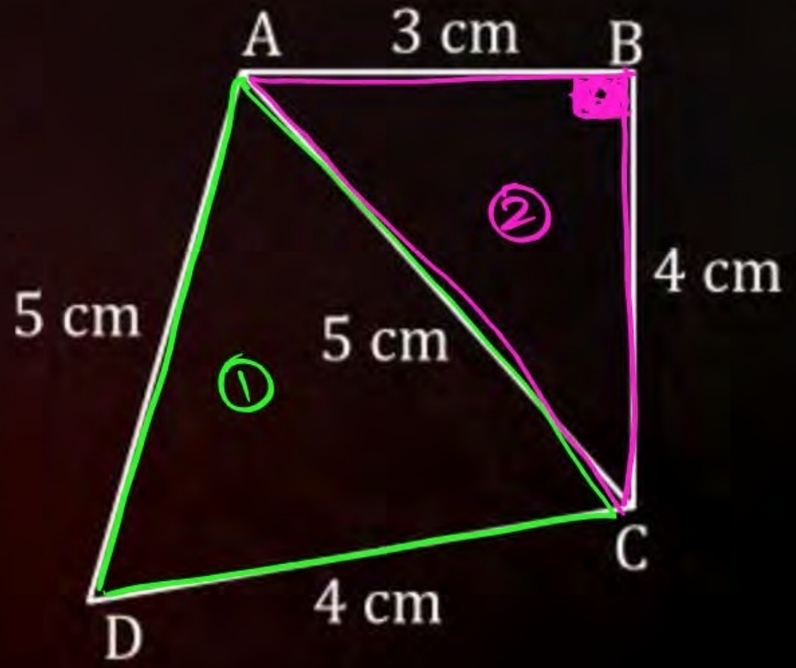


It can be used to determine the area of any irregular quadrilateral by converting the quadrilateral into triangles.

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

A quadrilateral whose sides are 3 cm, 4 cm, 4 cm, 5 cm and one of the diagonal is equal to 5 cm as per the figure given below. The area of the quadrilateral is



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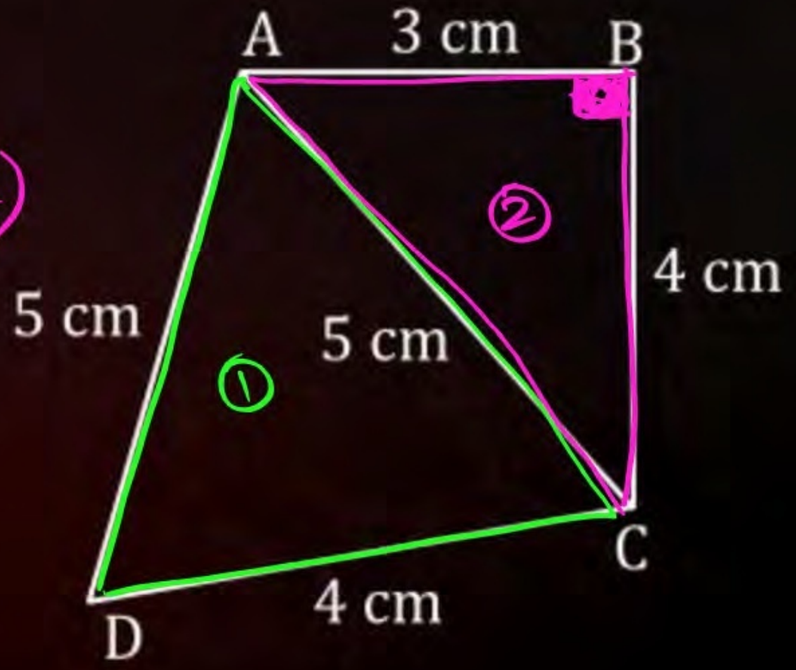

$$\text{Area of } \square ABCD = \text{ar}(1) + \text{ar}(2)$$

$$= \left(\frac{4}{2} \times \sqrt{5^2 - \frac{(4)^2}{4}}\right) + \left(\frac{1}{2} \times 3 \times 4\right)$$

