Q1

The point 'z' in Argand's plane moves such that $\operatorname{Re}\left(\frac{iz+1}{iz-1}\right) = 2,$ then locus of z is-

(1) straight line

(2) circle

(3) ellipse

(4) hyperbola

Q2

If $z \neq i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number, then, $z + \frac{1}{z}$ is

(1) any non-zero real number other than 1 .

(2) a purely imaginary number.

(3) 0

(4) any non-zero real number

Q3

Let $u = \frac{2z+i}{z-ki}, z = x + iy$ and $k > 0$. If the curve represented by $\operatorname{Re}(u) + \operatorname{Im}(u) = 1$ intersects the y-axis at points P and Q where $PQ = 5$ then the value of k is

(1) $\frac{3}{2}$

(2) $\frac{1}{2}$

(3) 4

(4) 2

Q4

The solution of the equation $|z| - z = 1 + 2i$ is

(1) $\frac{3}{2} + 2i$

(2) $\frac{3}{2} - 2i$

(3) $3 - 2i$

(4) None of these

Q5

For the complex number Z, the sum of all the solutions of $Z^2 + |Z| = \left(\overline{Z}\right)^2$ is equal to

Q6

Let z be a complex number satisfying the equation $\sqrt{2}|z - 1| + i + z = 0$. Find the number of such complex numbers.

Q7

The locus of point z, where $z = x + iy$, satisfying the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$, is

(1) The x - axis

(2) The straight line $y = 5$

(3) A circle passing through the origin

(4) None of these

Q8

If $Z = \cos\phi + i\sin\phi \left(\forall \phi \in \left(\frac{\pi}{3}, \pi\right)\right)$, then the value of $\arg\left(Z^2 - Z\right)$ is equal to (where, $\arg(Z)$ represents the argument of the complex number Z lying in the interval $(-\pi, \pi]$ and $i^2 = -1$)

(1) $\frac{3\phi+\pi}{2}$

(2) $\frac{3\phi}{2}$

(3) $\frac{3}{2}(\phi - \pi)$

(4) $\frac{3\phi-\pi}{2}$

Q9

If z and w are two non-zero complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then the value of $5i\bar{z}w$ is equal to

(1) – 5

(2) 5i

(3) 5

(4) – 5i

Q10

If z is a non-real complex number, then the minimum value of $\frac{Im\ z^5}{(Im\ z)^5}$ is ($Im\ z$ = Imaginary part of z)

(1) -2

(2) -4

(3) -5

(4) -1

Q11

If z and w are complex numbers satisfying $\bar{z} + i\bar{w} = 0$ and $\operatorname{amp}(zw) = \pi$, then $\operatorname{amp}(w)$ is equal to (where, $\operatorname{amp}(w) \in (-\pi, \pi]$)

(1) $\frac{\pi}{4}$

(2) $\frac{-\pi}{4}$

(3) $\frac{\pi}{2}$

(4) $\frac{3\pi}{4}$

Q12

Let α and β be the roots of $x^2 + x + 1 = 0$, then the equation whose roots are α^{2020} and β^{2020} is

(1) $x^2 + x + 1 = 0$

(2) $x^2 - x - 1 = 0$

(3) $x^2 + x - 1 = 0$

(4) $x^2 - x + 1 = 0$

Q13

If $z = \frac{1}{2}\left(\sqrt{3} - i\right)$ and the least positive integral value of n such that $\left(z^{101} + i^{109}\right)^{106} = z^n$ is k, then the value of $\frac{2}{5}k$ is equal to

Q14

The value of $\sum_{n=0}^{100} i^{n!}$ equals (where $i = \sqrt{-1}$)

(1) -1

(2) i

(3) $2i + 95$

(4) $97 + i$

Q15

If ω is the non-real cube root of unity, then the number of ordered pairs of integers (a, b), such that $|a\omega + b| = 1$, is equal to

Q16

The number of solutions of the equation $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$ (where, z is a complex number) are

(1) 2

(2) 3

(3) 6

(4) 5

Q17

$z \in C$ satisfies the condition $|z| \geq 3$. Then the least value of $\left|z + \frac{1}{z}\right|$ is

(1) $\frac{3}{8}$

(2) $\frac{8}{5}$

(3) $\frac{8}{3}$

(4) $\frac{5}{8}$

Q18

If m and M denotes the minimum and maximum value of $|2z + 1|$, where $|z - 2i| \leq 1$ and $i^2 = -1$, then the value of $(M - m)^2$ is equal to

(1) 17

(2) 34

(3) 51

(4) 16

Q19

For a complex number Z, if all the roots of the equation $Z^3 + aZ^2 + bZ + c = 0$ are unimodular, then

