Q1

$\because f(x)$ is continuous at $x=3$

$\therefore LHL = RHL = f(3)$

$\lambda\sqrt{2(3)+3} = \mu\left(3\right)+12 \Rightarrow \lambda = \mu + 4$ ......(1)

Also $f(x)$ is differentiable at $x=3$

$\therefore$ at $x=3$ $LHD = RHD$

$\frac{\lambda}{2\sqrt{2(3)+3}} \cdot 2 = \mu \Rightarrow \lambda = 3\mu$ ......(2)

By (1) and (2)

$\mu = 2, \lambda = 6$

Q2

The given function is clearly continuous at all points except possibly at $x=\pm 1$.

For $f(x)$ to be continuous at $x=1$, we must have

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{x\to 1^-} \alpha x^2+\beta = \lim_{x\to 1^+} \frac{1}{x^2}$

$\Rightarrow \alpha+\beta = 1 \ldots (1)$

Now, for $f(x)$ to be differentiable at $x=1$, we must have

$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$

$\Rightarrow \lim_{x\to 1^-} \frac{\alpha x^2+\beta-1}{x-1} = \lim_{x\to 1^+} \frac{\frac{1}{x^2}-1}{x-1}$ $(\because \alpha+\beta=1 \therefore \beta-1=-\alpha)$

$\Rightarrow \lim_{x\to 1^-} \frac{\alpha x^2-\alpha}{x-1} = \lim_{x\to 1^+} \frac{\frac{1}{x^2}-1}{x-1}$

$\Rightarrow \lim_{x\to 1^-} \alpha(x+1) = \lim_{x\to 1^+} \frac{-2}{x^3} \Rightarrow 2\alpha = -2$

$\Rightarrow \alpha = -1$

Putting $\alpha=-1$ in (1), we get $\beta=2$

Q3

Given that $f(x)$ is continuous in the interval $[-1,\ 1]$, so, it will be continuous at $x=0$.

Now $f\left(0^+\right) = \frac{-1}{4}$

$f\left(0^-\right) = \lim_{x\to 0^-} \frac{2ax}{x\left(\sqrt{4+ax}+\sqrt{4-ax}\right)}$ (rationalizing)

$= \lim_{x\to 0^-} \frac{2a}{4} = \frac{a}{2}$

Now $f\left(0^+\right) = f\left(0^-\right)$ $(\because f(x)$ is continuous at point $0)$

$\Rightarrow \quad \frac{a}{2} = \frac{-1}{4} \Rightarrow \quad a = \frac{-1}{2}$

Q4

If $\mathrm{f(x)}$ is continuous at $\mathrm{x}=0$

$\mathrm{f(x)} = \lim_{\mathrm{x}\to 0} \mathrm{f(x)}$

$= \lim_{\mathrm{x}\to 0} \frac{1-\cos(1-\cos \mathrm{x})}{\mathrm{x}^4} \times \frac{1+\cos(1-\cos \mathrm{x})}{1+\cos(1-\cos \mathrm{x})}$

$$= \lim_{x \to 0} \frac{\sin^2(1-\cos x)}{x^4 \cdot (1+\cos(1-\cos x))} \cdot \frac{(1-\cos x)^2}{(1-\cos x)^2}$$

$$= \lim_{x \to 0} \left[\frac{\sin(1-\cos x)}{(1-\cos x)}\right]^2 \times \lim_{x \to 0} \left(\frac{1-\cos x}{x^2}\right)^2 \times \lim_{x \to 0} \frac{1}{1+\cos(1-\cos x)}$$

$$= (1)^2 \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$\therefore k = \frac{1}{8} \Rightarrow 10k = \frac{5}{4} = 1.25$$

Q5

For the function $f(x)$ to be continuous at $x = 2\pi$

$$\lim_{x \to 2\pi} f(x) = f(2\pi)$$

Now, $\lim_{x \to 2\pi} \frac{1-\cos x}{(2\pi - x)^2} \frac{\sin^2 x}{\log\left\{1+(2\pi-x)^2\right\}} = \lambda$

