12/8/25

Chapter-7 (Motion)

* Objectives of the chapter-

- Define Motion
- Identify instances of Motion encountered in our everyday life.
- Analyse motion in a straight line.
- Identify uniform & non-uniform motion
- Calculate speed & avg. speed
- Define velocity.
- Calculate velocity of a moving object
- Explain scalar & vector quantity
- Define acceleration
- Calculate acceleration of a moving obj.

- What is Motion?

⇒ Motion is a state of object in which there is a change in its position with respect to its surroundings and time.

- The states of motion and rest are relative.
- Absolute states of rest or motion do not exist.

⇒ Types of Motion

→ Linear/One dimensional
The object moves along a straight line
for eg- a racing car moving along a straight path

→ Two-dimensional:
The obj moves in two directions at the same time: horizontal & vertical.
For eg: a golf ball from being hit to reaching its golf ~~curse~~ coast

→ Three-dimensional:
An obj moves in three directions at the same time. for eg: a plane taking off from a runway.

Distance & Displacement:

⇒ Distance: Distance is the actual length of the path covered by a moving object irrespective of the direction of motion.

- It is a scalar quantity (having only magnitude)
- It can be positive or zero, but never (-)ve.
- SI unit = m

⇒ Displacement: Displacement is the shortest distance between two points irrespective of the path between the points.

- It is a vector quantity (having both magnitude and direction)
- It can be (+)ve, zero or (-)ve.
- SI unit = m.

Uniform & Non-Uniform Motion:

⇒ Uniform Motion:
- An object is said to be in uniform motion if it shows equal displacements in equal intervals of time.

- The d-t graph for a body moving in uniform motion is a straight line (slope)
- for eg- rotation & revolution of earth.

⇒ Non Uniform Motion
- In non-uniform motions, if an object travels unequal distances in equal intervals of time.

- The d-t graph for a body moving in nonuniform motion is a curved line. (non-straight-line).
- for eg- a car moving in a crowded place.

13/8/25

Chapter 7 (Motion)

Speed: The speed of an object is the rate at which the object covers a given distance

- Speed is calculated by taking the ratio of the distance travelled to the time taken.

$$\text{Speed (m/s)} = \frac{\text{distance (m)}}{\text{time (sec)}} \text{ or } \boxed{s = \frac{d}{t}}$$

→ High Speed - Object covers a large distance in a small amount of time.
→ Low Speed - Object covers a small distance in a longer duration.

→ No speed: Object at rest

$$s = \frac{d}{t} = \frac{cm}{s} = cms^{-1}$$

System		Unit
CGS	–	cms^{-1}
SI	–	ms^{-1}
General Unit	–	Km/h

⇒ Average Speed
- Generally, the motion of an obj is non-uniform
- To determine the speed at which the obj. covers the total distance, we calculate the average of all the instantaneous speeds.

- Formula ⇒ Ratio of the total distance to the total time, $s = \frac{d_1 + d_2 + d_3 \dots d_n}{t_1 + t_2 + t_3 \dots t_n}$

$$\therefore \boxed{s_{avg} = \frac{d_{total}}{t_{total}}}$$

→ Uniform motion → Uniform speed
→ Non-uniform motion → Non-uniform speed

~~Speed with Direction - Velocity~~
~~Velocity: • The speed of an object moving in a definite direction.~~
~~• The rate of change in position of a body.~~
~~• The rate of displacement~~

To be Done later!

Numericals of unit conversions:

Km/h ⟶ m/s: Multiply by $\frac{5}{18}$

m/s ⟶ Km/h: Multiply by $\frac{18}{5}$

1. $36\,cms^{-1} \longrightarrow Km/h$

$36cm/s = \frac{36}{100} m/s = 0.36 ms^{-1}$

$\frac{0.36}{100} \times \frac{18}{5} = \frac{324}{250} = \underline{1.296\ Km/h}$

2. $54\, m/s \longrightarrow Km/min$

$= \frac{54m}{1s} = \frac{54 \times 10^{-3} Km}{1sec}$

$= \frac{54 \times 10^{-3} \times 60\ Km/min}{1}$

$= 54 \times 6 \times 10^{-2}$

$= \underline{3.24\ Km/min}$ Ans.

3. $180\, cm/min \longrightarrow Km/sec$

$= \frac{180}{100} cm/min = \frac{1.8m}{1min} = \frac{1.8 \times 10^{-3} Km}{60 sec}$

$= 0.3 \times 10^{-4}\ Km/sec$

$= \underline{3 \times 10^{-5}\ Km/sec}$ Ans.

