VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

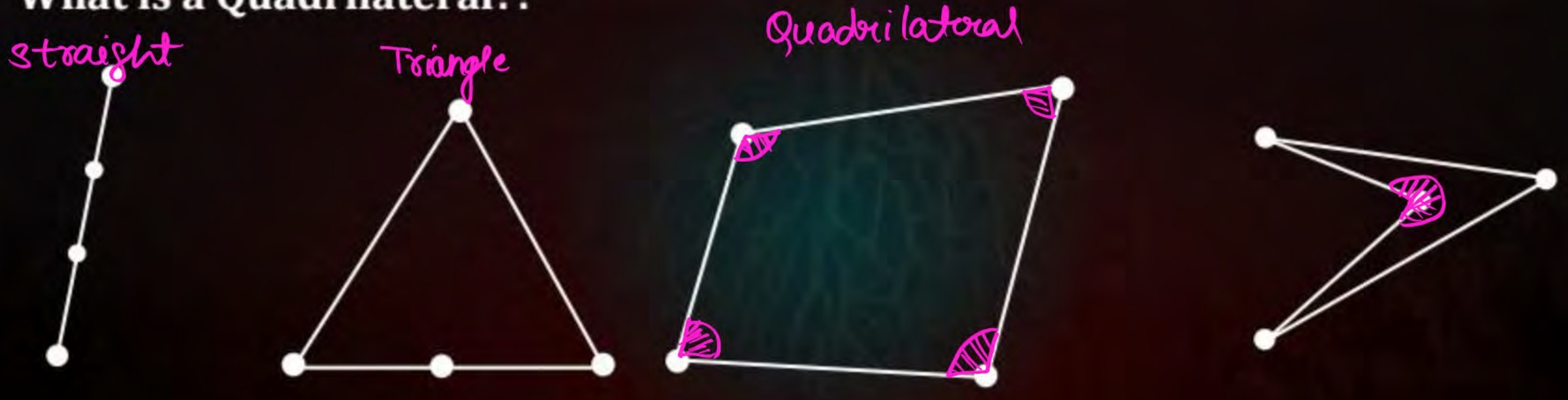
- **Subject –** Mathematics
- **Chapter –** Quadrilaterals

Today's Targets

1. Complete Chapter
2. Bonus Concepts
3. Practice Problems
4. Exam Centric Approach Kya Hana Chahiye ??

Introduction!!

What is a Quadrilateral??



Basic Terms Related To Quadrilaterals

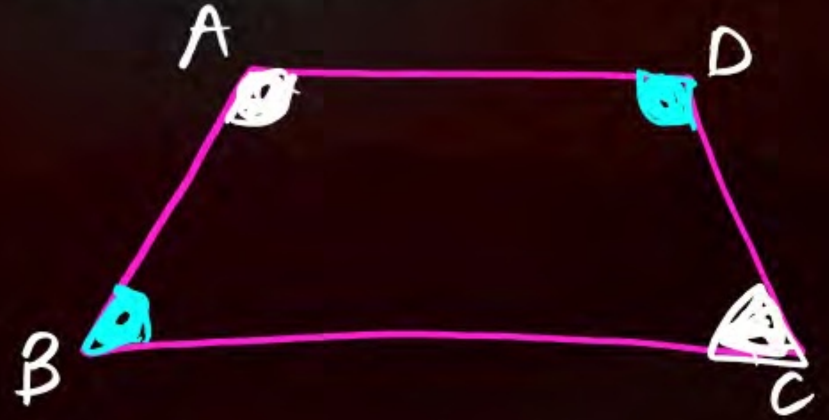
Adjacent/ Consecutive Angles of a Quadrilateral:

Two angles of a quadrilateral having common arm are called its adjacent angles.

$\angle A$ & $\angle D$
$\angle A$ & $\angle B$
$\angle B$ & $\angle C$
$\angle D$ & $\angle C$

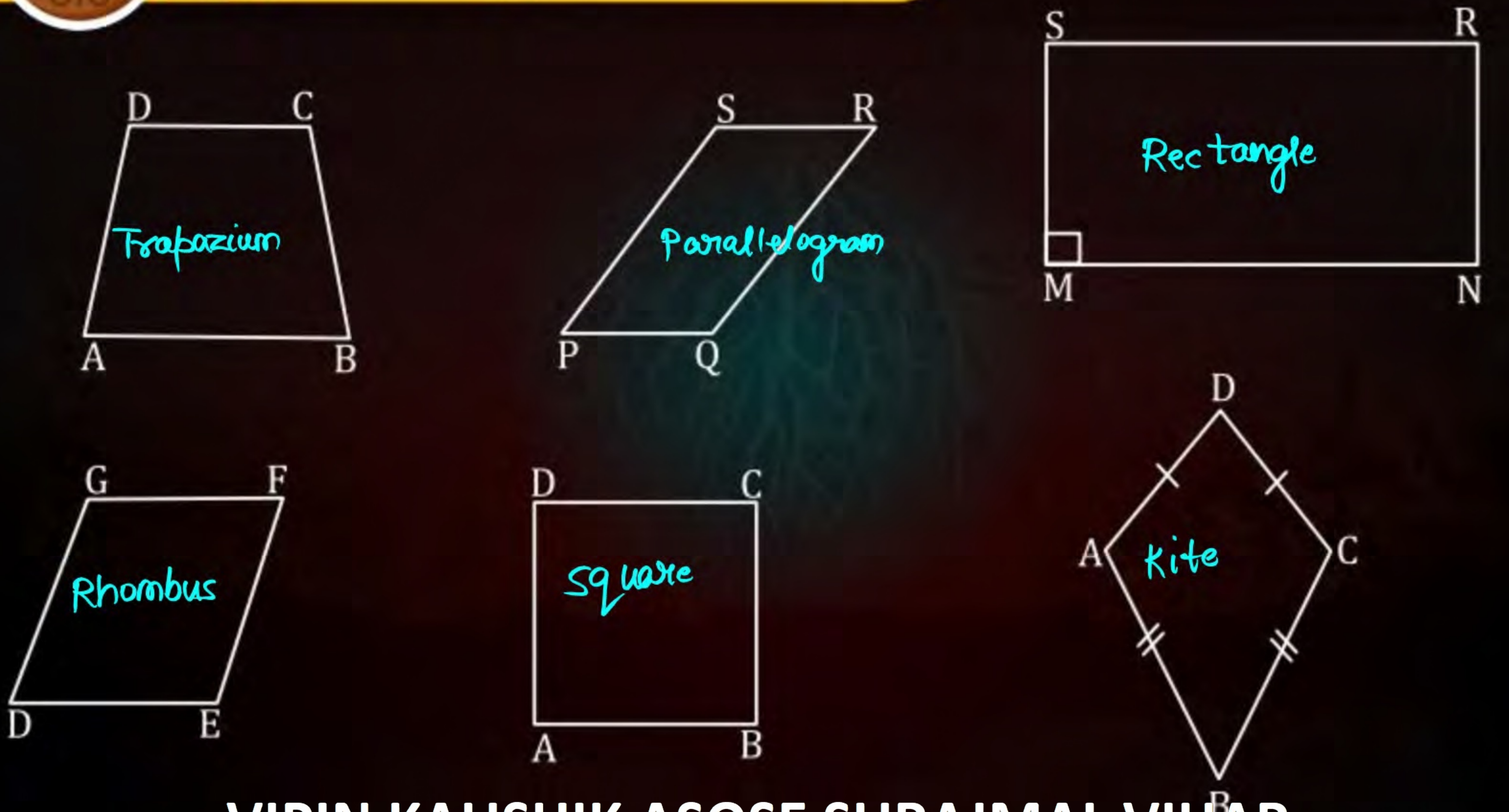
Opposite Angles of a Quadrilateral:

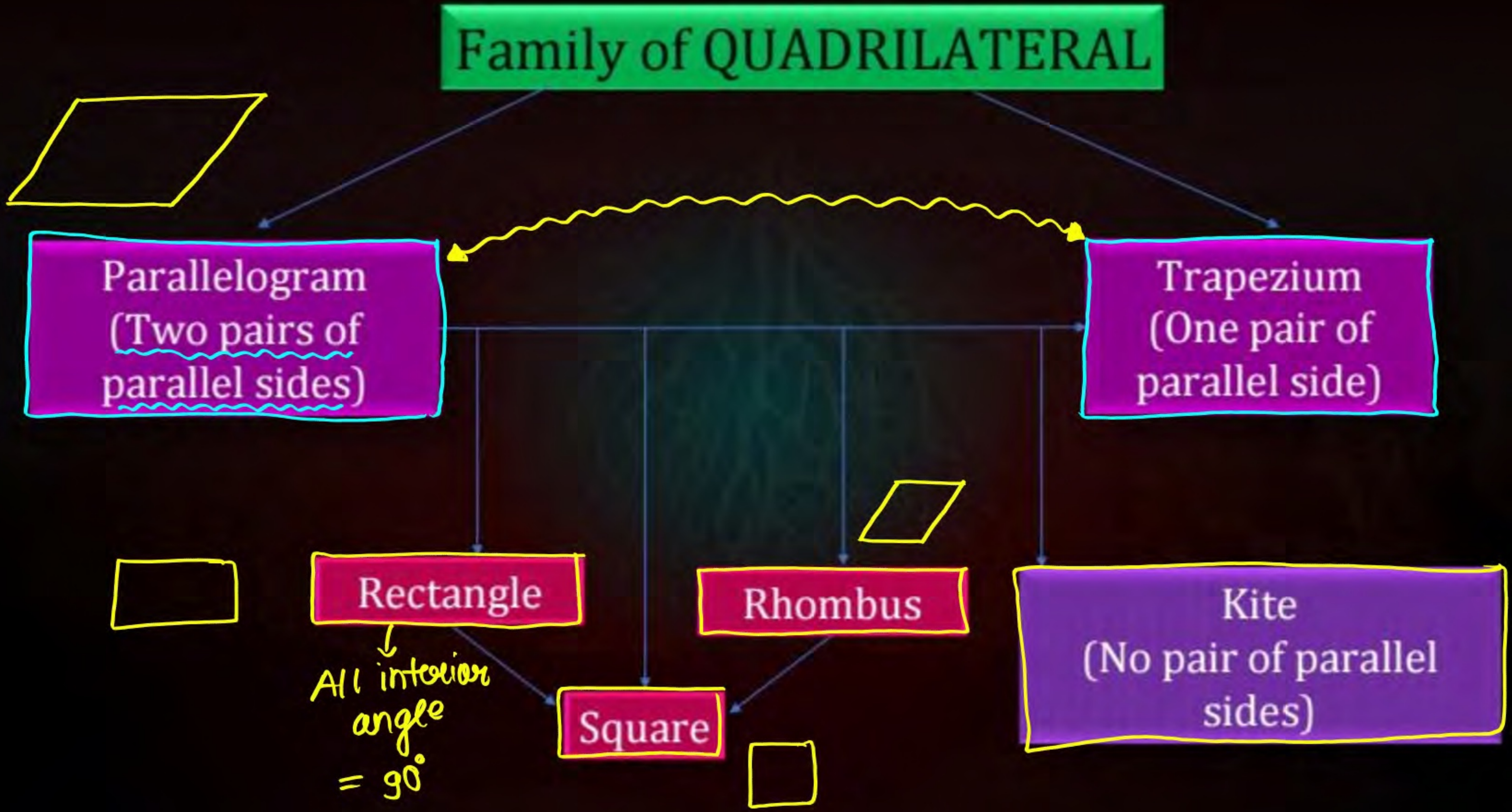
Two angles of a quadrilateral which are not adjacent angles are known as opposite angles.



$\angle A$ & $\angle C$
$\angle B$ & $\angle D$

Types of a Quadrilaterals




Family of QUADRILATERAL
Parallelogram (Two pairs of parallel sides)
Trapezium (One pair of parallel side)
Rectangle
Rhombus
Kite (No pair of parallel sides)
Square
All interior angle = 90°

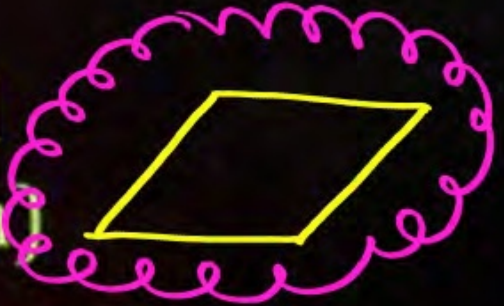
Ise Sides Ke According Samajhe Toh!!

Types of Quadrilaterals

- No parallel sides (Kite)

- One pair of parallel sides (Trapezium)

- Two pair of parallel sides (Parallelogram)



  - Rectangle

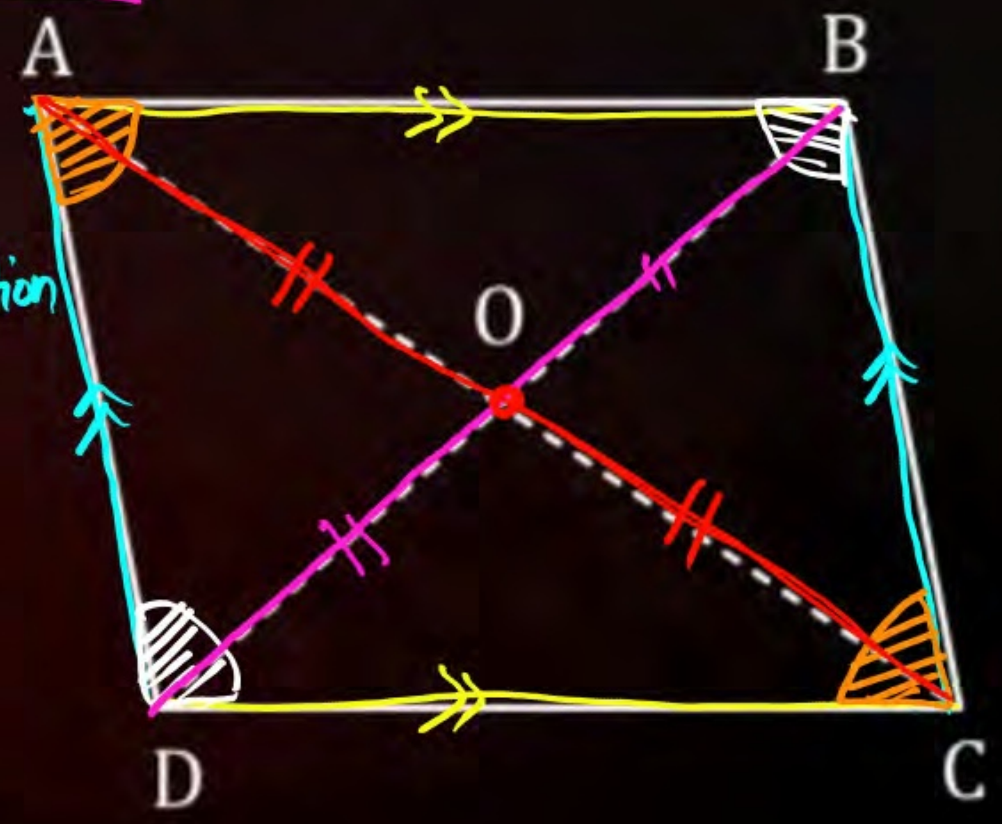
  - Rhombus

  - Square

Let's Talk About Parallelogram!!

In a quadrilateral, if both the pairs of opposite sides are parallel, then it is called a parallelogram.

- Opposite sides are equal i.e., $\boxed{AB = DC}$ and $\boxed{BC = AD}$. ✓
- Opposite sides are parallel i.e., $\boxed{AB \parallel DC}$ and $\boxed{BC \parallel AD}$. Definition
- Opposite angles are equal i.e., $\boxed{\angle A = \angle C}$ and $\boxed{\angle B = \angle D}$. ✓
- Sum of any two adjacent angles are supplementary i.e., $\boxed{\angle A + \angle B = 180° = \angle B + \angle C} = \angle C + \angle D = \angle A + \angle D$. ✓
- Diagonals bisect each other i.e., OA = OC and OB = OD. ✓
- Area of parallelogram = base × height.



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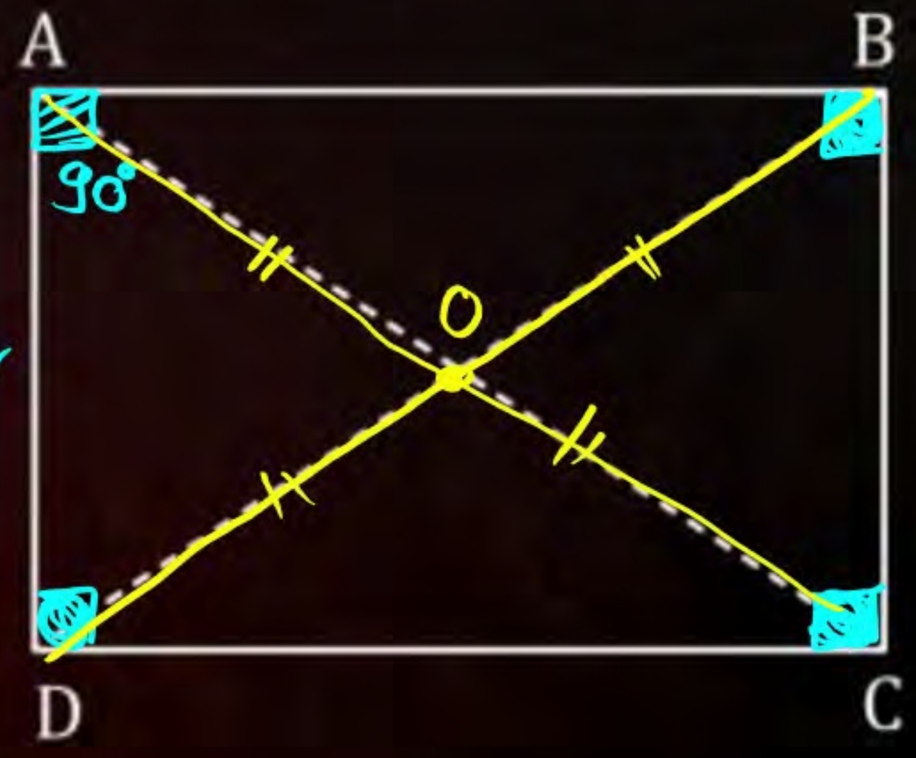
Let's Talk About Rectangle!!

A rectangle is a parallelogram in which all the four angles at the vertices are right angles, i.e., 90°.



- Opposite sides are equal i.e., AB = DC and BC = AD ✓
- Opposite sides are parallel i.e., AB || DC and BC || AD. ✓
- Each angle is a right angle i.e., $\boxed{\angle A = \angle B = \angle C = \angle D = 90°}$. ✓
- $\boxed{\text{Both diagonals are equal i.e., AC = BD}}$. ✓
- Diagonals bisect each other i.e., OA = OC and OB = OD

  ∴ $\boxed{OA = OB = OC = OD}$
- Sum of any two adjacent angles are supplementary i.e.,

  $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle A + \angle D = 180°$. ✓

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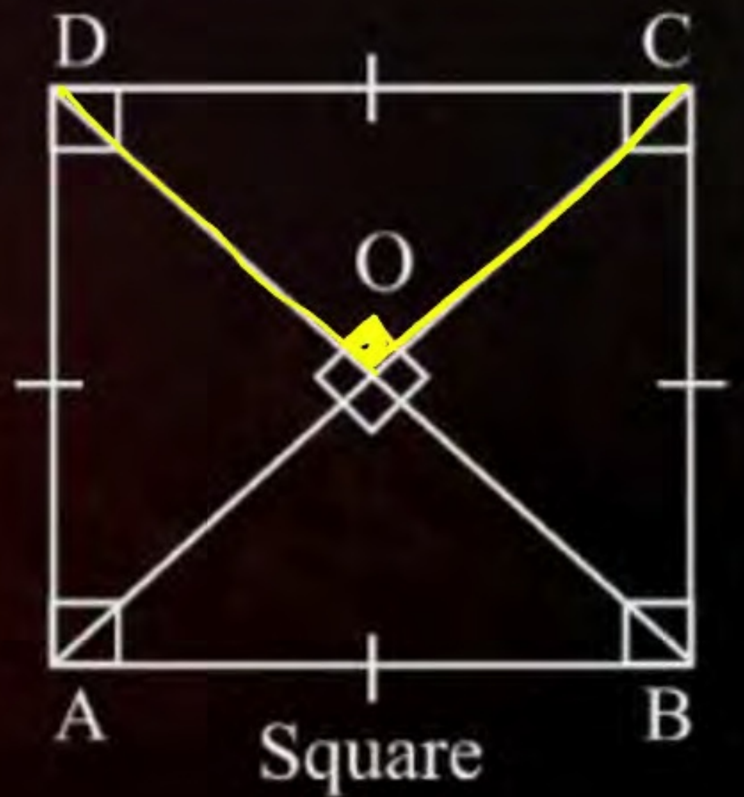
Let's Talk About Square!!

In a rectangle, if all the sides are equal, then it is called a square.

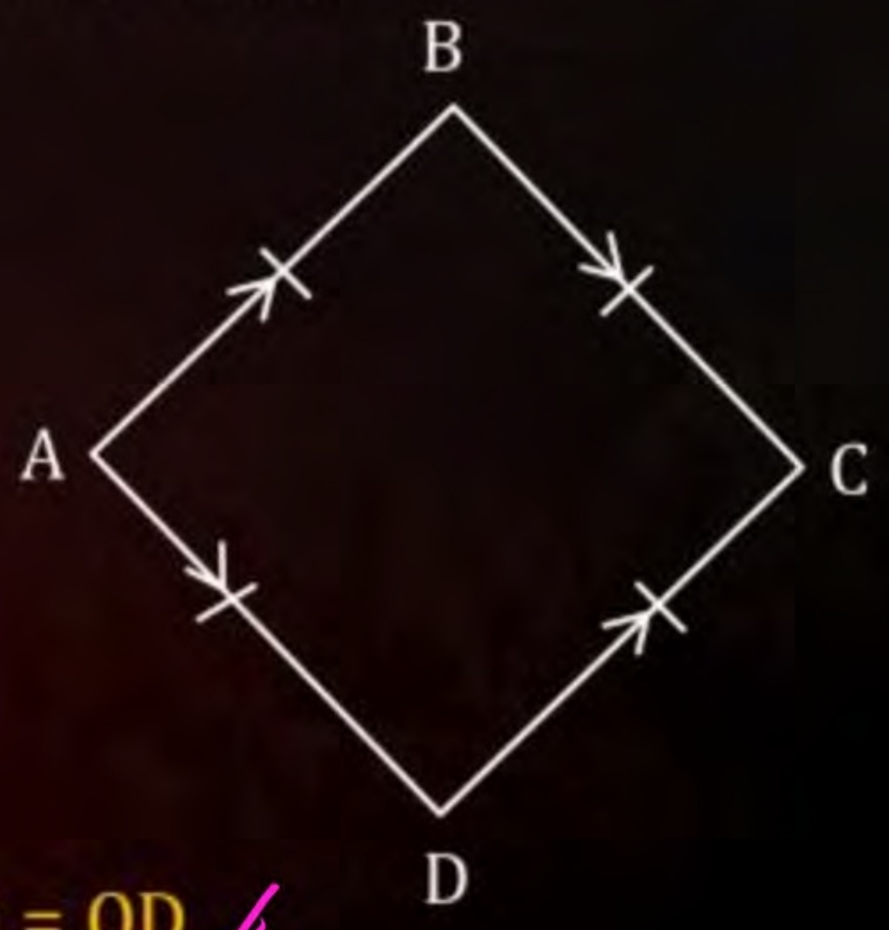
All four sides are equal i.e., $AB = DC = CD = AD$. ***



- Opposite sides are parallel i.e., $AB \parallel DC$ and $BC \parallel AD$.
- Each angle is a right angle i.e., $\angle A = \angle B = \angle C = \angle D = 90°$.
- Both diagonals are equal i.e., $AC = BD$.
- Diagonals bisect each other at right angle. i.e., $OA = OC$ and $OB = OD$ But diagonals $AC = BD$ $\therefore$ $OA = OB = OC = OD$ and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$. ($AC \perp BD$ or $BD \perp AC$).
- Sum of any two adjacent angles are supplementary i.e., $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle A + \angle D = 180°$.

Let's Talk About Rhombus!!

In a parallelogram, if all the sides are equal, then it is called a rhombus.



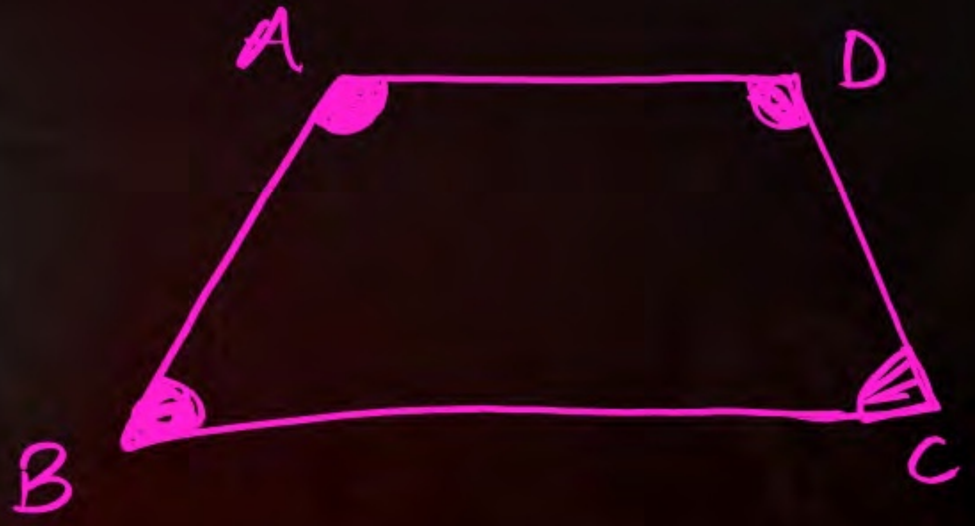
- All four sides are equal i.e., $AB = BC = CD = AD$.
- Opposite sides are parallel i.e., $AB || DC$ and $BC || AD$.
- Opposite angles are equal i.e., $\angle A = \angle C$ and $\angle B = \angle D$.
- Sum of any two adjacent angles are supplementary i.e.,

  $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle A + \angle D = 180°$
- Diagonals bisect each other at right angle i.e., $OA = OC$ and $OB = OD$ and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$ (i.e., $AC \perp BD$ or $BD \perp AC$).
- Area of rhombus = base × height or ½ (product of diagonals).

Angle Sum Property of a Quadrilateral

The sum of all the interior angles in a quadrilateral is 360°

$$\boxed{\angle A + \angle B + \angle C + \angle D = 360^\circ}$$



Question

Which of the following statements are true and which are false?

(i) The diagonals of a parallelogram are equal.

(ii) The diagonals of a parallelogram are perpendicular to each other.

(iii) The diagonals of a rhombus are equal.

(iv) Every rectangle is a square.

(v) Every square is a parallelogram.

(vi) Every rectangle is a parallelogram.

(vii) Every rhombus is a parallelogram.

Question

Which of the following statements are true and which are false?

(i) The diagonals of a parallelogram are equal. → $\boxed{\text{False}}$

(ii) The diagonals of a parallelogram are perpendicular to each other.

→ True

(iii) The diagonals of a rhombus are equal. → $\boxed{\text{False}}$

(iv) Every rectangle is a square. → False

(v) Every square is a parallelogram. → True

(vi) Every rectangle is a parallelogram. → True

(vii) Every rhombus is a parallelogram. → True

Question

ABCD is a rectangle with $\angle ABD = 40°$. Determine the measure of $(\angle DBC)/10$.

$50°$

$40°$

$5°$

$4°$

Question

ABCD is a rectangle with $\angle ABD = 40°$. Determine the measure of $(\angle DBC)/10$.

A 50°

B 40°

C 5°

D 4°

since $\square ABCD$ is a rectangle

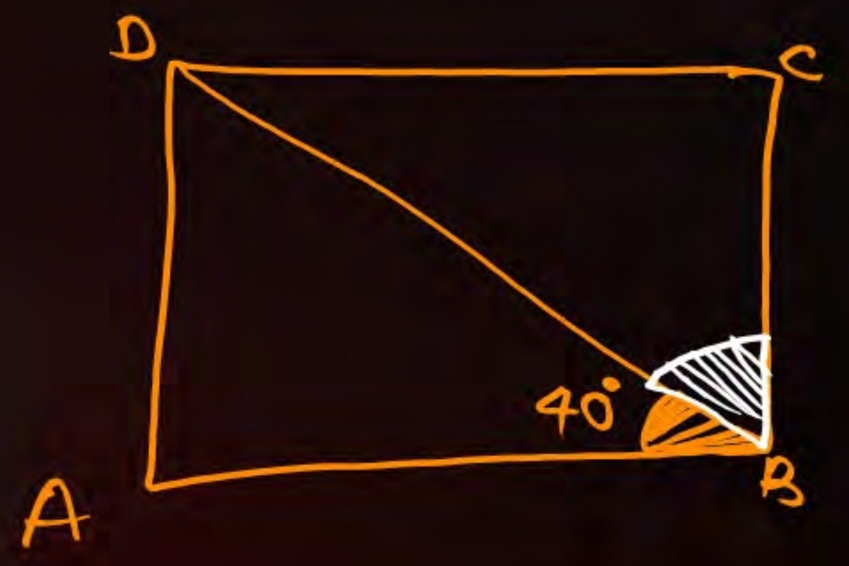
$\angle B = 90°$

$\angle ABD + \angle DBC = 90°$

$40° + \angle DBC = 90°$

$\boxed{\angle DBC = 50°}$

Now, $\frac{\angle DBC}{10} = \frac{50°}{10} = \boxed{5°}$



Question

The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm, what is the measure of twice of the shorter side?

4.5 cm

9 cm

15.5 cm

None of these

Question

The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm, what is the measure of twice of the shorter side?

