VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

- **Subject -** Mathematics
- **Chapter –** Circles

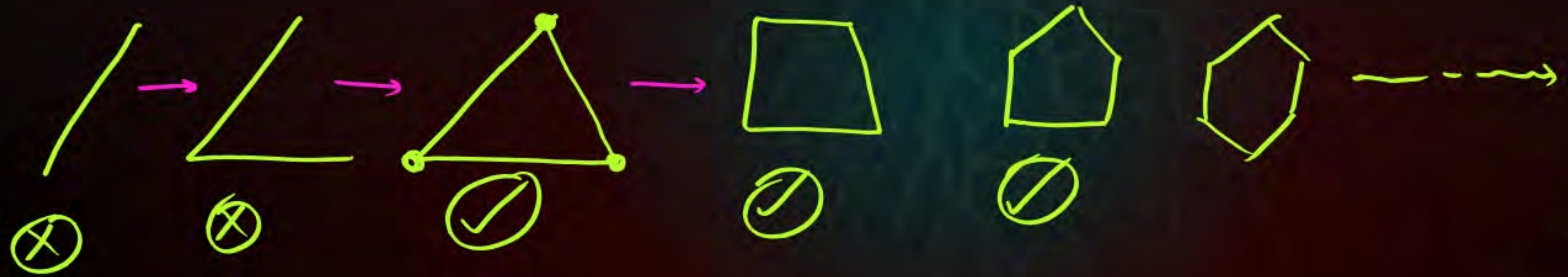
re

Today's Targets

1. Complete Chapter
2. Bonus Concepts
3. Practice Problems

Polygon and Regular Polygon with Infinite Sides

Polygon → closed figure + straight line



Regular Polygon

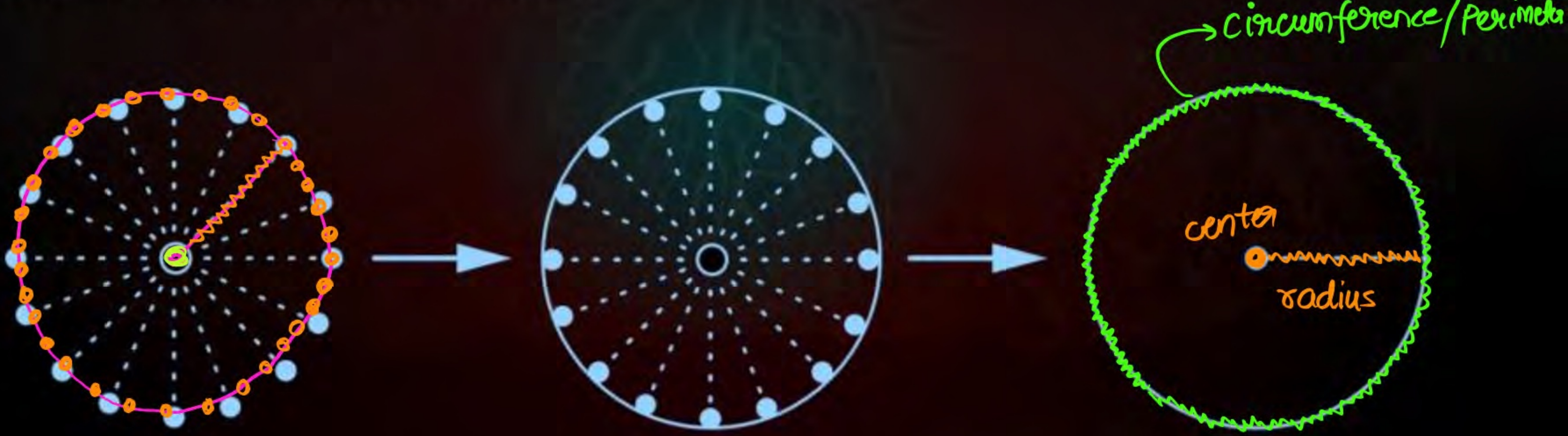
No. of sides → ∞

A......

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

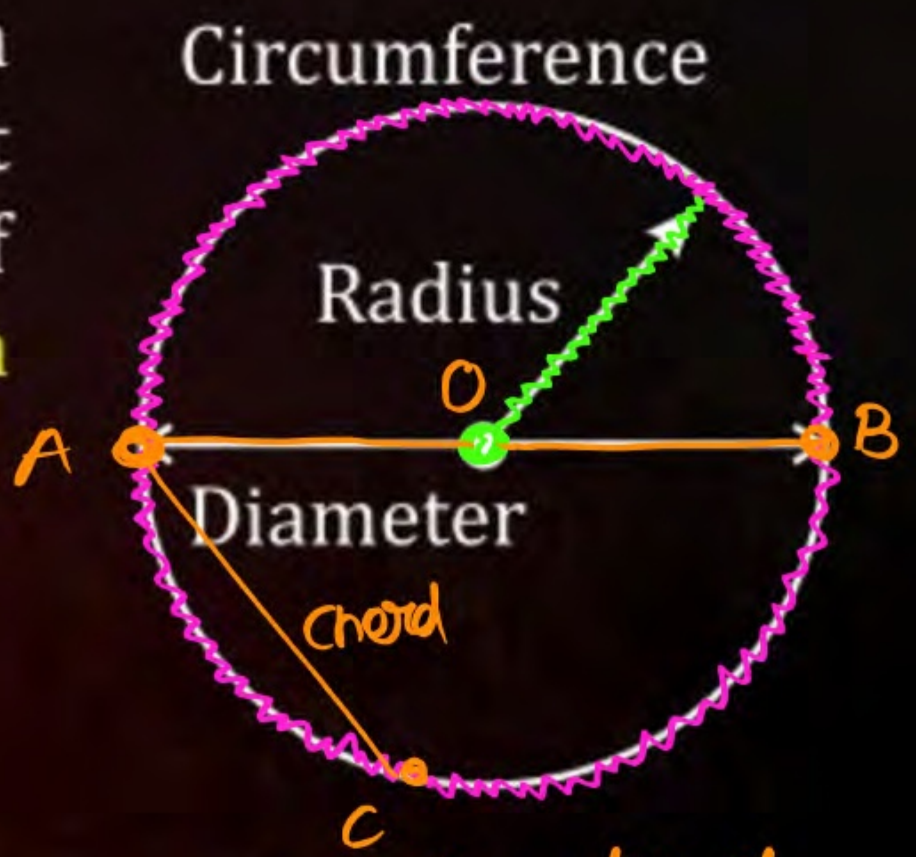
Introduction

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle.



Terms Related to Circle : Radius, Diameter and Circumference

- A circle can be described as the locus of a point moving in a plane, in such a way that its distance from a fixed point is always constant. The fixed point is called the centre of the circle and the constant distance between any point on the circle and its centre is called the radius.
- The circumference is the outside edge of the circle.
- A diameter is a straight line going through the centre of the circle and touching the circumference at each end.

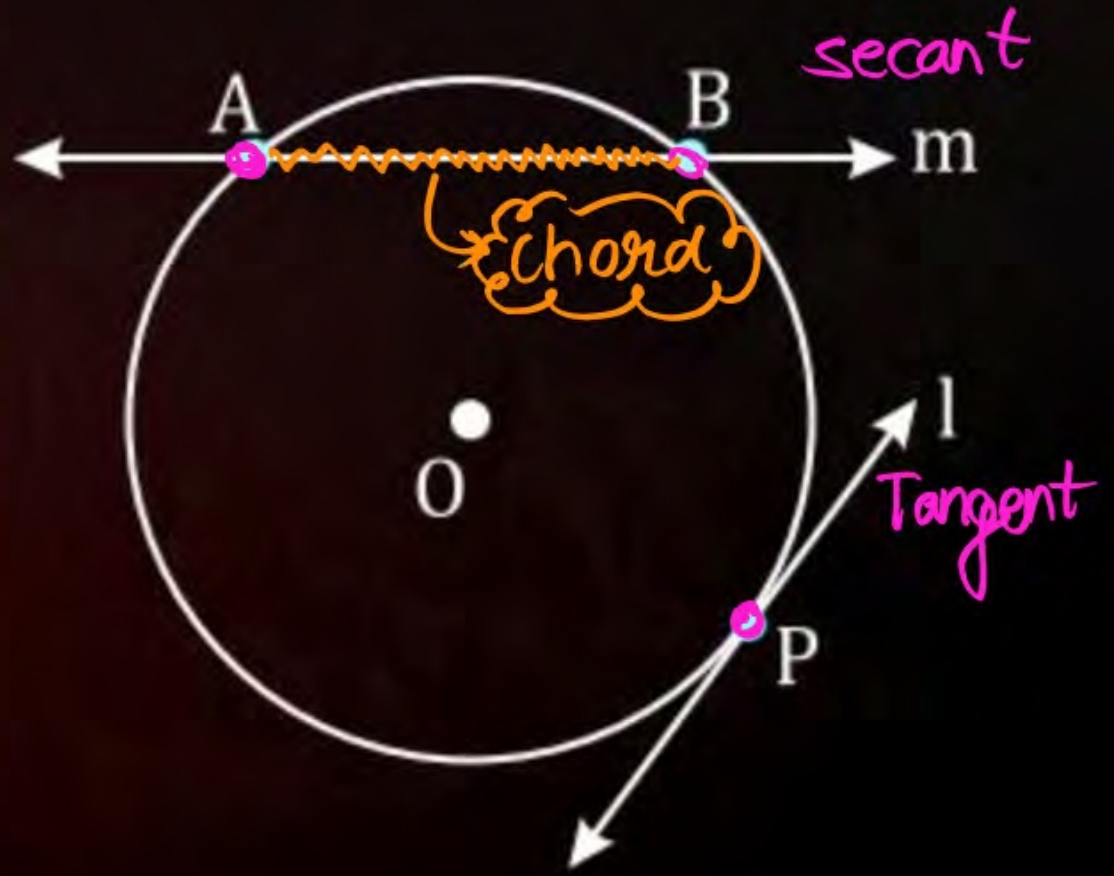


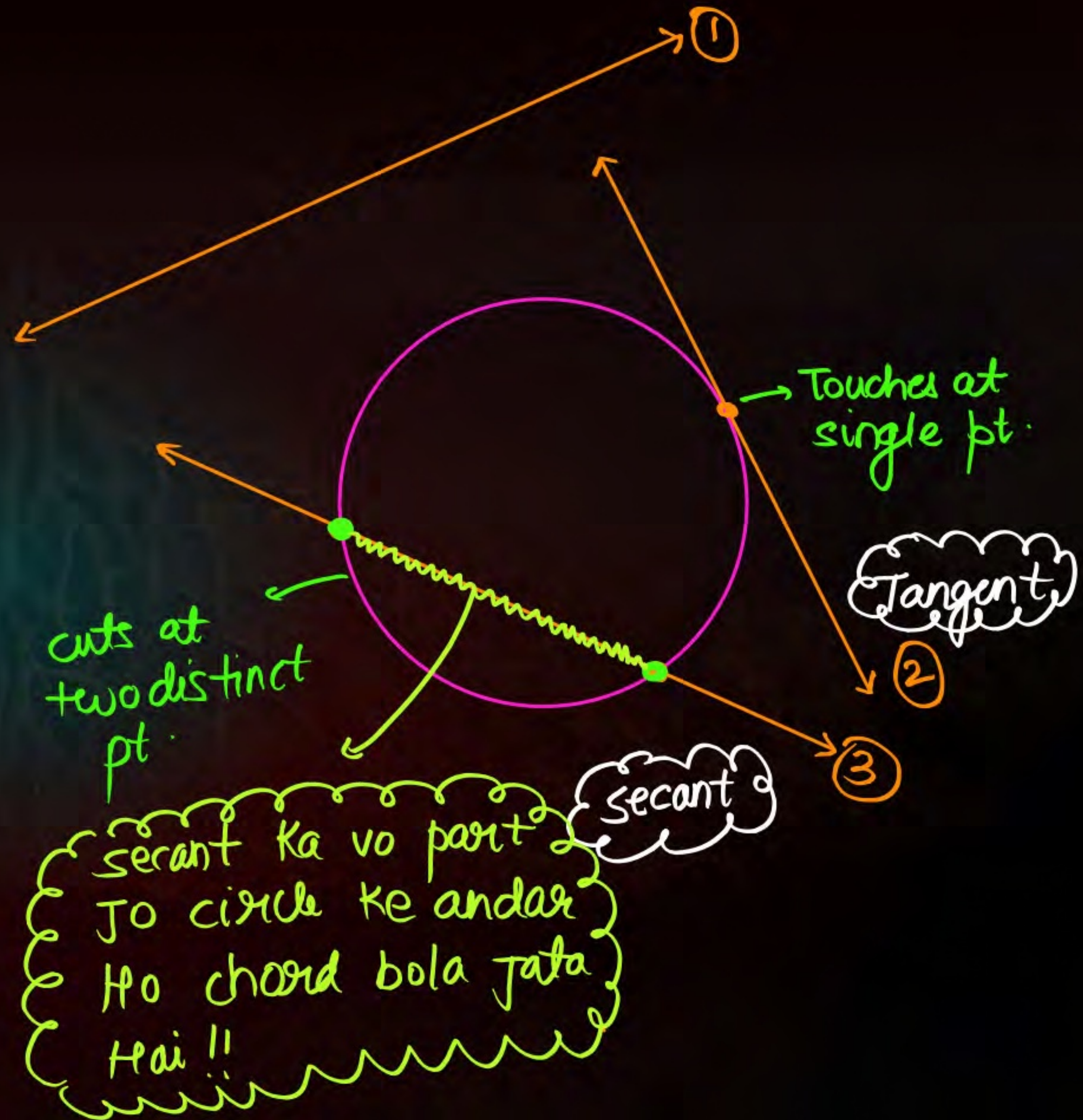
AB = diameter of circle with centre O.

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Terms Related to Circle: Tangent, secant and Chord

- A chord is a straight line joining any two parts of the circumference.
- If a line m intersects the circle at two distinct points A and B, then line m here is called secant of the circle. The line segment AB is known as the chord of the circle as its endpoints lie on the circumference of the circle.
- A tangent is a straight line that touches the circumference at a single point.

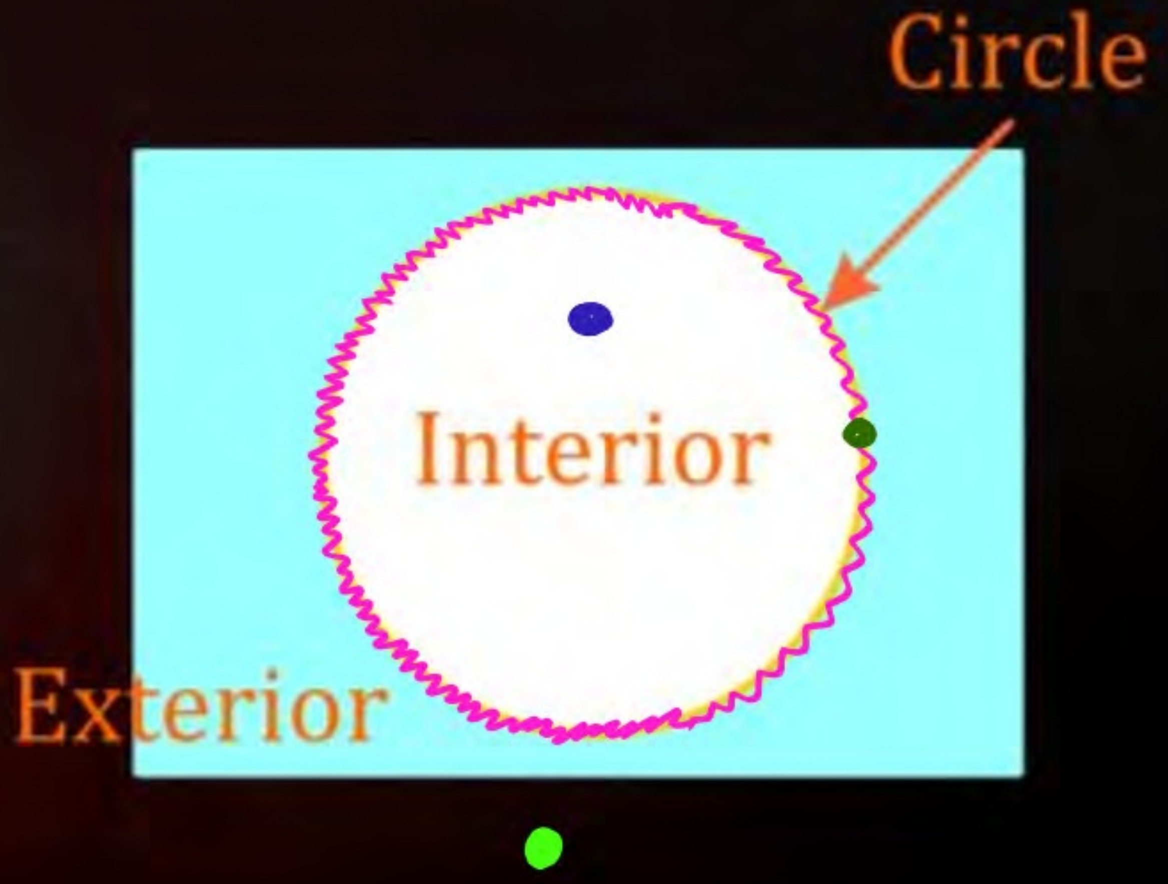



1
2
3
Touches at single pt.
Tangent
cuts at two distinct pt.
secant
secant ka vo part jo circle ke andar Ho chord bola jata Hai !!

Terms Related to Circle :Tangent, secant and Chord

A circle divides the plane on which it lies into three parts. They are:

- Inside the circle, which is also called the interior of the circle.
- On the circle
- Outside the circle, which is also called the exterior of the circle. The circle and its interior make up the circular region.



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

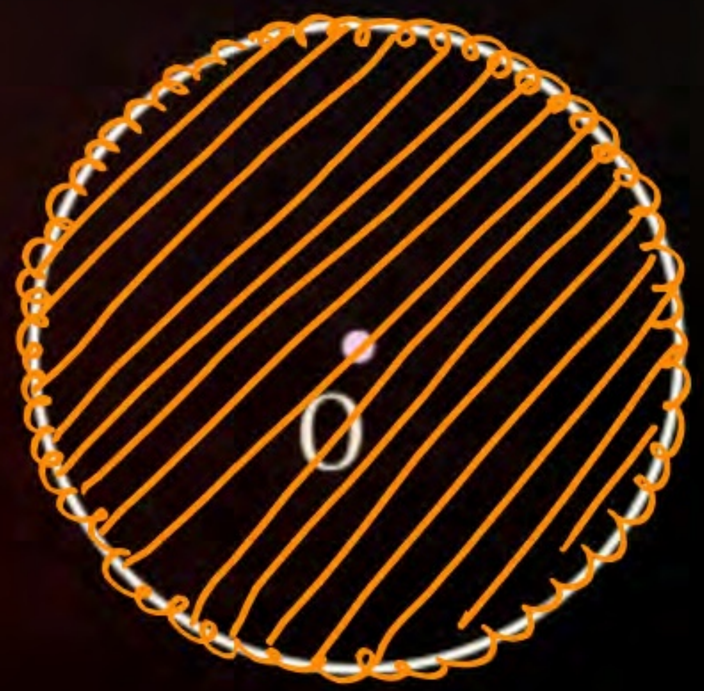
Circular Region

The region consisting of all points which are either on the circle or lie inside the circle is called the circular region or circular disc.

$\boxed{\text{Interior point + on the circle}}$

↓

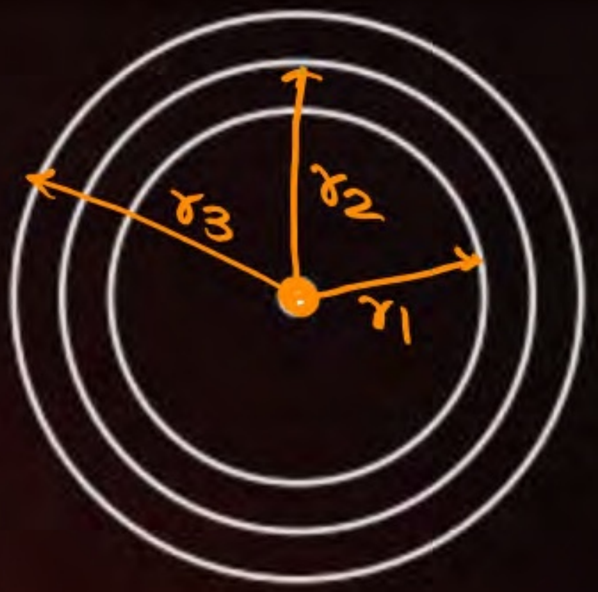
circular Region



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

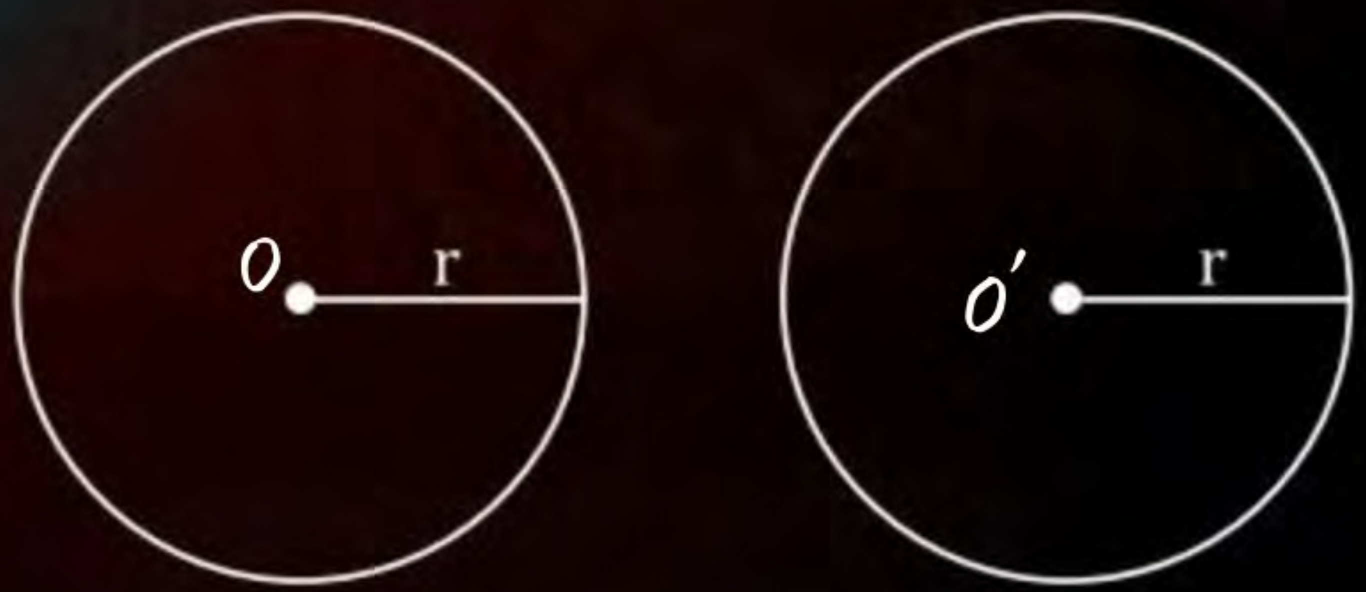
Terms Related to Circle

Concentric Circles : Circle which have the same centre and different radii are called concentric circles.



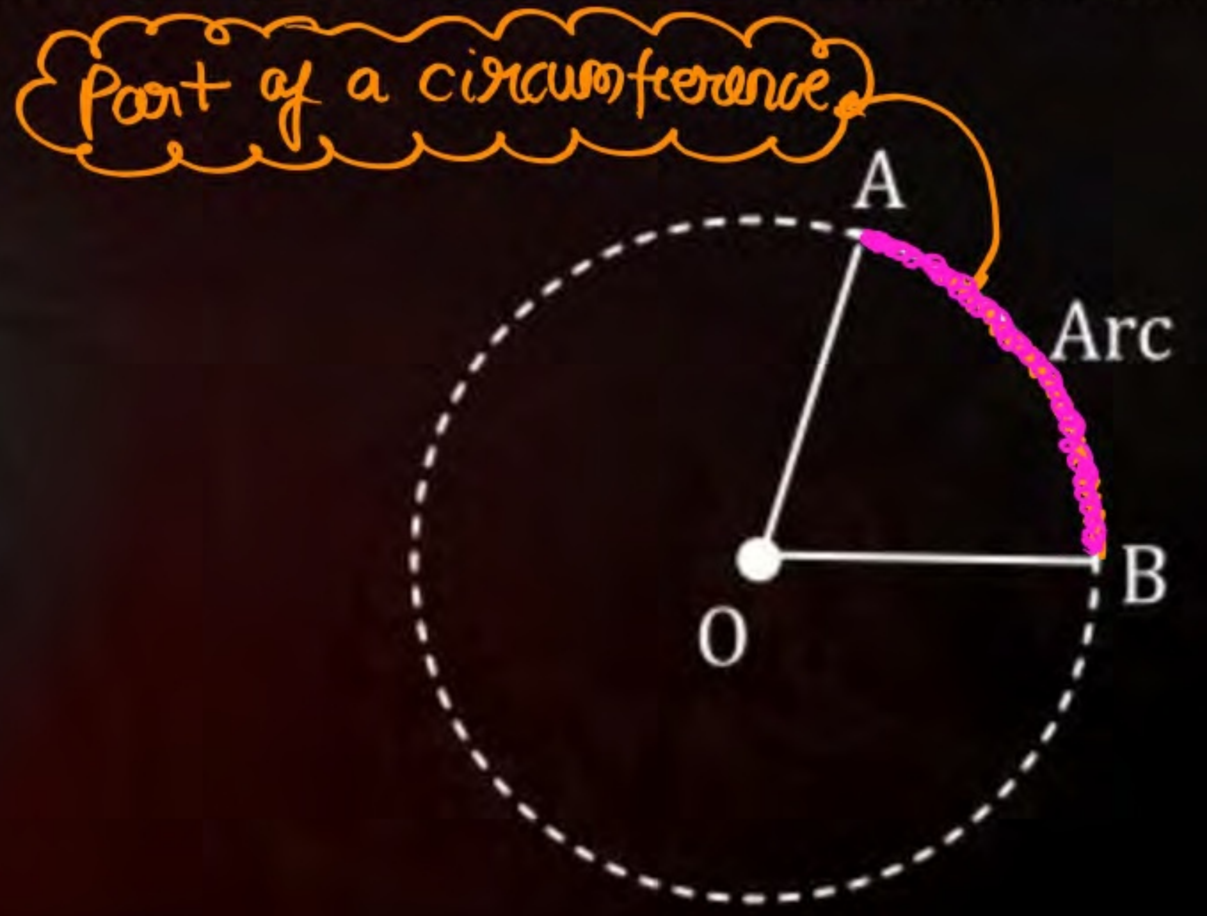
Congruent Circles :

Two circles C(O, r) and C(O′, s) are said to be congruent only when r = s.