Putting $x = 2\pi + t$, we get,

$$\lim_{t \to 0} \frac{1-\cos t}{t^2} \cdot \frac{\sin^2 t}{\log(1+t^2)} = \lambda$$

$$\lim_{t \to 0} \frac{1}{2} \cdot \frac{\sin^2 t}{t^2} \cdot \frac{t^2}{\log(1+t^2)} = \lambda$$

$$\frac{1}{2} \times 1 \times 1 = \lambda$$

$$\Rightarrow \lambda = \frac{1}{2}$$

Q6

Hints and Solutions **MathonGo**

$$f(x)=\begin{cases}\frac{e^{[x]+|x|}-1}{[x]+|x|} & x\neq 0\\ -1 & x=0\end{cases}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\frac{e^{[x]+|x|}-1}{[x]+|x|}=\frac{e^{-1}-1}{-1}=\frac{e-1}{e}$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{e^{[x]+|x|}-1}{[x]+|x|}=\lim_{x\to 0^+}\frac{e^x-1}{x}=1$$

$\because$ LHL $\neq$ RHL at $x=0\Rightarrow f(x)$ is discontinuous at $x=0$

Q7

We have,

$$\lim_{h\to 0^+} f(0-h)=\lim_{h\to 0^+}[1+|\sin(-h)|]^{\frac{l}{|\sin -h|}}$$

$$=\lim_{h\to 0^+}(1+\sin h)^{\frac{l}{\sin h}}$$

$$=\lim_{h\to 0^+}\left[(1+\sin h)^{\frac{1}{\sin h}}\right]^l=e^l$$

$$\lim_{h\to 0^+} f(0+h)=\lim_{h\to 0^+} e^{\frac{\tan 2h}{\tan 3h}}$$

$$=e^{\lim_{h\to 0^+}\left(\frac{\tan 2h}{2h}\times\frac{2}{3}\times\frac{3h}{\tan 3h}\right)}=e^{1\times\frac{2}{3}\times 1}=e^{\frac{2}{3}}$$

Also, $f(0)=m$

For $f(x)$ to be continuous at $x=0$, we must have

$$\lim_{h\to 0^+} f(0-h)=\lim_{h\to 0^+} f(0+h)=f(0)$$

$$\Rightarrow e^l=e^{\frac{2}{3}}=m$$

$$\Rightarrow l=\frac{2}{3}\text{ and } m=e^{\frac{2}{3}}$$

Q8

$\mathrm{f(x)}=\{\mathrm{x}\}\sin(\pi[\mathrm{x}])$

$=\{\mathrm{x}\}\sin$ (integral multiple of π)

$= \ 0$

Hence, $f(x)$ is continuous for all x.

Q9

$f(x) = \lim_{n \to \infty} \left\{\cos^2(\pi x)\right\}^n + [x]$

$= \begin{cases} [x] & : \ \cos^2(\pi x) \in [0\ ,1) \\ 1 + [x] & : \ \cos^2(\pi x) = 1 \end{cases}$

$= \begin{cases} [x] & : \ x \notin \mathrm{I} \\ 1 + [x] & : \ x \in \mathrm{I} \end{cases}$

For $x = 1$,

$f(1^-) = [1^-] = 0,\ f(1^+) = [1^+] = 1,\ f(1) = 1 + [1] = 2$

$\because f(1^+) \neq f(1^-) \Rightarrow f(x)$ is discontinuous at $x = 1$

For $x = \frac{3}{2}$,

$f\left(\frac{3}{2}^+\right) = \left[\frac{3}{2}^+\right] = 1,\ f\left(\frac{3}{2}^-\right) = \left[\frac{3}{2}^-\right] = 1,\ f\left(\frac{3}{2}\right) = \left[\frac{3}{2}\right] = 1$

$\because f\left(\frac{3}{2}^-\right) = f\left(\frac{3}{2}^+\right) = f\left(\frac{3}{2}\right)$

$\Rightarrow f(x)$ is continuous at $x = \frac{3}{2}$

Q10

$f(x) = -\left(x - \frac{1}{2}\right)^2 + \left(p + \frac{1}{4}\right)$

If $g(x)$ is discontinuous at $x = \frac{1}{2}$, then $f\left(\frac{1}{2}\right)$ should be an integer.

