Assignment 5

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1. DEA Analysis - Hope Valley Health Care Association

The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labour (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third-party sources and the number of patient-days reimbursed privately.

1) Formulating and Performing DEA Analysis

2) Peers and Lambdas under each assumption

```
library(Benchmarking)

## Loading required package: lpSolveAPI

## Loading required package: ucminf

## Loading required package: quadprog

x <- matrix(c(150,400,320,520,350,320,0.2,0.7,1.2,2.0,1.2,0.7),ncol = 2)
y <-
matrix(c(14000,14000,42000,28000,19000,14000,3500,21000,10500,42000,25000,150
00),ncol = 2)

colnames(x) <- c("Staff Hours per Day","Supplies per Day")
colnames(y) <- c("Reimbursed Patient-Days","Privately Paid Patient-Days")

Hope <- cbind(x,y)
Hope

##      Staff Hours per Day Supplies per Day Reimbursed Patient-Days
## [1,]                 150              0.2                   14000
## [2,]                 400              0.7                   14000
## [3,]                 320              1.2                   42000
## [4,]                 520              2.0                   28000
## [5,]                 350              1.2                   19000
## [6,]                 320              0.7                   14000
##      Privately Paid Patient-Days
```

```
## [1,]                           3500
## [2,]                          21000
## [3,]                          10500
## [4,]                          42000
## [5,]                          25000
## [6,]                          15000
```

Performing DEA analysis using FDH assumption

```
FDH <- dea(x,y,RTS = "fdh")
FDH

## [1] 1 1 1 1 1 1

# Peers under FDH assumption
peers(FDH)

##      peer1
## [1,]     1
## [2,]     2
## [3,]     3
## [4,]     4
## [5,]     5
## [6,]     6

# Lambdas under FDH assumption
lambda(FDH)

##      L1 L2 L3 L4 L5 L6
## [1,]  1  0  0  0  0  0
## [2,]  0  1  0  0  0  0
## [3,]  0  0  1  0  0  0
## [4,]  0  0  0  1  0  0
## [5,]  0  0  0  0  1  0
## [6,]  0  0  0  0  0  1
```

Performing DEA analysis using CRS assumption

```
CRS <- dea(x,y,RTS = "crs")
CRS

## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675

# Peers under CRS assumption
peers(CRS)

##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
# Lambdas under CRS assumption
lambda(CRS)

##             L1         L2 L3        L4
## [1,] 1.0000000 0.00000000  0 0.0000000
## [2,] 0.0000000 1.00000000  0 0.0000000
## [3,] 0.0000000 0.00000000  1 0.0000000
## [4,] 0.0000000 0.00000000  0 1.0000000
## [5,] 0.2000000 0.08048142  0 0.5383307
## [6,] 0.3428571 0.39499264  0 0.1310751
```

Performing DEA analysis using VRS assumption

```
VRS <- dea(x,y,RTS = "vrs")
VRS

## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963

# Peers under VRS assumption
peers(VRS)

##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5

# Lambdas under VRS assumption
lambda(VRS)

##             L1        L2 L3 L4        L5
## [1,] 1.0000000 0.0000000  0  0 0.0000000
## [2,] 0.0000000 1.0000000  0  0 0.0000000
## [3,] 0.0000000 0.0000000  1  0 0.0000000
## [4,] 0.0000000 0.0000000  0  1 0.0000000
## [5,] 0.0000000 0.0000000  0  0 1.0000000
## [6,] 0.4014399 0.3422606  0  0 0.2562995
```

Performing DEA analysis using IRS assumption

```
IRS <- dea(x,y,RTS = "irs")
IRS

## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963

# Peers under IRS assumption
peers(IRS)

##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
```

```
## [4,]     4    NA    NA
## [5,]     5    NA    NA
## [6,]     1     2     5
```

```
# Lambdas under IRS assumption
lambda(IRS)
```

```
##             L1        L2 L3 L4        L5
## [1,] 1.0000000 0.0000000  0  0 0.0000000
## [2,] 0.0000000 1.0000000  0  0 0.0000000
## [3,] 0.0000000 0.0000000  1  0 0.0000000
## [4,] 0.0000000 0.0000000  0  1 0.0000000
## [5,] 0.0000000 0.0000000  0  0 1.0000000
## [6,] 0.4014399 0.3422606  0  0 0.2562995
```

Performing DEA analysis using DRS assumption

```
DRS <- dea(x,y,RTS = "drs")
DRS
```

```
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
```

```
# Peers under DRS assumption
peers(DRS)
```

```
##      peer1 peer2 peer3
## [1,]     1    NA    NA
## [2,]     2    NA    NA
## [3,]     3    NA    NA
## [4,]     4    NA    NA
## [5,]     1     2     4
## [6,]     1     2     4
```

```
# Lambdas under DRS assumption
lambda(DRS)
```

```
##             L1         L2 L3        L4
## [1,] 1.0000000 0.00000000  0 0.0000000
## [2,] 0.0000000 1.00000000  0 0.0000000
## [3,] 0.0000000 0.00000000  1 0.0000000
## [4,] 0.0000000 0.00000000  0 1.0000000
## [5,] 0.2000000 0.08048142  0 0.5383307
## [6,] 0.3428571 0.39499264  0 0.1310751
```

