

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q1**

Let α and β be the roots of the equation $x^2 + ax + 1 = 0$, $a \neq 0$. Then the equation whose roots are $-\left(\alpha + \frac{1}{\beta}\right)$ and $-\left(\frac{1}{\alpha} + \beta\right)$ is

(1) $x^2 = 0$

(2) $x^2 + 2ax + 4 = 0$

(3) $x^2 - 2ax + 4 = 0$

(4) $x^2 - ax + 1 = 0$

Q2

If the roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then the value of $(a+b+c)^2$ is equal to

(1) $2b^2 - ac$

(2) Σa^2

(3) $b^2 - 4ac$

(4) $b^2 - 2ac$

Q3

The possible values of n for which the equation $nx^2 + (2n-1)x + (n-1) = 0$ has roots of opposite sign is/are given by

(1) no values of n

(2) all values of n

(3) $-1 < n < 0$

(4) $0 < n < 1$

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q4**

Consider the equation $x^2 + 2x - n = 0$, where $n \in \mathbb{N}$ and $n \in [5, 100]$. The number of different values of n so that the given equation has integral roots, is

Q5

If $-\pi < \theta < \pi$, the equation $(\cos 3\theta + 1)x^2 + (2\cos 2\theta - 1)x + (1 - 2\cos \theta) = 0$ has more than two roots for

- (1) no value of θ
- (2) one value of θ
- (3) two value of θ
- (4) all values of θ

Q6

Let α and β are the roots of equation $ax^2 + bx + c = 0$ ($a \neq 0$). If 1, $\alpha + \beta$, $\alpha\beta$ are in arithmetic progression and $a, 2, \beta$ are in harmonic progression, then the value of $\frac{\alpha^2 + \beta^2 - 2\alpha^2\beta^2}{2(\alpha^2 + \beta^2)}$ is equal to

- (1) 0
- (2) 0.5
- (3) 1
- (4) 1.5

Q7

The number of quadratic equations that are unchanged by squaring their roots is

- (1) 2
- (2) 4
- (3) 6

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(4) 8

Q8

If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n , then the

value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} \right)$ is equal to

Q9

Let α, β are the roots of the quadratic equation $2x^2 - 5x + 1 = 0$. If $S_n = (\alpha)^{2n} + (\beta)^{2n}$ then find the value of $\frac{4S_{2021} + S_{2019}}{S_{2020}}$

Q10

If $f(x) = \prod_{k=1}^{999} (x^2 - 47x + k)$, then product of all real roots of $f(x) = 0$ is

(1) 550!

(2) 551!

(3) 552!

(4) 999!

Q11

If $-3 < \frac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$ for all $x \in R$, then the value of λ belongs to

(1) (-1, 7)

(2) (-6, 2)

(3) (-1, 2)

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(4) (-6,7)

Q12

For the equation $|x^2 - 2x - 3| = b$, which of the following statements is true?

(1) For $b < 0$, there are no solutions(2) For $b = 0$, there are three solutions(3) For $0 < b < 4$, there are two solutions(4) For $b = 4$, there are four solutions**Q13**

If a, b, c are real numbers satisfying the condition $a + b + c = 0$, then the roots of the quadratic equation

 $3ax^2 + 5bx + 7c = 0$ are

(1) Positive

(2) negative

(3) real and equal

(4) distinct but not imaginary

Q14

If $a + b + c > \frac{9c}{4}$ and the equation $ax^2 + 2bx - 5c = 0$ has non-real complex roots, then

(1) $a > 0, c > 0$ (2) $a > 0, c < 0$ (3) $a < 0, c < 0$ (4) $a < 0, c > 0$ **Q15**

Sample Task**Questions****Questions with Answer Keys****MathonGo**

If the graph of the function $y = (a - b)^2 x^2 + 2(a + b - 2c)x + 1$ ($\forall a \neq b$) is strictly above the x -axis, then

(1) $a < b < c$

(2) $a < c < b$

(3) $b < a < c$

(4) $c < b < a$

Q16

The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first equation and the second equation are integers in the ratio 4:3. Then the common root is

(1) 4

(2) 3

(3) 2

(4) 1

Q17

The value of k for which both the roots of the equation $4x^2 - 20kx + (25k^2 + 15k - 66) = 0$ are less than 2, lies in