Q. Average speed $= \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

x Km, $t_2 = 1hr$, $d = (100-x) Km$

A 60Km/h B — x Km — C

Total distance = 100 Km
Total time = 2 hrs.

Average speed $= \frac{100}{2} = 50\ Km/h$

A → B
Distance = $(100 - x)$ Km
Time = $2 - 1 = 1hr$.
Speed $= \frac{d}{t} = \frac{x}{1} = x Km$

Q. On a 120 Km track, a train moves first 30Km at uniform speed of 30 Km/h. How fast the train travel in the next 90 Km so as to have avg. speed of 90 Km/h for the entire journey.

← 120Km →
A 30Km P 90Km B

$Sp_1 = 30\ Kmh^{-1}$ $Sp_2 = x\ Kmh^{-1}$

Average speed = 90 Km/h

avg. speed $= \frac{d_{total}}{t_{total}} \Rightarrow 90 = \frac{120}{\left(1+\frac{90}{x}\right)}$

$\Rightarrow \left(1+\frac{90}{x}\right) = \frac{120}{90} = \frac{4}{3}$

$\Rightarrow 1 + \frac{90}{x} = \frac{4}{3}$

$\Rightarrow \frac{90}{x} = \frac{1}{3}$

$\Rightarrow x = 90 \times 3 = \underline{270\ Km/h}$ Ans.

- Self - Assessment:

Q. A motorcyclist drives from A to B with a uniform speed of 30km/h and return back with a speed of 20km/h. Find the average speed.

⇒ Let distance from A to B be 'd' Km.

- Speed $A \rightarrow B$ = 30Km/h ⇒ Time $A \rightarrow B = \frac{d}{30}$ hr.
- Speed $B \rightarrow A$ = 20 Km/h ⇒ Time $B \rightarrow A = \frac{d}{20}$ hr

Total distance = d+d = 2d Km
Total time $= \frac{d}{30} + \frac{d}{20} = \frac{2d+3d}{60} = \frac{5d}{60} = \frac{d}{12}$ hr

Average speed $= \frac{\text{Total distance}}{\text{Total time}} = \frac{2d}{\frac{d}{12}}$

$= 2\cancel{d} \times \frac{12}{\cancel{d}} = \underline{24 Km/h}$ Ans

✱ Speed with direction - VELOCITY
- Velocity → The speed of an object moving in a definite direction.

→ The rate of change in position of a body (the rate of displacement)

⇒ Change in Velocity (How?)
By ① Changing speed in a particular direction
② Keeping the speed constant and changing direction

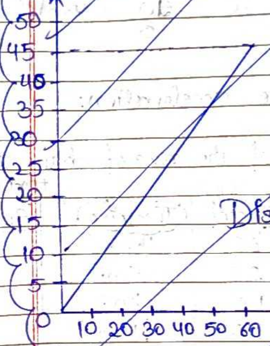
⇒ Average Velocity:
Avg. $\vec{v} = \frac{\vec{u}+\vec{v}}{2}$ (average of both the velocities)
($v_{avg} = \frac{S_{total}}{\ldots}$)

20.8.25

Chapter-7 (Motion)

⇒ Velocity - Time Graph (To be done later)
You can determine:
- The nature of the motion
- The velocity of the body at any instant
- Displacement of the body as area under the graph.

Acceleration = Rate of change of Velocity



= Change in velocity / Time Taken

⇒ $\boxed{\vec{a} = \frac{\vec{v}-\vec{u}}{t}}$

$= \frac{45-0}{60} = 45/60 = 0.75 m/s^2$

Displacement = Area under graph
$= \frac{1}{2} \times b \times h$
$= \frac{1}{\cancel{2}} \times \cancel{60}^{30} \times 45$
$= 30 \times 45$
$= 1350m$

⇒ ACCELERATION
It is the rate at which a moving object changes its velocity

$\vec{a} = \frac{\text{change in velocity}}{\text{Time Taken}} = \frac{\text{Final velocity} - \text{Initial v.}}{\text{Time Taken}}$

$\Rightarrow \boxed{\vec{a} = \frac{\vec{v}-\vec{u}}{t}} = \frac{m/s}{s} = ms^{-2}$ or m/s^2 (SI Unit)

→ Acceleration occurs when $v > u$.

