Q1

$S=a_1+a_2+a_3+\ldots+a_{23}+a_{24}$

$S=\frac{24}{2}(a_1+a_{24})=12(a_1+a_{24})$

Given that, $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$

$\Rightarrow a_1+a_1+\cancel{4d}+a_1+\cancel{9d}+a_{24}-\cancel{9d}+a_{24}-\cancel{4d}+a_{24}=225$

$\Rightarrow 3(a_1+a_{24})=225$

$\Rightarrow a_1+a_{24}=75$

$\Rightarrow S=12(a_1+a_{24})=12\times 75=900$

Q2

Let the numbers be $\mathrm{a}, \mathrm{ar}, \mathrm{ar}^2, \mathrm{ar}^3$,

$\Rightarrow \mathrm{a}-2, \mathrm{ar}-7, \mathrm{ar}^2-9$ and ar^3-5 are in A. P.

$\Rightarrow(\mathrm{a}-2)+(\mathrm{ar}^2-9)=2(\mathrm{ar}-7)$ and $(\mathrm{ar}-7)+(\mathrm{ar}^3-5)=2(\mathrm{ar}^2-9)$

$\Rightarrow \mathrm{a}+\mathrm{ar}^2=2\mathrm{ar}-3$ and $\mathrm{ar}+\mathrm{ar}^3=2\mathrm{ar}^2-6$

$\Rightarrow 1+\mathrm{r}^2=2\mathrm{r}-\frac{3}{\mathrm{a}}$ and $1+\mathrm{r}^2=2\mathrm{r}-\frac{6}{\mathrm{ar}}$

$\Rightarrow \frac{3}{\mathrm{a}}=\frac{6}{\mathrm{ar}} \Rightarrow \mathrm{r}=2$

$\Rightarrow \frac{3}{\mathrm{a}}=2\mathrm{r}-1-\mathrm{r}^2=4-1-4=-1 \Rightarrow \mathrm{a}=-3$

So, the numbers are $-3,-6,-12,-24$

Q3

$a+3c=2b$ and $b^2=4ac$

$\Rightarrow c=\frac{2b-a}{3}=\frac{b^2}{4a}$

$\therefore 8ab-4a^2=3b^2 \Rightarrow 4a^2-8ab+3b^2=0$

$4\frac{a^2}{b^2}-8\frac{a}{b}+3=0 \Rightarrow \frac{a}{b}=\frac{8\pm\sqrt{64-4\times4\times3}}{2\times4}$

$\frac{a}{b}=\frac{2\pm\sqrt{4-3}}{2}=\frac{2\pm1}{2}=\frac{3}{2},\frac{1}{2}.$

Q4

$a_2=a_1r, a_3=a_1r^2$

$a_3>a_2+2a_1$

$\Rightarrow a_1r^2>a_1r+2a_1$

$\Rightarrow r^2-r-2>0$

$\Rightarrow(r-2)(r+1)>0$

$\Rightarrow r<-1$ or $r>2$

Q5

As we know that $\frac{x+y}{1}\times\frac{x-y}{x-y}=\frac{x^2-y^2}{x-y}$ similarly $x^2+xy+y^2=\frac{x^3-y^3}{x-y}$

$\Rightarrow \frac{x^2-y^2}{x-y}+\frac{x^3-y^3}{x-y}+\frac{x^4-y^4}{x-y}+\ldots\infty$

$\Rightarrow \quad \frac{1}{x-y}\left[\left(x^2+x^3+\ldots\infty\right)-\left(y^2+y^3+\ldots\infty\right)\right]$

Now we will find the sum of infinite G. P.series

$\Rightarrow \quad \frac{1}{x-y}\left[\frac{x^2}{1-x}-\frac{y^2}{1-y}\right]$

$\Rightarrow\left[\frac{x+y-xy}{1-x-y+xy}\right]$

Q6

Case-I : $x < \frac{1}{3} \Rightarrow -3x + 1, 3, -x + 3$ are in $A.P.$

$\Rightarrow 6 = -3x + 1 - x + 3$

$\Rightarrow 4x = -2 \Rightarrow x = -\frac{1}{2}$

$\Rightarrow$ terms are $\frac{5}{2}$, 3 , $\frac{7}{2}$, 4 , $\frac{9}{2}$

$\Rightarrow$ sum $= \frac{35}{2}$

Case-II: $\frac{1}{3} \leq x < 3$

$3x - 1, 3, -x + 3$ are in A.P.