(1) $\left(\frac{4}{5}, 2\right)$

(2) $(0, 2)$

(3) $\left(-1, -\frac{4}{5}\right)$

(4) $(-\infty, -1)$

Q18

The range of a for which the equation $x^2 + ax - 4 = 0$ has its smaller root in the interval $(-1, 2)$ is

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- (1) $(-\infty, -3)$
 (2) $(0, 3)$
 (3) $(0, \infty)$
 (4) $(-\infty, -3) \cup (0, \infty)$

Q19

If $f(x)$ is a polynomial of degree four with the leading coefficient one satisfying $f(1) = 1, f(2) = 2$ and $f(3) = 3$

, then $\left[\frac{f(-1) + f(5)}{f(0) + f(4)} \right]$ (where $[\cdot]$ represents the greatest integer function) is equal to

- (1) 4
 (2) 5
 (3) 6
 (4) 7

Q20

Sum of the squares of all integral values of a for which the inequality $x^2 + ax + a^2 + 6a < 0$ is satisfied for all

$x \in (1, 2)$ must be equal to

- (1) 90
 (2) 89
 (3) 88
 (4) 91

Q21

The equations $kx^2 + x + k = 0$ and $kx^2 + kx + 1 = 0$ have exactly one root in common for

- (1) $k = -\frac{1}{2}, 1$

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(2) $k = 1$

(3) $k = -\frac{1}{2}$

(4) $k = \frac{1}{2}$

Q22

If the quadratic equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ have both the roots common, then $2r - p$ is equal to

(1) 0

(2) 1

(3) 2

(4) None of these

Q23

If α, β and γ are the roots of the equation $x^3 - 13x^2 + 15x + 189 = 0$ and one root exceeds the other by 2, then the value of $|\alpha| + |\beta| + |\gamma|$ is equal to

(1) 23

(2) 17

(3) 13

(4) 19

Q24

If equations $x^2 + ax + b = 0$ ($a, b \in R$) & $x^3 + 3x^2 + 5x + 3 = 0$ have two common roots, then value of $\frac{b}{a}$ is equal to

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q25**

If x is rational and $4\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) - 57 = 0$, then the product of all possible values of x is

(1) 4

(2) 3

(3) 2

(4) 1

Q26

The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is

(1) 6

(2) 5

(3) 3

(4) -4

Q27

If α and β are the real roots of $(\log_x 10)^3 - (\log_x 10)^2 - 6(\log_x 10) = 0$, then the value of $\left|\frac{1}{\log_{10} \alpha \beta}\right|$ is

Q28

The sum of the roots of the equation $2^{(33x-2)} + 2^{(11x+2)} = 2^{(22x+1)} + 1$ is

(1) $\frac{1}{11}$ (2) $\frac{2}{11}$ (3) $\frac{3}{11}$

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(4) $\frac{4}{11}$

Q29

The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to

Q30

If the equation in x given by $\left(2\left(\frac{1}{\cos^{-1}x}\right)\right)^{2\pi} - \left(a + \frac{1}{2}\right)\left(2\left(\frac{1}{\cos^{-1}x}\right)\right)^\pi - a^2 = 0$ has only one real solution then

exhaustive set of values of 'a' is

(1) $(-3, 1)$

(2) $(-\infty, -3] \cup [1, \infty)$

(3) $(-\infty, -3) \cup (1, \infty)$

(4) $[-3, \infty)$

Sample Task**Questions****Questions with Answer Keys****MathonGo****Answer Key****Q1 (3)** **Q2 (3)** **Q3 (4)** **Q4 (8)****Q5 (1)** **Q6 (4)** **Q7 (2)** **Q8 (1)****Q9 (21)** **Q10 (3)** **Q11 (3)** **Q12 (1)****Q13 (4)** **Q14 (2)** **Q15 (2)** **Q16 (3)****Q17 (4)** **Q18 (1)** **Q19 (2)** **Q20 (4)****Q21 (3)** **Q22 (1)** **Q23 (4)** **Q24 (1.50)****Q25 (1)** **Q26 (3)** **Q27 (6)** **Q28 (2)****Q29 (2)** **Q30 (2)****mathongo** **mathongo** **mathongo** **mathongo** **mathongo** **mathongo** **mathongo****mathongo** **mathongo** **mathongo** **mathongo** **mathongo** **mathongo** **mathongo****MathonGo**<https://www.mathongo.com>