Hence, p cannot be $\frac{1}{2}$

Q11

$f(x)=[x]\left(\left\{x^2\right\}+\left[x^2\right]\right)+\{x\}\left(\left[x^2\right]+\left\{x^2\right\}\right)$

$=([x]+\{x\})\left(\left[x^2\right]+\left\{x^2\right\}\right)$

$=x.x^2=x^3$

Hence, $f(x)$ is continuous $\forall x \in[0,10]$

Q12

$\text{L. H. L.}=\lim_{h\to 0}\left[(2-h)^2\right]+sgn(2-h)$

$=3+1=4$

$f(2)=4+1=5$

$\therefore f(x)$ is discontinuous.

Q13

$f'\left(3^+\right)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$

$=\lim_{h\to 0}\frac{\left(2-e^h\right)-1}{h}=-\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)=-1$

$f'\left(3^-\right)=\lim_{h\to 0}\frac{f(3-h)-f(3)}{-h}$

$=\lim_{h\to 0}\frac{\sqrt{10-(3-h)^2}-1}{-h}=-\lim_{h\to 0}\frac{\sqrt{1+\left(6h-h^2\right)}-1}{-h}$

$=\lim_{h\to 0}\frac{6h-h^2}{-h\left(\sqrt{1+6h-h^2}+1\right)}=\frac{-6}{2}=-3$

Hence, $f'\left(3^+\right)\neq f\left(3^-\right)$

Q14

Clearly, $x=1$ is a point of discontinuity of the function $f(x)=\frac{1}{1-x}$.

if $x\neq 1$, then

$(fof)(x) = f[f(x)] = f\left(\frac{1}{1-x}\right) = \frac{x-1}{x}$, which is discontinuous at $x = 0$.

If $x \neq 0$ and $x \neq 1$, then

$(fo\ fof)(x) = f[(fof)(x)] = f\left(\frac{x-1}{x}\right) = x$

Which is continuous everywhere.

Hence, $f^{30}(x) = x$, which is continuous everywhere.

So, the only points of discontinuity are $x = 0$ and $x = 1$

Q15

$\mu(x) = \frac{1}{x-1}$, which is discontinous at $x = 1$

$f(u) = \frac{1}{u^2+u-2} = \frac{1}{(u+2)(u-1)}$

which is discontinous at $u = -2,\ 1$

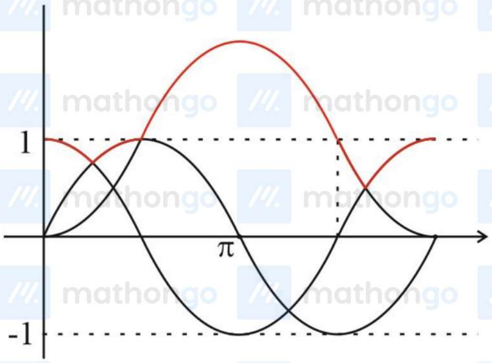
when $u = -2$, then $\frac{1}{x-1} = -2 \Rightarrow x = \frac{1}{2}$

when $u = 1$, then $\frac{1}{x-1} = 1 \Rightarrow x = 2$

Hence given composite function is discontinous at three points, $x = 1,\ \frac{1}{2}$ and 2 .

