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UNDERGRADUATE PROJECT II

System Identification using Moving Window Extended Kalman Filter

Author:
Vishal KASHYAP

Supervisor:
Dr. G M KAMATH

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CERTIFICATE

The project entitled "System Identification using Moving Window Extended Kalman Filter" was done by Vishal Kashyap (150815), Department of Aerospace Engineering, under my supervision towards the fulfillment of credits for UGP-II (AE471A).

Signature:
Dr G M Kamath
Associate Professor,
Department of Aerospace Engineering,IIT Kanpur

1 Introduction[1]

Kalman filter is used for optimally estimating system state from noisy measurements.It combines the results obtained from dynamic equation (linear) of the state and the noisy measurements obtained to give an optimal value of the state.The extended Kalman filter (EKF), is a generalization of Kalman filter to non linear equations.It has been used successfully applied in various areas. EKF operates in a recursive fashion in which it requires just the past estimate and current measurement to find the optimal value of future estimate.

However, classical EKF has several limitations when applied to structural system identification which we will discuss subsequently; thus, we will analyze a method MWEKF (Moving window extended kalman filter) proposed by Z. Lai [1] as a tool in comparison to EKF for real-time system identification and damage detection after a discussion on the problems associated with the application of classical EKF in time-variant systems. MWEKF uses the moving-window technique to estimate covariance and mean of several quantities making it robust and adaptive in real time damage detection compared with classical EKF.As pointed out in [1] :

(1) it is insensitive to the selection of the initial state vector;

(2) it exhibits more accurate system parameter identification; and

(3) it is immune to the inaccurate assumption of noise levels because measurement and process noise levels are estimated in this approach.

The above three features of MWEKF are put to test and compared with classical EKF through numerical simulations of time- variant spring mass damper system. Results demonstrate that MWEKF is a robust and effective tool for system identification and damage detection in structures.

1.1 Motivation

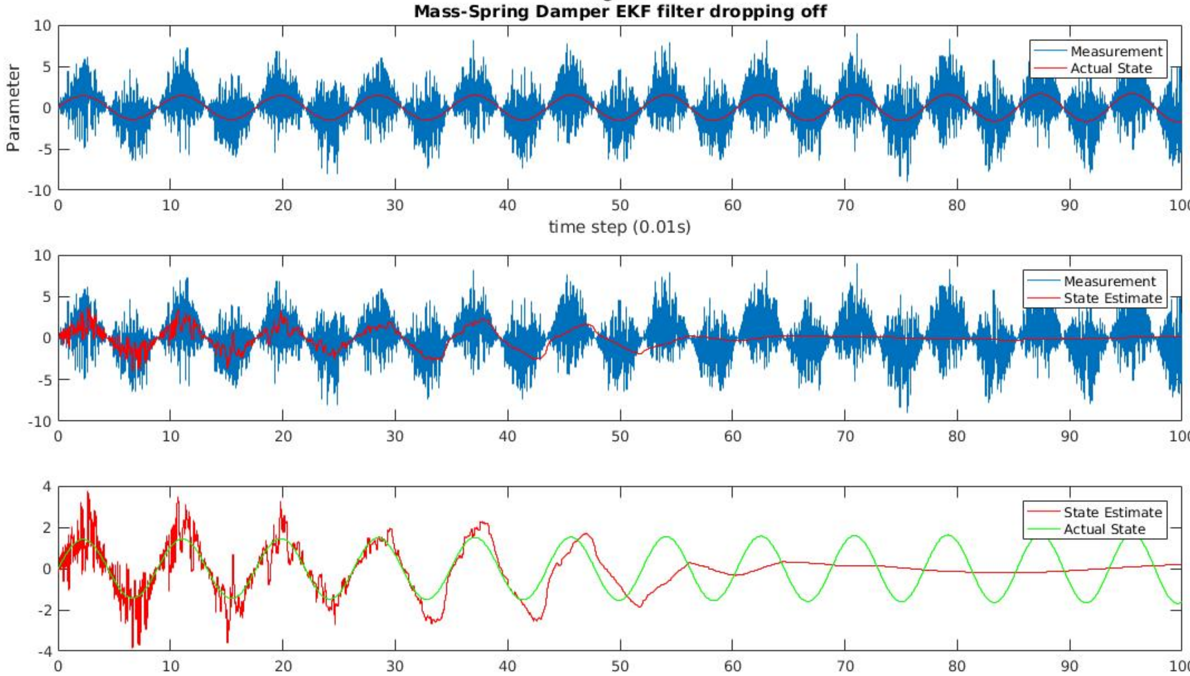
We will first start with discussing the limitations of EKF and then visit how these limitations can be overcome with MWEKF

