

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q1**

The domain of definition of the function $f(x) = \sqrt{\log_{(|x|-1)}(x^2 + 4x + 4)}$, is

- (1) $[-3, -1] \cup [1, 2]$
- (2) $(-2, -1) \cup [2, \infty)$
- (3) $(-\infty, -3] \cup (-2, -1) \cup (2, \infty)$
- (4) $[-2, -1] \cup [2, \infty)$

Q2

If domain of $f(x)$ is $[1, 3]$, then find the domain of $f(\log_2(x^2 + 3x - 2))$

- (1) $[-5, -4] \cup [1, 2]$
- (2) $\left[-13, -2\right] \cup \left[\frac{3}{2}, 5\right]$
- (3) $[-4, 1] \cup [2, 7]$
- (4) $[-3, 2]$

Q3

If $f(x) = \tan^{-1}\sqrt{x^2 + 4x} + \sin^{-1}\sqrt{x^2 + 4x + 1}$, then

- (1) domain of $f(x)$ contains 3 integers only
- (2) range of $f(x)$ has two elements only
- (3) $f(x)$ is a constant function $\forall x \in R$
- (4) $f(x)$ contains only two elements in its domain

Q4

Sample Task**Questions****Questions with Answer Keys****MathonGo**

If $f(x) = \frac{x^2 - [x^2]}{1+x^2 - [x^2]}$ (where $[.]$ represents the greatest integer part of x), then the range of $f(x)$ is

- (1) $[0,1]$
- (2) $(-1,1)$
- (3) $(0, \infty)$

$$(4) \left[0, \frac{1}{2} \right)$$

Q5

If $f(x) = \sin^4 x + \cos^4 x - \frac{1}{2}\sin 2x$ then the range of $f(x)$ is

- (1) $\left[0, \frac{3}{2} \right]$
- (2) $\left[-\frac{1}{2}, \frac{7}{2} \right]$
- (3) $\left[0, \frac{9}{8} \right]$
- (4) $\left[\frac{3}{4}, \frac{7}{8} \right]$

Q6

Domain and range of the function $f(x) = \sqrt{\sin^{-1}(3x)} + \frac{\pi}{3}$ is $\left[\frac{a}{\sqrt{3}}, \frac{b}{3} \right]$ and $\left[c, d\sqrt{5\pi} \right]$ respectively, then $2a + b + c + 6d$ is equal to

- (1) 1
- (2) $2\sqrt{3}$

Sample Task**Questions****Questions with Answer Keys****MathonGo**(3) $\sqrt{6}$

(4) none of these

Q7

The number of integral value(s) of x which satisfying the equation $\left| \log_4(2x^2 - x) + \log_2(2 - x^2) + x^2 + 2x + 2 \right| = x^2 + 2x + 2 + \left| \log_4(2x^2 - x) \right| + \left| \log_2(2 - x^2) \right|$

(1) 0

(2) 1

(3) 2

(4) 4

Q8

$[x]$ and $\{x\}$ represent the greatest integer function and fractional part function respectively. Let

$$f(x) = [x] + \sum_{i=1}^{2020} \frac{\{x+r\}}{2020}$$
. Find the value of $f(-1000)$
Q9

The function $f(x) = \sec \left[\log \left(x + \sqrt{1+x^2} \right) \right]$ is

(1) an odd function

(2) an even function

(3) neither an odd nor an even function

(4) a constant function

Q10

Sample Task**Questions****Questions with Answer Keys****MathonGo**

Which of the following is a function whose graph is symmetrical about the origin?

(1) $f(x) = (2^x + 2^{-x})$

(2) $f(x) = [\log(x + \sqrt{1+x^2})]^2$

(3) $f(x+y) = f(x) + f(y) \forall x, y \in R$

(4) None of these

Q11

If the graph of the function $f(x) = ax^3 + x^2 + bx + c$ is symmetric about the line $x = 2$, then the value of $a + b$ is equal to