$$= \left(2 \times \sqrt{25 - 4}\right) + 6$$

$$= \boxed{2 \times \sqrt{21} + 6}$$ ✓✓

Question

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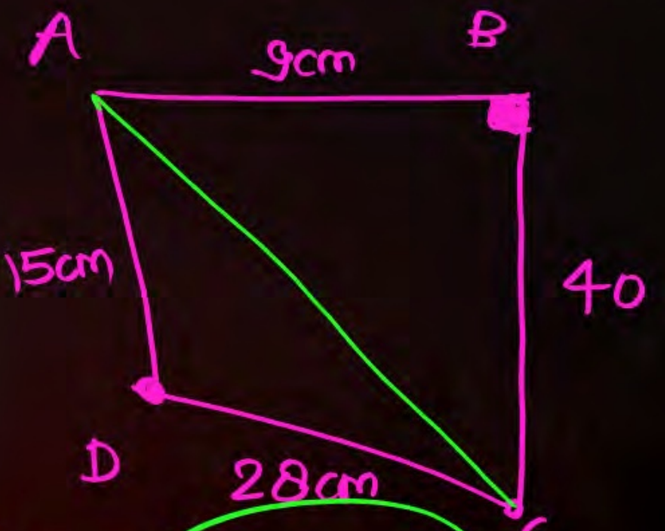
In ΔABC,

