1. **(Computer Center Staffing)** You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center. It is open from 8 am until midnight. You have monitored the usage of the center at various times of the day and determined that the following numbers of computer consultants are required.

| Time of day | Minimum number of consultants required to be on duty |
|---|---|
| 8 am–noon | 4 |
| Noon–4 pm | 8 |
| 4 am–8 pm | 10 |
| 8 am–midnight | 6 |

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 am – 4 pm), afternoon (noon – 8 pm), and evening (4 pm – midnight). Full-time consult- ants are paid $14 per hour.

Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, at least one full-time consultant must be on duty for every part-time consultant on duty.

A. a) Based on the above data, the following information can be deduced.

Salary of Full-time consultants = $14

Salary of Part-time consultants = $1

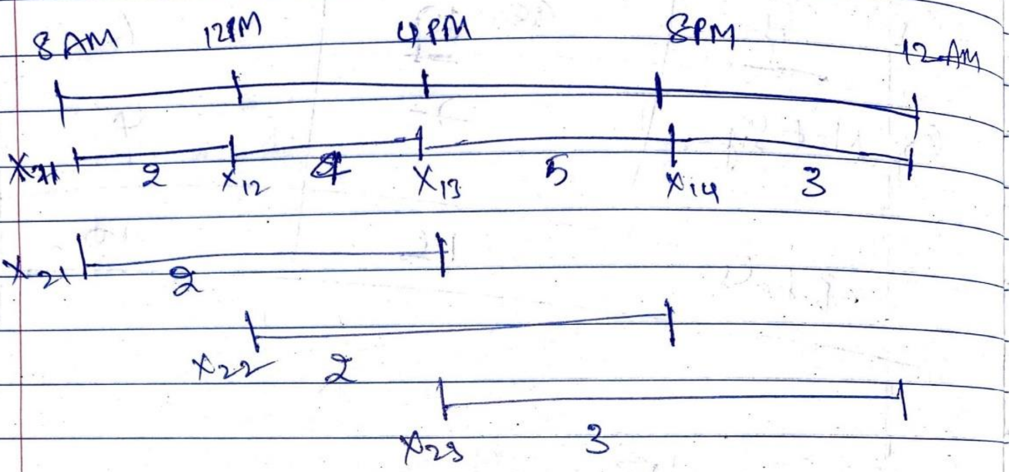
Working hours of Full-time workers = 8

Working hours of Part-time workers = 4

a) Determine a minimum-cost staffing plan for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part time? What is the minimum cost?

Computer Center Staffing

1 (a)




$$\text{Min } Z = 14 \times 8 \times [X_{21} + X_{22} + X_{23}] + 12 \times 4 [X_{11} + X_{12} + X_{13} + X_{14}]$$

Where,

$X_{11}$ → Part-time workers in 8AM – 12PM shift.

$X_{12}$ → 12PM – 4PM

$X_{13}$ → 4PM – 8PM

$X_{14}$ → 8PM – 12AM

$X_{21}$ → Full-time workers in 8AM – 4PM shift.

$X_{22}$ → 12PM – 8PM

$X_{23}$ → 4PM – 12AM

Constraints :

$$X_{11} + X_{21} \geq 4$$
$$X_{12} + X_{22} + X_{21} \geq 8$$
$$X_{13} + X_{22} + X_{23} \geq 10$$
$$X_{14} + X_{23} \geq 6$$

$$X_{21}, X_{22}, X_{23} > 0$$
$$X_{11}, X_{12}, X_{13}, X_{14} \geq 0$$

Time Constraints

$$X_1 \leq X_2 \quad ; \quad X_1 = 4 \quad ; \quad X_2 = 8$$

For minimum cost using the above constraints the values for the above variables are as follows:

$X_{11} = 2$ ; $X_{21} = 2$.
$X_{12} = 4$ ; $X_{22} = 2$
$X_{13} = 5$ ; $X_{23} = 3$.
$X_{14} = 3$.

