Sample Task Solutions

MathonGo **Hints and Solutions** mathongo /// mathongo $\lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{(\pi-2x)^3} \frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{(\pi-2x)^3} \frac{(\pi-2x)^3}{(\pi-2x)^3} \frac{(\pi-2x)^3}{($ Let, $x = \frac{\pi}{2} + y$ $\Rightarrow \lim_{y \to 0} = \frac{\tan\left(\frac{-y}{2}\right)(1-\cos y)}{(-2y)^3} \quad \text{ongo} \quad \text{///} \quad \text{mathongo} \quad \text{///}$ $=\lim_{y\to 0}\frac{-\tan\frac{y}{2}\cdot 2\sin^2\frac{y}{2}}{(-8)y^3}=\lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}\cdot \left[\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right]^2$ $+\tan\frac{y}{2} \left(\sin\frac{y}{2}\right)^2 =\lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}\cdot \left[\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right]^2$ $+\tan\frac{y}{2} \left(\sin\frac{y}{2}\right)^2$ $+\sin\frac{y}{2}\left(\sin\frac{y}{2}\right)^2$ $+\sin\frac{y}{2}\left($ $=\lim_{y\to 0}\frac{1}{32}\times\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}\cdot\left(\lim_{y\to 0}\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2$ ## mathongo ## mathon $\Rightarrow \frac{1}{32} \times 1 \times 1^{2}$ mathongo /// mathongo // mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathon /// mathongo Let, $\sin x = t$ go ///. mathongo ///. t $\Rightarrow \lim_{t o 0} rac{1-\cos^3 t}{t \sin t \cos t}$ $\Rightarrow \lim_{t \to 0} \frac{(1-\cos t)}{|t|^2} \times \left(\frac{t}{\sin t}\right) \times \frac{(1+\cos t + \cos^2 t)}{\cos t} \text{ mathongo } \text{ mathongo$ /// mathongo /// О3 $\frac{\text{mathor}_{x}\frac{\tan(\frac{1}{x})}{(\frac{1}{x})}}{-x\sqrt{4-\frac{1}{x}+\frac{1}{x^2}}} = \frac{1}{-\sqrt{4}} = -\frac{1}{2}$ mathongo /// m $-x\sqrt{4-x}+x^2$ $-v^4$ mathongo /// mathongo ///Q4 $\lim_{x \to \infty} \frac{3}{2} \left[\frac{1}{1 - \left(\cos x\right)^{\frac{3}{2}}} \right] = \lim_{x \to \infty} \frac{1}{1 - \left(\cos x\right)^{\frac{3}{2}}} \lim_{x \to \infty} \frac{1}$ $=\lim_{x\to 0}\frac{\sqrt{\int \lfloor \cdot \cdot \cdot \rfloor}}{\operatorname{athong}} \frac{\sqrt{\int \lfloor \cdot \cdot \cdot \rfloor}}{\sqrt{\int \int \int \int \int \int \partial x}} \frac{1}{x^2} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $= 1^{\frac{3}{2}} \cdot \lim_{x\to 0}\frac{1-\cos^3 x}{x^2} \cdot \frac{1}{1+\left(\cos x\right)^{\frac{3}{2}}} \left(\operatorname{Rationalizing}\right)$ $=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot \left(1+\cos x+\cos^2 x\right)\cdot \frac{1}{1+(\cos x)^{\frac{3}{2}}}$ $=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot \left(1+\cos x+\cos x\right)\cdot \frac{1}{1+(\cos x)^{\frac{3}{2}}}$ $=\frac{1}{2}\cdot\frac{1}{2}(1+1+1)=\frac{3}{4}$. $\lim_{x\to 0} \frac{1-\cos x}{x^2} \cdot \frac{(3+\cos 2x)}{1} \cdot \frac{n2x}{\tan 2x} \cdot \frac{1}{2}$ /// mathongo // mathongo $=\frac{1}{2}$ $\text{rr}\left(4\right)$ $\odot\frac{1}{2}$ = 1 m mathongo \text

