Data Assimilation Applications for Radiation Belts

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1.Abstract

The near-Earth radiation environment can be hazardous to satellites and humans in space. The sparsity of satellite observations complicates the understanding and prediction of plasma dynamics in this region. Data assimilation helps to reconstruct the state of the system using various measurements, to issue real-time predictions and to identify missing physical processes. We present applications of the Kalman Filter **(KF)** and of the Ensemble Kalman Filter **(EnKF)** for the outer electron radiation belts **(RB)**, including a satellite intercalibration approach.

2.Data assimilation

In order to reconstruct the global state of the near-Earth radiation environment, we can combine satellite data from missions at various orbits with the 3-D Versatile Electron Radiation Belt (VERB-3D) code using data assimilation techniques.

The VERB-3D code [4] solves the 3D Fokker-Planck equation to calculate the non-adiabatic evolution of electrons in the radiation belts:

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\Big|_{\mu,J} \frac{1}{L^2} D_{LL} \frac{\partial f}{\partial L}\Big|_{\mu,J} + \frac{1}{p^2}\frac{\partial}{\partial p}\Big|_{\alpha,L} p^2 \cdot \left(D_{pp}\frac{\partial}{\partial p}\Big|_{\alpha,L} f + D_{p\alpha}\frac{\partial}{\partial \alpha}\Big|_{p,L} f\right) + \frac{1}{T(\alpha)\sin(2\alpha)}\frac{\partial}{\partial \alpha}\Big|_{p,L} \cdot T(\alpha)\sin(2\alpha)\left((D_{\alpha\alpha}\frac{\partial}{\partial \alpha}\Big|_{p,L} f + D_{\alpha p}\frac{\partial}{\partial p}\Big|_{\alpha,L} f\right) - \frac{f}{\tau}$$

where f is the electron PSD, μ and J are the first and second adiabatic invariants, L^* is inversely proportional to the third adiabatic invariant Φ, τ is the electron's lifetime, and D_{LL}, D_{pp}, D_{aa}, $D_{p\alpha}$, and $D_{\alpha p}$ are diffusion coefficients. This equation accounts for **radial**, **energy**, and **pitch-angle** diffusion.

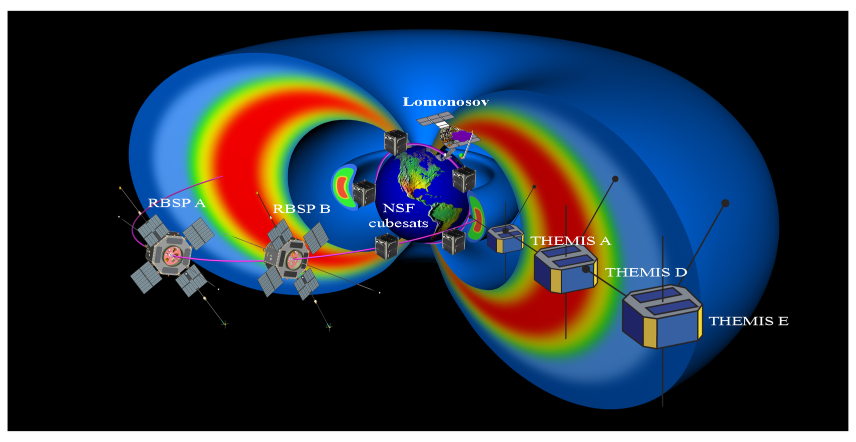

Figure 1: Radiation belts and orbits of satellites including Van Allen Probes, THEMIS, and NSF-supported Lomonosov mission.

Using the split-operator Kalman Filter technique [3], we can blend satellite data into the VERB-3D code. The state vector $\mathbf{x}_k^f$ calculated by the VERB-3D code is the electron PSD (f) at numerical grid locations in L^*. We use a standard Kalman filter to propagate the state vector in discrete time intervals Δt and update the forecast error $\mathbf{P}_k^f$. This adjustment of the model predictions to the satellite data, avoids accumulation of model errors with time.

References

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3.Convergence of EnKF to KF

- We first test the setup and convergence of our EnKF [2] in a controlled environment.
- We run a simulation of VERB-code with fixed parameters to create a "True-state"(figure 2,b)), from which we extract "synthetic" data [1].
- We run the VERB-code with different fixed parameters and use it as the "model-state" (figure 2,a)), in which the synthetic data is assimilated.