(1) $|a| > 3$ and $|c| = 1$

(2) $|a| \leq 3$ and $|c| = 3$

(3) $|a| > 3$ and $|c| = \frac{1}{3}$

(4) $|a| \leq 3$ & $|c| = 1$

Q20

Let z and w be non-zero complex numbers such that $zw = \left|z^2\right|$ and $\left|z - \bar{z}\right| + \left|w + \bar{w}\right| = 4$. If w varies, then the perimeter of the locus of z is

(1) $8\sqrt{2}$ units

(2) $4\sqrt{2}$ units

(3) 8 units

(4) 4 units

Q21

The straight line $(1 + 2i)z + (2i - 1)\bar{z} = 10i$ on the complex plane, has intercept on the imaginary axis equal to

(1) 5

(2) $\frac{5}{2}$

(3) $-\frac{5}{2}$

(4) -5

Q22

If a complex number z lie on a circle of radius $\frac{1}{2}$ units, then the complex number $\omega = -1 + 4z$ will always lie on a circle of radius k units, where k is equal to

Q23

Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z+3i|$ is

(1) $\sqrt{10}$
(2) $\frac{7}{2}$
(3) $\frac{15}{4}$
(4) $2\sqrt{3}$

Q24

A complex number z satisfies $arg\left(\frac{z}{z-i}\right) = \frac{\pi}{3}$ and $|z| = |z-i|$, then evaluate $[Re\ (2z-i)]$ where $[\ \cdot\ ]$ represents the greatest integer function.

Q25

Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and Re (w) has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to ______.

Q26

The complex number z, satisfying the equation $z^3 = \bar{z}$ and $\arg(z+1) = \frac{\pi}{4}$ simultaneously, is (where, $i^2 = -1$)

(1) i

(2) $1 + 2i$

(3) $2 + 3i$

(4) $3 + 4i$

Q27

The real part of the complex number z satisfying $|z - 1 - 2i| \leq 1$ and having the least positive argument, is

(1) $\frac{4}{5}$

(2) $\frac{8}{5}$

(3) $\frac{6}{5}$

(4) $\frac{7}{5}$

Q28

If a and b are two real numbers lying between 0 and 1, such that $Z_1 = a + i$, $Z_2 = 1 + bi$ and $Z_3 = 0$ form an equilateral triangle, then

(1) $a = 2 + \sqrt{3}$

(2) $b = 4 - \sqrt{3}$

(3) $a = b$

(4) $a = 2, b = \sqrt{3}$

Q29

If the locus of the complex number z given by $\arg(z + i) - \arg(z - i) = \frac{2\pi}{3}$ is an arc of a circle, then the length of the arc is

(1) $\frac{4\pi}{3}$

(2) $\frac{4\pi}{3\sqrt{3}}$

(3) $\frac{2\sqrt{3}\pi}{3}$

(4) $\frac{2\pi}{3\sqrt{3}}$

Q30

Let the locus of any point $P(z)$ in the argand plane is $\arg\left(\frac{z-5i}{z+5i}\right)=\frac{\pi}{4}$. If O is the origin, then the value of $\frac{\max.(OP)+\min.(OP)}{2}$ is

(1) $5\sqrt{2}$

(2) $5+\frac{5}{\sqrt{2}}$

(3) $5+5\sqrt{2}$

(4) $10-\frac{5}{\sqrt{2}}$

Answer Key

Q1 (2)	**Q2** (4)	**Q3** (4)	**Q4** (2)
Q5 (0)	**Q6** (0)	**Q7** (1)	**Q8** (3)
Q9 (3)	**Q10** (2)	**Q11** (1)	**Q12** (1)
Q13 (4)	**Q14** (3)	**Q15** (6)	**Q16** (4)
Q17 (3)	**Q18** (4)	**Q19** (4)	**Q20** (1)
Q21 (1)	**Q22** (2)	**Q23** (2)	**Q24** (1)
Q25 (4)	**Q26** (1)	**Q27** (2)	**Q28** (3)
Q29 (2)	**Q30** (2)		

Sample Task **Solutions**

Hints and Solutions **MathonGo**

Q1

We have,

$\operatorname{Re}\left(\frac{iz+1}{iz-1}\right)=2$

Let $z=x+iy$, then

$\operatorname{Re}\left(\frac{i(x+iy)+1}{i(x+iy)-1}\right)=2$

$\Rightarrow \operatorname{Re}\left(\frac{ix+i^2y+1}{ix+i^2y-1}\right)=2$

$\Rightarrow \operatorname{Re}\left(\frac{1-y+ix}{-1-y+ix}\right)=2$

$\Rightarrow \operatorname{Re}\left\{\left(\frac{1-y+ix}{-1-y+ix}\right)\times\left(\frac{-1-y-ix}{-1-y-ix}\right)\right\}=2$

$\Rightarrow \operatorname{Re}\left\{\frac{(1-y+ix)((-1-y)-ix)}{(-1-y)^2+x^2}\right\}=2$

$\Rightarrow \left\{\frac{(1-y)(-1-y)+x^2}{(-1-y)^2+x^2}\right\}=2$

$\Rightarrow y^2-1+x^2=2y^2+2x^2+2+4y$

$\Rightarrow x^2+y^2+4y+3=0$

Hence, the locus is a circle.