Semicircle

Arc of a circle : A continuous piece of a circle is called an arc of the circle. An arc is a section of the circumference



Semicircle

Semicircle : A diameter of a circle divides it into two equals arcs. Each of these two arcs is called a semicircle.

$\overline{BCA}$

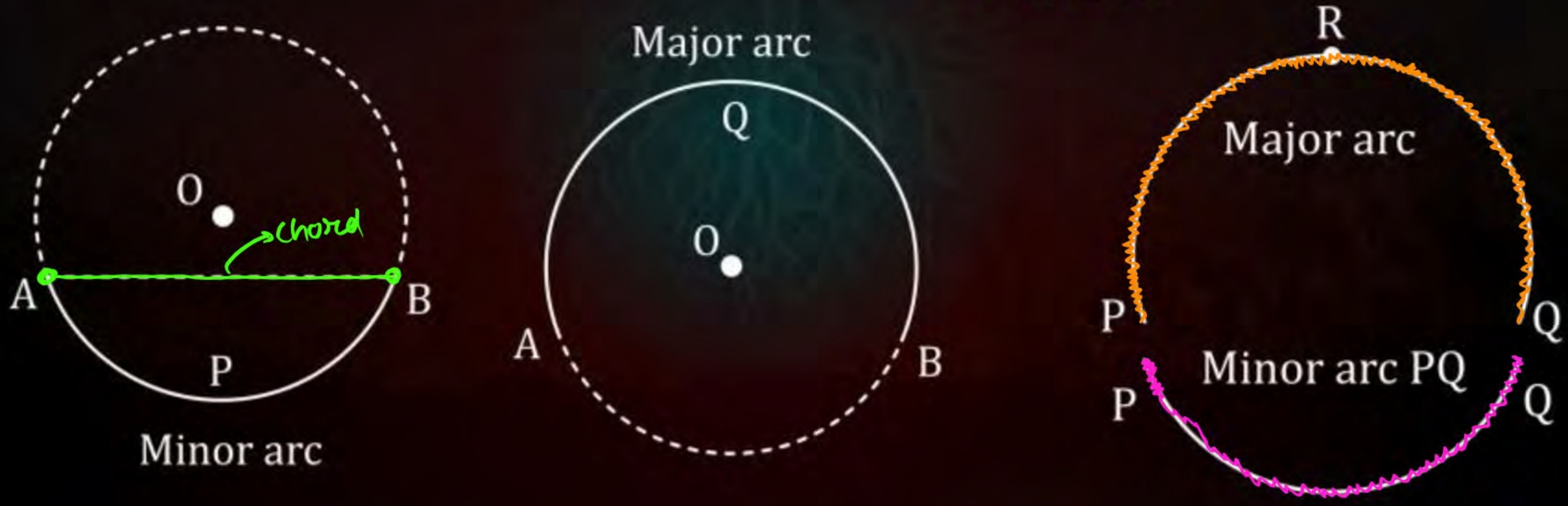
$\overset{\frown}{ACB}$

In the given figure, $\overset{\frown}{BCA}$ and $\overset{\frown}{BDA}$ are two semicircles.

The degree measure of a semicircle is 180°.

Minor and Major Arcs of a Circle

If the length of an arc is less than the length of the arc of the semicircle then it is called a minor arc. Otherwise, it is a major arc. → circumference



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Terms Related to Circle : Segment of a Circle

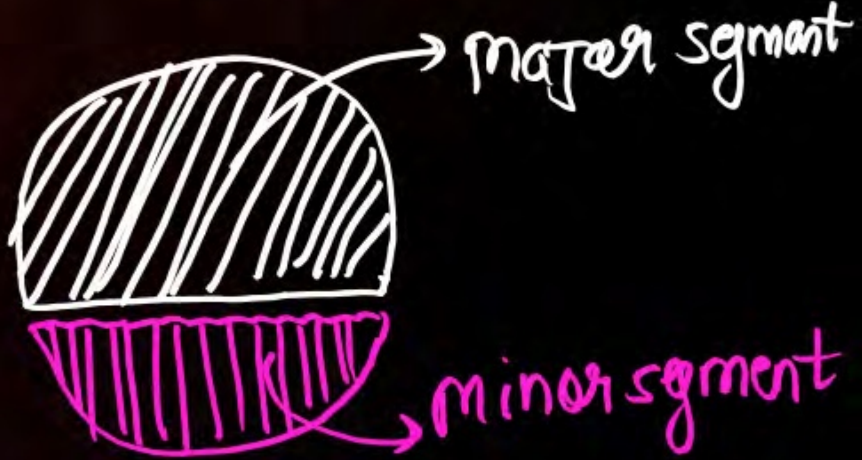
Segment of a Circle :

circle

The part of the circular region bounded by an arc and a chord, including the arc and the chord, is called a segment of the circle. The figure given below depicts the major and minor segments of the circle.

Alternate Segments of a Circle :

The minor and major segments of a circle are called the alternate segments of the circle.



Terms Related to Circle : Sector of a Circle

Sector of a Circle :

The region enclosed by an arc of a circle and its two bounding radii is called a sector of the circle.

Question

Fill in the blanks:

(i) A point, whose distance from the centre of a circle is greater than its radius lies in ___________ of the circle. (exterior/ interior)

(ii) The longest chord of a circle is a ___________ of the circle. (radius/ diameter)

(iii) An arc is a ___________ when its ends are the ends of a diameter.

(iv) Segment of a circle is the region between an arc and ___________ of the circle.

(v) A circle divides the plane, on which it lies, in ___________ parts.

Question

Fill in the blanks:

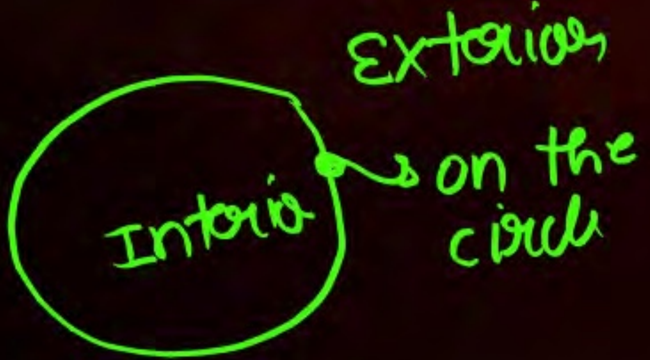
(i) A point, whose distance from the centre of a circle is greater than its radius lies in Exterior ________ of the circle. (exterior/ interior)

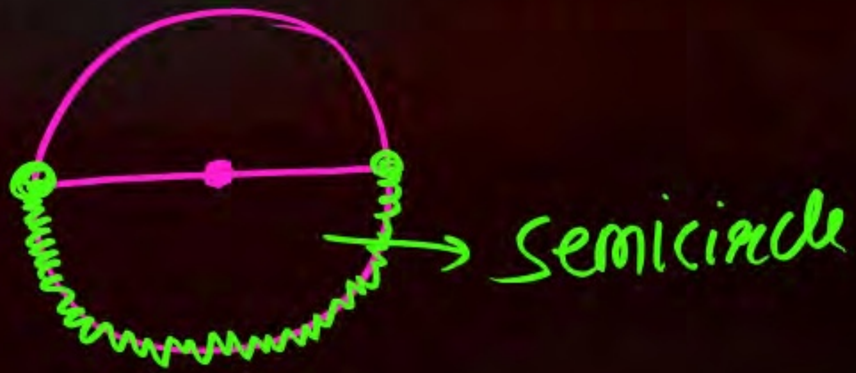
(ii) The longest chord of a circle is a diameter ________ of the circle. (radius/ diameter)

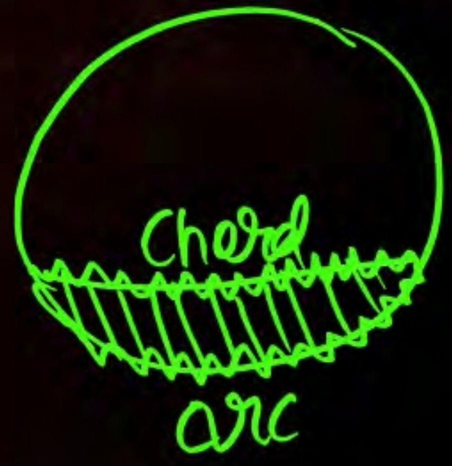
(iii) An arc is a semicircle ________ when its ends are the ends of a diameter.

(iv) Segment of a circle is the region between an arc and chord ________ of the circle.

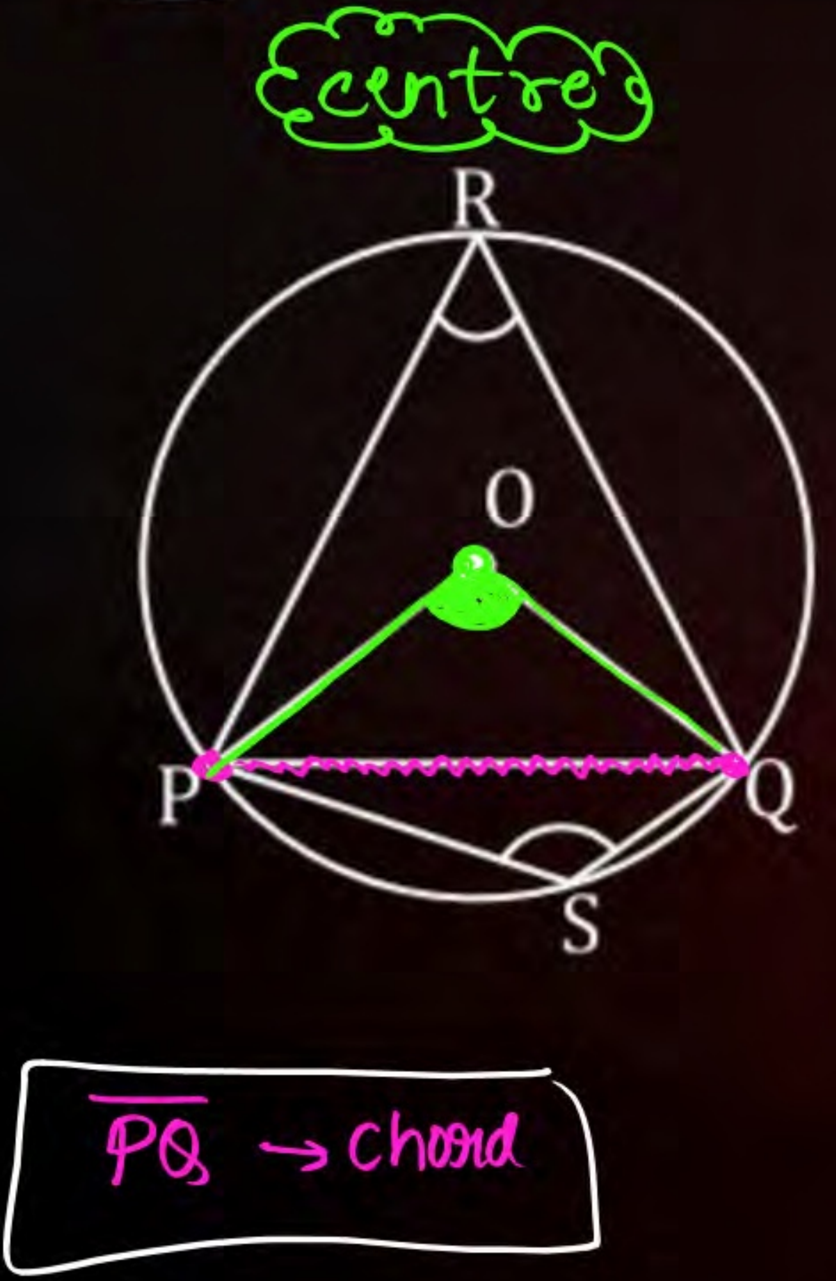
(v) A circle divides the plane, on which it lies, in three ________ parts.

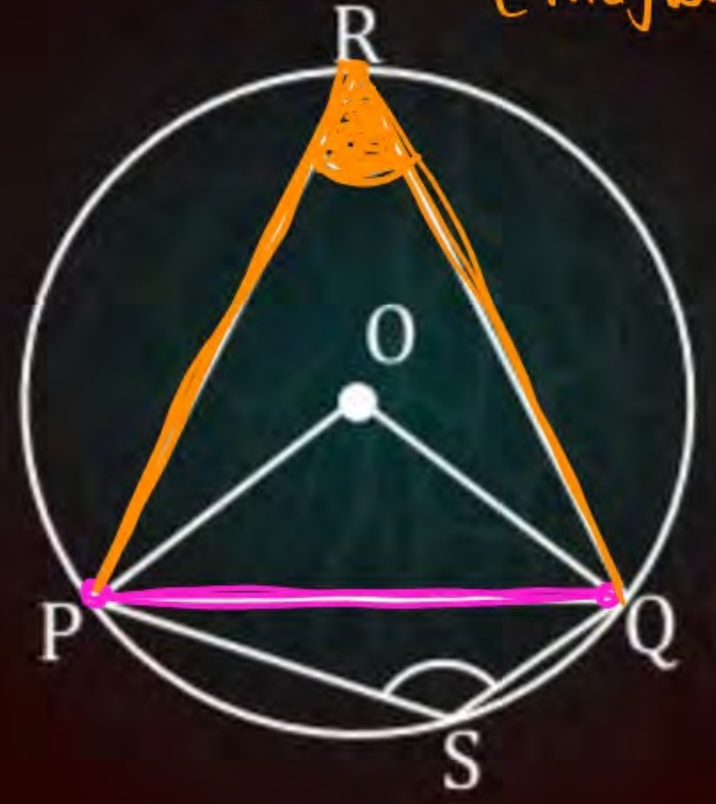


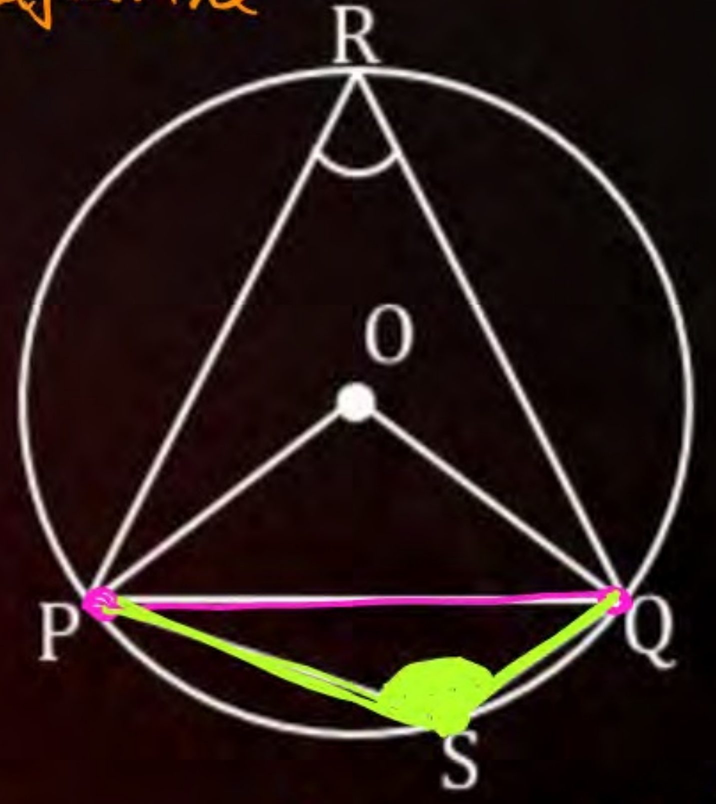




Angle Subtended by a Chord at a point







VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Theorems related to Circle

Theorem 1: Equal chords of a circle subtend equal angles at the centre.

$OA = OB = OC = OD$ (Radius)

considering $\triangle OAB$ & $\triangle OCD$

$$\left\{\begin{array}{l} AB = CD \text{ (given)} \\ OA = OD \\ OB = OC \end{array}\right.$$

$\boxed{\text{SSS congruence criteria}}$

Using SSS congruence rule, $\boxed{\triangle OAB \cong \triangle ODC}$

By CPCT, $\boxed{\angle AOB = \angle COD}$

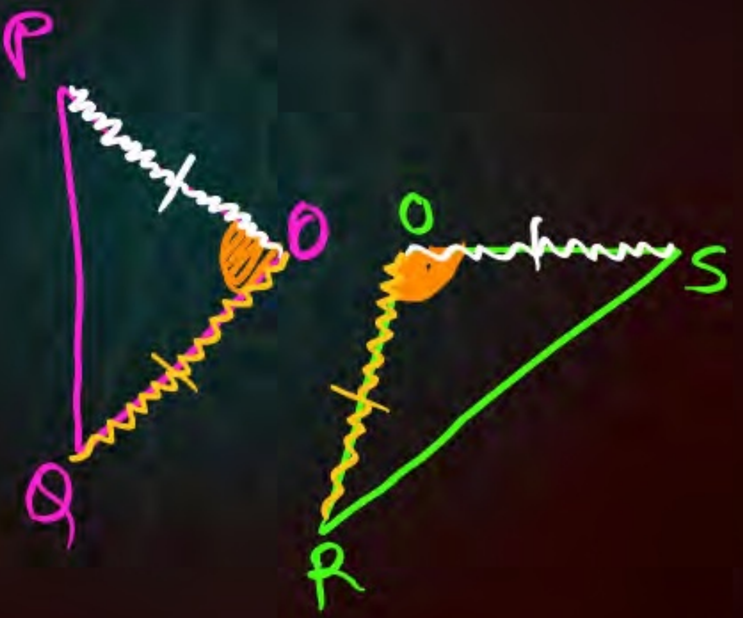
Hence, proved !!

Theorems related to Circle

Theorem 2: If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Given: $\angle POQ = \angle ROS$

Also, $OP = OQ = OR = OS$ (Radius)



In $\triangle OPQ$ & $\triangle OSR$

congruence Rule SAS

$$\begin{cases} OP = OS & \text{(Radius)} \\ \angle POQ = \angle SOR & \text{(given)} \\ OQ = OR & \end{cases}$$

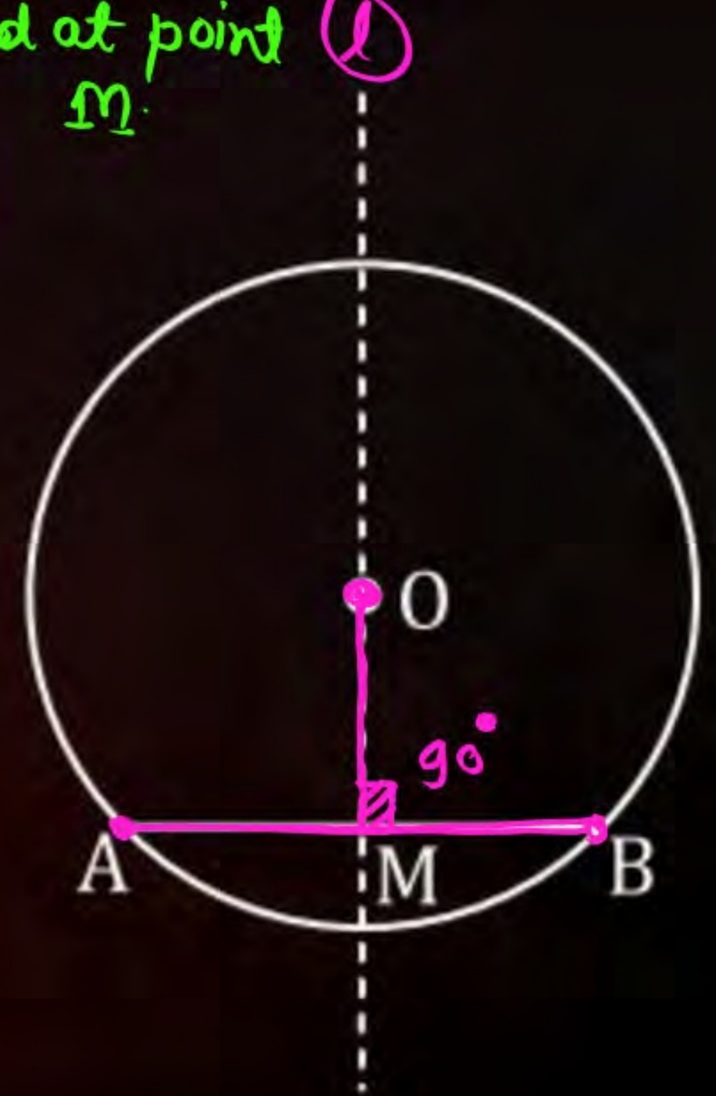
Using SAS rule, $\boxed{\triangle OPQ \cong \triangle OSR}$. By CPCT, $\boxed{PQ = RS}$ Hence proved!!

Perpendicular from the Centre to a Chord

$\boxed{l \perp AB}$ $\Rightarrow$ $\boxed{OM \perp AB}$

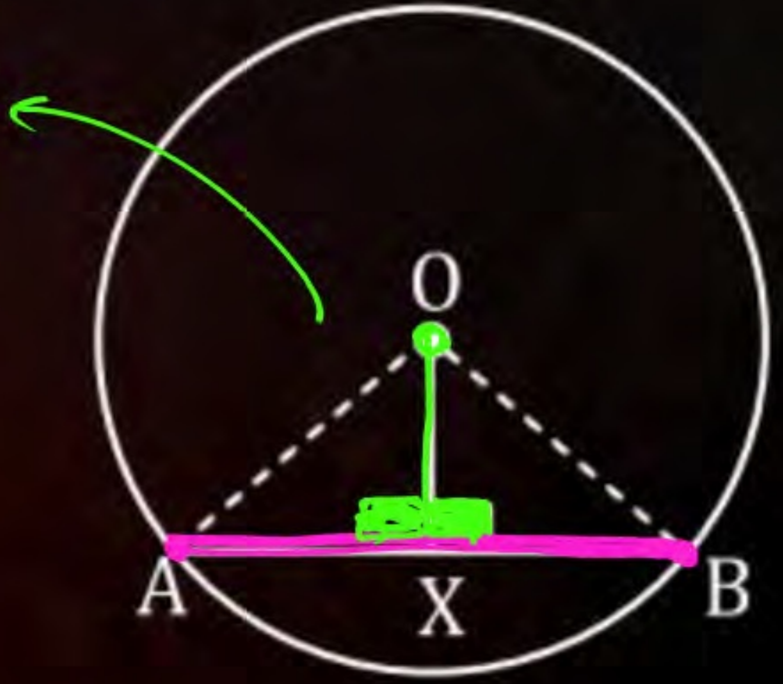
- AB will be bisected at point M.
- OM will be known as distance of chord from centre.

$\boxed{AM = BM}$ Bisected at point M.



Theorems related to Circle

Theorem 3: The perpendicular from the centre of a circle to a chord bisects the chord.



In $\triangle OXA$ & $\triangle OXB$

R — $\angle OXA = \angle OXB$ (Both are 90°)

H — $OA = OB$ (Radius)

S — $OX = OX$ (common)

Rule

Therefore, $\triangle OXA \cong \triangle OXB$

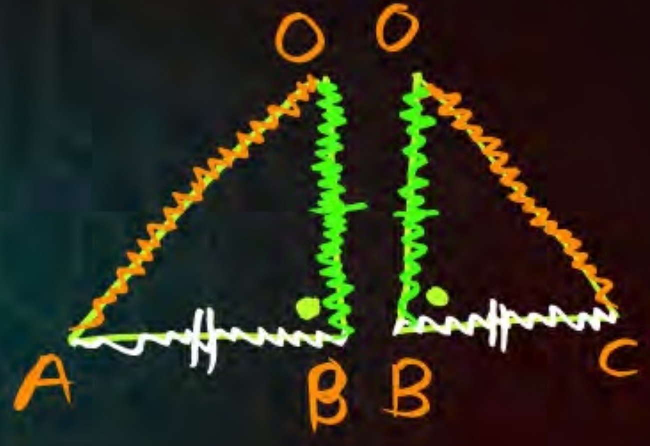
By CPCT, $\boxed{AX = XB}$

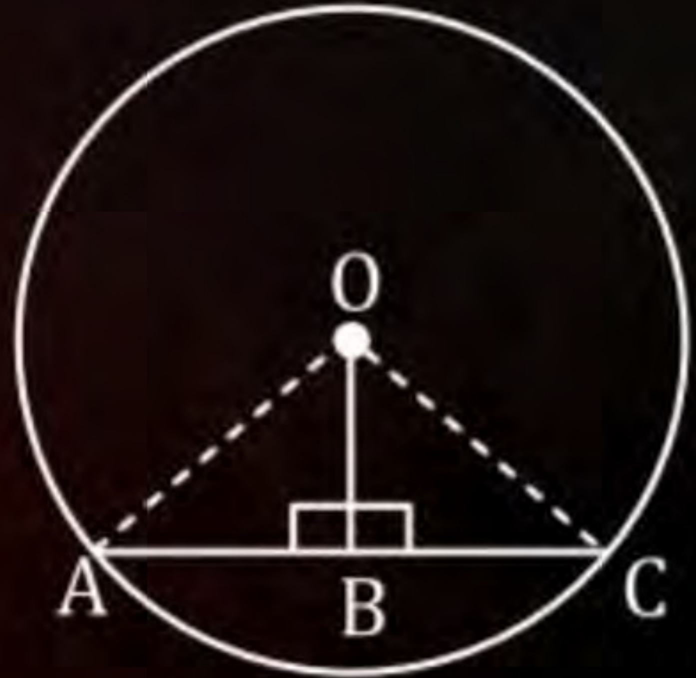
↳ chord AB bisected at point X.