(1) 10

(2) -4

(3) 16

(4) -10

Q12

The fundamental period of the function $f(x) = |\sin x| + |\cos x|$ is

(1) π

(2) $\pi/2$

(3) 2 π

(4) None of these

Q13

Sample Task**Questions****Questions with Answer Keys****MathonGo**

A function f is defined as $f(x) = \frac{1}{2} \left(\frac{|\sin x|}{\cos x} + \frac{\sin x}{|\cos x|} \right)$. If the fundamental period of function f is $m\pi$, then the value of m is

Q14

If $f: R \rightarrow A$ defined as $f(x) = \tan^{-1} \left(\sqrt{4(x^2 + x + 1)} \right)$ is surjective, then A is equal to

(1) $\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$

(2) $\left[0, \frac{\pi}{2} \right)$

(3) $\left[\frac{\pi}{3}, \frac{\pi}{2} \right)$

(4) $\left(0, \frac{\pi}{3} \right]$

Q15

Which of the following function is surjective but not injective?

(1) $f: R \rightarrow R$, $f(x) = x^4 + 2x^3 - x^2 + 1$

(2) $f: R \rightarrow R$, $f(x) = x^3 + x + 1$

(3) $f: R \rightarrow R^+$, $f(x) = \sqrt{1+x^2}$

(4) $f: R \rightarrow R$, $f(x) = x^3 + 2x^2 - x + 1$

Q16

If $f: R \rightarrow R$ be defined as $f(x) = \frac{e^{2x} - e^{-2x}}{2}$, then

(1) f is many-one

Sample Task**Questions****Questions with Answer Keys****MathonGo**(2) f is into

$$(3) f^{-1}(x) = \frac{1}{2} \left[\log\left(x - \sqrt{x^2 + 1}\right) \right]$$

$$(4) f^{-1}(x) = \frac{1}{2} \left[\log\left(x + \sqrt{x^2 + 1}\right) \right]$$

Q17

If $f: R \rightarrow R$ be a function such that $f(x) = x^3 + x^2 + 4x + \sin x$. Then, the function $f(x)$ is

- (1) one-one and onto
- (2) one-one and into
- (3) many-one and onto
- (4) many-one and into

Q18

Let $f: R \rightarrow R$ be a function defined by

$$f(x) = \begin{cases} x + \frac{1}{x}, & x > 0 \\ e^x, & x \leq 0 \end{cases}$$

then f is

- (1) both one-one onto
- (2) one-one but not onto
- (3) onto but not one-one
- (4) neither one-one nor onto

Q19

Sample Task**Questions****Questions with Answer Keys****MathonGo**

A function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ is defined as $f(n) = \begin{cases} n+1 & n \in \text{odd integer} \\ \frac{n}{2} & n \in \text{even integer} \end{cases}$. If $k \in \mathbb{Z}$ is an odd integer and $f(f(f(k))) = 33$, then

the sum of the digits of k is

- (1) 7
- (2) 5
- (3) 9
- (4) 8

Q20

If $A = \{1, 2, 3, 4\}$ and $f: A \rightarrow A$, the total number of invertible functions, 'f', such that

$f(2) \neq 2$, $f(4) \neq 4$, $f(1) = 1$ is equal to

- (1) 1
- (2) 2
- (3) 3
- (4) None of these

Q21

Given two real sets $A = \{a_1, a_2, a_3, \dots, a_{2n}\}$ and $B = \{b_1, b_2, \dots, b_n\}$. If $f: A \rightarrow B$ is a function such that every element of B has an inverse image and $f(a_1) \leq f(a_2) \leq f(a_3) \leq f(a_4) \dots \leq f(a_{2n})$, then the number of such mappings are

- (1) ${}^{2n}C_n$
- (2) ${}^{2n}C_{n-1}$
- (3) ${}^{2n-1}C_{n-1}$

Sample Task**Questions****Questions with Answer Keys****MathonGo**

(4) ${}^{2n+1}C_n$

Q22

Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then the number of bijective functions $f: A \rightarrow A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

Q23

If $f: R \rightarrow R$, $f(x) = \frac{\sin([x]\pi)}{x^2+2x+3} + 2x - 1 + \sqrt{x(x-1)} + \frac{1}{4}$ (where $[x]$ denotes greatest integral value less than or

equal to x) denotes a function, then number of real solutions of equation $f(x) = f^{-1}(x)$ is

