Q1

If f(x) = $\begin{cases} \lambda\sqrt{2x+3}, & 0 \le x \le 3 \\ \mu x + 12, & 3 < x \le 9 \end{cases}$ is differentiable at $x = 3$, then the value of $\lambda + \mu$ is equal to

Q2

Let $f(x) = \begin{cases} \frac{1}{x^2} & : |x| \ge 1 \\ \alpha x^2 + \beta & : |x| < 1 \end{cases}$. If $f(x)$ is continuous and differentiable at any point, then

(1) $\alpha = 2$, $\beta =$ - 1

(2) $\alpha =$ - 1 , $\beta = 2$

(3) $\alpha = 1$, $\beta = 0$

(4) $\alpha =$ - 2 , $\beta = 3$

Q3

If $f(x) = \begin{cases} \frac{\sqrt{4+ax} - \sqrt{4-ax}}{x}, & -1 \le x < 0 \\ \frac{3x+2}{x-8}, & 0 \le x \le 1 \end{cases}$ is continuous in $[-1, 1]$, then the value of a is

(1) 1

(2) - 1

(3) $\frac{1}{2}$

(4) - $\frac{1}{2}$

Q4

The value of f(0) so that the function f(x) = $\frac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere is k, then value of 10 k is

Q5

Let $f(x) = \begin{cases} \left(\frac{1-\cos x}{(2\pi-x)^2}\right)\left(\frac{\sin^2 x}{\log\left(1+4\pi^2-4\pi x+x^2\right)}\right) & : x \neq 2\pi \\ \lambda & : x = 2\pi \end{cases}$ is continuous at $x = 2\pi$, then the value of λ is equal to

Q6

If $f(x) = \begin{cases} \frac{e^{[x]+|x|}-1}{[x]+|x|} & : x \neq 0 \\ -1 & : x = 0 \end{cases}$ (where [.] denotes the greatest integer function), then

(1) $f(x)$ is continuous at $x = 0$

(2) $\lim_{x\to 0^+} f(x) = -1$

(3) $\lim_{x\to 0^-} f(x) = 1$

(4) $\lim_{x\to 0^+} f(x) = 1$

Q7

Let $f(x) = \begin{cases} (1+|\sin x|)^{\frac{l}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \\ m, & x = 0 \end{cases}$ is continuous at $x = 0$. Then, the values of l and m are

(1) $l = -\frac{2}{3}, m = e^{\frac{2}{3}}$

(2) $l = \frac{2}{3}, m = e^{-\frac{2}{3}}$

(3) $l = \frac{2}{3}, m = e^{\frac{2}{3}}$

(4) None of these

Q8

The function f(x) = {x} sin (π[x]), where [.] denotes the greatest integer function and {.} is the fractional part function, is discontinuous at

(1) all x

(2) all integer points

(3) no x

(4) x which is not an integer

Q9

The function $f(x) = \lim_{n \to \infty} \cos^{2n}(\pi x) + [x]$ is (where, [.] denotes the greatest integer function and $n \in N$)

(1) continuous at $x = 1$ but discontinuous at $x = \frac{3}{2}$

(2) continuous at $x = 1$ and $x = \frac{3}{2}$

(3) discontinuous at $x = 1$ and $x = \frac{3}{2}$

(4) discontinuous at $x = 1$ but continuous at $x = \frac{3}{2}$

Q10

Let $f(x) = - x^2 + x + p,$ where p is a real number. If $g(x) = [f(x)]$ and $g(x)$ is discontinuous at $x = \frac{1}{2}$, then p cannot be (where [.] represents the greatest integer function)

(1) $\frac{1}{2}$

(2) $\frac{3}{4}$

(3) $\frac{7}{4}$

(4) $-\frac{1}{4}$

Q11

Let $f(x) = [x]\left\{x^2\right\} + [x]\left[x^2\right] + \{x\}\left[x^2\right] + \{x\}\left\{x^2\right\}$, $\forall x \in [0,10]$ (where $[\cdot]$ and $\{\cdot\}$ are the greatest integer and fractional part functions respectively). The number of points of discontinuity of $f(x)$ is

Q12

Consider function $f(x) = \begin{cases} \frac{\tan\{2x-3\}}{x-2}, & x \in (2, \infty) \\ \left[x^2\right] + sgn(x), & x \in (-\infty, 2] \end{cases}$, then at $x = 2$

[Note: {k} & [k] denote fractional part & greatest integer function less than or equal to k respectively and sgn denotes signum part of function.]

(1) $f(x)$ is continuous

(2) $f(x)$ is discontinuous

(3) $f(x)$ is differentiable, but $f'(x)$ is discontinuous

(4) None of these

Q13

Given $f(x) = \begin{cases} \sqrt{10 - x^2} & \text{if } -3 < x < 3 \\ 2 - e^{x-3} & \text{if } x \geq 3 \end{cases}$ The graph of $f(x)$ is -

(1) continuous and differentiable at $x = 3$

(2) continuous but not differentiable at $x = 3$

(3) differentiable but not continuous at $x = 3$

(4) neither differentiable nor continuous at $x = 3$

Q14

If $f(x) = \frac{1}{1-x}$, then the points of discontinuity of the function $f^{30}(x)$ where $f^{n}(x) = fof......of$ (n times) are

(1) $x = 2,1$

(2) $x = 0,1$

(3) $x = 1,2$

(4) no points of discontinuity

Q15

Let f be a composite function of x defined by

$f(u) = \frac{1}{u^2+u-2}$, $u(x) = \frac{1}{x-1}$.