Case II : If $u > v$, then $\vec{a} = \frac{\vec{v}-\vec{u}}{t} = (-)ve$

Retardation = Acceleration

Ex: ⇒ $u = 20\ m/s$, $v = 0\ m/s$, $t = 5\ sec$.

$\vec{a} = \frac{\vec{v}-\vec{u}}{t} = \frac{0-20}{5} = -20/5 = -4\ m/s^2$

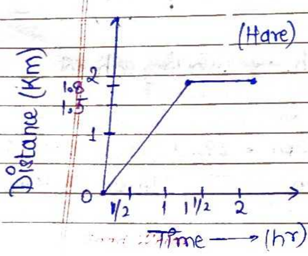
⇒ $u = 20\ m/s$, $v = 60\ m/s$, $t = 5\ sec$.

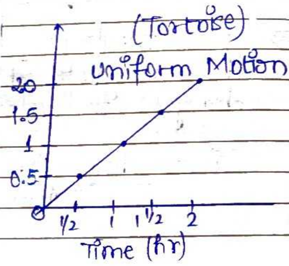
$\vec{a} = \frac{\vec{v}-\vec{u}}{t} = \frac{60-20}{5} = \frac{40}{5} = 8\ m/s^2$

Reference data to practice acceleration:

(From the observations of the race of hare & tortoise.)

Time	Hare (Km)	Speed	Tortoise (Km)	Speed
00	0.0	0.0	$1 Kmh^{-1}$	
30	1.0	0.5	$1 Kmh^{-1}$	
60	1.8	1.0	$1 Kmh^{-1}$	
90	1.8	1.5	$1 Kmh^{-1}$	
120	1.8	2.0	$1 Kmh^{-1}$	





	Time	Velocity	Acceleration
(i)	1/2 hr = 30 mins	$u = 0 Kmh^{-1}$, $v = 2 Kmh^{-1}$	$3 \times 10^{-4}\ Km/h^2$
(ii)	1/2 hr = 30 mins	$u = 2 Kmh^{-1}$, $v = 1.8 Km^{-1}$	$-0.4 Km/h^2$

(i) $u = 0\ Kmh^{-1}$, $v = 2\ Km/h$, $t = 1/2\ hr$.

$a = \frac{v-u}{t} = \frac{2-0}{1/2} = 4\ Kmh^{-2} = \frac{4 \times 1000}{(3600)^2}$

$= \cancel{\frac{4^2 \times 5}{789}} = \cancel{10/9}$

$= \frac{0.4}{36 \times 36} = \frac{0.1}{9 \times 36}$

$= \frac{10^{-1}}{9 \times 36} = \frac{1.11 \times 10^{-3}}{.36} = \frac{0.111 \times 10^{-1}}{36}$

$= 0.3 \times 10^{-3} = 3 \times 10^{-4}$

(ii) $u = 2 Kmh^{-1}$, $v = 1.8\ Kmh^{-1}$
$t_2 - t_1 = 1 - 1/2 = 1/2\ hr$.

$a = \frac{v-u}{t} = \frac{1.8-2}{0.5}$
$= -0.2/0.5 = -0.4 Km/h$

⇒ Uses of Graphical Representation of Motion

You can:

- Infer the nature of Motion.
- Determine its position at any intermediate level
- Calculate its acceleration, displacement, velocity at any instant
- Derive equations of Motion.

⇒ Commonly used Motion Graps:

Distance-Time (d-t) graph

Velocity-Time (v-t) graph

Types of Quantities (plotted on a Motion graph)

- Independent Quantities are taken along ~~y~~ x-axis. for example - time in (v-t) | (d-t) graph.
- Dependant Quantities are taken along ~~y~~ axis. for eg- distance or ~~a~~ velocity in a (d-t) | (v-t) graph.

Let say:

→ The graph of "Runs Scored by a cricket team in a One-day Match."

→ The graph of "Mass & Velocity showing Momentum".

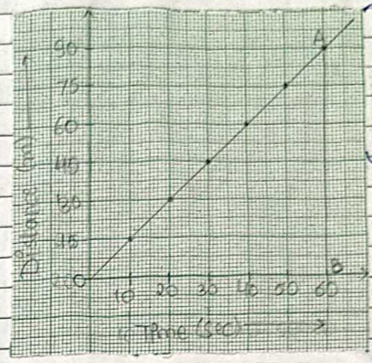
Here,	Independent	Dependant
	~~Time~~ Overs	Runs
	Mass	Velocity

⇒ Distance-Time Graphs

A distance-time graph represents the change in the position of an object with time.