Classical EKF exhibits poor performance when identifying non-stationary systems. Estimation error covariance decreases rapidly during the recursive process of classical EKF, and the estimated parameters gradually converge to steady-state values. During this process as the estimation error covariance matrix becomes

[1] All code available at : https://github.com/vkashyap10/mwekf

too small its ability to track fast and sudden changes in system parameter becomes less and less effective. This situation may lead to inaccurate estimation of changed and unchanged parameters. As the above process takes place, it so happens that Kalman Gain matrix which controls the trade-off between solution of dynamic equations and the measurements takes a constant value. But in a non stationary system which has changing covariance matrices, the value of kalman gain obtained previously no longer gives optimal solution. This Kalman gain does not change as the measurement covariance matrix which controls it has already approached zero. Due to this the system estimates get stuck resulting in inadaptibility of EKF.This problem is referred to as “filter dropping off”.

fig 1 :
Mass-Spring Damper EKF filter dropping off



One solution suggested for the above problem in literature is to keep resetting the estimation error covariance matrix. But this leads to loss in accuracy as the system has to converge again and again each time we change the estimation error. Another practical constraint of EKF is related to guaranteeing optimality. This algorithm requires an estimation of the model which is can be approximated to generate the process and measurement noise covariance matrices that are often unavailable in applications.In classical EKF, these two matrices are often assumed to be constant. They are not only difficult to evaluate in advance, but may change during the course of experiment and hence have to be kept track of. Inaccurate evaluation of these two covariance matrices of process and measurement noises may considerably degrade EKF performance. Furthermore, the estimation errors introduced by the inaccurate noise covariance matrices may accumulate over time, thus causing the estimated state and parameters to deviate significantly from actual values.

Solution : In this study, we test the MWEKF in the above mentioned two problems. The statistical properties of process and measurement noises are calculated using a moving-window estimation technique. The structural response and parameters, as well as process and measurement noise covariance, are simultaneously evaluated in the recursive process of the filter. Compared with classical EKF, MWEKF (1) Does not require an accurate estimation of process and measurement covariance noise, (2) avoids difficult selection of the initial estimation error covariance and the "filter drop- ping off" problem, and (3) the above two make MWEKF robust and adaptive and can be used in non-stationary systems.The numerical and experimental results clearly demonstrate the effectiveness of MWEKF in different structural damage detection situations.

2 Literature review and objectives of work

In literature, EKF and its versions have been tested several times for various applications.MWEKF has been proposed and tested for sudden parameter change detection in civil structures in Z. Lai et al [1]. We will have the following objectives:

1. Test MWEKF for its adaptability and robust nature with a spring mass damper system.

2. Application of MWEKF for estimation of non-stationary system parameters.

3. Lay down a method for backlash/free-play detection in a system using MWEKF and apply it to spring mass damper system with free-play.

3 Numerical Method

The following section initially lays down the working framework for the Extended Kalman filter and points out the equations that lead to the limitations listed in previous section. We then will look at substitutes for these equations as proposed in MWEKF. We will also look at the corresponding equations for our application of estimation of spring constant.

3.1 Extended Kalman Filter

Consider a spring mass damper system with mass m, spring constant k and damping coefficient c. The dynamic equation for such a system can be written as :

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{1}$$

Each of the coefficients mentioned above can be represented as a vector of parameters according to the system.The state X can then be

$$X = [x, \dot{x}, \theta] \tag{2}$$

Here θ is the vector of unknowns (system parameters) that we want to estimate using the filter.Notice that in such a case the dynamic equation above becomes non linear and therefore we need to employ EKF instead of kalman filter. The state equation :

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ -m^{-1}c\dot{x} - m^{-1}kx + m^{-1}f \\ 0 \end{bmatrix} + w(t) \tag{3}$$

More generalized form of the above equations is $\dot{X} = g(X, f, t) + w(t)$, where $w(t)$ is the process noise caused by model uncertainty. Similarly we can relate observation from sensor data to the quantities we want to measure (present in our state matrix).