Hence, proved!!

Theorems related to Circle

Theorem 4: The line drawn through the centre of a circle to bisect a cord is perpendicular to the chord.





In $\triangle OBA$ & $\triangle OBC$

S S S Rule

$AB = BC$ (OB bisect AC at point B)

$OA = OC$ (Radius)

$OB = OB$ (common)

Thus, $\boxed{\triangle OBA \cong \triangle OBC}$

By CPCT, $\angle OBA = \angle OBC = x$ (let)

since chord AC is a straight line

$\angle OBA + \angle OBC = 180^\circ \Rightarrow x + x = 180^\circ \Rightarrow 2x = 180^\circ \Rightarrow x = \boxed{90^\circ = \angle OBA = \angle OBC}$

$\boxed{OB \perp AC}$

Question

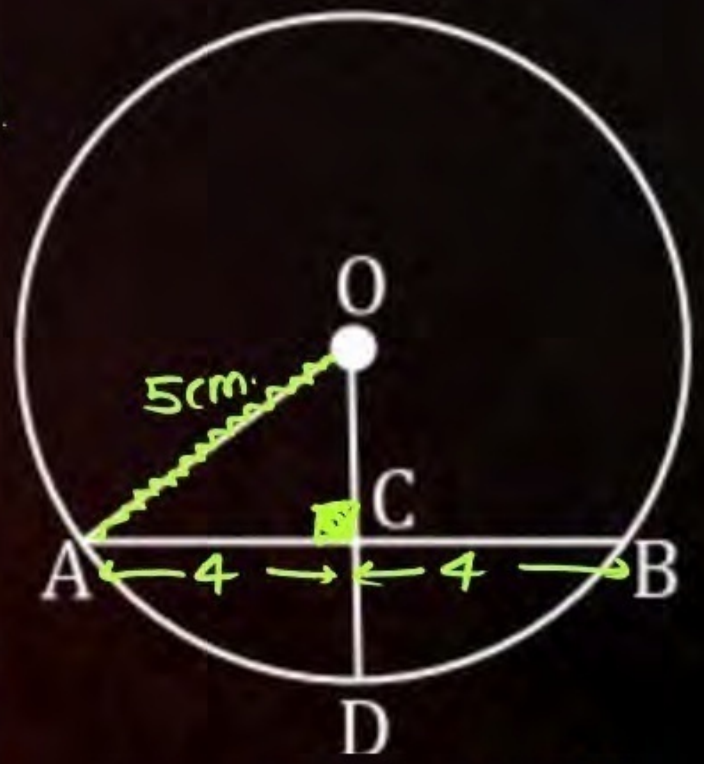
In the given figure, If OA = 5 cm, AB = 8 cm and OD = is perpendicular to AB, then CD is equal to

2

3

4

5



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In the given figure, If OA = 5 cm, AB = 8 cm and OD = is perpendicular to AB, then CD is equal to

A 2 ✓

B 3

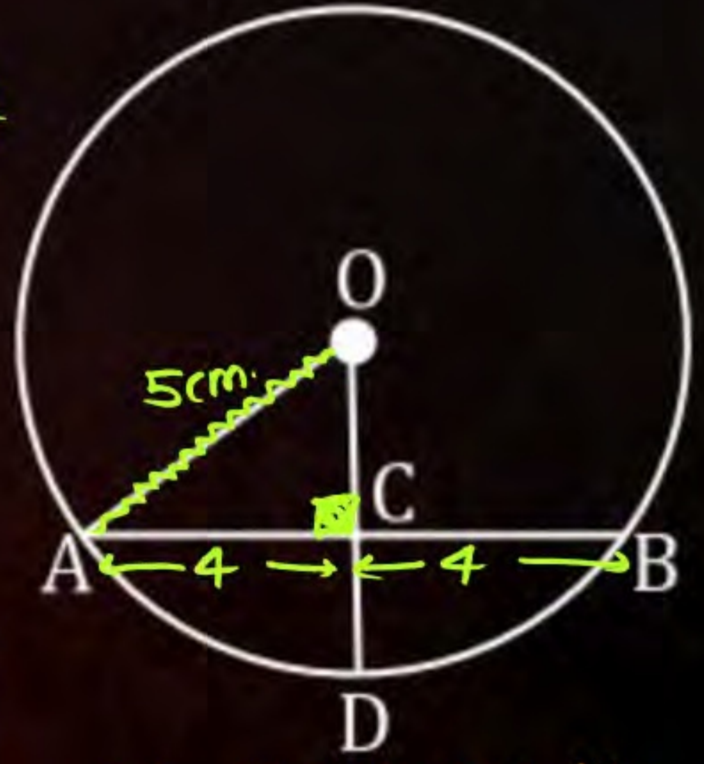
C 4

D 5

Given ⇒ $\boxed{OA = 5\text{cm.}}$

Since $OD \perp AB$ or $\boxed{OC \perp AB}$ → AB will be bisected at at point C.

Therefore, $\boxed{AC = 4\text{ cm}}$



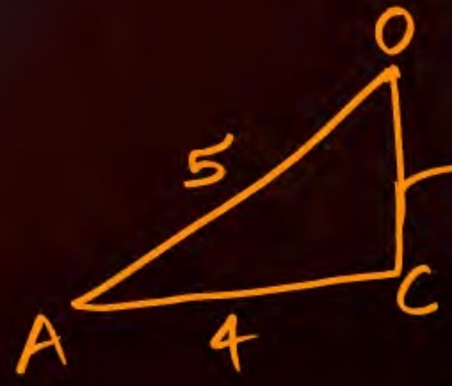
since $OD = OA$ (radius) $= 5\text{cm.}$

Therefore

$CD = OD - OC$

$= 5 - 3$

$= \boxed{2\text{cm}}$



→ Using Pythagoras theorem

$OC = \sqrt{5^2 - 4^2} = \sqrt{9}$

$\boxed{OC = 3\text{cm}}$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

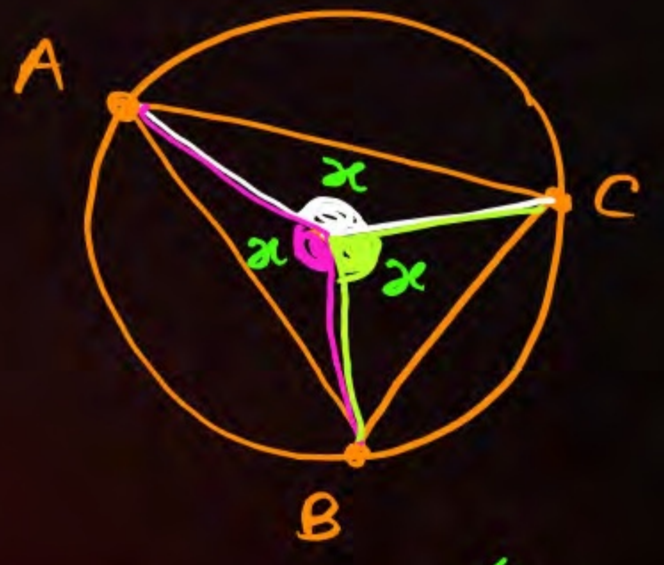
Question

If A, B and C are three points on a circle such that AB = BC = CA and O is the centre of the circle, then find the angle subtended by the chords AB, BC and CA at the centre O.

Question

If A, B and C are three points on a circle such that AB = BC = CA and O is the centre of the circle, then find the angle subtended by the chords AB, BC and CA at the centre O.

$AB = BC = CA$ (chords are of equal lengths)



Over the centre,

$x + x + x = 360°$

$3x = 360°$

$\boxed{x = 120°} \Rightarrow \boxed{\angle AOB = \angle BOC = \angle AOC = 120°}$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

AB is a chord of a circle having centre O. If $\angle AOB = 60^\circ$. Then prove that the chord is of radius length.

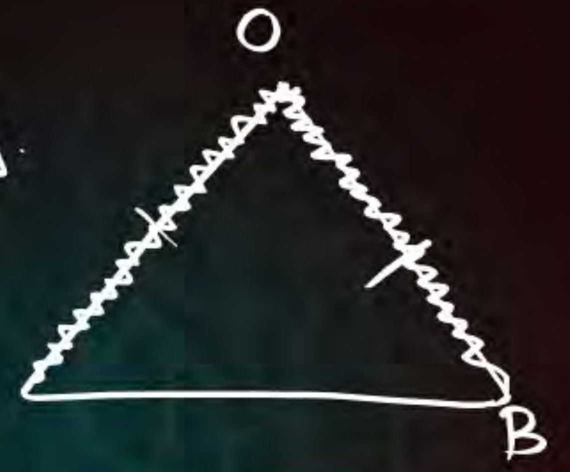
Question

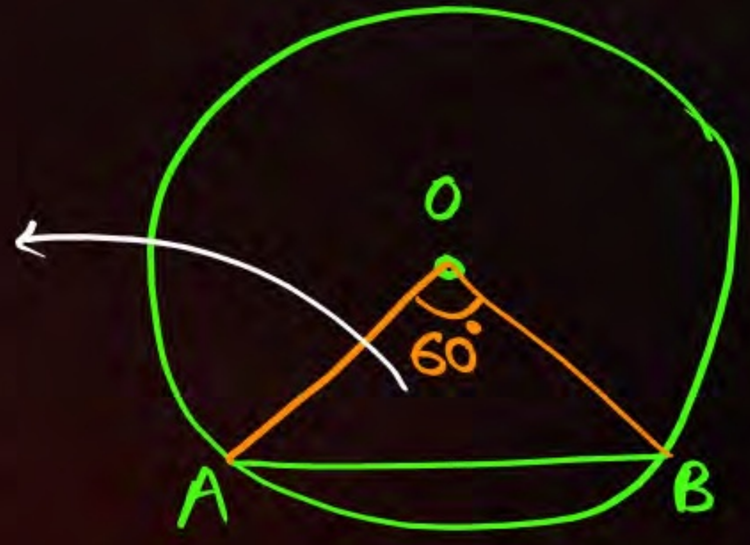
AB is a chord of a circle having centre O. If ∠AOB = 60°. Then prove that the chord is of radius length.

since $OA = OB$ (radius)

Therefore, ΔOAB will be isosceles Δ.

Thus, $\angle OAB = \angle OBA = y$ (let)





In ΔOAB, using angle sum property

$$y + y + 60^\circ = 180^\circ \Rightarrow 2y = 120^\circ \Rightarrow \boxed{y = 60^\circ}$$

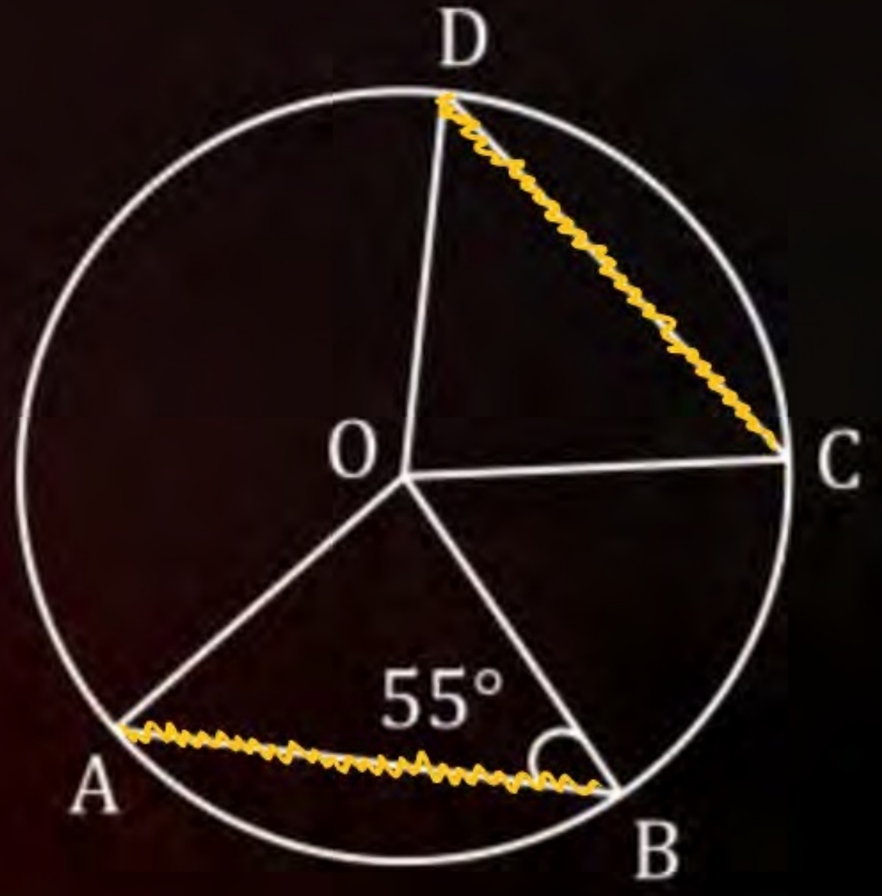
since all the interior angle of given Δ is equal, thus it would be equi. Δ.

$$\boxed{OA = AB = OB = \text{radius}}$$

Hence, proved!!

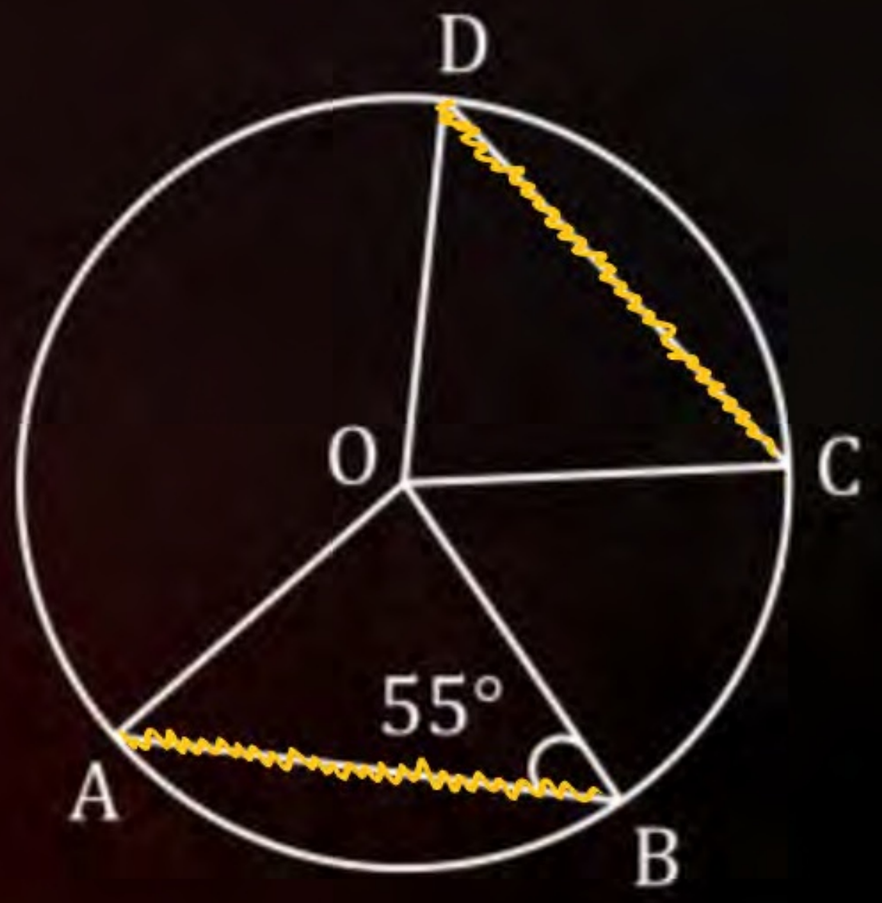
Question

In the given figure, chords AB and CD are equal. If $\angle OBA = 55°$, then find the measure of $\angle COD$.



Question

In the given figure, chords AB and CD are equal. If $\angle OBA = 55°$, then find the measure of $\angle COD$.



since, AB and CD are of equal length

$\angle AOB = \angle COD$ ---(1)

Now, $OA = OB$ (radius)

Thus, $\triangle OAB$ will be isosceles triangle

Therefore, $\boxed{\angle OAB = \angle OBA = 55°}$

using angle sum property in $\triangle OAB$

$\angle AOB + 55° + 55° = 180° \Rightarrow \angle AOB = 70°$

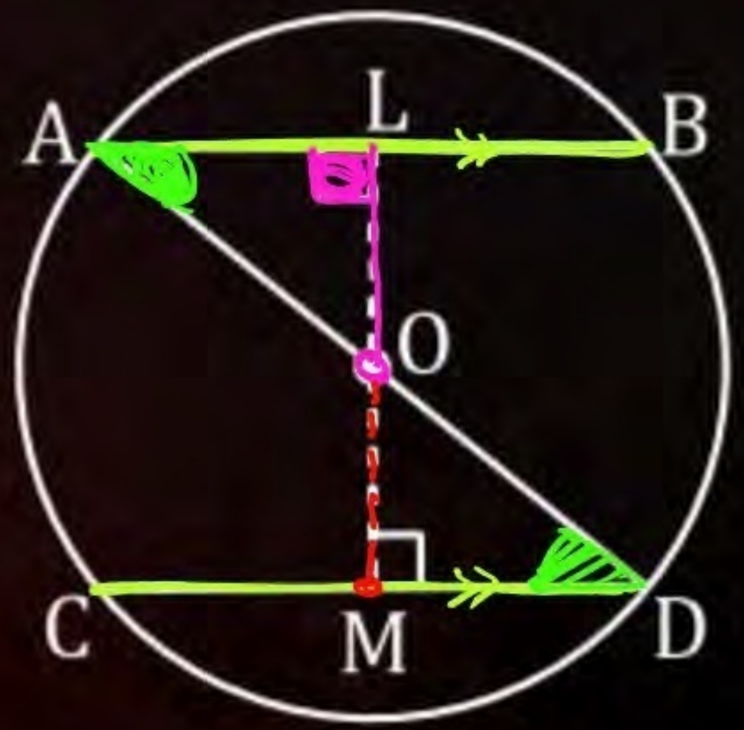
From eqn (1)

$\boxed{\angle COD = \angle AOB = 70°}$ ✓

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

AB and CD are two parallel chords of a circle whose diameter is AD. Prove that AB = CD.



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

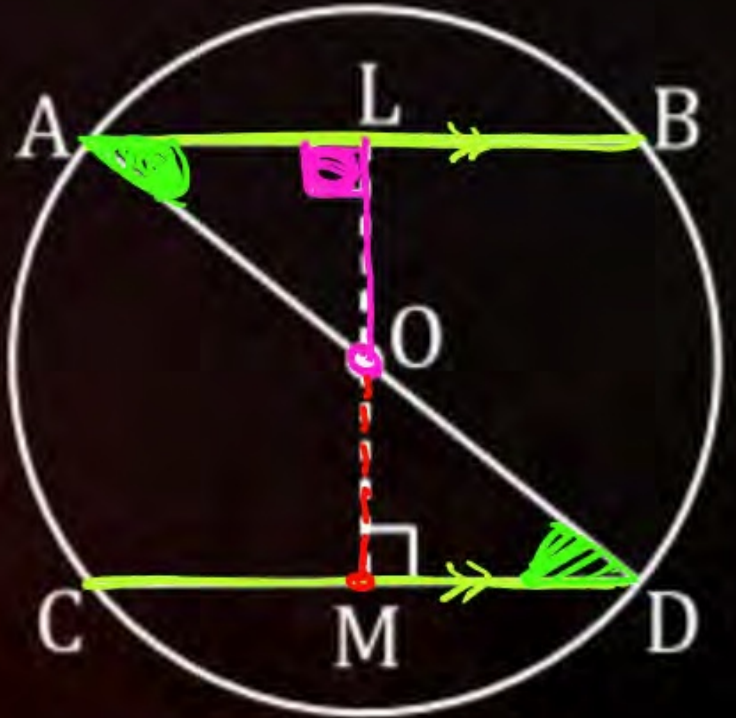
Question

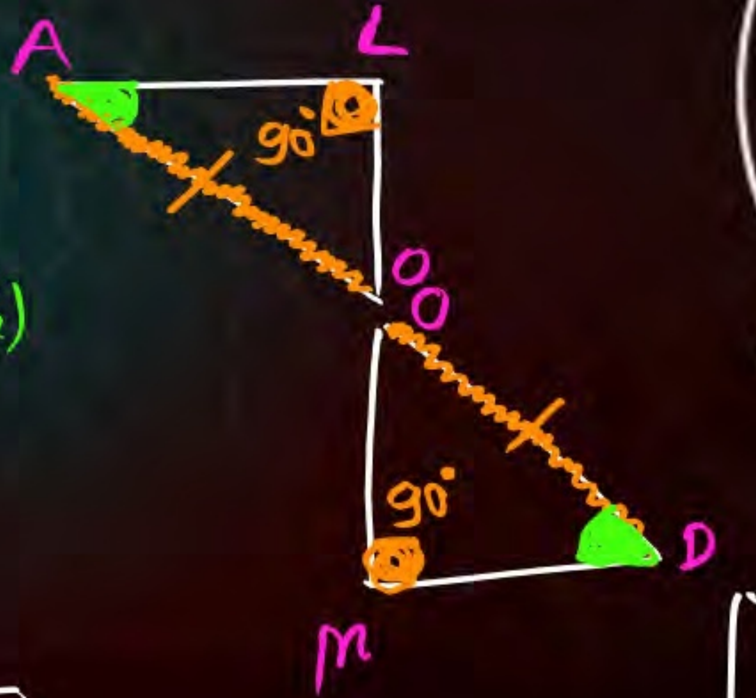
AB and CD are two parallel chords of a circle whose diameter is AD. Prove that AB = CD.

Given:- $\boxed{AB \parallel CD} \rightarrow \angle LAO = \angle ODM$ (Alternate Interior angle)

contruction: $\boxed{OL \perp AB}$

since $\boxed{AB \parallel CD}$, therefore, $\boxed{OL \perp CD} \rightarrow \boxed{OM \perp CD}$





In $\triangle OLA$ and $\triangle OMD$

$\boxed{\begin{matrix} A \\ A \\ S \end{matrix}}$ congruence Rule
$\left\{ \begin{array}{ll} \angle OLA = \angle OMD & (90^\circ \text{ by construction}) \\ \angle LAO = \angle MDO & (\text{Alternate Interior angle}) \\ OA = OB & (\text{Radius}) \end{array} \right.$

Thus, $\boxed{\triangle OLA \cong \triangle OMD}$

By CPCT, $\boxed{AL = MD} \rightarrow \boxed{2AL = 2MD} \rightarrow \boxed{AB = CD}$ Hence proved!!

$OL \perp AB \Rightarrow$ AB bisected at L

$\Rightarrow \boxed{2AL = AB}$

$OM \perp CD \Rightarrow$ CD bisected at M

$\boxed{2MD = CD}$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Equal Chords and their Distances from the Centre

Statement : The length of the perpendicular from a point to a line is the distance of the line from the point.

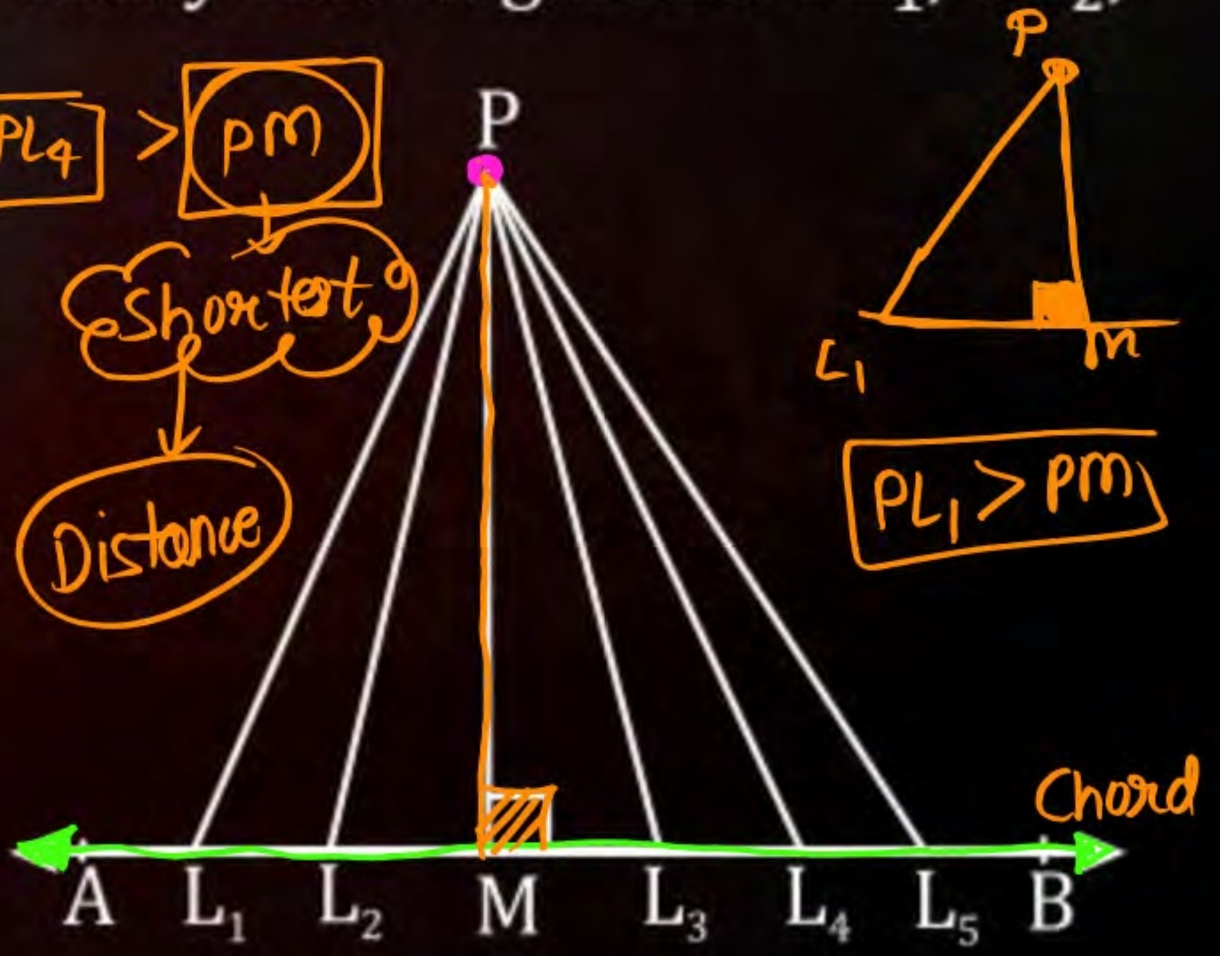
Let P be any point and AB be any line, as shown in the figure. There are infinite numbers of points on a line. We will have infinitely many line segments PL_1, PL_2, PM, PL_3, PL_4, etc., by joining these points to P.