$\Rightarrow 6 = 3x - 1 - x + 3 = 2x + 2 \Rightarrow x = 2$

$\Rightarrow$ terms are 5 , 3 , 1 , -1 , -3

$\Rightarrow$ sum $= 5$

Case-III: $x \geq 3$

$\Rightarrow$ terms are $3x - 1, 3, x - 3$

$\Rightarrow 6 = 3x - 1 + x - 3 = 4x - 4$

$\Rightarrow 4x = 10 \Rightarrow x = \frac{5}{2}$ not possible

Q7

$$(16x)^2 + (4y)^2 + (z)^2 - (16x)(4y) - (4y)(z) - (z)(16x) = 0$$

$$\Rightarrow \frac{1}{2}\left[(16x - 4y)^2 + (4y - z)^2 + (z - 16x)^2\right] = 0 \Rightarrow 16x = 4y = z$$

Q8

Let the first term of the given sequence be a

First 11 terms are: $a, \ a + 2, \ldots a + 20$

Since, eleventh term of the sequence is the first of last 11 terms of this sequence.

Last 11 terms are: $a + 20, \ (a + 20). 2, \ldots (a + 20) \cdot 2^{10}$

Thus, the sequence obtained is as follows:

$a,\ a+2,\ \ldots,\ a+20,\ (a+20)2,\ (a+20)2^2,\ \ldots\ldots,\ (a+20)2^{10}$

Now middle term of the AP is the $\left(\frac{(11+1)}{2}\right)^{\text{th}}$ term, i.e., 6^{th} term and the same goes for GP.

So, $(a+10)=(a+20)2^5$

$\Rightarrow 10-20\cdot 2^5=a(2^5-1) \Rightarrow 10\frac{(1-2^6)}{2^5-1}=a$

Middle term of the whole sequence $=T_{11}=a+20=10\frac{(1-2^6)}{2^5-1}+20$

$=10\left[\frac{-1}{2^5-1}\right]=\frac{10}{1-2^5}$

Q9

$\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+\ldots\left(\frac{1}{a_9}-\frac{1}{a_{10}}\right)=\frac{9\text{ d}}{64}$

$\Rightarrow \frac{a_{10}-a_1}{a_1a_{10}}=\frac{9\text{ d}}{64}$

$\Rightarrow a_1a_{10}=64$

Also $(a_1+a_{10})\left(\frac{1}{a_1a_{10}}+\ldots\frac{1}{a_{10}a_1}\right)=\frac{a_1+a_{10}}{10}\left(\frac{1}{a_1}+\ldots\frac{1}{a_{10}}\right)$

$\Rightarrow 2\left(\frac{1}{a_1}+\ldots\frac{1}{a_{10}}\right)=\frac{a_1+a_{10}}{10}\left(\frac{1}{a_1}+\ldots\frac{1}{a_{10}}\right)$

$\Rightarrow a_1+a_{10}=20$

$a_1a_{10}=64$ ...(1)

$a_1+a_{10}=20$ ...(2)

From (1) & (2)

$a_1=4$ & $a_{10}=16$

$4\left(\frac{a_1}{a_{10}}+\frac{a_{10}}{a_1}\right)=4\left(\frac{4}{16}+\frac{16}{4}\right)=17$

Sum of digits of $17=1+7=8$

Q10

$a+b+c=25; 2a=2+b; c^2=18b$

$\Rightarrow 2a+2b+2c=50$

$\Rightarrow 2+b+2b+2c=50$

$\Rightarrow 3b+2c=48 \Rightarrow \frac{c^2}{6}+2c=48$

$\Rightarrow c^2+12c-48\times 6=0 \Rightarrow c^2+12c-24\times 12=0$

$\Rightarrow (c+24)(c-12)=0$

$\Rightarrow c=12,-24 \Rightarrow c=12$ (between 2 and 18)