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Sample Task Solutions

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$$\lim_{x \to 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$$
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$$= \lim_{x \to 0} \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{\sinh x \cos x \text{ athongo } \text{ mathongo }$$

$$= \lim_{x \to 0} \frac{2 \sin^2\left(\frac{x}{2}\right)}{x \cdot 2 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)} \times \frac{\left(1 + \cos x + \cos^2 x\right)}{\cosh x}$$
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$$=\lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{2\left(\frac{x}{2}\right)}\times\frac{1+\cos x+\cos^2 x}{\cos\left(\frac{x}{2}\right)\cos x}=\frac{1}{2}\times 3=\frac{3}{2}$$

$$\lim_{x\to 0} \left(\frac{\sin 3x}{x^3} + \frac{a}{x^2} + b\right) = \lim_{x\to 0} \frac{\sin 3x + ax + bx^3}{x^3 \text{ mathongo } \text{mathongo } \text{m$$

limit exists if
$$a = 0$$
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or
$$a = 43$$
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$$\therefore L = \lim_{x \to 0} \frac{\sin 3x - 3x + bx^3}{x^3} = 27 \left(\lim_{t \to 0} \frac{\sin t - t}{t^3} + b \right) = 0$$

$$\text{Multiply mathenge} \text{ Multiply mathenge$$

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$$\frac{1}{12}$$
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$$\lim_{x\to 0} \frac{f(x^2+x+1)-f(1)}{f(x^4-x^2+2x+4)-f(4)} \quad \left(\frac{0}{0} \text{ form}\right)^{\prime\prime\prime} \quad \text{mathongo} \quad \prime\prime\prime \quad \text{m$$

$$\lim_{x\to 0} \frac{\max(2x+1)f^1(x^2+x+1)}{(4x^3-2x+2)f^1(x^4-x^2+2x+4)} \hspace{1cm} \text{mathongo} \hspace{1cm}$$

$$=\frac{f'(1)}{2f'(4)}$$
 = 4 (Applying L' Hospital's Rule) athongo /// mathongo /// mathongo /// mathongo /// mathongo ///

$$\lim_{x \to \infty} \left(\sqrt{x^2 + x + 2} - ax - b \right) = 2$$
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$$a = 1; b = \frac{-3}{2}$$
 required equation is $(x - 1)^2 + (y + 3)^2 = 1$

Q10

$$x = \frac{1}{t}$$
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$$\lim_{t \to 0} \left(\frac{\ln(1+t)}{t^3} + \frac{1}{2t} - \frac{1}{t^2}\right) = \lim_{t \to 0} \frac{2 \ln(1+t) + t^2 - 2t}{2t^3}$$
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MathonGo **Hints and Solutions**

$$\frac{1}{2} \frac{1}{4} \frac{1$$

$$= \lim_{t \to 0} \left(\frac{1}{3} - \frac{t}{4} + \frac{t^2}{5} \dots \right) = \frac{1}{3} = \frac{1}{m} \implies m = 3$$
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$$\lim_{n\to\infty} \frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}$$
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$$=\lim_{n\to\infty}\frac{5^n\left(6.\left(\frac{2}{5}\right)^n-20\right)}{5^n\left(5.\left(\frac{2}{5}\right)^n+7\right)}=\frac{20}{7}\left(\because\lim_{n\to\infty}\left(\frac{2}{5}\right)^n=0\right)$$
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$$\lim_{x o 0} \frac{\ln(2-\cos 15x)}{\ln^2(\sin 3x+1)}$$
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$$= \lim_{x \to 0} \frac{\ln^2(1+\sin 3x)}{\ln^2(1+\sin 3x)}$$
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$$=\lim_{x\to 0}\frac{1-\cos 15x}{\left(\sin 3x\right)^2}\left(\text{Applying}\lim_{x\to 0}\frac{\ln \left(1+x\right)}{x}=1\right)$$

$$=\lim_{x\to 0}\frac{1-\cos 15x}{(\sin 3x)^2} \text{ (Applying}\lim_{x\to 0}\frac{\ln (1+x)}{x}=1\text{)}$$
"Mathongo "Mathongo

$$\frac{1}{2\times 9} \left(\frac{1}{2\times 9}\right) = 12.5$$
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$$f(x)=a\sin\left(rac{\pi[x]}{2}
ight)+[2-x], a\in\mathbb{R}$$
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Now given
$$\lim_{x\to -1} f(x)$$
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So
$$\lim_{x\to -1^+} a \sin\left(\pi \frac{1}{2}\right) + [2-x] = -a + 2$$

