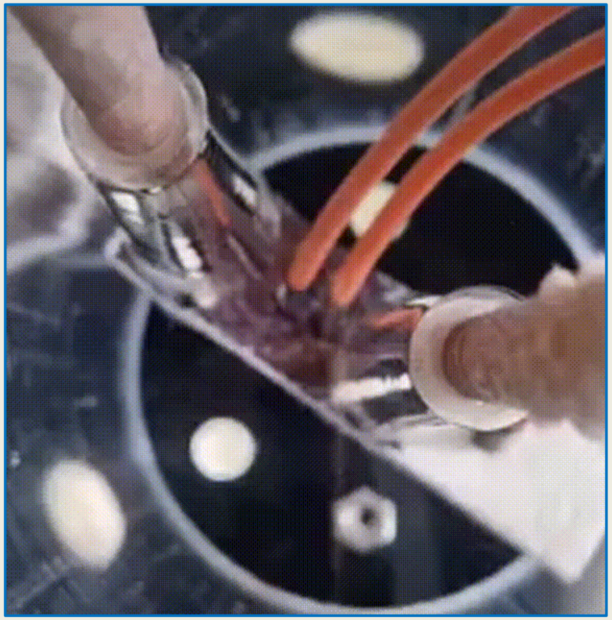
Quantifying the Dynamics of Bridge Formation in Crystal Growth Phenomena

During the process of **crystallization** between two crystallization centers grows a **"bridge"**. The aim of research is to investigate the **dynamics** of “bridge” formation and a **shape** of bridge



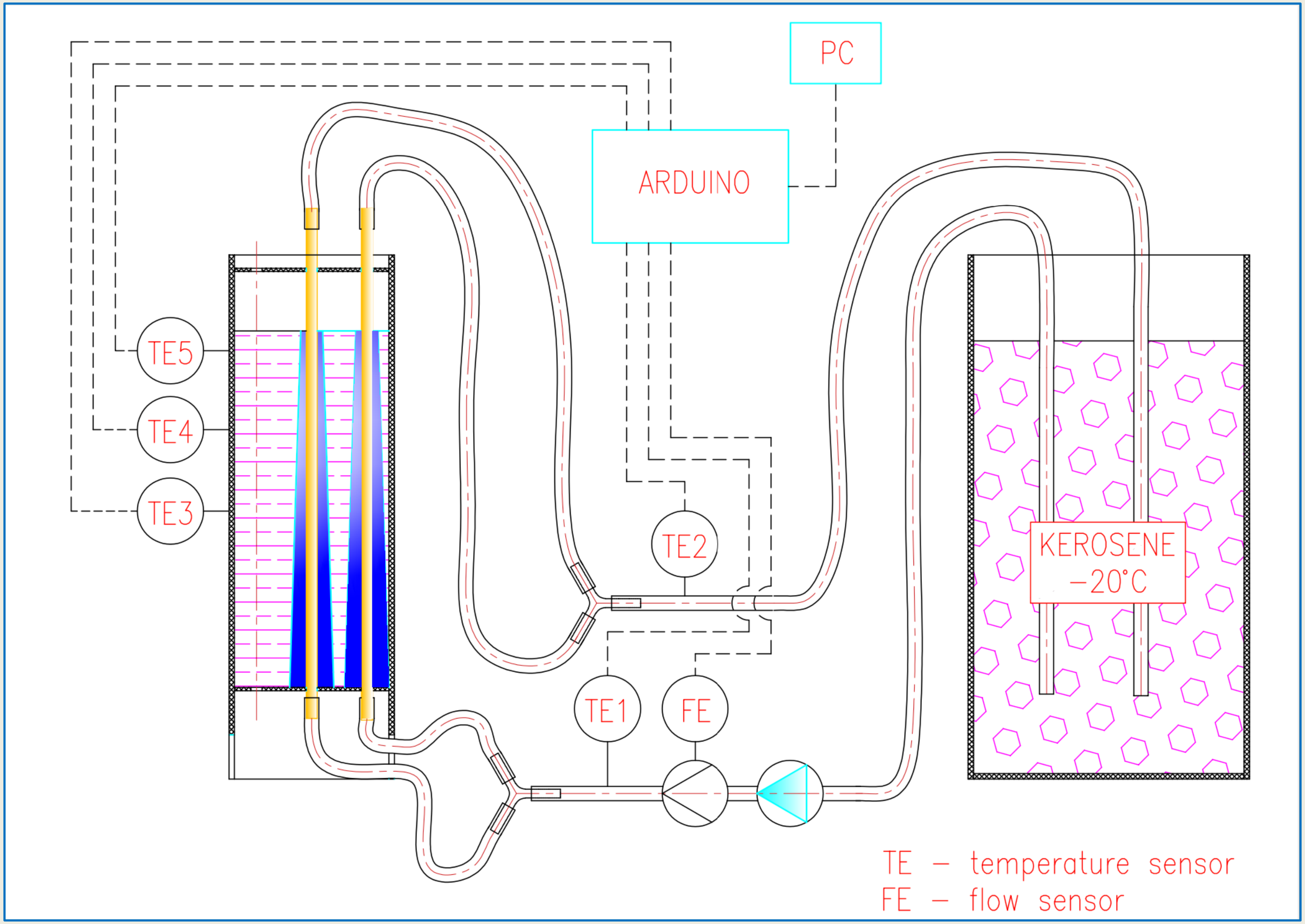
Scientific advisor : Alexander Svetlichnyi