A 4.5 cm

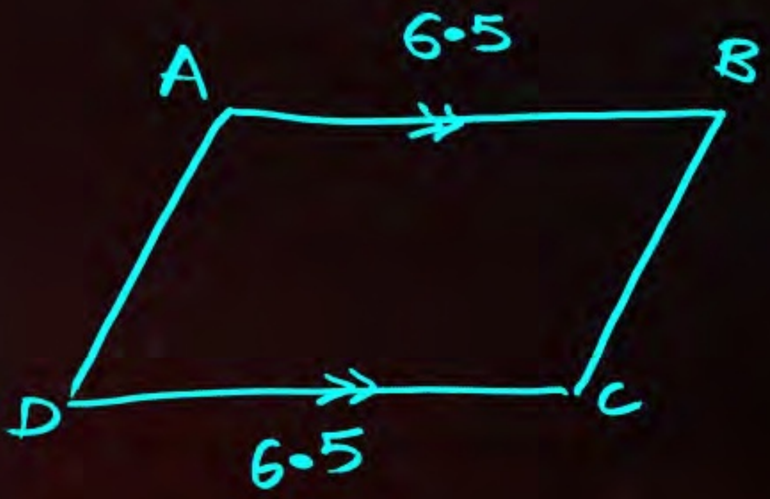
B 9 cm ***

C 15.5 cm

D None of these

$AB \parallel DC \Rightarrow AB = DC$

$AD \parallel BC \Rightarrow AD = BC$



A/Q $\quad AB + BC + CD + AD = 22$

$6.5 + BC + 6.5 + BC = 22$

$2BC + 13 = 22$

$2 \times BC = 9$

$BC = \frac{9}{2} = \boxed{4.5\text{ cm} = AD = \text{smaller side}}$

Therefore, twice of shorter side $= 2 \times 4.5 = \boxed{9\text{cm}}$ Ans.

Question

If an angle of a parallelogram is two-third of its adjacent angle, then the smallest angle of the parallelogram is

$108°$

$72°$

$90°$

None of these

Question

If an angle of a parallelogram is two-third of its adjacent angle, then the smallest angle of the parallelogram is

- **A** 108°
- **B** 72°
- **C** 90°
- **D** None of these

In a parallelogram,

sum of adjacent angle $= 180^\circ$

$$\frac{\theta}{1} + \frac{2}{3}\theta = 180^\circ$$

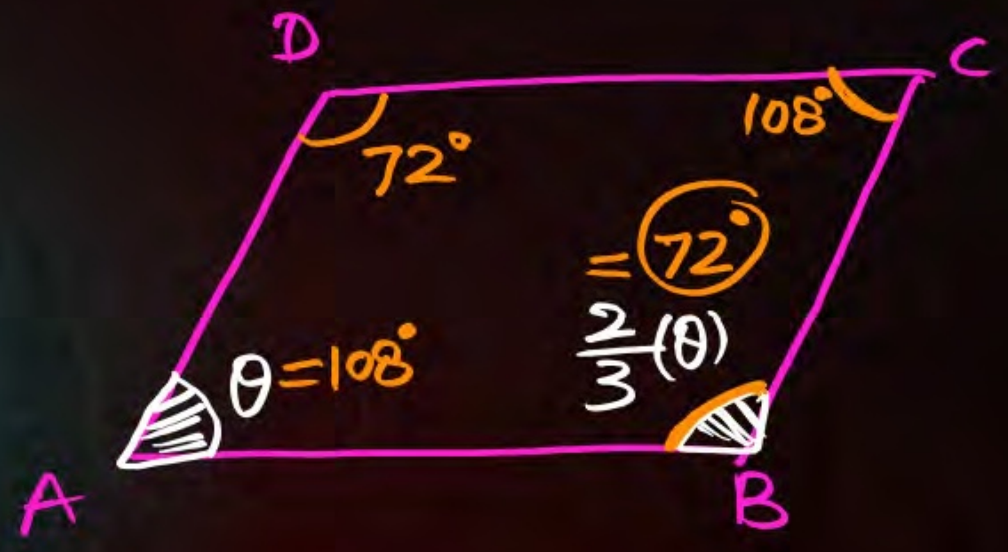
$$\Rightarrow \frac{3\theta + 2\theta}{3} = 180^\circ$$

$$\Rightarrow 5\theta = 180^\circ \times 3$$

$$\Rightarrow \theta = \frac{180^\circ \times 3}{5}$$

$$\Rightarrow \theta = 36^\circ \times 3$$

$$\Rightarrow \boxed{\theta = 108^\circ}$$



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Question

The sides of a rectangle are in the ratio 5 : 4 and its perimeter is 90 cm. Find the length and breadth.

Length = 50cm and Width = 40 cm

Length = 45 cm and Width = 45 cm

Length = 25 cm and Width = 20 cm

None of these

Question

The sides of a rectangle are in the ratio 5 : 4 and its perimeter is 90 cm. Find the length and breadth.

A Length = 50cm and Width = 40 cm

B Length = 45 cm and Width = 45 cm

C Length = 25 cm and Width = 20 cm

D None of these

Perimeter = $2(l+b)$

$90 = 2(5x+4x)$

$\frac{90}{2} = 9x \Rightarrow x = \frac{90}{9\times2} \Rightarrow \boxed{x=5}$

Length $= 5\times5 = \boxed{25}$

width $= 4\times5 = \boxed{20}$

Question

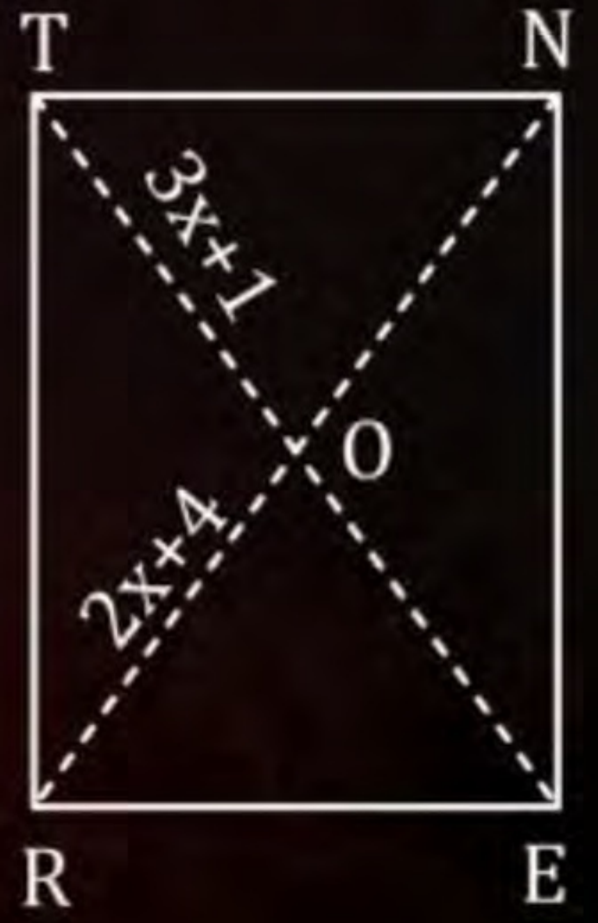
RENT is a rectangle. Its diagonal meet at O. Find x, if OR = 2x + 4 and OT = 3x + 1.

$x = 5$

$x = 3$

$x = \frac{3}{5}$

None of these



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Question

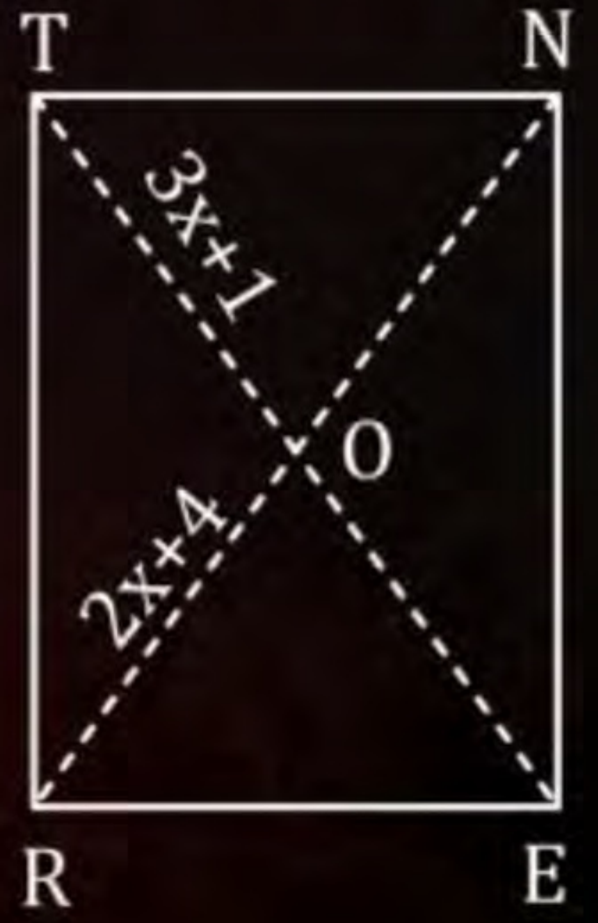
RENT is a rectangle. Its diagonal meet at O. Find x, if OR = 2x + 4 and OT = 3x + 1.

A $x = 5$

B $x = 3$ ✓

C $x = \frac{3}{5}$

D None of these



Diagonal of Rectangle are equal in length and bisect each other.

$$TE = RN \Rightarrow \frac{TE}{2} = \frac{RN}{2}$$

$$\Rightarrow OT = OR$$

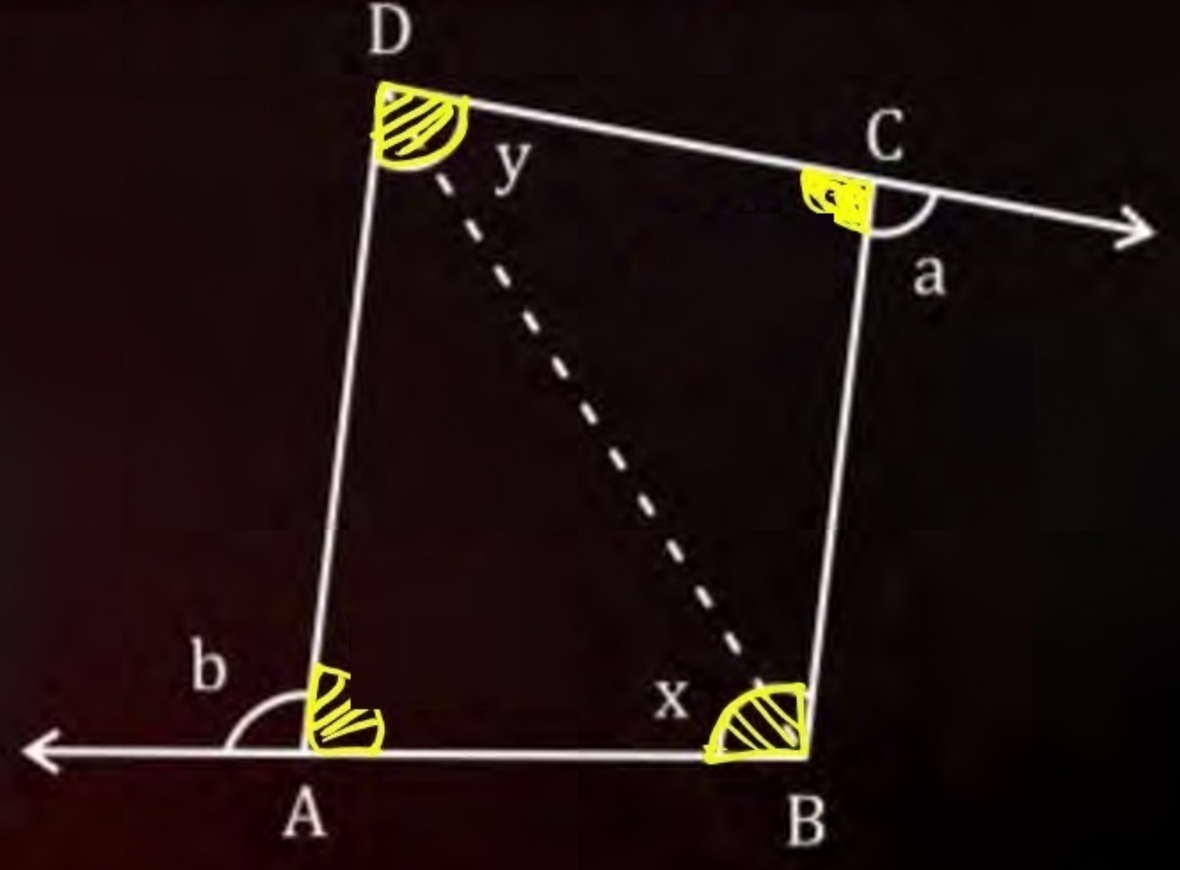
$$\Rightarrow 3x+1 = 2x+4$$

$$\Rightarrow 3x-2x = 4-1$$

$$\Rightarrow \boxed{x=3}$$

Question

The sides BA and DC of a quadrilateral ABCD are produced as shown in fig. Prove that $a + b = x + y$



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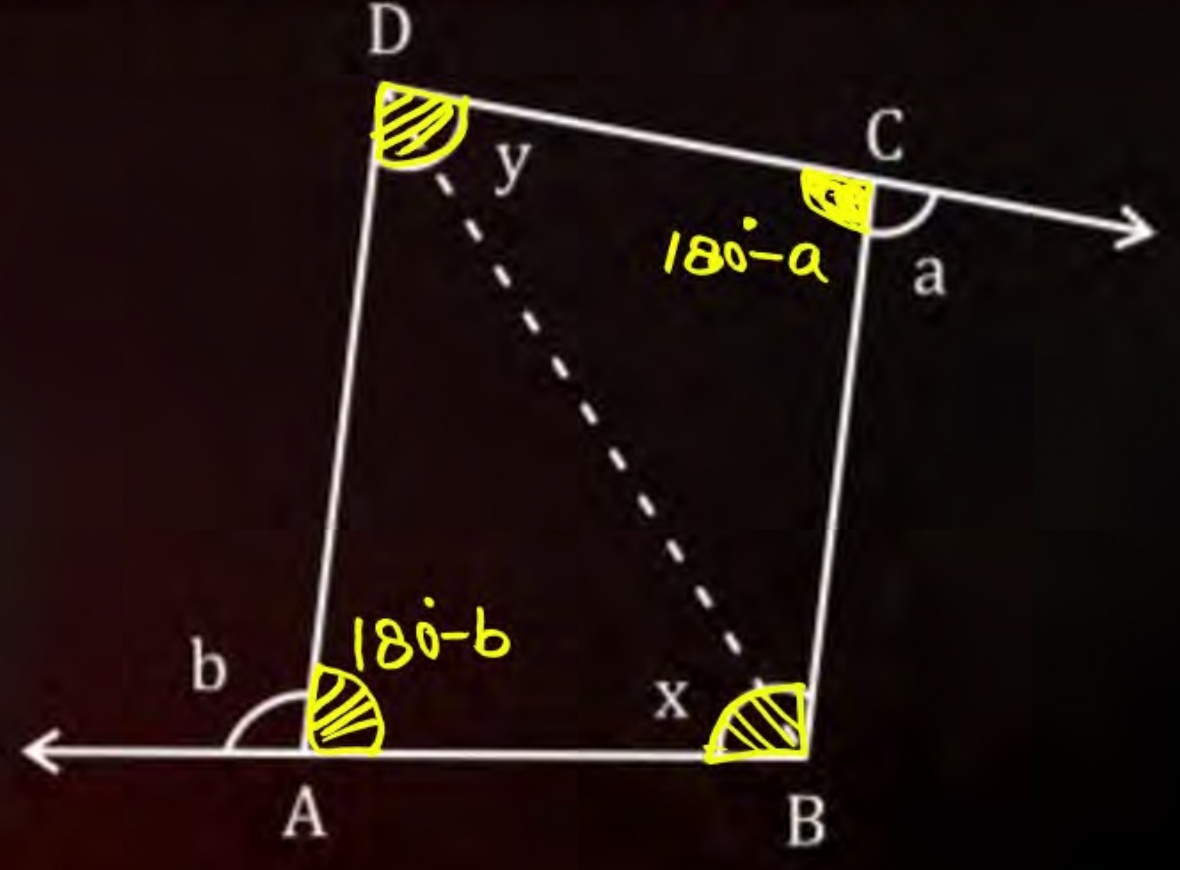

Angle sum property of Quadrilaterals

$$(180^\circ - b) + x + (180^\circ - a) + y = 360^\circ$$

$$360^\circ + x + y - a - b = 360^\circ$$

$$x + y = \cancel{360^\circ} - \cancel{360} + a + b$$

$$\boxed{x + y = a + b}$$

Hence proved!!

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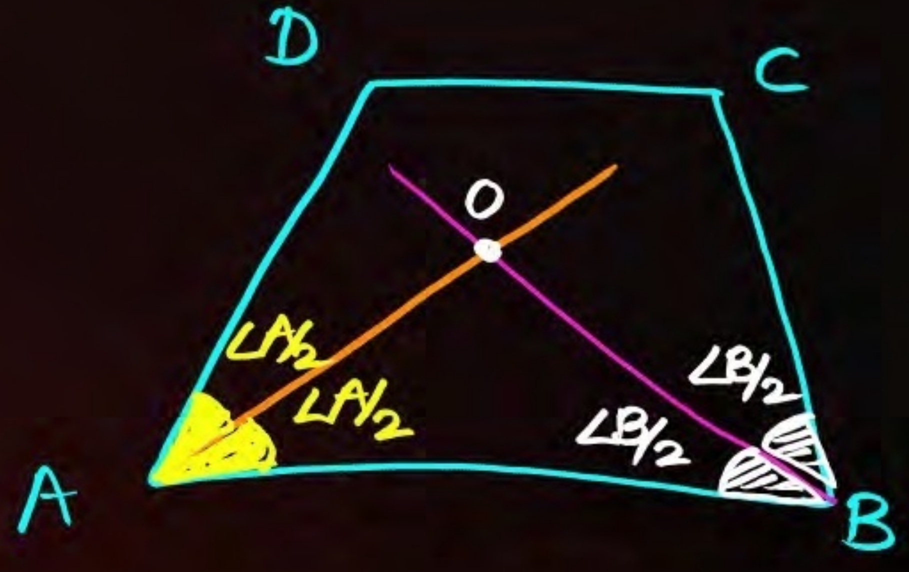
Question

In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$. (Imp.)



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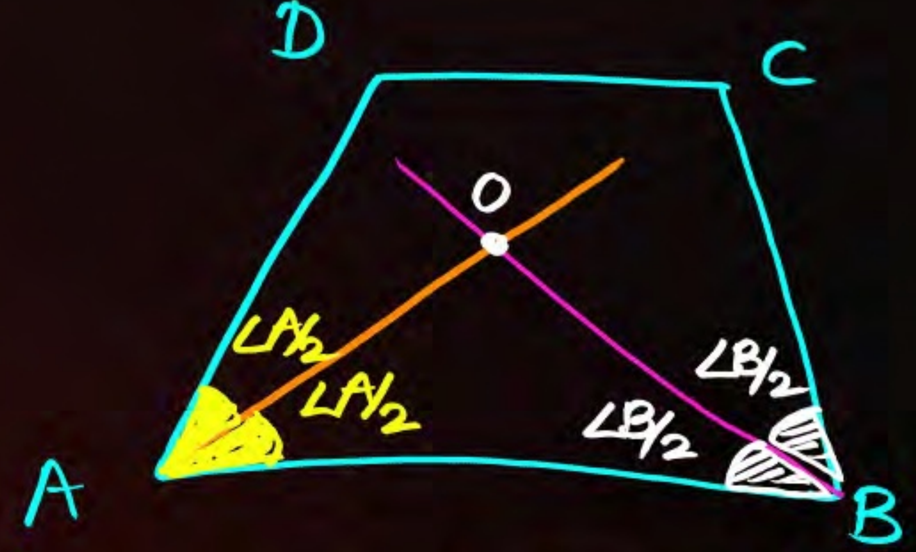
Question

In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$. (Imp.)



since □ABCD is a quadrilateral,
using angle sum property,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\angle A + \angle B = 360^\circ - (\angle C + \angle D)$$

In $\triangle$ AOB, angle sum property

$$\angle AOB + \frac{\angle A}{2} + \frac{\angle B}{2} = 180^\circ$$

$$\boxed{\angle AOB = 180^\circ - \frac{1}{2}(\angle A + \angle B)}$$

$$\angle AOB = 180^\circ - \frac{1}{2}(360^\circ - (\angle C + \angle D))$$

$$\angle AOB = \cancel{180^\circ} - \cancel{180^\circ} + \frac{1}{2}(\angle C + \angle D)$$

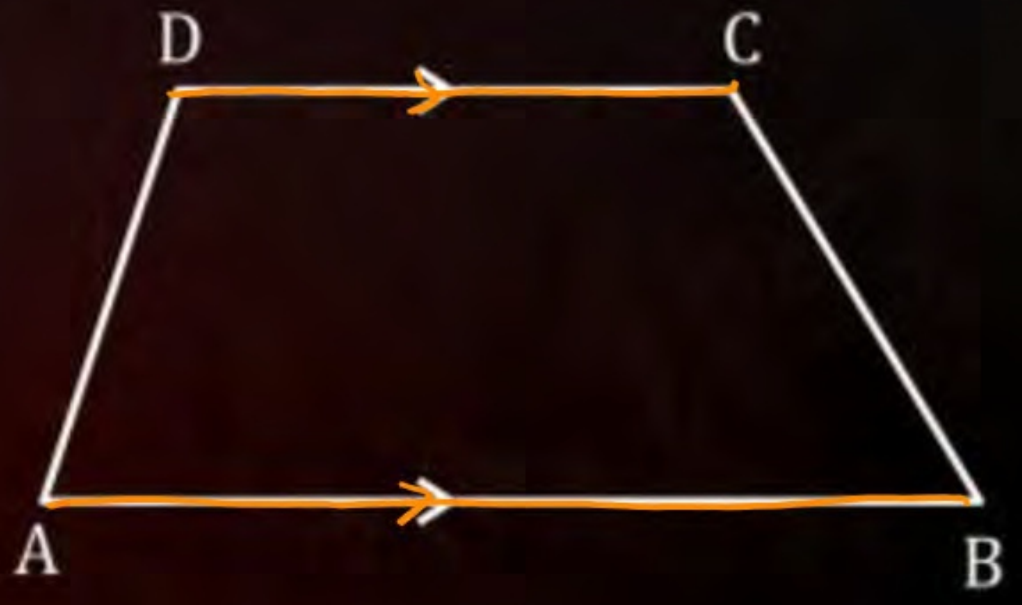
$$\boxed{\angle AOB = \frac{1}{2}(\angle C + \angle D)}$$

Hence, proved!!

Trapezium

A quadrilateral having exactly one pair of parallel sides is called a trapezium. In the given figure, ABCD is a trapezium in which AB || DC.

$\boxed{AB \parallel CD \ \& \ AD \nparallel BC}$



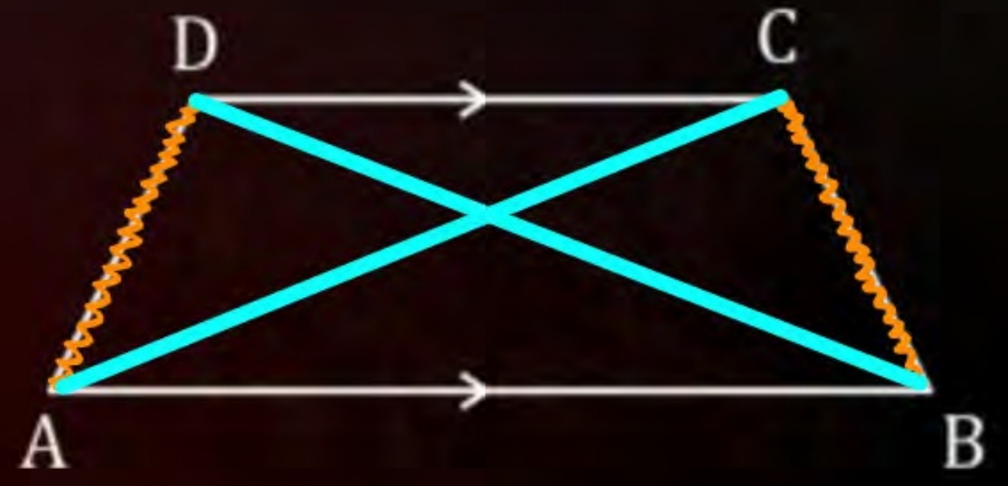
Isosceles Trapezium

A trapezium is said to be an isosceles trapezium, if its non parallel sides are equal.

In the figure, quadrilateral ABCD is an isosceles trapezium in which AB || DC and AD = BC

$\boxed{AB \parallel CD}$ & $\boxed{AD = BC}$

Diagonals are of equal length $\Rightarrow BD = AC$



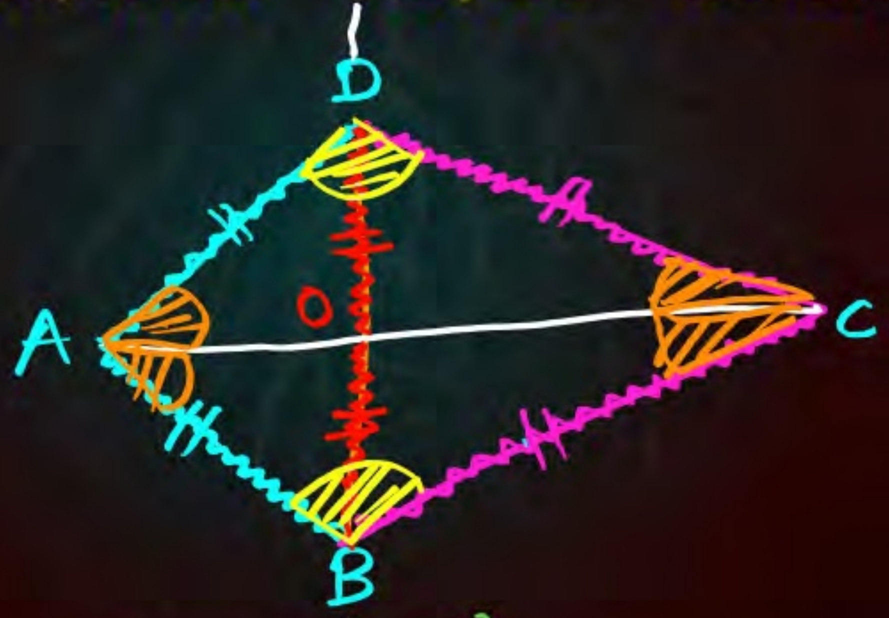
Kite

A quadrilateral is a kite if it has two pairs of equal adjacent side and unequal opposite sides.

Thus, a quadrilateral ABCD is a kite, if AB = AD, BC = CD and AD ≠ BC and AB ≠ CD

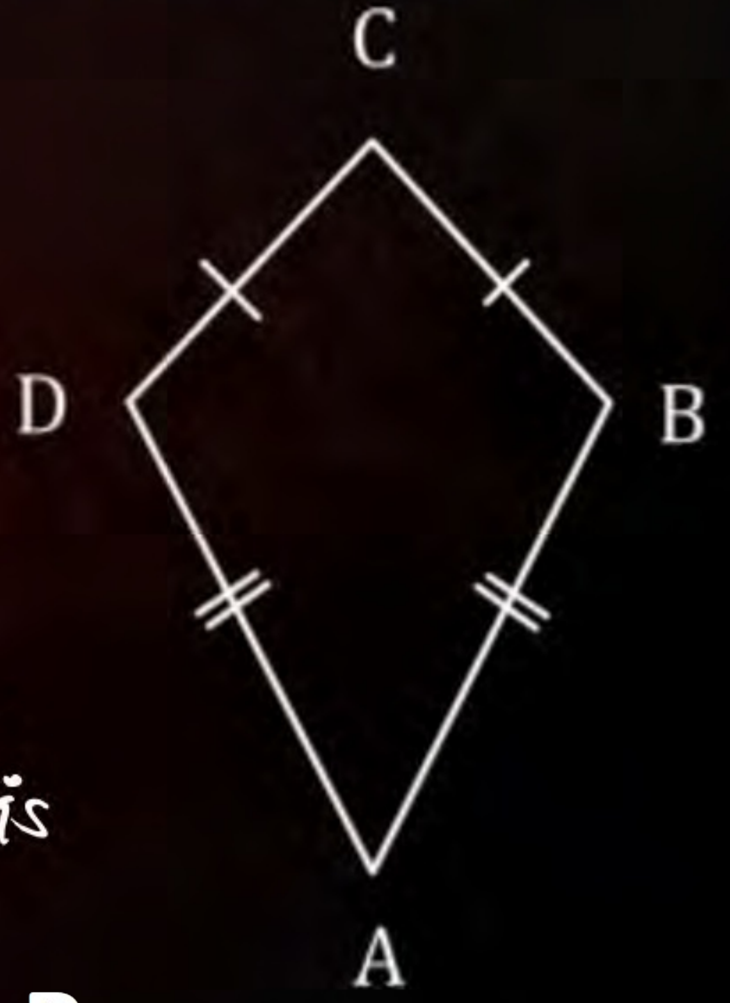
$\boxed{AB = AD}$

Longer diagonal ke Raste me Jo aayega sab bisect ho Jayega.



$\boxed{OD \neq OB}$

smaller diagonal ke raste me Jo kuch bhi aayega voh as it is rahega !!



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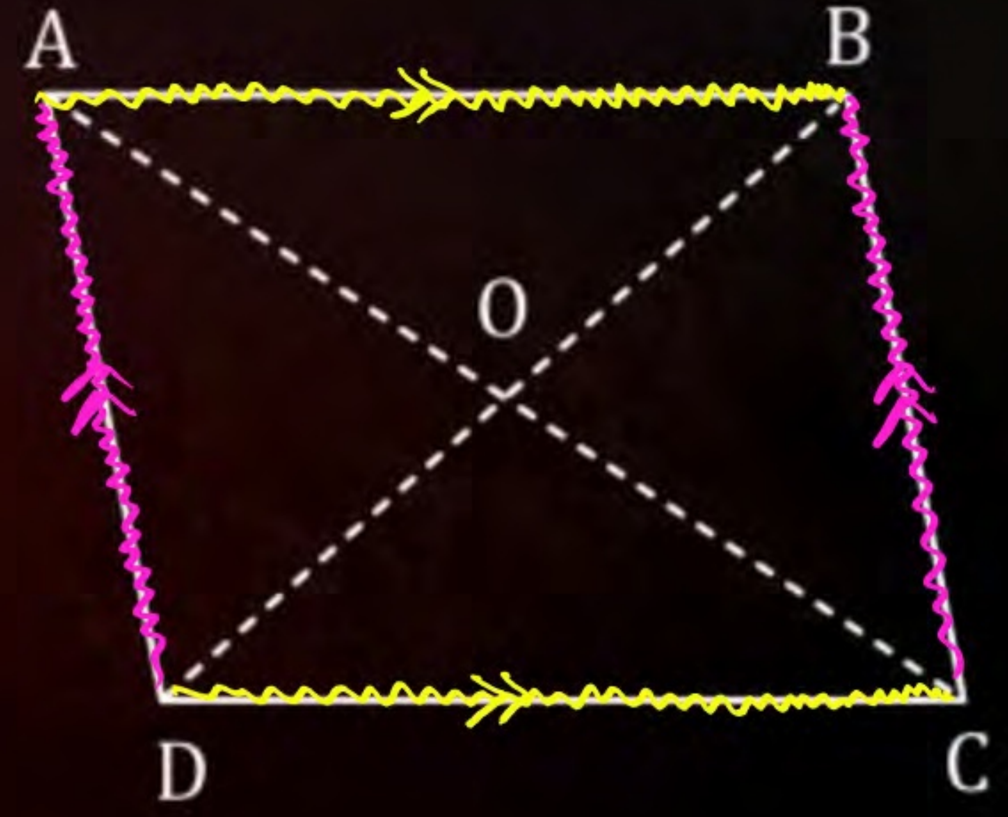
Definition of Parallelogram

In a quadrilateral, if both the pairs of opposite sides are parallel, then it is called a parallelogram.