$$Y = h(X, f, t) + v(t) \tag{4}$$

Here $v(t)$ is the measurement noise and $h(.)$ is the observation matrix.We will be dealing with the case where our observation matrix will have just be the displacement of the mass from the rest position.Equations 3 and 4 can be discretized and expressed as follows:

$$X(t_{k+1}) = X(t_k) \int_{t_k}^{t_{k+1}} g(X(t_k), f(t), t)dt + w_k \tag{5}$$

$$Y_{k+1} = h(X_{k+1}, f_{k+1}, t_{k+1}) + v_{k+1} \tag{6}$$

In the general EKF the noise and process covariance matrices are considered to be random Gaussian noise with a guess of paramters for the Gaussian distribution (mean and covariance). This may lead to an inaccurate model.Without accurate information for initial state and system parameters, the extended state can only be approximately estimated at step k as

$$\tilde{X}_{k+1} = \hat{X}_k + \int_{t_k}^{t_{k+1}} g(\tilde{X}_k, f(t), t)dt \tag{7}$$

where $\hat{x}$ is the posterior estimate and $\tilde{x}$ is the priori estimate. Given the priori estimate at step k, the optimal posterior can be obtained at step k+1 using the kalman gain H and the measurements.

$$\hat{X}_{k+1} = \tilde{X}_{k+1} + K_{k+1}(Y_{k+1} - H_{k+1}\tilde{X}_{k+1}) \tag{8}$$

Equation (7) and (8) are the system prediction and measurement update equations.The measurement update equation combines the system prediction obtained using dynamic equation of the system with the measurements obtained form sensor data optimally using the Kalman gain. The linearized observation and Kalman gain matrices are updated as follows

$$H_{k+1} = \left[\frac{\partial h_i(\hat{X}_{k+1})}{\partial \hat{x}_j} \right] \tag{9}$$

$$K_{k+1} = \tilde{P}_{k+1} H_{k+1}^T (H_{k+1} \tilde{P}_{k+1} H_{k+1}^T + R_{k+1})^{-1} \tag{10}$$

P_{k+1} is the covariance matrix of the estimation error at step k + 1. Eq 9 is the Jacobian matrix of the partial derivative of h(.) with respect to each element in X.

$$\tilde{P}_{k+1} = \phi_k P_k \phi_k^T + Q_k \tag{11}$$

$$\phi_k = e^{A(X_k)\delta t} I + \delta t(A(X_k)) \tag{12}$$

$$A = \left[\frac{\partial g_i(\hat{X}_k)}{\partial \hat{x}_j} \right] \tag{13}$$

$$P_{k+1} = (I - K_{k+1}H_{k+1})\tilde{P}_{k+1} \tag{14}$$

For the above iterative scheme to work it has to be initially fed with the process and measurement noise covariance matrices. EKF assumes these to be gaussian with zero mean and considers the noise to be constant throughout time. This may not be true and the inaccuracy in the system which estimate using process noise or the measurement noise may change properties. This inefficiency in calculation of initial parameters and their static nature assumption makes the filter non optimal.

3.2 Moving Window Extended Kalman Filter

MWEKF was introduced to overcome the difficulty of estimation of process noise and measurement noise covariance matrices. This method uses a specified number of past data points to estimate the covariance matrices. Thus the estimates are no longer static and the system is no longer dependent on the initial estimates fed to it. The measurement noise is estimated as follows:

$$\epsilon_k = Y_k - Y_k^{true} \tag{15}$$

Where Y_k^{true} is the actual value of system parameter being observed. We estimate this with the moving average of past data points.One can also estimate this with exponential or weighted average depending on the system.

$$Y_k^{smooth} = (1-\lambda)Y_{k-1} + \lambda Y_k \tag{16}$$

$$\epsilon_k^{'} = Y_k - Y_k^{smooth} \tag{17}$$

The mean and covariance of the process noise can then be easily obtained as follows:

$$E[\epsilon_j^{'}] = \frac{1}{N_r} \sum_{j=k-N_r+1}^{k} \epsilon_j^{'} \tag{18}$$

$$R_k^{'} = \frac{1}{N_r - 1} \sum_{j=k-N_r+1}^{k} \left[(\epsilon_j^{'} - E[\epsilon_j^{'}])(\epsilon_j^{'} - E[\epsilon_j^{'}])^T \right] \tag{19}$$

where N_r is the number of data points being considered , that is the size of window taken for estimation. Substituting in (16) we get

$$\epsilon_k^{'} = (1-\lambda)\left[Y_k^{true} - Y_{k-1}^{true} + (\epsilon_k - \epsilon_{k-1}) \right] \tag{20}$$