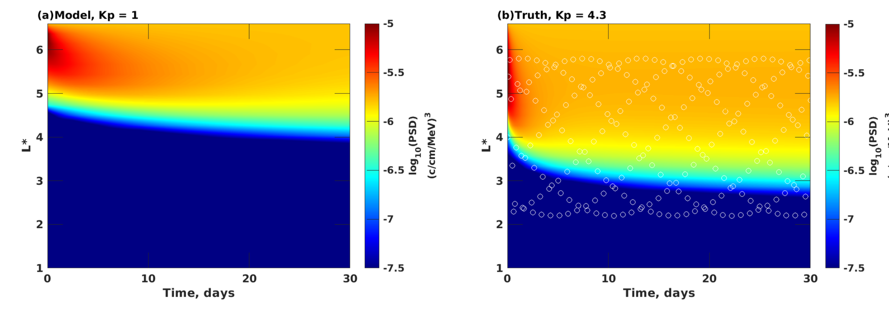

Figure 2: Model and true states for fraternal experiment : Electron PSD at $\mu = 700$ MeV/G and $K = 0.11$ G$^{0.5}$ Re. a) Model, using constant Kp = 1, b) true state of the system, using constant Kp= 4.3 (white circles are the orbit of the synthetic satellite, only 1pt per 3hours plotted)

- EnKF converges to KF solution with increasing number of ensemble members [1].