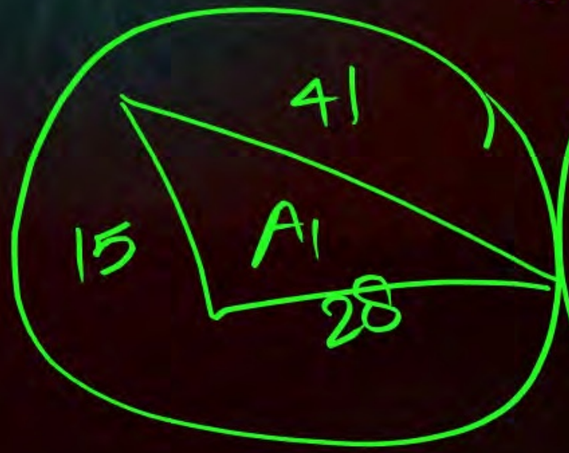
$$AC = \sqrt{(9)^2 + (40)^2}$$

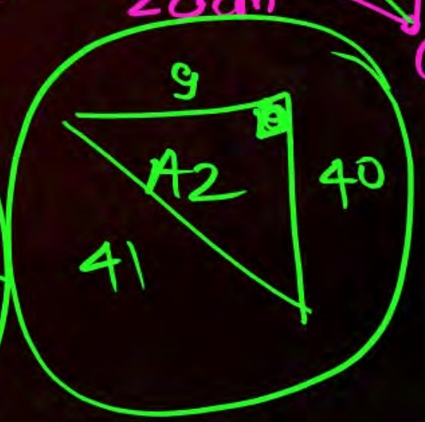
$$= \sqrt{81 + 1600}$$

$$= \sqrt{1681}$$

$$= 41 \text{ cm}$$

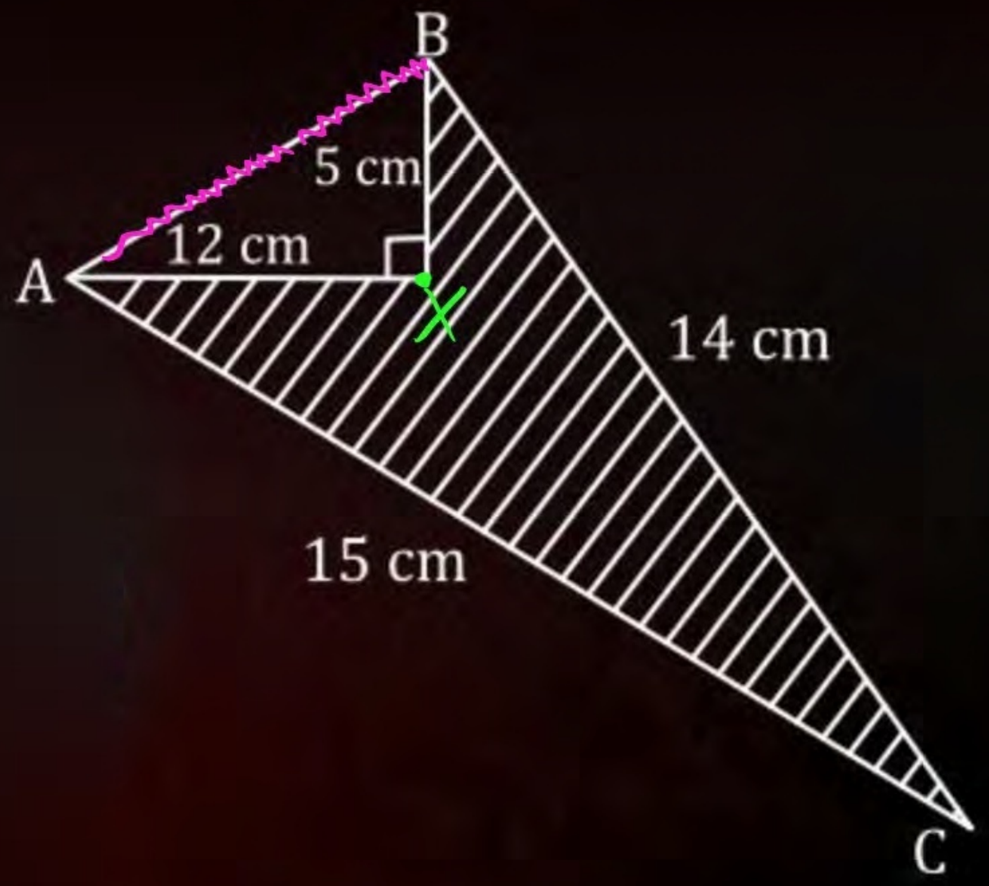






Question

Calculate the area of the shaded region



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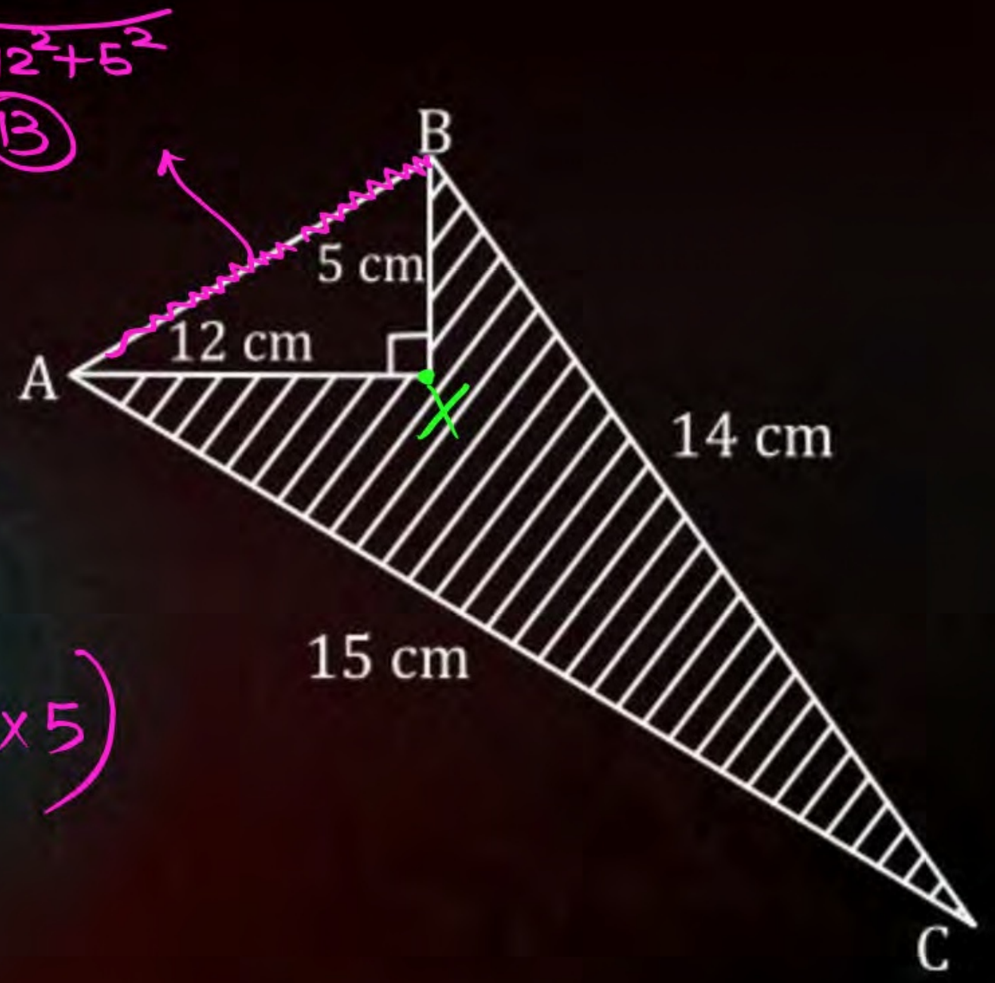