Q16

The graph of $\mathrm{f(x)} = \max.\{\sin \mathrm{x}, \cos \mathrm{x}, 1 - \cos \mathrm{x}\}$ is



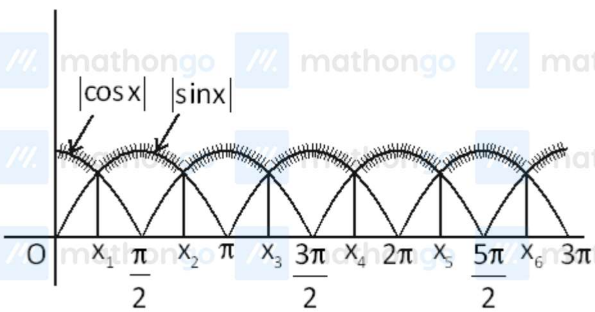
$\Rightarrow$ f(x) is not differentiable at x $= \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{3}$.

$\Rightarrow$ f(x) is not differentiable at 3 points

Q17

Using graph of $|\sin x| \& |\cos x|$, we get,

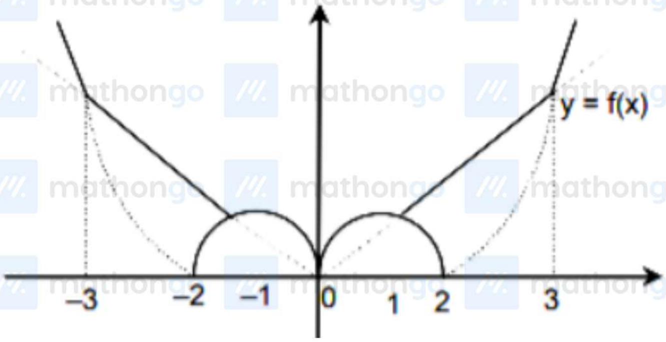
$f(x)$ is non differentiable at



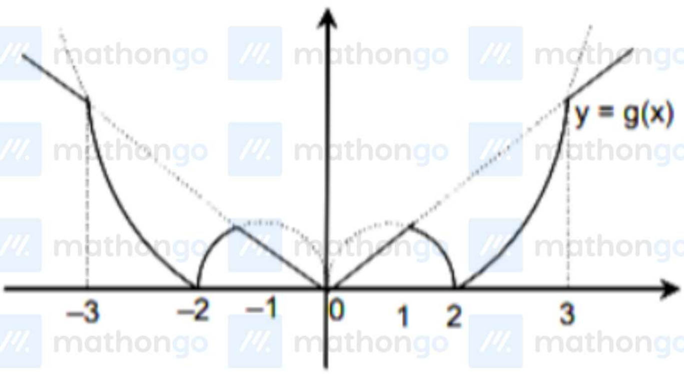
$x_1 = \frac{\pi}{4},\ x_2 = \frac{3\pi}{4},\ x_3 = \frac{5\pi}{4}, x_4 = \frac{7\pi}{4},\ x_5 = \frac{9\pi}{4},\ x_6 = \frac{11\pi}{4}$

Hence, $\lambda = 6$

Q18



$\mathrm{f(x)}$ is not differentiable function at $x=-3,-1,0,1,3$



$g(x)$ is not differentialble at $\mathrm{x}=-3,-2,-1,0,1,2,3$

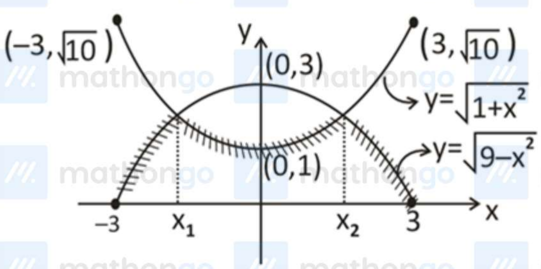
Q19

Using graph of

$\mathrm{y}=\sqrt{9-\mathrm{x}^2} \Rightarrow \mathrm{x}^2+\mathrm{y}^2=9$,

$\mathrm{y}=\sqrt{1+\mathrm{x}^2} \Rightarrow \mathrm{y}^2-\mathrm{x}^2=1$

In the figure, we can see the graph of $\mathrm{f(x)}$



Clearly, $\mathrm{f(x)}$ is non-differentiable at

$\mathrm{x}=-3,\ \mathrm{x}_1,\ \mathrm{x}_2,\ 3$ (4 points)

because there are sharp points at $\mathrm{x}_1,\ \mathrm{x}_2$ and at $\mathrm{x}=3$, $\mathrm{x}=-3$ there are vertical tangents.