$\Rightarrow b=\frac{c^2}{18}=\frac{144}{18}=8$

$\Rightarrow a=\frac{b+2}{2}=5$

$\Rightarrow a=5,\ b=8$ and $c=12$

$\Rightarrow c-a=7$

Q11

Given, harmonic mean $H=4$

We know that, $G^2=AH$

Since, $2A+G^2=27$

$\Rightarrow 2A+AH=27$

$\Rightarrow 2A+4A=27$

$\Rightarrow A=\frac{27}{6}=\frac{9}{2}=\frac{a+b}{2} \Rightarrow a+b=9$

$G^2=AH=\frac{9}{2}\times 4=18$

$\Rightarrow ab=18$

$|a-b|=\sqrt{(a+b)^2-4ab}=\sqrt{81-4\times 18}=3$

$\Rightarrow \alpha=9,\ \beta=3$

$\Rightarrow \frac{\alpha}{\beta}=3$

Q12

Let, $A_1, A_2, A_3, \ldots\ldots, A_{11}$ be 11 A.M.'s between 28 and 10

$\Rightarrow 28, A_1, A_2, \ldots\ldots, A_{11}, 10$ are in A.P

Let, d be the common difference of A.P.

Also, the number of terms $= 13$

$\Rightarrow 10 = \mathrm{T}_{13} = \mathrm{T}_1 + 12\mathrm{d} = 28 + 12\mathrm{d}$

$\Rightarrow d = \frac{10-28}{12} = \frac{-18}{12} = \frac{-3}{2}$

$\Rightarrow$ Number of integral A.M.'s are 5

Q13

In the n^{th} set of observation, the total observations are 'n'

and the first observation of n^{th} set is

$1 + 2 + 3 + \ldots . + (n-1) + 1 = \frac{(n-1)n}{2} + 1 = a$

Sum of all the n observations in the n^{th} set

$= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}\left[2\left\{\frac{(n-1)n}{2} + 1\right\} + (n-1)\right]$

$= \frac{n}{2}\left[n^2 - n + 2 + n - 1\right] = n\left(\frac{n^2+1}{2}\right)$

Mean $= \frac{n\frac{\left(n^2+1\right)}{2}}{n} = \frac{n^2+1}{2}$

Hence, for $n = 13$

Mean $= \frac{13^2+1}{2} = \frac{170}{2} = 85$

Q14

$S_1 \equiv 3,\ 7,\ 11,\ 15, \ldots\ldots \Rightarrow\ c.d = 4 \Rightarrow d_1 = 4$

$S_2 \equiv 1,\ 6,\ 11,\ 16, \ldots\ldots \Rightarrow\ c.d = 5 \Rightarrow d_2 = 5$

$\Rightarrow$Every 5^{th} term of S_1 and 4^{th} term of S_2 will be same

$\Rightarrow$ Terms common to both AP will have $a = 11$ and $d = 20$

Hence, $S_{20} = \frac{20}{2}[(2 \times 11) + (20 - 1)\,20]$

$= 10 \times 402$

$= 4020$

Q15

$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots + \infty$ ... (1)

$\frac{S}{5} = 0 + \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \ldots + \infty$ ... (2)

On subtracting both the eq (1) and (2), we get,

$\frac{4}{5}S = 1 + \frac{3}{5} + \frac{3}{5^2} + \ldots + \infty$

$S = \frac{35}{16}$

Q16

$S = \frac{7}{17} + \frac{77}{17^2} + \frac{777}{17^3} + \ldots\ldots.$

$\frac{S}{17} = \frac{7}{17^2} + \frac{77}{17^3} + \frac{777}{17^4} + \ldots\ldots..$