Case (i): Uniform Motion

Time (sec)	Distance (m)
0	0
10	15
20	30
30	45
40	60
50	75
60	90



Scale:
On x-axis 1 unit = 10 sec
On y-axis 1 unit = 15 m

Speed = Slope of the line

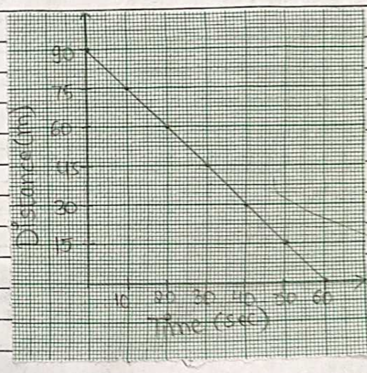
= $\frac{\text{Perpendicular (distance)}}{\text{Base (time)}}$

= $\frac{90}{60}$ = 1.5 m/s

Case (ii):

Retardation (Uniform Motion)

Time(s)	Dist. (m)
0	90
10	75
20	60
30	45
40	30
50	15
60	0



at 40 sec,

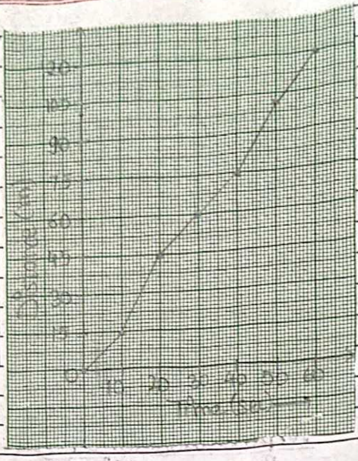
speed = P/b

= 30/40

= 0.75 m/s

Non-Uniform Motion Case (iii):

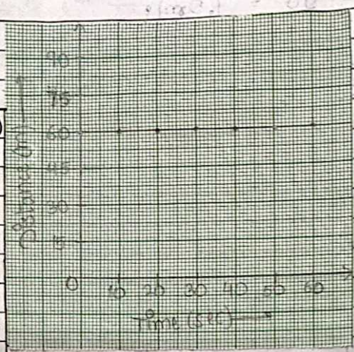
Time (s)	Dist. (m)
0	0
10	15
20	45
30	60
40	75
50	100
60	120



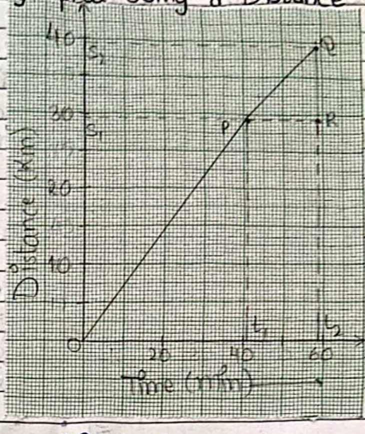
Curve = Fractional slope of diff. intervals (no accurate value)

Case (iv): Body at Rest

Time (s)	Dist (m)
0	60
10	60
20	60
30	60
40	60
50	60
60	60



⇒ Calculating Speed Using a Distance-Time Graph



$PR = t_2 - t_1$

$QR = s_2 - s_1$

$\boxed{v = \frac{s}{t}}$

$v = \frac{s_2 - s_1}{t_2 - t_1}$

⇒ Velocity-Time Graphs

Using v-t graphs, u can determine:

→ The nature of the motion

→ The velocity of the body at any instant.

→ Displacement of the body as the area under the graph.

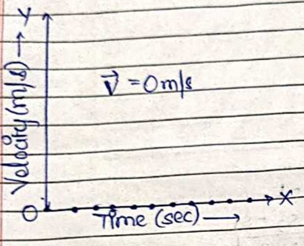
→ The instantaneous acceleration.

→ The equations of motion along a straight line.