Q20

For $g(x)$ to be continuous and differentiable $\forall x \in R,\ f(x)$ must be continuous and differentiable $\forall x \in R$

Since, $f(x)$ is continuous for all x, therefore, it is continuous at $x=1$ also.

$\therefore f(1)=\lim_{h\to 0^+} f(1-h)=1=\lim_{h\to 0^+}\left[a(1-h)^2+b\right]$

$\Rightarrow a+b=1 \ldots (1)$

Also, $f(x)$ is differentiable at $x=1 \Rightarrow f'\left(1^-\right)=f'\left(1^+\right)$

$\Rightarrow \lim_{h\to 0^+}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}$

$\Rightarrow \lim_{h\to 0^+}\frac{a(1-h)^2+b-1}{-h}=\lim_{h\to 0^+}\frac{\frac{1}{|1+h|}-1}{h}$

$\Rightarrow \lim_{h\to 0^+}\frac{(a+b-1)+\left(h^2-2h\right)a}{-h}=\lim_{h\to 0^+}\frac{1-1-h}{h(1+h)}$

$\Rightarrow 2a=-1$ (Using $a+b=1$)

$\therefore a=\frac{-1}{2}$

Hence, $a+b=1 \Rightarrow b=1-a=\frac{3}{2}$.

Q21

$f(x)=n+[p\sin x]$

As we know the greatest integer function is discontinuous at integer.

So, $f(x)$ is discontinuous when $p\sin x$ is integer.

In $\left(0,\ \frac{\pi}{2}\right)$. $f(x)$ is discontinuous when $p\sin x=1,\ 2,\ \ldots\ldots.p-1$

and in $\left(\frac{\pi}{2},\ \pi\right)$, $f(x)$ is discontinuous when $p\sin x=1,\ 2,\ \ldots\ldots.p-1$

and at $\frac{\pi}{2}$, $p\sin x=p$

So total number of points of discontinuity is $p-1+p-1+1=2p-1$

As we know function is not differentiable at a point when function is not continuous at that point.

So total number of point of where $f(x)$ is not differentiable is $2p-1$

Q22

The curve will have sharp corner at $x = 0.5$ and at $x = 1$ and a point of discontinuity at $x=\frac{\pi}{2}$.

Hence, total number of points where the function is not differentiable is 3.

Q23

$f(f(x))=10-|f(x)-5|=10-|10-|x-5|-5|$
$=10-|5-|x-5||\Rightarrow 10-||x-5|-5|$

Points where $f(f(x))$ is non-differentiable are

$x-5=0$ & $|x-5|=5$
$x=5$ & $x=0,10$

Q24

Given, $f(x)=\begin{cases}-2, & -2\le x<0\\ x^2-2, & 0\le x\le 2\end{cases}$

$\therefore |f(x)|=\begin{cases}2, & -2\le x<0\\ 2-x^2, & 0\le x<\sqrt{2}\\ x^2-2, & \sqrt{2}\le x\le 2\end{cases}$

And $f(|x|)=x^2-2,\ x\in[-2,2]$

$\therefore g(x)=\begin{cases}x^2, & -2\le x<0\\ 0, & 0\le x<\sqrt{2}\\ 2\left(x^2-2\right), & \sqrt{2}\le x\le 2\end{cases}$

$g'(x)=\begin{cases}2x & -2<x<0\\ 0 & 0<x<\sqrt{2}\\ 4x & \sqrt{2}<x<2\end{cases}$