(1) 0

(2) 1

(3) 2

(4) 3

Q24

Equation $|x - 4| + |x + 4| = ax + 8$ has

(1) no solution if $a \in (-\infty, -2] \cup [2, \infty)$.(2) exactly one solution if $a \in (-2, 2)$.(3) exactly two solutions if $a \in (-2, 0) \cup (0, 2)$.(4) exactly two solutions if $a = 0$ **Q25**

The total number of solution(s) of the equation $2x + 3\tan x = \frac{5\pi}{2}$ in $x \in [0, 2\pi]$ is/are equal to

(1) 1

Sample Task**Questions****Questions with Answer Keys****MathonGo**

(2) 2

(3) 3

(4) 4

Q26

If $2f(xy) = (f(x))^y + (f(y))^x$ for all $x, y \in R$ and $f(1) = 3$, then the value of $\sum_{r=1}^{10} f(r)$ is equal to

(1) $\frac{3}{2}(3^{10} - 1)$

(2) $\frac{3}{2}(3^9 - 1)$

(3) $\frac{3^{10} - 1}{2}$

(4) $\frac{1}{2}(3^{10} - 1)$

Q27

Let f be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$ where $x \in R - \{0, 1\}$, then the value of $f(2)$ must be

Q28

Let $f(x)$ be a function defined as $f: R \rightarrow R$ such that $f(x+2) + f(x-2) = f(x)$ and $f(1) = 3$ then the value of the expression $\sum_{r=0}^{15} f(1+12r)$ is equal to

Q29

If $f(x+y) = f(x) + f(y) - xy - 1$, $\forall x, y \in R$ and $f(1) = 1$, then the number of solutions of $f(n) = n$, $n \in N$, is

(1) one

(2) two

(3) no solution

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(4) three

Q30

If $f(x)$ is a real valued function such that $f(x+6) - f(x+3) + f(x) = 0$, $\forall x \in R$, then period of $f(x)$ is

(1) 6

(2) 12

(3) 18

(4) 24

Sample Task**Questions****Questions with Answer Keys****MathonGo****Answer Key****Q1 (3)****Q2 (1)****Q3 (4)****Q4 (4)****Q5 (3)****Q6 (3)****Q7 (3)****Q8 (10.5)****Q9 (2)****Q10 (3)****Q11 (2)****Q12 (2)****Q13 (2)****Q14 (3)****Q15 (4)****Q16 (4)****Q17 (1)****Q18 (4)****Q19 (2)****Q20 (3)****Q21 (3)****Q22 (720)****Q23 (2)****Q24 (3)****Q25 (3)****Q26 (1)****Q27 (3)****Q28 (48.00)****Q29 (1)****Q30 (3)**

Sample Task**Solutions****Hints and Solutions****MathonGo****Q1**

$$\log_{(|x|-1)}(x^2 + 4x + 4) \geq 0$$

Case 1: $0 < |x| - 1 < 1$ i.e., $1 < |x| < 2$, then

$$0 < x^2 + 4x + 4 \leq 1 \Rightarrow x^2 + 4x + 3 \leq 0 \text{ & } (x+2)^2 > 0$$

$$\Rightarrow -3 \leq x \leq -1 \text{ & } x \neq -2$$

So, $x \in (-2, -1)$ Case 2: $|x| - 1 > 1$ i.e., $|x| > 2$

$$x^2 + 4x + 4 \geq 1 \Rightarrow (x+1)(x+3) \geq 0 \Rightarrow x \in (-\infty, -3] \cup [-1, \infty)$$

