Q1

If $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then the sum of the first 24 terms of the arithmetic progression $a_1,\ a_2,\ a_3.\ldots..$ is equal to

(1) 450

(2) 675

(3) 900

(4) 1200

Q2

If $2,\ 7,\ 9$ and 5 are subtracted respectively from four numbers in geometric progression, then the resulting numbers are in arithmetic progression. The smallest of the four numbers is

(1) – 24

(2) – 12

(3) 6

(4) 3

Q3

If $a,\ b$ & $3c$ are in arithmetic progression and $a,\ b$ & $4c$ are in geometric progression, then the possible values of $\frac{a}{b}$ are

(1) $\left\{\frac{2}{3}, 2\right\}$

(2) $\left\{\frac{3}{2}, \frac{1}{2}\right\}$

(3) $\left\{\frac{2}{3}, \frac{3}{2}\right\}$

(4) $\left\{\frac{1}{2}, 2\right\}$

Q4

Let a_1, a_2, a_3 be three positive numbers which are in geometric progression with common ratio r. The inequality $a_3 > a_2 + 2a_1$ holds true if r is equal to

(1) 2

(2) 1.5

(3) 0.5

(4) 2.5

Q5

If $|x| < 1,\ |y| < 1,$ the sum to infinity of the series $(x + y),\ \left(x^2 + xy + y^2\right),\ \left(x^3 + x^2y + xy^2 + y^3\right), \ldots$ Is -

(1) $\frac{x+y-xy}{1-x-y+xy}$

(2) $\frac{x+y+xy}{1-x-y+xy}$

(3) $\frac{x}{1-x} + \frac{y}{1-y}$

(4) $\frac{(x-y)(x+y-xy)}{1-x-y+xy}$

Q6

If $|3x - 1|, 3, |x - 3|$ are the first three terms of an arithmetic progression, then the sum of the first five terms can be

(1) 5

(2) 10

(3) 20

(4) 30

Q7

If $x, y, z \in R^+$ and $16\left(16x^2+y^2-4xy\right)=z\left(16x+4y-z\right)$, then

(1)

y, z, x are in $A.P.$

(2)

y, z, x are in $G.P.$

(3)

x, y, z are in $A.P.$

(4) x, y, z are in $G.P.$

Q8

In a sequence of 21 terms first 11 terms are in A.P. with common difference 2 and last 11 terms are in G.P. with common ratio 2. If middle term of A.P. is equal to middle term of G.P. then, middle term in the complete sequence is

(1) $\frac{10}{1-2^5}$

(2) $\frac{10\left(1-2^{11}\right)}{\left(2^{10}-1\right)}$

(3) $\frac{1-2^{11}}{2^{10}-1}+10$

(4) $\frac{20}{2^{10}-1}$

Q9

If $a_1, a_2, \ldots\ldots a_{10}$ are positive numbers in an arithmetic progression such that $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \ldots. + \frac{1}{a_9 a_{10}} = \frac{9}{64}$

and $\frac{1}{a_1 a_{10}} + \frac{1}{a_2 a_9} + \ldots. \frac{1}{a_{10} a_1} = \frac{1}{10}\left(\frac{1}{a_1} + \ldots + \frac{1}{a_{10}}\right)$, then sum of digits of $\left(4\left(\frac{a_1}{a_{10}} + \frac{a_{10}}{a_1}\right)\right)$ is

Q10

Three numbers $a,\ b$ and c are in between 2 and 18 such that $2,\ a,\ b$ are in arithmetic progression and $b,\ c,\ 18$ are in geometric progression. If $a + b + c = 25$, then the value of $c - a$ is

(1) 4

(2) 3

(3) 7

(4) 0

Q11

The harmonic mean of two positive numbers a and b is $4,$ their arithmeitc mean is A and the geometric mean is $G.$ If $2A + G^2 = 27,\ \ a + b = \alpha$ and $|a - b| = \beta$, then the value of $\frac{\alpha}{\beta}$ is equal to