Q2

Let $\mathrm{z}=\mathrm{x}+\mathrm{iy}$

$\frac{\mathrm{z}-\mathrm{i}}{\mathrm{z}+\mathrm{i}}$ is a purely imaginary number

$\Rightarrow \frac{\mathrm{x}+\mathrm{i}(\mathrm{y}-1)}{\mathrm{x}+\mathrm{i}(\mathrm{y}+1)}\times\frac{\mathrm{x}-\mathrm{i}(\mathrm{y}+1)}{\mathrm{x}-\mathrm{i}(\mathrm{y}+1)}$ is a purely imaginary

$\Rightarrow \frac{(\mathrm{x}^2+\mathrm{y}^2-1)-\mathrm{i}(2\mathrm{x})}{\mathrm{x}^2+(\mathrm{y}+1)^2}$ is purely imaginary

$\Rightarrow x^2+y^2-1=0 \Rightarrow x^2+y^2=1\ldots(i)$

$z+\frac{1}{z}=x+iy+\frac{1}{x+iy}$

$=(x+iy)+\frac{1}{(x+iy)}\times\frac{(x-iy)}{(x-iy)}$

$=(x+iy)+\frac{(x-iy)}{x^2+y^2}=2x$

$y\neq\pm1$ so $x\neq0$ (from (i) and since, z won't be an imaginary number)

Q3

$u=\frac{2(x+iy)+i}{(x+iy)-ki}=\frac{2x+(2y+1)i}{x+(y-k)i}\times\frac{x-(y-k)i}{x-(y-k)i}$

Real part of $u=Re(u)=\frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$

Imaginary part of $u=\operatorname{Im}(u)\frac{x(2y+1)-2x(y-k)}{x^2+(y-k)^2}$

Now $\operatorname{Re}(u)+\operatorname{Im}(u)=1$

$\frac{2x^2+(2y+1)(y-k)+x(2y+1)-2x(y-k)}{x^2+(y-k)^2}=1$

for y-axis put $x=0$

$\Rightarrow\frac{(2y+1)(y-k)}{(y-k)^2}=1$

$\Rightarrow(y-k)(y+1+k)=0$

$y=k,\ -(1+k)$

Now point $P(0,\ k),\ Q(0,\ -(1+k))$

$PQ=|2K+1|=5$

$2k+1=\pm5$

$2k=4,-6$

$k=2,-3$

Hence, $k=2\quad(k>0)$.

Q4

Let $z=x+iy$

Given, $|z|-z=1+2i$

$\Rightarrow\quad\sqrt{x^2+y^2}-(x+iy)=1+2i$

$\Rightarrow\quad\sqrt{x^2+y^2}-x=1,\quad y=-2$

$\Rightarrow\quad\sqrt{x^2+4}-x=1$

$\Rightarrow\quad x^2+4=(1+x)^2$

$\Rightarrow\quad 2x=3\Rightarrow\quad x=\frac{3}{2}$

$\therefore\quad z=\frac{3}{2}-2i$

Q5

$Z^2+|Z|=\left(\overline{Z}\right)^2\ \ldots(1)$

Taking conjugate,

$\left(\overline{Z}\right)^2+|Z|=Z^2\ \ldots(2)$

Adding (1) & (2), we get,

$\Rightarrow 2|Z|=0 \Rightarrow Z=0$

Q6

$\sqrt{2}\,|z-1|=-(i+z)$

$\Leftrightarrow\ \sqrt{2}\sqrt{(x-1)^2+y^2}=-[x+i(y+1)] \quad \ldots(i)$

$L.\ H.\ S.\ \geq 0 \Rightarrow R.\ H.\ S.$ must be real

$\Rightarrow y=-1$

(i) reduces to $\sqrt{2}\,\sqrt{(x-1)^2+1}=-x \qquad \ldots(ii)$

$L.\ H.\ S.$ of $(ii) \geq 0 \Rightarrow x \leq 0 \qquad \ldots(iii)$

Squaring (ii), we get $2\left[x^2-2x+2\right]=x^2$

$\Leftrightarrow x^2-4x+4=0$

$\Leftrightarrow x=2$

(rejected) $\ldots$[From (iii)]

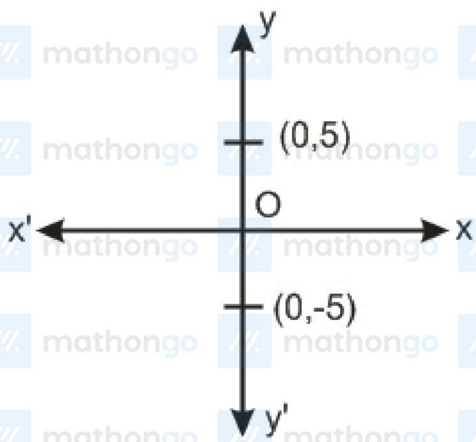
$\Rightarrow$ There is no z that satisfies the given equation. Thus, there are 0 such complex numbers.