Opyakin O. Lishik K. Vikultsev D. May 2024

Plan of Research

1. Develop theory:
   - 1 – center system
   - 2 – centers system
2. Assemble experimental setup
3. Conduct experiments for both systems
4. Analyze the results

Experimental Setup



THEORY & EXPERIMENTS

Considering supercooled layer

According to [6] **supercooled layer** may appear due to **diffusion** of crystallization centers. Should it be considered?

Possible temperature:

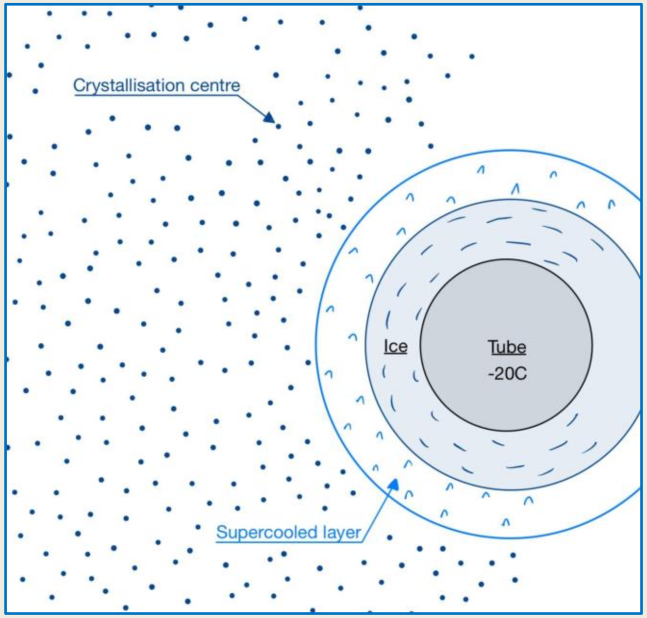
$$\Delta T \geqslant \frac{2\sigma T_{phase} v^{(1)}}{\lambda R_{dust}} \approx 0.2K$$

Temperature distribution:

$$T(r) = \frac{T_1 \ln r_2 - T_2 \ln r_1}{\ln \frac{r_2}{r_1}} + \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} \ln r$$

$$\downarrow \downarrow$$

$$\boxed{R_{supercooled} \approx 1 \cdot 10^{-3} m}$$



supercooled layer won't be considered

1 - CENTER SYSTEM

Theory : 1 center (first approach) , [3]