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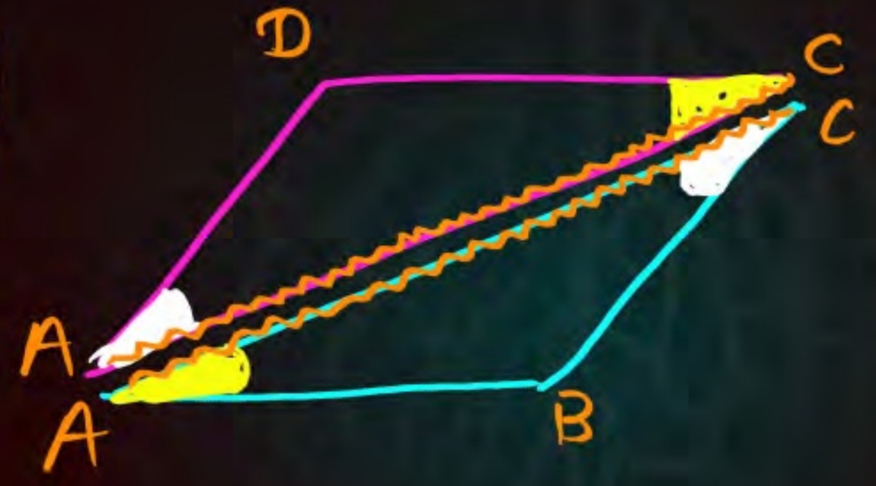
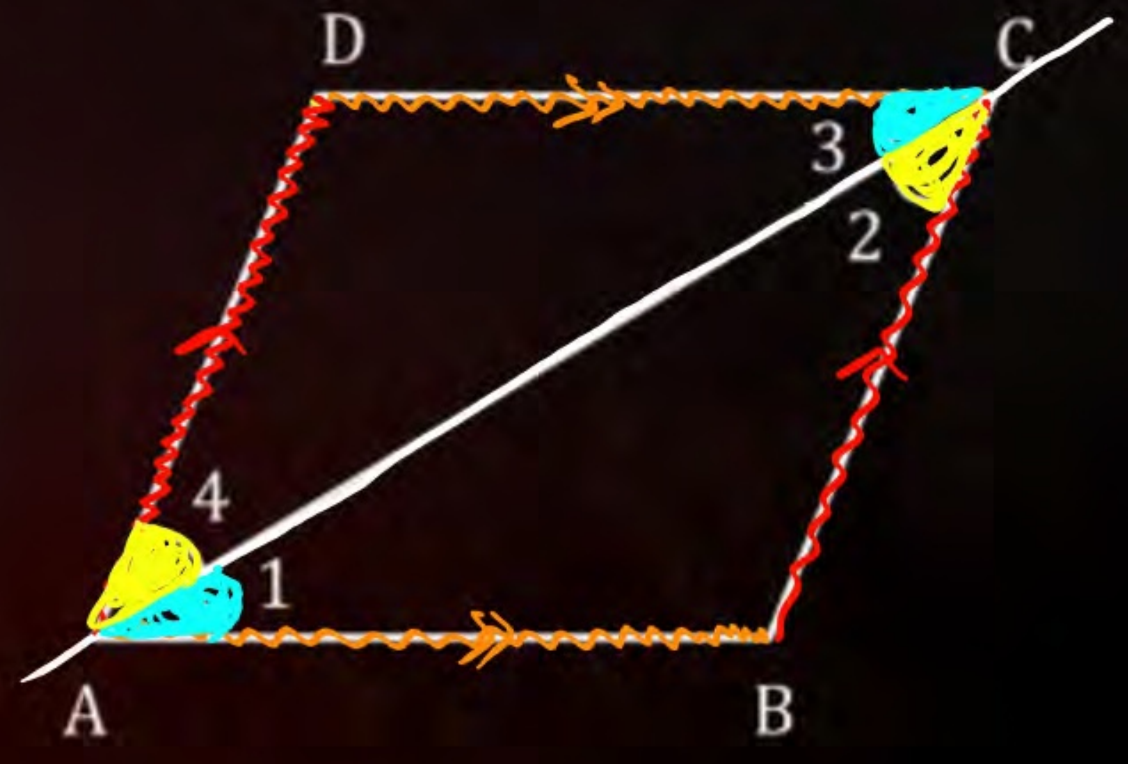
Theorem – 1

Statement : A diagonal of parallelogram divides it into two congruent triangles.

$\boxed{A - S - A \text{ criteria}}$

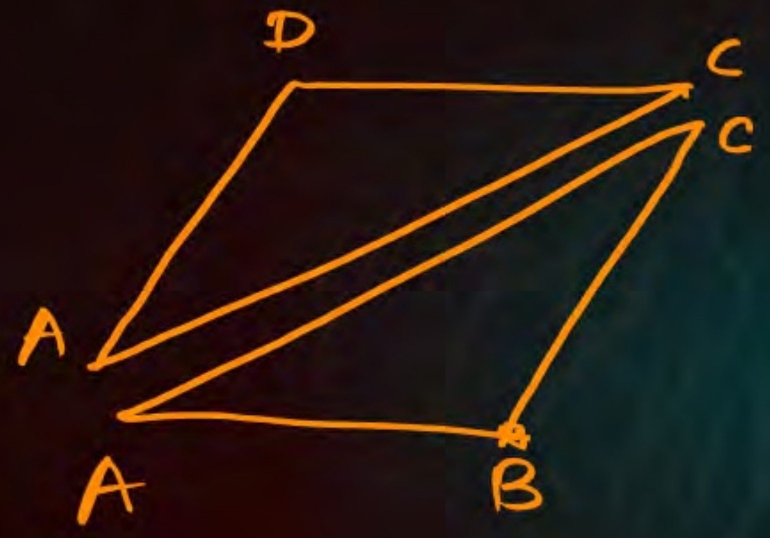
$\boxed{\Delta ADC \cong \Delta CBA}$

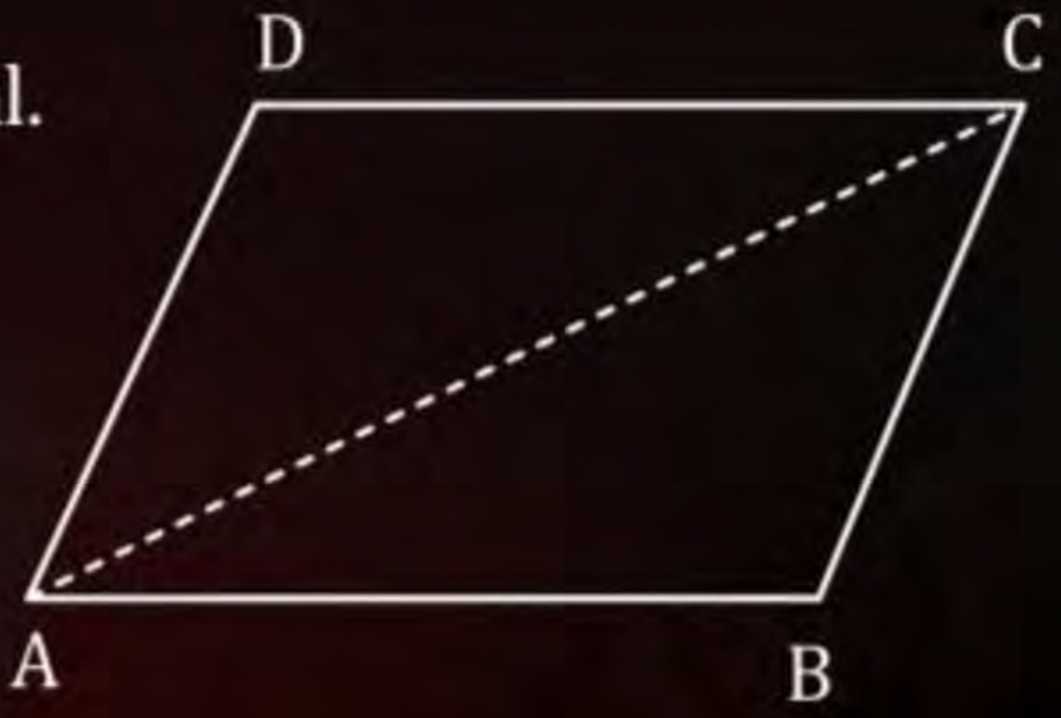




Theorem – 2

Statement : In a parallelogram, opposite sides are equal.





Since,

$\boxed{\triangle ADC \cong \triangle CBA}$

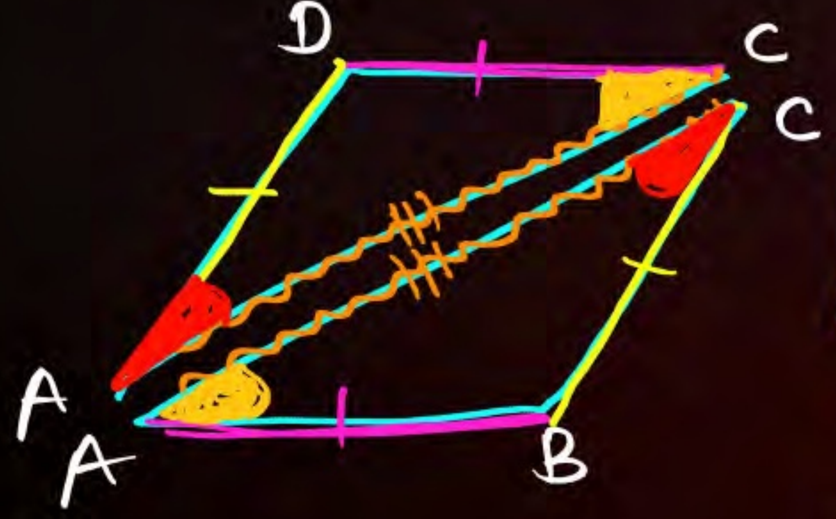
By CPCT

$\boxed{AB = CD}$ ✓

$\boxed{AD = BC}$ ✓

Theorem – 3

Statement (Converse of Theorem 2): If each pair of opposite sides of a quadrilateral are equal, then it is a parallelogram.



S–S–S criteria

$\triangle ADC \cong \triangle CBA$

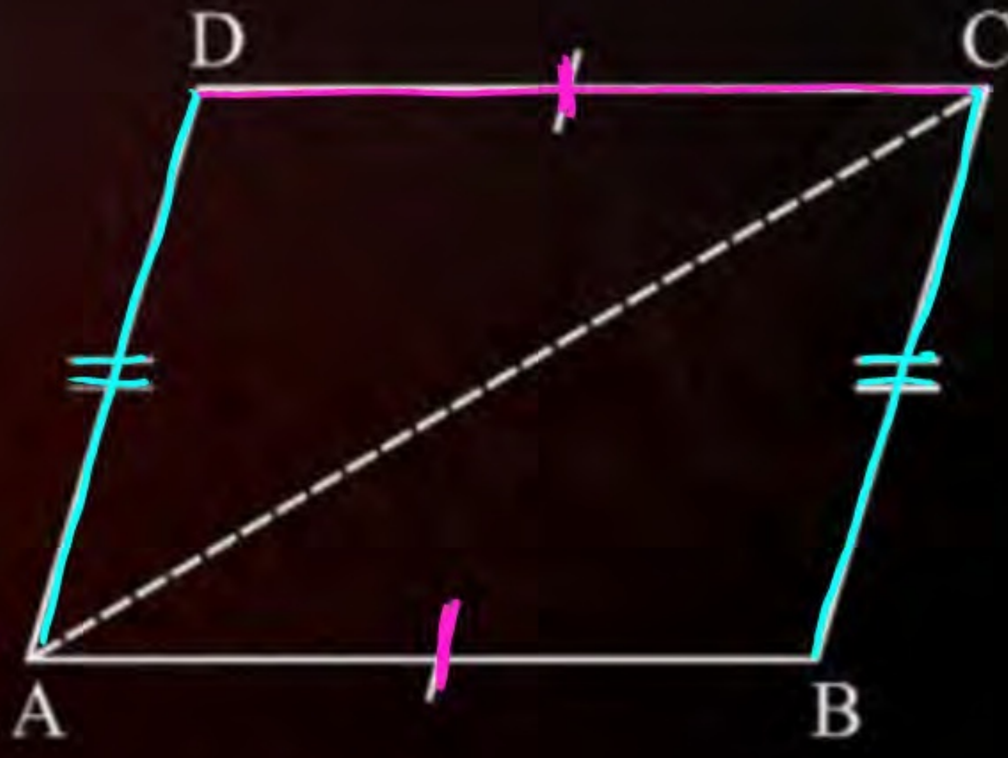
By CPCT,

$\angle DCA = \angle BAC$

$\hookrightarrow AB \parallel DC$ — ①

$\angle DAC = \angle BCA$

$\hookrightarrow AD \parallel BC$ — ②



From ① & ②

$\square ABCD$ is a $\parallel^{gram}$

Theorem – 4

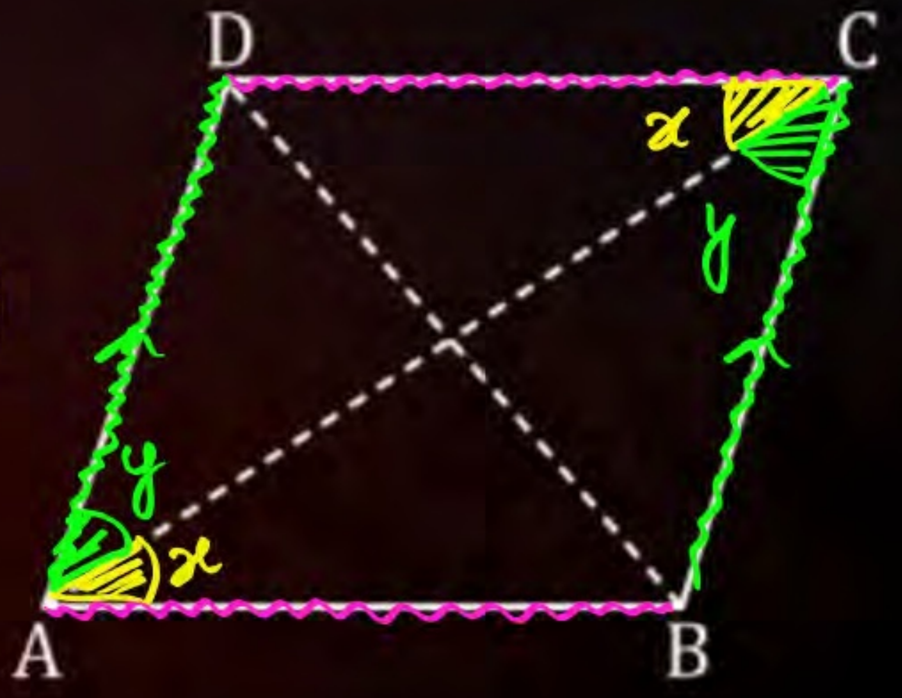
Statement : In a parallelogram, opposite angles are equal.

since □ABCD is a ||gram

$\boxed{AB \parallel CD}$

Therefore, using AC as transversal line

$\boxed{\angle DCA = \angle BAC = x}$ (let say) —①



$\boxed{AD \parallel BC}$

Therefore, using AC as a transversal

$\boxed{\angle BCA = \angle DAC = y}$ (let say) —②

Adding eq① & ② $\boxed{\angle C = \angle A}$, similarly $\boxed{\angle B = \angle D}$

Hence, proved!!

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Theorem-5

Statement (Converse of Theorem 4): If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

$\angle 1 = \angle 3$ & $\angle 2 = \angle 4$

Using angle sum property,

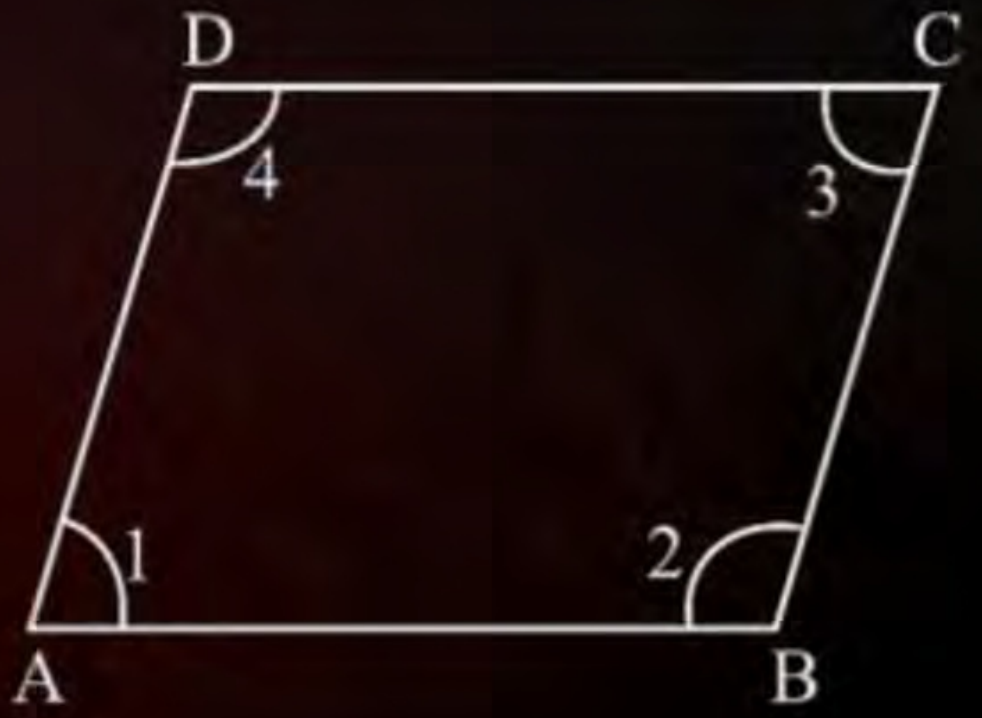
$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$

$\angle 1 + \angle 2 + \angle 1 + \angle 2 = 360^\circ$

$2(\angle 1 + \angle 2) = 360^\circ$

$\boxed{\angle 1 + \angle 2 = 180^\circ}$

$\boxed{AD \parallel BC}$ --- (1)



Now,

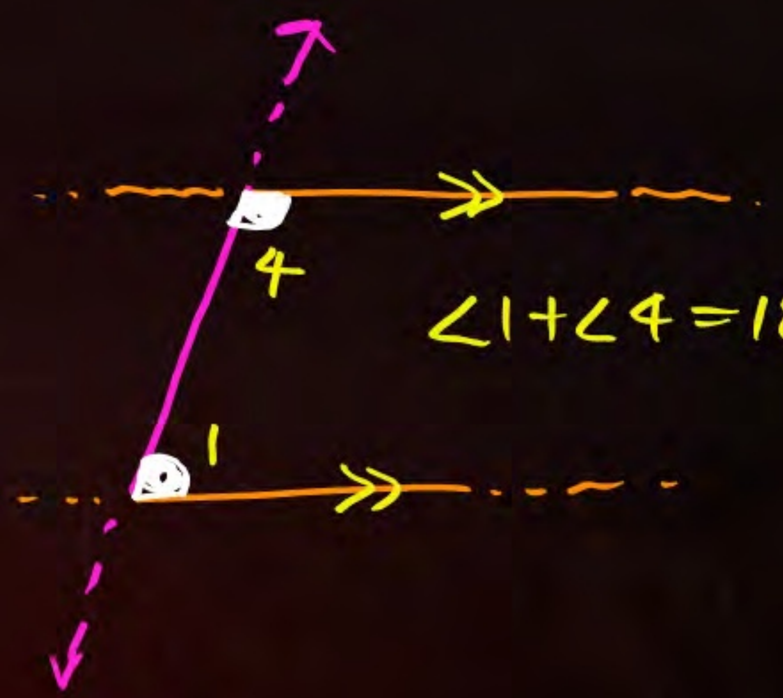
$$\angle 1+\angle 2+\angle 3+\angle 4=360^\circ$$

$$\angle 1+\angle 4+\angle 1+\angle 4=360^\circ$$

$$2(\angle 1+\angle 4)=360^\circ$$

$$\boxed{\angle 1+\angle 4=180^\circ}$$

$\hookrightarrow$ $\boxed{AB \parallel CD}$ — (2)



$\angle 1+\angle 4=180^\circ$

From Eq^n (1) & (2)

If and only if $\square ABCD$ is a $\parallel^{gram}$

Question

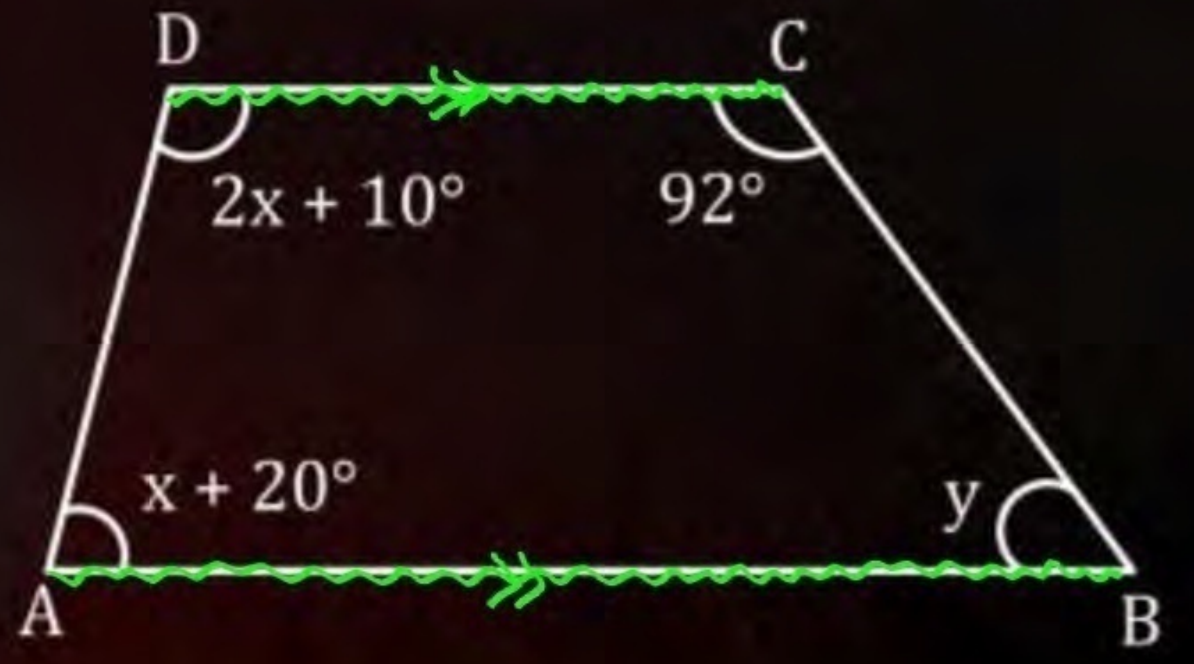
In figure, ABCD is a trapezium. Find the value of x and y.

$x = 50°, y = 80°$

$x = 50°, y = 88°$

$x = 80°, y = 50°$

None of these



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In figure, ABCD is a trapezium. Find the value of x and y.

A $x = 50°, y = 80°$

B $x = 50°, y = 88°$

C $x = 80°, y = 50°$

D None of these

since □ABCD is a trapezium, where $AB \parallel CD$

sum of Adjacent = 180°

$\Rightarrow (x+20°)+(2x+10°) = 180°$

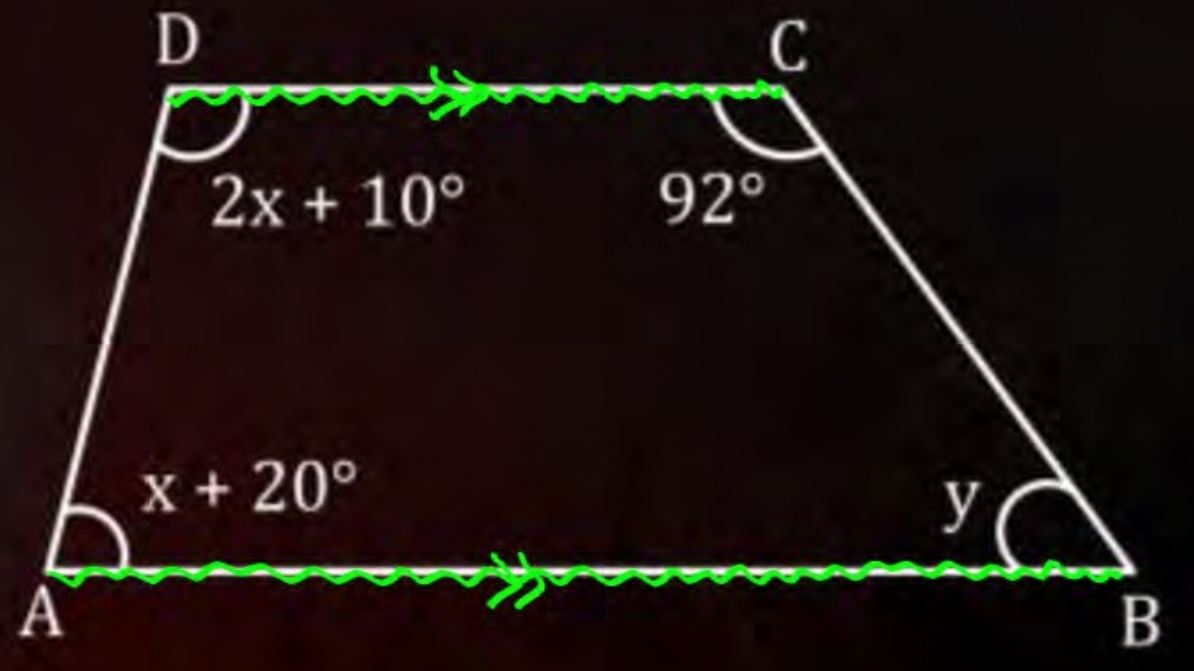
$3x+30° = 180°$

$3x = 150° \Rightarrow \boxed{x = 50°}$

$\Rightarrow 92°+y = 180°$

$y = 180°-92°$

$\boxed{y = 88°}$



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

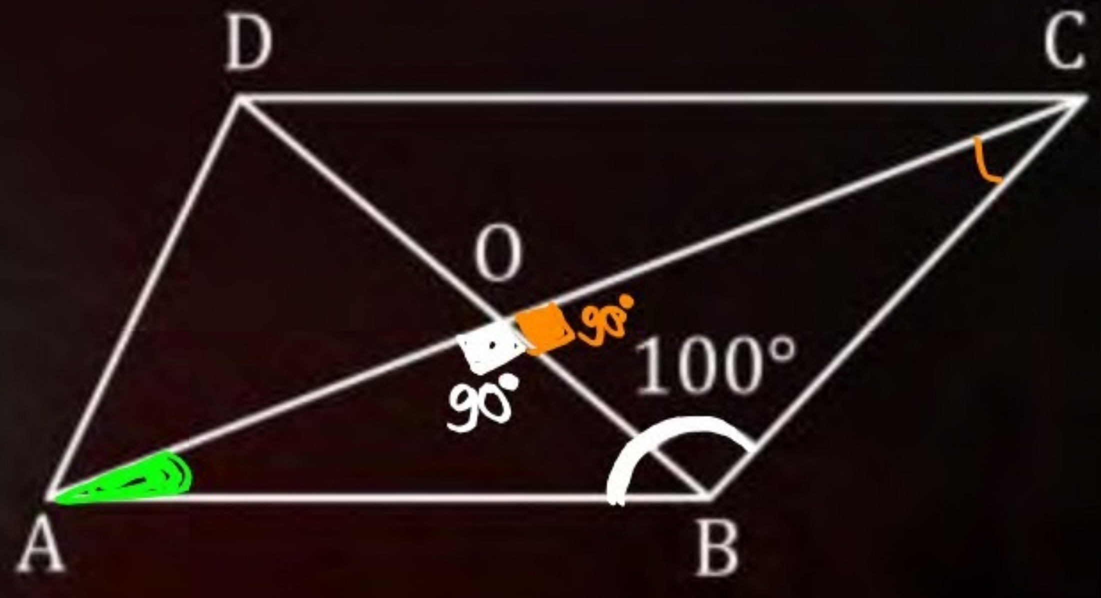
In the given rhombus ABCD, the measure of ∠OAB is

A 90°

B 50°

C 40°

D 45°



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

If one angle of a parallelogram is 50°, then other angles are

130°, 50°, 130°

60°, 125°, 125°

70°, 120°, 120°

Cannot be determined

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

If one angle of a parallelogram is 50°, then other angles are

A 130°, 50°, 130° ✓

B 60°, 125°, 125°

C 70°, 120°, 120°

D Cannot be determined

Question

In the given rectangle PQRS, the measure of ∠POQ is

40°

120°

100°

80°

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In the given rectangle PQRS, the measure of $\angle POQ$ is