$g'\left(0^{-}\right)=0=g'\left(0^{+}\right)$

$g'\left(\sqrt{2}^{-}\right)=0, g'\left(\sqrt{2}^{+}\right)=4\sqrt{2}$

So, $g(x)$ is not differentiable at $x=\sqrt{2}$

Q25

$\because\ x-2\ \&\ \left|x^2-3x+2\right|$ both are continuous $\forall x\in R$

$\therefore f(x)$ is continuous $\forall x\in R$

Now, $\left|x^2-3x+2\right|=|(x-1)(x-2)|$

$$\therefore\ f(x)=\begin{cases}(x-2)(x-1)(x-2) & : & x\in(-\infty,1)\cup(2,\infty)\\ -(x-2)(x-1)(x-2) & : & x\in[1,2]\end{cases}$$

$$f(x)=\begin{cases}(x-1)(x-2)^2 & : & x\in(-\infty,1)\cup(2,\infty)\\ -(x-1)(x-2)^2 & : & x\in[1,2]\end{cases}$$

$$f'(x)=\begin{cases}(x-1)2(x-2)+(x-2)^2 & : & x\in(-\infty,1)\cup(2,\infty)\\ -(x-1)2(x-2)-(x-2)^2 & : & x\in(1,2)\end{cases}$$

Clearly, $f'\left(2^{-}\right)=f'\left(2^{+}\right)=0\Rightarrow f(x)$ is differentiable at $x=2$

and $f'\left(1^{-}\right)=1, f'\left(1^{+}\right)=-1\Rightarrow f(x)$ is non-differentiable at $x=1$

Q26

$x=2020\pi, 2\pi$ are points of non-differentiability

Q27

$$\mathrm{g(x)}=\begin{cases}\frac{\mathrm{ax^2+bx}}{(\mathrm{cotx})^{\mathrm{n}}}+\mathrm{c}\\ \frac{4}{1+\frac{4}{(\mathrm{cotx})^{\mathrm{n}}}}=\mathrm{c}\quad\mathrm{if}\quad\mathrm{x}<\frac{\pi}{4}\\ \frac{\frac{\mathrm{sinx+cosx+1}}{(\mathrm{tanx})^{\mathrm{n}}}}{\frac{1}{(\mathrm{tanx})^{\mathrm{n}}}+\mathrm{c}}=\frac{1}{\mathrm{c}}\quad\mathrm{if}\quad\mathrm{x}>\frac{\pi}{4}\end{cases}$$

limit to exist $\mathrm{c}=\frac{1}{\mathrm{c}}\Rightarrow\mathrm{c}^2=1,\ \mathrm{c}=\pm1$

Q28

$f(x) = \cos\ x$

$f'(x) = -\sin\ x < 0$

So, function is decreasing in x $\in [0, \pi]$

$g(x)=\begin{cases} f(x), & x \in [0,\pi] \\ \sin\ x - 1, & x > \pi \end{cases}$

$\therefore$ L. H. L. = R. H. L. $= f(\pi) = -1$.

Q29

$f'(x)= \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h \to 0} \frac{f(x)+f(h)+2xh-f(x)}{h}$

$f'(x)= \lim\limits_{h \to 0} \left(\frac{f(h)}{h} + 2x\right)= 210 + 2x$

$f(x)= 210x + x^2 + C$

$f(0)= 0 \Rightarrow C = 0$

$f(x)= 210x + x^2$

$\therefore f(2)= 424$

Q30

We have, $f\left(\frac{x+y}{3}\right)= \frac{f(x)+f(y)}{3}$, $f(0)= 0$ and $f'(0)= 5$

$f'(x)= \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h \to 0} \frac{f\left(\frac{3x+3h}{3}\right)-f\left(\frac{3x+0}{3}\right)}{h}$

$= \lim\limits_{h \to 0} \frac{\frac{f(3x)+f(3h)}{3}-\frac{f(3x)+f(0)}{3}}{h}$

Hints and Solutions **MathonGo**

$= \lim_{h \to 0} \frac{f(3h)-f(0)}{3h} = f'(0) = 5$

$\therefore f(x) = 5x + c$

$\therefore f(0) = 0 \Rightarrow c = 0$

Hence, $f(x) = 5x$