$$\Rightarrow x \in (-\infty, -3] \cup (2, \infty)$$

Hence, domain is $(-\infty, -3] \cup (-2, -1) \cup (2, \infty)$ **Q2**

$$1 \leq \log_2(x^2 + 3x - 2) \leq 3 \Rightarrow 2 \leq (x^2 + 3x - 2) \leq 8$$

On solving them, we get $-5 \leq x \leq -4$ and $1 \leq x \leq 2$

Sample Task**Solutions****Hints and Solutions****MathonGo****Q3**

$$\therefore x^2 + 4x \geq 0 \text{ for the first term}$$

$$\text{and } 0 \leq x^2 + 4x + 1 \leq 1 \text{ for the second term}$$

\therefore by both the results, there is only one possibility

$$x^2 + 4x = 0$$

$$\Rightarrow x = 0, -4$$

$$\therefore f(x) = \tan^{-1}(0) + \sin^{-1}(1) = \frac{\pi}{2}$$

Hence, $f(x)$ contains only two elements in its domain

Q4

$$x^2 - [x^2] = \{x^2\}$$

$$\therefore f(x) = \frac{\{x^2\}}{1+\{x^2\}} = \frac{1+\{x^2\}-1}{1+\{x^2\}} = 1 - \frac{1}{1+\{x^2\}}$$

$$\therefore 0 \leq \{x^2\} < 1 \Rightarrow 1 \leq \{x^2\} + 1 < 2$$

$$\frac{1}{2} < \frac{1}{\{x^2\}+1} \leq 1 \Rightarrow -\frac{1}{2} > \frac{-1}{1+\{x^2\}} \geq -1$$

$$\frac{1}{2} > 1 + \frac{(-1)}{1+\{x^2\}} \geq 0 \Rightarrow \text{Range of } f(x) \in \left[0, \frac{1}{2}\right)$$

Q5

$$\text{Let } f(x) = \sin^4 x - \sin x \cos x + \cos^4 x$$

$$f(x) = 1 - \frac{1}{2}\sin 2x - \frac{1}{2}\sin^2 2x$$

$$\Rightarrow f(x) = g(t) = 1 - \frac{1}{2}t - \frac{1}{2}t^2 \quad t \in [-1, 1]$$

$$\text{Hence } f(x) \in \left[0, \frac{9}{8}\right]$$

Sample Task**Solutions****Hints and Solutions****MathonGo****Q6**

$$\sin^{-1}(3x) + \frac{\pi}{3} \geq 0 \text{ and } 3x \in [-1, 1]$$

$$\sin^{-1}(3x) \geq -\frac{\pi}{3}$$

$$1 \geq 3x \geq -\frac{\sqrt{3}}{2}$$

$$\frac{1}{3} \geq x \geq -\frac{\sqrt{3}}{6}$$

domain is $\left[\frac{-1}{2\sqrt{3}}, \frac{1}{3} \right] \therefore a = \frac{-1}{2} \text{ & } b = 1.$

For Range put $x = \frac{-1}{2\sqrt{3}}$ and $x = \frac{1}{3}$

$$\text{at } x = \frac{-1}{2\sqrt{3}}, b\left(\frac{-1}{2\sqrt{3}}\right) = 0$$

$$\text{at } x = \frac{1}{3}, b\left(\frac{1}{3}\right) = \sqrt{\frac{5\pi}{6}}$$

$$\text{So } C = 0 \text{ and } d = \frac{1}{\sqrt{6}}$$

$$2a + b + c + 6d = \sqrt{6}$$

Q7

Since $|a + b| = |a| + |b| \Rightarrow ab \geq 0$

$$\text{So, } \log_4(2x^2 - x) \geq 0 \Rightarrow 2x^2 - x \geq 1$$

$$\Rightarrow (2x+1)(x-1) \geq 0 \Rightarrow x \leq -\frac{1}{2} \text{ or } x \geq 1$$

$$\text{and } \log_2(2-x^2) \geq 0 \Rightarrow 2-x^2 \geq 1$$

$$\Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$$

Hence, $x \in \left[-1, \frac{-1}{2}\right] \cup \{1\}$

i.e. 2 integral values.

Q8

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$\begin{aligned}
 f(x) &= [x] + \sum_{r=1}^{2020} \frac{x+r}{2020} \\
 &= [x] + \sum_{r=1}^{2020} \frac{x+r}{2020} \\
 &= [x] + \frac{2020}{2020} \{x\} + \frac{1+2+3+\dots+2020}{2020} \\
 &= [x] + \{x\} + \frac{2020 \times 2021}{2 \times 2020} \\
 &= x + \frac{2021}{2}
 \end{aligned}$$

Q9

$$f(-x) = \sec(\log(-x + \sqrt{1+x^2}))$$

$$= \sec(\log(x + \sqrt{1+x^2})^{-1})$$

$$\left(\because \sqrt{1+x^2} - x = \frac{1}{\sqrt{1+x^2}+x} \right)$$

$$= \sec(-\log(x + \sqrt{1+x^2}))$$

$$= \sec(\log(x + \sqrt{1+x^2})) \quad (\because \sec(-\theta) = \sec(\theta))$$

$$= f(x)$$

Hence $f(x)$ is an even function.