PL_1 or PL_2 or PL_3 or PL_4 > $\boxed{PM}$

Shortest

Distance

From all of these line segments, the perpendicular (PM) from P to AB is the least in length. The line segment having the least length (PM) gives the distance of the point P from the line AB. The perpendicular to a line from the point gives the distance of the line from the point.



$\boxed{PL_1 > PM}$

Theorems related to Circle

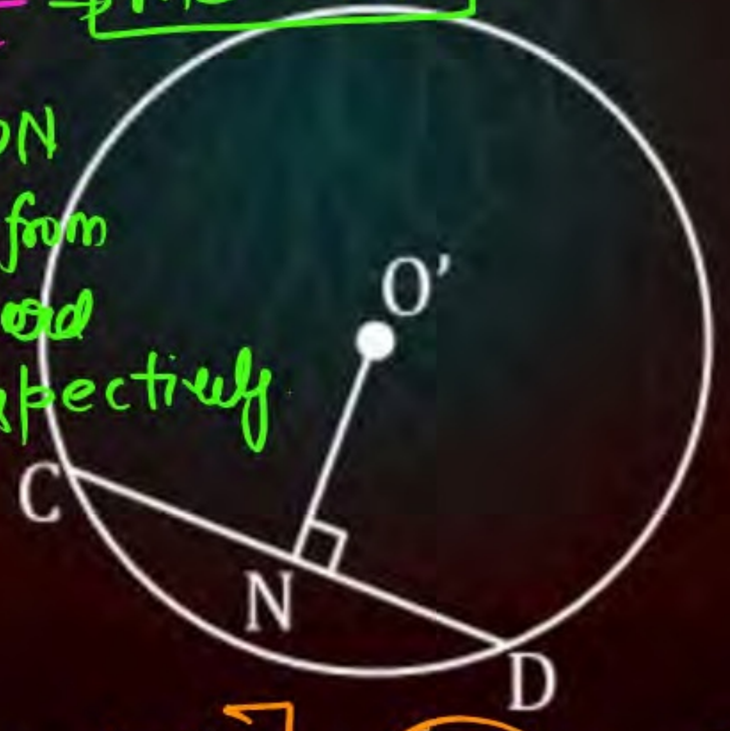
Theorem 5: Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres). To prove: $\boxed{OM = ON}$, $\boxed{O'N = OM}$

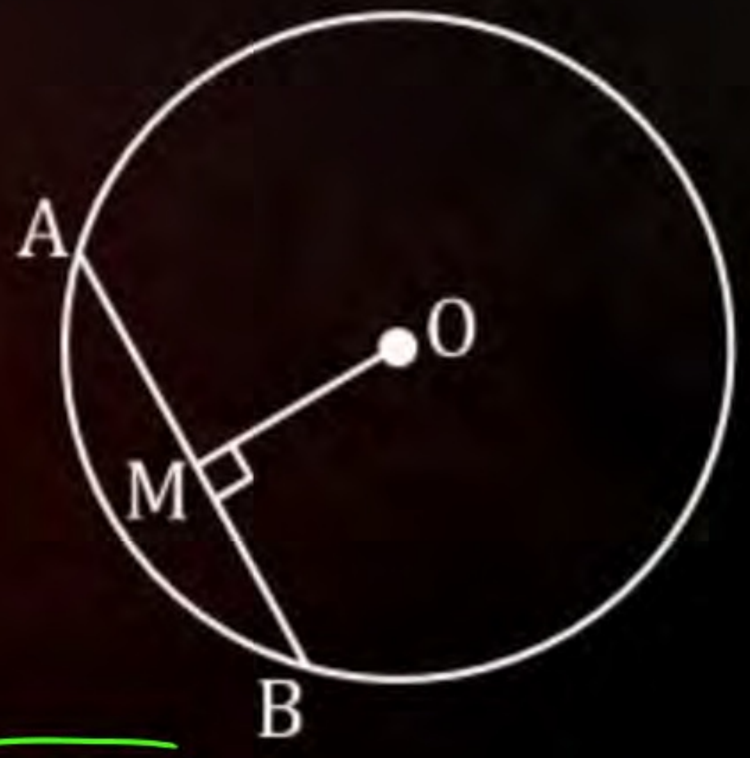
Given: $AB = CD$

$\Rightarrow \frac{AB}{2} = \frac{CD}{2} \Rightarrow \boxed{MB = ND}$

since, OM and ON are distance from centre to chord AB & CD respectively

$$\boxed{\begin{array}{l} OM \perp AB \\ ON \perp CD \end{array}}$$





In ΔOMD & ΔOND → $\angle OMB = \angle OND$ (90°)
$MB = ND$
$OB = OB$ (radius)

RHS $\Rightarrow \boxed{\Delta OMB \cong \Delta OND}$

By CPCT, $\boxed{OM = ON}$ Hence, proved!!

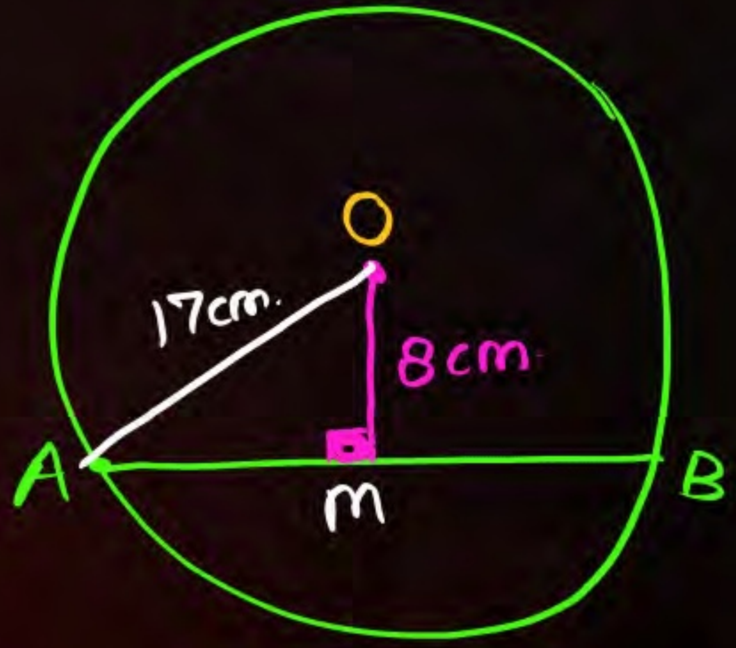
Question

Find the length of a chord which is at a distance of 8 cm from the center of a circle of radius 17 cm.

Question

Find the length of a chord which is at a distance of 8 cm from the center of a circle of radius 17 cm.

Let O is centre of circle having chord as AB



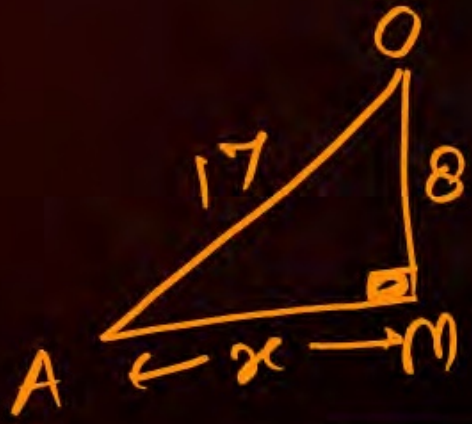
since OM is distance of chord AB from centre.

$\boxed{OM \perp AB}$

Thus, chord AB will be bisected by OM at point M.

$\boxed{AM = MB = x}$ (Let say)

Using pythagoras theorem in ΔOAM



$$(17)^2 = (8)^2 + (x)^2$$

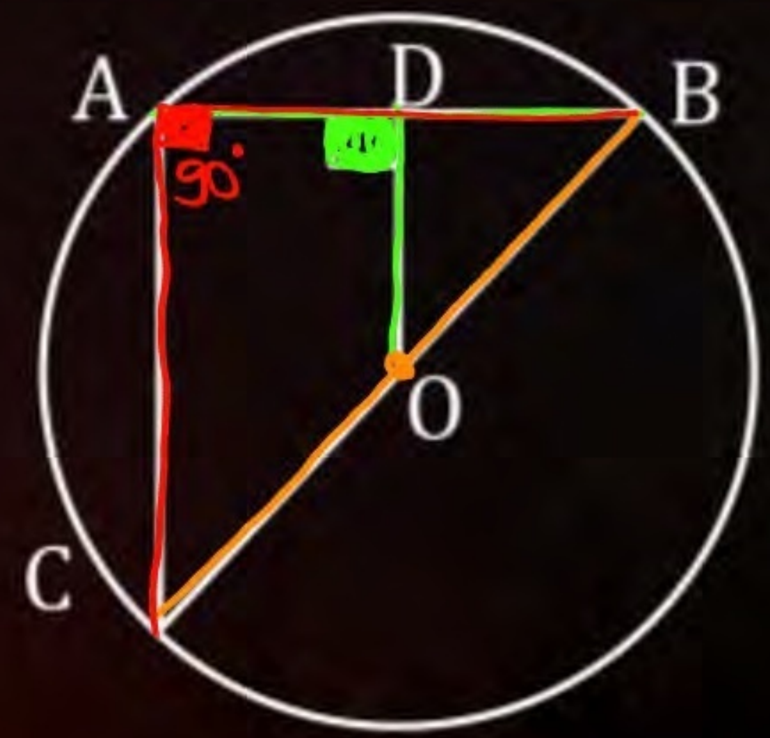
$$x = \sqrt{(17)^2 - (8)^2}$$

$$x = \sqrt{225} \Rightarrow \boxed{x = 15\,cm}$$

Required chord length $AB = 2x = 2 \times 15 = \boxed{30\,cm}$.

Question

In the fig. OD is perpendicular to the chord AB of a circle whose centre is O. If BC is a diameter, show that CA = 2(OD).

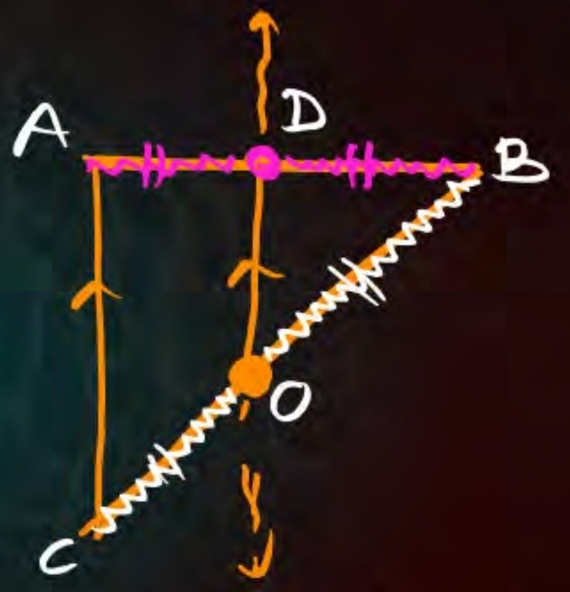


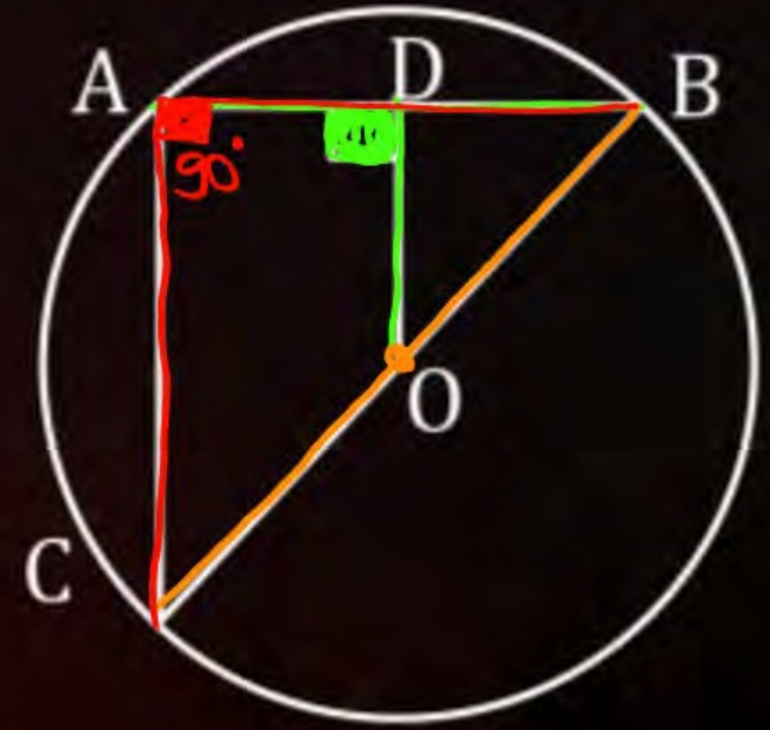
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In the fig. OD is perpendicular to the chord AB of a circle whose centre is O. If BC is a diameter, show that CA = 2(OD).

$\boxed{OD \parallel AC}$





converse of mid-pt theorem.

$\boxed{OB = OC \rightarrow O \text{ is mid point of } BC}$

$\downarrow$

$\boxed{AD = DB}$

using mid-point theorem. $OD \parallel AC$ & O and D are mid points of BC & AB respectively.

$$OD = \frac{1}{2} AC$$

$$\Rightarrow \boxed{2\,OD = CA}$$

Question

In fig. O is the centre of the circle of radius 5 cm. OP$\perp$ AB, OQ$\perp$CD, $\boxed{AB||CD}$, AB = 6 cm and CD = 8 cm. Determine PQ.

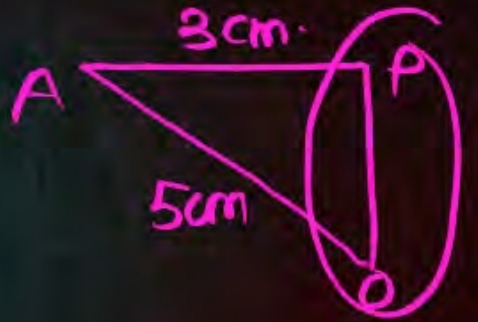
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In fig. O is the centre of the circle of radius 5 cm. OP⊥ AB, OQ⊥CD, $\boxed{AB || CD}$, AB = 6 cm and CD = 8 cm. Determine PQ.

given: $\boxed{OP \perp AB}$ → chord AB will be bisected at pt. P.

$AP = PB = \frac{1}{2} AB \Rightarrow \boxed{AP = \frac{6}{2} = 3\,cm.}$

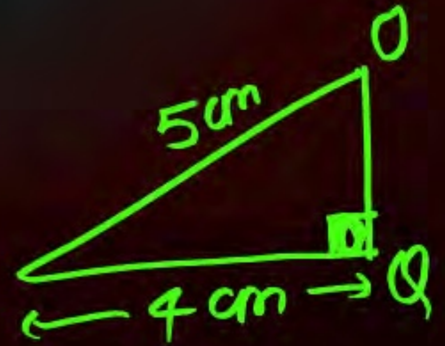


using pythagoras theorem $OP = \sqrt{5^2 - 3^2} = 4\,cm$

$\Rightarrow \boxed{OP = 4\,cm}$

similarily, $OQ = \sqrt{5^2 - 4^2} = 3\,cm$

$\boxed{OQ = 3\,cm}$



AB || CD & OP⊥AB & OQ⊥CD

POQ is a straight line.

Therefore $PQ = 4 + 3 = \boxed{7\,cm}$ Ans.

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

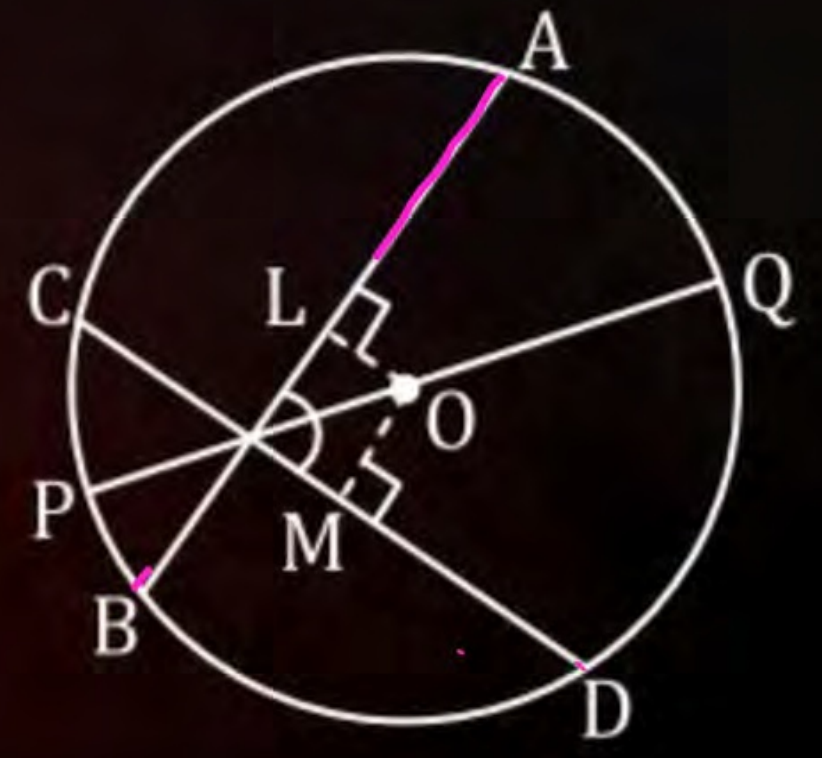
Theorems related to Circle

Theorem 6 : Chords equidistant from the centre of a circle are equal in length.

perpendicular vali distance

$\boxed{OL = OM}$

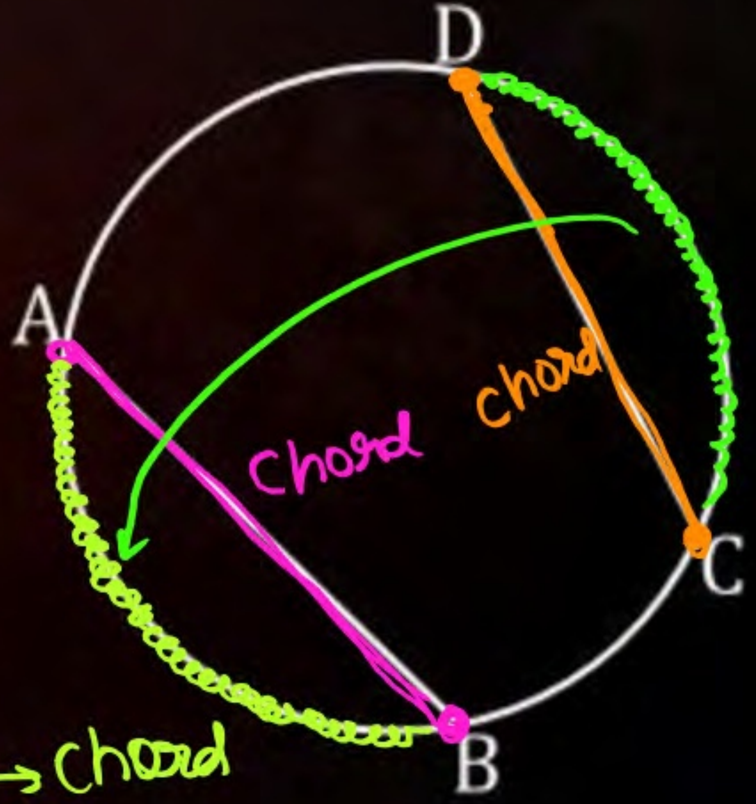
$\boxed{\begin{matrix} OL \perp AB \\ OM \perp CD \end{matrix}} \Longrightarrow \boxed{AB = CD}$



How Chords and Respective Arcs are Related??

A chord other than diameter of a circle cuts it into two arcs – one major and other minor. If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.

We can verify this fact by cutting the arc, corresponding to the chord CD from the circle along CD and put it on the corresponding arc made by equal chord AB. You will find that the arc CD superimpose the arc AB completely (see Fig.). This shows that equal chords make congruent arcs and conversely congruent arcs make equal chords of a circle.



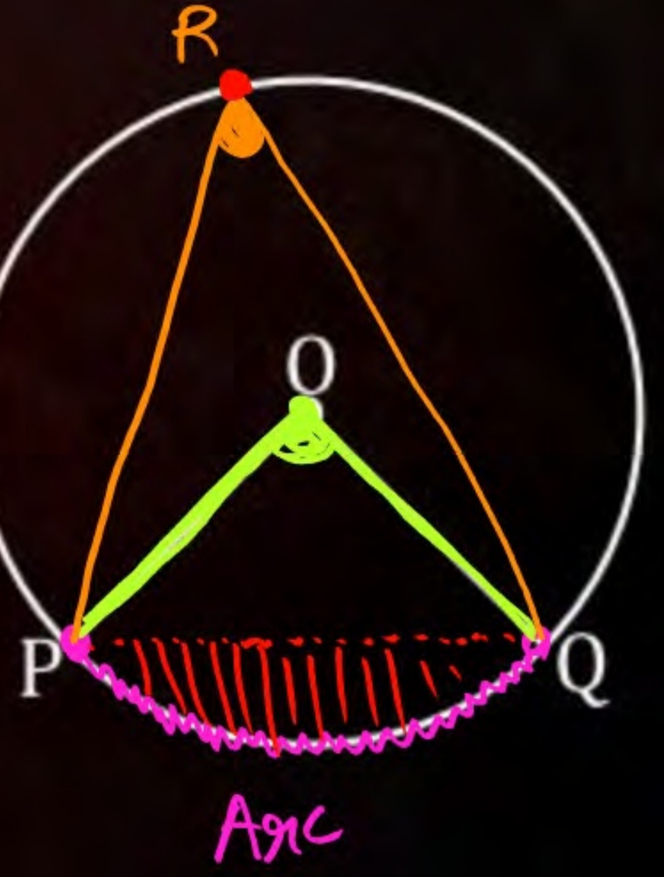
$\overline{AB} \rightarrow$ chord

$\overset{\frown}{AB} \rightarrow$ Arc

Angle Subtended by an Arc of a Circle

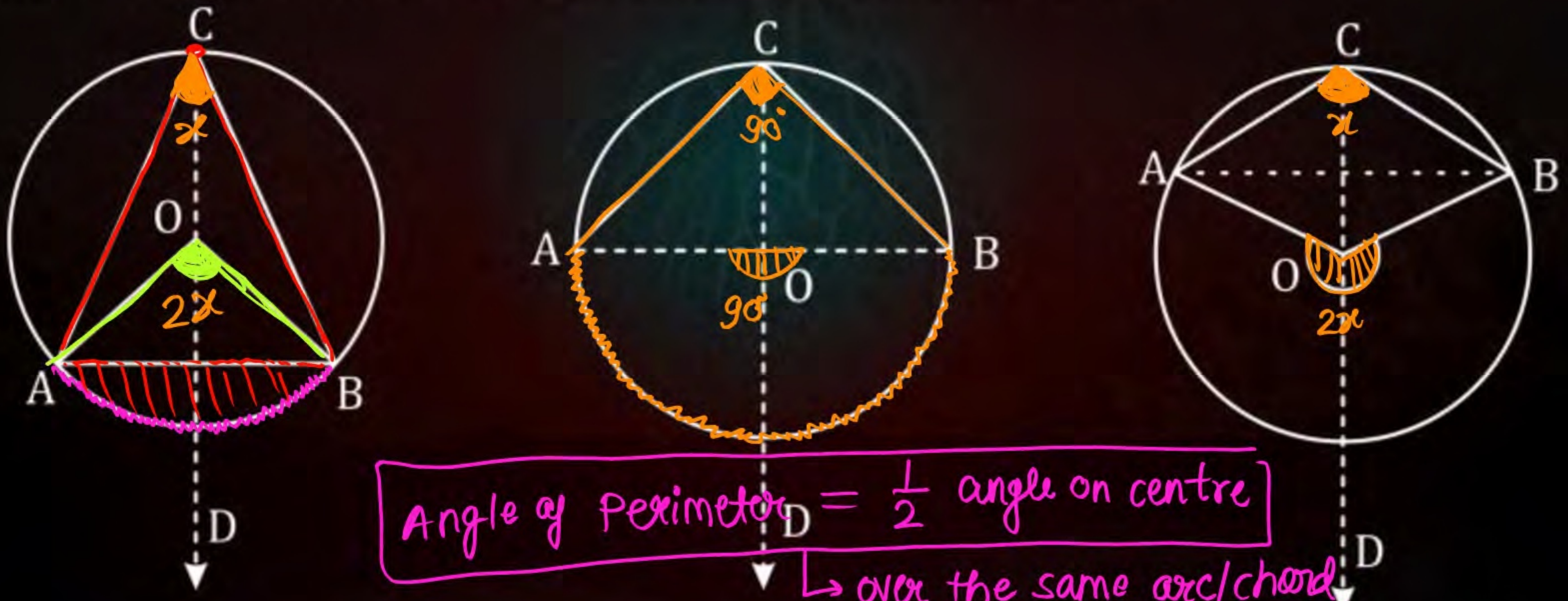
The angle subtended by an arc at the centre is defined to be angle subtended by the corresponding chord at the centre in the sense that the minor arc subtends the angle and the major arc subtends the reflex angle. Therefore, in below figure, the angle subtended by the minor arc PQ at O is ∠POQ and the angle subtended by the major arc PQ at O is reflex angle POQ.

Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre. Therefore, the angle subtended by a chord of a circle at its centre is equal to the angle subtended by the corresponding (minor) arc at the centre. The following theorem gives the relationship between the angles subtended by an arc at the centre and at a point on the circle.



Theorems related to Circle

Theorem 7: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

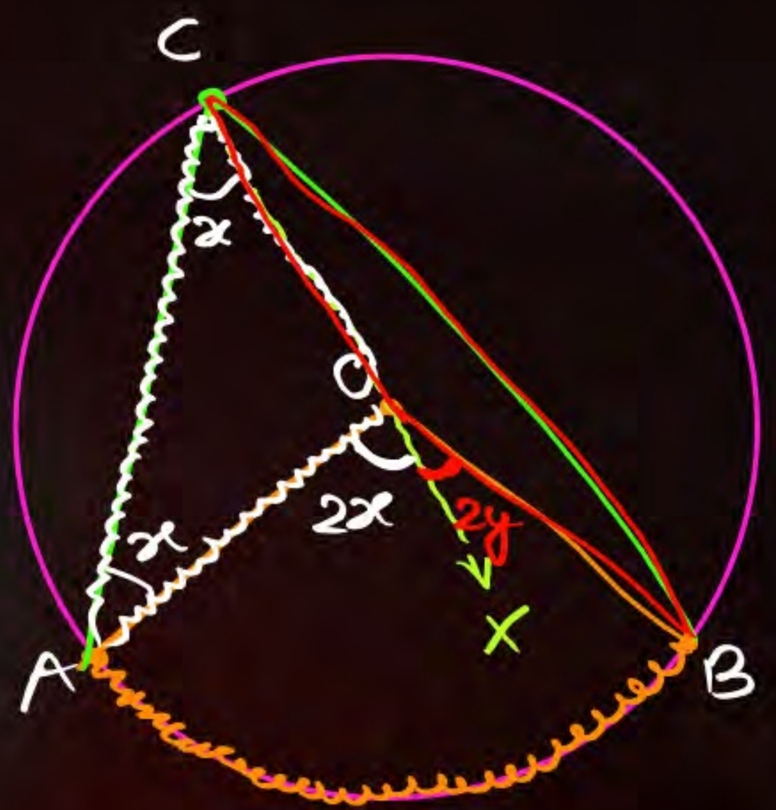


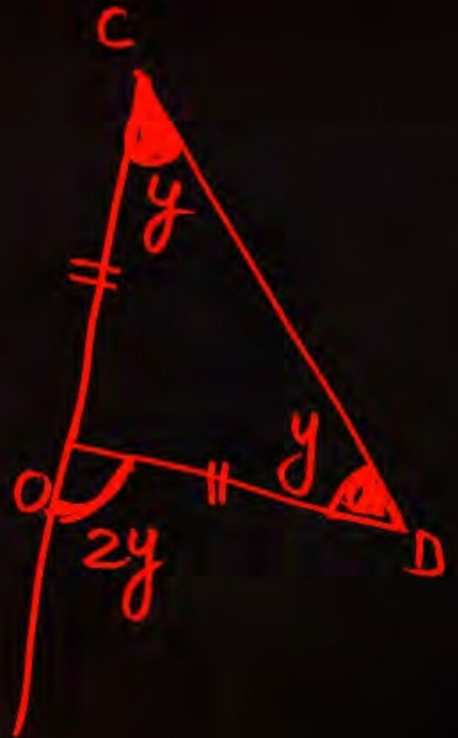
$\boxed{\text{Angle of perimeter} = \frac{1}{2} \text{ angle on centre}}$

↳ over the same arc/chord

Proof:- Joint C to O and extend till X.

Exterior = sum of opposite Interior,





Now, $\angle ACB = (x+y)$

$\angle AOB = 2x+2y$

$\angle AOB = 2(x+y)$

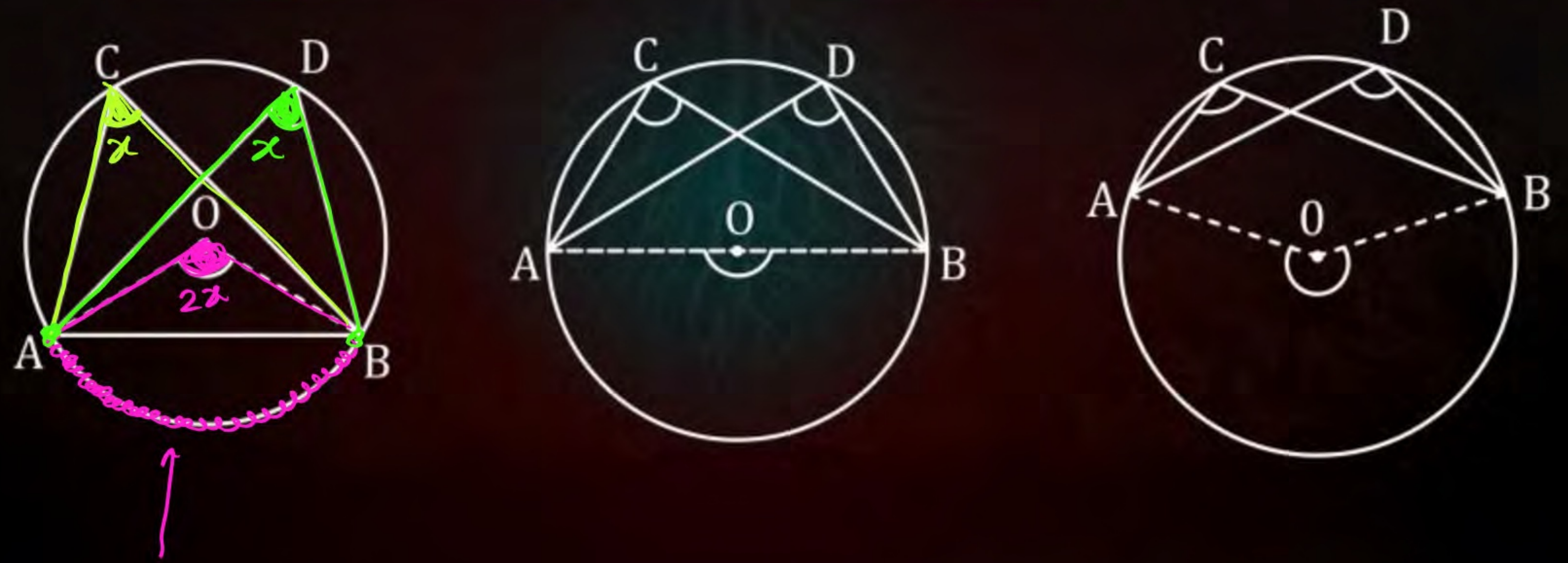
$\angle AOB = 2\angle ACB$

$\Rightarrow \boxed{\angle ACB = \frac{1}{2}\angle AOB}$

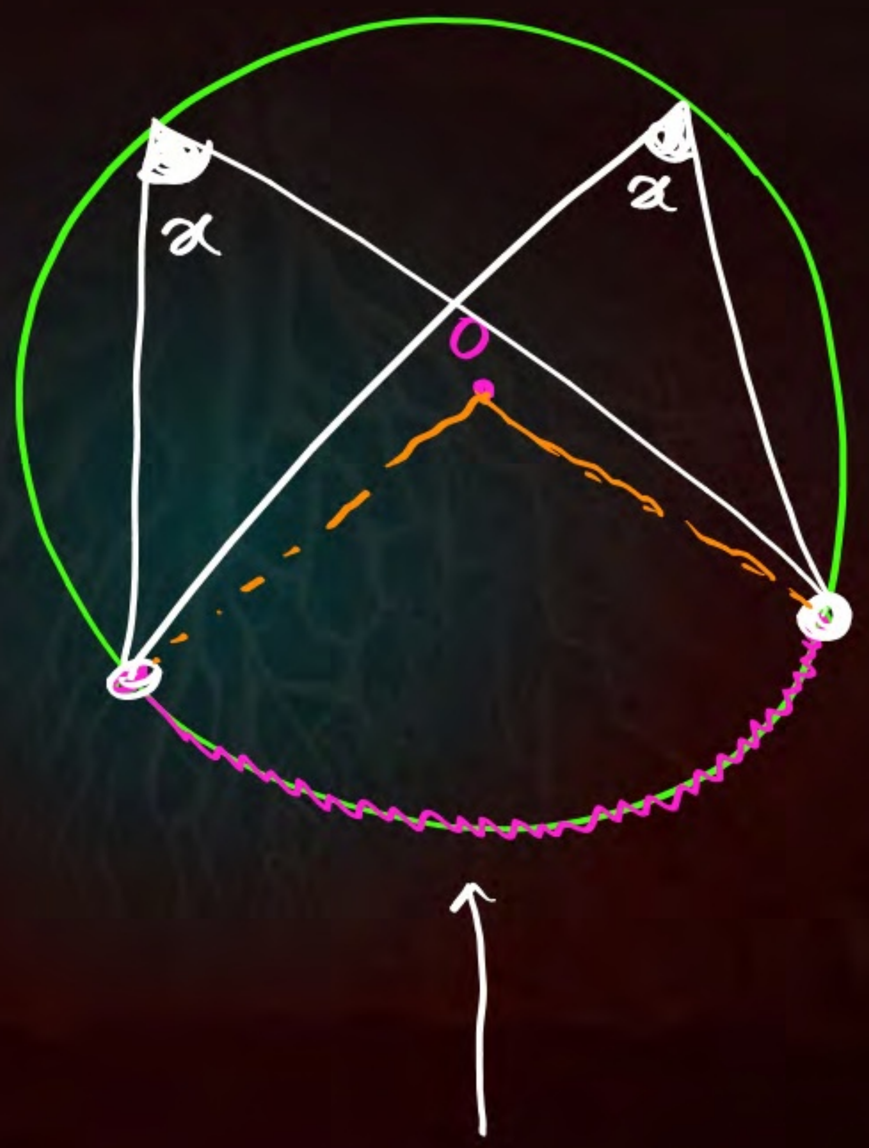
angle on circumference $= \frac{1}{2}$ (Angle on centre) → over the same arc

Theorems related to Circle

Theorem 8: $\boxed{\text{Angles in the same segment of a circle are equal}}$ → over the same arc/chord.



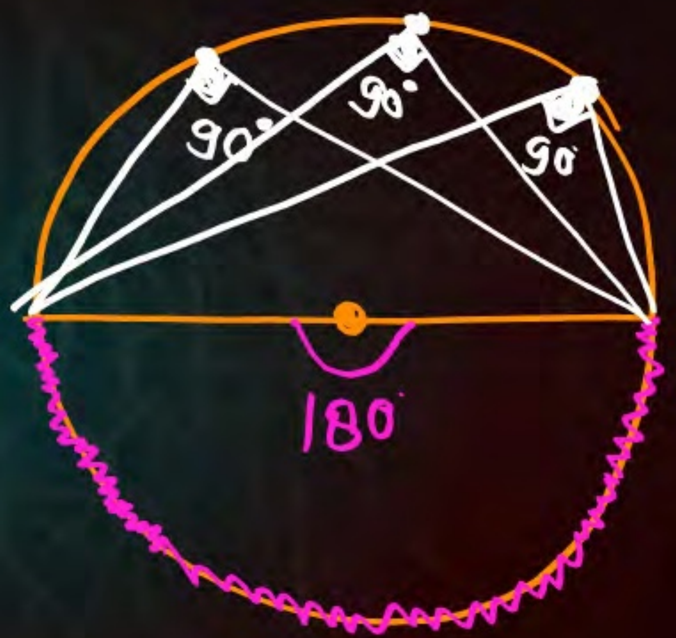
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR



x
x
O

Theorems related to Circle

Theorem (Result of Theorem 8): Angle in a semicircle is a right angle



Question

In the given figure, calculate the measure of minor $\angle AOC$.

30°

40°

70°

140°

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In the given figure, calculate the measure of minor $\angle AOC$.

A 30°

B 40°

C 70°

D 140°

$\angle ABC = 30° + 40° = 70°$

$\angle ABC = \frac{1}{2} \angle AOC$ (minor)

$\angle AOC_{minor} = 2 \angle ABC$

$= 2 \times 70°$

$= \boxed{140°}$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

A Chord of a circle is equal to the radius of the circle find the angle subtended by the chord at a point on the minor arc and the major arc respectively.

120°, 60°

150°, 30°

60°, 60°

120°, 120°

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

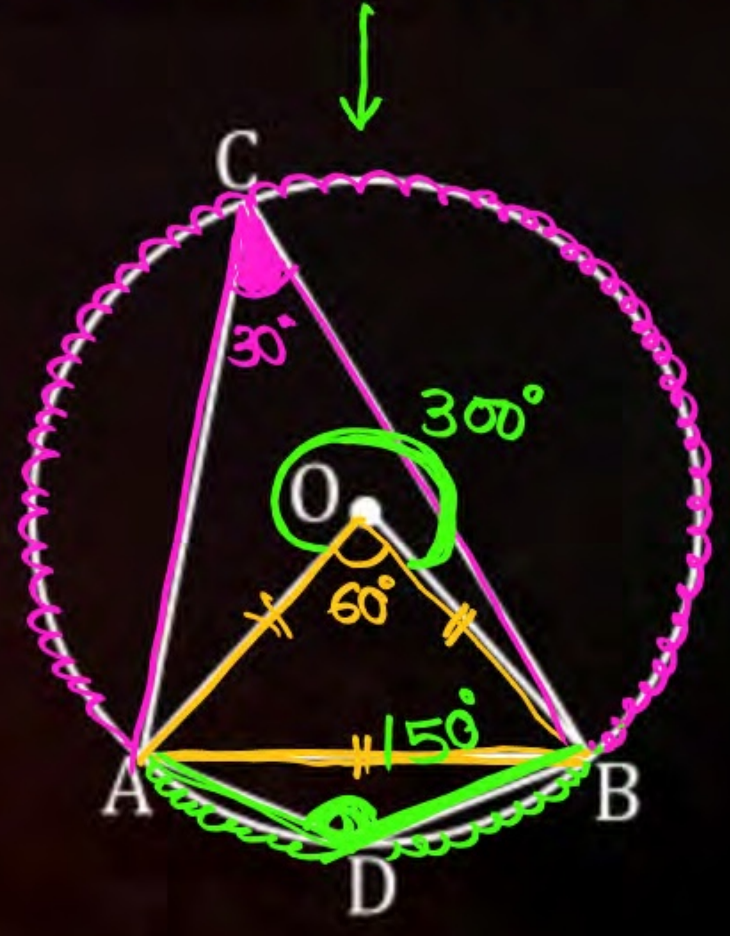
A Chord of a circle is equal to the radius of the circle find the angle subtended by the chord at a point on the minor arc and the major arc respectively.

A. 120°, 60°

B. 150°, 30°

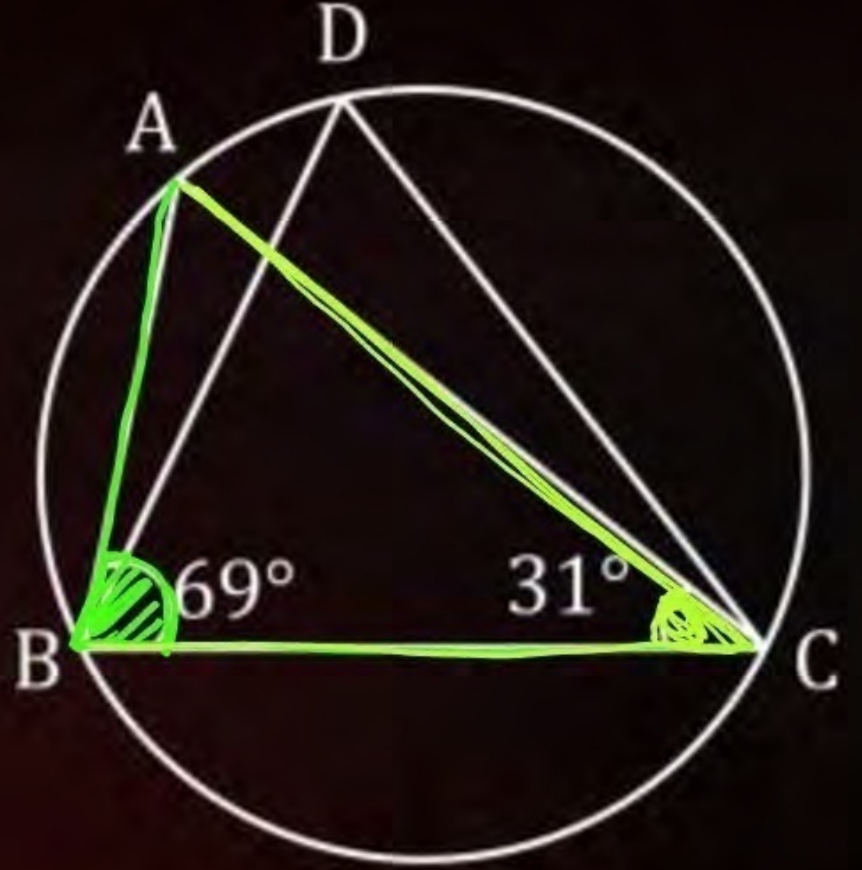
C. 60°, 60°

D. 120°, 120°



Question

In fig. $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



90°

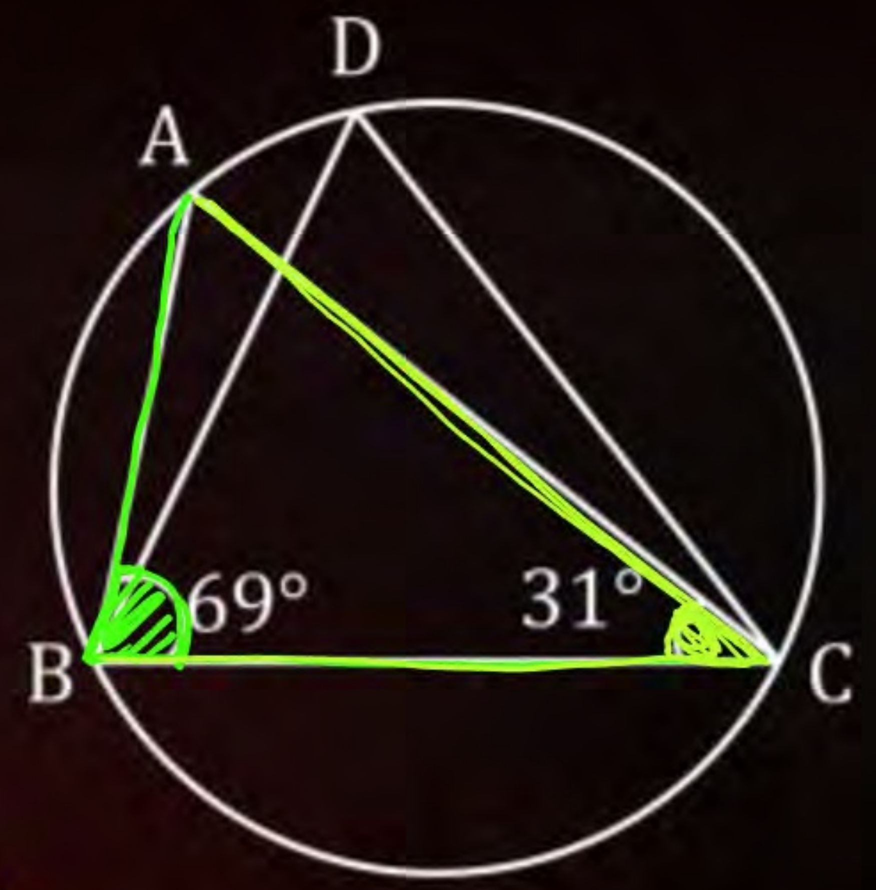
80°

69°

31°

Question

In fig. $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



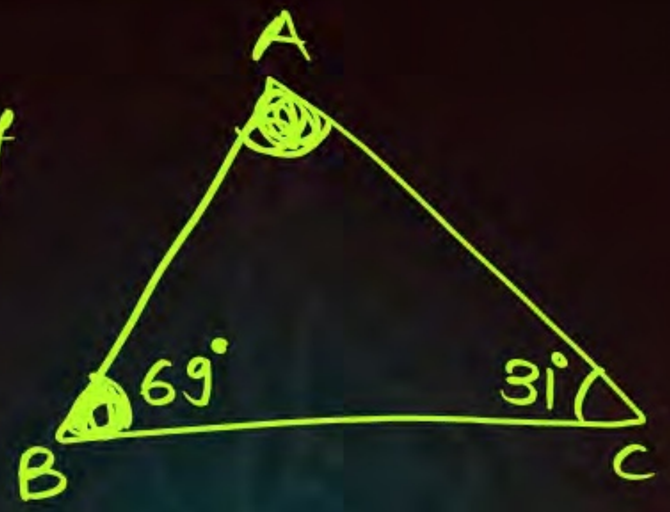
- A 90°
- B 80° ✓
- C 69°
- D 31°

using angle sum property

$\angle BAC = 180° - 69° - 31°$

$= 180° - 100$

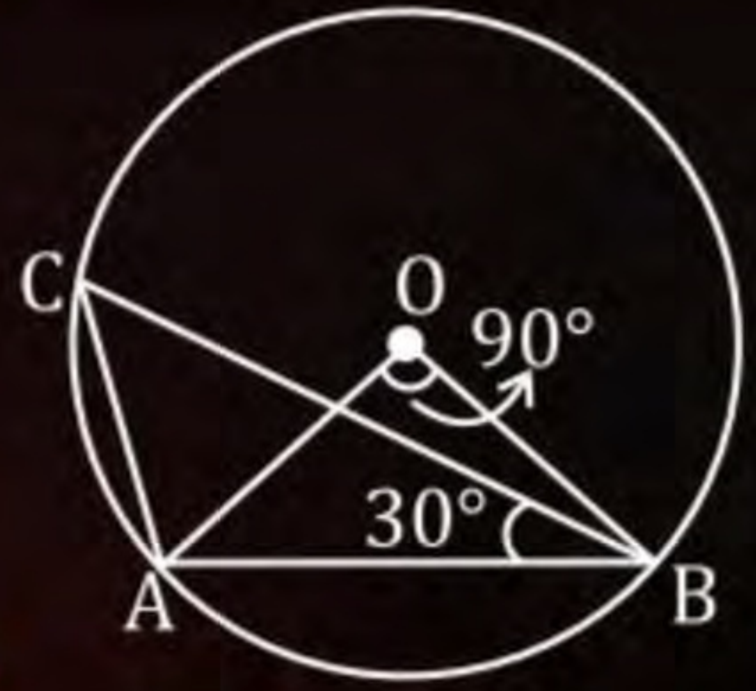
$= \boxed{80°}$



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

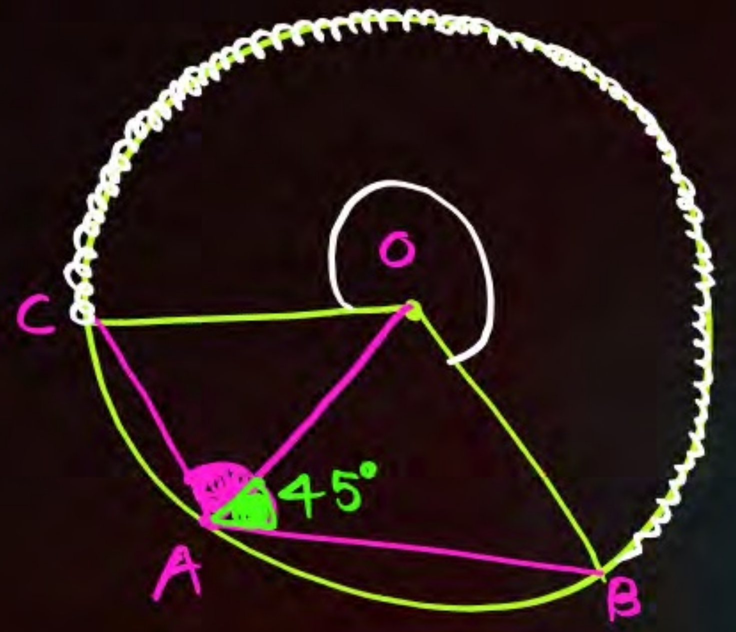
In the given figure, $\angle AOB = 90°$ and $\angle ABC = 30°$, then find $\angle CAO$