- **A** 40°
- **B** 120°
- **C** 100°
- **D** 80°

Question

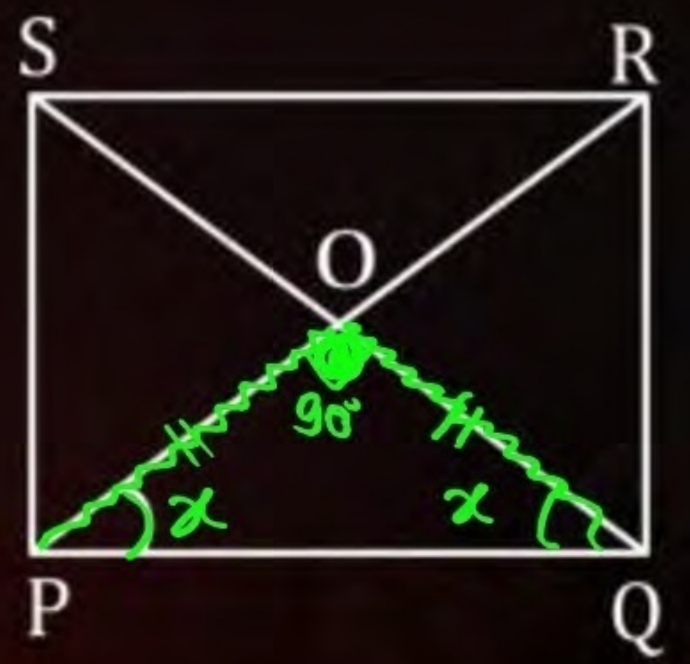
In the given square PQRS, the measure of ∠POQ is

90°

45°

60°

Cannot determined



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In the given square PQRS, the measure of $\angle POQ$ is

A 90°

B 45°

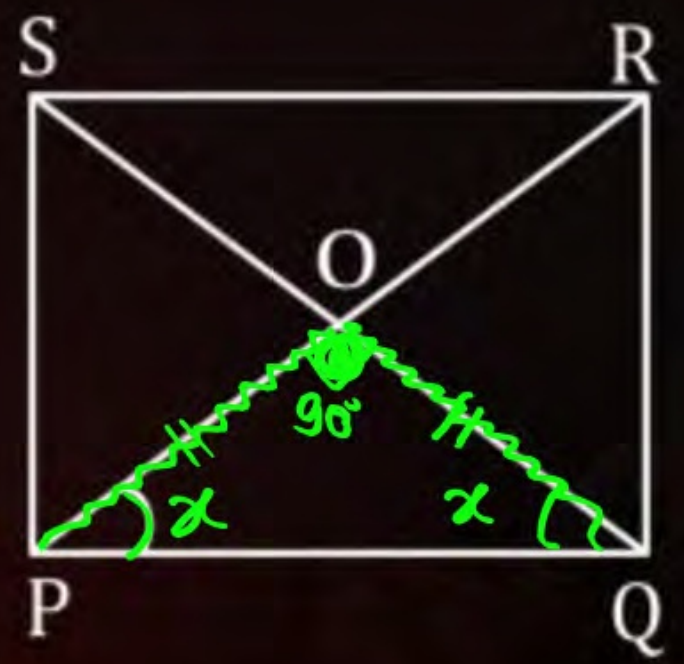
C 60°

D Cannot determined

$x + x + 90^\circ = 180^\circ$

$2x = 90^\circ$

$\boxed{x = 45^\circ}$



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

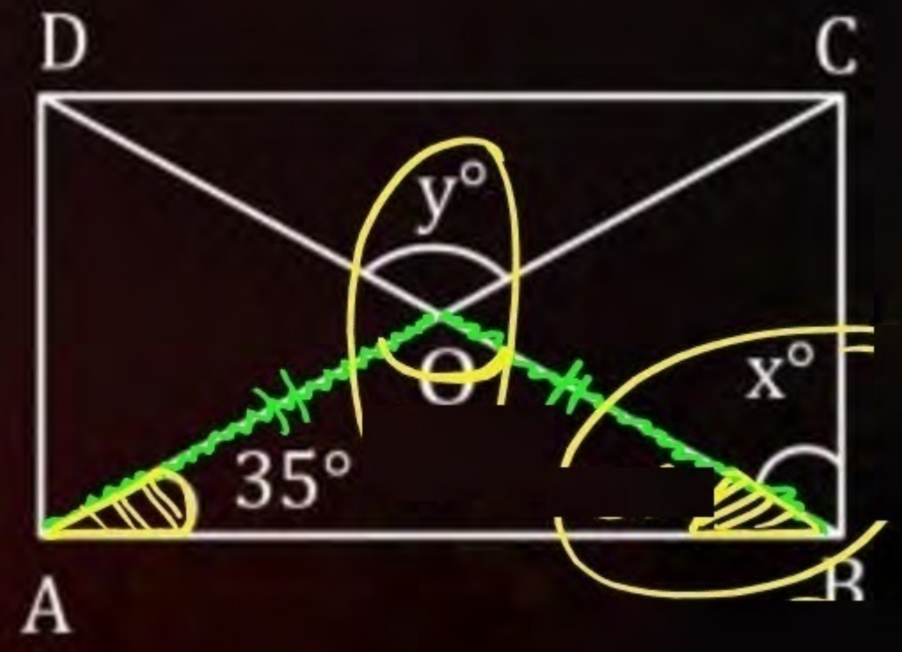
In the given rectangle ABCD, find the value of x and y.

$x = 110°, y = 55°$

$x = 55°, y = 135°$

$x = 70°, y = 55°$

$x = 55°, y = 110°$



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

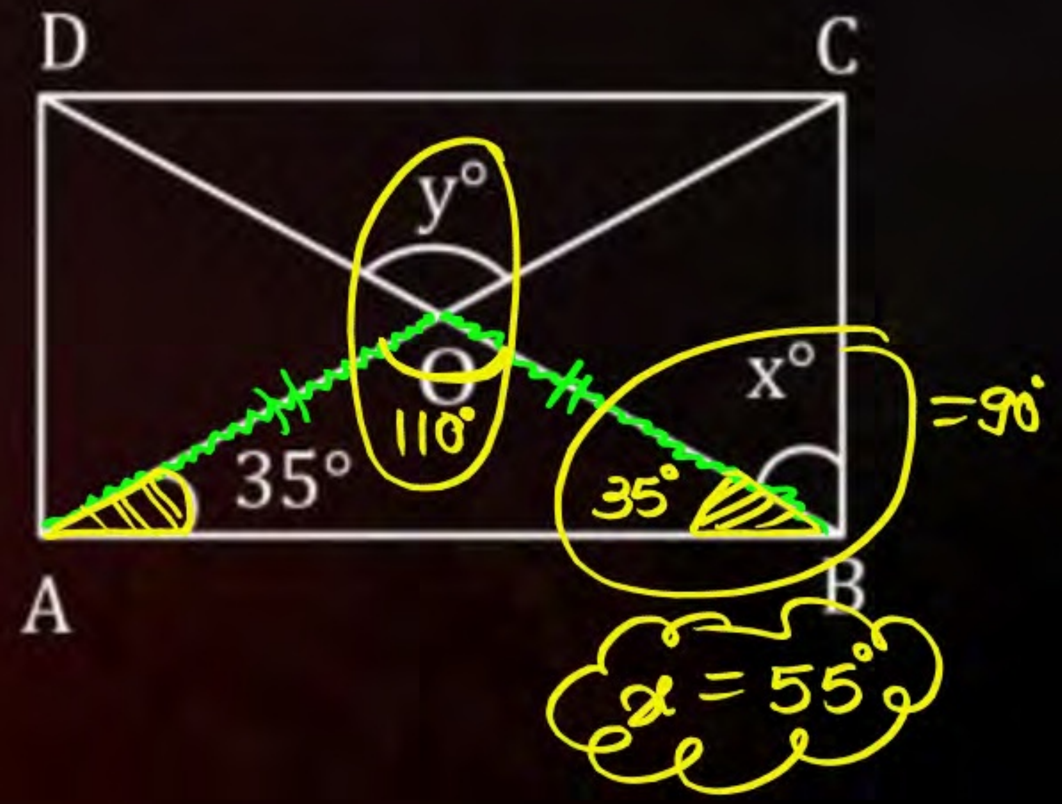
In the given rectangle ABCD, find the value of x and y.

A. $x = 110°, y = 55°$

B. $x = 55°, y = 135°$

C. $x = 70°, y = 55°$

D. $\boxed{x = 55°, y = 110°}$ ✓



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Assertion and Reason Type Problem

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R).

Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

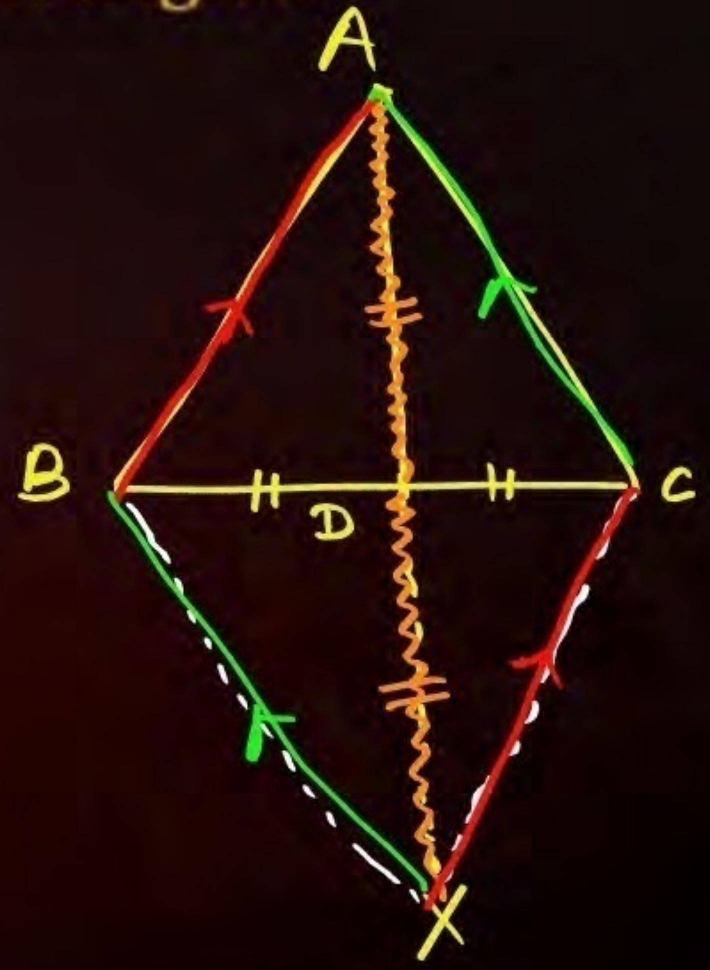
(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Question

Assertion: In ΔABC, median AD is produced to X such that AD = DX. Then ABXC is a parallelogram.

Reason: Diagonals AX and BC bisect each other at right angles.



Question

Assertion: In ΔABC, median AD is produced to X such that AD = DX. Then ABXC is a parallelogram. → True

Reason: Diagonals AX and BC bisect each other at right angles. → False

$\boxed{\Delta ADB \cong \Delta XDC}$ → S-A-S

By CPCT, $\boxed{\angle CXD = \angle BAD}$

→ AB ∥ CX ... (I)

$\boxed{\Delta ADC \cong \Delta XDB}$ → S-A-S

$\boxed{\angle CAD = \angle BXD}$

→ $\boxed{AC \parallel BX}$ ... (II)

From (I) & (II)

□ABXC → ∥gm

$\boxed{\text{option (c)}}$

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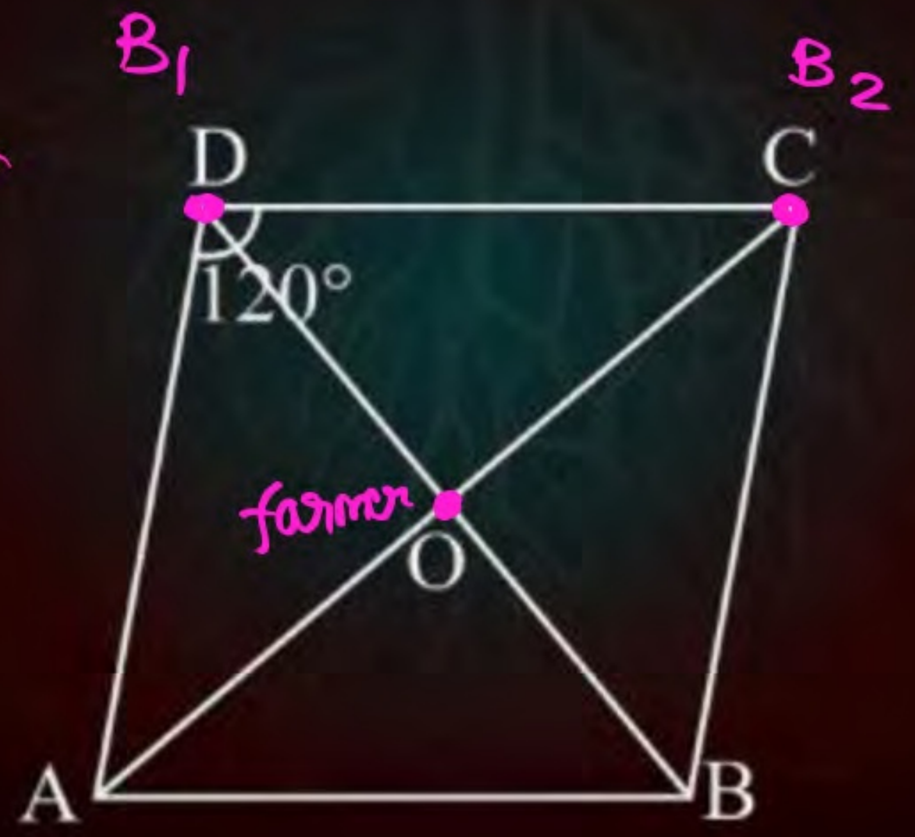
Cased-Based Type Questions

A farmer has a field namely ABCD in the shape of rhombus in which $\angle ADC = 120°$. He has two buffaloes and tied them at C and D and he himself stand at O.



Question

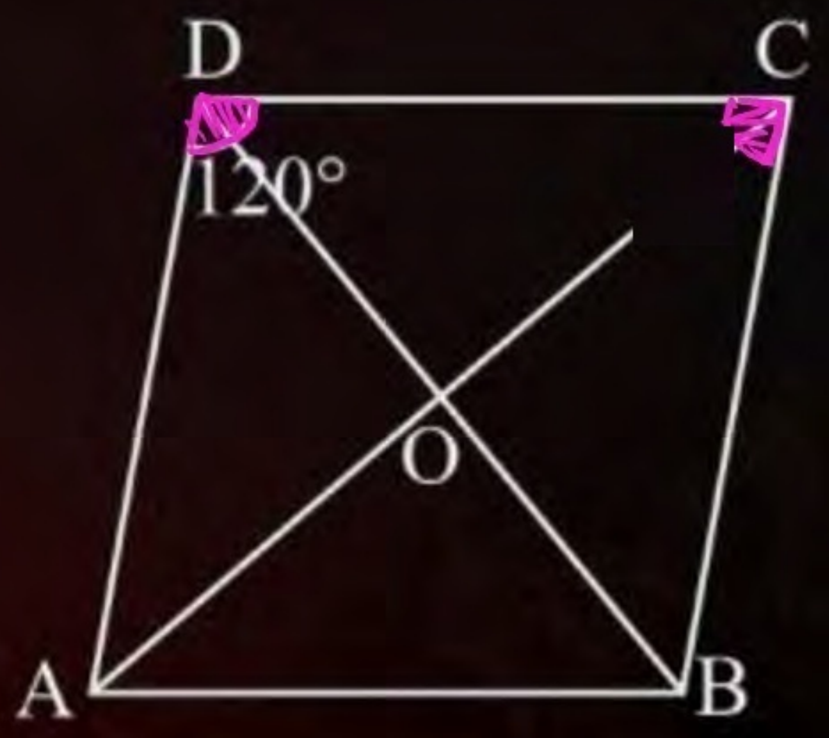
The measure of $\angle DCB$ is

$120°$

$60°$

$30°$

$240°$



Question

The measure of $\angle DCB$ is

A $120°$

B $60°$

C $30°$

D $240°$

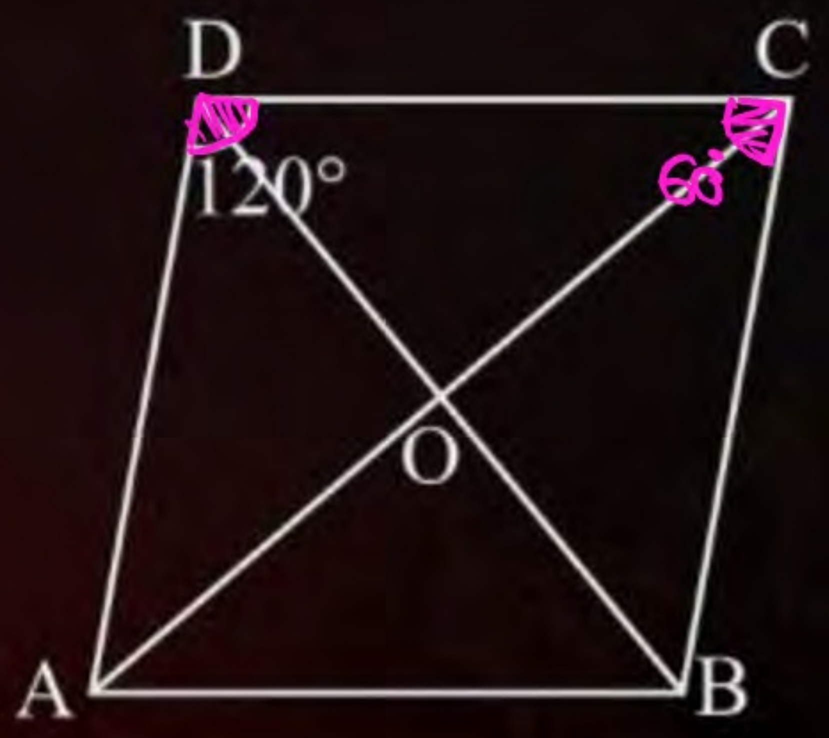
since $\square$ ABCD is a Rhombus

$\boxed{AD \parallel BC}$

sum of adjacent $= 180°$

$120° + \angle DCB = 180°$

$\boxed{\angle DCB = 60°}$



Question

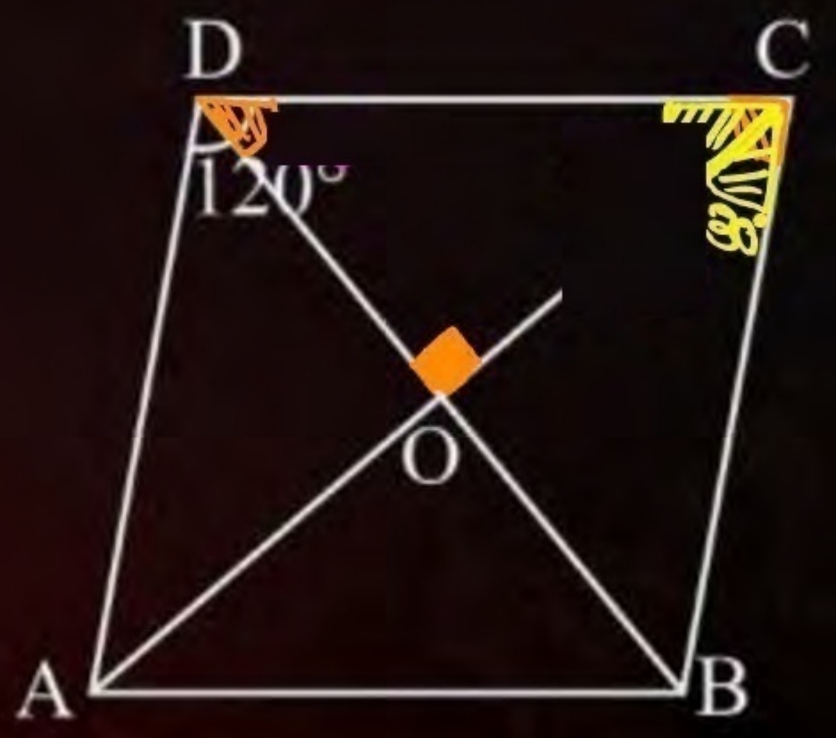
The measure of $\angle CDO$ and $\angle DCO$ are _ and _ respectively.

$120°, 60°$

$30°, 60°$

$60°, 30°$

$60°, 120°$



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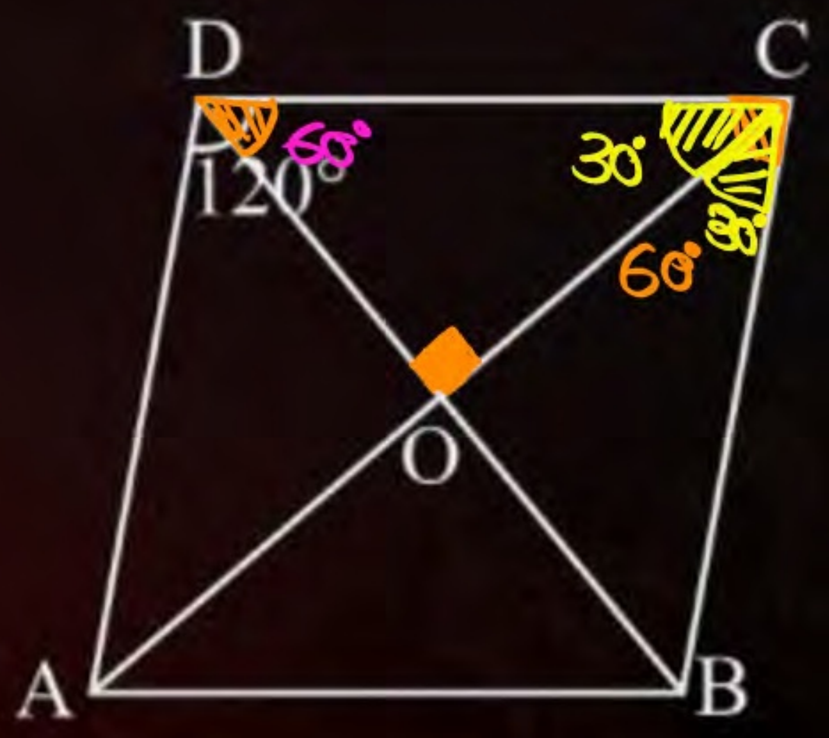
Question

The measure of ∠CDO and ∠DCO are $\boxed{60^\circ}$ and $\boxed{30^\circ}$ respectively.

- **A** 120°, 60°
- **B** 30°, 60°
- **C** 60°, 30°
- **D** 60°, 120°

since AC is longest diagonal

AC bisect ∠C



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

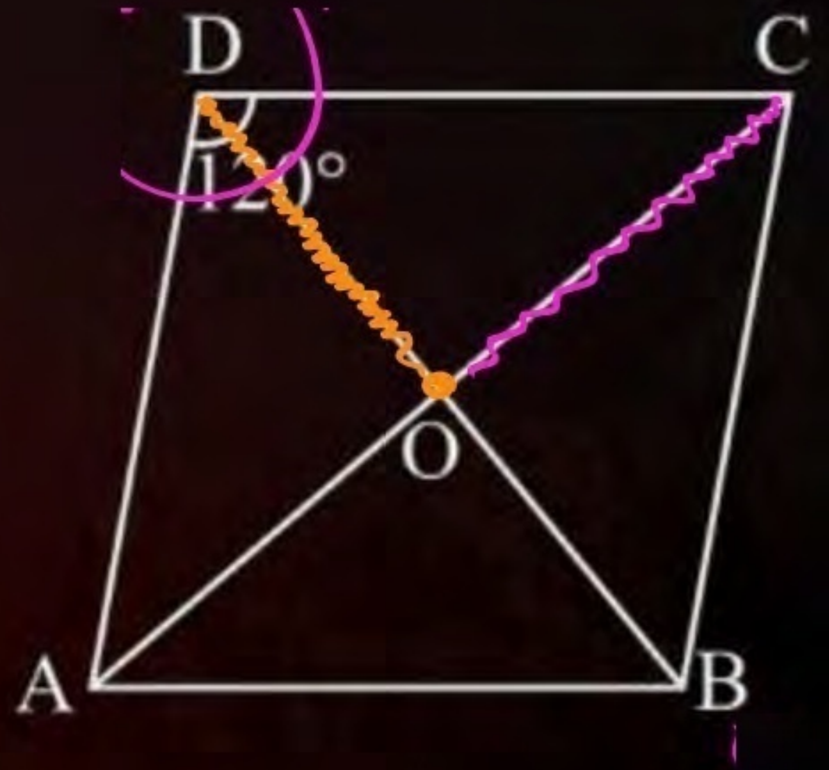
Out of two buffaloes, which one can reach the farmer early? [Assuming they both walk at same speed

Buffalo at C

Buffalo at D

Both will reach at same time

Can't Say



Question

Out of two buffaloes, which one can reach the farmer early? [Assuming they both walk at same speed

$DB < AC \Rightarrow \frac{DB}{2} < \frac{AC}{2}$

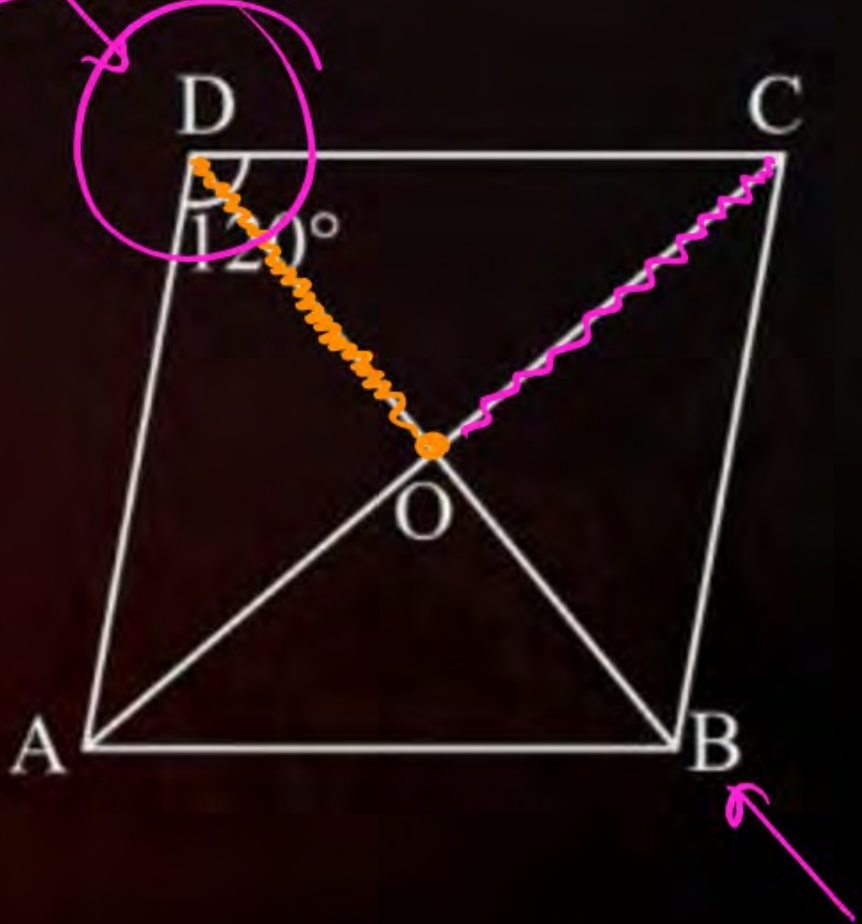
$\boxed{OD < OC}$

A Buffalo at C

$\boxed{\textbf{B}\ \text{Buffalo at D}}$ ✓

C Both will reach at same time

D Can't Say



Question

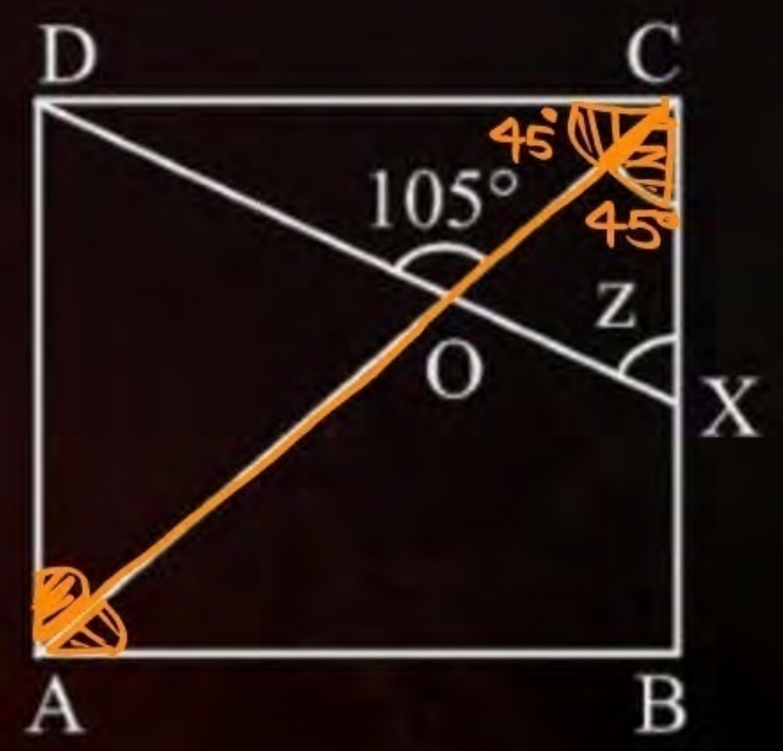
In the given figure, if ABCD is a square, then the value of z is

45°

60°

70°

95°



Question

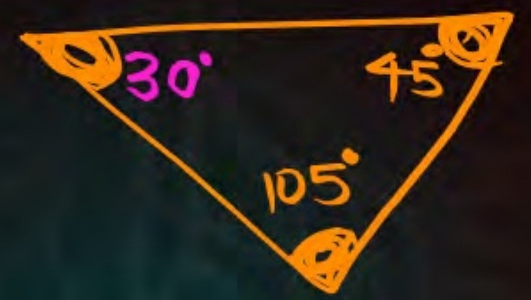
In the given figure, if ABCD is a square, then the value of z is