Q10

A function whose graph is symmetrical about the origin must be odd.

$(2^x + 2^{-x})$ is an even function.

Since, $\log(x + \sqrt{1+x^2})$ is an odd function,

Sample Task**Solutions****Hints and Solutions****MathonGo**

$\therefore [\log(x + \sqrt{1 + x^2})]^2$ is an even function.

If $f(x + y) = f(x) + f(y) \forall x, y \in R$, then

Put $x = y = 0 \Rightarrow f(0) = 0$

Now, put $y = -x \Rightarrow f(x) + f(-x) = 0$

$\therefore f(x)$ is an odd function

Q11

$f(x)$ is symmetric about the line $x = 2$

$\therefore f(2 + x) = f(2 - x)$

$$a(2 + x)^3 + (2 + x)^2 + b(2 + x) + c = a(2 - x)^3 + (2 - x)^2 + b(2 - x) + c$$

$$a\{(2 + x)^3 - (2 - x)^3\} + \{(2 + x)^2 - (2 - x)^2\} + b\{(2 + x) - (2 - x)\} = 0$$

$$a\{(8 + 12x + 6x^2 + x^3) - (8 - 12x + 6x^2 - x^3)\} + 2(4x) + b(2x) = 0$$

$$a(24x + 2x^3) + 8x + 2bx = 0$$

$$2ax^3 + (24a + 2b + 8)x = 0$$

Which must be true $\forall x \in R$

\therefore it is an identity

$$\therefore 2a = 0, 24a + 2b + 8 = 0$$

$$a = 0, 2b + 8 = 0$$

$$b = -4$$

Q12

$$f\left(\frac{\pi}{2} + x\right) = |\sin\left(\frac{\pi}{2} + x\right)| + |\cos\left(\frac{\pi}{2} + x\right)|$$

$$= |\cos x| + |\sin x| \text{ for all } x.$$

Hence, $f(x)$ is periodic with period $\frac{\pi}{2}$.

Sample Task**Solutions****Hints and Solutions****MathonGo****Q13**

$$f(x) = \frac{1}{2} \left(\frac{|\sin x|}{\cos x} + \frac{\sin x}{|\cos x|} \right)$$

$$= \frac{1}{2} \left(\frac{\sin x}{\cos x} + \frac{\sin x}{\cos x} \right), 0 \leq x < \frac{\pi}{2}$$

$$= \frac{1}{2} \left(\frac{\sin x}{\cos x} - \frac{\sin x}{\cos x} \right), \frac{\pi}{2} \leq x < \pi$$

$$= \frac{1}{2} \left(\frac{-\sin x}{\cos x} + \frac{\sin x}{\cos x} \right), \frac{3\pi}{2} \leq x < 2\pi$$

$$\Rightarrow f(x) = \tan x, 0 \leq x < \frac{\pi}{2}$$

$$= 0, \frac{\pi}{2} \leq x < \pi$$

$$= -\tan x, \pi \leq x < \frac{3\pi}{2}$$

$$= 0, \frac{3\pi}{2} \leq x < 2\pi$$

$\Rightarrow f$ is periodic with fundamental period $2\pi \Rightarrow m = 2$

Q14

$$\because x^2 + x + 1 = \left(x + \frac{1}{2} \right)^2 + \frac{3}{4}$$

$$\therefore 4(x^2 + x + 1) \geq 3$$

$$\Rightarrow \sqrt{4(x^2 + x + 1)} \geq \sqrt{3}$$

$$\Rightarrow \tan^{-1} \left(\sqrt{4(x^2 + x + 1)} \right) \geq \tan^{-1} \left(\sqrt{3} \right)$$

$$\Rightarrow f(x) \geq \frac{\pi}{3}$$

$$\therefore \text{Range of } f(x) \text{ is } \left[\frac{\pi}{3}, \frac{\pi}{2} \right)$$

Sample Task**Solutions****Hints and Solutions****MathonGo****Q15**

$f(x) = x^4 + 2x^3 - x^2 + 1$ is polynomial of even degree hence its range can't be \mathbb{R} . Hence not surjective.