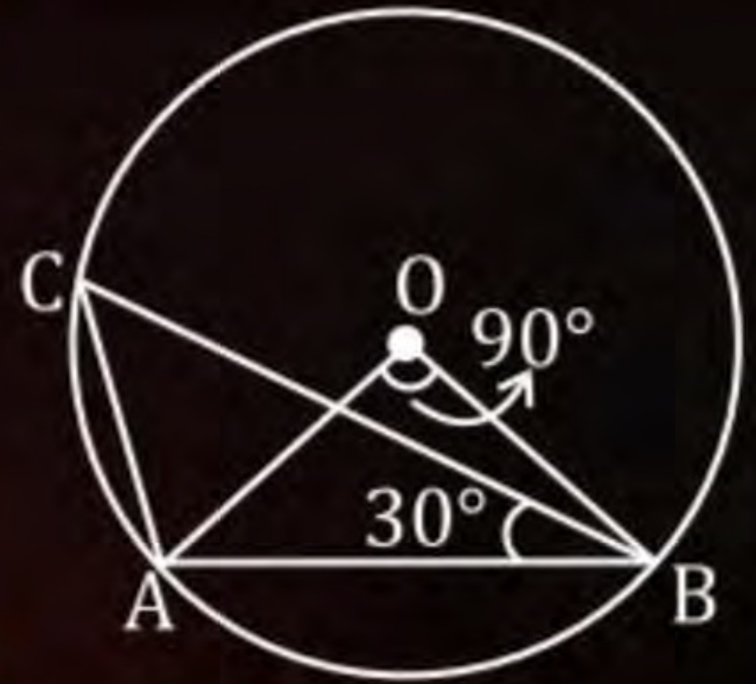


VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

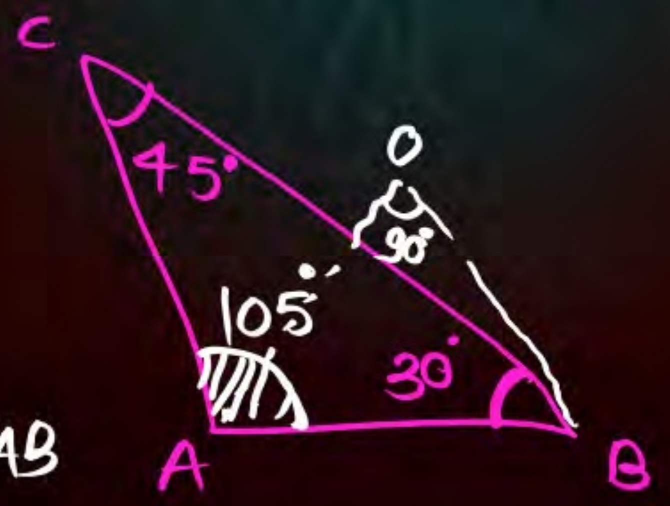
In the given figure, $\angle AOB = 90°$ and $\angle ABC = 30°$, then find $\angle CAO$





$\angle CAB = 180° - 45° - 30°$

$= \boxed{105°}$



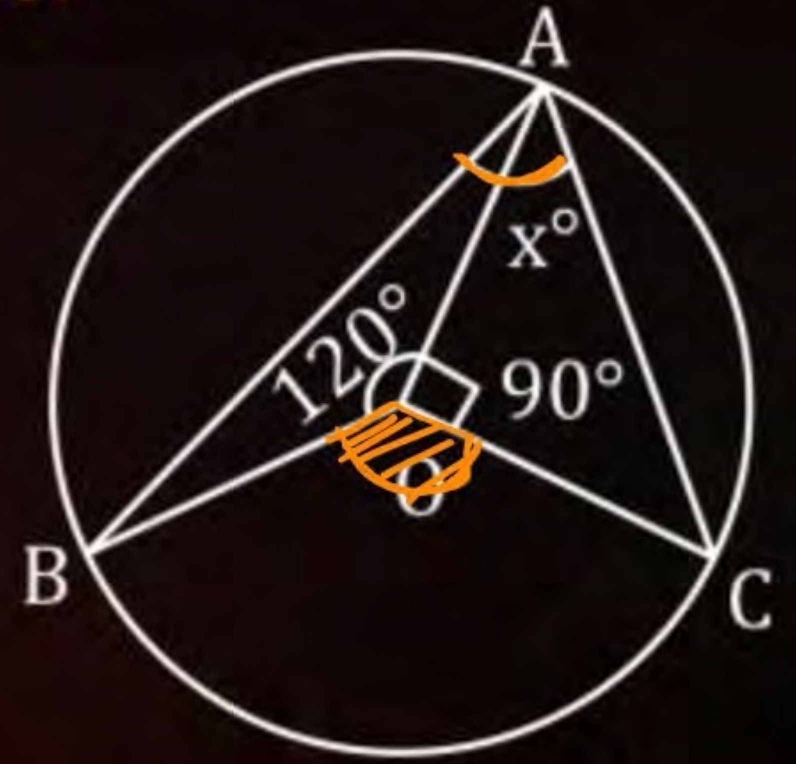
$\angle CAO = \angle CAB - \angle OAB$

$= 105° - 45°$

$= \boxed{60°}$ Ans

Question

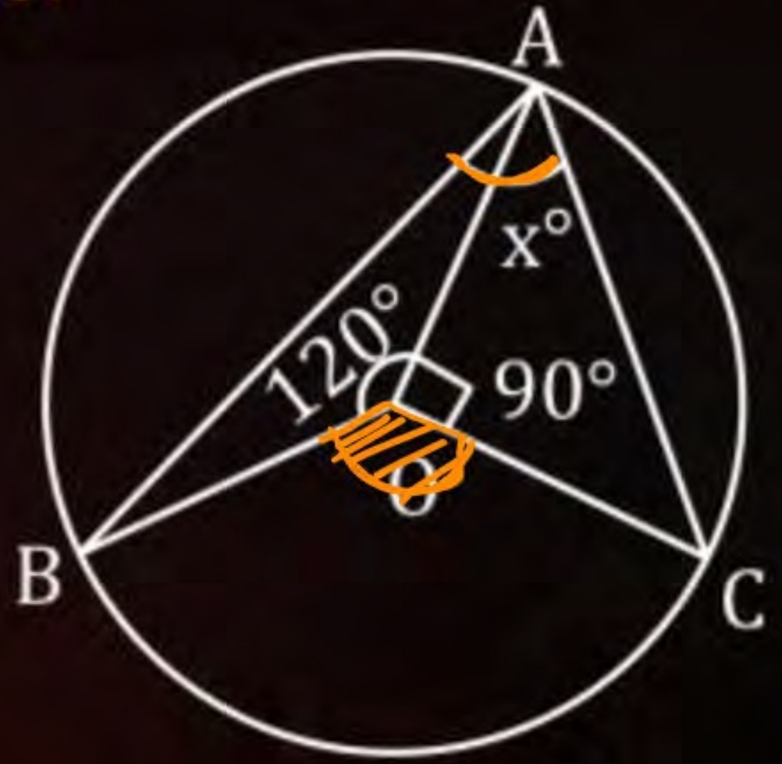
If O is the centre, find the value of x in each of the following figure:



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

If O is the centre, find the value of x in each of the following figure:



$\angle BOC = 360° - 120° - 90°$

$= 360° - 210°$

$\angle BOC = 150°$

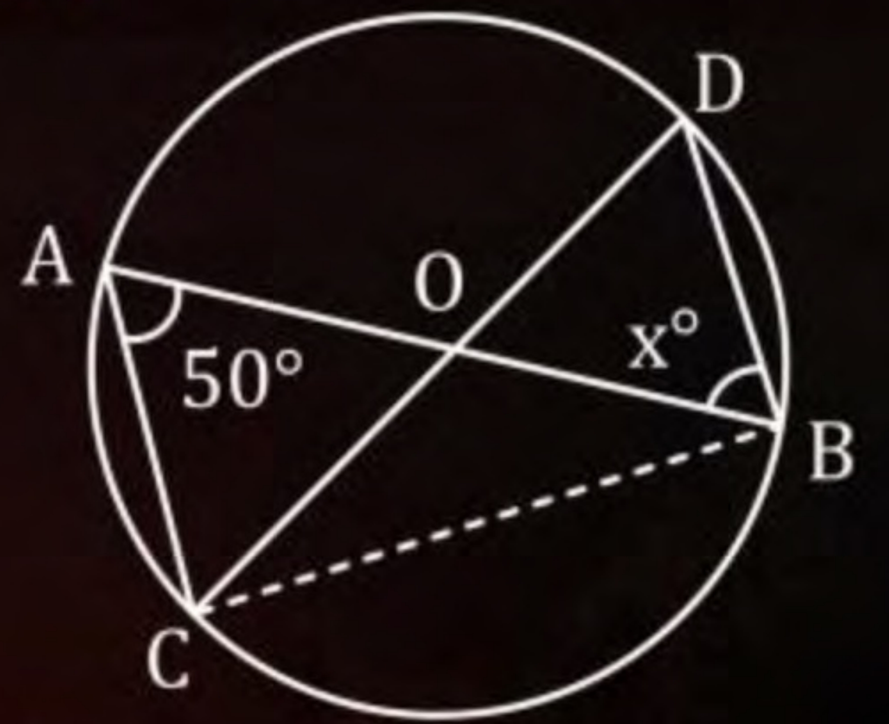
$x = \frac{1}{2}\angle BOC = \frac{1}{2} \times 150°$

$= 75°$

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

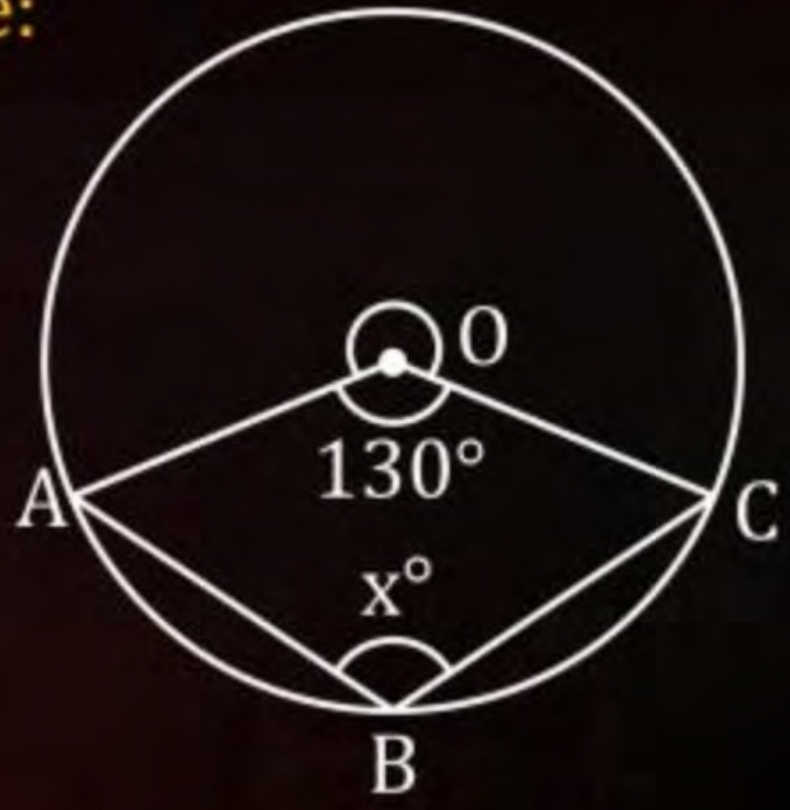
Question

If O is the centre, find the value of x in each of the following figure:



Question

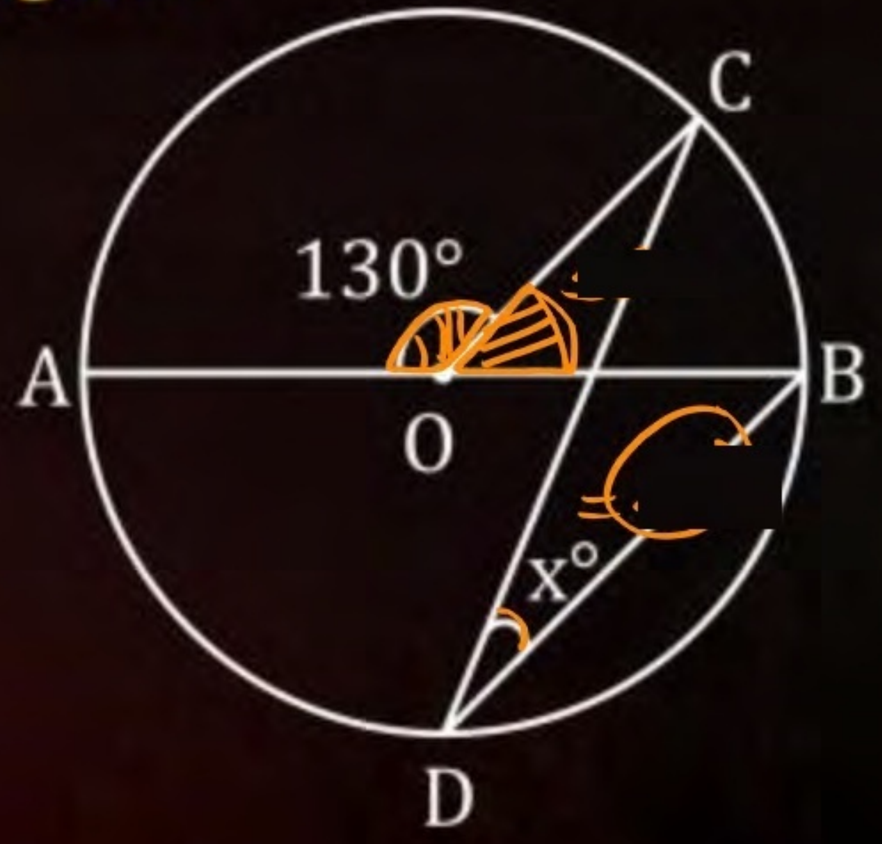
If O is the centre, find the value of x in each of the following figure:



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

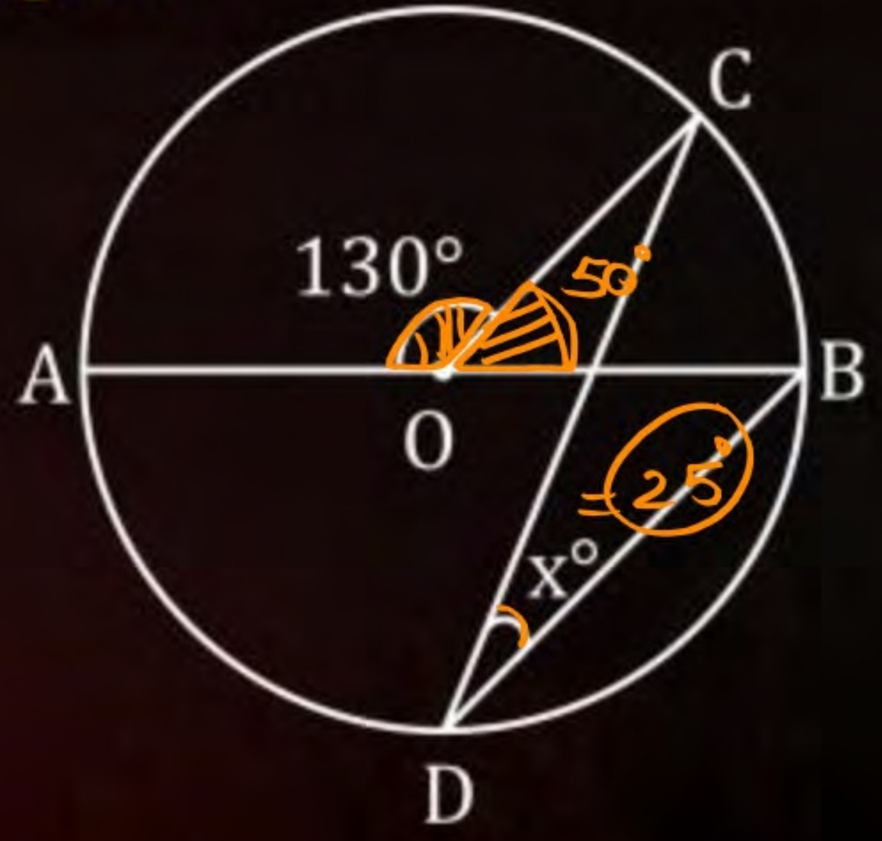
Question

If O is the centre, find the value of x in each of the following figure:



Question

If O is the centre, find the value of x in each of the following figure:



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

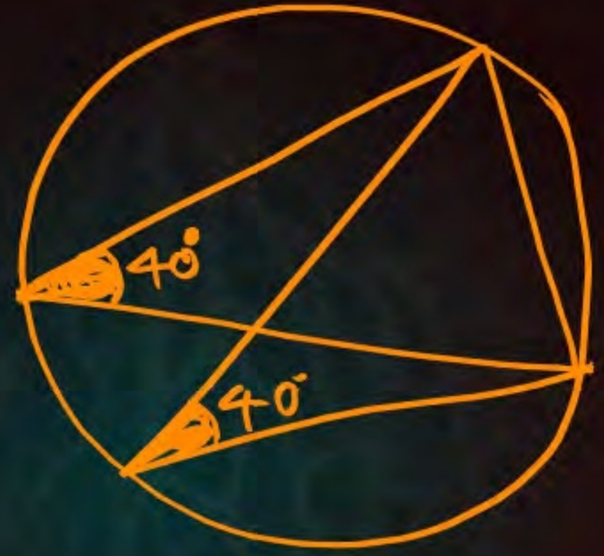
Question

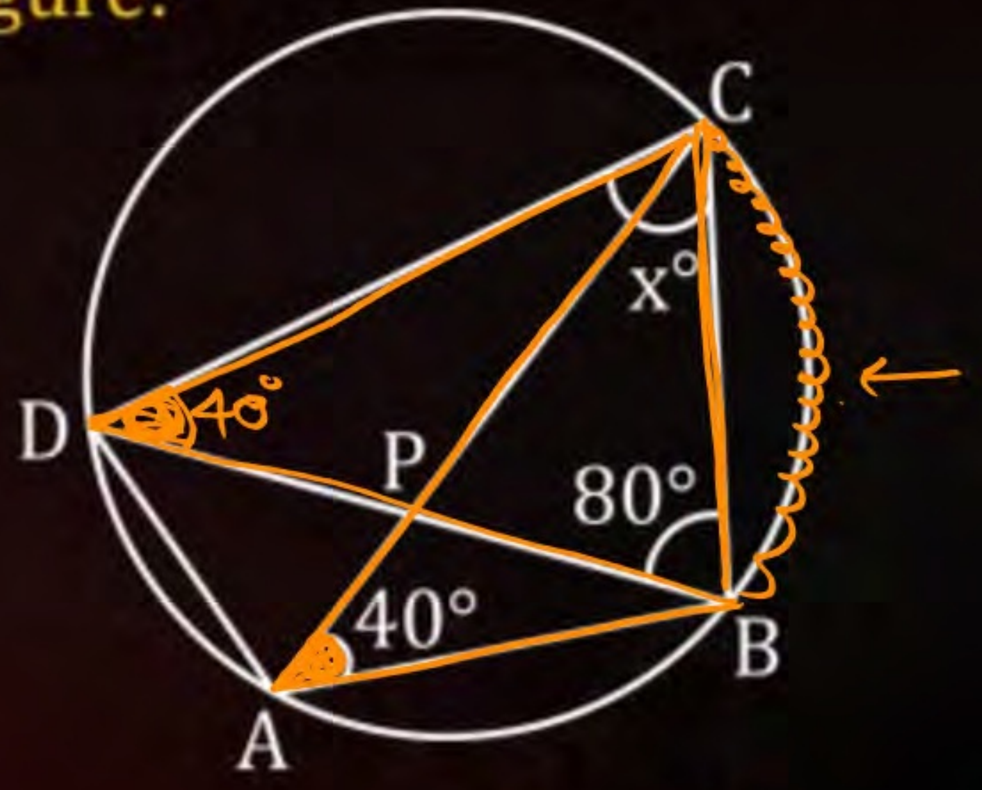
If O is the centre, find the value of x in each of the following figure:

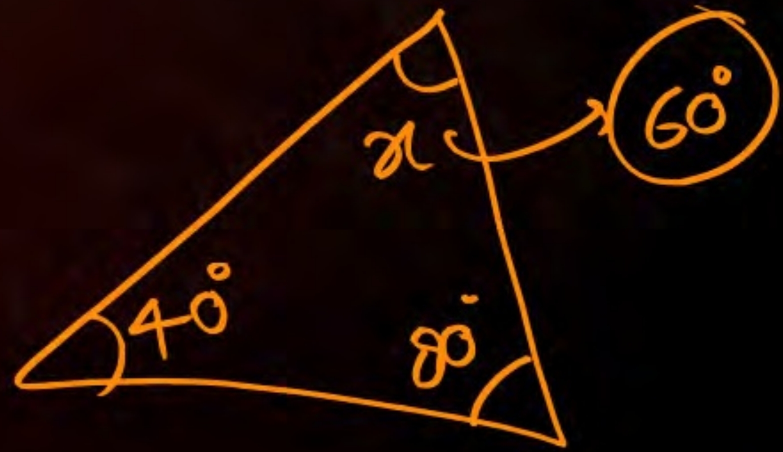
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

If O is the centre, find the value of x in each of the following figure:

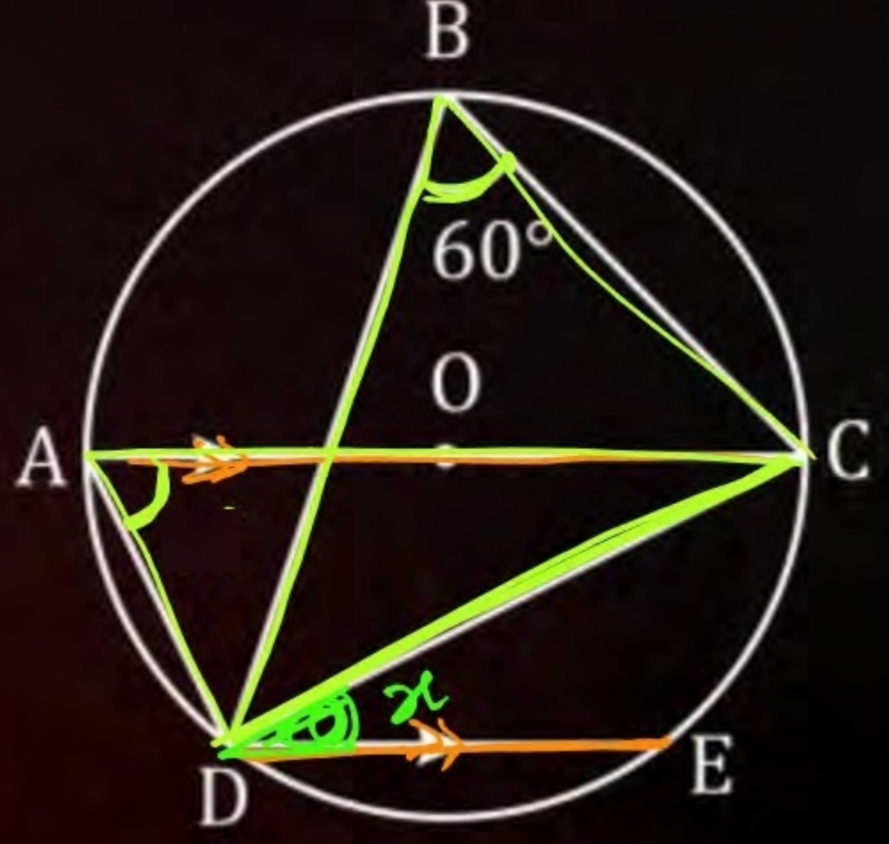






Question

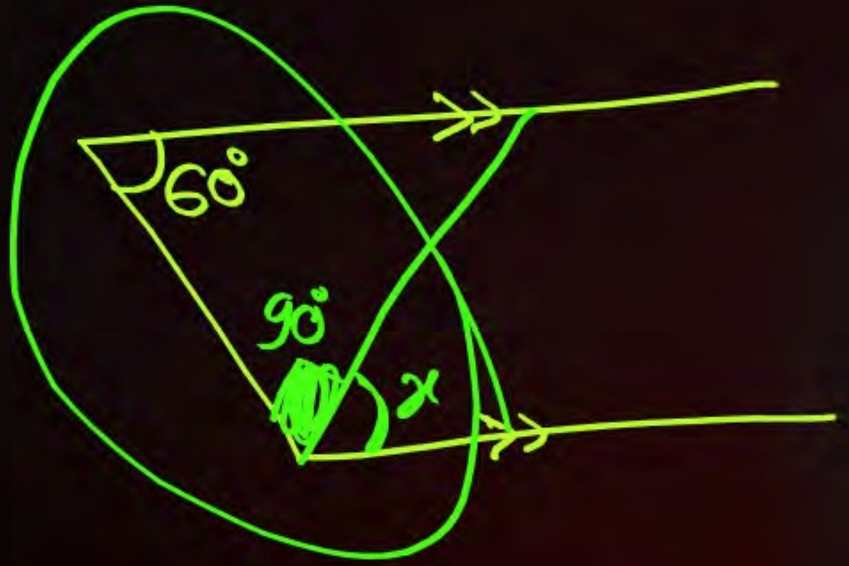
In the adjoining figure, DE is a chord parallel to diameter AC of the circle with centre O. If $\angle CBD = 60°$, calculate $\angle CDE$.