Now, the minimum cost of the problem is

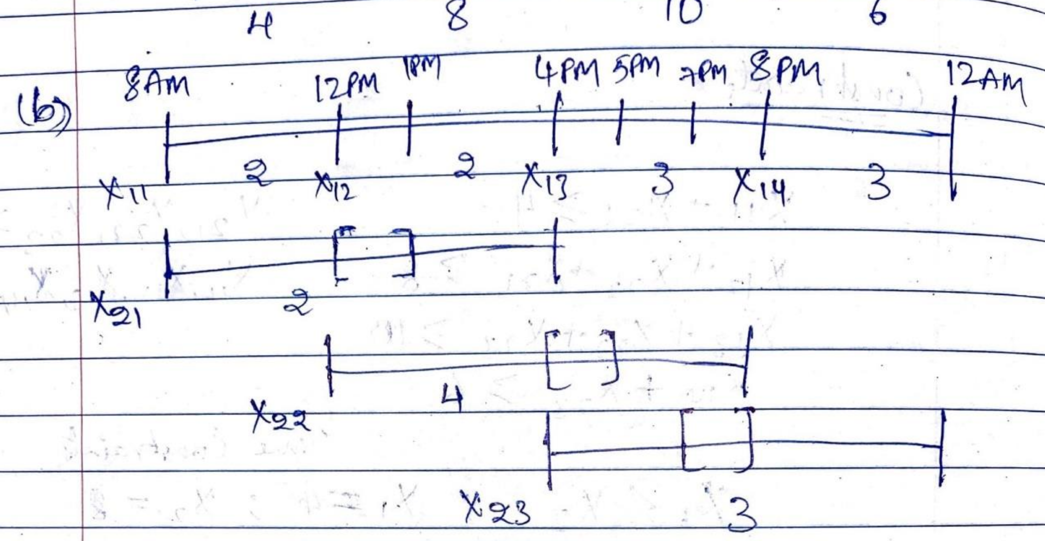
140
28
112

$$Z = 112\left[X_{21} + X_{22} + X_{23}\right] + 48\left[X_{11} + X_{12} + X_{13} + X_{14}\right]$$
$$= 112\left[2 + 2 + 3\right] + 48\left[2 + 4 + 5 + 3\right]$$
$$= 112 \times 7 + 48 \times 14 = \$1456$$

with 7 full-time workers and 14 Part-time workers

b) After thinking about this problem for a while, you have decided to recognize meal breaks explicitly in the scheduling of full-time consultants. In particular, full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

Computer Center Staffing - With Lunch




For the minimum cost including lunch breaks for full-time workers, the min function remains the same with change in values of X. Assuming the workers ~~g~~not get paid for lunch,

$$\min Z = 14 \times 7[X_{21}+X_{22}+X_{23}] + 12 \times 4 [X_{11}+X_{12}+X_{13}+X_{14}]$$

For the above problem with inclusion of lunch breaks, the constraints change as follows;

The shifts would be divided into 3-parts each for full-time workers to accomodate lunch.

Constraints

$$X_{11} + X_{21} \geq 4 \qquad \text{8AM to 12PM}$$

$$X_{21} + X_{12} + X_{22} \geq 8 \qquad \text{12PM to 4PM}$$

$$X_{13} + X_{22} + X_{23} \geq 10 \qquad \text{4PM to 8PM}$$

$$X_{14} + X_{23} \geq 6. \qquad \text{8PM to 12AM}$$

$$X_1 \leq X_2$$

$X_{21}, X_{22}, X_{23} > 0$ ; $X_{11}, X_{12}, X_{13}, X_{14} \geq 0$.

For minimum cost, along with the above constraints, the best possible values for the variables are as follows;

$X_{11} = 2$ ; $X_{12} = 2$ ; $X_{13} = 3$ ; $X_{14} = 3$

$X_{21} = 2$ ; $X_{22} = 4$ ; $X_{23} = 3$

Hence, the minimum cost function applied to the variables is,

$$Z = 98[2+4+3] + 48[2+2+3+3]$$

$$= 98 \times 9 + 48 \times 10$$

$$= 882 + 480$$

$$= \$1362$$

with 9 Full-time workers and 10 Part-time workers

2. Consider the problem from the previous assignment.
Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. Solve this problem graphically.

Back Savers Production

(2) The mathematical formulation of the given problem is as below.

Max: $Z = 32X_1 + 24X_2$

where, $X_1$ = no. of collegiate backpacks
$X_2$ = no. of Mini backpacks.
$Z$ = profit per week.

The constraints are as follows;