Sample Task**Solutions****Hints and Solutions****MathonGo****Q1**

$$\alpha + \beta = -a \text{ and } \alpha\beta = 1$$

Let S and P be the sum and product of the roots of the required equation. Then,

$$S = -\alpha - \frac{1}{\beta} - \frac{1}{\alpha} - \beta = -(\alpha + \beta) - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= -(\alpha + \beta) - \left(\frac{\alpha + \beta}{\alpha\beta}\right) = -(-a) - \left(\frac{-a}{1}\right) = 2a$$

$$P = -\left(\alpha + \frac{1}{\beta}\right)\left(-\left(\frac{1}{\alpha} + \beta\right)\right)$$

$$= 1 + \alpha\beta + \frac{1}{\alpha\beta} + 1 = 1 + 1 + 1 + 1 = 4$$

So, the required equation is

$$x^2 - Sx + P = 0$$

$$\text{i.e. } x^2 - 2ax + 4 = 0$$

Q2

$$\text{We have } \frac{k+1}{k} + \frac{k+2}{k+1} = \frac{-b}{a} \dots\dots (i)$$

$$\text{and } \frac{k+1}{k} \cdot \frac{k+2}{k+1} = \frac{c}{a}$$

$$\Rightarrow \frac{k+2}{k} = \frac{c}{a}$$

$$\text{or } \frac{2}{k} = \frac{c}{a} - 1 = \frac{c-a}{a}$$

$$\text{or } k = \frac{2a}{c-a} \dots\dots (ii)$$

Now eliminate k putting the value of k in 1st relation, we get

$$\frac{c+a}{2a} + \frac{2c}{c+a} = \frac{-b}{a}$$

$$\Rightarrow (c+a)^2 + 4ac = -2b(a+c)$$

$$\Rightarrow (a+c)^2 + 2b(a+c) = -4ac$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

Adding b^2 on both sides,

$$(a + b + c)^2 = b^2 - 4ac$$

Q3

$$D = (2n-1)^2 - 4n(n-1) = 4n^2 + 1 - 4n - 4n^2 + 4n = 1 > 0$$

$$\text{Product of roots} = \frac{n-1}{n} < 0$$

$$\Rightarrow n(n-1) < 0$$

$$\Rightarrow n \in (0, 1)$$

Q4

$$x^2 + 2x - n = 0; n \in [5, 100]$$

D will be a perfect square,

$$D = 4 + 4n = 4(1 + n)$$

$\Rightarrow 1 + n$ is a perfect square

$$\Rightarrow 1 + n = 9, 16, 25, 36, 49, 64, 81, 100$$

$$\Rightarrow n = 8, 15, 24, 35, 48, 63, 80, 99$$

Therefore, 8 values are possible.

Q5

Given equation has more than two roots if it is an identity

$$\Rightarrow \cos 3\theta + 1 = 0; 2 \cos 2\theta - 1 = 0 \text{ and } 1 - 2 \cos \theta = 0$$

$$\Rightarrow \cos 3\theta = -1 \Rightarrow \theta = \pm \frac{\pi}{3} \text{ which does not satisfy } 2 \cos 2\theta - 1 = 0$$

Hence, no value possible

Q6

Sample Task**Solutions****Hints and Solutions****MathonGo**

$1, \alpha + \beta, \alpha\beta$ are in A.P. $\Rightarrow 1, \frac{-b}{a}, \frac{c}{a}$ are in A.P.

