

# Response to Reviewer Comments

November 24, 2013

## Reviewer-1

1. The paper presents a rigorous formulation and analysis of economic MPC for the problem of inventory management in supply chains. A novel cost function is presented; the approach enables the integration of the scheduling and inventory control tasks as well. A number of illustrative examples are presented. I do not see issues of technical correctness with the paper; however, the paper is hampered somewhat by a lack of accessibility in the presentation that may be acceptable in other journals, but I would hope not be the case in CACE. This is illustrated in Section 2, for example. A real problem will likely have 1) longer (and uncertain) lead times and 2) uncertain demand. In these scenarios, I would perhaps not be so eager to have MPC with an economic objective function, as having guarantees of stability would not be sufficient. Somehow the economic objective function would have to allow a means to incorporate potentially significant robustness margins in the response, while still maintaining meaningfulness as a metric. Can the authors justify why would the proposed approach be better than a well-coordinated hierarchical approach, which would maintain a separation between the control and scheduling layers?

A minor issue: some of the references need to be updated, particularly those that involve the existence of journal versions of papers cited from conferences. This applies to References [2] and [13], for example.

**Response:** The reviewer raises two related but distinguishable points: (i) the suitability of the economic objective function, and (ii) the choice of the solution approach. Regarding the objective function, we believe that the selection of an economic metric is the natural one in the context of supply chain (SC), where managers are interested in minimizing costs and not necessarily maintaining certain levels of inventory (tracking problem). Now, given the adoption of an economic objective, our goal is to develop new results for EMPC in systems with integrators, something that has not been done in the past. In other words, we aim to expand the scope of these methods in new classes of problems. Of course, a number of other approaches could be used to address the same problems (hierarchical decomposition is one of them). However, no theoretical results are available for the closed-loop performance of these approaches, even when all parameters are known deterministically. Thus, although they may appear promising, there is no guarantee they will work; in fact, counter examples showing how these methods fail can be constructed.

We have made references in the text to papers detailing Model predictive control. A complete introduction to MPC was out of the scope of the paper. We have also made the corrections to the references.

## REVIEWER-2

1. Since the authors are dealing with inventory management in supply chains, which often have integer units (books, parts, etc.), they should at least discuss the limits of their continuous variable formulation for these problems. While the difference between 50.5 and 51 units/day will not radically change a solution, presumably 1.25 units/day would require 3 days of 1 unit and 1 day of 2 units.

**Response:** The reviewer raises two interesting points. The first concerns the dynamic model of the supply chain (section 3) and the second the production model at each manufacturing node. Regarding the first, please note that our primary focus is the supply chain of process industries, where the major products are commodity chemicals and fluids (often shipped using pipelines, trucks, etc). Thus, we believe that the treatment of the variables in the dynamic model as continuous variables is reasonable. Note that even if products are produced in discrete amounts (e.g., due to batch size restrictions), the orders and shipments, and thus inventory levels, can be continuous variables. Regarding the second point, we note that the model can be easily modified to enforce production of discrete amounts by simply setting  $\underline{B}_i = \bar{B}_i$  in eq. (24); i.e. without defining variable  $B_i$  as an integer. However, the scheduling model does have binary  $X_{(i,t)}$  and  $Z_{(i,i,t)}$ .

2. While the authors discuss periodic supply chains, I would like to better understand how this relates to a typical Federal Express or UPS problem, where deliveries are made daily, with perhaps Saturday mornings but no Saturday afternoon or Sunday service. **Response:** Restrictions on transportation and production can be easily handled through constraints on the corresponding variables. For example, in a problem with a 14 day horizon divided into 336 1-hour periods, shipments between Saturday 12:00 pm and Monday 6:00 am can be forbidden by setting  $S_{i,t} = 0$  for  $t=132, \dots, 174$  (assuming zero time delay). Similar restrictions can be placed on orders and production (e.g., setting scheduling variables to zero during weekends).