Then the number of points x where f is discontinuous is :

(1) 4

(2) 3

(3) 2

(4) 1

Q16

In $(0, 2\pi)$, the total number of points where f(x) = max. {sinx, cosx, 1– cosx } is not differentiable, are equal to

(1) 3

(2) 4

(3) 5

(4) 6

Q17

Consider the function $f(x) = \max\{|\sin x|, |\cos x|\}, \ \forall x \in [0,3\pi]$. If λ is the number of points at which $f(x)$ is non-differentiable, then the value of $\frac{\lambda^3}{5}$ is

Q18

Let $f(x) = \max.\left\{\left|x^2 - 2|x|\right|, |x|\right\}$ and $g(x) = \min.\left\{\left|x^2 - 2|x|\right|, |x|\right\}$ then if $f(x)$ is not differentiable at 'p' number of points and g(x) is non differentiable at 'q' number of points, then find |p - q|.

Q19

If $f(x) = \min\left\{\sqrt{9 - x^2}, \sqrt{1 + x^2}\right\}, \ \forall x \in [-3,3]$, then the number of point(s) where f(x) is non-differentiable is/are

(1) 4

(2) 3

(3) 2

(4) 0

Q20

Consider a function $g(x) = f(x - 2),\ \ \forall x \in R,$ where $f(x) = \begin{cases} \frac{1}{|x|} & : & |x| \geq 1 \\ ax^2 + b & : & |x| < 1 \end{cases}$. If $g(x)$ is continuous as well as differentiable for all x, then

(1) $a = \frac{-1}{2}, b = \frac{3}{2}$

(2) $a = \frac{1}{2}, b = \frac{3}{2}$

(3) $a = \frac{-1}{2}, b = \frac{-3}{2}$

(4) None of these

Q21

Let $f(x) = [n + p\sin x], x \in (0, \pi), n \in I$ and p is a prime number. The number of points where $f(x)$ is not differentiable is

(Here $[x]$ represents the greatest integer less than or equal to x)

(1) $p - 1$

(2) $p + 1$

(3) $2p + 1$

(4) $2p - 1$

Q22

The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ is not differentiable in the interval $(0,\ 2)$ is/are

(1) 1

(2) 2

(3) 3

(4) 4

Q23

Let $f(x) = 10 - |x - 5|, x \in R$, then the set of all values of x at which $f(f(x))$ is not differentiable is

(1) {0,5, 10}

(2) {5,10}

(3) {0,5, 10,15}

(4) {5,10,15}

Q24

Consider $f(x) = \begin{cases} -2, & -2 \leq x < 0 \\ x^2 - 2, & 0 \leq x \leq 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2)$, $g(x)$ is

(1) not differentiable at one point

(2) differentiable at all points

(3) not continuous

(4) not differentiable at two points

Q25

Consider the function $f(x) = (x - 2)\left|x^2 - 3x + 2\right|$, then the incorrect statement is

(1) $f(x)$ is continuous at $x = 1$

(2) $f(x)$ is continuous at $x = 2$

(3) $f(x)$ is differentiable at $x = 1$

(4) $f(x)$ is differentiable at $x = 2$

Q26

The number of points where the function, $f(x) = \cos\left|2018\pi - x\right| + \sin\left|2020\pi - x\right| + (x - \pi)\left|x^2 - 3\pi x + 2\pi^2\right|$ is non-differentiable is/are

(1) 0

(2) 1

(3) 2

(4) 3

Q27

Let $g(x) = \begin{cases} \dfrac{ax^2 + bx + c(\cot x)^n}{4 + (\cot x)^n}, & x \in \left(0, \dfrac{\pi}{4}\right) \\ 1, & \text{at } x = \dfrac{\pi}{4} \\ \dfrac{\sin x + \cos x + (\tan x)^n}{1 + c(\tan x)^n}, & x \in \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right) \end{cases}$, where a, b, c are real constants and

$f(x) = \lim\limits_{n \to \infty} g(x)$ If $\lim\limits_{x \to \frac{\pi}{4}} f(x)$ exists, then c may be equal to

(1) 2

(2) $\frac{1}{2}$

(3) 3

(4) -1

Q28

Let $f\left(x\right) = \cos x, g\left(x\right) = \begin{cases} \min. \{f(t): 0 \leq t \leq x\}, & x \in [0, \pi] \\ \sin x - 1, & x > \pi \end{cases}$ then

(1) $g(x)$ is discontinuous at $x = \pi$

(2) $g(x)$ is continuous for $x \in (0, \infty)$

(3) $g(x)$ is differentiable at $x = \pi$

(4) $g(x)$ is differentiable for $x \geq 0$

Q29

Let $f(x)$ is a differentiable function such that $f(x + y) = f(x) + f(y) + 2xy \, \forall x, \; y \in R$ and $\lim_{x \to 0} \frac{f(x)}{x} = 210$, then $f(2)$ is equal to

(1) 20

(2) 105

(3) 424

(4) none of these

Q30

Let $f: R \to R$ be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}, \; f(0) = 0$ and $f'(0) = 5, \;$ then

(1) $f(x)$ is a quadratic function

(2) $f(x)$ is continuous but not differentiable

(3) $f(x)$ is differentiable in R

(4) $f(x)$ is bounded in R

Answer Key

Q1 (8)	**Q2** (2)	**Q3** (4)	**Q4** (1.25)
Q5 (0.5)	**Q6** (4)	**Q7** (3)	**Q8** (3)
Q9 (4)	**Q10** (1)	**Q11** (0)	**Q12** (2)
Q13 (2)	**Q14** (2)	**Q15** (2)	**Q16** (1)
Q17 (43.20)	**Q18** (2)	**Q19** (1)	**Q20** (1)
Q21 (4)	**Q22** (3)	**Q23** (1)	**Q24** (1)
Q25 (3)	**Q26** (3)	**Q27** (4)	**Q28** (2)
Q29 (3)	**Q30** (3)		