$$\Rightarrow 1 + \frac{c}{a} = \frac{-2b}{a} \Rightarrow a + c + 2b = 0 \dots (1)$$

$$\frac{1}{\alpha}, \frac{1}{2}, \frac{1}{\beta} \text{ are in A.P.} \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = 1 \Rightarrow \alpha + \beta = \alpha\beta$$

$$\Rightarrow \frac{-b}{a} = \frac{c}{a} \Rightarrow b + c = 0 \dots (2)$$

From (1) & (2) we get,

$$a = -b = c$$

$\Rightarrow \alpha, \beta$ are roots of equation $x^2 - x + 1 = 0$

$$\text{Now, } \frac{\alpha^2 + \beta^2 - 2\alpha^2\beta^2}{2(\alpha^2 + \beta^2)} = \frac{1}{2} - \frac{(\alpha\beta)^2}{(\alpha + \beta)^2 - 2\alpha\beta}$$

$$= \frac{1}{2} - \frac{(1)^2}{(1)^2 - 2(1)} = \frac{1}{2} + 1 = 1.5$$

Q7

$$\alpha + \beta = \alpha^2 + \beta^2 \text{ & } \alpha\beta = \alpha^2\beta^2 \Rightarrow \alpha\beta = 0 \text{ or } 1.$$

If $\alpha\beta = 0$, then let, $\alpha = 0 \Rightarrow \beta = 0$ or 1.

$$\text{If } \beta = \frac{1}{\alpha},$$

$$\alpha + \frac{1}{\alpha} = \alpha^2 + \frac{1}{\alpha^2} = \left(\alpha + \frac{1}{\alpha}\right)^2 - 2$$

$$\Rightarrow \left(\alpha + \frac{1}{\alpha}\right)^2 - \left(\alpha + \frac{1}{\alpha}\right) - 2 = 0 \Rightarrow \alpha + \frac{1}{\alpha} = 2 \text{ or } -1 \Rightarrow \alpha = 1 \text{ or } \omega, \omega^2$$

Hence number of such equations are four $(0, 0), (0, 1), (1, 1) \& (\omega, \omega^2)$

Q8

$$\text{Given, } x^2 + 5\sqrt{2}x + 10 = 0$$

$$\text{and } P_n = \alpha^n - \beta^n$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$\text{Now } \frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2} = \frac{P_{17}(P_{20}+5\sqrt{2}P_{19})}{P_{18}(P_{19}+5\sqrt{2}P_{18})}$$

$$\frac{P_{17}(\alpha^{20}-\beta^{20}+5\sqrt{2}(\alpha^{19}-\beta^{19}))}{P_{18}(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2}))}$$

Since $\alpha + 5\sqrt{2} = -10/\alpha \dots (1)$

and $\beta + 5\sqrt{2} = -10/\beta \dots (2)$

Now put there values in above expression

$$\frac{P_{17}(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2}))}{P_{18}(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2}))} = -\frac{10P_{17}P_{18}}{-10P_{18}P_{17}} = 1$$

Q9

Given α, β are the roots of the quadratic equation $2x^2 - 5x + 1 = 0$

Let us find an equation with roots α^2 and β^2 , let $y = x^2$, so $x = \sqrt{y}$

$$2y - 5\sqrt{y} + 1 = 0$$

$$\Rightarrow 2y + 1 = 5\sqrt{y}$$

$$\Rightarrow 4y^2 + 4y + 1 = 25y$$

$$\Rightarrow 4y^2 - 21y + 1 = 0 \quad \langle d$$

Put $\alpha^2 = c$ and $\beta^2 = d$

Now, $S_n = (c)^n + (d)^n$

Sample Task**Solutions****Hints and Solutions****MathonGo**

Consider

$$\begin{aligned} 4S_{2021} + S_{2019} &= 4(c^{2021} + d^{2021}) + c^{2019} + d^{2019} \\ &= c^{2019}(4c^2 + 1) + d^{2019}(4d^2 + 1) \\ &= c^{2019}(21c) + d^{2019}(21d) \end{aligned}$$

$$= 21S_{2020}$$

$$\text{Hence, } \frac{4S_{2021} + S_{2019}}{S_{2020}} = 21$$

Q10

$$\text{Consider } x^2 - 47x + k = 0$$

$$\text{For real roots, } 47^2 - 4k \geq 0 \Rightarrow k \leq 552$$

$$\therefore k = 1, 2, 3, \dots, 552$$

$$\text{Product of real roots} = 1 \times 2 \times 3 \times 4 \times \dots \times 552 = 552!$$