Q7

Given, $\left|\frac{z-5i}{z+5i}\right|=1$

$\Rightarrow |z-5i|=|z+5i|$

(if $|z-z_1|=|z-z_2|$, then it is a perpendicular bisector of the line segment joining points z_1 and z_2)



and perpendicular bisector of z_1 $(0,\ 5)$ and z_2 $(0,\ \ -5)$ is $x-axis$,

Therefore, z will lie on the $x-$ axis.

Q8

$\left(Z^2-Z\right)=Z(Z-1)$

$=(\cos\phi+i\sin\phi)(\cos\phi-1+i\sin\phi)$

$=(\cos\phi+i\sin\phi)\left(-2\sin^2\frac{\phi}{2}+i2\sin\frac{\phi}{2}\cos\frac{\phi}{2}\right)$

$=2i\sin\frac{\phi}{2}\left(\cos\frac{3\phi}{2}+i\sin\frac{3\phi}{2}\right)$

$=2\sin\frac{\phi}{2}\left(\cos\frac{3\phi+\pi}{2}+i\sin\frac{3\phi+\pi}{2}\right)$

Now, $3\phi\in(\pi,3\pi)\Rightarrow\frac{3\phi+\pi}{2}\in(\pi,2\pi)$

But, the argument lies in $(-\pi,\pi]$, hence

$\arg\left(Z^2-Z\right)=\frac{3\phi+\pi}{2}-2\pi=\frac{3}{2}(\phi-\pi)$

Q9

$\arg\left(\frac{z}{w}\right)=\frac{\pi}{2} \Rightarrow \frac{z}{w}=\left|\frac{z}{w}\right|e^{i\pi/2}$

$\Rightarrow \frac{z}{w}=\left|\frac{z}{w}\right|i$

$\Rightarrow w=z\frac{|w|}{|z|}(-i)$

$\Rightarrow w\bar{z}=\frac{\bar{z}z|w|}{|z|}(-i)$

$\Rightarrow 5i\bar{z}w=5\left(-i^2\right)\frac{|z|^2|w|}{|z|}$

$=5(1)|z||w|$

$=5$

Q10

Let, $z=r(\cos\theta+i\sin\theta)$

$z^5=r^5(\cos 5\theta+i\sin 5\theta)$

$Im\left(z^5\right)=r^5\sin 5\theta$

and $(Im\ z)^5=r^5\sin^5\theta$

$\frac{Im\left(z^5\right)}{(Im\ z)^5}=\frac{\sin 5\theta}{\sin^5\theta}=\text{A (Let)}$

$\Rightarrow \frac{dA}{d\theta}=\frac{5\sin^5\theta\cos 5\theta-5\sin 5\theta\sin^4\theta\cos\theta}{\left(\sin^5\theta\right)^2}$

$\Rightarrow \frac{dA}{d\theta}=\frac{5\sin^4\theta(\sin\theta\cos 5\theta-\sin 5\theta\cos\theta)}{\sin^{10}\theta}$

$\Rightarrow \frac{dA}{d\theta}=\frac{5}{\sin^6\theta}[\sin(-4\theta)]=0$

$\Rightarrow \theta=\frac{\pi}{4}$

The minimum value of A will be at $\theta = \frac{\pi}{4}$.

$\Rightarrow \frac{\sin 5\frac{\pi}{4}}{\left(\sin\frac{\pi}{4}\right)^5}$
$= \frac{\frac{-1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^5}$

$= -\left(\sqrt{2}\right)^4 = -4$

Q11

Given $\overline{z} + i\overline{w} = 0 \Rightarrow \overline{z} = -i\overline{w}$ or $z = iw$ or $\frac{z}{w} = i$

$\text{amp}(z) - \text{amp}(w) = \text{amp}\, i = \frac{\pi}{2} \ldots .(\text{i})$
also $\text{amp}(zw) = \pi$

$\text{amp}(z) + \text{amp}(w) = \pi \ldots ..(\text{ii})$

Adding (i) & (ii), we get,

$2\text{amp}(z) = \frac{3\pi}{2}$
$\Rightarrow \text{amp}(z) = \frac{3\pi}{4}$
Also, $\text{amp}(w) = \frac{\pi}{4}$

Q12

$\mathrm{x}^2 + \mathrm{x} + 1 = 0$

$\Rightarrow \mathrm{x} = \omega, \omega^2$

$\Rightarrow \alpha = \omega$ and $\Rightarrow \beta = \omega^2$

$\Rightarrow \alpha^{2020} = \omega^{2020} = \left(\omega^3\right)^{673} . \omega = \omega$