$f(x) = x^3 + x + 1$, $f'(x) = 3x^2 + 1 \Rightarrow$ monotonic hence bijective.

$f(x) = \sqrt{1 + x^2}$, neither surjective nor injective

$f(x) = x^3 + 2x^2 - x + 1$, $f'(x) = 3x^2 + 4x - 1 \Rightarrow D = 16 + 12 > 0$

Hence $f(x)$ is non-monotonic cubic polynomial hence surjective but not injective.

Q16

Let us check for invertibility of $f(x)$

(A) one-one: we have, $f(x) = \frac{e^{2x} - e^{-2x}}{2}$
 $\Rightarrow f'(x) = \frac{e^{4x} + 1}{e^{2x}}$, which is strictly increasing as $e^{4x} > 0$ for all x .

Thus, f is one-one

(B) Onto: Let $y = f(x)$

$\Rightarrow \frac{dy}{dx} = e^{2x} + e^{-2x}$, where y is strictly monotonic

Hence, the range of $f(x) = (f(-\infty), f(\infty))$

\Rightarrow range of $f(x) = (-\infty, \infty)$

So, the range of $f(x)$ = co-domain

Hence, $f(x)$ is one-one and onto

(C) To find f^{-1} : $y = \frac{e^{4x} - 1}{2e^{2x}}$

$$\Rightarrow e^{4x} - 2e^{2x}y - 1 = 0$$

$$\Rightarrow e^{2x} = \frac{2y \pm \sqrt{4y^2 + 4}}{2}$$

$$\Rightarrow 2x = \log(y \pm \sqrt{y^2 + 1})$$

$$\Rightarrow f^{-1}(y) = \frac{\log(y \pm \sqrt{y^2 + 1})}{2}$$

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Since, $e^{f^{-1}(x)}$ is always positive, so neglecting negative sign.

$$\text{Hence, } f^{-1}(x) = \frac{\log(x + \sqrt{x^2 + 1})}{2}$$

Q17

$$f(x) = x^3 + x^2 + 4x + \sin x$$

$$\Rightarrow f'(x) = 3x^2 + 2x + 4 + \cos x$$

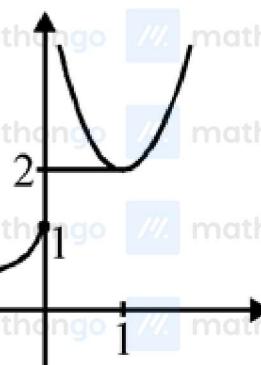
$$f''(x) = 3\left[\left(x + \frac{1}{3}\right)^2 + \frac{11}{9}\right] - (-\cos x) > 0 \text{ as } 3\left[\left(x + \frac{1}{3}\right)^2 + \frac{11}{9}\right]_{\min} = \frac{11}{3}$$

and $-\cos x$ has the maximum value 1.

$\Rightarrow f(x)$ is strictly increasing and hence it is one-one

Also, $\lim_{x \rightarrow \infty} f(x) \Rightarrow \infty$ and $\lim_{x \rightarrow -\infty} f(x) \Rightarrow -\infty$.

Thus, the range of $f(x)$ is R , hence it is onto.