Subtracting both the above results, we get,

$\frac{16}{17}S = \frac{7}{17} + \frac{70}{17^2} + \frac{700}{17^3} + \ldots\ldots.$
$\left(\text{Infinite G.P. with r } = \frac{10}{17}\right)$
$\frac{16}{17}S = \frac{\frac{7}{17}}{1-\frac{10}{17}} = \frac{\frac{7}{17}}{\frac{7}{17}} = 1$
$S = \frac{17}{16}$

Q17

$x = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots.\infty$

$= \left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty\right) - \left(\frac{1}{2^4} + \frac{1}{4^4} + \ldots\infty\right)$

$= \frac{\pi^4}{90} - \frac{1}{16}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots..\infty\right)$

$= \frac{\pi^4}{90} - \frac{1}{16}.\frac{\pi^4}{90} = \frac{\pi^4}{96}$.

Q18

$S = \sum_{r=1}^{25} r(26-r) = 26\sum_{r=1}^{25} r - \sum_{r=1}^{25} r^2$
$= 26 \times \frac{25\times26}{2} - \frac{25\times26\times51}{6}$
$= \frac{25\times26}{2}\left(26 - \frac{51}{3}\right) = \frac{25\times \cancel{26}13}{\cancel{2}} \times 9$
$\Rightarrow \frac{S}{900} = \frac{13}{4} = 3.25$

Q19

$0.2 + 0.22 + 0.222 + \ldots n$ terms

$= 2(0.1 + 0.11 + 0.111 + \ldots n\,\text{terms})$

$$= 2\left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n\,\text{terms}\right)$$

$$= \frac{2}{9}\left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots n \text{ terms}\right)$$

$$= \frac{2}{9}\left(1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \ldots n \text{ terms}\right)$$

$$= \frac{2}{9}\left[n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots n\right)\right]$$

$$= \frac{2}{9}\left[n - \frac{1}{10}\frac{\left\{1 - \left(\frac{1}{10}\right)^n\right\}}{\left(1 - \frac{1}{10}\right)}\right]$$

$$= \frac{2}{9}\left[n - \frac{1}{10} \times \frac{10}{9} \cdot \left(\frac{10^n - 1}{10^n}\right)\right]$$

$$= \frac{2}{9}\left[n - \frac{1}{9}\left(1 - 10^{-n}\right)\right]$$

Q20

$$\mathrm{S}_{20} = \frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots. + \text{up to } 20 \text{ terms}$$

$$= \sum_{\mathrm{r}=1}^{20} \frac{(2\mathrm{r}+1)}{1^2+2^2+3^2+\ldots.\ \mathrm{r}^2}$$

$$= \sum_{\mathrm{r}=1}^{20} \frac{2\mathrm{r}+1}{\frac{\mathrm{r}}{6}(\mathrm{r}+1)(2\mathrm{r}+1)}$$

$$= 6\sum_{\mathrm{r}=1}^{20} \frac{1}{\mathrm{r}(\mathrm{r}+1)}$$

$$= 6\sum_{\mathrm{r}=1}^{20}\left(\frac{1}{\mathrm{r}} - \frac{1}{\mathrm{r}+1}\right)$$

$$= 6\left\{\left[\frac{1}{1} - \frac{1}{2}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] + \left[\frac{1}{3} - \frac{1}{4}\right] + ... + \left[\frac{1}{20} - \frac{1}{21}\right]\right\}$$

$$= 6\left(1 - \frac{1}{21}\right)$$

$$= \frac{6\times 20}{21}$$

$$= \frac{120}{21}$$

$$= \frac{\mathrm{k}}{21}$$

$\mathrm{k} = 120$

Q21

Given, $S = \sum_{n=1}^{9999} \frac{1}{\left(\sqrt{n}+\sqrt{n+1}\right)\left(\sqrt[4]{n}+\sqrt[4]{n+1}\right)}$

$$= \sum_{n=1}^{9999} \frac{1}{\left(\sqrt{n}+\sqrt{n+1}\right)\left(\sqrt[4]{n}+\sqrt[4]{n+1}\right)} \times \frac{\left(\sqrt[4]{n+1}-\sqrt[4]{n}\right)}{\left(\sqrt[4]{n+1}-\sqrt[4]{n}\right)}$$