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$$^{\prime\prime\prime}$$
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$$\int_{0}^{4} f(x)dx = \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{4} f(x)dx$$
 mathong mathong mathong mathong mathong mathong mathons $\int_{0}^{4} f(x)dx = \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{4} f(x)dx$ mathong mathong mathong mathons $\int_{0}^{4} f(x)dx = \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{4} f(x)dx$ mathong mathons $\int_{0}^{4} f(x)dx + \int_{1}^{4} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{4} f(x)dx +$

$$\Rightarrow \int_{0}^{4} f(x) dx = \int_{0}^{1} -\sin\left(\frac{\pi[x]}{2}\right) + [2-x] dx + \int_{1}^{2} -\sin\left(\frac{\pi[x]}{2}\right) + [2-x] dx + \int_{2}^{3} -\sin\left(\frac{\pi[x]}{2}\right) + [2-x] dx + \int_{3}^{4} -\sin\left(\frac{\pi[x]}{2}\right) + [2-x] dx$$

$$=\int_0^1 (0+1)dx + \int_1^2 (-1+0)dx + \int_2^3 (0-1)dx + \int_3^4 (1-2)dx$$

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Sample Task **Solutions**

Hints and Solutions MathonGo

$$\frac{(\cos x {-}1)(\cos x {-}e^x)}{}$$

$$=\frac{1}{x^n}\left(\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)-1\right)\left(\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)^{m-1}\right)\left(\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)^{m-1}\right)$$

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$$\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}...\right)$$
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$$=\frac{1}{x^n}\left[\left(-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right)\left(-x-x^2-\frac{x^3}{3!}\ldots\right)\right]_{\text{hongo}} \text{ mathongo } \text{ matho$$

$$x^{n-3} \left[\left(\begin{array}{cc} 2! & 4! \\ \end{array} \right) \left(\begin{array}{cc} 3! & 1 \end{array} \right]$$

$$\therefore \text{ For } \lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n} \text{ to exist as a nonzero number we must have } n - 3 = 0 \Rightarrow n = 3.$$

$$\lim_{x\to\infty} \left(1+\frac{a}{x}\right)_{00}^{\frac{x}{b}} = e^{\lim_{x\to\infty} \frac{x}{b} \left(1+\frac{a}{x}-1\right)}$$
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Q16

$$l = \lim_{n \to \infty} \frac{n^2 - 1}{n} \left[\frac{2n^2 - 3 - 2n^2 + n - 1}{2n^2 - n + 1} \right]$$
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$$= \lim_{n \to \infty} \frac{n^2 \left(1 - \frac{1}{n^2}\right) - n \left(1 - \frac{4}{n}\right)}{n \cdot n^2 \left(2 - \frac{1}{n} + \frac{1}{2}\right)} = \frac{1}{2}$$
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Let,
$$3^{rac{x}{2}}=t, x o 2\Rightarrow t o 3$$

Let,
$$3^2=t, x \Rightarrow 2 \Rightarrow t \Rightarrow 3$$

/// $\max_{t \neq 2} \frac{2^2}{t-12}$ /// $\max_{t \neq 2} \frac{2^2}{t-12} = \lim_{t \to 3} \frac{t^2+27-12t^2}{t-3}$ /// $\max_{t \neq 2} \frac{1}{t-3} = \lim_{t \to 3} \frac{t^2+27-12t^2}{t-3}$

$$\lim_{t \to 3} \frac{(t^2-3)(t+3)(t-3)}{(t-3)} = 6 \times 6 = 36$$
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Let,
$$f(x)=rac{1+ an x}{1+\sin x}$$

and
$$g(x) = \frac{1}{\sin x}$$
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and
$$g(x) = \frac{n^2}{\sin x}$$
 and $g(x) = \frac{n^2}{\sin x}$ mathongo we were also we were