$\sqrt{12^2+5^2} = 13$

Shaded = ar(ΔABC) − ar(ΔABX)

= (13, 14, A, 15) − $\left(\frac{1}{2} \times 12 \times 5\right)$

= A − 30 =

Question

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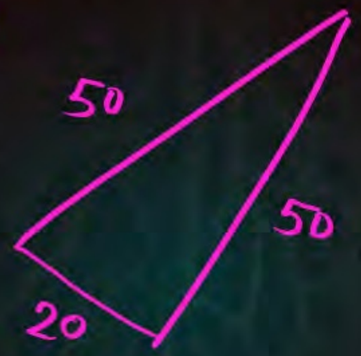
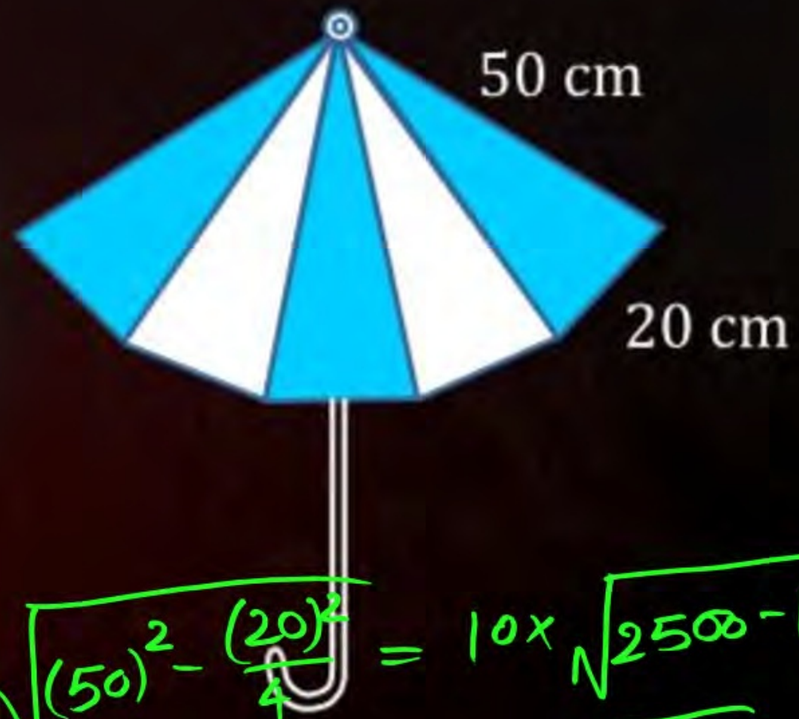
5 Blue triangular piece → $5 \times 200\sqrt{6}$

+

5 white triangular piece → $5 \times 200\sqrt{6}$





Area of each piece

$$= \frac{b}{2} \times \sqrt{a^2 - \frac{b^2}{4}} = \frac{20}{2} \times \sqrt{(50)^2 - \frac{(20)^2}{4}} = 10 \times \sqrt{2500 - 100}$$

$$= 10 \times \sqrt{2400}$$

$$= 10 \times \sqrt{2 \times 2 \times 3 \times 2 \times 2 \times 3 \times 10 \times 10}$$

$$= 200\sqrt{6}$$

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Question

The sides of a triangle are 11 m, 60 m and 61 m. The altitude to the smallest side is