Q11

$$-3 < \frac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$$

$$\Rightarrow -3x^2 - 3x - 3 < x^2 - \lambda x - 2 < 2x^2 + 2x + 2 \quad (\text{since } x^2 + x + 1 > 0, \forall x \in R)$$

$$\Rightarrow 4x^2 + x(3 - \lambda) + 1 > 0, x^2 + x(2 + \lambda) + 4 > 0$$

$$(i) 4x^2 - x(\lambda - 3) + 1 > 0$$

$$\Rightarrow D < 0 \Rightarrow (\lambda - 3)^2 - 4 \times 4 \times 1 < 0$$

$$\Rightarrow (\lambda - 3 + 4)(\lambda - 3 - 4) < 0$$

$$\Rightarrow (\lambda + 1)(\lambda - 7) < 0 \Rightarrow \lambda \in (-1, 7)$$

$$(ii) x^2 + x(\lambda + 2) + 4 > 0$$

$$\Rightarrow D < 0 \Rightarrow (\lambda + 2)^2 - 4 \times 4 < 0$$

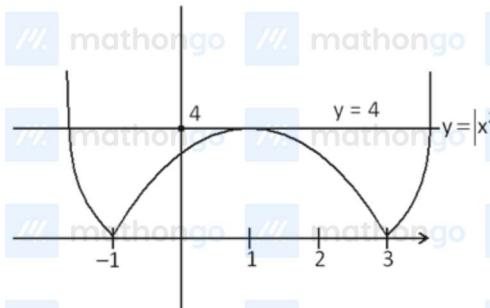
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$$(\lambda + 2 - 4)(\lambda + 2 + 4) < 0$$

$$(\lambda - 2)(\lambda + 6) < 0 \Rightarrow \lambda \in (-6, 2)$$

Taking the intersection of the solutions of (i) and (ii), we get,

$$\lambda \in (-1, 2)$$

Q12

$b < 0 \rightarrow$ no solution

$b = 0 \rightarrow$ two solutions

$0 < b < 4 \rightarrow$ four solutions

$b = 4 \rightarrow$ three solutions

Q13

$$D = 25b^2 - 4 \times 3a \times 7c$$

$$= 25(-a - c)^2 - 84ac$$

$$= 25(a^2 + c^2 + 2ac) - 84ac$$

$$= 25(a^2 + c^2) - 34ac$$

$$= 17(a^2 + c^2 - 2ac) + 8(a^2 + c^2)$$

$$= 17(a - c)^2 + 8(a^2 + c^2)$$

$D > 0 \Rightarrow$ Roots are real and distinct

Q14

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$$4a + 4b + 4c - 9c > 0$$

$$4a + 4b - 5c > 0$$

$$f(x) = ax^2 + 2bx - 5c, D < 0$$

$$f(2) = 4a + 4b - 5c > 0$$

$a > 0$ and

$$\begin{aligned} -5c > 0 \\ \Rightarrow c < 0 \end{aligned}$$

Q15

$$D = B^2 - 4AC$$

$$= (2(a + b - 2c))^2 - 4(a - b)^2$$

$$= 4\{(a + b - 2c)^2 - (a - b)^2\}$$

$$= 4(a + b - 2c - a + b)(a + b - 2c + a - b)$$

$$= 4(2b - 2c)(2a - 2c)$$

$$= 16(b - c)(a - c)$$

$$= 16(c - b)(c - a)$$

If c lies between a and b , then D is negative. Hence, the roots will be imaginary and the graph will be entirely above the x -axis as the coefficient of x^2 is positive.

Hints and Solutions

MathonGo

Q16

Let, α be the common root and the other roots of the equations be 4β and 3β respectively. Then,

$$\alpha + 4\beta = 6, \quad \alpha(4\beta) = a$$

$$\alpha + 3\beta = c, \quad \alpha(3\beta) = 6 \Rightarrow \frac{4}{3} = \frac{a}{6} \Rightarrow a = 8$$

The first equation is $x^2 - 6x + 8 = 0$

Whose roots are 2 and 4

If $\alpha = ? \Rightarrow \beta = 1$

So roots of first equation is 2, 4 and that of second equation is 2, 3

$$\text{If } \alpha = 4 \Rightarrow \beta = \frac{1}{2} \Rightarrow 3\beta = \frac{3}{2}, 4\beta = 2$$