$\Rightarrow \beta^{2020}=\left(\omega^2\right)^{2020}=\left(\omega^3\right)^{2\times 673}\times\omega^2=\omega^2$

$\Rightarrow$ The required equation is $\mathrm{x}^2+\mathrm{x}+1=0$

Q13

$z=\frac{-1}{2}i\left(1+i\sqrt{3}\right)=i\omega^2$

$z^{101}=i\omega$

$\left(z^{101}+i^{109}\right)^{106}=(i\omega+i)^{106}=\left(i\left(-\omega^2\right)\right)^{106}=-\omega^2$

as given that $\left(z^{101}+i^{109}\right)^{106}=z^n$

$\therefore -\omega^2=\left(i\omega^2\right)^n=i^n\omega^{2n}$

$\omega^{2n-2}i^n=-1$

this is possible only when $n=4r+2$ and $2n-2$ is a multiple of 3 i.e.,

$2(4r+2)-2$ is a multiple of 3

i.e., $8r+2$ is a multiple of $3 \Rightarrow r=2$

$\therefore n=10 \quad \therefore \frac{2}{5}k=4$

Q14

Let, $S=\sum_{n=0}^{100}(i)^{n!}$ (where $i=\sqrt{-1}$)

$= (i)^{0!} + (i)^{1!} + (i)^{2!} + (i)^{3!} + (i)^{4!} + \ldots + (i)^{100!}$

$= (i)^{1} + (i)^{1} + (i)^{2} + (i)^{6} + (i)^{24} + \ldots . + (i)^{100!}$

(since, $i^{4n} = 1$, $i^{4n+1} = i$, $i^{4n+2} = -1$ and $i^{4n+3} = -i$, where $n \in N$)

From i^{24} term onwards, every term is of the form i^{4n} which equals 1,

Simplifying above expression, we get

$= i + i - 1 - 1 + 1 + 1 + \ldots . + 1$

$= 2i - 2 + 97 = 2i + 95$

Q15

We have, $|a\omega + b|^2 = 1$

$\Rightarrow (a\omega + b)\left(a\bar{\omega} + b\right) = 1$

$\Rightarrow a^2 + ab\left(\omega + \bar{\omega}\right) + b^2 = 1$

$\Rightarrow a^2 - ab + b^2 = 1$

$\Rightarrow (a-b)^2 + ab = 1 \ldots . .$(i)

$\left(\text{As, } 1 + \omega + \omega^2 = 0\right)$

When $(a-b)^2 = 0$ and $ab = 1$ then $(1,1);(-1,-1)$

When $(a-b)^2 = 1$ and $ab = 0$ then $(0,1);(1,0);(0,-1);(-1,0)$

Hence, $(0,1); (1,0); (0,-1); (-1,0); (1,1); (-1,-1)$ i.e., 6 ordered pairs.

Q16

Let, $z = re^{i\theta}$

$\Rightarrow r^3 e^{i3\theta} + 3re^{-i2\theta} = 0$

$\Rightarrow r^2 e^{i5\theta} = -3$

$\Rightarrow r^2 = 3$ and $e^{i5\theta} = -1$

$\Rightarrow r = \sqrt{3}$ and $\theta = \frac{\pi}{5} + \frac{2k}{5}$ where, $k = 0, 1, 2, 3, 4$

$\Rightarrow 5$ solutions

Q17

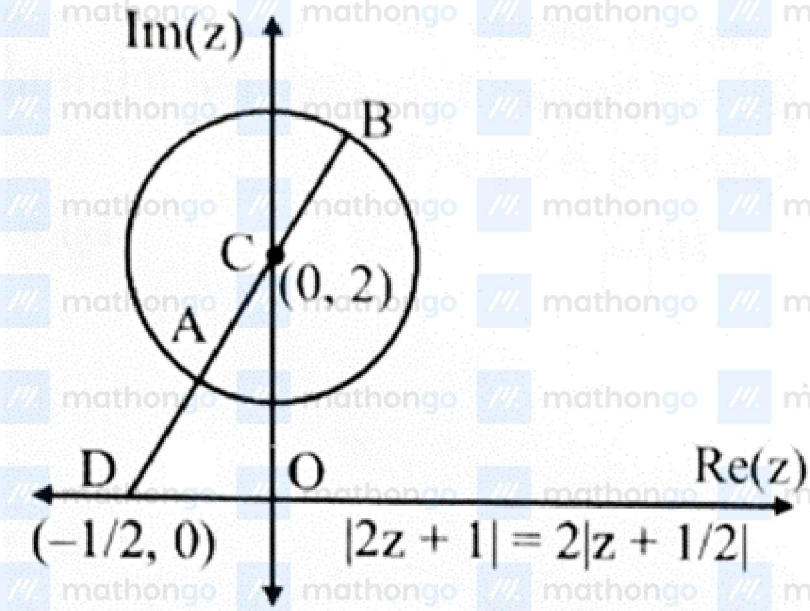
From triangle inequality we know that $|z_1 + z_2| \geq ||z_1| - |z_2||$

hence $\left|z + \frac{1}{z}\right| = \left|z - \left(-\frac{1}{z}\right)\right| \geq |z| - \left|-\frac{1}{z}\right| \geq 3 - \frac{1}{3} = \frac{8}{3}$

Hence $\frac{8}{3}$ is the correct answer.