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B 66 m

C 50 m

D 60 m

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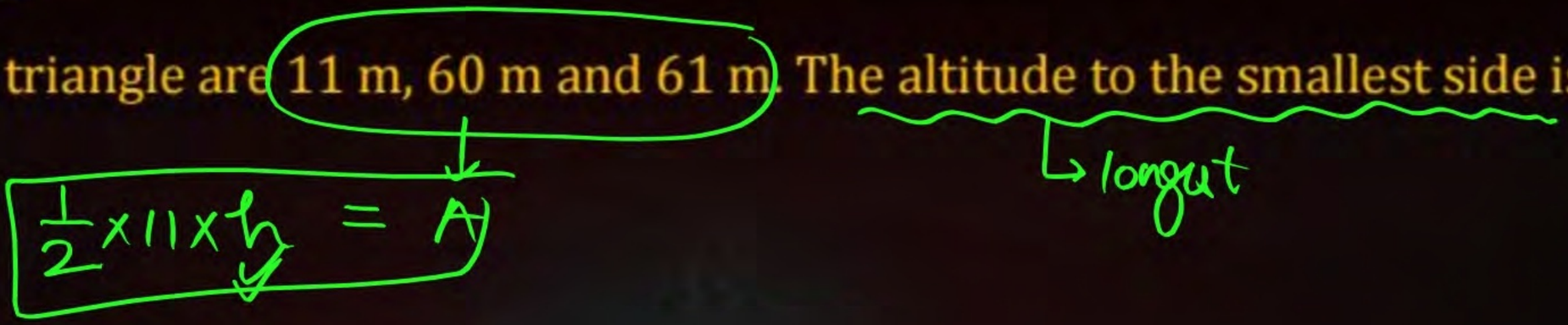

A 11 m

B 66 m

C 50 m

D 60 m

Question

If the area of an isosceles right triangle is 8 cm^2, what is the perimeter of the triangle?

$9 + \sqrt{2}$ cm

$8 + 4\sqrt{2}$ cm

$4 + 8\sqrt{2}$ cm

$13\sqrt{2}$ cm

Question

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B $8 + 4\sqrt{2}$ cm

C $4 + 8\sqrt{2}$ cm

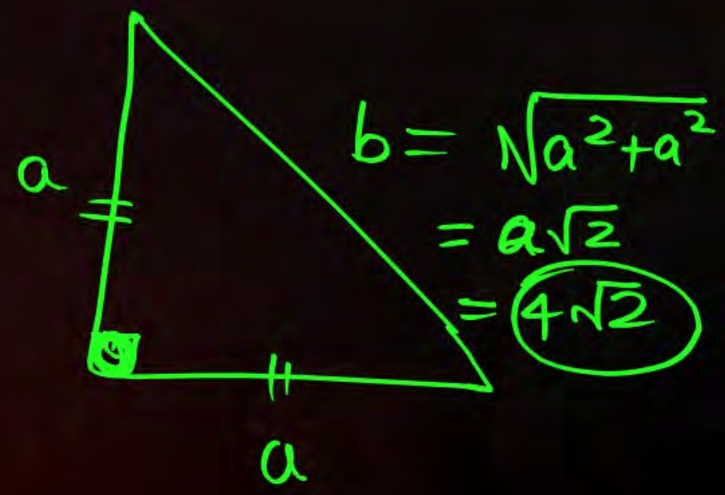
D $13\sqrt{2}$ cm

$A = \frac{1}{2} \times a \times a$

$8 = \frac{1}{2} \times a \times a \Rightarrow a^2 = 16$

$\Rightarrow a^2 = (4)^2$

$\Rightarrow \boxed{a = 4}$



perimeter $= a + a + b$

$= 4 + 4 + 4\sqrt{2}$

$= 8 + 4\sqrt{2}$

Question

A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field. (Imp.)

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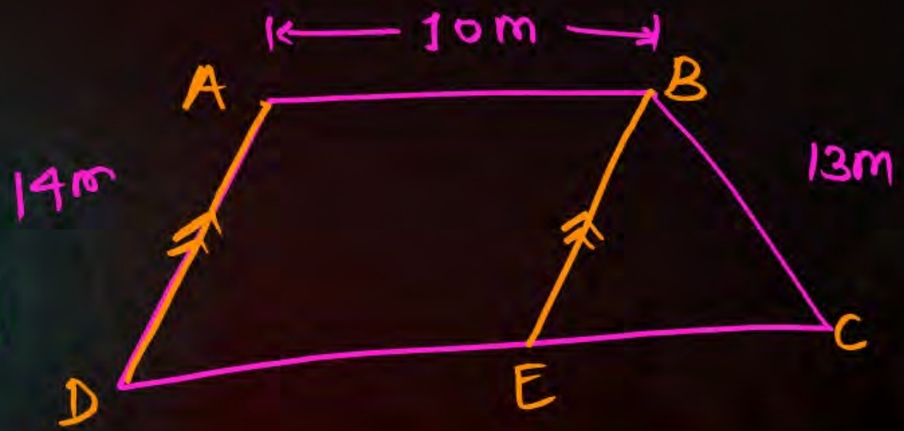
A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field. (Imp.)