(1) 1

(2) 3

(3) $\frac{5}{2}$

(4) 5

Q12

If 11 arithmetic means are inserted between 28 and $10,$ then the number of integral arithmetic means are

(1) 5

(2) 6

(3) 7

(4) 8

Q13

There are n sets of observations given as $(1),\ (2, 3),\ (4, 5, 6),\ (7, 8, 9, 10), \ldots\ldots$ The mean of the 13^{th} set of observations is equal to

(1) 70

(2) 80

(3) 75

(4) 85

Q14

The sum of the first 20 terms common between the series $3 + 7 + 11 + 15 + \ldots\ldots$ and $1 + 6 + 11 + 16 + \ldots\ldots$ is

(1) 4000

(2) 4200

(3) 4220

(4) 4020

Q15

The sum to infinity of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$ is

(1) $\frac{16}{25}$

(2) $\frac{11}{5}$

(3) $\frac{35}{16}$

(4) $\frac{8}{11}$

Q16

The sum (upto two decimal places) of the infinite series $\frac{7}{17}+\frac{77}{17^2}+\frac{777}{17^3}+\ldots\ldots.$ is

(1) 1.06

(2) 2.06

(3) 3.06

(4) 4.06

Q17

It is given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots.\infty=\frac{\pi^4}{90}$ then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots+\infty$ is equal to -

(1) $\frac{\pi^4}{96}$

(2) $\frac{\pi^4}{45}$

(3) $\frac{89\pi^4}{90}$

(4) None of these

Q18

If $S=1(25)+2(24)+3(23)+\ldots\ldots+24(2)+25(1)$, then the value of $\frac{S}{900}$ is equal to

Q19

$0.2+0.22+0.222+\ldots$ upto n terms is equal to

(1) $\left(\frac{2}{9}\right)-\left(\frac{2}{81}\right)\left(1-10^{-n}\right)$

(2) $n\left(\frac{1}{9}\right)\left(1-10^{-n}\right)$

(3) $\left(\frac{2}{9}\right)\left[n-\left(\frac{1}{9}\right)\left(1-10^{-n}\right)\right]$

(4) $\left(\frac{2}{9}\right)$

Q20

If the sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots..+$ up to 20 terms is equal to $\frac{k}{21}$, then k is equal to

(1) 240

(2) 120

(3) 60

(4) 180

Q21

If $S=\sum_{n=1}^{9999}\frac{1}{\left(\sqrt{n}+\sqrt{n+1}\right)\left(\sqrt[4]{n}+\sqrt[4]{n+1}\right)}$, then the value of S is equal to

(1) 9

(2) 99

(3) 999

(4) 9999

Q22

If $S = \sum_{r=1}^{80} \frac{r}{\left(r^4 + r^2 + 1\right)}$, then the value of $\frac{6481S}{1000}$ is

Q23

For the series $S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \ldots\ldots$, if the sum of the first 10 terms is K, then $\frac{4K}{101}$ is equal to

Q24

Let the sum $\sum_{n=1}^{9} \frac{1}{n(n+1)(n+2)}$, written in the rational form be $\frac{p}{q}$ (where p and q are co-prime), then the value of $\left[\frac{q-p}{10}\right]$ is, (where [.] is the greatest integer function)

Q25

If $S_n = \left(1^2 - 1 + 1\right)\left(1!\right) + \left(2^2 - 2 + 1\right)\left(2!\right) + \ldots + \left(n^2 - n + 1\right)(n!)$, then S_{50} is:

(1) $52!$

(2) $1 + 49 \times 51!$

(3) $52! - 1$

(4) $50 \times 51! - 1$

Q26

Let $a_1, a_2, \ldots\ldots, a_n$ be real numbers such that

$\sqrt{a_1} + \sqrt{a_2 - 1} + \sqrt{a_3 - 2} + \ldots + \sqrt{a_n - (n-1)} = \frac{1}{2}\left(a_1 + a_2 + \ldots + a_n\right) - \frac{n(n-3)}{4}$. Compute the value of $\sum_{i=1}^{100} a_i$.