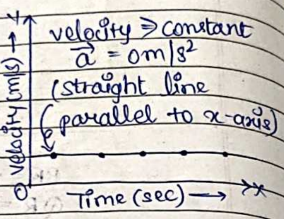
Cases of Velocity - Time Graph

(i) Body at rest
(ii) Body with uniform velocity
(iii) Body with uniform acceleration
(iv) Body with non-uniform acceleration
(v) Body with uniform retardation

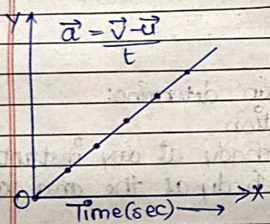
⇒ Case 1: Body at Rest



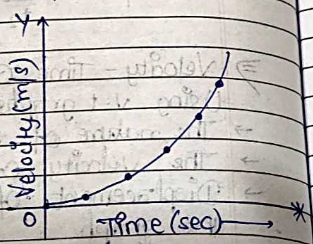
Case 2: Body with Uniform Velocity



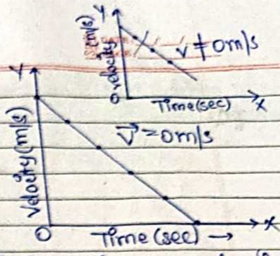
Case 3: Body with Uniform Acceleration



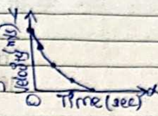
Case 4: Body with non uniform acceleration



Case 5: Body with uniform retardation

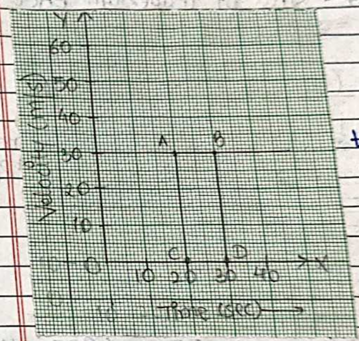


Case 6: Body with non-uniform Retardation



⇒ The area under a velocity - time graph gives you the displacement of the body in a given time

⇒ Calculating Displacement Using a Velocity - Time Graph



$t = AB = CD = t_2 - t_1 = 10$ sec

$v = AC = BD = 30$ m/s

$v = s/t$

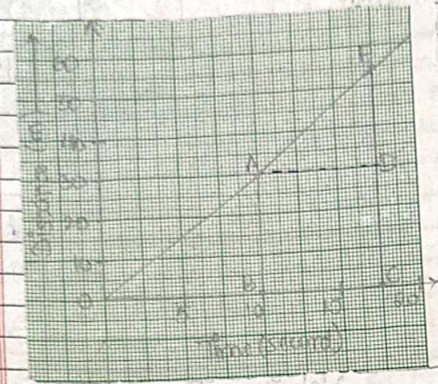
$\Rightarrow s = vt$

Subsituting v & t in: $s = vt$

$s = AC \times AB = 30 \times 10$

$= \underline{300\text{ m}}$

#Q.



(i) Disp. from 0 → 17.5 sec.

(ii) Acc. from 10 → 17.5 sec.

(iii) Disp. from 10 → 17.5 sec.

③ Displacement (s) = Area of trapezium BACE

$$= \frac{\text{sum of} \parallel \text{sides} \times \text{height}}{2}$$

$$= \frac{(30+55)}{2} \times 7.5$$

$$= \frac{\cancel{85}^{17} \times 75}{2 \times \cancel{10}_2} = \frac{1275}{4} = 319\,m$$ (Approx)

② $$\vec{a} = \frac{\vec{v}-\vec{u}}{t_2-t_1} = \frac{55-30}{17.5-10} = \frac{250}{7.5} = 3.33\,m/s^2$$

① Area of $\Delta OEC = \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times \frac{17.5 \times \cancel{55}^{11}}{\cancel{10}_2} = \frac{1,925}{4} = 481\,m$$ (Approx)

OR

$$\boxed{s = (AB \times BC) + \frac{AD \times DE}{2}}$$

Equations of Motion: Equations that define the relationship between velocity and acceleration of a body moving in a straight line.

1. $v = u + at$
2. $s = ut + \frac{1}{2}at^2$
3. $2as = v^2 - u^2$

These equations are used to determine the position, velocity and acceleration of an object.