Q18

$z - 2i = 1$ represents a circle with centre at $(0, 2)$ and radius 1 unit



$|2z + 1| = 2\left|z - \left(\frac{-1}{2}\right)\right|$ represents twice the distance from the point $\left(\frac{-1}{2}, 0\right)$

Hence from the diagram,

$m = 2AD = 2(CD - AC)$

$m = 2\left(\frac{\sqrt{17}}{2} - 1\right) = \sqrt{17} - 2$

$M = 2BD = 2(CD + BC)$

$M = 2\left(\frac{\sqrt{17}}{2} + 1\right) = \sqrt{17} + 2$

$\Rightarrow (M - m)^2 = 4^2 = 16$

Q19

Given, $|Z_1| = |Z_2| = |Z_3| = 1$

Now, $|Z_1 + Z_2 + Z_3| \leq |Z_1| + |Z_2| + |Z_3|$

$\Rightarrow |-a| \leq 1 + 1 + 1$

$\Rightarrow |a| \leq 3$

Also, $|Z_1 Z_2 Z_3| = |Z_1| \times |Z_2| \times |Z_3|$

$\Rightarrow |-c| = 1 \times 1 \times 1$

$\Rightarrow |c| = 1$

Q20

Given, $\mathrm{zw} = |\mathrm{z}|^2 \Rightarrow \mathrm{zw} = \mathrm{z}\,\overline{\mathrm{z}}$

$\Rightarrow \mathrm{w} = \overline{\mathrm{z}}\ \{\mathrm{z} \neq 0\}$

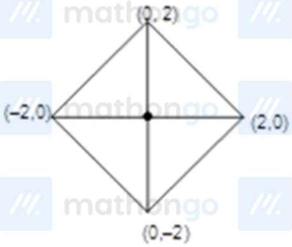
Now, $\left|\mathrm{z} - \overline{\mathrm{z}}\right| + \left|\mathrm{w} + \overline{\mathrm{w}}\right| = 4$

$\Rightarrow \left|\mathrm{z} - \overline{\mathrm{z}}\right| + \left|\mathrm{z} + \overline{\mathrm{z}}\right| = 4$

Let, $\mathrm{z} = \mathrm{x} + i\mathrm{y}$, then we get,

$|\mathrm{x}| + |\mathrm{y}| = 2$

which represents a square of side length equal to $2\sqrt{2}$



$\Rightarrow$ The perimeter of the locus is $8\sqrt{2}$ units

Q21

Let $z = x + iy$

$\Rightarrow(1+2i)z+(2i-1)\bar{z}=10i$

$\Rightarrow(1+2i)(x+iy)+(2i-1)(x-iy)=10i$

$\Rightarrow(x-2y)+i(2x+y)+(-x+2y)+i(2x+y)=10i$

$\Rightarrow 2i(2x+y)=10i$

Or $2x+y=5$

For interception on imaginary axis

Put $x=0$

So, we get $y=5$

Intercept on imaginary axis $=5$

Q22

Let us assume that z lies on a circle with centre z_0 (fixed point) and radius $\frac{1}{2}$ units.

$\Rightarrow \quad |z - z_0| = \frac{1}{2}$

Now, $\omega = -1 + 4z \Rightarrow \omega + 1 = 4z$

$\Rightarrow \quad \omega + 1 - 4z_0 = 4z - 4z_0$

Now, taking modulus on both sides, we get,

$|\omega + 1 - 4z_0| = 4|z - z_0| \Rightarrow |\omega + 1 - 4z_0| = 2$

Locus of ω represents the circle having centre $(-1 + 4z_0)$ and radius 2 units.

Q23

Given, $|z - i| = |z + 2i|$

$\Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\Rightarrow -2y + 1 = 4y + 4$

$\Rightarrow 6y = -3 \Rightarrow y = -\frac{1}{2}$

From, $|z| = \frac{5}{2}$, we get,

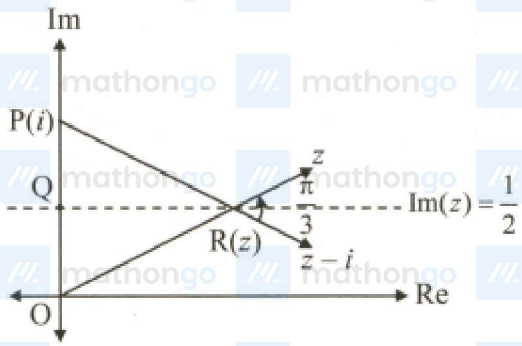
$x^2 + y^2 = \frac{25}{4} \Rightarrow x^2 = \frac{24}{4} = 6$