Here the roots are not integers $\Rightarrow \alpha = 2$

017

Let, $f(x) = 4x^2 - 20kx + (25k^2 + 15k - 66) = 0$ (i)

Let the roots of $f(x) = 0$ be α, β

Since α, β are real,

• $\Delta \geq 0$

$$\Rightarrow 400k^2 - 4.4(25k^2 + 15k - 66) \geq 0$$

We have $\alpha, \beta < 2$

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$$\therefore \alpha + \beta < 4$$

$$\Rightarrow -\frac{(-20k)}{4} < 4 \Rightarrow k < \frac{4}{5} \dots\dots\dots(iii)$$

$$f(x) = 4(x - \alpha)(x - \beta)$$

$$\therefore f(2) = 4(2 - \alpha)(2 - \beta) = 4(+)(+) = +ve$$

$$\therefore f(2) = 16 - 40k + (25k^2 + 15k - 66) > 0$$

$$\Rightarrow 25k^2 - 25k - 50 > 0 \Rightarrow k^2 - k - 2 > 0$$

$$\Rightarrow (k+1)(k-2) > 0 \Rightarrow k < -1 \text{ or } k > 2 \dots\dots(iv)$$

Combining (ii), (iii) & (iv), we get $k \in (-\infty, -1)$

Q18

Clearly, $f(-1) > 0$, $f(2) < 0$

since, $f(0) = -4 < 0$

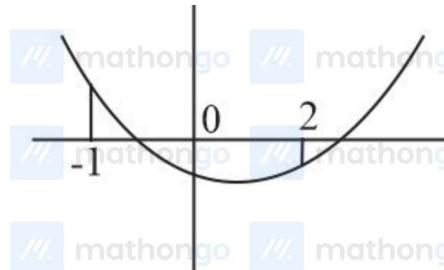
$$\Rightarrow 1 - a - 4 > 0 \Rightarrow -a - 3 > 0 \Rightarrow -a > 3$$

or $a < -3$ and $4 + 2a - 4 < 0$

$$\Rightarrow a < 0$$

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$\Rightarrow a \in (-\infty, -3)$.

**Q19**

The roots of $f(x)-x=0$ are 1, 2 and 3.

So, we get,

$$f(x)-x=(x-1)(x-2)(x-3)(x-a)$$

For $x = -1$, we get,

$$f(-1)+1=(-2)(-3)(-4)(-1-a)=24(1+a)$$

For $x = 5$, we get,

$$f(5)-5=4\cdot 3\cdot 2(5-a)=24(5-a)$$

$$f(-1)+f(5)=(23+24a)+(125-24a)=148$$

For $x = 0$, we get,

$$f(0)-0=(-1)(-2)(-3)(-a)=6a$$

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For $x = 4$, we get,

$$f(4) - 4 = 3 \cdot 2 \cdot 1 \cdot (4 - a) = 24 - 6a$$

$$f(0) + f(4) = 28$$

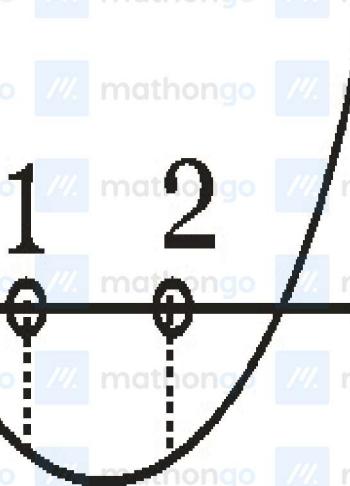
$$\left[\frac{f(-1) + f(5)}{f(0) + f(4)} \right] = \left[\frac{148}{28} \right]$$

$$\left[\frac{f(-1) + f(5)}{f(0) + f(4)} \right] = 5$$

Q20

$$\text{Let, } f(x) = x^2 + ax + a^2 + 6a$$

$$\therefore f(1) \leq 0$$



$$\Rightarrow a^2 + 7a + 1 < 0$$

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or $\frac{-7-3\sqrt{5}}{2} < a < \frac{-7+3\sqrt{5}}{2}$ (i)

$$f(2) \leq 0$$

$$\Rightarrow a^2 + 8a + 4 < 0$$

or $-4 - 2\sqrt{3} < a < -4 + 2\sqrt{3}$ (ii)

and $D > 0$

$$\Rightarrow a^2 - 4 \cdot 1(a^2 + 6a) > 0$$

$$\Rightarrow a^2 + 8a < 0$$

or $-8 < a < 0$ (iii)