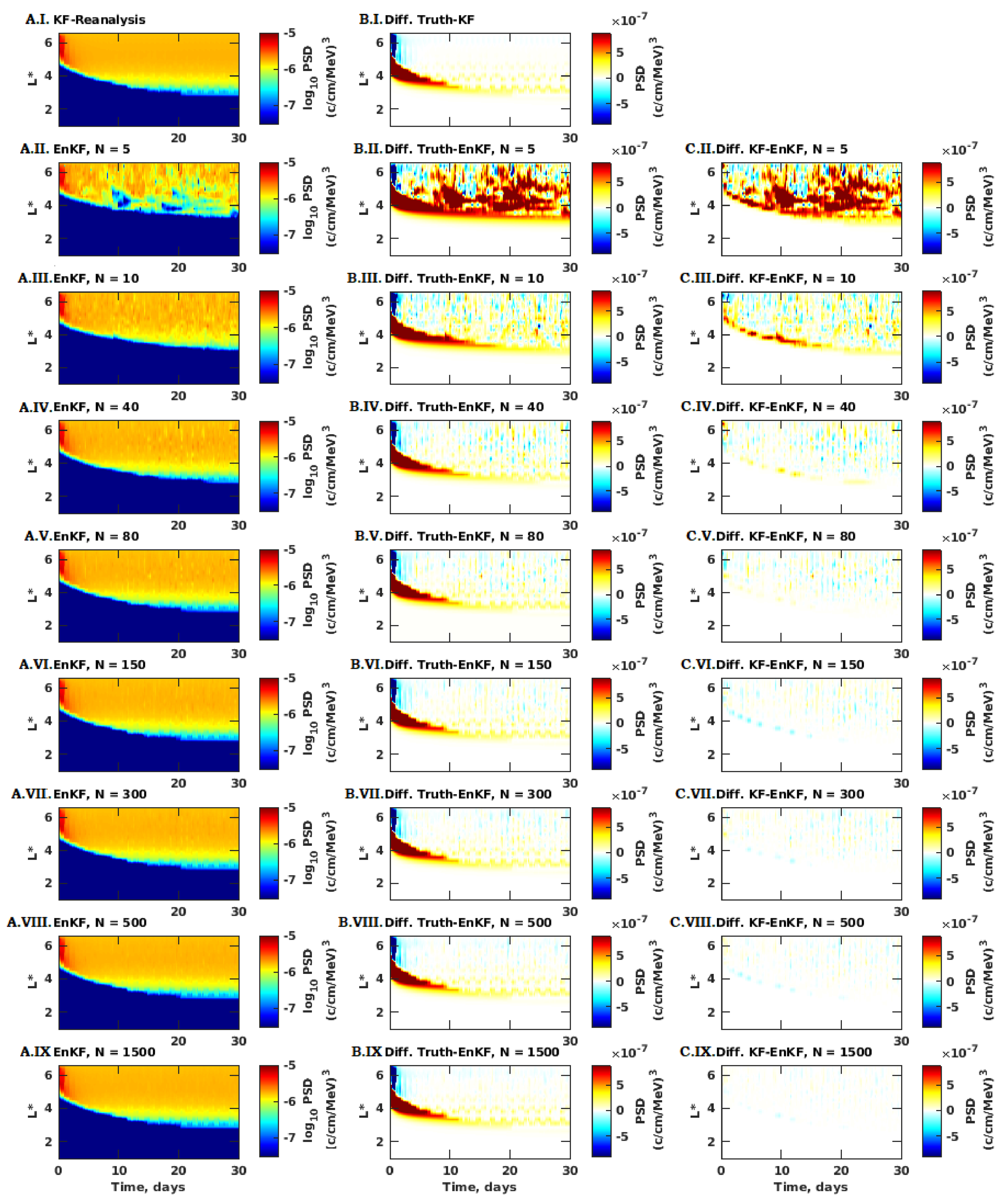

Figure 3: Fraternal experiment results: Electron PSD at $\mu = 700$ MeV/G and $K = 0.11$ G$^{0.5}$ Re. Column A depicts the assimilation results of KF(A.I) and EnKF (A.II to A.IX) for ensemble sizes $N_ens = 5, 10, 40, 80, 150, 300, 500, 1500$. Column (B) shows the difference between the true state and the assimilation results in column (A). Column (C) displays the difference between the estimate of KF and the estimate of EnKF

- For ensemble sizes larger 150, the convergence to the KF becomes so slow that even 1500 members do not lead to a significant difference.

6.Future Work

The implementation of DA techniques for space weather problems shows encouraging results. We additionally would like to:

1) implement localization and/or inflation.
2) perform parameter estimation.
3) improve the initial ensemble.
4) implement other ensemble based filters, s.a. square root filter or even hybrid methods.
5) extend of pitch-angle distributions for POES-MetOp data set.
6) perform long-term reanalysis of the RB using calibrated POES-MetOp data and compare it to electron flux data from ARASE.
7) extend the implementation of ensemble filters for other regions of the near Earth space

4.Satellite data intercalibration

Large data sets from polar-orbiting satellites, s.a. the meteorological MetOp and the Polar-orbiting Operational Environmental Satellite (POES) fleets, are particularly interesting to perform data assimilation studies of the radiation belts. However, these observations are often difficult to use due to proton contamination, instrumental errors and the need for background correction.

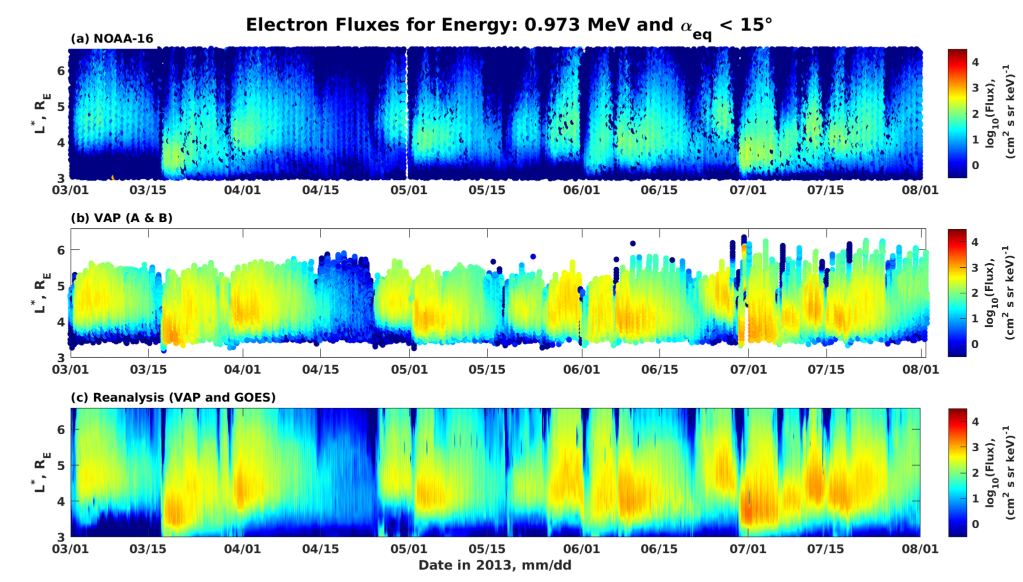

Figure 4: Datasets: Electron fluxes for the period of March till July, 2013 for $E = 1$ MeV, $\alpha_0 < 15°$ for a) NOAA-16; b) VAP (probes A and B); c) Reanalysis using VAP and GOES

We propose a cross-calibration approach for the POES-MetOp data sets using the reanalysis of electron fluxes obtained with Van Allen Probe (VAP) and Geostationary Operational Environmental Satellite (GOES) observations.