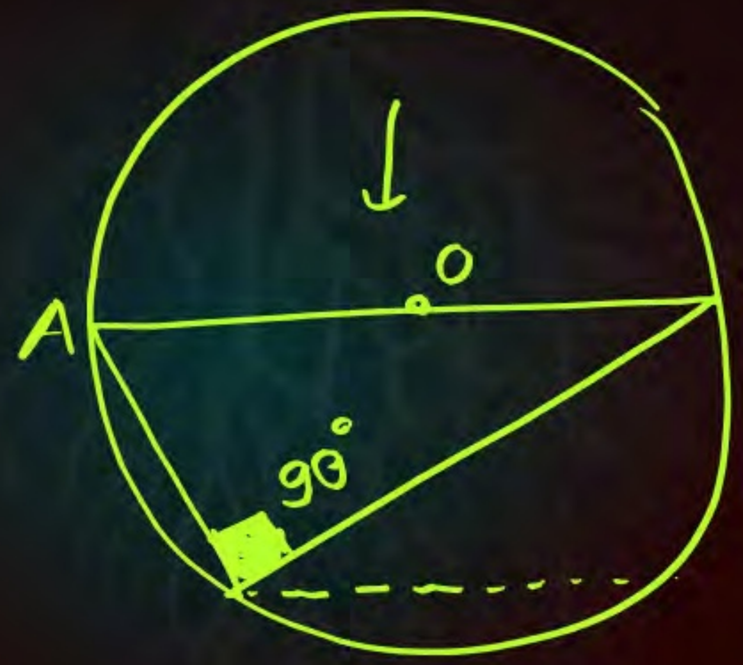


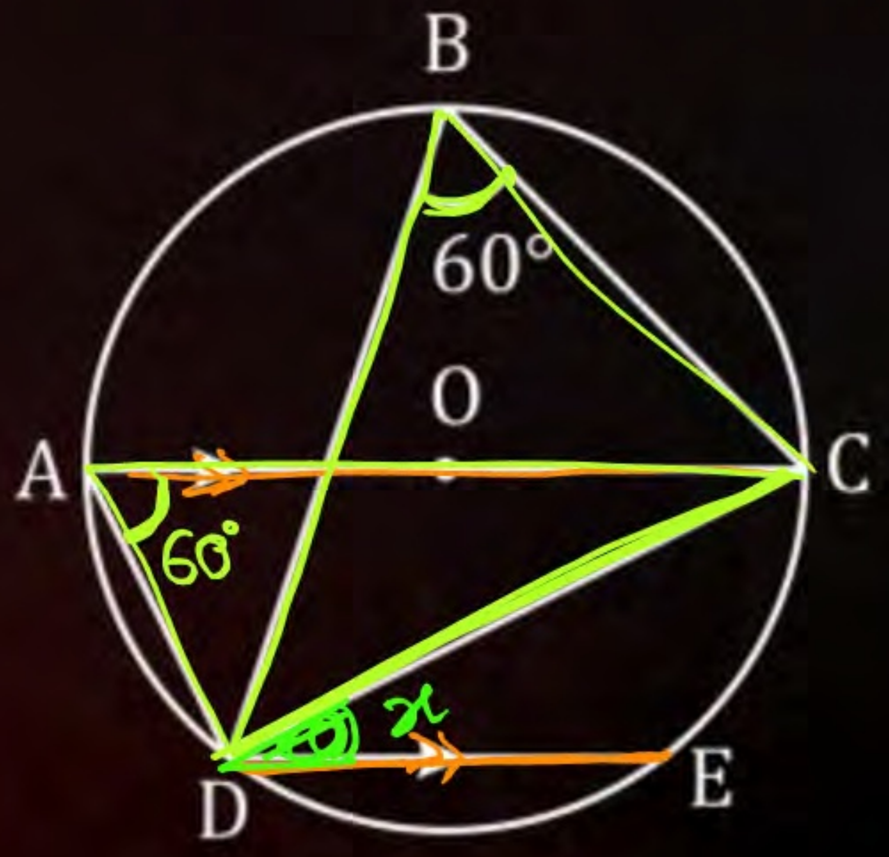
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

In the adjoining figure, DE is a chord parallel to diameter AC of the circle with centre O. If $\angle CBD = 60°$, calculate $\angle CDE$.







$$60° + 90° + x = 180°$$

$$\boxed{x = 30°}$$

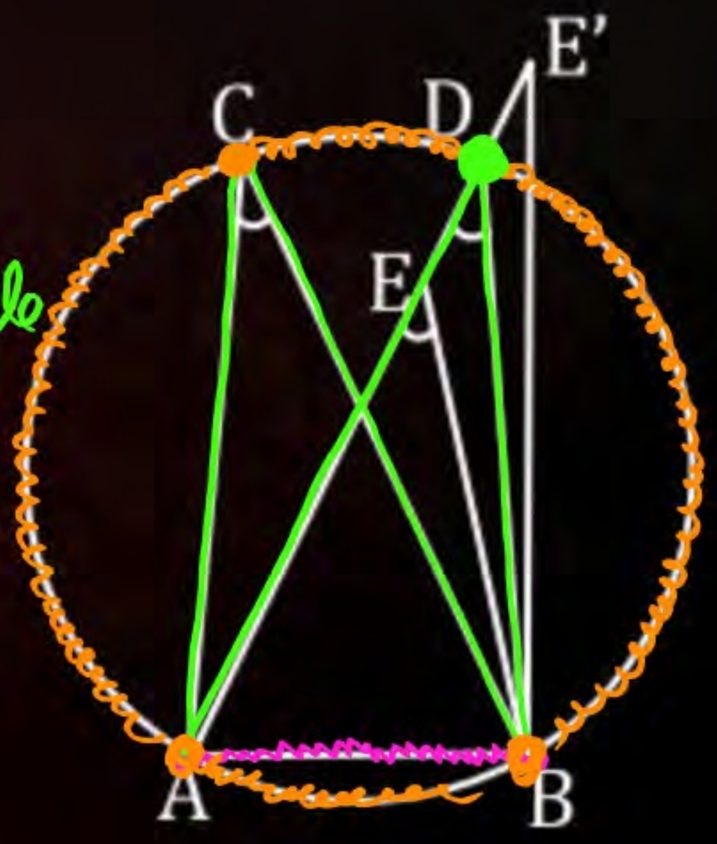
Theorems related to Circle

Theorem 9: If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

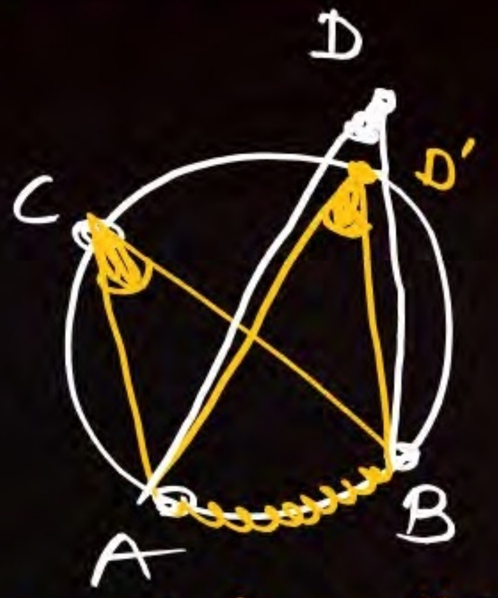
$\boxed{A, B, C \& D}$ → concylic points

⇒ To proved → point D is over the circle

Given:- $\boxed{\angle ACB = \angle ADB}$



Case-(I)



Let D′ is on the circle

$\angle ACB = \angle AD'B$

$\downarrow$

$\angle ADB = \angle AD'B$

↳ If and only if

D′ coincide with D

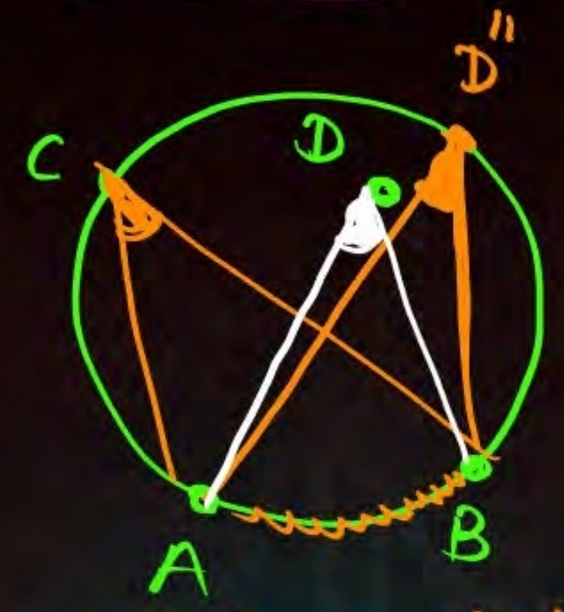
Both D′ and D are same point.

since D′ was over the circle, D will be also over the circle.

case-(II)

✓

case-(III)



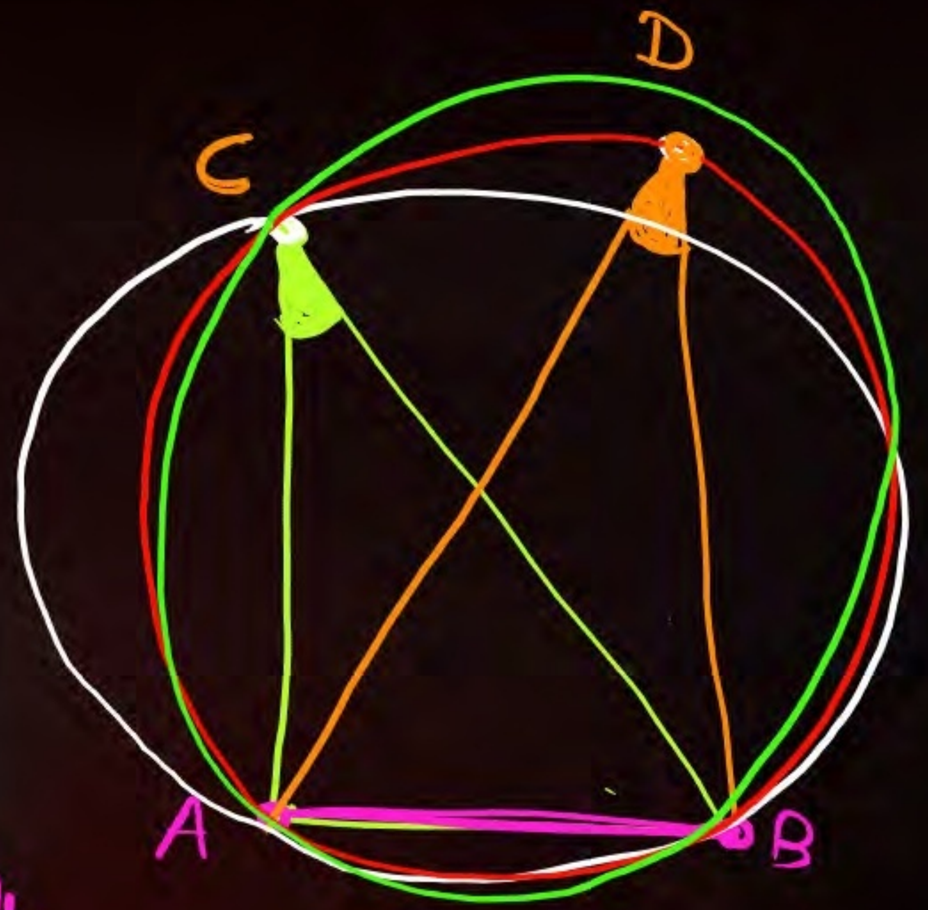
Let D″ is on circle

$\angle ACB = \angle AD''B$

$\boxed{\angle ADB = \angle AD''B}$

↳ only possible by D coincide with D″

It mean D is over the circle & concylic



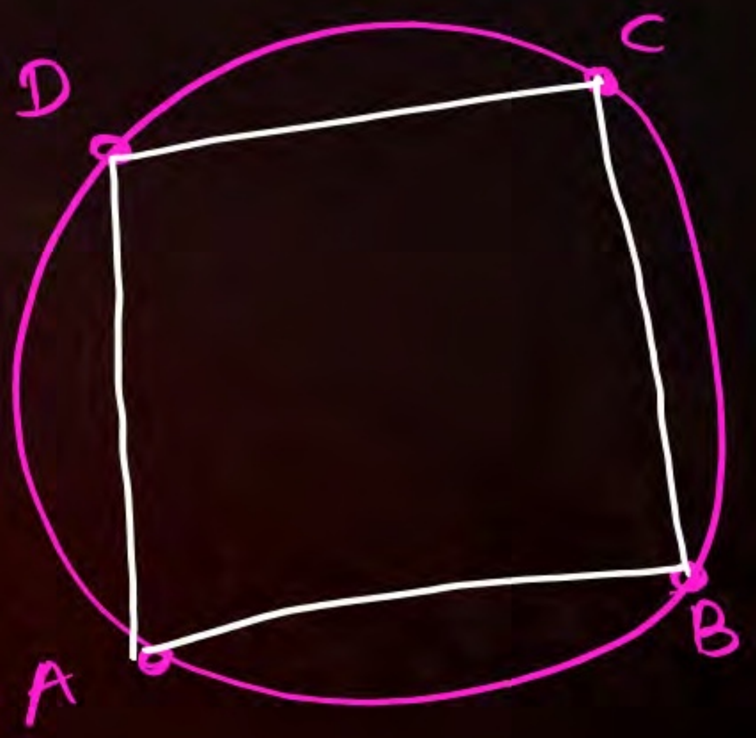
VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Cyclic Quadrilateral

A quadrilateral is called cyclic if all the four vertices of it lie on the circle.

A, B, C and D are concylic point

Thus, $\square ABCD$ will be $\boxed{\text{cyclic quadrilatral}}$



Theorems related to Circle

Theorem 10: The sum of either pair of opposite angles of a cyclic quadrilateral is 180°

Now, over the centre,

$$\boxed{x+y=360^\circ} \quad \text{— (1)}$$

Since □ABCD is cyclic quadrilateral,

$$\text{sum of opposite angle} = \angle DCB + \angle DAB$$
$$= \frac{x}{2} + \frac{y}{2}$$
$$= \frac{x+y}{2}$$
$$= \frac{360^\circ}{2} = \boxed{180^\circ}$$

$$\boxed{\angle 1 + \angle 2 = 180^\circ}$$

↳ only holds if quadrilateral is cyclic

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Theorems related to Circle

Theorem 11: If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

$\boxed{\angle A + \angle D = 180°}$

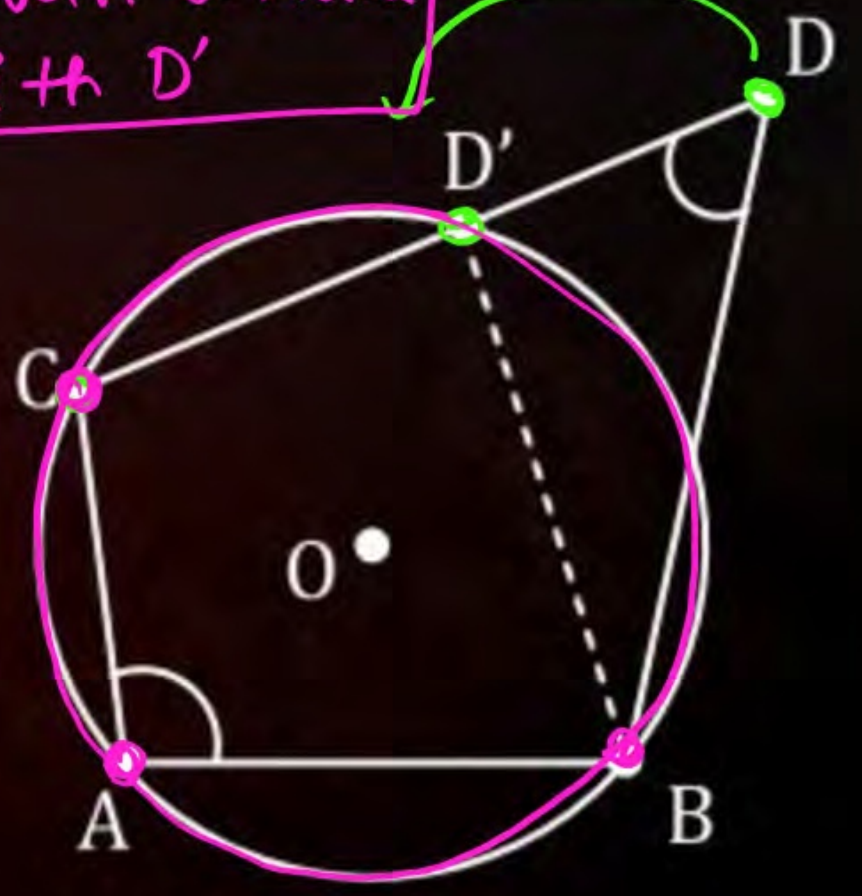
$\boxed{\angle CDB = \angle CD'B}$

concyclic Point A, B, C & D

quadrilateral

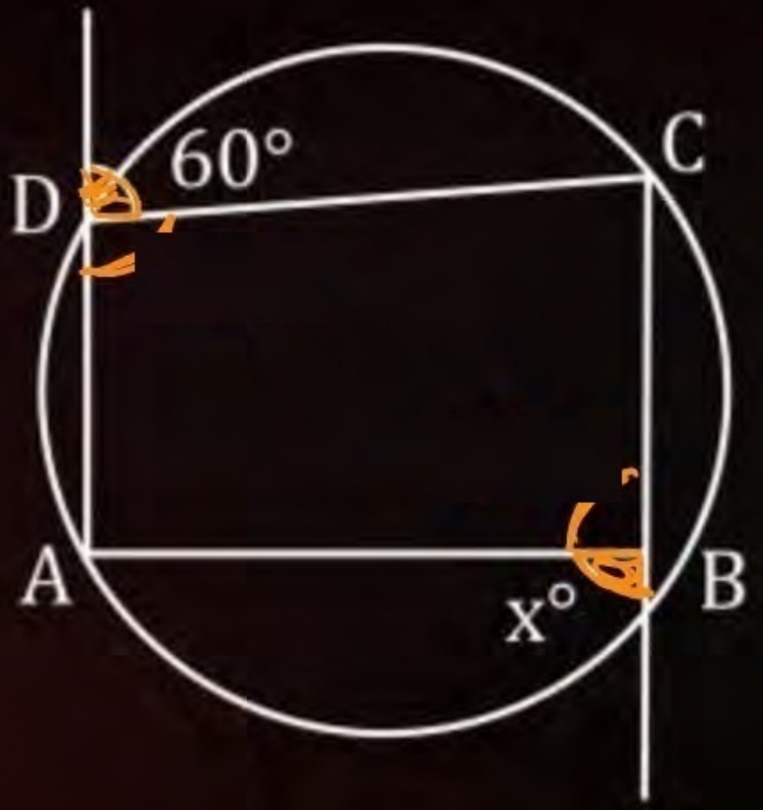
cyclic

D will coincide with D'



Question

Find the value of x in the given figure.



120°

100°

140°

None of these

Question

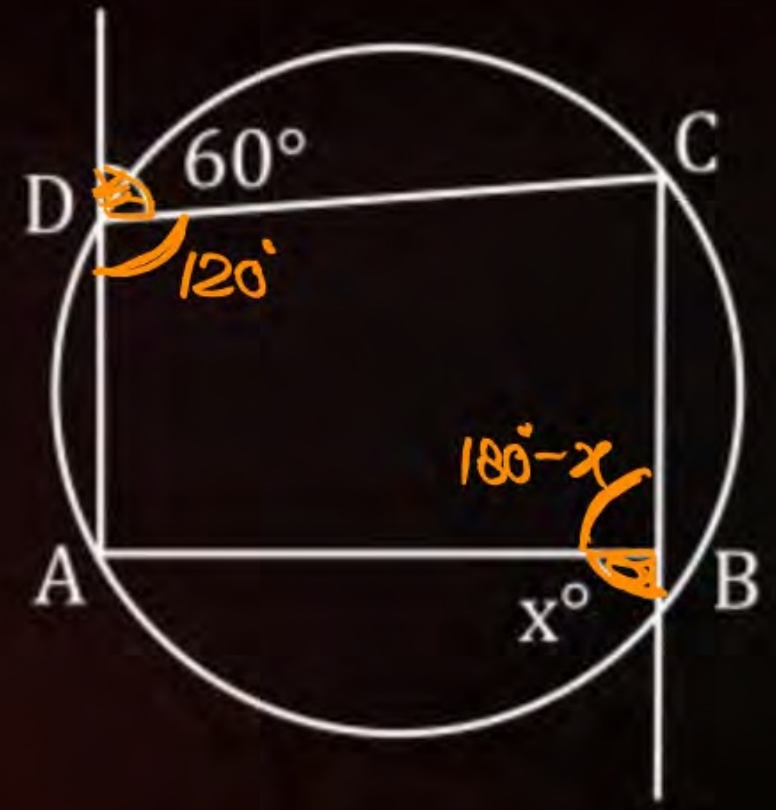
Find the value of x in the given figure.

A 120°

B 100°

C 140°

D None of these



□ ABCD is cyclic quadrilateral

sum of opposite angles = 180°

$\angle ADC + \angle ABC = 180°$

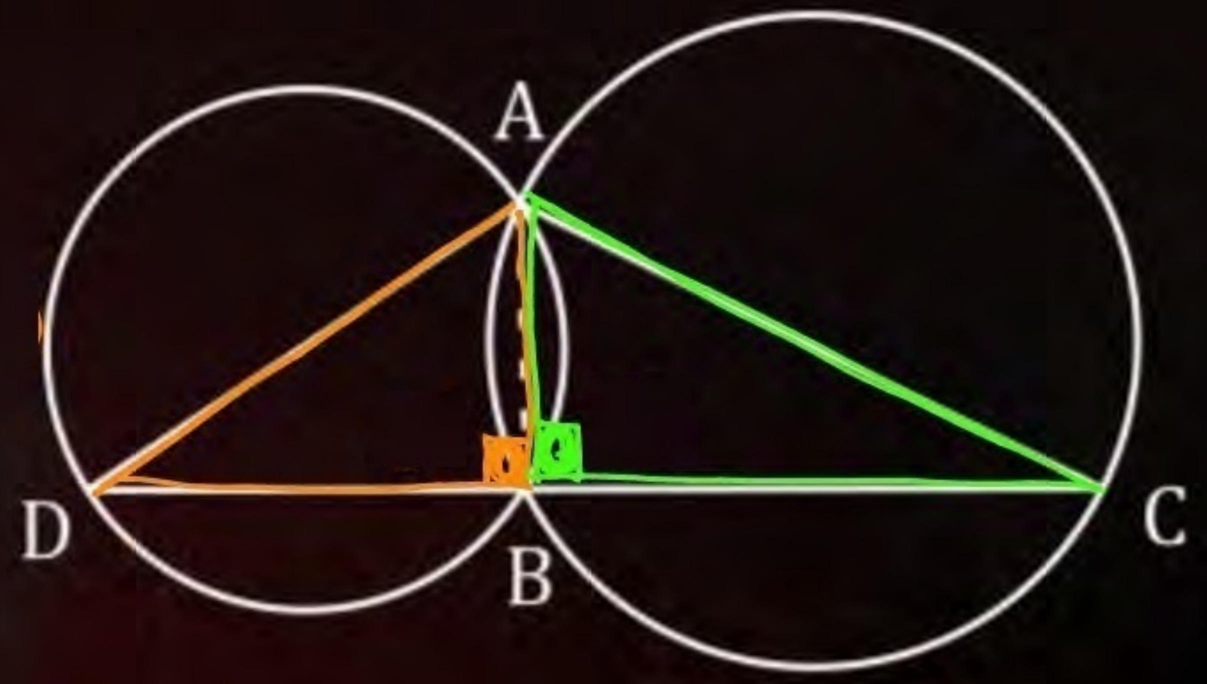
$120° + 180° - x = 180°$

$120° + \cancel{180°} - \cancel{180°} = x$

$\boxed{x = 120°}$

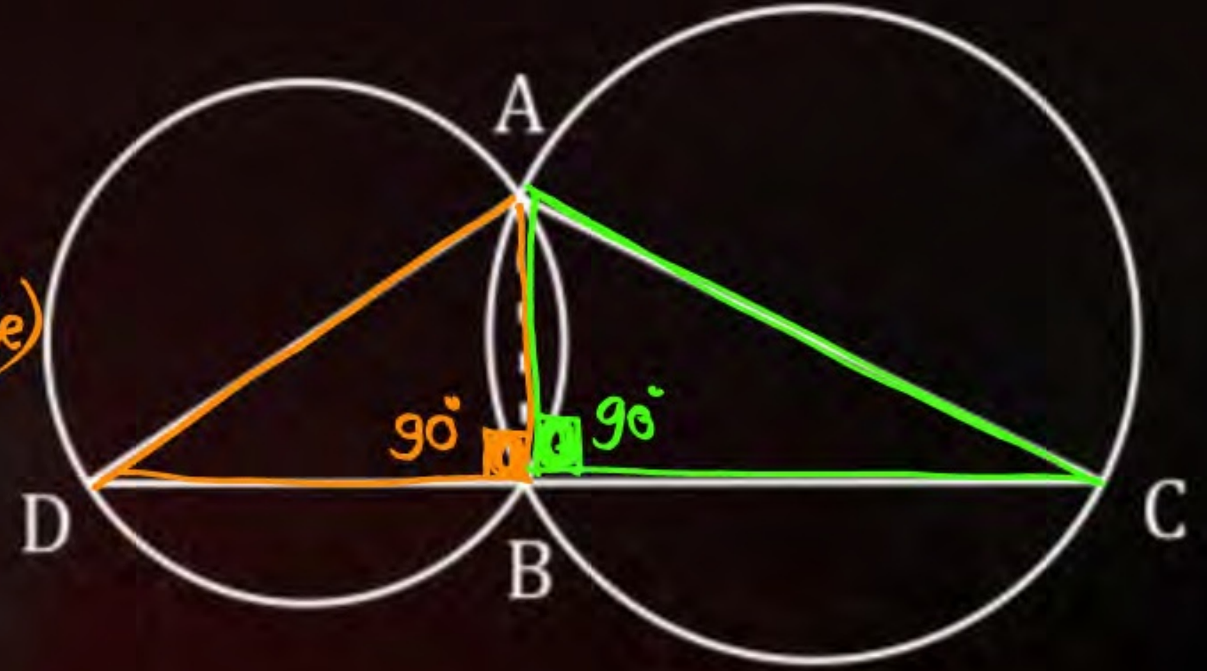
Question

Two circles intersects at two points A and B. AD and AC diameters to the two circles. Prove that B lies on the line segment DC.