From Eqs. (i), (ii) and (iii), we get,

$$\frac{-7-3\sqrt{5}}{2} \leq a \leq -4 + 2\sqrt{3}$$

Hence, integral values of a are $-6, -5, -4, -3, -2, -1$

Required Sum

$$= (-6)^2 + (-5)^2 + (-4)^2 + (-3)^2 + (-2)^2 + (-1)^2$$

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= 91.

Q21

Let α be the common root

$$\Rightarrow k\alpha^2 + \alpha + k = 0$$

$$k\alpha^2 + k\alpha + 1 = 0$$

On solving, we get,

$$\begin{aligned}\frac{\alpha^2}{1-k^2} &= \frac{\alpha}{k^2-k} = \frac{1}{k^2-k} \\ \frac{\alpha^2}{\alpha} &= \frac{1-k^2}{k^2-k} \text{ and } \frac{\alpha}{1} = \frac{k^2-k}{k^2-k} \\ \Rightarrow \alpha &= \frac{1-k^2}{k^2-k} = 1 \Rightarrow k^2 - k = 1 - k^2\end{aligned}$$

$$\Rightarrow 2k^2 - k - 1 = 0 \Rightarrow k = -\frac{1}{2}, 1$$

For $k = 1$, equations are identical, thus not possible

Hence, $k = -\frac{1}{2}$

Q22

$$(6k+2)x^2 + rx + 3k - 1 = 0$$

$$(12k+4)x^2 + px + 6k - 2 = 0$$

both roots common.

So,

$$\frac{6k+2}{12k+4} = \frac{r}{p} = \frac{3k-1}{6k-2}$$

$$\Rightarrow \frac{r}{p} = \frac{1}{2} \Rightarrow 2r - p = 0$$

Q23

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Let, the roots be $\alpha, \beta, \alpha + 2$.

$$S_1 = \alpha + \beta + \alpha + 2 = 2\alpha + \beta + 2 = 13 \Rightarrow 2\alpha + \beta = 11 \Rightarrow \beta = 11 - 2\alpha$$

$$S_2 = \alpha\beta + \beta(\alpha + 2) + (\alpha + 2)\alpha = 15$$

$$\Rightarrow \beta(\alpha + \alpha + 2) + \alpha(\alpha + 2) = 15$$

$$\Rightarrow (11 - 2\alpha)(2\alpha + 2) + \alpha(\alpha + 2) = 15$$

$$\Rightarrow 22\alpha + 22 - 4\alpha^2 - 4\alpha + \alpha^2 + 2\alpha = 15$$

$$\Rightarrow 3\alpha^2 - 20\alpha - 7 = 0 \Rightarrow (\alpha - 7)(3\alpha + 1) = 0$$

$$\Rightarrow \alpha = 7 \text{ or } -\frac{1}{3}.$$

$$\alpha = 7, \beta = 11 - 2\alpha = 11 - 14 = -3, \gamma = \alpha + 2 = 9$$

$$\alpha = -\frac{1}{3}, \beta = 11 - 2\alpha = 11 + \frac{2}{3} = \frac{35}{3}, \gamma = \alpha + 2 = \frac{5}{3}.$$

Since, $\alpha\beta\gamma = -189$, hence we will take the first case.