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Solutions

Sample Task MathonGo **Hints and Solutions** Clearly, $f(x) \to 1$ and $g(x) \to \infty$ as $x \to 0$ athongo /// mathongo /// mathongo /// mathongo /// mathongo $\lim_{x\to 0} \left(\frac{1+\tan x}{1+\sin x}\right)^{\frac{2}{\sin x}} = e^{\lim_{x\to 0} \frac{2}{\sin x} \left(\frac{1+\tan x}{1+\sin x}-1\right)}$ ///. mathongo ///. mathongo ///. mathongo ///. mathongo $\big\{ \text{ using } \lim_{x \to a} [f(x)]^{g(x)} = e^{\lim_{x \to a} g(x)[f(x)-1]} \text{ for } 1^{\infty} \text{ form } \big\}$ $= e^{\lim_{x \to 0} \frac{2}{\sin x} \left(\frac{\tan x - \sin x}{1 + \sin x}\right)} = e^{\lim_{x \to 0} \frac{2(1 - \cos x)}{\cos x(1 + \sin x)}} = e^{\lim_{x \to 0} \frac{2(1 - \cos x)}{\cos x(1 + \sin x)}}$ mathongo /// math $= e^0 = 1$ mathongo /// mathongo $x o 0^{+}$ mathong $x o 0^$ $= \lim_{x \to 0^+} \frac{1}{\tan x} + \lim_{x \to 0^+} \frac{\left(\frac{1}{x}\right)}{\left(-\frac{1}{2}\right)} = 1 + \lim_{x \to 0^+} (-x)^{\text{hongo}} \text{ /// mathongo // mathongo /// mathongo // mathongo /$ mathongo /// math $\frac{\pi}{6}$ at nongo $\frac{\pi}{8}$ mathongo $\frac{\pi}{8}$ ma $\therefore \lim_{\mathbf{x} \to \frac{\pi}{2}} \frac{\lfloor \frac{\pi}{2} \rfloor}{\ln(\sin \mathbf{x})} = 0$ $\frac{\pi}{2}$ $\ln(\sin x)$ mathongo /// mathongo Q21 $\frac{1}{2}$ mathongo $\frac{1}{2}$ ma $\frac{1}{x^3} - 1 \le \left[\frac{1}{x^3}\right] \le \frac{1}{x^3}$ $\frac{1}{x^3} + \frac{1}{x^3} = \frac{1}{x^3} = \frac{1}{x^3}$ $\frac{1}{x^3} + \frac{1}{x^3} = \frac{1}{x^3$ $x^{8}\left(rac{1}{x^{3}}-1
ight)\leq x^{5}\left[rac{1}{x^{3}}
ight]\leq x^{5} ext{ for all }x$ But $\lim_{x \to 0} x^5 = 0 = \lim_{x \to 0} \left(x^5 - x^8 \right)$ so athongo we mathongo we mathon we mathongo we mathon with the mathon we will be a supplication with the mathon we m $\lim_{x o 0} x^8 \left|rac{1}{x^3}
ight| = 0 \in \mathbf{I} \subseteq \mathbf{Q}$ Q22 For this limit to exists finitely, $\lim_{x\to 0}\frac{2\cos x+a}{x^2} = \text{finite} \quad \text{mathongo} \quad \text{"". mathongo} \quad \text{"". mathong$ \therefore It must be $\frac{0}{0}$ form mathongo ///. ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///.

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localhost:3002/solution 5/9 Sample Task Solutions