$\Rightarrow z = \pm\sqrt{6} - \frac{i}{2}$

Hence, $|z + 3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$

$|z + 3i| = \frac{7}{2}$

Q24



$|z|=|z-i|$

$\Leftrightarrow|PR|=|OR|$

$arg\ \frac{z}{z-i}=\frac{\pi}{3}$

$\Leftrightarrow\angle PRO=\frac{\pi}{3}$

$\Rightarrow\Delta POR$ is an equilateral triangle.

$\Rightarrow QR$ is angle bisector as well as median.

$\Rightarrow\angle PRQ=\frac{\pi}{6},\ |PQ|=\frac{1}{2}$

$\Rightarrow Im\ (z)=\frac{1}{2},\ Re(z)=|QR|$

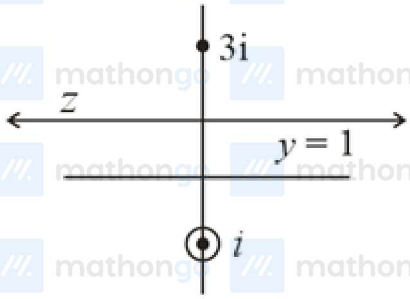
$=|PQ|\ cot\ \frac{\pi}{6}$

$=\frac{1}{2}\left(\sqrt{3}\right)$

$z=\frac{\sqrt{3}}{2}+\frac{i}{2}\Rightarrow 2z-i=\sqrt{3}$

$\Rightarrow\ [Re\ (2z-i)]=[1.73]=1$

Q25



$\omega = z\bar{z} - 2z + 2$

$\left|\frac{z+i}{z-3i}\right| = 1$

$\Rightarrow |z+i| = |z-3i|$

$\Rightarrow z = x + i,\ x \in R$

$\omega = (x+i)(x-i) - 2(x+i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$\text{Re}(\omega) = x^2 - 2x + 3$

For min $(\text{Re}(\omega)), x = 1$

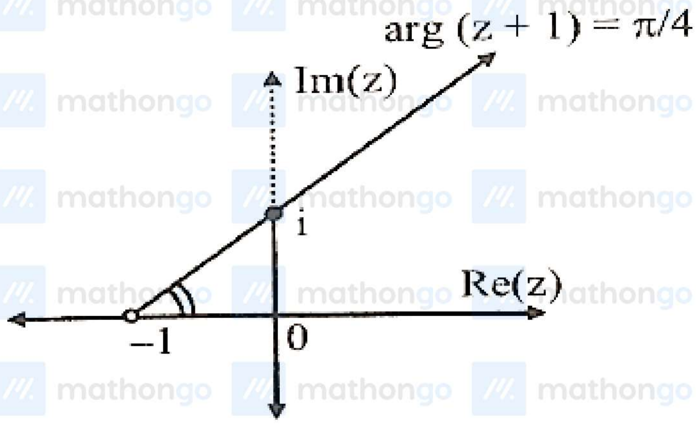
$\Rightarrow \omega = 2 - 2i = 2(1-i) = 2\sqrt{2}e^{-i\frac{\pi}{4}}$

$\omega^n = \left(2\sqrt{2}\right)^n e^{-i\frac{n\pi}{4}}$

For real and minimum value of n,

$n = 4$

Q26



$z^3 = \bar{z} \Rightarrow |z| = 0$ or 1

$|z| = 0 \Rightarrow z = 0$

$|z| = 1 \Rightarrow z^4 = 1$

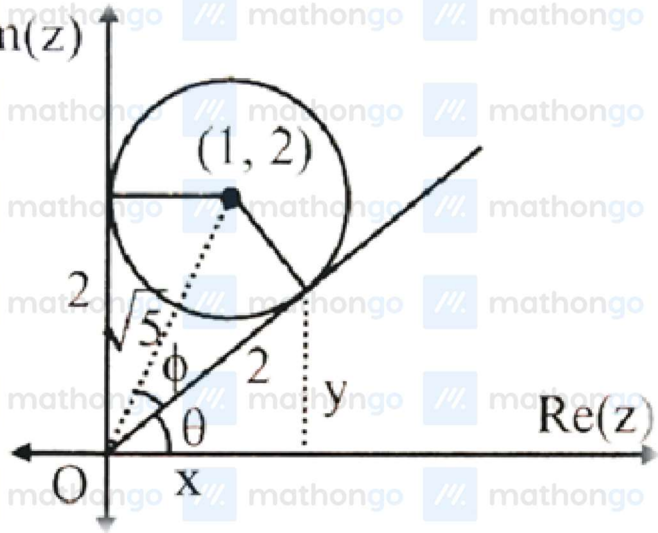
$\Rightarrow z = \pm 1, \pm 1$

Only $z = i$ satisfies $\arg(z+1) = \frac{\pi}{4}$

Q27

Here, $|z - 1 - 2i| = 1$ represents a circle with centre $(1, 2)$ and radius 1 unit.