Non-linear differential heat equation:

$$c_{ice}\rho_{ice}\frac{\partial T}{\partial r} = \left[\frac{\partial}{\partial r}\lambda_{ice}(T)\frac{\partial T}{\partial r}\right] + \frac{\lambda_0}{r}\frac{\partial T}{\partial r}$$

where

$c_{ice}-$ heat capacity

$\lambda_{ice}-$ enthalpy of fusion

$\rho_{ice}-$ density

Considering water interaction:

$$\lambda\frac{\partial T}{\partial r}|_r = \alpha(T_{out} - T_{phase}) + \rho L\frac{d\eta}{dr}$$

$$\downarrow\downarrow$$

$$c_{ice}\rho_{ice}\frac{dT}{d\nu} = \frac{d}{d\nu}\left[\frac{K}{T}\frac{dT}{d\nu}\right] + \frac{1}{\nu\sqrt{\tau}+r_0}\frac{K}{T}\frac{dT}{d\nu}$$

$$\downarrow\downarrow$$

Taylor's series to the second order

$$T_{wall} = T_{phase} - \left(\frac{\rho L\boxed{\beta}}{2\lambda} + \frac{\alpha\sqrt{\tau}}{\lambda}(T_{out} - T_{phase})\right)\boxed{\beta}$$

Lame-Clayperon Substitution:

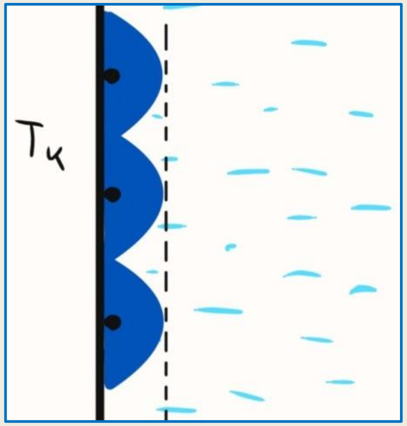
$$\vartheta = \frac{r - r_0}{\sqrt{\tau}}$$

$$T(r;\tau) = T(\vartheta)$$

$$(\eta - r_0) = \beta\sqrt{\tau}$$

Theory : 1 center (second approach)

Front of crystallization:



Heat flow from a unit area:

$$q = -\varkappa S \frac{dT}{dr}$$

integration

$$Q = 2\varkappa \Delta T \pi r = \boxed{m}\lambda = \lambda \rho S v$$

$$\boxed{m = \rho V = \rho S v dt, dt = 1}$$

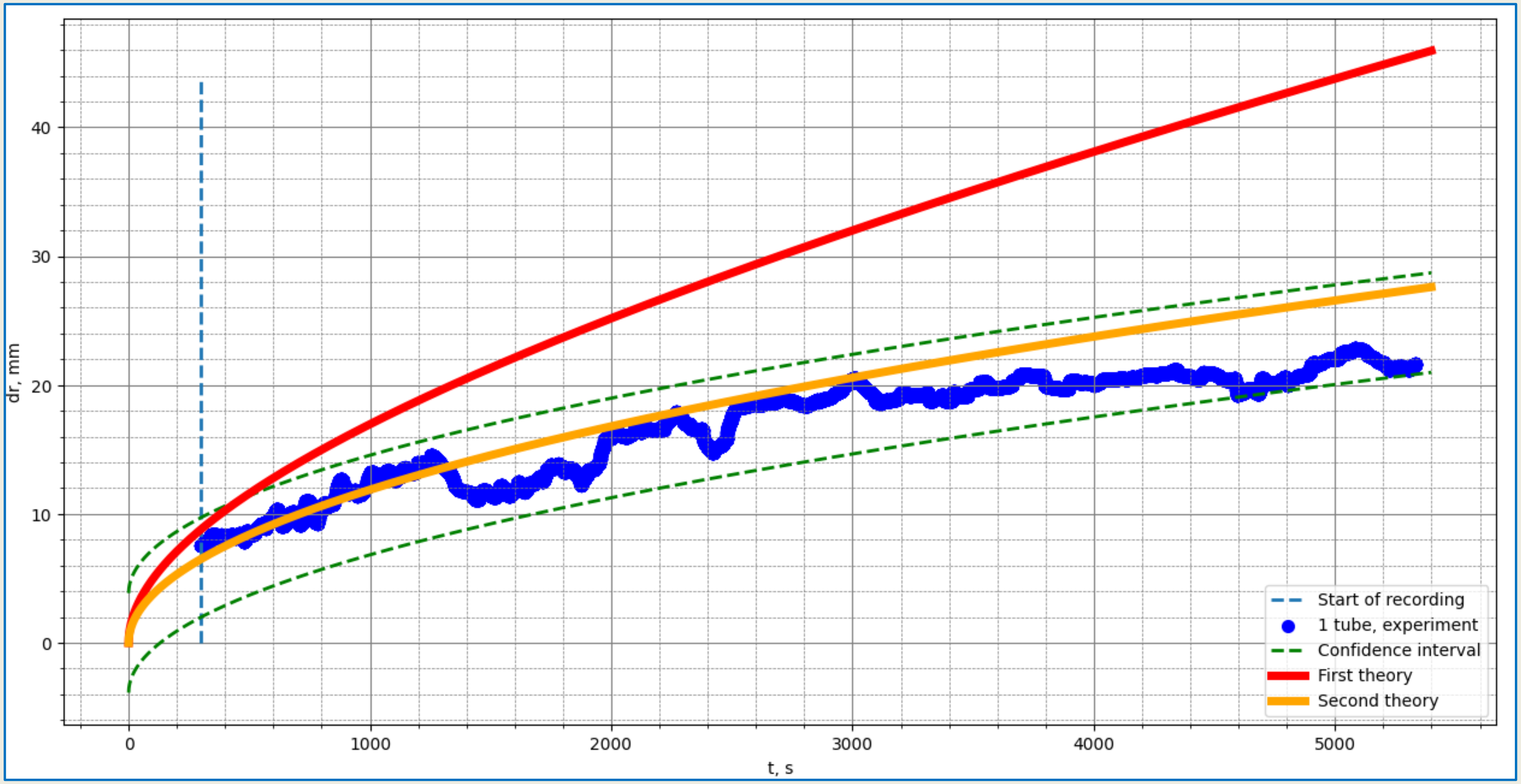
Speed of crystallization:

$$v = \frac{\varkappa \Delta T}{\lambda \rho r} = \frac{dr}{dt} \quad \Rightarrow \int_o^r r\, dr = \frac{\varkappa \Delta T}{\lambda \rho} \int_o^t t\, dt$$

Final solution:

$$\Rightarrow \boxed{r(t) = \sqrt{\frac{2\varkappa \Delta T}{\lambda \rho} t}}$$

Results : 2 theories



2 - CENTERS SYSTEM

Numerical Solution

Thermal conductivity equation:

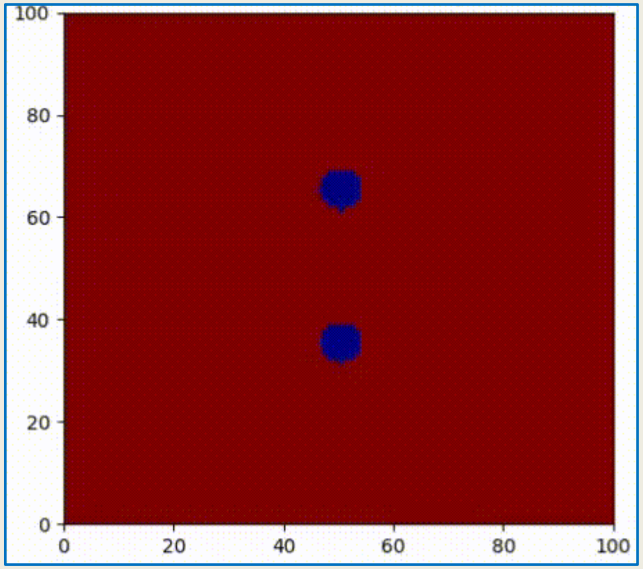
$$\frac{\partial T}{\partial t} = -\chi(\nabla^2 T)$$

time step

$$T_{i,j}^{n+1} = \frac{\chi\tau}{h^2}(T_{i-1,j}^{n} + T_{i+1,j}^{n} + T_{i,j-1}^{n} + T_{i,j+1}^{n} - 4T_{i,j}^{n}) \quad \text{where} \quad T^{\boxed{t}}_{\boxed{x,y}}$$

coordinates

Python implementation:



Theory : 2 centers

Heat flow towards one center:

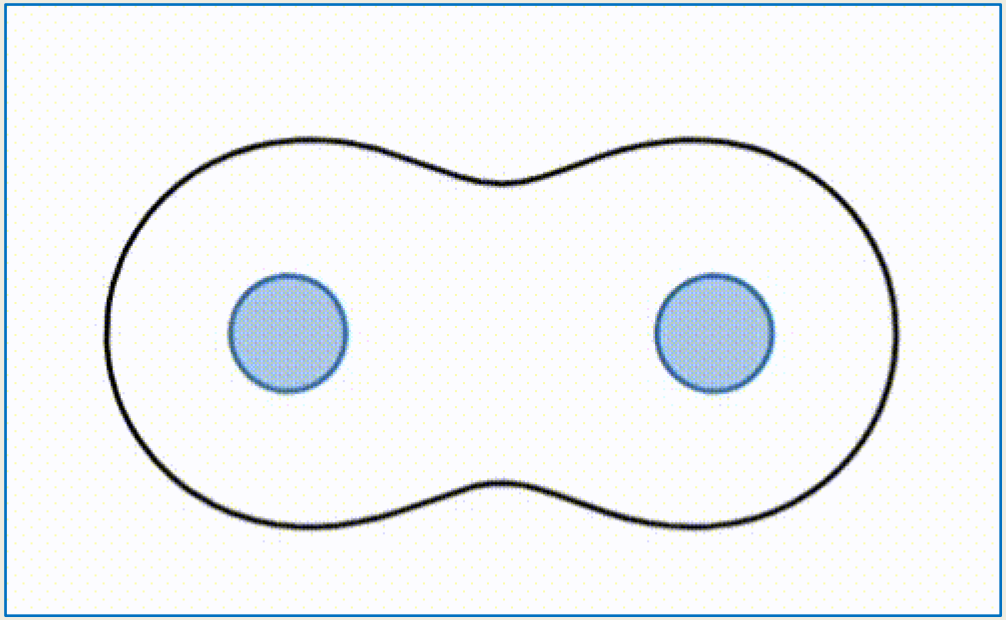
$$j = -\varkappa(\nabla T)$$

$$\rho_1 = \sqrt{x^2 + y^2} - r_0$$

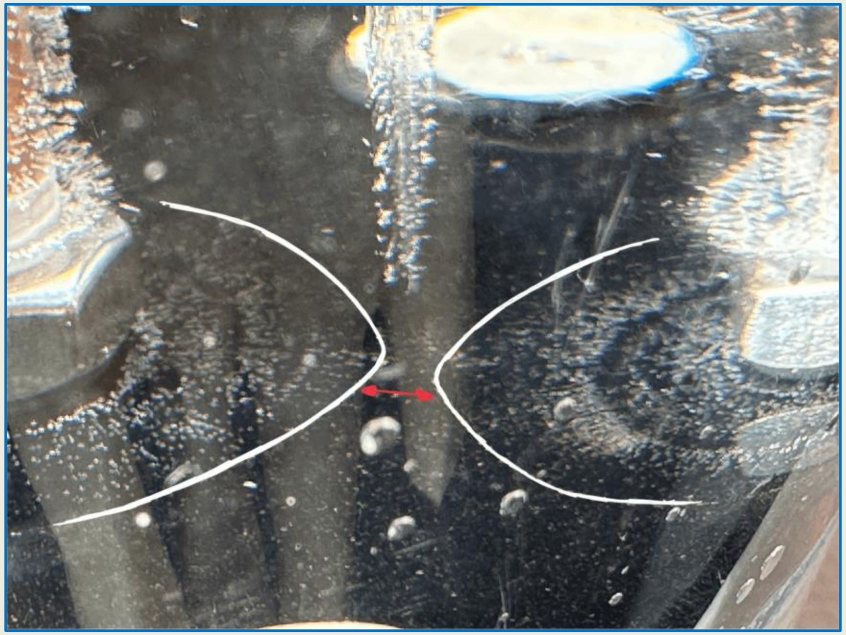
$$\rho_2 = \sqrt{(l - x)^2 + y^2} - r_0$$