MathonGo **Hints and Solutions** by sandwich theorem mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo $\lim_{x \to 0} g(x) = \lim_{x \to 0} h(x) = \frac{5}{3}$ mathongo /// $\therefore \lim_{x\to 0} f(x) = \frac{5}{3}$ mathongo /// Let the given expression be y.ongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// Then, $y = \lim_{n \to \infty} n^2$ mathongo /// mathongo // mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// matho $\sqrt{\left(1-\cos\frac{1}{n}\right)\sqrt{\left(1-\cos\frac{1}{n}\right)\sqrt{\left(1-\cos\frac{1}{n}\right)}}}$ On putting $\frac{1}{n} = \theta \dots \infty$ hongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo So that, $n \to \infty \Rightarrow \theta \to 0$ mathongo /// mathongo // matho Thus, mathongo || matho $=\lim_{ heta o 0}\Bigl(rac{1-\cos\, heta}{ heta^2}\Bigr)$ /// mathongo /// $= \lim_{\theta \to \infty} \frac{2\sin^2\theta/20}{\theta^2}$ /// mathongo // mathongo /// mathongo // mathongo /// mathongo /// mathongo // mathongo // $=\lim_{ heta o0} 2\Big(rac{\sin heta/2}{ heta/2}\Big)^2 imes rac{1}{4}$ mathongo $ilde{\prime\prime\prime}$ mathongo $ilde{\prime\prime\prime}$ mathongo $ilde{\prime\prime\prime}$ mathongo $ilde{\prime\prime\prime}$ mathongo $ilde{\prime\prime\prime}$ $=2:1^{2}:1^{2}:\frac{1}{4}:=\frac{1}{2}$ /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// Let $f(x) = \frac{[x] + [2^2x] + [3^2x] + ... + [n^2x]}{2}$ $1^2+2^2+3^2+...+n^2$ Now, we have, $f(x) \le rac{x+2^2x+3^2x+...+n^2x}{1^2+2^2+3^2+...+n^2} = x$ mathongo /// $\frac{\text{and, } f(x) > \frac{(x-1) + (2^2x-1) + (3^2x-1) + \ldots + (n^2x-1)}{1^2 + 2^2 + 3^2 + \ldots + n^2} }{\text{mathongo } \text{ mathongo } \text{ ma$ $=\frac{x\Sigma n^2-n}{\Sigma n^2}=x-\frac{6}{(n+1)\left(2n+1\right)}\left(\because x-1\leq [x]< x,\ \forall x\in R\right)$ mathongo /// mathongo // mathongo //

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Sample Task Solutions MathonGo **Hints and Solutions** Thus, we have, $\frac{6}{(n+1)(2n+1)} < f(x) \le x$ mathongo /// mathongo // mathongo $\lim_{n \to \infty} x - \frac{6}{(n+1)(2n+1)} = x \& \lim_{n \to \infty} x = x$ mathongo /// mathongo // m Hence, by Sandwich Theorem, we have mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// $\lim_{n\to\infty} f(x) = x \operatorname{go}$ ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. mathongo ///. Q26 mathongo | ///. $\lim_{x \to 0} \frac{x^2 \tan(\alpha x)}{\beta x - \tan(2x)} = 1 \to \lim_{x \to 0} \frac{x^2 \tan(\alpha x)}{\beta x - \left\{2x + \frac{(2x)^3}{3} + \frac{2(2x)^5}{15}\right\}_{\text{ngo}}} = 1$ /// mathongo // ightarrow eta = 2 and $rac{3lpha}{8} = -1$, nathongo $rac{3lpha}{8} = -1$, nathongo $rac{3lpha}{8} = -1$, mathongo $rac{$ So, $5\beta \pm 3\alpha \equiv 2$ /// mathongo /// Q27 mathongo /// Consider $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ mathon $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} \left(\frac{0}{0} \text{ form}\right)$ $\lim_{x \to 0} \frac{2 \sin \left(\frac{\sin x + x}{2}\right) \cdot \sin \left(\frac{x - \sin x}{2}\right)}{x^4}$ mathongo /// mathongo // mathongo /// mathongo /// mathongo /// mathongo /// mathongo // mathong $=\lim_{x\to 0}2\left[\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\right]\left[\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right]\times \left(\frac{\sin x+x}{2}\right)\times \left(\frac{x-\sin x}{2}\right)\times \frac{1}{x^4}$ mathongo /// mathongo // mathong $\lim_{x\to 0} 2 \left\lceil \frac{\sin\left(\frac{\sin x + x}{2}\right)}{\cos\left(\frac{\sin x + x}{2}\right)} \right\rceil \left\lceil \frac{\sin\left(\frac{x - \sin x}{2}\right)}{\left(\frac{x - \sin x}{2}\right)} \right\rceil \times \left(\frac{x^2 - \sin^2 x}{4x^4}\right) \right\rceil = 1 \times \left(\frac{x^2 - \sin^2 x}{4x^4}\right)$ mathongo /// mathongo // mathongo $\lim_{x\to 0} 2 \times \left(\frac{x^2 - \sin^2 x}{4x^4}\right) \cdot \left(\frac{0}{0} \text{ form}\right) / \left(\because \lim_{t\to 0} \frac{\sin t}{t} = 1\right) \text{ mathongo } / / \text{ matho$ Applying L'Hospital's Rule,