(1) 1010

(2) 505

(3) 2525

(4) 5050

Q27

Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}\ \forall\ k = 3, 4, \ldots\ldots, 11$. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

Q28

$\frac{1.2}{1!} + \frac{2.3}{2!} + \frac{3.4}{3!} + \frac{4.5}{4!} + \ldots\infty =$

(1) $2e$

(2) $3e$

(3) $3e - 1$

(4) e

Q29

The minimum value of sum of real numbers a^{-6}, $2a^{-4}$, $2a^{-3}$, 1 and $2a^{10}$ with $a > 0$ is equal to

(1) 1

(2) 2

(3) 4

(4) 8

Q30

If $a + b + c = 3$ (where $a, b, c > 0$), then the greatest value of $a^2b^3c^2$ is

(1) $\frac{3^{10}2^4}{7^7}$

(2) $\frac{3^9 2^4}{7^7}$

(3) $\frac{3^8 2^4}{7^7}$

(4) $\frac{3^9 2^3}{7^6}$

Answer Key

Q1 (3)	**Q2** (1)	**Q3** (2)	**Q4** (4)
Q5 (1)	**Q6** (1)	**Q7** (4)	**Q8** (1)
Q9 (8)	**Q10** (3)	**Q11** (2)	**Q12** (1)
Q13 (4)	**Q14** (4)	**Q15** (3)	**Q16** (1)
Q17 (1)	**Q18** (3.25)	**Q19** (3)	**Q20** (2)
Q21 (1)	**Q22** (3250)	**Q23** (5)	**Q24** (8)
Q25 (2)	**Q26** (4)	**Q27** (0)	**Q28** (2)
Q29 (4)	**Q30** (1)		

Q1

$S = a_1 + a_2 + a_3 + \ldots + a_{23} + a_{24}$

$S = \frac{24}{2}(a_1 + a_{24}) = 12(a_1 + a_{24})$

Given that, $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow a_1 + a_1 + \cancel{4d} + a_1 + \cancel{9d} + a_{24} - \cancel{9d} + a_{24} - \cancel{4d} + a_{24} = 225$

$\Rightarrow 3(a_1 + a_{24}) = 225$

$\Rightarrow a_1 + a_{24} = 75$

$\Rightarrow S = 12(a_1 + a_{24}) = 12 \times 75 = 900$

Q2

Let the numbers be $\mathrm{a}, \mathrm{ar}, \mathrm{ar}^2, \mathrm{ar}^3$,

$\Rightarrow \mathrm{a} - 2, \mathrm{ar} - 7, \mathrm{ar}^2 - 9$ and $\mathrm{ar}^3 - 5$ are in A. P.

$\Rightarrow (\mathrm{a} - 2) + (\mathrm{ar}^2 - 9) = 2(\mathrm{ar} - 7)$ and $(\mathrm{ar} - 7) + (\mathrm{ar}^3 - 5) = 2(\mathrm{ar}^2 - 9)$

$\Rightarrow \mathrm{a} + \mathrm{ar}^2 = 2\mathrm{ar} - 3$ and $\mathrm{ar} + \mathrm{ar}^3 = 2\mathrm{ar}^2 - 6$

$\Rightarrow 1 + \mathrm{r}^2 = 2\mathrm{r} - \frac{3}{\mathrm{a}}$ and $1 + \mathrm{r}^2 = 2\mathrm{r} - \frac{6}{\mathrm{ar}}$

$\Rightarrow \frac{3}{\mathrm{a}} = \frac{6}{\mathrm{ar}} \Rightarrow \mathrm{r} = 2$

$\Rightarrow \frac{3}{\mathrm{a}} = 2\mathrm{r} - 1 - \mathrm{r}^2 = 4 - 1 - 4 = -1 \Rightarrow \mathrm{a} = -3$