Q18

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$\because k \in \text{odd integer}$

$f(k) = k + 1$ which is even

$$\therefore f(f(k)) = \frac{k+1}{2}$$

If $\frac{k+1}{2}$ is odd $\Rightarrow 33 = \frac{k+1}{2} + 1 \Rightarrow k = 63$ not possible (because $\frac{k+1}{2}$ is even)

If $\frac{k+1}{2}$ is even $\Rightarrow 33 = \frac{\frac{k+1}{2}}{2} \Rightarrow 33 = \frac{k+1}{4} \Rightarrow k = 131$

Sum of digits = 5

Q20

If $f(2) = 3 \Rightarrow f(4) = 2, f(3) = 2$

If $f(2) = 4 \Rightarrow f(4) = 2, f(3) = 3$

or

$f(4) = 3, f(3) = 2$

Hence there are 3 such functions

Q21

Sample Task**Solutions****Hints and Solutions****MathonGo**

There is no loss of generality in considering the order of b_i 's as $b_1 < b_2 < b_3 \dots < b_n$ also given

that $f(a_1) \leq f(a_2) \cdot f(a_{2n})$. Now suppose number of preimages of every b_i are x_i in numbers. Therefore $x_1 + x_2 + \dots + x_n = 2n$ where $1 \leq x_i \leq n+1 \rightarrow (1)$

Number of solutions of (1) is ${}^{2n-1}C_{n-1}$ or ${}^{2n-1}C_n$

Q22

Given the set $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

And, also a function $f : A \rightarrow A$ satisfying $f(1) + f(2) = 3 - f(3)$

$$\Rightarrow f(1) + f(2) + f(3) = 3$$

Since, the range of the function is also A , hence the only possibility satisfying the given condition is:

$$0 + 1 + 2 = 3$$

We know that, the number of arrangements of n objects at n places is $n!$.

Since, the given function is bijective i.e. one-one and onto, hence, the elements 1, 2, 3 in the domain can be mapped with only 0, 1, 2 in the co-domain in $3!$ ways and the remaining 5 elements 0, 4, 5, 6, 7 in the domain can be mapped with any of the remaining 5 elements 3, 4, 5, 6, 7 in $5!$ ways.

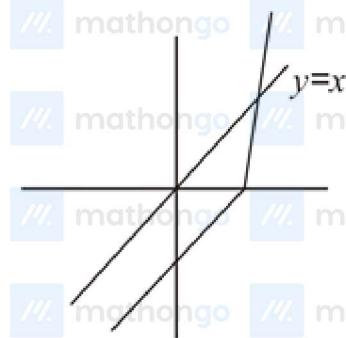
So, the number of bijective functions are $= 3! \times 5! = 6 \times 120 = 720$.

Q23

$$f(x) = 2x - 1 + |x - \frac{1}{2}|$$

$$= \begin{cases} x - \frac{1}{2} & x < \frac{1}{2} \\ 3x - \frac{3}{2} & x \geq \frac{1}{2} \end{cases}$$

As $f(x)$ is increasing, solution of equation $f(x) = f^{-1}(x)$ is same as $f(x) = x$.

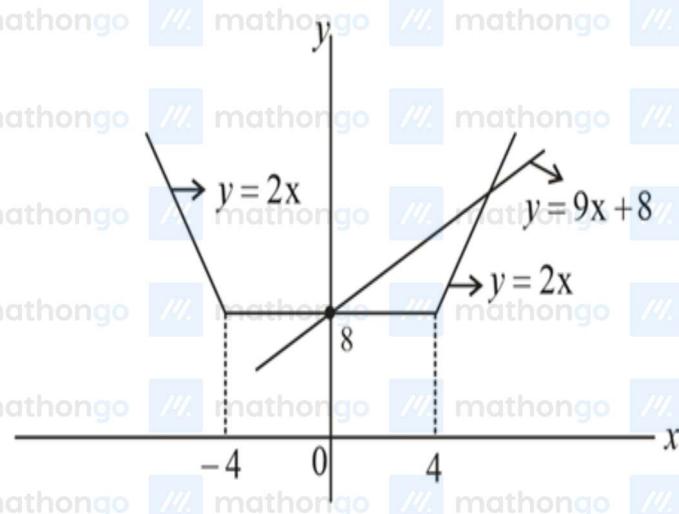
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∴ No. of solutions of $f(x) = f^{-1}(x)$ is one.