⇒ Derivations of Equations of Motion

1) $v = u + at$

$$\vec{a} = \frac{\text{Change in velocity}}{\text{Time}}$$

$$\vec{a} = \frac{\vec{v}-\vec{u}}{t}$$

$$\boxed{\vec{v} = \vec{u} + at}$$

2) $s = ut + \frac{1}{2}at^2$ ∵ $\vec{v} = \vec{u} + \vec{a}t$

$$\frac{s}{t} = \text{avg. speed/velocity}$$

$$\frac{s}{t} = \frac{\vec{v}+\vec{u}}{2}$$

$$s = \frac{v+u(t)}{2} = \frac{[(u+at)+u]t}{2}$$

$$s = \frac{(2u+at)t}{2}$$

$$s = \frac{\cancel{2}ut}{\cancel{2}} + \frac{at^2}{2}$$

$$\Rightarrow \boxed{s = ut + \frac{at^2}{2}}$$

Good Write

Case I: If body starts from rest, $\vec{u}=0\,m/s$ $\Rightarrow \vec{v}=\vec{a}t \Rightarrow \boxed{s=\frac{at^2}{2}}$

Case II: If body comes to rest, $\vec{v}=0\,m/s$
$\vec{u}=-at$

3) $2as=v^2-u^2$ $\qquad v=u+at$ (1st Eq^n of Motion)

$$t=\frac{v-u}{a} \quad —①$$

$$s=ut+\frac{at^2}{2} \quad —②$$

From Eq^n ① & ②, we get:

$$s=u\left(\frac{v-u}{a}\right)+\frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$s=\frac{uv-u^2}{a}+\frac{1}{2}\cancel{a}\frac{(v^2+u^2-2uv)}{a^{\cancel{2}}}$$

$$s=\frac{uv-u^2}{a}+\frac{v^2+u^2-2uv}{2a}$$

$$s=\frac{2(uv-u^2)+v^2+u^2-2uv}{2a}$$

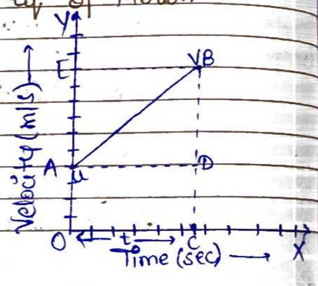
$$2as=\cancel{2uv}-2u^2+v^2+u^2-\cancel{2uv}$$

$$\boxed{2as=v^2-u^2}$$

Graphical Derivation of Eq^n of Motion

Velocity-Time Relation

1) OA = u (Initial Velocity)
BC = v (final velocity)
OC = t (Time)
~~BD = BC − DC~~
~~⇒ BC = BD + DC~~ —①



Substituting BC = v and OA
Body accelerated from A to B.
Therefore,

Acc. = Slope of Line AB

$$= \frac{\text{Perp.}}{\text{Base}} = \frac{BD}{AD} = \frac{\vec{v}-\vec{u}}{t}$$

$$a=\frac{\vec{v}-\vec{u}}{t} \Rightarrow at=\vec{v}-\vec{u} \Rightarrow \boxed{v=u+at}$$

Position-Time Relation

2) The total distance travelled by the body from A to B = Area of the Trapezium OADC

$$s=\frac{\text{Sum of } \parallel \text{ sides} \times h}{2}$$

$$s=\frac{(OA+BC)\times OC}{2}$$

$$s=\frac{(u+v)t}{2}$$

$$s=\frac{(u+u+at)t}{2}$$

$$s=\frac{2ut+at^2}{2}$$

$$\Rightarrow s=\frac{\cancel{2}ut}{\cancel{2}}+\frac{at^2}{2}$$

$$\Rightarrow \boxed{s=ut+\frac{at^2}{2}}$$

3) $s=\frac{(u+v)t}{2}$
$s=\frac{(u+u+at)t}{2}$ ✗

Position - Velocity Relation

s = Area of Trapezium OABC

$$\text{Area}=\frac{1}{2}(OA+BC)\times OC$$

$$s=\frac{1}{2}(u+v)\times t \quad —①$$

Velocity - Time Relation

$$v = u + at \quad \text{--- (2)}$$

$$t = \frac{v-u}{a}$$

Substuting (2) in (1):

$$s = \frac{(v+u)(v-u)}{2a}$$

$$\boxed{2as = v^2 - u^2} \quad [(a+b)(a-b) = a^2 - b^2]$$

Uniform Circular Motion

An object moves in a circular path with uniform speed, subtending equal angles at the centre of the circle in equal intervals

Here, s (distance) $= 2\pi R$ (Circumference)

v (speed) $= s/t$

$$\therefore \boxed{v = \frac{2\pi r}{t}}$$

Examples: • Rotation/Revolution of Earth around the sun.

- a person running on a circular track.
- motion of an election around the nucleus