Question

Two circles intersects at two points A and B. AD and AC diameters to the two circles. Prove that B lies on the line segment DC.



since AD is diameter,

$\angle ABD = 90^\circ$ (angle in semi-circle)

Also, AC is diameter

$\angle ABC = 90^\circ$ (Angle is semicircle

Now, $\boxed{\angle ABD + \angle ABC = 90^\circ + 90^\circ = \boxed{180^\circ}}$

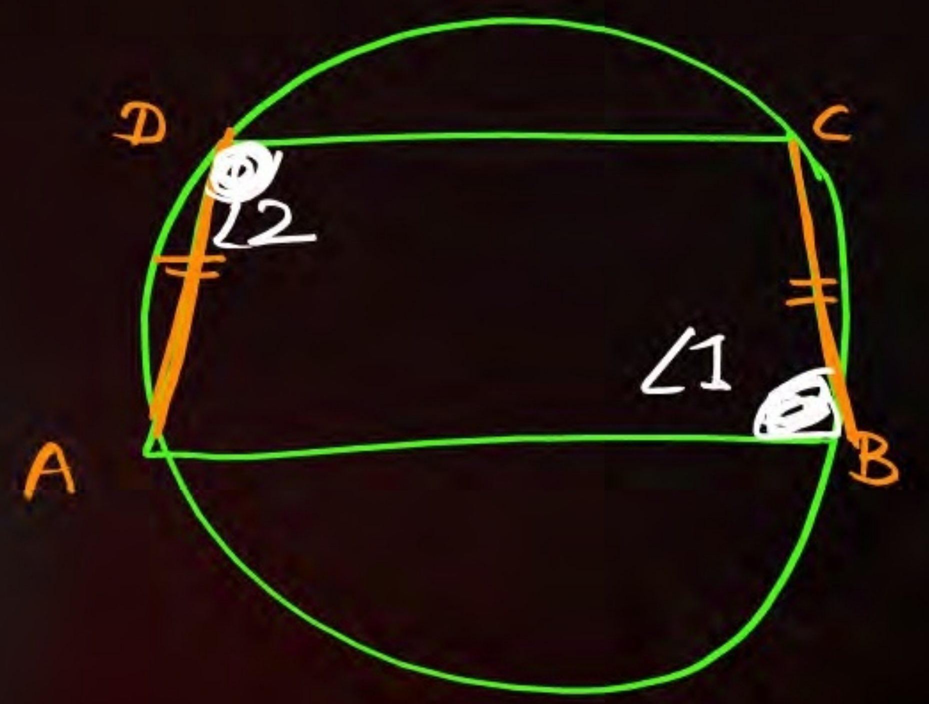
↳ D, B & C form a line

↳ DC will be a line segment and B lie over DC

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

since non-parallel sides are equal

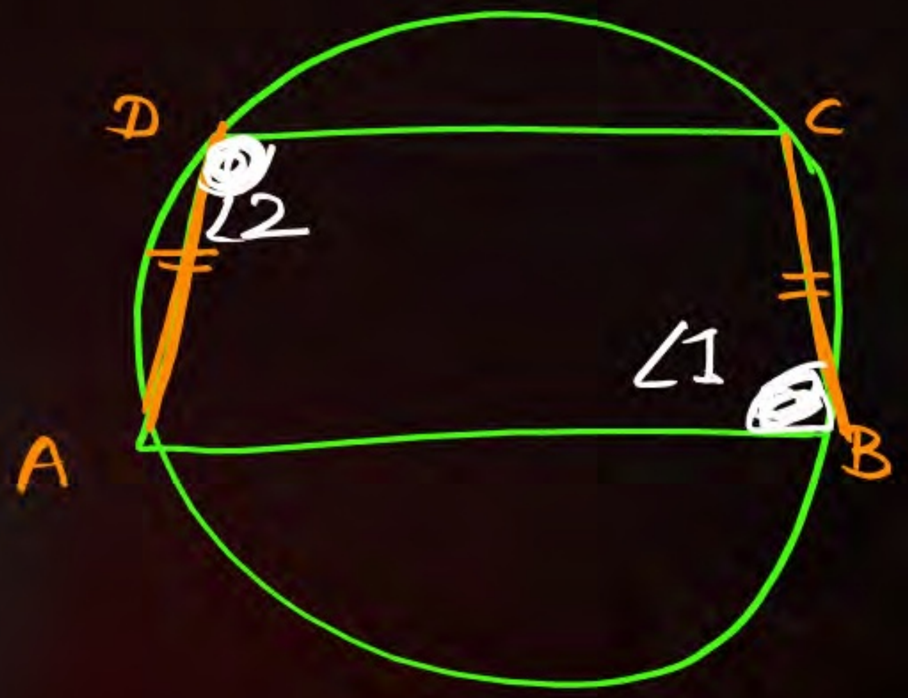
↳ Isosceles trapazium

$\angle DAB = \angle CBA = \angle 1$ (let)

$\boxed{DC \parallel AB}$

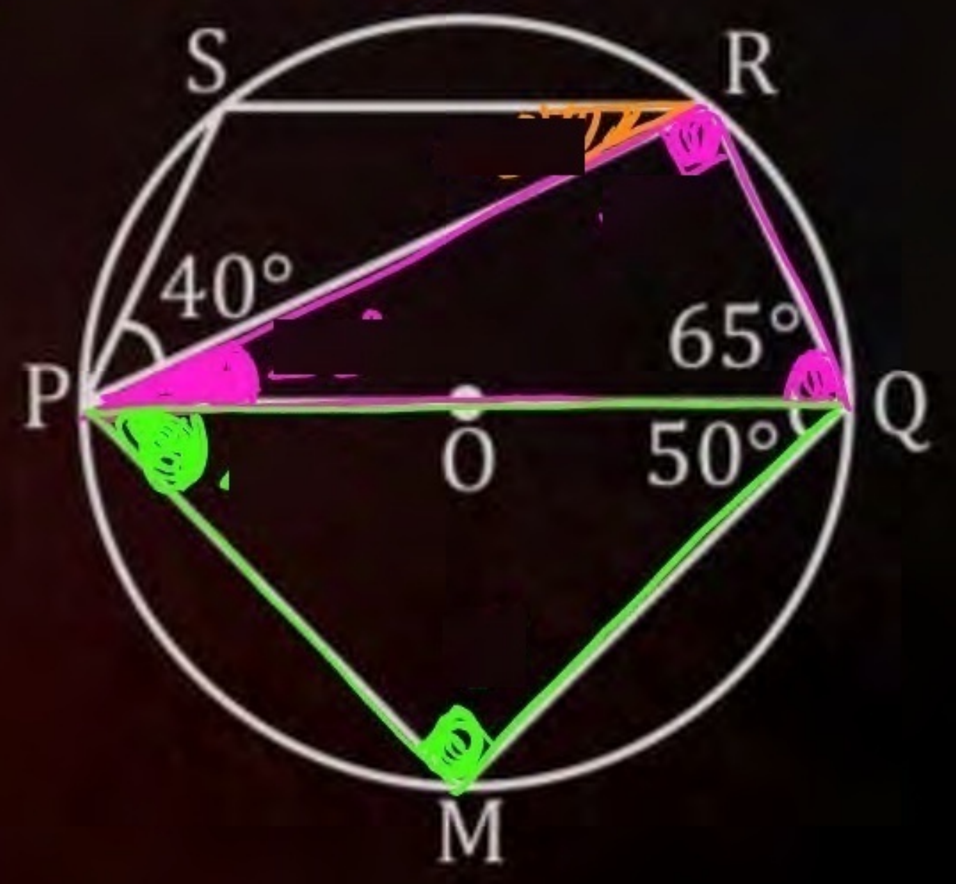
$\boxed{\angle 2 + \angle 1 = 180^\circ}$

↳ □ABCD will be cyclic



Question

In the given figure, PQ is a diameter of a circle with centre 'O'. If $\angle PQR = 65°$, $\angle SPR = 40°$ and $\angle PQM = 50°$, find $\angle QPR$, $\angle QPM$ and $\angle PRS$.



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

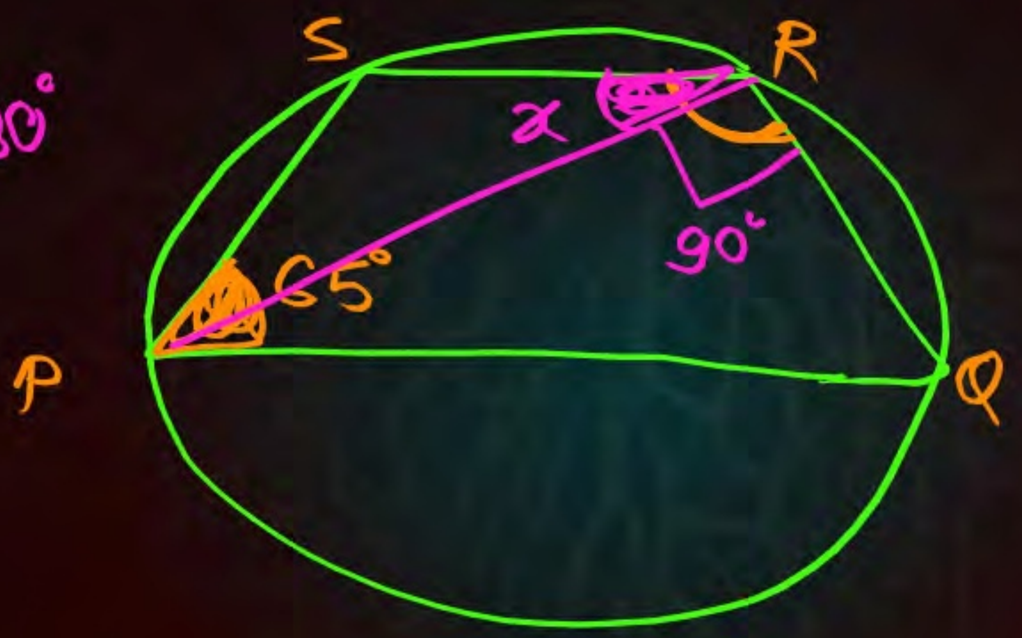
Question

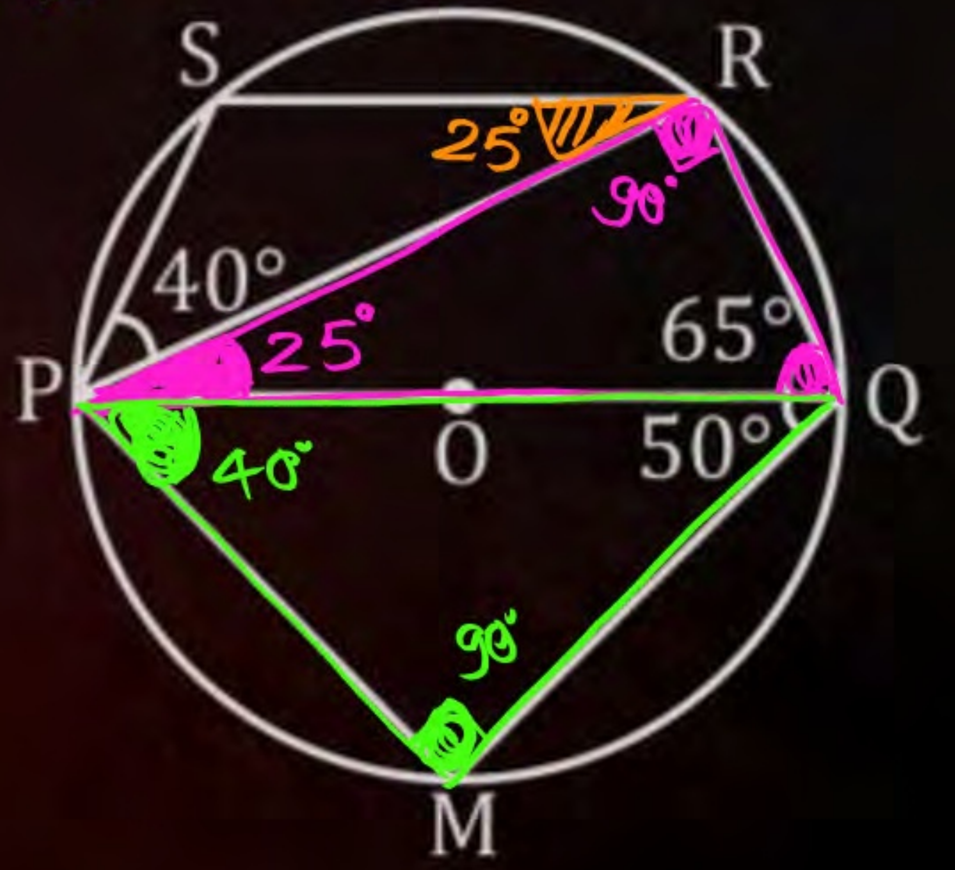
In the given figure, PQ is a diameter of a circle with centre 'O'. If $\angle PQR = 65°$, $\angle SPR = 40°$ and $\angle PQM = 50°$, find $\boxed{\angle QPR}$, $\boxed{\angle QPM}$ and $\boxed{\angle PRS}$.

25° 40° 25°

$(65°) + (x + 90°) = 180°$

$\boxed{x = 25°}$

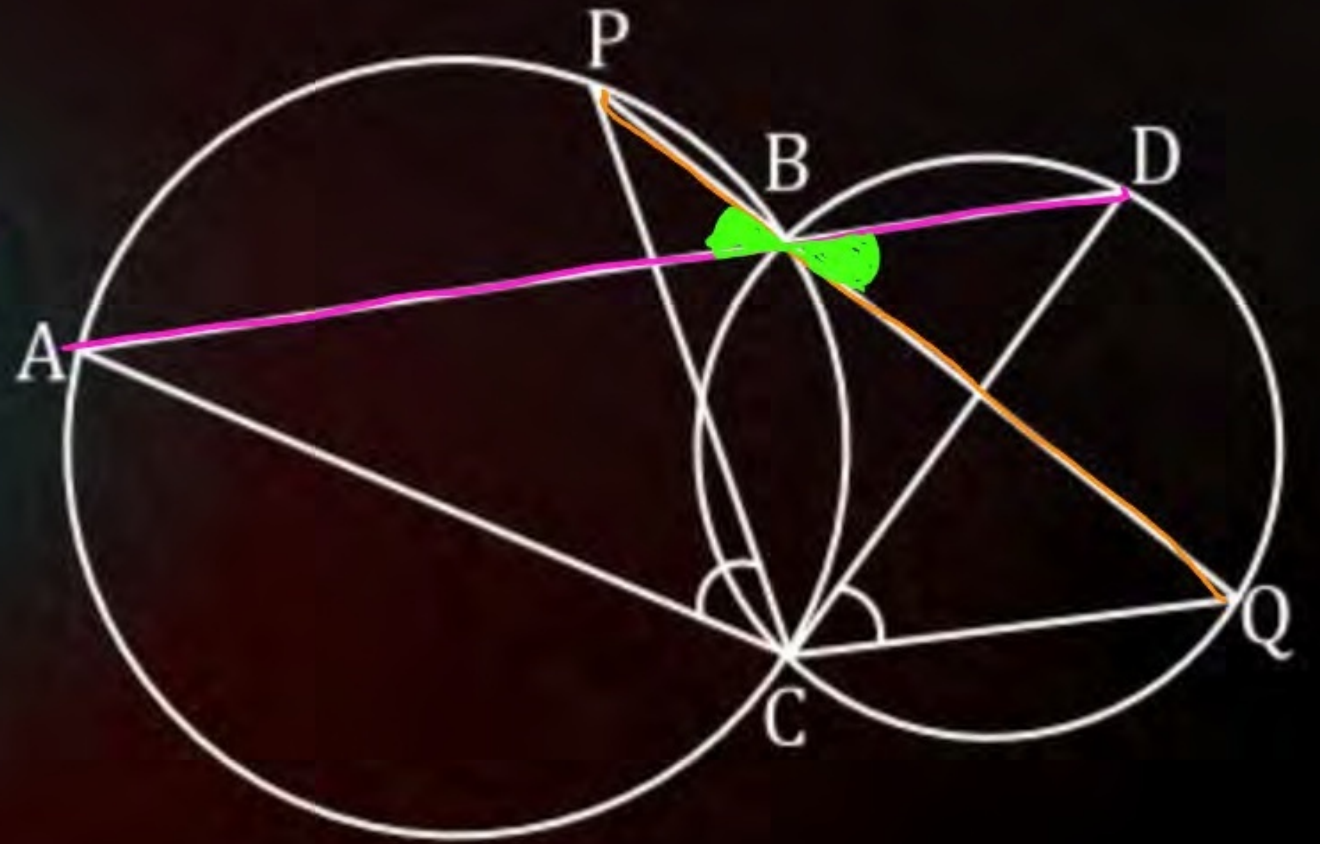




VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

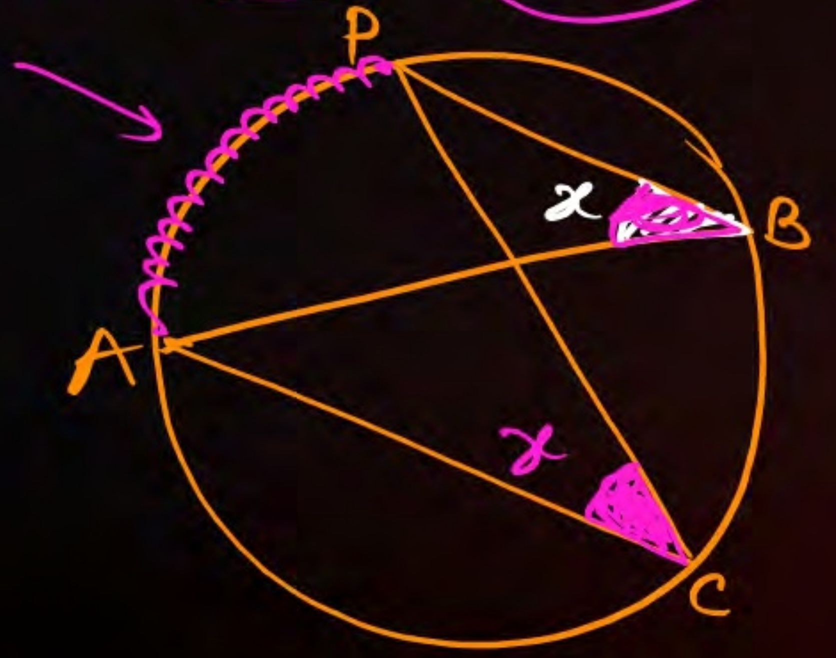
Question

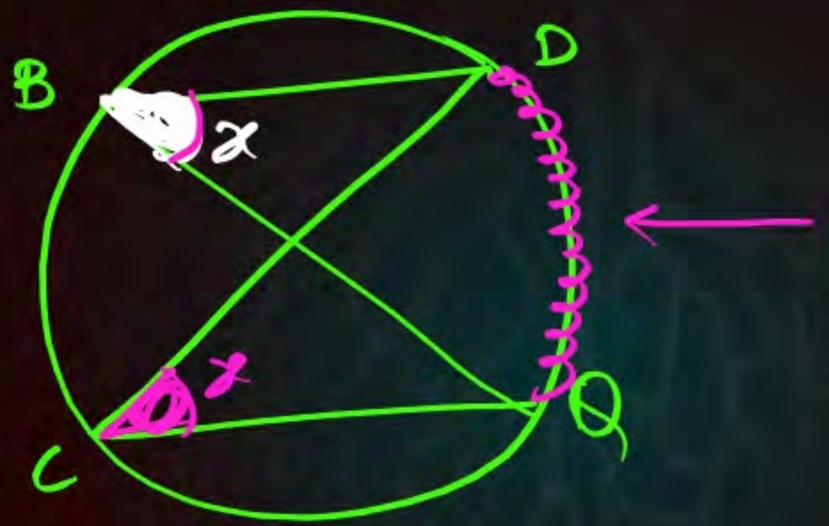
Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that $\angle ACP = \angle QCD$

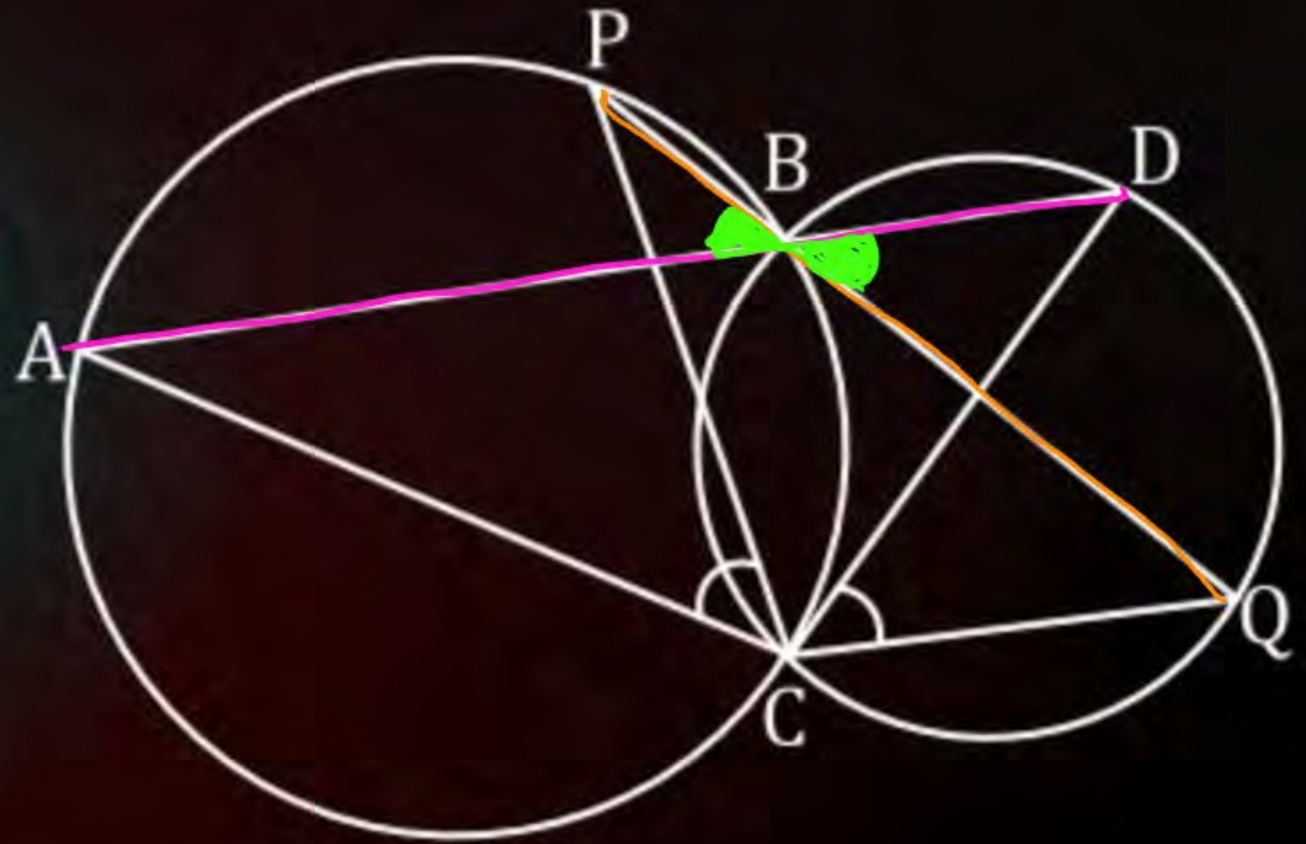


Question

Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that ∠ACP = ∠QCD







Question

In the given, AOB is a diameter of a circle and CD||AB. If ∠BAD = 30°, then ∠CAD =

30°

60°

45°

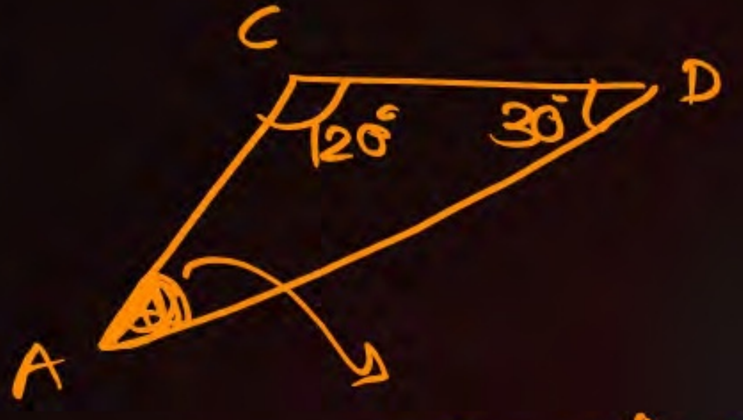
50°

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

Question

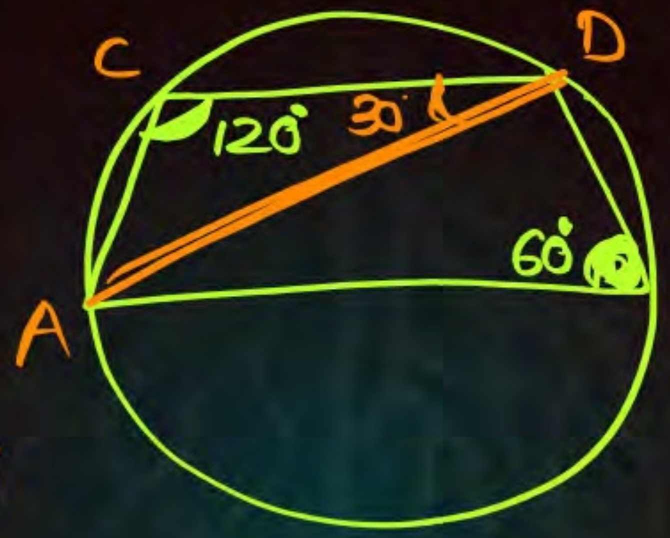
In the given, AOB is a diameter of a circle and CD||AB. If ∠BAD = 30°, then ∠CAD =

- **A** 30°
- **B** 60°
- **C** 45°
- **D** 50°



$= 180° - 120° - 30°$

$= \boxed{30°}$



VIPIN KAUSHIK ASOSE SURAJMAL VIHAR

VIPIN KAUSHIK ASOSE SURAJMAL VIHAR