$$= \sum_{n=1}^{9999} \frac{\left(\sqrt[4]{n+1}-\sqrt[4]{n}\right)}{\left(\sqrt{n}+\sqrt{n+1}\right)\left(\sqrt{n+1}-\sqrt{n}\right)}$$

$$= \sum_{n=1}^{9999}\left(\sqrt[4]{n+1} - \sqrt[4]{n}\right)$$

$$= \left(2^{\frac{1}{4}} - 1\right) + \left(3^{\frac{1}{4}} - 2^{\frac{1}{4}}\right) + \left(4^{\frac{1}{4}} - 3^{\frac{1}{4}}\right) + \ldots + \left(10000^{\frac{1}{4}} - 9999^{\frac{1}{4}}\right)$$

$$= 10000^{\frac{1}{4}} - 1 = 10 - 1 = 9$$

Q22

$$T_r = \frac{1}{2}\left\{\frac{\left(r^2+r+1\right)-\left(r^2-r+1\right)}{\left(r^2+r+1\right)\left(r^2-r+1\right)}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{r^2-r+1} - \frac{1}{\left(r+1\right)^2-\left(r+1\right)+1}\right\}$$

$$= \frac{1}{2}\{f(r) - f(r+1)\}$$

Hints and Solutions **MathonGo**

$S=\Sigma T_r=\sum_{r=1}^{80}\frac{1}{2}(f(\alpha)-f(\alpha+1))$

$=\frac{1}{2}\left\{f(1)-f\cancel{(2)}+f\cancel{(2)}-f\cancel{(3)}\ldots\ldots\ldots+\cancel{f(80)}-f(81)\right\}$

$=\frac{1}{2}(f(1)-f(81))=\frac{1}{2}\left(1-\frac{1}{81^2-81+1}\right)$

$=\frac{1}{2}\left(1-\frac{1}{6481}\right)\Rightarrow 6481S=\frac{6480}{2}=3240$

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Q23

r^{th} term, $\mathrm{T}_r=\frac{(1+2+3+\ldots\ldots+r)^2}{(1+3+5+\ldots+(2r-1))}$

$=\frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}=\frac{(r+1)^2}{4}=\frac{r^2+2r+1}{4}$

$S_{10}=\mathrm{K}=\frac{1}{4}\sum_{r=1}^{10}\left(r^2+2r+1\right)$

$\Rightarrow 4\mathrm{K}=\sum_{r=1}^{10}r^2+2\sum_{r=1}^{10}r+\sum_{r=1}^{10}1$

$=\frac{10\times11\times21}{6}+2\frac{10\times11}{2}+10$

$=385+110+10=505$

$\frac{4\mathrm{K}}{101}=5$

Q24

$\sum_{n=1}^{9}\frac{1}{2}\frac{n+2-n}{n(n+1)(n+2)}$

$=\frac{1}{2}\left(\left(\frac{1}{1.2}-\frac{1}{2.3}\right)+\left(\frac{1}{2.3}-\frac{1}{3.4}\right)+\ldots+\left(\frac{1}{9\times10}-\frac{1}{10\times11}\right)\right)$

$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{110}\right)=\frac{1}{2}\left(\frac{55-1}{110}\right)=\frac{27}{110}$

$\Rightarrow\quad q-p=110-27=83$

Q25

Given,

$S_n=\left(1^2-1+1\right)(1!)+\left(2^2-2+1\right)(2!)+\cdots+\left(n^2-n+1\right)(n!)$

Its general term t_r is given as,

$\left(r^2-r+1\right)r!$

$t_r=\left[\left(r^2-1\right)-(r-2)\right](r!)$

$t_r=(r-1)(r+1)!-(r-2)(r)!$

$S_n=\sum_{r=1}^n t_r$

$S_n=(0-(-1))+(3!-0)+(2(4)!-3!)+\ldots+(n-1)(n+1)!-(n-2)n!$

$S_{50}=1+49(51)!$.