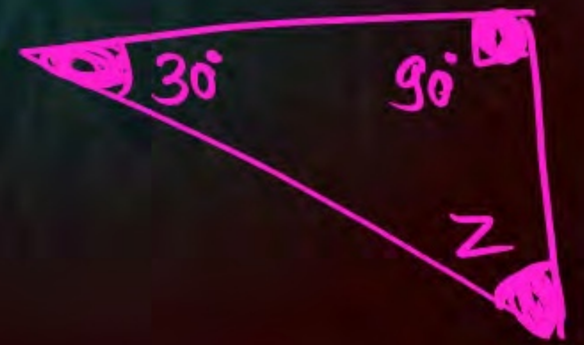
A 45°

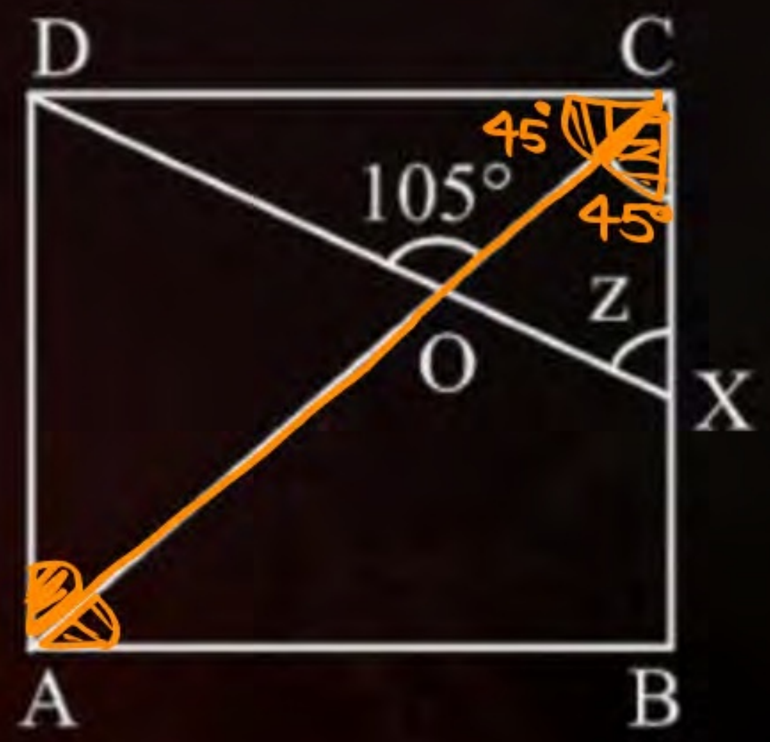
B 60°

C 70°

D 95°







Question

In the given figure, lines are drawn through A, B and C parallel to the sides BC, CA and AB respectively to form ΔPQR. Show that $BC = \frac{1}{2} QR$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

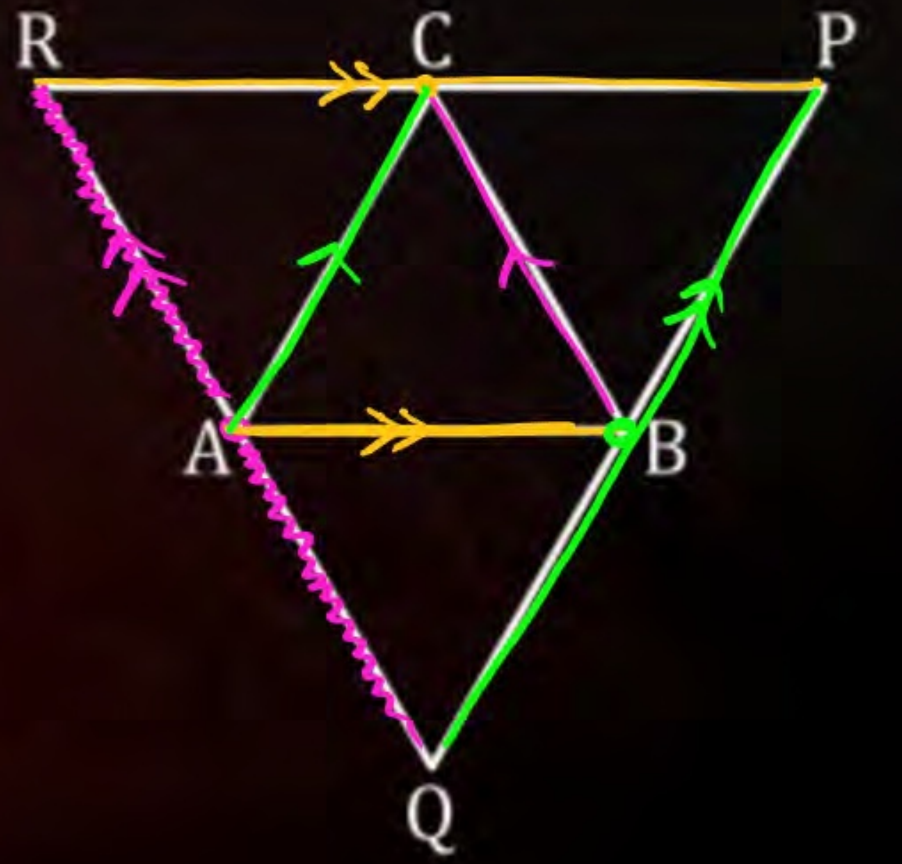
Question

In the given figure, lines are drawn through A, B and C parallel to the sides BC, CA and AB respectively to form ΔPQR. Show that $BC = \frac{1}{2} QR$

From point A, $RQ \parallel BC \Rightarrow \boxed{RA \parallel BC} \Rightarrow \boxed{AQ \parallel BC}$ ✓

From point C, $RP \parallel AB \Rightarrow \boxed{RC \parallel AB} \Rightarrow \boxed{CP \parallel AB}$ ✗

From point B, $PQ \parallel AC \Rightarrow \boxed{BQ \parallel AC} \Rightarrow \boxed{BP \parallel AC}$ ✗



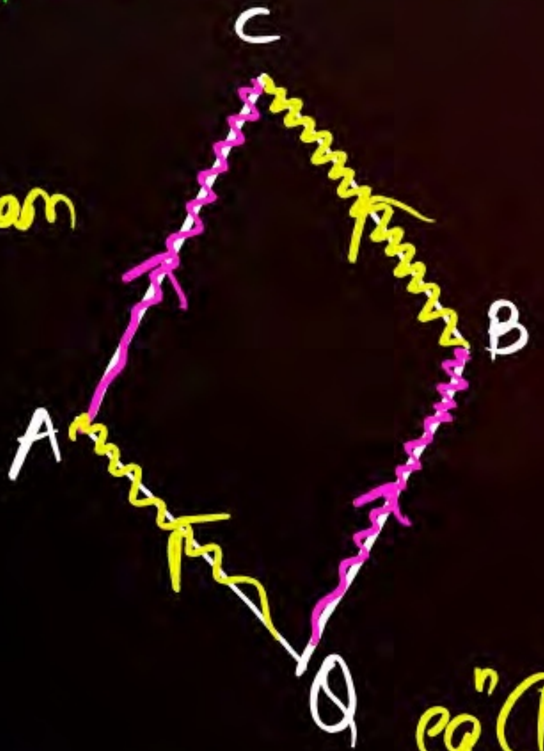
$\square ABCR \rightarrow \parallel^{gram}$

$\boxed{BC = AR}$ ---- (1)

If

$\square AQBC \rightarrow \parallel^{gram}$

$\boxed{BC = AQ}$ $\rightarrow$ (2)



eqⁿ (1) + (11) $2BC = RQ \rightarrow \boxed{BC = \frac{1}{2} QR}$ Hence, proved!!

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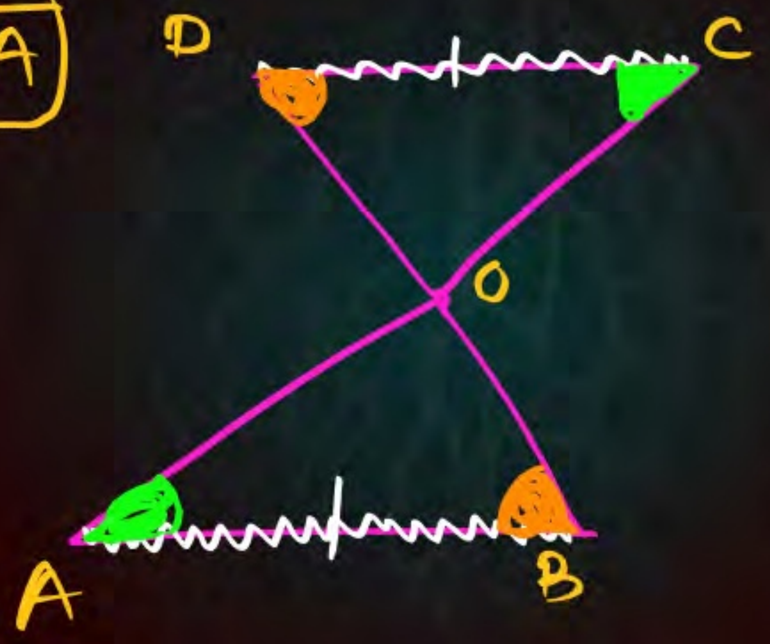
Theorem - 6

Statement: The diagonal of a parallelogram bisect each other.

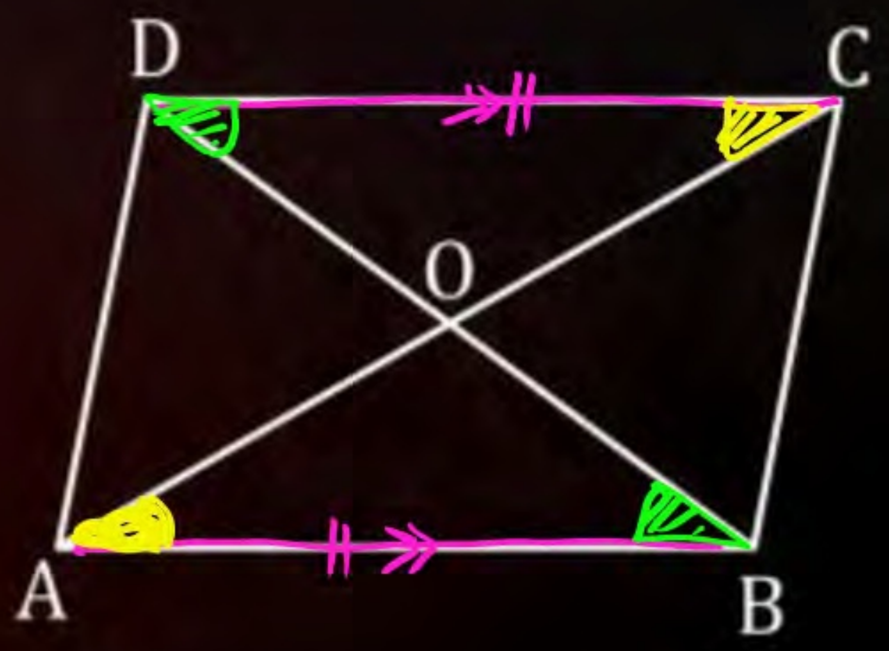
$\boxed{A-S-A}$ $\boxed{\Delta ODC \cong \Delta OBA}$



By CPCT,

$$\boxed{\begin{array}{c} OB = OD \\ \hline OA = OC \end{array}}$$

↳ Diagonal bisect each other



Theorem - 7

Statement (Converse of Theorem 6): If the diagonals of a quadrilateral bisects each other, then the quadrilateral is parallelogram.

$\boxed{S-A-S}$

$\boxed{\triangle ODC \cong \triangle OBA}$

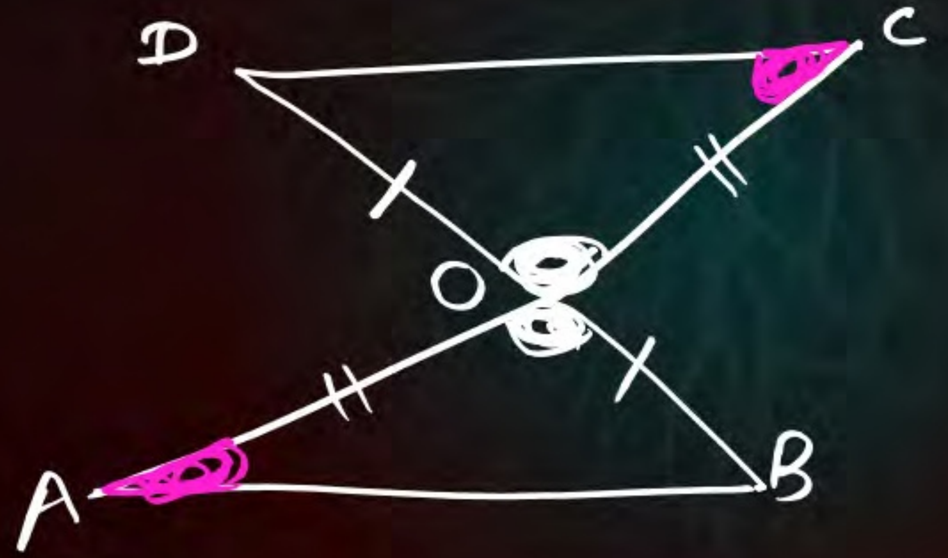
By CPCT

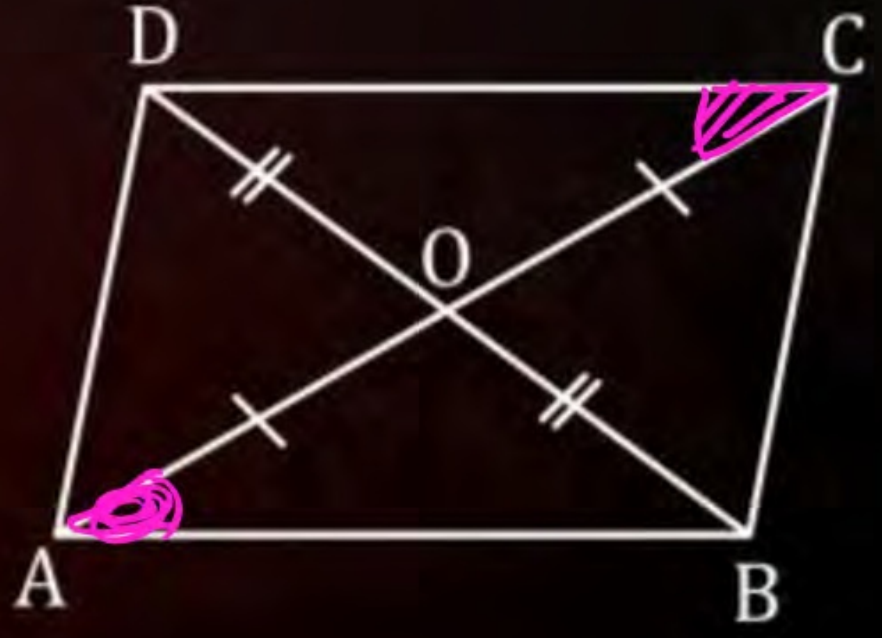
$AB \parallel CD$

$\boxed{\angle OAB = \angle OCD} \rightarrow CD \parallel AB$

similarily $\rightarrow AD \parallel BC$

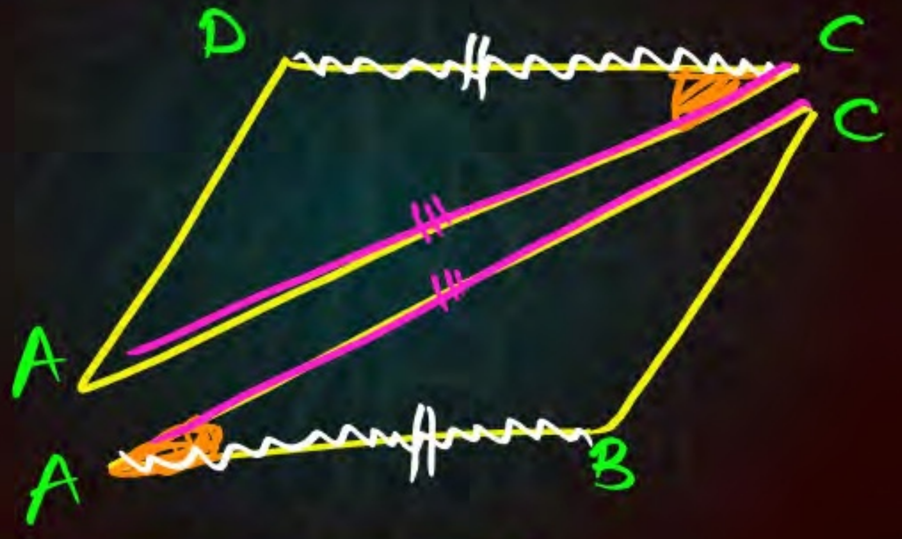
$\rightarrow \square ABCD \rightarrow \parallel^{gm}$

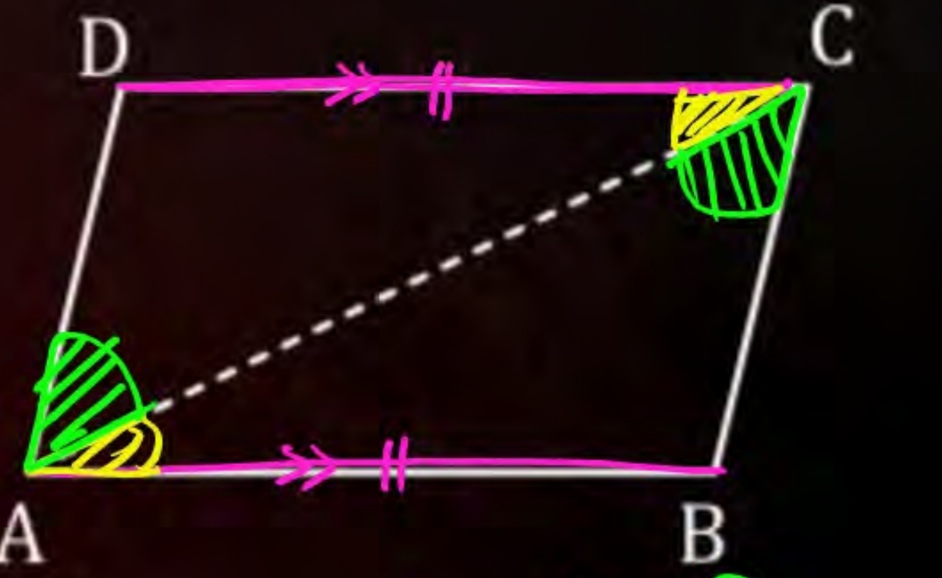




Theorem - 8

Statement : A quadrilateral is a parallelogram, if its one pair of opposite sides is equal and parallel. → Given $\boxed{CD \parallel AB}$ — ①





$\boxed{S-A-S} \rightarrow \boxed{\triangle DCA \cong \triangle BAC}$

By CPCT,

$CB = AD$

$\boxed{\angle BCA = \angle DAC} \rightarrow \boxed{AD \parallel BC} \rightarrow$ ②

From ① & ②

$\square ABCD \rightarrow \parallel$gram

Properties of Special Parallelogram

Rectangle:

Diagonals of a rectangle are equal.

If diagonals of a parallelogram are equal, then the parallelogram is a rectangle.

Rhombus:

Diagonal of a rhombus are perpendicular to each other.

If diagonals of a parallelogram are perpendicular to each other, the parallelogram is a rhombus.

Properties of Special Parallelogram

Square:

Diagonal of a square are equal and perpendicular to each other .

If diagonals of a parallelogram are equal and intersect each other at right angles, then the parallelogram is a square.

Quadrilateral
Polygon with four sides

Parallelogram
Both pair of opposite sides are parallel and equal.
Diagonals bisect each other
Both pairs of opposite angles are equal
Consecutive angle are supplementary

Kite
- Both pairs of consecutive sides are equals.
- Opposite sides are not equal.

Trapezium
- One pair of opposite sides are parallel.

Rectangle
All the properties of parallelogram
Has four right angles.
Diagonals are equal

Rhombus
- All properties of parallelogram
- All sides are equal
- Diagonals bisects each other at right angle.

Kite
- Both pairs of consecutive sides are equal
- Opposite sides are not equal

Isosceles Trapezium
All the properties
Non parallel sides are equal
Diagonals are equal
Base angles are equals.

Square
All the properties of parallelogram
All the properties of Rectangle
All the properties of rhombus

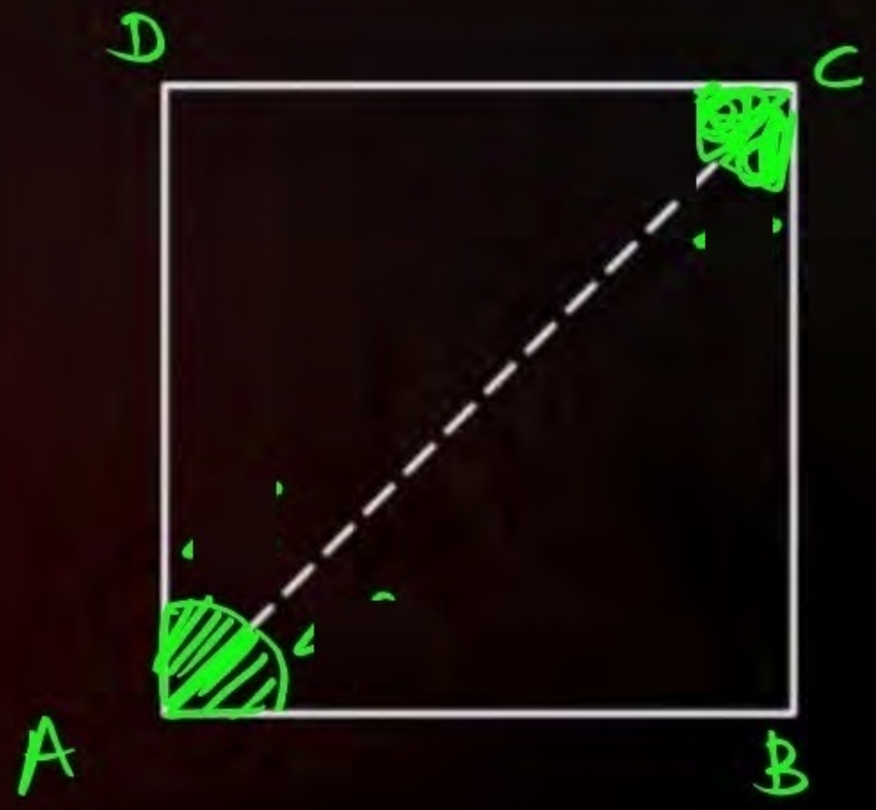
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that

(i) ABCD is a square

(ii) Diagonal BD bisects $\angle B$ as well as $\angle D$.



Question

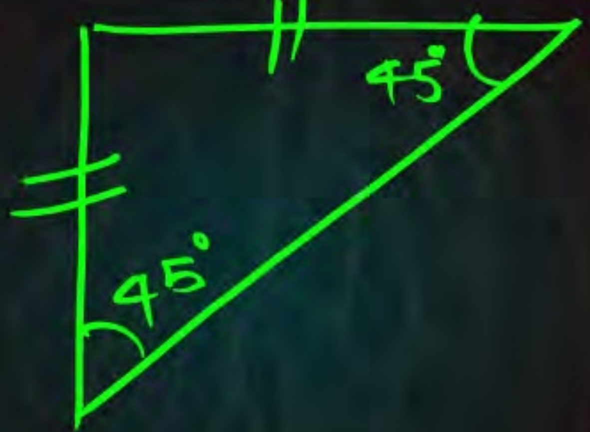
ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that

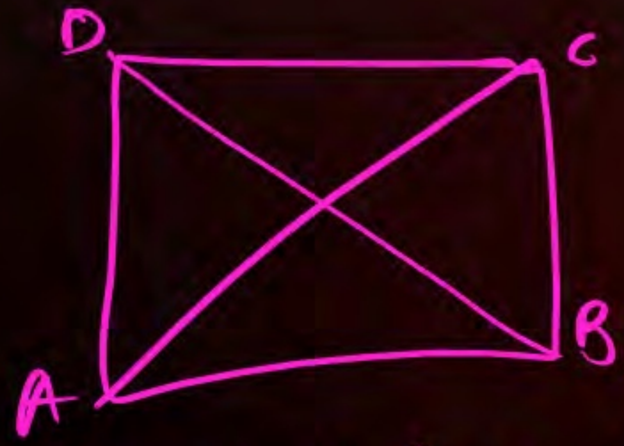
(i) ABCD is a square

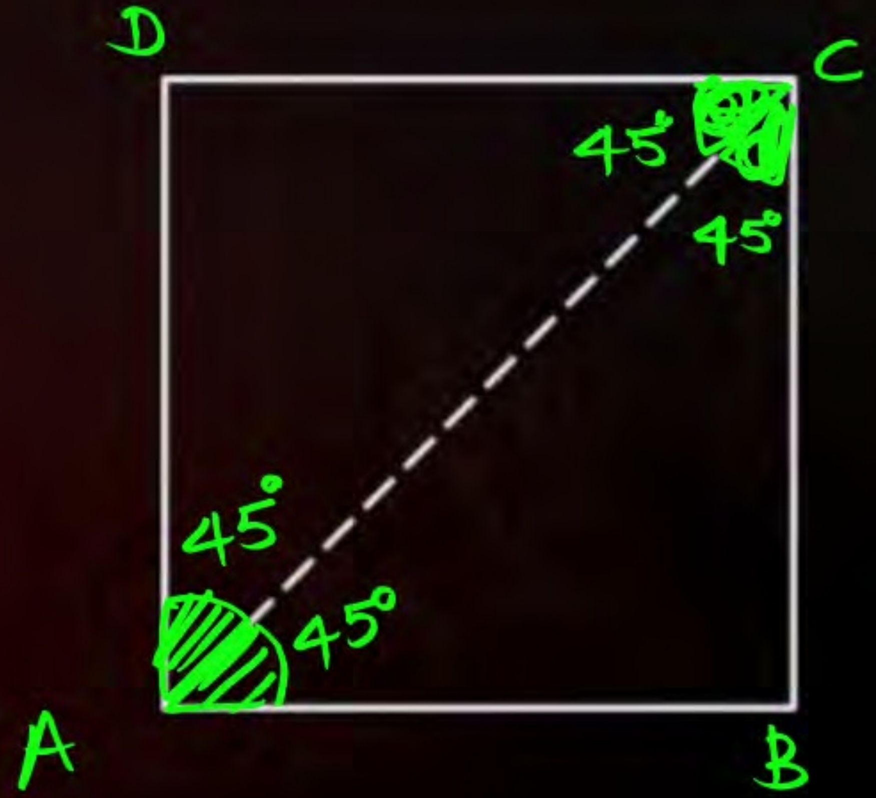
(ii) Diagonal BD bisects ∠B as well as ∠D.