Q24

We have,

$$|x + 4| + |x - 4| = \begin{cases} -2x & x < -4 \\ 8 & -4 \leq x \leq 4 \\ 2x & x > 4 \end{cases}$$



It is clear from the above straight line $y = ax + 8$ cuts the curve $y = |x + 4| + |x - 4|$ at exactly one point of $a \in (-\infty, -2] \cup [2, \infty)$ exactly two points of $a \in (-2, 0) \cup (0, 2)$ more than two points if

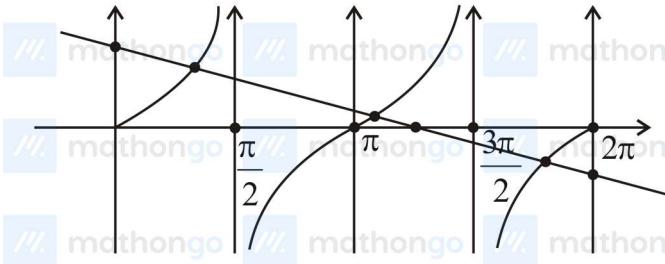
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$a = 0$. Hence, option (C) is correct.

Q25

$$2x + 3 \tan x = \frac{5\pi}{2} \Rightarrow \tan x = -\frac{2}{3}x + \frac{5\pi}{6}$$

$$y = \tan x \text{ and } y = \frac{5\pi}{6} - \frac{2x}{3}$$



Both the graphs meet exactly three times in $[0, 2\pi]$.

Thus, there are 3 solutions.

Q26

From given functional equation, $2f(xy) = (f(x))^y + (f(y))^x$, $\forall x, y \in \mathbb{R}$

putting $y = 1$,

$$2f(x) = f(x) + (f(1))^x$$

$$f(x) = 3^x$$

$$\therefore \sum_{r=1}^{10} f(r) = \sum_{r=1}^{10} 3^r = \frac{3(3^{10}-1)}{3-1} = \frac{3}{2}(3^{10}-1)$$

Q27

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Replacing x by $\frac{1}{(1-x)}$, we obtain $f\left(\frac{1}{1-x}\right) + f\left(1 - \frac{1}{x}\right) = -2x + \frac{2}{x}$

Again, replacing x by $1 - \frac{1}{x}$ and solve

Q28

$$\therefore f(x+2) + f(x-2) = f(x) \dots (i)$$

$$\text{Replace } x \text{ by } x+2 : f(x+4) + f(x) = f(x+2) \dots (ii)$$

$$\text{from eq (i) and (ii)} : f(x-2) + f(x+4) = 0$$

$$\text{Replace } x \text{ by } x+2 : f(x) + f(x+6) = 0 \dots (iii)$$

$$\text{Replace } x \text{ by } x-2 \text{ in eq (i)} : f(x) + f(x-4) = f(x-2) \dots (iv)$$

$$\text{from eq (i) and eq (iv)} : f(x+2) + f(x-4) = 0$$

$$\text{Replace } x \text{ by } x-2 : f(x) + f(x-6) = 0 \dots (v)$$

$$\text{from (iii) and (v)} : f(x+6) = f(x-6)$$

$\therefore f(x)$ is periodic with period 12.

$$\therefore \sum_{r=0}^{15} f(1 + 12r) = f(1) + f(13) + f(25) + \dots$$

$$= 16 \times f(1)$$

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$$= 16 \times 3$$

$$= 48$$

Q29

Given, $f(x+y) = f(x)+f(y)-xy - 1, \forall x, y \in R$

$$\therefore f(x+1) = f(x)+f(1)-x - 1 \quad [\text{putting } y = 1]$$

$$\Rightarrow f(x+1) = f(x)-x \quad [\because f(1)=1]$$

$$\therefore f(n+1) = f(n)-n < f(n)$$

$$\Rightarrow f(n+1) < f(n)$$

$$\text{So, } f(n) < f(n-1) < f(n-2) < \dots < f(3) < f(2) < f(1) = 1$$

$$\therefore f(n) = n \text{ holds only for } n = 1$$

Q30

In given equation, putting $x \rightarrow x + 3$

$$f(x+9) - f(x+6) + f(x+3) = 0$$

On adding both equations, we get,

$$f(x+9) + f(x) = 0 \dots (\text{i})$$

now, putting $x \rightarrow x + 9$

$$f(x+18) + f(x+9) = 0 \dots (\text{ii})$$

Equation (ii) – Equation (i)

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$$f(x + 18) = f(x)$$

∴ Period = 18

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