 $\lim_{x\to 0} 2\times \left(\frac{2x-2\sin x\cos x}{4\cdot 4x^3}\right) = \lim_{x\to 0} \left(\frac{2x-\sin 2x}{8x^3}\right) \quad \ \left(\frac{0}{0} \text{ form}\right)$ mathongo /// $\lim_{x \to 0} \left(\frac{2 - 2\cos 2x}{24x^2} \right) \left(\frac{0}{0} \text{ form} \right)$

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$$\lim_{x\to 0} \left(\frac{4\sin 2x}{48x}\right) = \frac{1}{6} \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right) = \frac{1}{6} \quad \text{mathongo} \quad \text{mathongo$$

We know that
$$r \le [r] < r+1$$
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$$2r \le [2r] < 2r + 1$$
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$$3r \le [3r] < 3r + 1$$

$$r+2r+\ldots+nr \leq [r]+[2r]+\ldots+[nr] < r+2r+\ldots nr+n$$
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$$rac{n(n+1)}{2}\cdot r \leq [r]+[2r]+\ldots+[nr]<rac{n(n+1)}{2}\cdot r+n$$

$$\frac{r_{n(n+1)}}{r_{n}} = \frac{r_{n(n+1)}}{r_{n}} = \frac{r_{n(n+1)}}{r_{n}}$$

$$\underset{n \to \infty}{\text{Now, }} \lim_{n \to \infty} \frac{n(n+1) \cdot r}{2 \cdot n^2} = \lim_{n \to \infty} \frac{n^2 \left(1 + \frac{1}{n}\right) \cdot r''}{2 \cdot n^2} = \frac{mathongo}{2} \quad \text{mathongo} \quad \text{''' mathongo} \quad$$

$$\inf_{n\to\infty}\frac{m_{n+1}}{n^2}\cdot r+n \text{ mat } \frac{n^2\left\{\left(1+\frac{1}{n}\right)\cdot r+\frac{1}{n}\right\}\text{ athongo }}{2\cdot n^2} = \lim_{n\to\infty}\frac{n^2\left\{\left(1+\frac{1}{n}\right)\cdot r+\frac{1}{n}\right\}\text{ athongo }}{2\cdot n^2} = \frac{r}{2}$$

$$\min_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)\cdot r+\frac{1}{n}}{2\cdot n^2} = \frac{r}{2}$$

$$\min_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)\cdot r+\frac{1}{n}}{2\cdot n^2} = \frac{r}{2}$$

$$\min_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)\cdot r+\frac{1}{n}}{2\cdot n^2} = \frac{r}{2}$$

$$\lim_{n\to\infty}\frac{[r]+[2r]+...+[nr]}{\ln \operatorname{div}_{n^2}}\equiv\frac{r}{2}\text{-hongo} \quad \text{///} \quad \text{mathongo} \quad \text{/$$

Using expansion,
$$\frac{1}{1+\left(-\frac{x^2}{2}\right)+\frac{\left(\frac{x^4}{4}\right)}{2!}+\dots\right)} = \left(1-\frac{x^2}{2!}+\frac{x^2}{4!}-\dots\right) = 1$$
 mathongo $\frac{x}{2!}$ mathongo $\frac{x}{2!}$ mathongo $\frac{x}{2!}$ mathongo $\frac{x}{2!}$ mathongo $\frac{x}{2!}$ mathongo $\frac{x}{2!}$

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Hints and Solutions MathonGo Using expansions, we get, $\lim_{x\to 0} \frac{(a+b+c)+(a+d)x+\left(\frac{a-b}{2}\right)x^2+\frac{a}{6}x^3+\dots}{x^3\left(1-\frac{x^2}{3!}+\dots\right)^2} \equiv 3_{mathongo} \text{ /// mathongo // mat$: in the denominator lowest power of x is 3 For the limit to be finite, the numerator should also have the least power of x as 3 // mathongo // ma $\therefore \mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \quad \dots (1)$ /// mathongo /// $\mathrm{a}+\mathrm{d}=0.\,.\,.(2)$ $\frac{\mathrm{a-b}}{2}=0...(3)$ Now, $\frac{\left(\frac{a}{6}\right)}{1} = 3 \Rightarrow a = 18$ From (1), (2), (3), we get, thongo /// mathongo /// mathongo /// mathongo /// mathongo /// mathongo /// $\frac{\text{abd}}{\text{c}^3} = \frac{-(18)^3}{-8(18)^3} = \frac{1}{8}$

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