$\boxed{BC = AD = CD = AB}$

↳ □ABCD → square







since □ABCD is a square, therefore BD will bisect ∠B & ∠D

Question

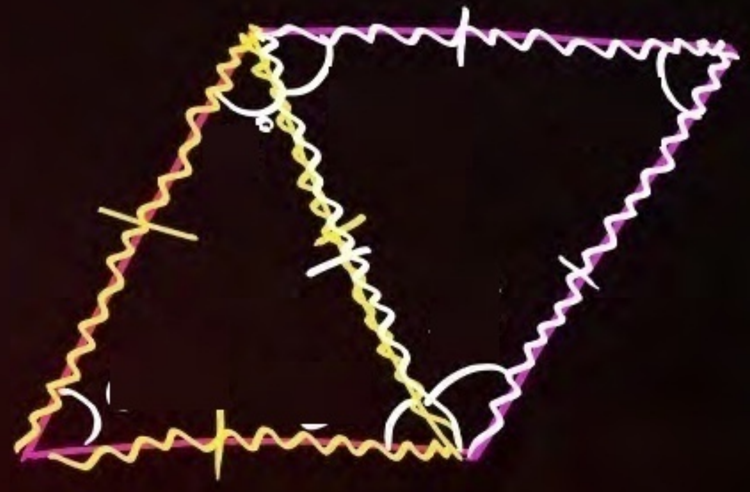
One of the diagonals of a rhombus is equal to a side of the rhombus. The distinct angle of the rhombus are

$70°$ and $110°$

$60°$ and $120°$

$80°$ and $100°$

$120°$ and $240°$



Question

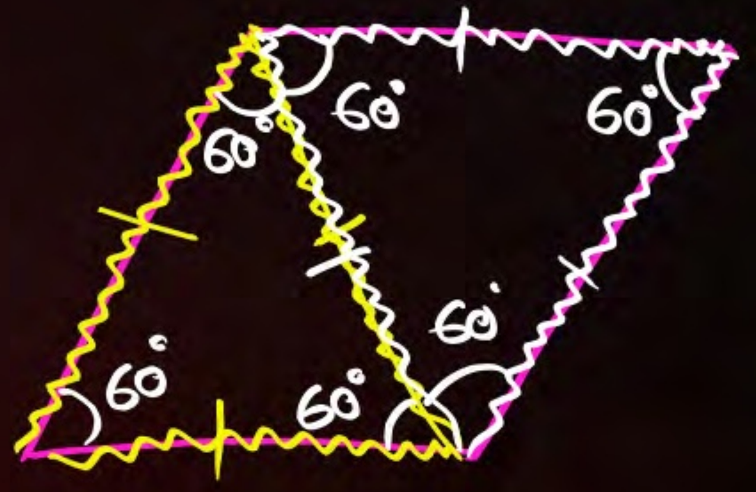
One of the diagonals of a rhombus is equal to a side of the rhombus. The distinct angle of the rhombus are

A. $70°$ and $110°$

B. $\boxed{60° \text{ and } 120°}$ ✓

C. $80°$ and $100°$

D. $120°$ and $240°$



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Question

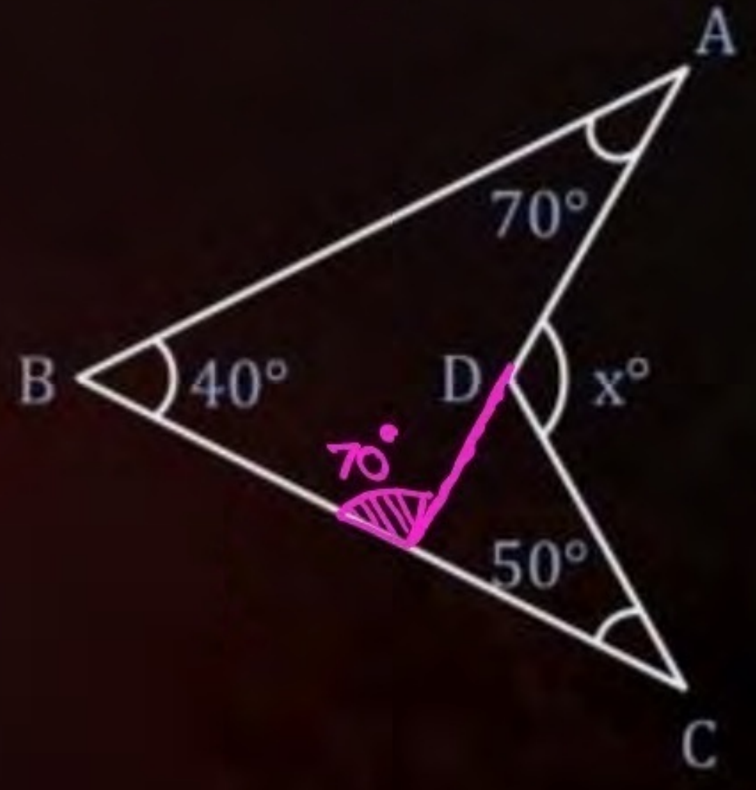
In the given figure, the value of x is

60°

80°

160°

180°



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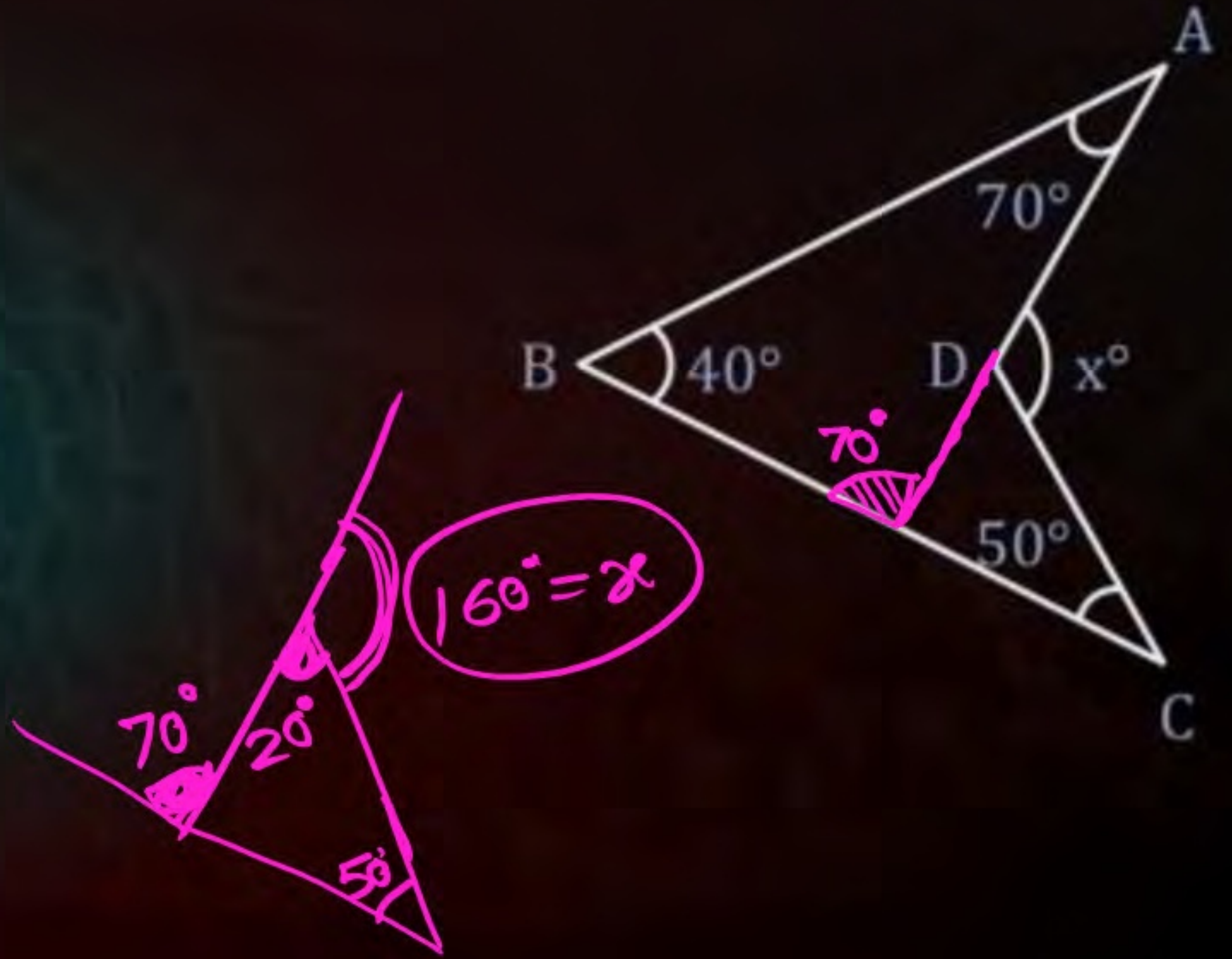
Question

In the given figure, the value of x is



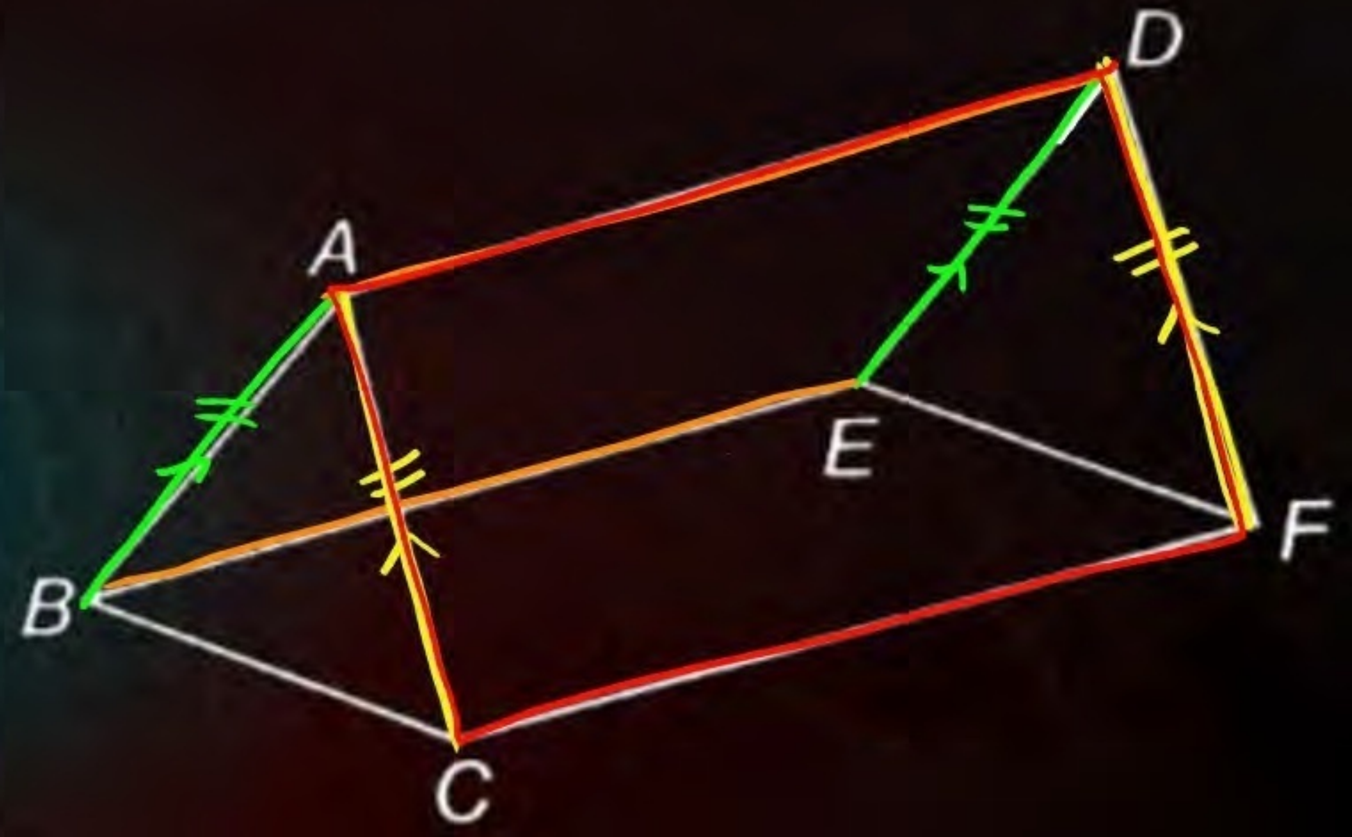
- **A** 60°
- **B** 80°
- **C** 160°
- **D** 180°

Important result obtained from Parallelogram

In a quadrilateral, if any pair of side is equal and parallel then it must be parallelogram only (Square and Rhombus are special type of parallelogram) because it will automatically makes the other pair of opposite sides equal and parallel.

Question

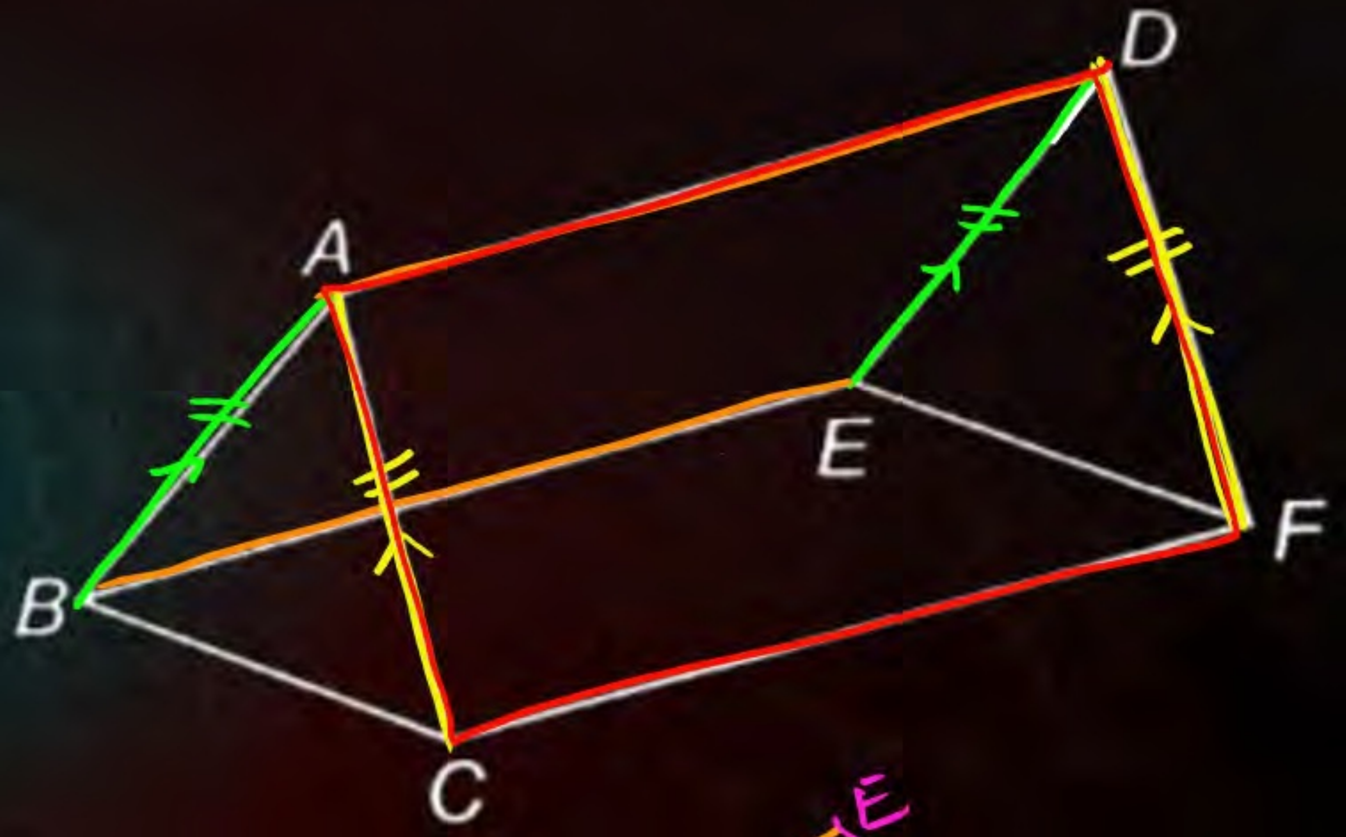
In Fig., AB || DE, AB = DE, AC || DF and AC = DF. Prove that BC || EF and BC = EF.



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In Fig., AB || DE, AB = DE, AC || DF and AC = DF. Prove that BC || EF and BC = EF.



Since, AB || DE & AD = DE

Therefore, □ABED must be a ||gram

$\boxed{AD = BE \ \& \ AD \parallel BE}$ —— ①

Also, AC || DF & AC = DF

Therefore, □ACFD must be a ||gram

$\boxed{AD = CF \ \& \ AD \parallel CF}$ —— ②

$\boxed{BE = CF}$ CF || AD || BE ⟹ $\boxed{CF \parallel BE}$

only possible if □BCFE is a ||gram

Therefore, $\boxed{BC \parallel EF, \ BC = EF}$

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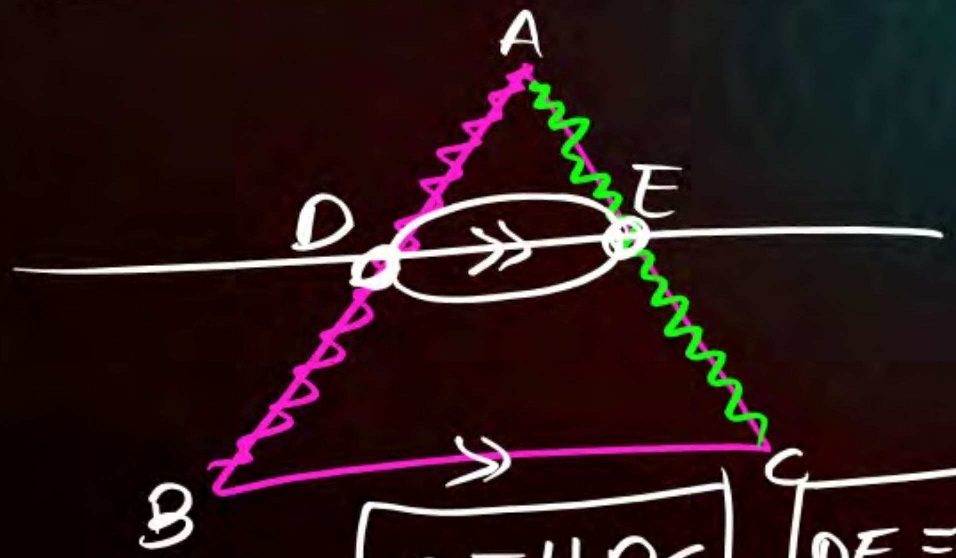
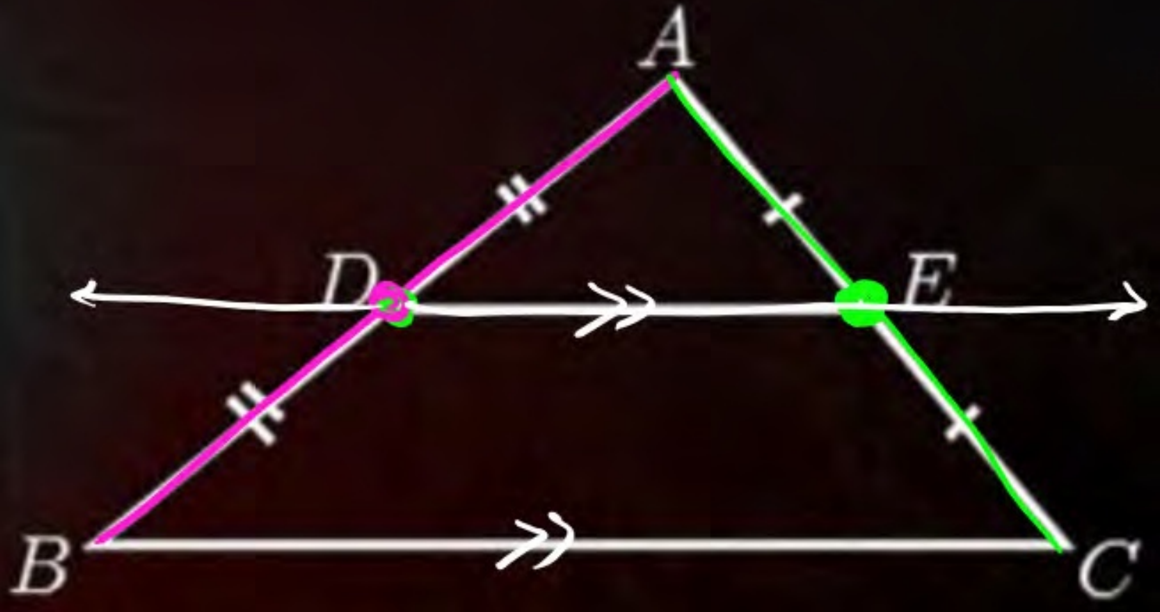
Mid-Point theorem

Theorem : The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

$DE \parallel BC$ & $DE = \frac{1}{2}BC$



$\boxed{DE \parallel BC}$, $\boxed{DE = \frac{1}{2}BC}$



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Proof of Mid-Point theorem

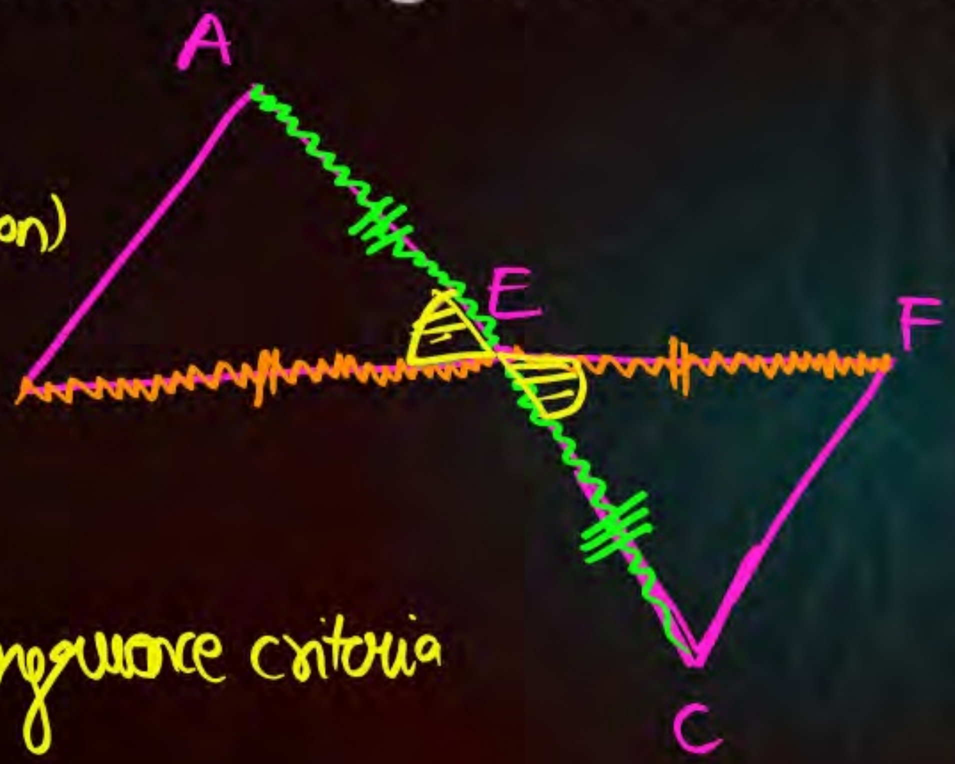
Construction: Produce line segment DE to F such that $\boxed{DE = EF}$. Join FC.

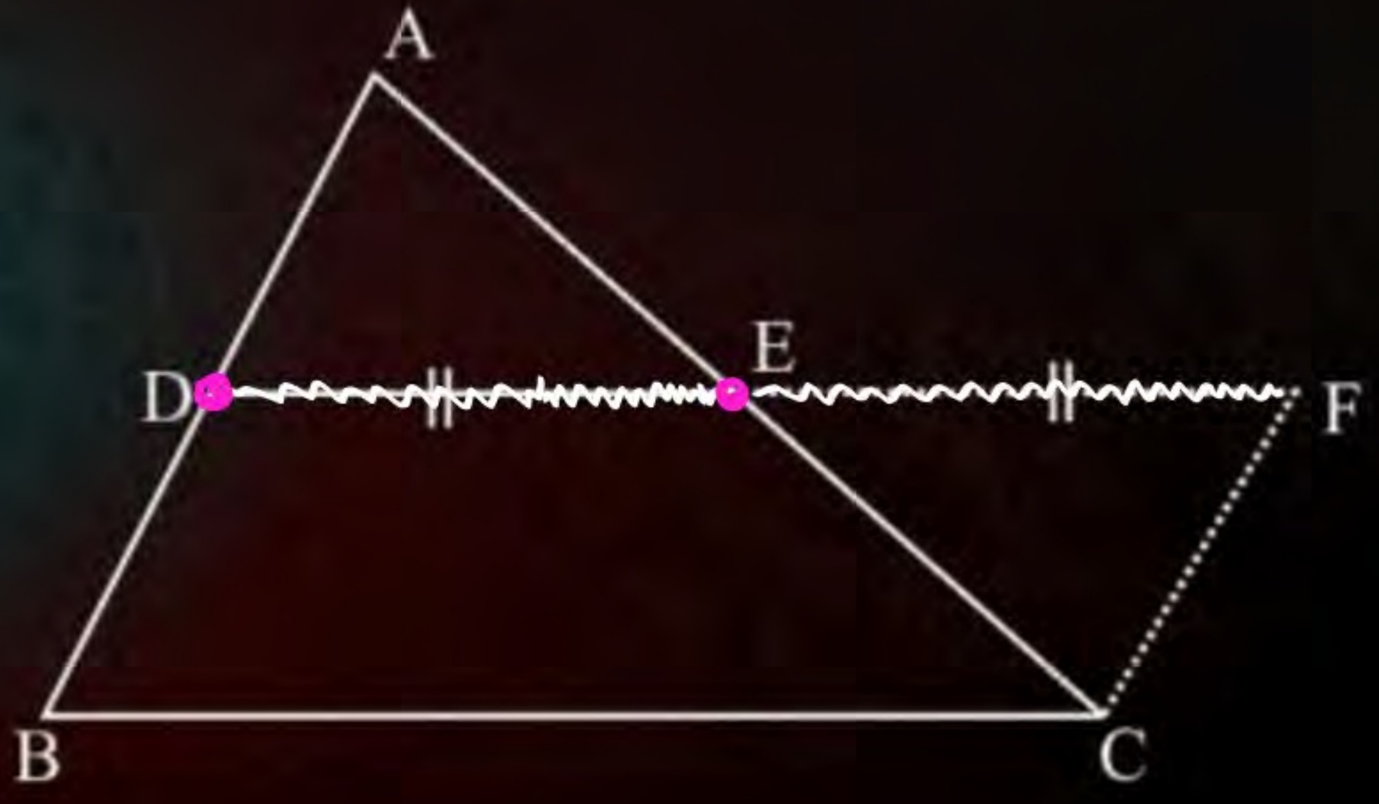
In $\triangle ADE$ & $\triangle CFE$

$DE = EF$ (By construction)

$\angle AED = \angle CEF$ (V.O.)

$AC = EC$ (E is mid pt of AC)





By using $\boxed{S-A-S}$ congruence criteria

$\boxed{\triangle AED \cong \triangle CEF}$

By CPCT, $\boxed{AD = CF}$ & $\boxed{\angle ADE = \angle CFE}$

$\Downarrow$

$\boxed{AD \parallel CF}$

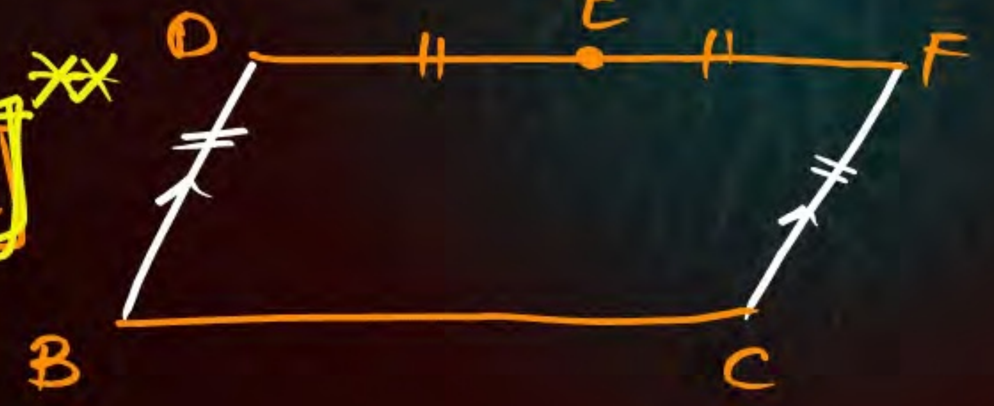
since D is mid point

$AD = CF$ & $AD \parallel CF$

$\boxed{BD = CF}$ & $AB \parallel CF \Rightarrow \boxed{DB \parallel CF}$

$\boxed{\square BDFC} \rightarrow \parallel$gram.

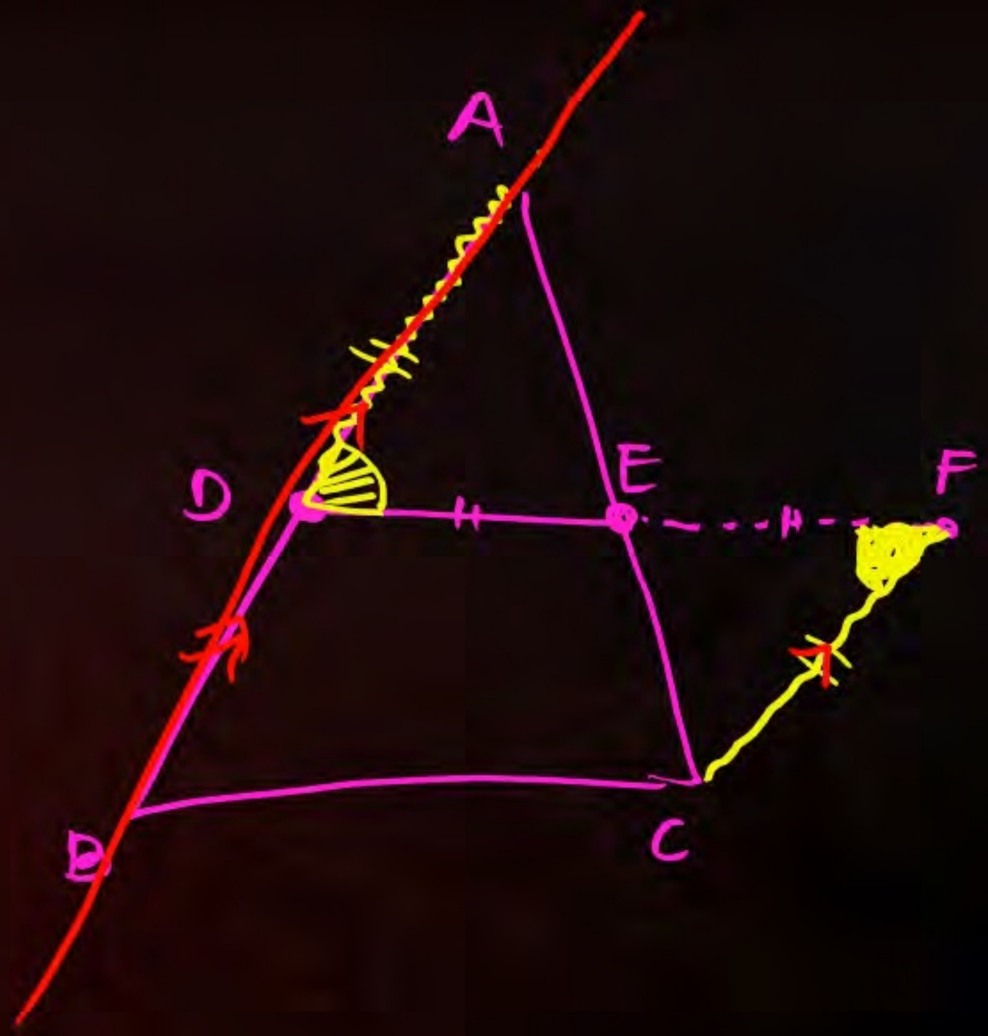
$DF \parallel BC$ or $\boxed{DE \parallel BC}$ **



$DF = BC$

$2DE = BC$

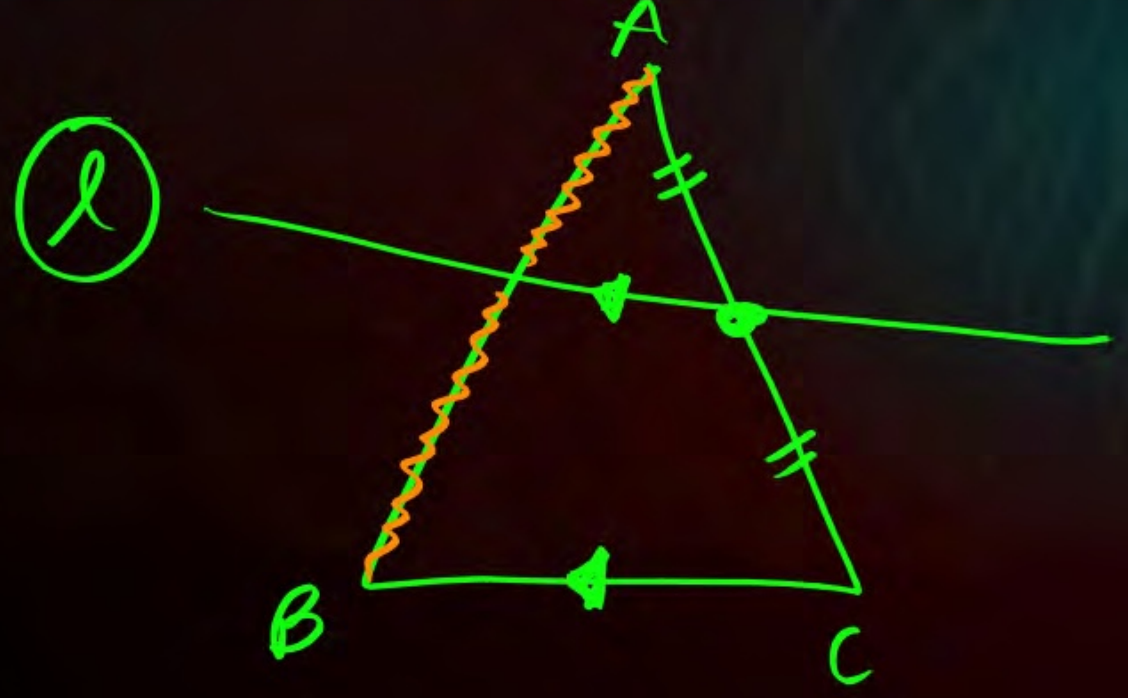
$\boxed{DE = \frac{1}{2} BC}$ **

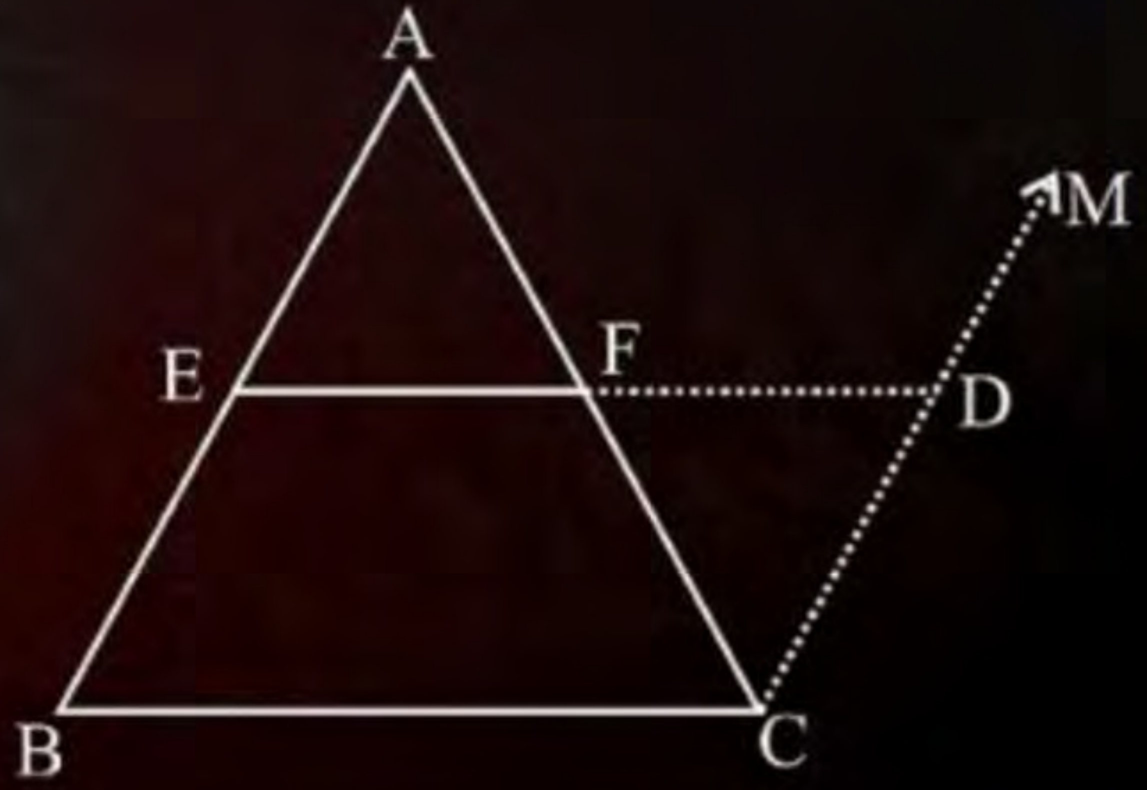


Converse of mid point theorem

Statement : The line drawn through the mid point of one side of a triangle, parallel to another side, intersects the third side at its mid-point.