Let the given trapezium $\square ABCD$

contruction :- $AO \parallel BE$



In ΔBEC,

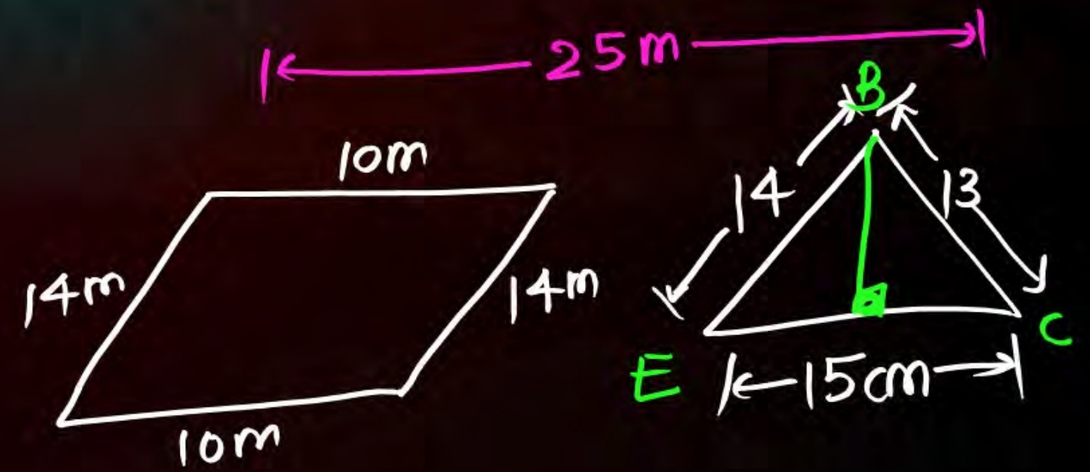
$$S = \frac{13+14+15}{2} = \frac{42}{2} = 21\ m$$

$$A = \sqrt{21 \times 7 \times 6 \times 8}$$

$$A = \sqrt{3 \times 7 \times 7 \times 3 \times 2 \times 2 \times 2 \times 2}$$

$$= 7 \times 3 \times 2 \times 2 = 21 \times 4$$

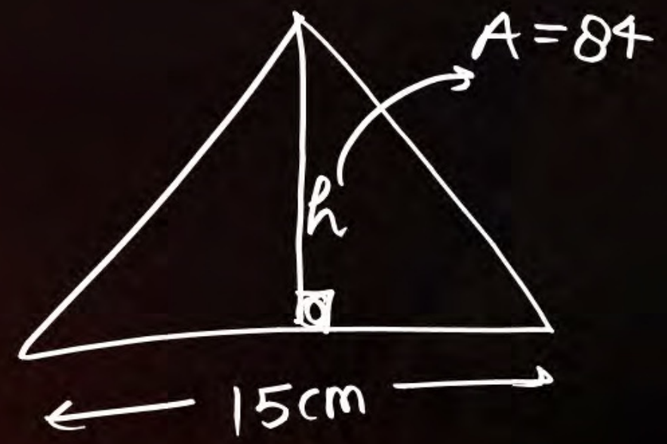
$$= 84\ m^2$$



$$A = \frac{1}{2} \times 15 \times h$$

$$84 = \frac{1}{2} \times 15 \times h$$

$$h = \frac{84 \times 2}{15} \text{ m} = 11.2 \text{ m}$$



Area of 11^{gram} = b × h

Total Area = area(11^{gram}) + 84

$= (10 \times 11.2) + 84$

$= 112 + 84 = 196 \text{ m}^2$


THANK YOU