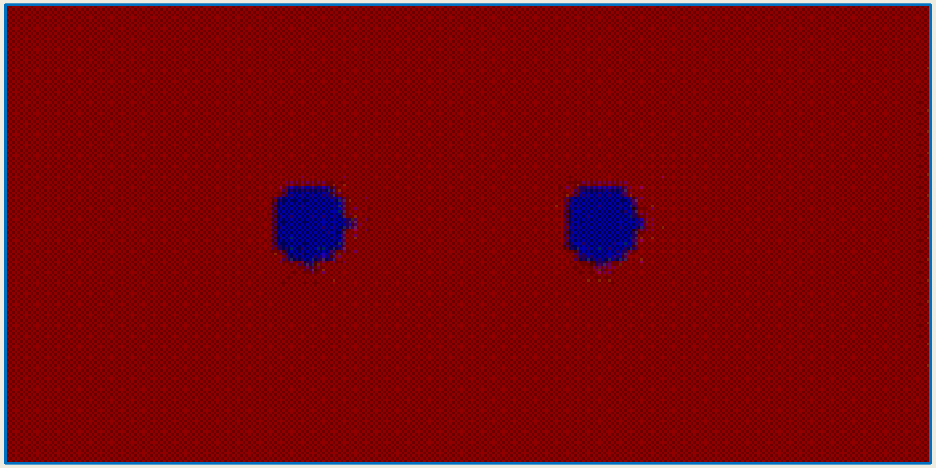
Considering 2 pipes:

$$Q(x, y) = q_1(x, y) + q_2(x, y) = -\varkappa(\nabla T_1 + \nabla T_2) = \boxed{-\varkappa \frac{\rho_1 + \rho_2}{\rho_1 \rho_2}}$$