Construction: Through C draw CM || AB





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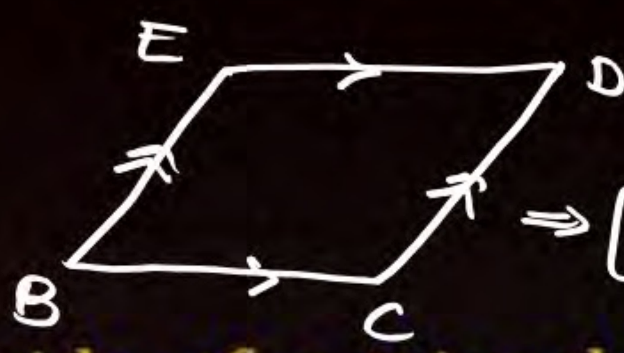
Converse of mid point theorem



$\Rightarrow \boxed{CD = EB}$

Statement : The line drawn through the mid point of one side of a triangle, parallel to another side, intersects the third side at its mid-point.

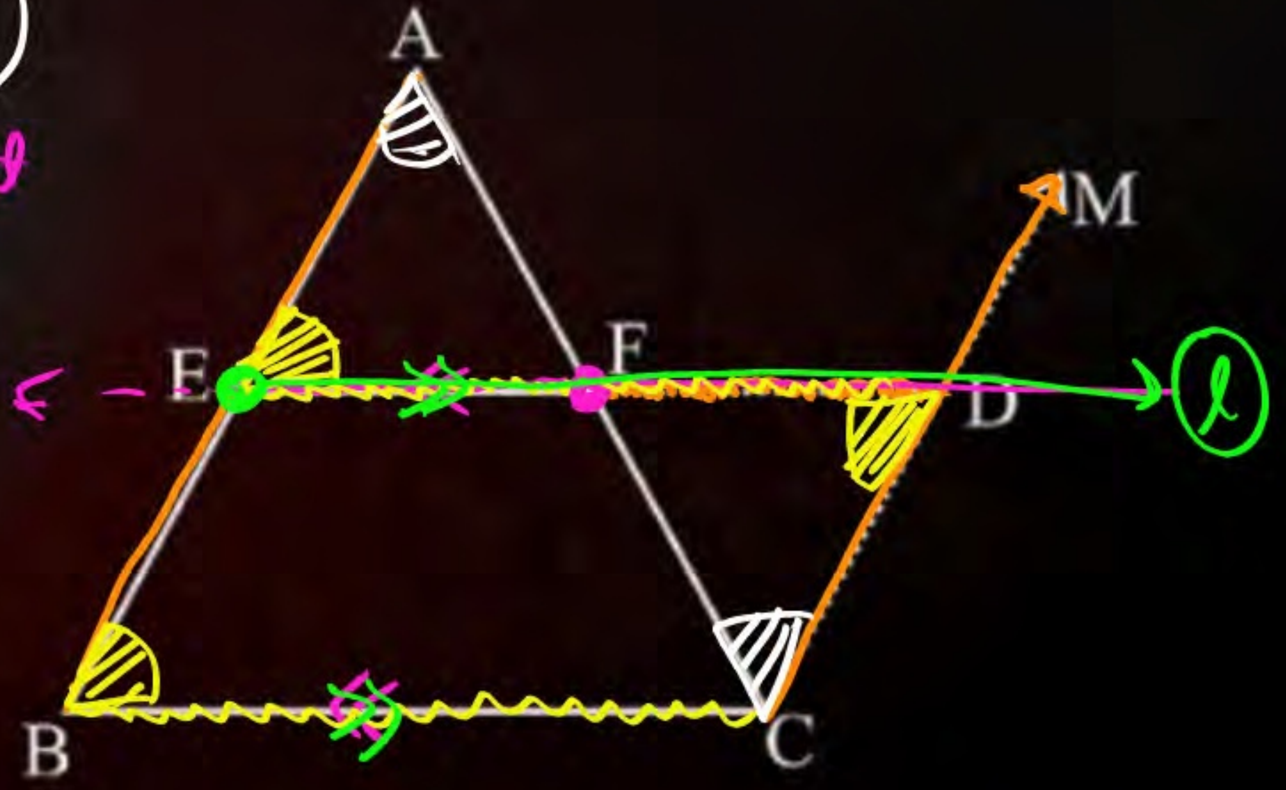
given, $EF \parallel BC \Rightarrow \boxed{ED \parallel BC}$

Construction: Through C draw CM || AB $\Rightarrow \boxed{CD \parallel EB}$

Draw a line through E such that E is mid point of AC.

Also, $\boxed{FE \parallel BC}$

Now, $ED \parallel BC$, therefore $\boxed{\angle CBA = \angle AEF}$ — (1)



$AB \parallel CM$

$\boxed{\angle AEF = \angle CDE}$ — (2)

From (1) & (2) $\boxed{\angle CBA = \angle CDE}$

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since E is mid point of AB

$EB = AC$

$\Rightarrow \boxed{CD = AE}$

ΔAEF & ΔCDF

A $\angle AEF = \angle CDF$

S $AE = CD$

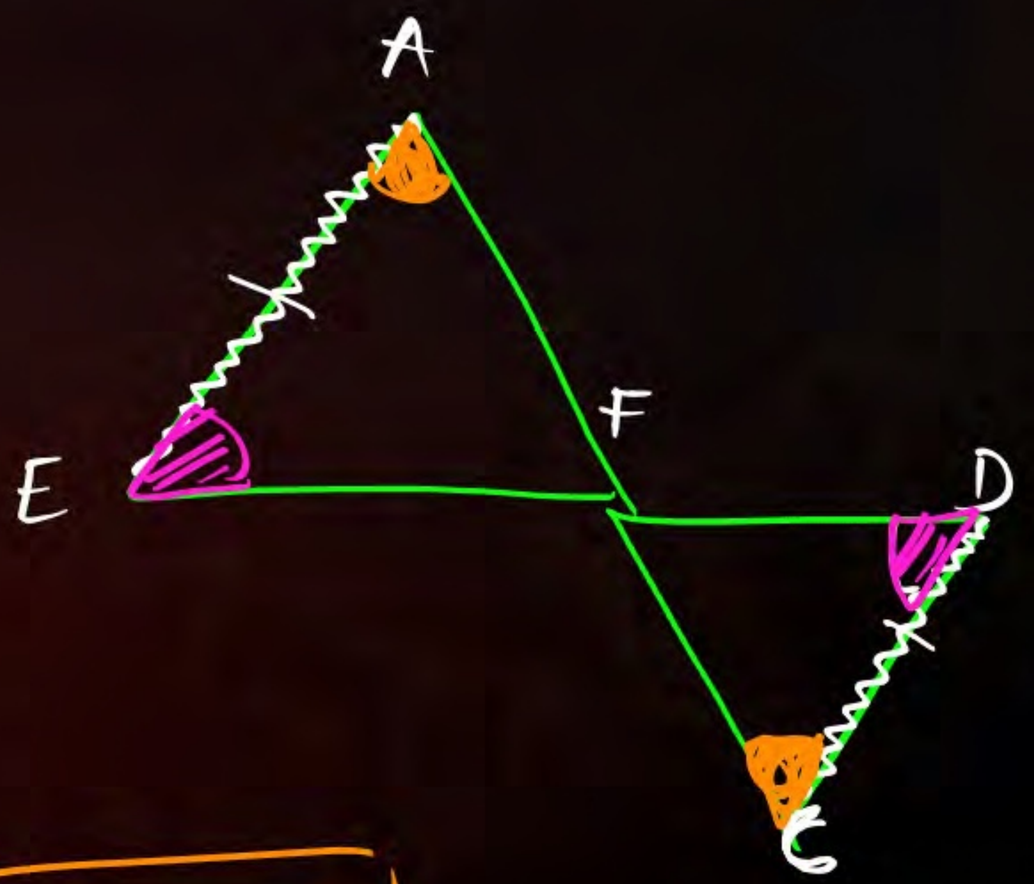
A $\angle EAF = \angle DCF$

$\boxed{\Delta AEF \cong \Delta CDF}$

By CPCT,

$\boxed{AF = FC}$

$\boxed{F \text{ is mid pt of } AC}$



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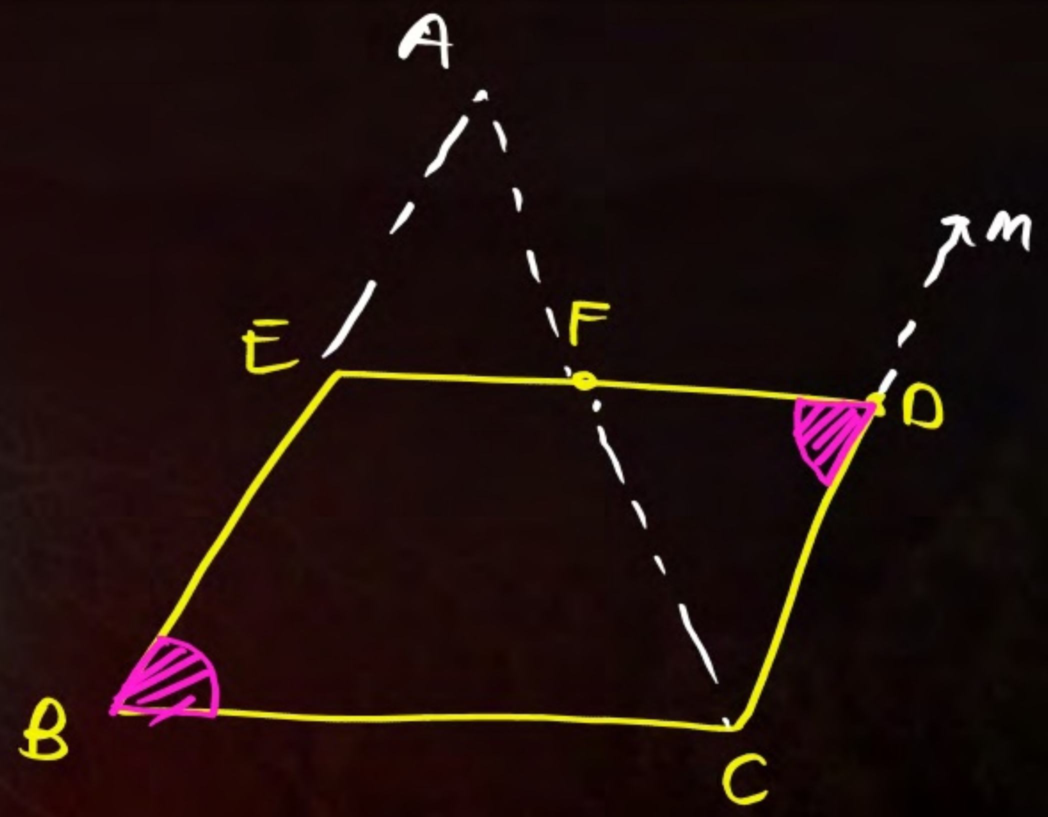
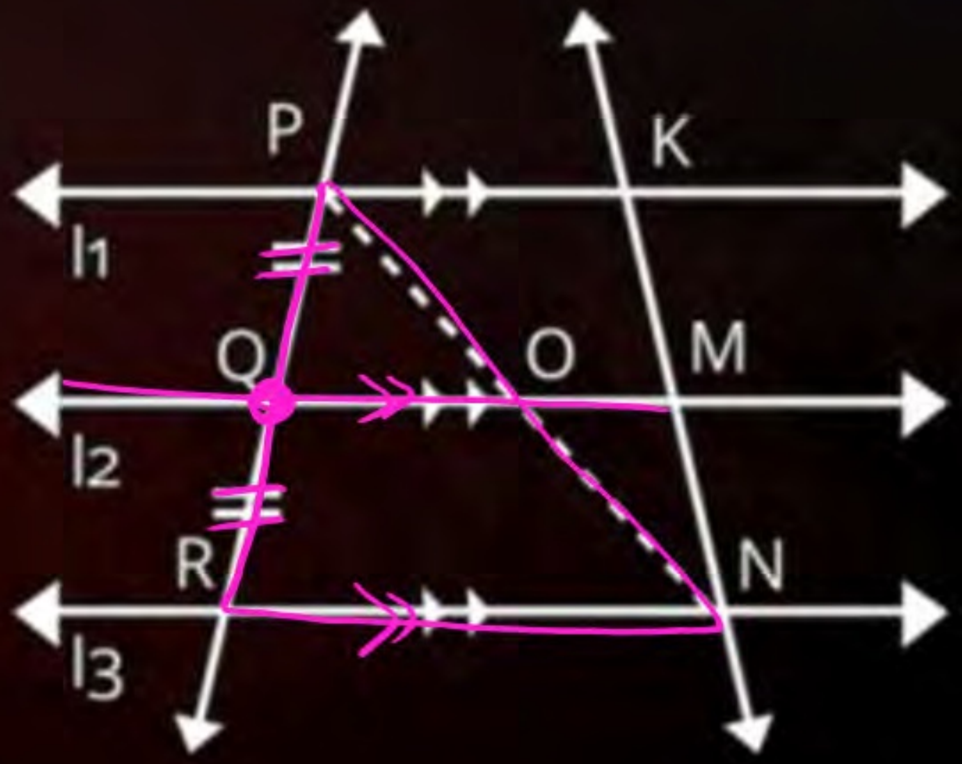

A
m
E
F
D
B
C

Equal Intercept Theorem

Theorem : If there are three or more parallel lines and the intercepts made by them on one transversal are equal then the intercepts on any other transversal are also equal.



Q is mid point of PR & QO || RN

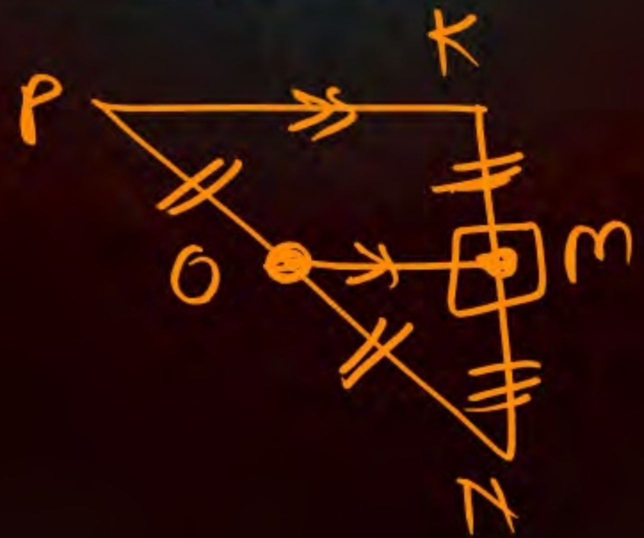
↳ converse of mid-point theorem

O must be mid-point of PN

Using converse of mid-pt theorem

$\boxed{KM = MN}$

↳ M is mid pt.



Question

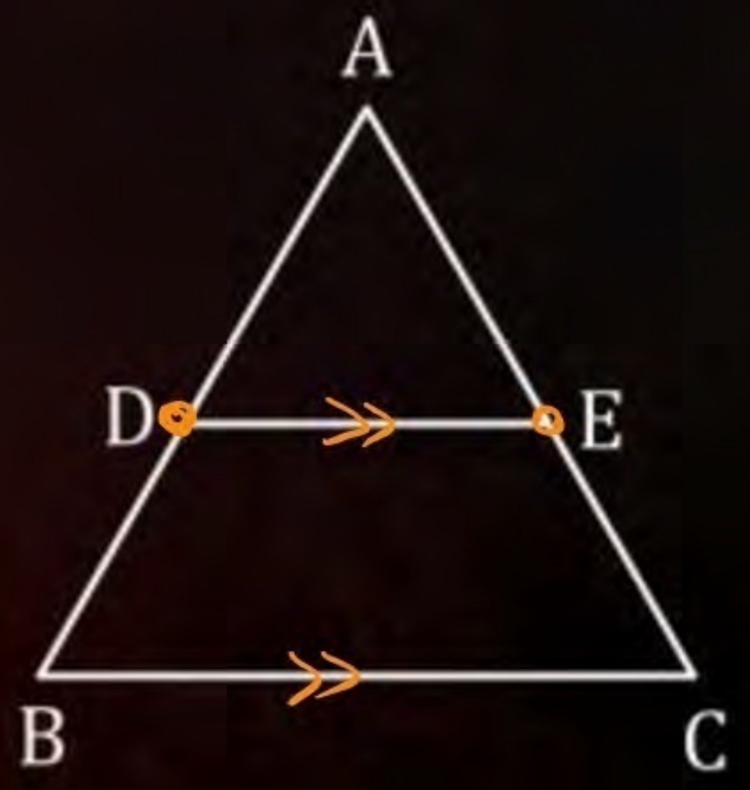
In the following figure, D and E are the mid points of sides AB and AC respectively. If DE = 5 cm, then the length of BC is

7.5 cm

5 cm

10 cm

25 cm



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Question

In the following figure, D and E are the mid points of sides AB and AC respectively. If DE = 5 cm, then the length of BC is

A 7.5 cm

B 5 cm

C 10 cm ✓

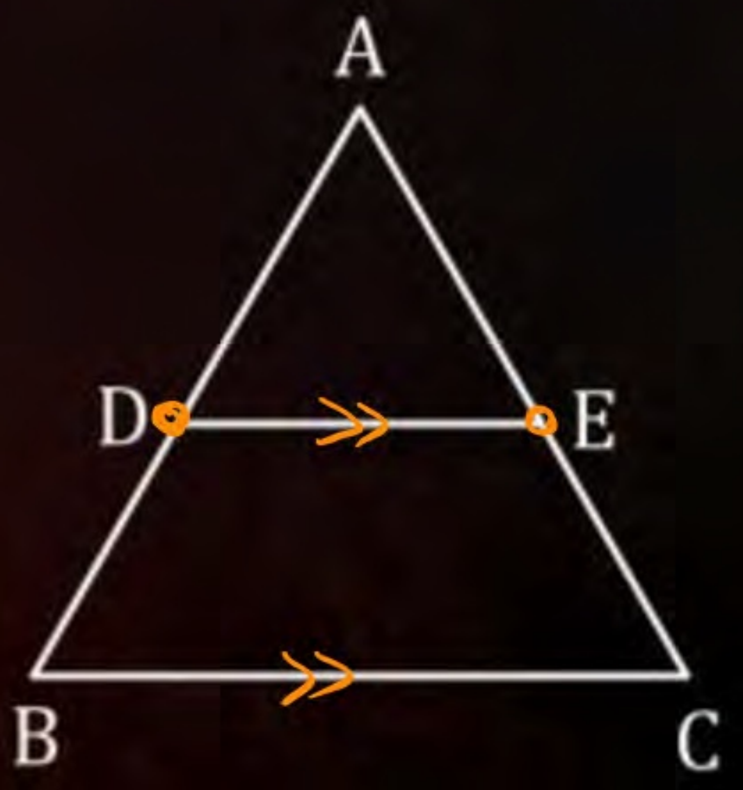
D 25 cm

since D & E are mid-point of AB & AC → $DE \parallel BC$ & $DE = \frac{1}{2} BC$

Therefore, using mid point theorem

$$5 = \frac{1}{2} \times BC$$

$$\boxed{BC = 10 \text{ cm}}$$



Question

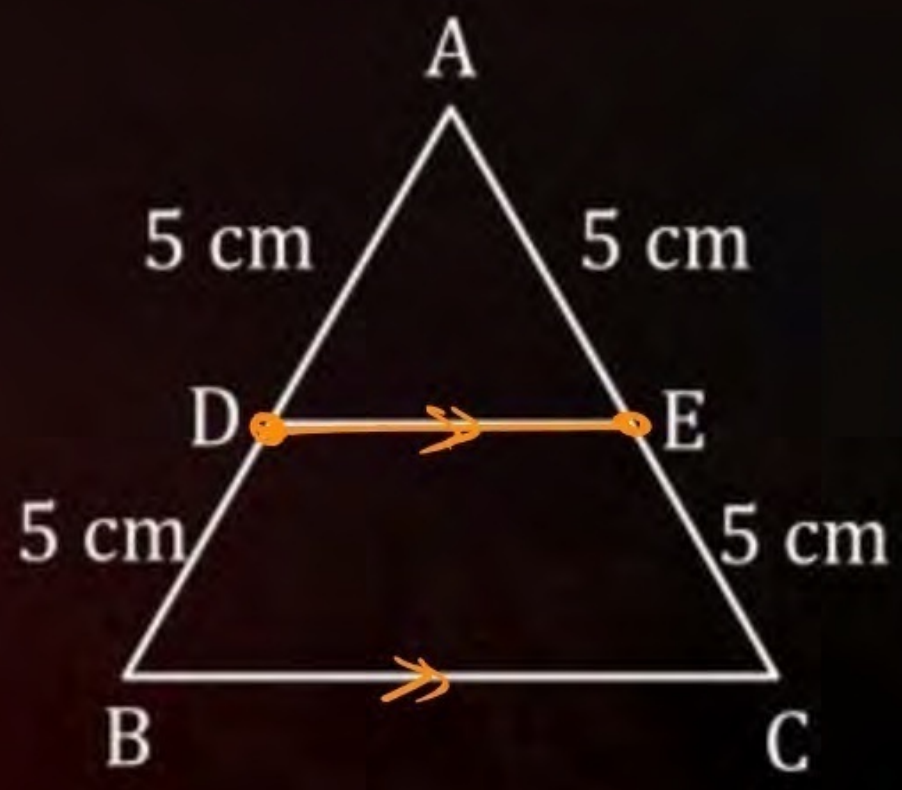
In the given figure, which of the following is true?

$DE = BC$

$DE = BC/2$

$2DE = 3BC$

$3DE = 2BC$



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Question

In the given figure, which of the following is true?

A) DE = BC

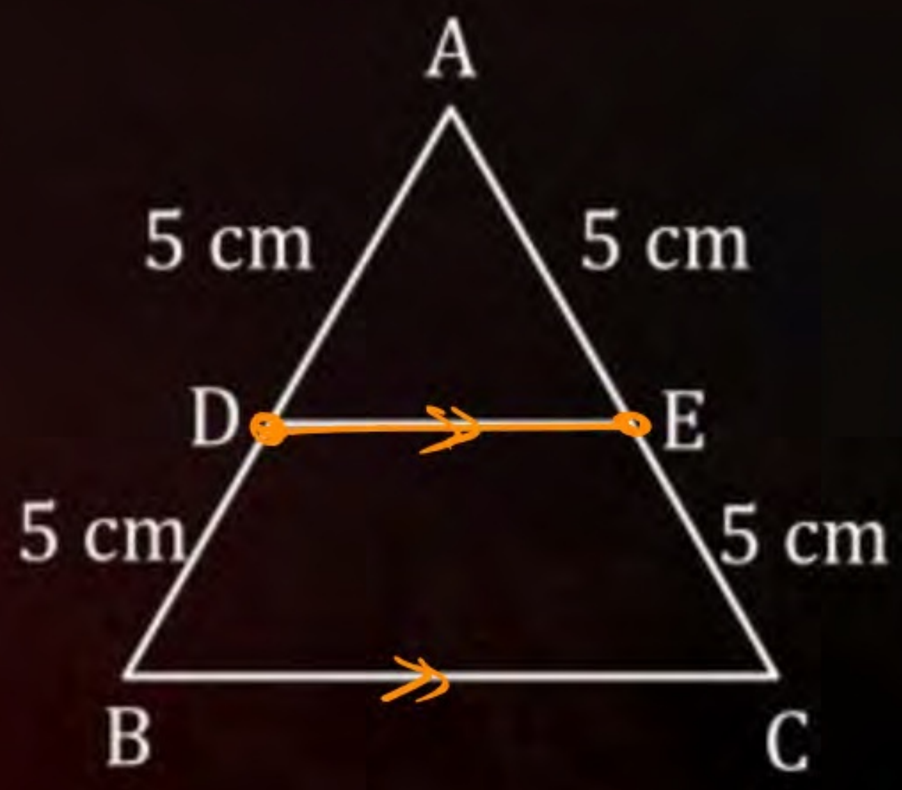
B) DE = BC/2 ✓

C) 2DE = 3BC

D) 3DE = 2BC

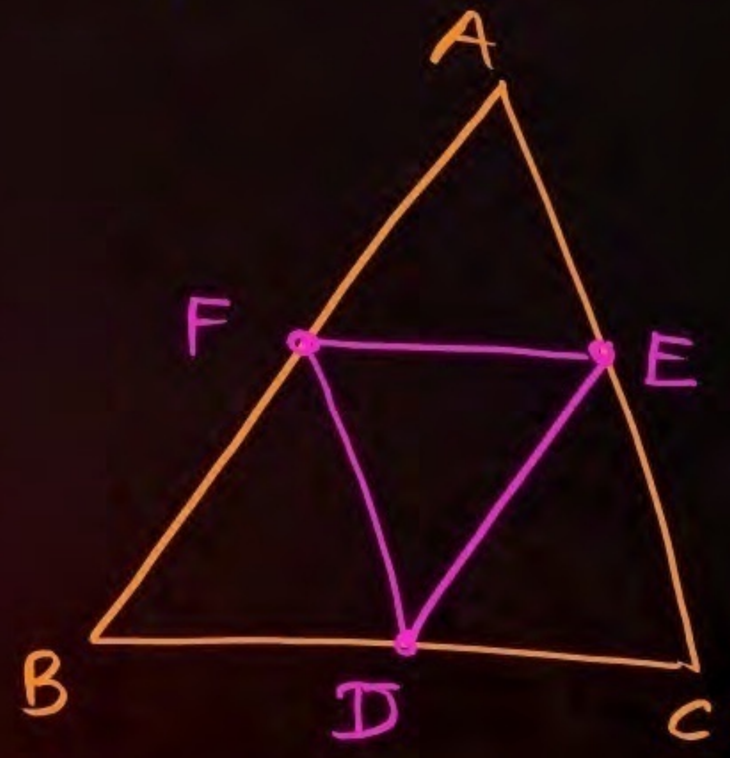
$DE \parallel BC$

$DE = \frac{1}{2} BC$



Question

In a ΔABC, D, E and F are respectively, the mid points of BC, CA and AB. If the lengths of sides AB, BC and CA are 7 cm, 8 cm and 9 cm respectively, find the semi-perimeter of ΔDEF.