Performing DEA analysis using FRH assumption

```
FRH <- dea(x,y,RTS = "add")
FRH
```

```
## [1] 1 1 1 1 1 1
```

```
# Peers under FRH assumption
peers(FRH)
```

```
##      peer1
## [1,]     1
## [2,]     2
## [3,]     3
## [4,]     4
## [5,]     5
## [6,]     6

# Lambdas under FRH assumption
lambda(FRH)

##      L1 L2 L3 L4 L5 L6
## [1,]  1  0  0  0  0  0
## [2,]  0  1  0  0  0  0
## [3,]  0  0  1  0  0  0
## [4,]  0  0  0  1  0  0
## [5,]  0  0  0  0  1  0
## [6,]  0  0  0  0  0  1
```

3) Summary of DEA analysis under all assumptions

```
Summary <- data.frame(FDH=c(1, 1, 1, 1, 1, 1), CRS=c(1, 1, 1, 1, 0.9775,
0.8675), VRS=c(1, 1, 1, 1, 1, 0.8963), IRS=c(1, 1, 1, 1, 1,0.8963), DRS=c(1,
1, 1, 1, 0.9775, 0.8675), FRH=c(1, 1, 1, 1, 1, 1))
DEA <- cbind(Hope, Summary)
DEA

##   Staff Hours per Day Supplies per Day Reimbursed Patient-Days
## 1                 150              0.2                   14000
## 2                 400              0.7                   14000
## 3                 320              1.2                   42000
## 4                 520              2.0                   28000
## 5                 350              1.2                   19000
## 6                 320              0.7                   14000
##   Privately Paid Patient-Days FDH    CRS    VRS    IRS    DRS FRH
## 1                        3500   1 1.0000 1.0000 1.0000 1.0000   1
## 2                       21000   1 1.0000 1.0000 1.0000 1.0000   1
## 3                       10500   1 1.0000 1.0000 1.0000 1.0000   1
## 4                       42000   1 1.0000 1.0000 1.0000 1.0000   1
## 5                       25000   1 0.9775 1.0000 1.0000 0.9775   1
## 6                       15000   1 0.8675 0.8963 0.8963 0.8675   1
```

4) Analysis comparision

DMUs 1, 2, 3 and 4 have efficiency value of 1 under all the above assumptions upon DEA analysis.

DMU5 has efficiency value of 1 under FDH, VRS, IRS and FRH assumptions but the efficiency value is reduced to 0.9775 under CRS and DRS assumptions.

DMU6 has efficiency value of 1 under FDH and FRH assumptions, 0.8675 under CRS and DRS assumptions and value of 0.8963 under VRS and IRS assumptions.

2. Goal Programming - Emax Corporation

The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings next year from the $75 million achieved this year. In particular, using the units given in the following table, they want to

Maximize Z = P - 6C - 3D, where

P = total (discounted) profit over the life of the new products,
C = change (in either direction) in the current level of employment,
D = decrease (if any) in next year's earnings from the current year's level.

The amount of any increase in earnings does not enter into Z, because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.)

Based on the problem statement, the goal is to:

Maximize Z = P - 6C - 3D

P = total (discounted) profit over the life of the new products, C = change (in either direction) in the current level of employment, D = decrease (if any) in next year's earnings from the current year's level.

Subject to:

Total Profit: P = 20X1 + 15X2 + 25X3

Employment Level: 6X1 + 4X2 + 5X3 = 50

Earnings Next Year: 8X1 + 7X2 + 5X3 >= 75

As a result, the auxiliary variables become: Y1 = 6X1 + 4X2 + 5X3 - 50 Y2 = 8X1 + 7X2 + 5X3 - 75 (Y1P - Y1M) = 6X1 + 4X2 + 5X3 - 50 (Y2P - Y2M) = 8X1 + 7X2 + 5X3 - 75

Therefore, the problem statement is:

Maximize Z = 20X1 + 15X2 + 25X3 - 6Y1P - 6Y1M - 3Y2M

Where, 6X1 + 4X2 + 5X3 - (Y1P - Y1M) = 50

8X1 + 7X2 + 5X3 - (Y2P - Y2M) = 75

X1, X2, X3, Y1P, Y1M, Y2P, Y2M >= 0

Lastly, we will run this problem in R as a linear programming model and discuss the results.

Reading data

```
library(lpSolve)
library(lpSolveAPI)
Goal <- read.lp("vkatta_5.lp")
```

Solving the LP

```
solve(Goal)

## [1] 0

get.objective(Goal)

## [1] 225

get.variables(Goal)

## [1] 375  25   0   0   0   0  15   0
```

Based on the output of the linear programming model, we can conclude the below;

X1 = 0; X2 = 0; X3 = 15; Y1P = 25; Y1M = 0; Y2M = 0; Y2P = 0.

Therefore, we can conclude that the product mix should only contain product 3. With this mix, there would be an object value of 225 units. The goal for earnings for next year is fully met. However, the employment level goal will be exceeded by 25 units, which is 2,500 employees and a penalty of 150 units to the objective function.