The above values of expected measurement noise and its covariance can be used instead of its static counterparts as proposed in EKF. As proposed by Z lai [1] the following equations can be used to estimate the process covariance mean and expected value with $\tau = 0$.

$$Q_{k+1} = Q_k e^{a^T H^T (C_0 - R - H\tilde{P}H^T) H a} \tag{21}$$

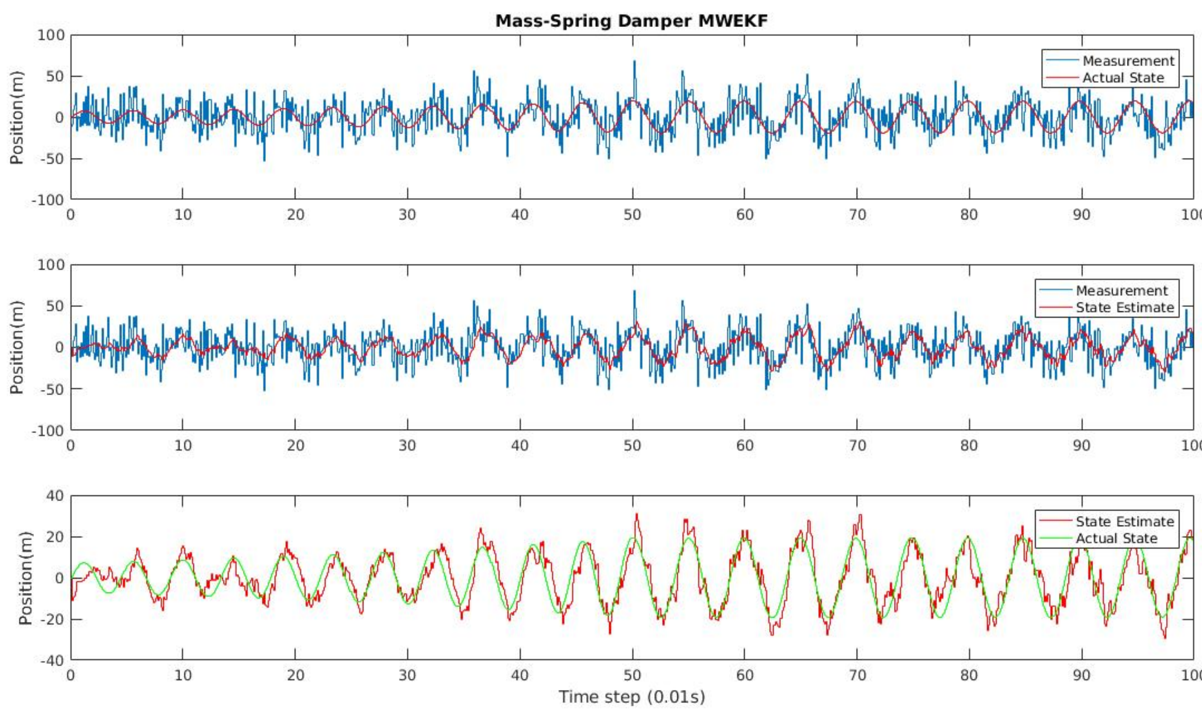
$$C_\tau = \frac{1}{N_q - 1} \sum_{j=k-N_q+1}^{k} e_j e_{j-\tau}^T \tag{22}$$

Notice that MWEKF relies on the past window of data points unlike EKF which just needs one past data point. An optimal value of window length has to be selected. A small value may result in inaccurate estimation of the statistical properties, a large value may cause the filter to lose its fast response to change in system properties.

4 Results and Discussion

Simulated data which will be taken as true state of the system was obtained in the following manner: The true system state at each point in time was first calculated by solving the system equations forward in time and adding process noise with mean [00], covariance $[0.40.4; 0.40.4]$ and unit magnitude. Sensor measurement(position in this case) was taken to be the sum of true state and measurement noise with mean 0 ,variance 0.4 and magnitude 5. The damping constant c was kept to be 0.1 ,mass of the spring was taken to be 5 Kg. This above description gives us the true state which we will try to achieve using MWEKF which takes as input initial guess of the process and measurement covariance matrices and their mean,spring constant (θ paramter to be estimated),step rate for process covariance calculation,window size, λ and covariance matrix of estimation error.