24 cm

12 cm

6 cm

None of these

Question

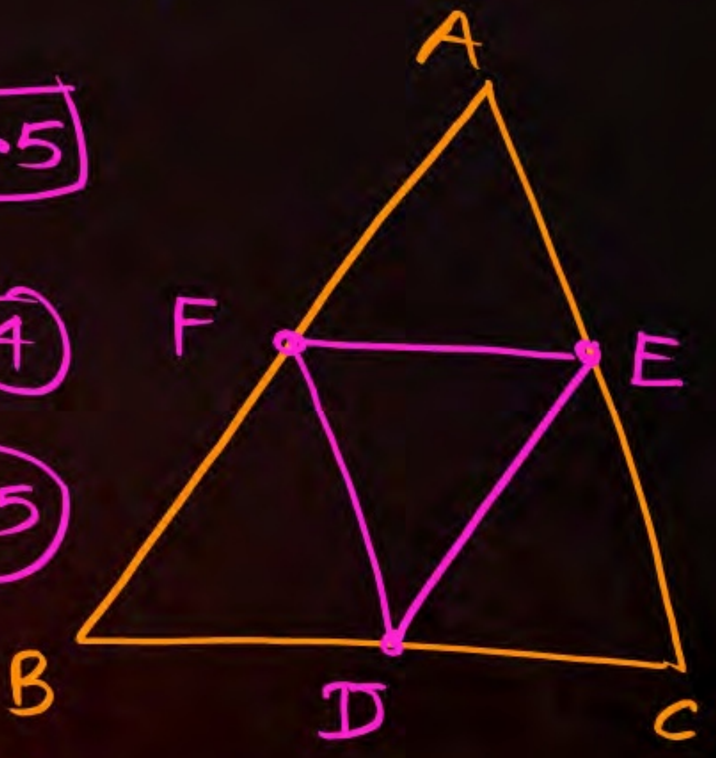
In a ΔABC, D, E and F are respectively, the mid points of BC, CA and AB. If the lengths of sides AB, BC and CA are 7 cm, 8 cm and 9 cm respectively, find the semi-perimeter of ΔDEF.

D & E are mid-pt.

$AB = 7 \Rightarrow DE = \frac{1}{2} AB \Rightarrow \boxed{DE = 3.5}$

$\Rightarrow FE = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$

$\Rightarrow DF = \frac{1}{2} \times AC = \frac{1}{2} \times 9 = 4.5$



A. 24 cm

B. 12 cm

C. 6 cm

D. None of these

$= \frac{\text{Perimeter } \Delta DEF}{2} = \frac{3.5 + 4 + 4.5}{2}$

$= \frac{12}{2} = 6 \text{ cm}$

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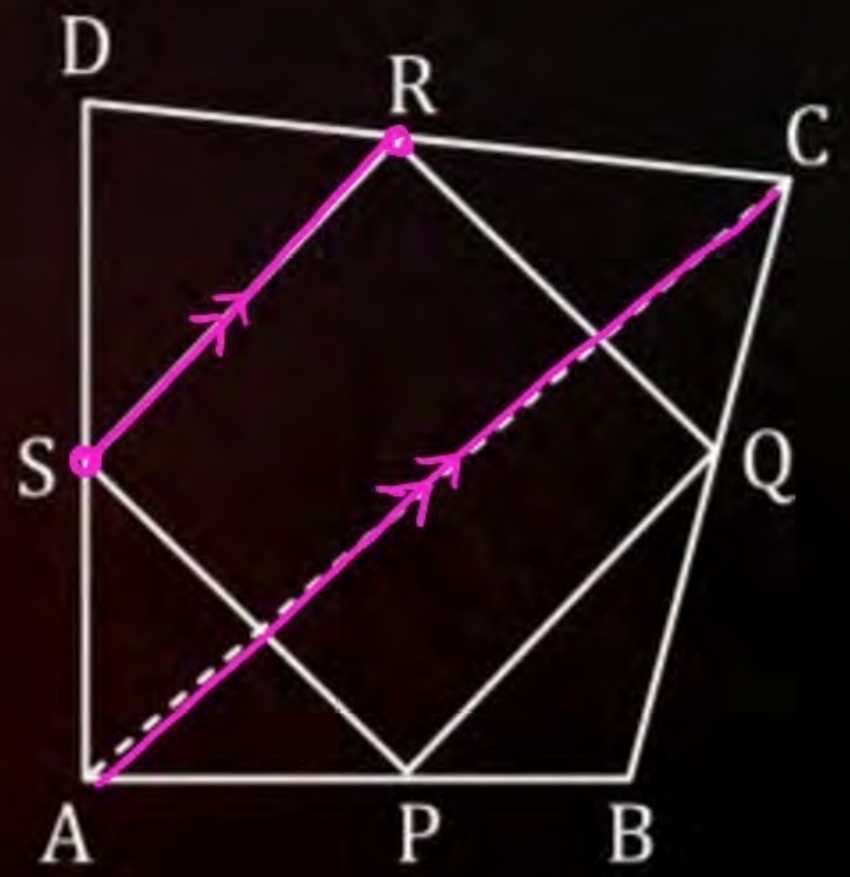
Question

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see given figure). AC is a diagonal Show that:

(i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$

(ii) $PQ = SR$ ✓

(iii) PQRS is a parallelogram



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

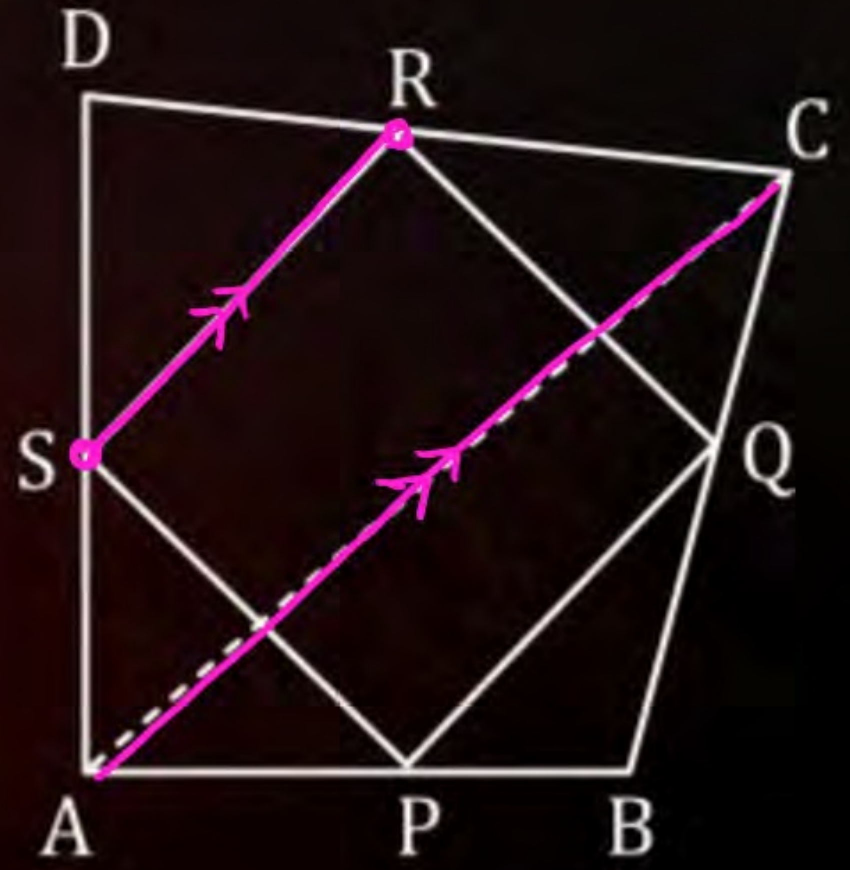
ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see given figure). AC is a diagonal Show that:

(i) $\boxed{SR \parallel AC \text{ and } SR = \frac{1}{2}AC}$

(ii) $\boxed{PQ = SR}$ ✓

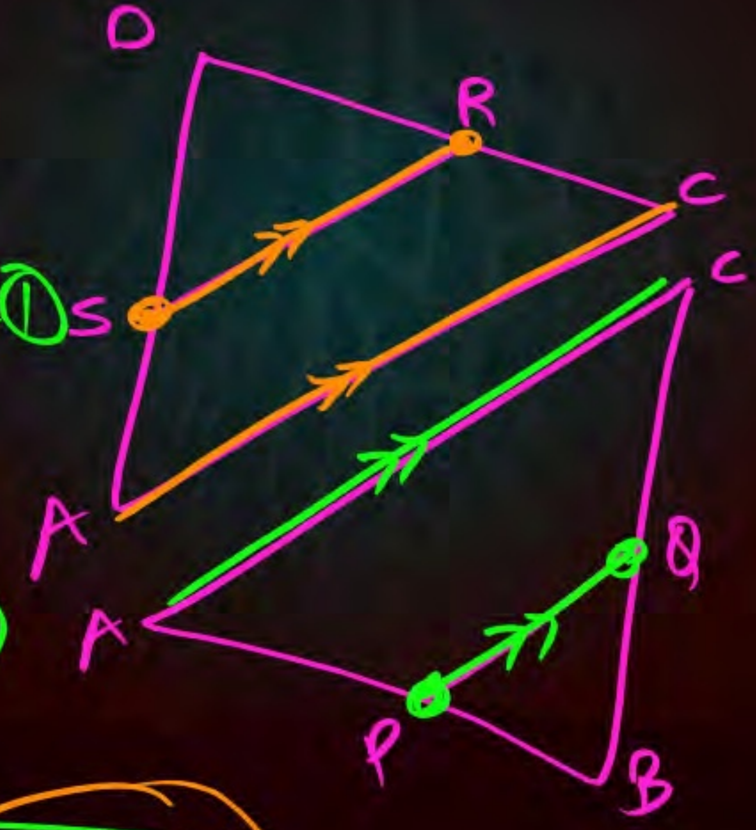
(iii) PQRS is a parallelogram

since S & R are mid point of AD & DC
Therefore, using mid-pt. theorem



(I) since S & R are mid-point

SR || AC & $\boxed{SR = \frac{1}{2}AC}$ —(1)



(II) Also, P & Q are mid-points

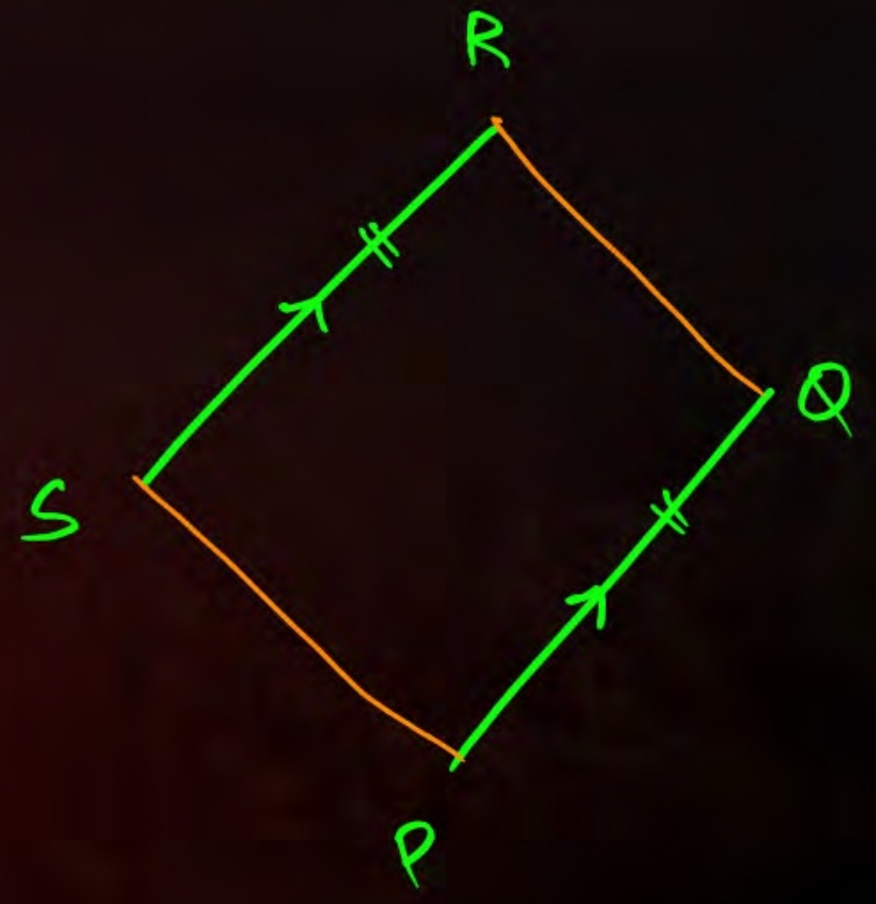
PQ || AC ⇒ $\boxed{PQ = \frac{1}{2}AC}$ —(2)

From (1) & (2) $\boxed{SR \parallel PQ \ \& \ SR = PQ}$

(iii) since $SR \parallel PQ$ and $SR = PQ$

This holds only in parallelogram

therefore, $\square PQRS$ will be a parallelogram



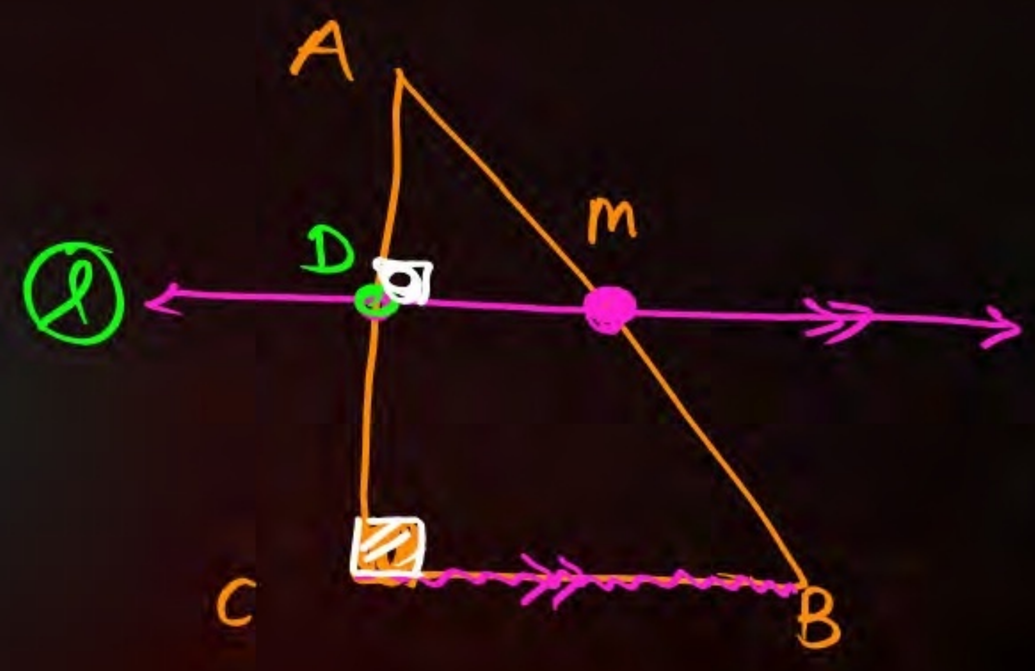
Question

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2}AB$



Question

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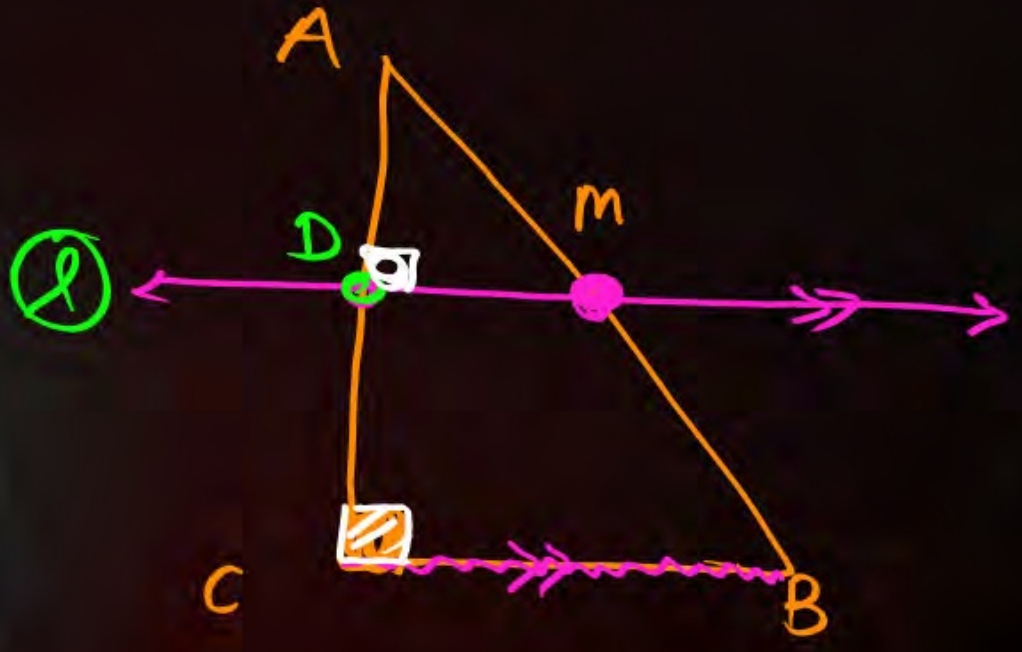


(i) using converse of mid-pt. theorem

M is mid point of AB & DM || BC

Therefore, D will be mid-point of AC

(ii) Now, MD || BC

Therefore, $\angle ACB = \angle ADM$ (corresponding Angle)

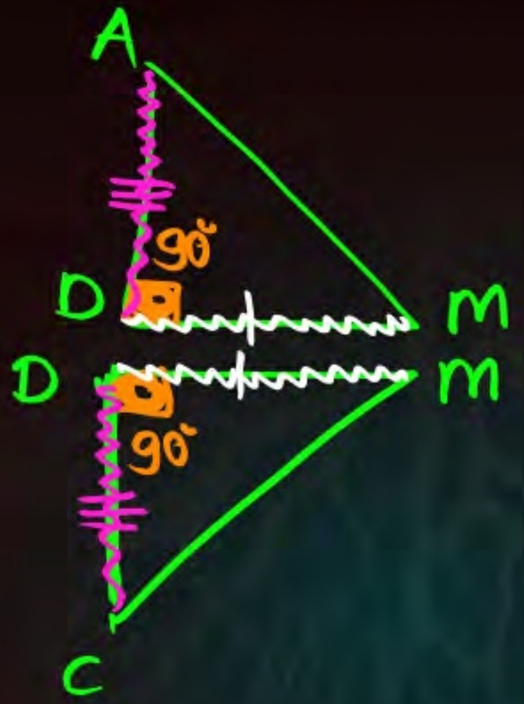
$= 90^\circ$

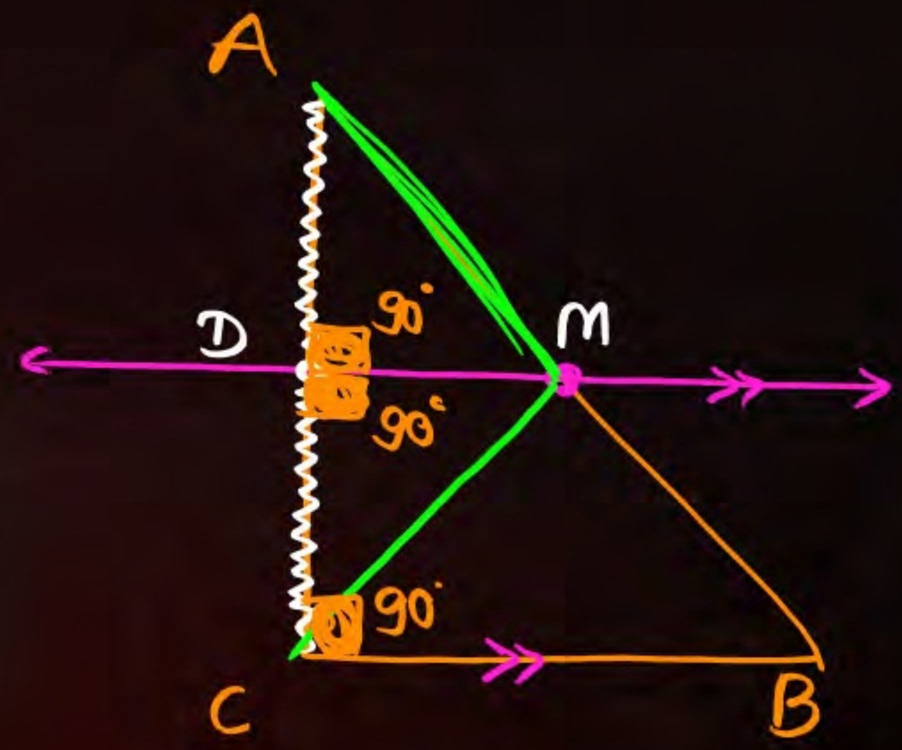
Thus, $MD \perp AD \Rightarrow \boxed{MD \perp AC}$

(iii)

since, D is mid point of AC

therefore, $\boxed{AD = DC}$





Now, $\triangle ADM$ & $\triangle CDM$

S A S

$AC = CD$ (proved)

$\angle ADM = \angle CDM$ (90°)

$DM = DM$ (common)

By SAS criteria,

$\boxed{\triangle ADM \cong \triangle CDM}$ $\rightarrow$ By CPCT, $\boxed{CM = AM}$

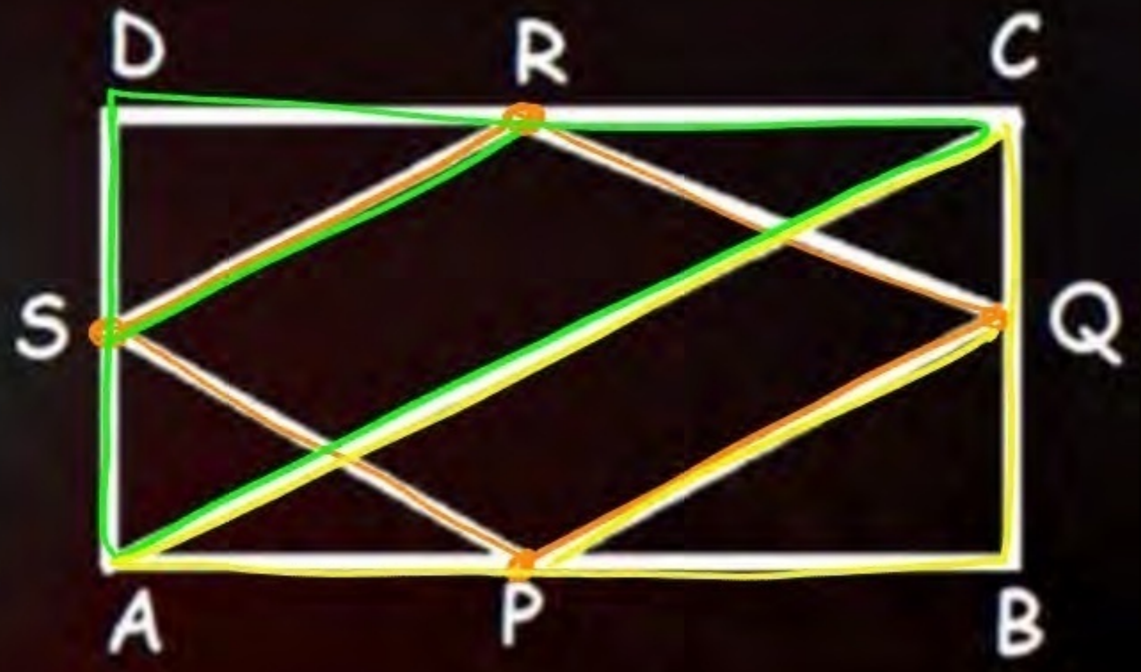
Now, M is mid point of AB

Thus $AM = \frac{1}{2}AB$

therefore, $\boxed{CM = AM = \frac{1}{2}AB}$

Question

Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a rectangle is a rhombus.



Question

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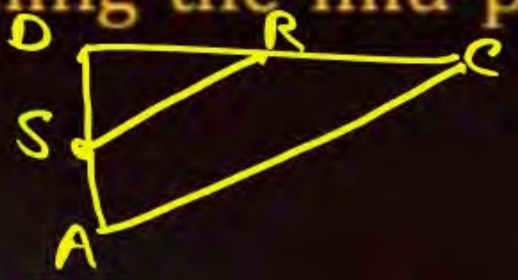
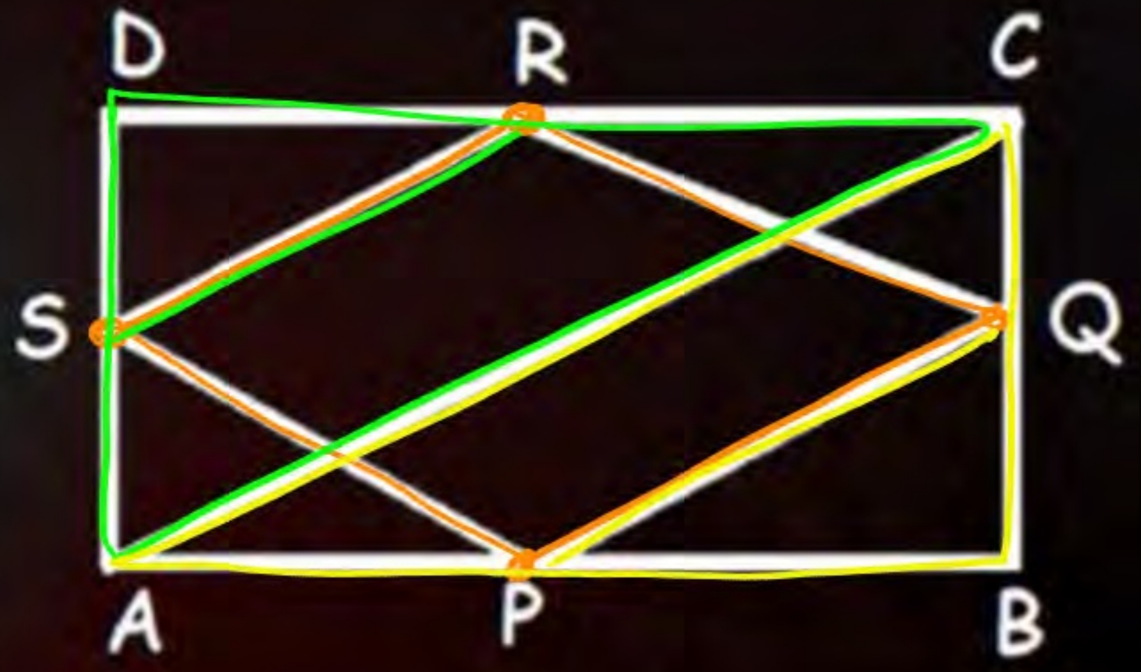


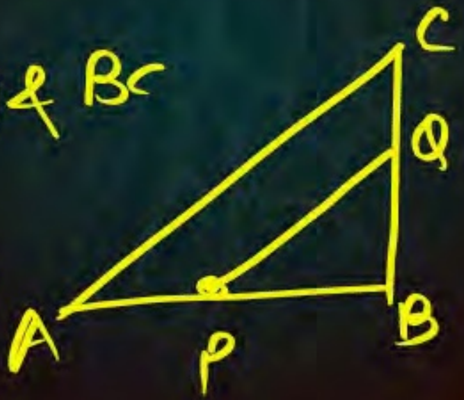


since S & R are mid point of DA & DC

using mid point theorem,

$\boxed{SR \parallel AC \Rightarrow SR = \frac{1}{2} AC}$ — ①

since, P & Q are mid-point of AB & BC



Using mid-pt theorem

$\boxed{PQ \parallel AC \Rightarrow PQ = \frac{1}{2} AC}$ — ②

From ① & ②

$\boxed{SR \parallel PQ \ \& \ SR = PQ}$

$\Rightarrow$ □ PQRS → ||gram

$\boxed{SP \parallel RQ \ \& \ SP = RQ}$

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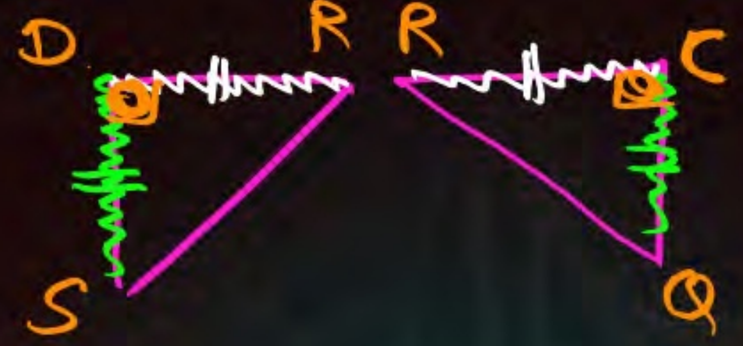
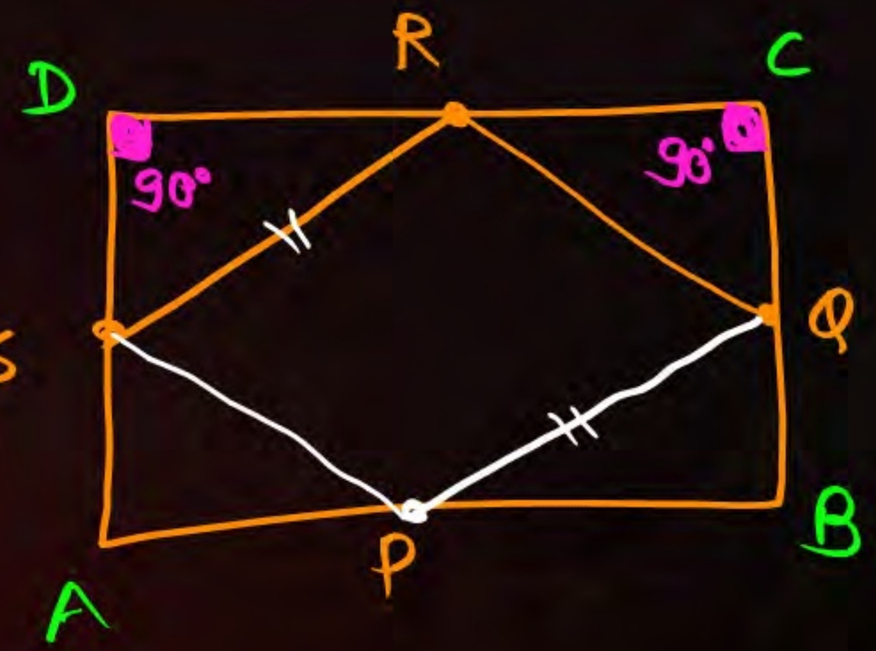
since ABCD is rectangle

$$DA = BC$$



$$\Rightarrow \frac{DA}{2} = \frac{BC}{2} \Rightarrow \boxed{DS = CQ}$$



Now, $\boxed{\triangle DSR \ \& \ \triangle CQR}$ $\rightarrow$

$DR = RC$ $\rightarrow$ R mid pt.

$\angle D = \angle C$ $(90°)$

$DS = CQ$ (calculated)

By $\boxed{S\text{-}A\text{-}S}$ $\boxed{\triangle DSR \cong \triangle CQR}$

By CPCT, $SR = RQ$ But $SR = PQ$ & $RQ = SP$

$$\Rightarrow \boxed{SR = RQ = PQ = SP} \rightarrow \text{Rhombus}$$