Q26

Let $\sqrt{a_1}=b_1$;

$\sqrt{a_2-1}=b_2$;

$\sqrt{a_3-2}=b_3$;

$\sqrt{a_n-(n-1)}=b_n$

$\therefore$ LHS $= b_1+b_2+\ldots+b_n=\frac{1}{2}\left[b_1^2+\left(b_2^2+1\right)+\left(b_3^2+2\right)+\ldots+\left(b_n^2+(n-1)\right)\right]-\frac{n(n-3)}{4}$

$\therefore\ \Sigma b_i=\frac{1}{2}\left[\left(b_1^2+b_2^2+b_3^2+\ldots+b_n^2\right)+(1+2+3+\ldots+(n-1))\right]-\frac{n(n-3)}{4}$

$\Rightarrow 2\Sigma b_i=\Sigma b_i^2+\frac{n(n-1)}{2}-\frac{n(n-3)}{2}$

$\Rightarrow 2\Sigma b_i=\Sigma b_i^2+n$

$\therefore \ \Sigma b_i^2 - 2\Sigma b_i + \Sigma 1 = 0$

$\therefore \sum_{i=1}^{n} (b_i - 1)^2 = 0$

$b_1 - 1 = 0 \Rightarrow b_1^2 = a_1 = 1$

$b_2 - 1 = 0 \Rightarrow b_2^2 = a_2 - 1 = 1 \Rightarrow a_2 = 2$

$b_3 - 1 = 0 \Rightarrow b_3^2 = a3 - 2 = 1 \Rightarrow a_3 = 3$ and so on.

Hence, $a_n = n$.

$\therefore \sum_{i=1}^{100} a_i = 1 + 2 + 3 + \ldots + 100 = 5050$

Q27

$a_k = 2a_{k-1} - a_{k-2}$

$\Rightarrow a_1, a_2, \ldots, a_{11}$ are in AP with let common difference be d

$\therefore \ \frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11\mathrm{d}^2 + 110ad}{11} = 90$

$\Rightarrow 225 + 35\mathrm{d}^2 + 150\mathrm{d} = 90$

$35\mathrm{d}^2 + 150\mathrm{d} + 135 = 0$

$\Rightarrow \mathrm{d} = -3, -\frac{9}{7}$

Given, $a_2 < \frac{27}{2} \therefore \mathrm{d} = -3$ and $\mathrm{d} \neq -\frac{9}{7}$

Hence, $a_1 + a_2 + \ldots\ldots a_{11} = \frac{11}{2}[2 \times 15 + 10(-3)] = 0$

Q28

$\frac{1.2}{1!} + \frac{2.3}{2!} + \frac{3.4}{3!} + \frac{4.5}{4!} + \ldots\infty,$

Here, $T_n = \frac{n(n+1)}{n!} = \frac{n+1}{(n-1)!} = \frac{(n-1)+2}{(n-1)!}$

$= \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$

$\Rightarrow \quad S = \Sigma T_n = e + 2e = 3e$

Q29

$\because \; A.M. \geq G.M.$

$\Rightarrow \frac{a^{-6}+a^{-4}+a^{-4}+a^{-3}+a^{-3}+1+a^{10}+a^{10}}{8} \geq 1$

$\therefore$ Minimum value $= 8$

Q30

Given, $a + b + c = 3$

$\Rightarrow 2 \cdot \frac{a}{2} + 3 \cdot \frac{b}{3} + 2 \cdot \frac{c}{2} = 3$

$\frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}{7} \geq \left(\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2\right)^{\frac{1}{7}}$

$\frac{a+b+c}{7} \geq \left(\frac{a^2b^3c^2}{2^4 3^3}\right)^{\frac{1}{7}}$

$\left(\frac{3}{7}\right)^7 \geq \frac{a^2b^3c^2}{2^4 3^3}$

Hints and Solutions **MathonGo**

$\Rightarrow a^2b^3c^2 \leq \frac{3^7}{7^7} \times 2^4 \times 3^3 = \frac{2^4 \times 3^{10}}{7^7}$

Hence, maximum value $= \frac{2^4 \times 3^{10}}{7^7}$