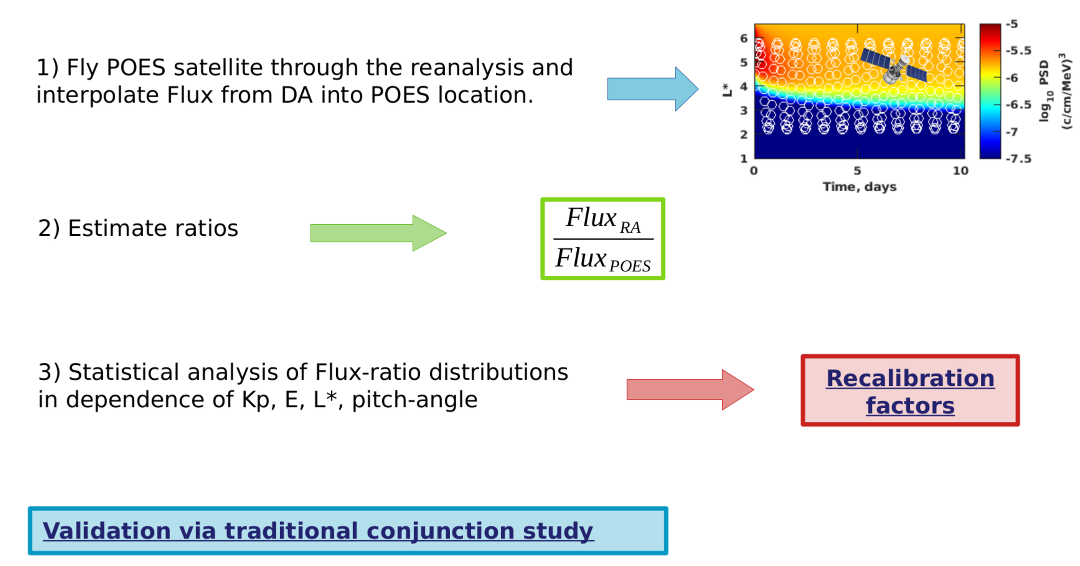

Figure 5: Method rationale: 1) A reanalysis of the global state of the radiation belts is performed using the VERB-3D code and measurements of RBSP and GOES. 2) The reanalysis of RBSP and GOES is interpolated along POES orbit. 3) PSD-ratios between the reanalysis and measured POES-MetOp allow us to estimate energy dependent intercalibration factors. 5) Using the estimated factors, we rescale the POES-MetOp dataset.

Intercalibration coefficients: We obtain energy-dependent distributions of the calculated PSD-ratios between the DA-modelled and measured POES/MetOp. From these distributions, energy-dependent intercalibration factors are estimated.

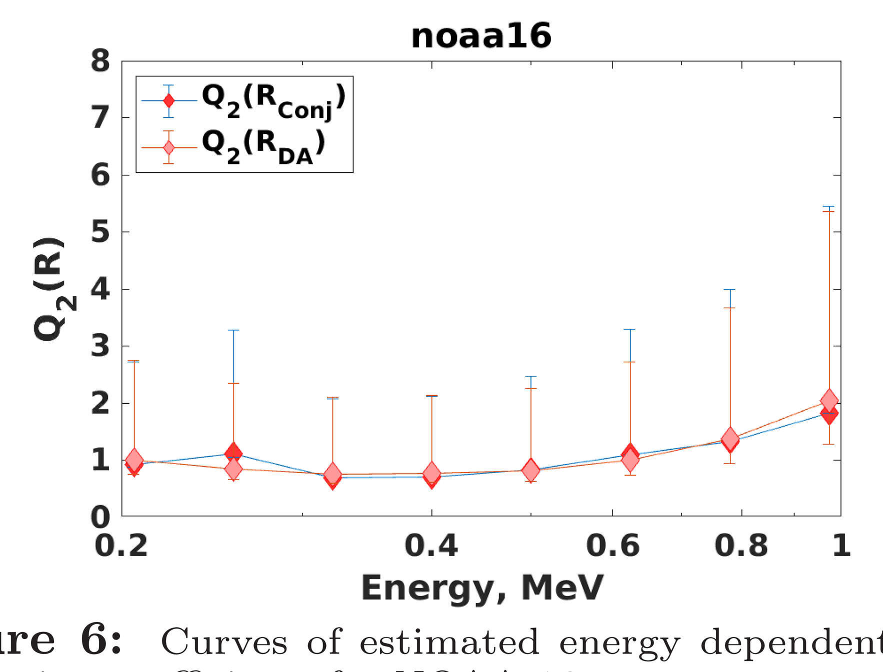

Figure 6: Curves of estimated energy dependent intercalibration coefficients for NOAA-16

5.Summary

- We have successfully implemented a perturbed EnKF on RB diffusion model and assured convergence to the KF solution.
- satellite data intercalibration using data assimilative reanalysis delivers reliable results that are in good agreement with traditional conjunction.