Starting with the guess of $k = 0$, window size 10,$\lambda = 0.5$ and the parameters: P = [0.1 0.1 0.1; 0.1 0.1 0.1;0.1 0.1 0.1]; Q = [0.01 0.01 0.01; 0.01 0.01 0.01 ; 0.01 0 0]; R = 21; a = [1e-1 1e-1 1e-1] where P,Q,R and a are covariance matrix of estimation error, process covariance, measurement covariance and step rate coefficient matrix respectively.Notice the accuracy difference from fig 1.



4.1 Linear-Variation Parameter Estimation

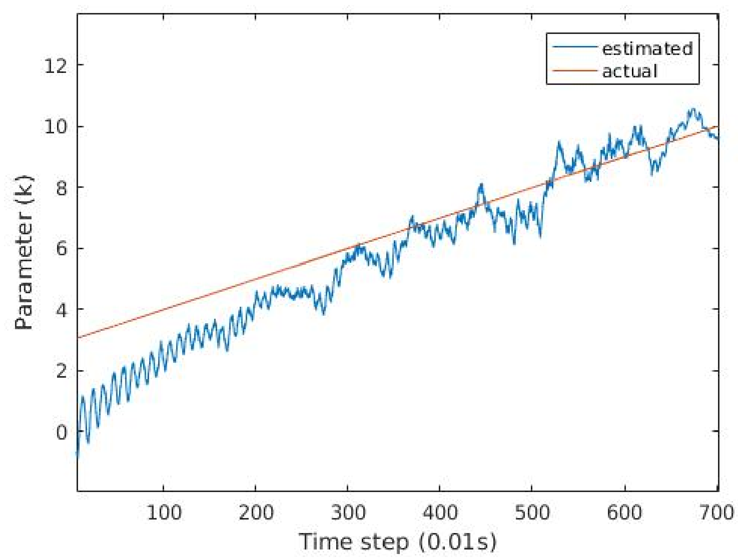


Figure 3 : Linear change in parameter.

System Parameter identification has been successfully attempted earlier using EKF for systems whose initial parameters can be accurately estimated.But it fails dealing with non-stationary parameter estimation. In this study, given the robust nature of MWEKF we tested its ability to estimate a linearly changing parameter. The spring constant k was given an initial value of 3 N/m and varied with a slope of 0.01 with each time step.The spring mass m was set to be 5 Kg. MWEKF successfully captures the trend as it is not limited by "dropping off" of the filter as in the case of EKF. Such continuous real time evaluation of system parameters finds many applications in structural health monitoring as one can make a good prediction about the behaviour of the system in advance.

4.2 Sudden change Parameter Detection

A spring mass damper system subjected to abrupt change in spring constant k is studied numerically.Such a sudden change can not be explained by the dynamic equations of motion of the system and requires a feedback system that keeps correcting itself. EKF assumes constant parameters and loses its ability to show rapid changes in the system. MWEKF on the other hand keeps refining the parameters fed to it and the parameters it has to estimate using the past window of data. Thus any change in the system reflects in the past data points which are used to generate the current estimates. Giving the initial spring k to be 3 N/m with a slope of 0.01 per time step, a sudden change of +3 units was introduced at time step 500 in k.

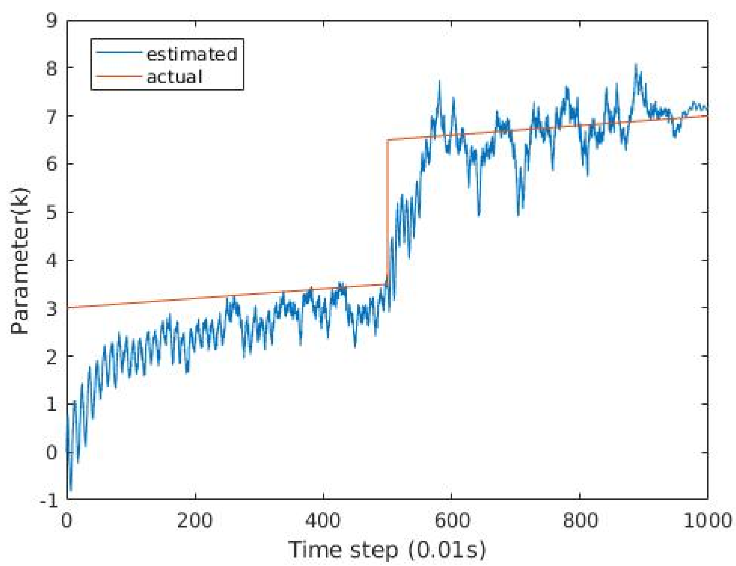


Figure 4 : Sudden change in parameter detection.