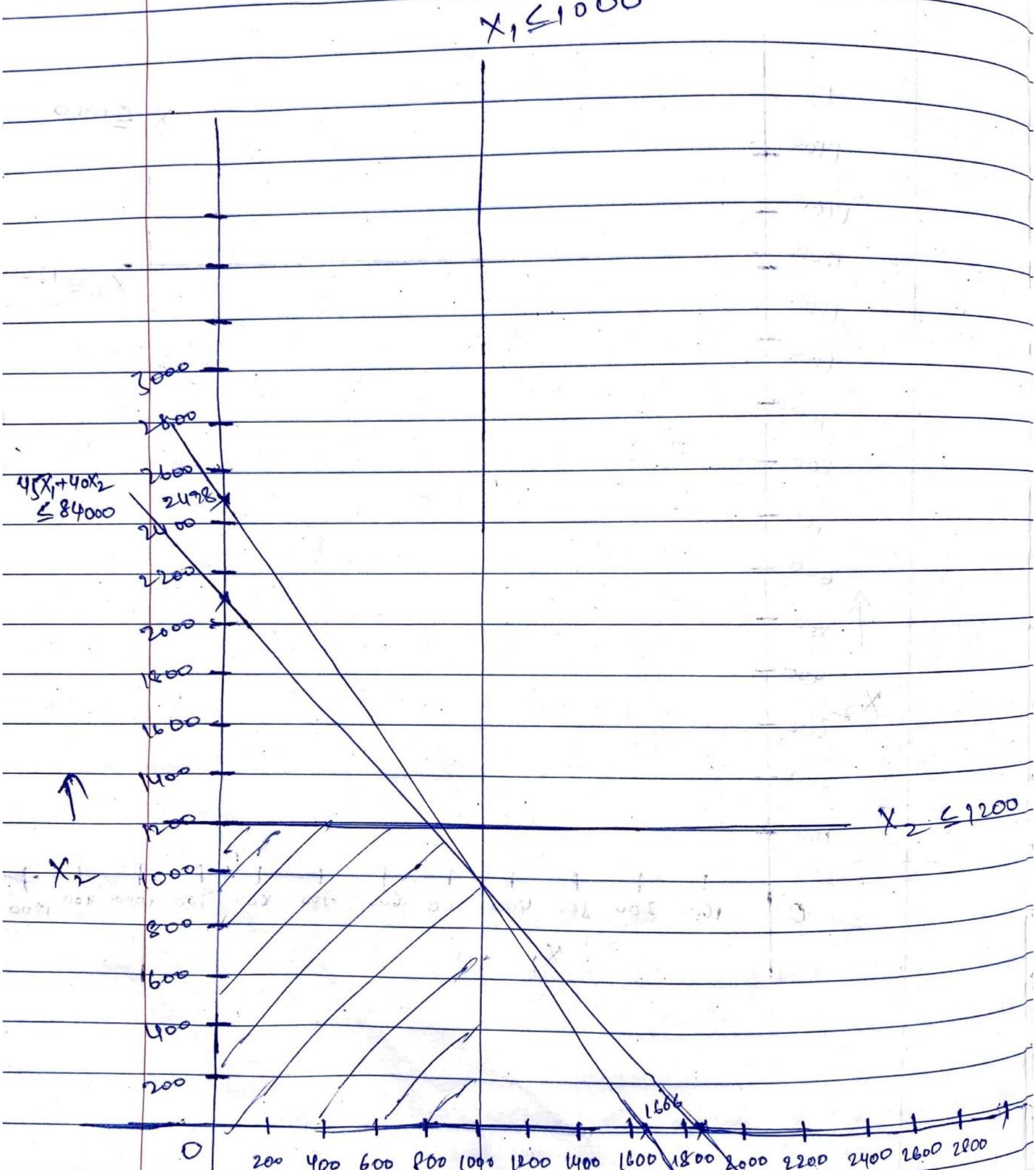
$45X_1 + 40X_2 \leq 84000$

$3X_1 + 2X_2 \leq 5000$

$X_1, X_2 \geq 0$; $X_1 \leq 1000$; $X_2 \leq 1200$

The profit maximization values of $X_1$ and $X_2$ from the above functions is as follows;

$X_1 = 1000$; $X_2 = 975$.



$X_1 \le 1000$
$45X_1 + 40X_2 \le 84000$
2498
$X_2 \le 1200$
$X_2$
1666
0
200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800
$X_1 \longrightarrow$
$3X_1 + 2X_2 \le 5000$
200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000

3. (Weigelt Production)

a. Define the decision variables
b. Formulate a linear programming model for this problem.
c. Solve the problem using *lpsolve*, or any other equivalent library in R.

Weighit Production.

3. The given information from the problem is as below;

let $X_{11} \rightarrow$ large units produced by plant 1.
$X_{12} \rightarrow$ " 2.
$X_{21} \rightarrow$ Medium units produced by plant 1
$X_{22} \rightarrow$ " 2.
$X_{31} \rightarrow$ Small units produced by plant 1
$X_{32} \rightarrow$ " 2.

$X_{13} \rightarrow$ Large $\rightarrow$ plant 3.
$X_{23} \rightarrow$ Medium $\rightarrow$ plant 3.
$X_{33} \rightarrow$ Small $\rightarrow$ plant 3.

Based on above assumptions, the profit function is as below;

$$\text{Max: } Z = 420\times(X_{11}+X_{12}+X_{13}) + 360(X_{21}+X_{22}+X_{23}) + 300(X_{31}+X_{32}+X_{33}).$$

where, the following constraints are applicable.

Constraints

Qty $\quad X_{11} + X_{21} + X_{31} \le 750$

$X_{12} + X_{22} + X_{32} \le 900$

$X_{13} + X_{23} + X_{33} \le 450$

Space $\quad 20X_{11} + 15X_{21} + 12X_{31} \le 13000$

$20X_{12} + 15X_{22} + 12X_{32} \le 12000$

$20X_{13} + 15X_{23} + 12X_{33} \le 5000$

Sales $\quad X_{11} + X_{12} + X_{13} \le 900$.

$X_{21} + X_{22} + X_{23} \le 1200$

$X_{31} + X_{32} + X_{33} \le 750$

$X_{11}, X_{12}, X_{13}, X_{21}, X_{22}, X_{23}, X_{31}, X_{32}, X_{33} \ge 0$.