$$|\alpha| + |\beta| + |\gamma| = |7| + |-3| + |9| = 19$$

Q24

$$x^3 + 3x^2 + 5x + 3 = 0 \text{ has one root } x = -1$$

$$\therefore x^3 + 3x^2 + 5x + 3 = (x + 1)(x^2 + 2x + 3)$$

$$\Rightarrow a = 2, b = 3$$

Q25

Given equation,

$$4\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) - 57 = 0$$

$$\text{Let, } x + \frac{1}{x} = y; x^2 + \frac{1}{x^2} = y^2 - 2$$

$$\Rightarrow 4y^2 + 16y - 65 = 0$$

$$\Rightarrow y = -\frac{13}{2} \text{ or } \frac{5}{2}$$

$$\text{When, } y = \frac{5}{2}$$

$$x + \frac{1}{x} = \frac{5}{2} \Rightarrow x = 2 \text{ or } \frac{1}{2}$$

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When, $y = -\frac{13}{2}$

$$\Rightarrow x + \frac{1}{x} = -13/2$$

$$\Rightarrow 2x^2 + 13x + 2 = 0$$

$$\Rightarrow x = \frac{-13 \pm \sqrt{153}}{4}$$

Since x is rational, $x = 2$ or $\frac{1}{2}$

Q26

The given equation will be true in 3 cases.

Case 1: when $x^2 - 5x + 5 = 1$

$$\Rightarrow x^2 - 5x + 4 = 0$$

$$\Rightarrow x = 1, 4$$

Case 2: when $x^2 + 4x - 60 = 0$

$$\Rightarrow x = 6, -10$$

Case 3: when $x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60 \in$ even integers

Now, $x^2 - 5x + 5 = -1$

$$\Rightarrow x = 2, 3$$

Only $x = 2$ satisfies the given condition,

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Hence, sum of all real values of x is $1 + 4 + 6 - 10 + 2 = 3$

Q27

Let $\log_x 10 = t$

$$\therefore t^3 - t^2 - 6t = 0$$

$$\Rightarrow t(t^2 - t - 6) = 0$$

$$\Rightarrow t = 0, -2, 3$$

$$\Rightarrow \log_x 10 = 0, -2, 3$$

$$\Rightarrow 10 = x^0, x^{-2}, x^3$$

$$\Rightarrow x = 10^{-\frac{1}{2}}, 10^{\frac{1}{3}}$$

Let $\alpha = 10^{-\frac{1}{2}}$ and $\beta = 10^{\frac{1}{3}}$

$$\text{Now, } \left| \frac{1}{\log_{10} \alpha \beta} \right| = \left| \frac{1}{\log_{10} 10^{-\frac{1}{6}}} \right|$$

$$\Rightarrow \left| \frac{1}{\log_{10} \alpha \beta} \right| = \left| \frac{-6}{\log_{10} 10} \right| = 6$$

Q28

Let, $t = 2^{11x}$

$$\begin{aligned} \Rightarrow \frac{(2^{11x})^3}{2^2} + 2^{11x} \cdot 2^2 &= (2^{11x})^2 \cdot 2 + 1 \\ \Rightarrow \frac{t^3}{4} + 4t &= 2t^2 + 1 \\ \Rightarrow t^3 - 8t^2 + 16t - 4 &= 0 \end{aligned}$$

Cubic in t has roots t_1, t_2, t_3

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i.e. $t_1 t_2 t_3 = 4 \Rightarrow 2^{11x_1} \cdot 2^{11x_2} \cdot 2^{11x_3} = 4$

$\Rightarrow 2^{11(x_1+x_2+x_3)} = 2^2$

$\Rightarrow 11(x_1 + x_2 + x_3) = 2 \Rightarrow x_1 + x_2 + x_3 = \frac{2}{11}$

Q29

We have, $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$

Let $e^x = t$

$$t^4 - t^3 - 4t^2 - t + 1 = 0$$

$$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\Rightarrow \alpha^2 - \alpha - 6 = 0, \alpha = t + \frac{1}{t} \geq 2$$

$$\Rightarrow \alpha = 3, -2 \text{ (reject)}$$

$$\Rightarrow t + \frac{1}{t} = 3$$

\Rightarrow The number of real roots = 2.

Q30

$$\text{Let } \frac{\pi}{2^{\cos^{-1} x}} = t \Rightarrow t \geq 2$$

$$\text{equation becomes } t^2 - \left(a + \frac{1}{2}\right)t - a^2 = 0$$

has one root 2 or greater than 2 and other root less than 2, $f(2) \leq 0$

$$\Rightarrow 4 - \left(a + \frac{1}{2}\right)2 - a^2 \leq 0$$

$$a^2 + 2a - 3 \geq 0$$

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$$(a + 3)(a - 1) \geq 0$$

$$a \leq -3 \text{ or } a \geq 1$$