The complex number $z = \mathrm{x} + i\mathrm{y}$ satisfying the given inequality and having the least positive argument is the point of contact of the tangent from the origin to the circle with the least positive slope.



From the diagram,

$\tan \phi = \frac{1}{2}$

$\therefore \tan 2\phi = \frac{2 \tan \phi}{1 - \tan^2 \phi} = \frac{4}{3}$

For the least positive argument, $\arg(z) = \theta$ (let)

$\tan \theta = \tan\left(\frac{\pi}{2} - 2\phi\right) = \cot 2\phi = \frac{3}{4}$

Also, from the diagram,

$x^2 + y^2 = 4$ and $\tan \theta = \frac{y}{x} = \frac{3}{4}$

i.e. $y = \frac{3}{4} x$

$\Rightarrow x^2 + \frac{9x^2}{16} = 4 \Rightarrow x = \frac{8}{5}$

Hence, for the least positive argument, the real part of z is equal to $\frac{8}{5}$

Q28

If $Z_1,\ Z_2,\ Z_3$ form an equilateral triangle, then we know that,

$Z_1^2+Z_2^2+Z_3^2=Z_1Z_2+Z_2Z_3+Z_3Z_1$
$\Rightarrow a^2-1+2ai+1-b^2+2bi-a+b-i-abi=0$

$\Rightarrow(a-b)(a+b-1)+(2a+2b-ab-1)i=0$

Case-I:

$a=b\ \&\ 2a+2b-ab-1=0$

$\Rightarrow a=b\ \&\ a^2-4a+1=0\Rightarrow a=b=2-\sqrt{3}$

Case-II:

$a+b-1=0\ \&\ 2a+2b-ab-1=0$

$\Rightarrow ab=1$ (not possible because $a,b\in(0,1)$)

$\Rightarrow a=b=2-\sqrt{3}$ is the only solution

Q29

Given, $\arg\left(\frac{z+i}{z-i}\right)=\frac{2\pi}{3}$

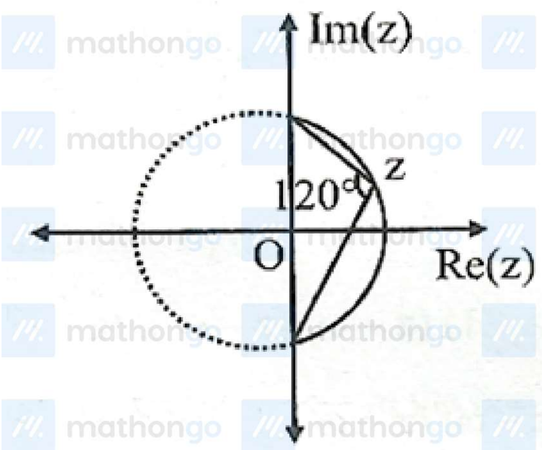
Let, $z=x+iy$

$\Rightarrow \frac{x+i(y+1)}{x+i(y-1)}=\frac{x^2+(y^2-1)+2ix}{x^2+(y-1)^2}$

$\Rightarrow \tan^{-1}\frac{2x}{x^2+y^2-1}=\frac{2\pi}{3}$

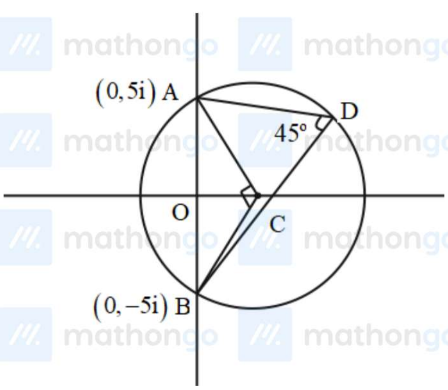
$\Rightarrow x^2+y^2+\frac{2}{\sqrt{3}}x-1=0$

Hence, the given locus is a circle with centre $\left(-\frac{1}{\sqrt{3}}, 0\right)$ and radius $\frac{2}{\sqrt{3}}$ units



$\Rightarrow$ Length of the arc of the circle is $\frac{2\pi}{3} \times \left(\frac{2}{\sqrt{3}}\right) = \frac{4\pi}{3\sqrt{3}}$ units

Q30



$r^2 + r^2 = 10^2 \Rightarrow r = 5\sqrt{2}$

max.$(OP) = OC +$ radius $= 5 + 5\sqrt{2}$

and min.$(OP) = OA = 5$

Required value $= \frac{5+5+5\sqrt{2}}{2} = 5 + \frac{5}{\sqrt{2}}$