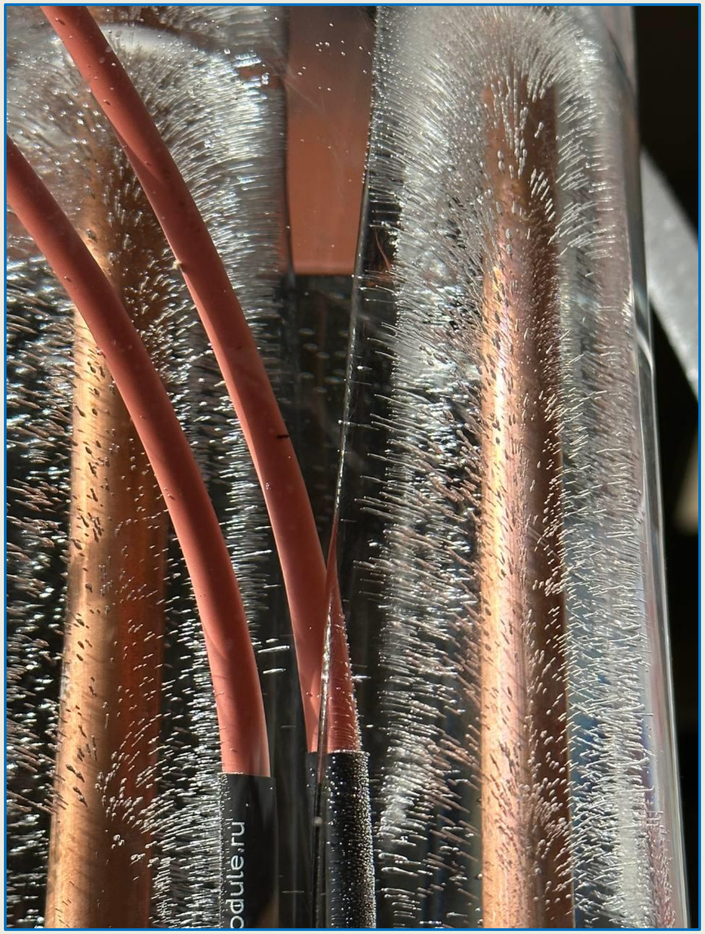


Results : 2 centers

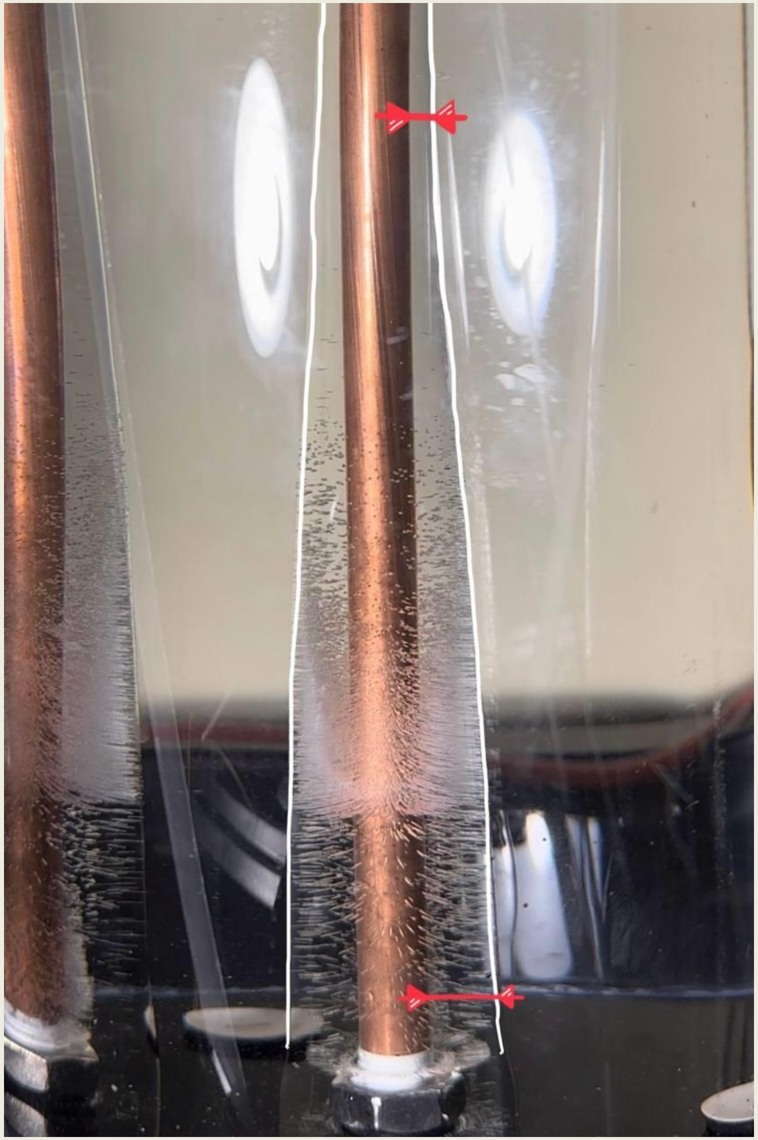




Interesting effects : needle ice



Interesting effects : cone shape



Results

1. Considered 2 theories on 1 – center case :
   – Our theory fits the experimental data
   – Other theory does not
2. Shape of the bridge has been determined analytically, numerically and proofed by experiment

Literature

- [1] M. G. Worster, "Convection in mushy layers", 1997.
- [2] F. Roters, P. Eisenlohr, T. R. Bieler, and D. Raabe, "Crystal Plasticity Finite Element Methods".
- [3] M. A. Ugolnikova, "Modeling Of Heat Transfer Processes at Freezing of Water Ice on Uninsulated Elements of Low-Temperature Equipment", Moscow, 2017.
- [4] D. V. Sivukhin, “Thermodynamics and Molecular Physics,” 5th Edition, FITMAZLIT, Moscow, 2005.
- [5] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, "Statistical Physics", Butterworth-Heinemann, Oxford, 1999.
- [6] D. V. Alexandrov, L. V. Toporova, "Towards the theory of how a constitutional supercooling layer appears ahead of the planar crystallization front", 2023.
- [7] L.D. Landau, E.M. Lifshitz , "Mechanics and Molecular Physics", 1960.