We can observe that the system reacts in approximately 100 time steps which is 1 second. This can be adjusted by varying the window length and step size in process covariance estimation, but as already mentioned there is a trade-off between accuracy and delay.

4.3 Free-play/Backlash detection

Citing wikipedia :In mechanical engineering, backlash, sometimes called lash or play, is a clearance or lost motion in a mechanism caused by gaps between the parts. It can be defined as ”the maximum distance or angle through which any part of a mechanical system may be moved in one direction without applying appreciable force or motion to the next part in mechanical sequence. In this subsection a method to calculate free-play is being proposed using MWEKF due to its robust nature and quick response to parameter change. Free-play can be modeled as a system with $k(spring constant) \approx 0$, keeping all other parameters constant.This will result in parameter identification as shown in the figure below.

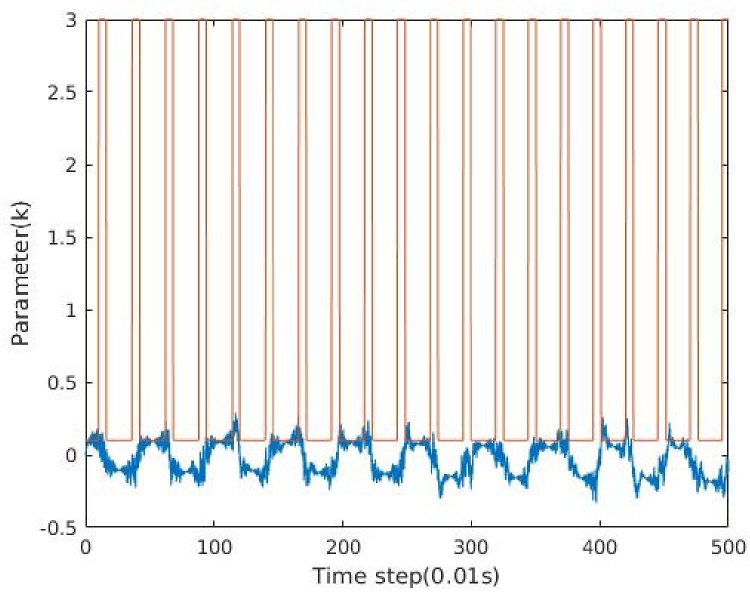


Figure 5 : Backlash detection

The method being proposed below can be applied to a non stationary system by following the below steps on a window of data points as in the MWEKF. It is important for the backlash to depend on the paramter being estimated for this method to apply. Steps to detect free-play (backlash):

1. Store the height,width and corresponding time of change in the parameter.
2. Select a threshold depending on the system and merge all the stored values in step 1 whose difference of height and width is below the threshold.
3. Now MWEKF can be used to calculate any system parameter corresponding to all the times at which backlash occur.
4. Take the average of the backlashes that repeat after equal interval of time. This will give a value corresponding to the backlash, for each backlash in the system.

In the current example to model free-play condition the spring constant was kept at a value of 0.1 for $abs(x) < 0.5$, where x is the position of the spring. The free-play was then calculated to be 0.693.

5 Concluding Remarks and Future Work

We tested the MWEKF for its ability to perform with inaccurate initial covariance matrices. It proves to be adaptable and robust and deals with the ”filter dropping off” problem which results in much more accurate state estimation than classical EKF. We tested MWEKF succesfully in estimating a non-stationary system paramter. We also proposed a scheme to find backlash in a non-stationary system with multiple backlashes which uses MWEKF and relies on its fast response and accuracy in estimation of some system parameter which in-turn relates to the backlash.

Further study can concentrate on testing the approach suggested for backlash estimation on other non-linear, non-stationary system and testing the limit on time of response of parameter change.

6 References

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