So, the numbers are $-3, -6, -12, -24$

Q3

$a + 3c = 2b$ and $b^2 = 4ac$

$\Rightarrow c = \frac{2b-a}{3} = \frac{b^2}{4a}$

$\therefore 8ab - 4a^2 = 3b^2 \Rightarrow 4a^2 - 8ab + 3b^2 = 0$

$4\frac{a^2}{b^2} - 8\frac{a}{b} + 3 = 0 \Rightarrow \frac{a}{b} = \frac{8 \pm \sqrt{64-4\times4\times3}}{2\times4}$

$\frac{a}{b} = \frac{2\pm\sqrt{4-3}}{2} = \frac{2\pm1}{2} = \frac{3}{2}, \frac{1}{2}.$

Q4

$a_2 = a_1 r, a_3 = a_1 r^2$

$a_3 > a_2 + 2a_1$

$\Rightarrow a_1 r^2 > a_1 r + 2a_1$

$\Rightarrow r^2 - r - 2 > 0$

$\Rightarrow (r-2)(r+1) > 0$

$\Rightarrow r < -1$ or $r > 2$

Q5

As we know that $\frac{x+y}{1} \times \frac{x-y}{x-y} = \frac{x^2-y^2}{x-y}$ similarly $x^2 + xy + y^2 = \frac{x^3-y^3}{x-y}$

$\Rightarrow \frac{x^2-y^2}{x-y} + \frac{x^3-y^3}{x-y} + \frac{x^4-y^4}{x-y} + \ldots \infty$

$\Rightarrow \quad \frac{1}{x-y}\left[\left(x^2 + x^3 + \ldots \infty\right) - \left(y^2 + y^3 + \ldots \infty\right)\right]$

Now we will find the sum of infinite G. P.series

$\Rightarrow \quad \frac{1}{x-y}\left[\frac{x^2}{1-x} - \frac{y^2}{1-y}\right]$

$\Rightarrow \left[\frac{x+y-xy}{1-x-y+xy}\right]$

Q6

Case-I : $x < \frac{1}{3} \Rightarrow -3x + 1, 3, -x + 3$ are in $A.P.$

$\Rightarrow 6 = -3x + 1 - x + 3$

$\Rightarrow 4x = -2 \Rightarrow x = -\frac{1}{2}$

$\Rightarrow$ terms are $\frac{5}{2}$, 3 , $\frac{7}{2}$, 4 , $\frac{9}{2}$

$\Rightarrow$ sum $= \frac{35}{2}$

Case-II: $\frac{1}{3} \leq x < 3$

$3x - 1, 3, -x + 3$ are in A.P.

$\Rightarrow 6 = 3x - 1 - x + 3 = 2x + 2 \Rightarrow x = 2$

$\Rightarrow$ terms are 5 , 3 , 1 , -1 , -3

$\Rightarrow$ sum $= 5$

Case-III: $x \geq 3$

$\Rightarrow$ terms are $3x - 1, 3, x - 3$

$\Rightarrow 6 = 3x - 1 + x - 3 = 4x - 4$

$\Rightarrow 4x = 10 \Rightarrow x = \frac{5}{2}$ not possible

Q7

$$(16x)^2 + (4y)^2 + (z)^2 - (16x)(4y) - (4y)(z) - (z)(16x) = 0$$

$$\Rightarrow \frac{1}{2}\left[(16x - 4y)^2 + (4y - z)^2 + (z - 16x)^2\right] = 0 \Rightarrow 16x = 4y = z$$

Q8

Let the first term of the given sequence be a

First 11 terms are: $a,\ a + 2, \ldots a + 20$

Since, eleventh term of the sequence is the first of last 11 terms of this sequence.

Last 11 terms are:$a + 20,\ (a + 20). 2, \ldots (a + 20) \cdot 2^{10}$

Thus, the sequence obtained is as follows:

$a,\ a+2,\ \ldots,\ a+20,\ (a+20)2,\ (a+20)2^2,\ \ldots\ldots,\ (a+20)2^{10}$

Now middle term of the AP is the $\left(\frac{(11+1)}{2}\right)^{\text{th}}$ term, i.e., 6^{th} term and the same goes for GP.

So, $(a+10)=(a+20)2^5$

$\Rightarrow 10-20\cdot 2^5=a(2^5-1)\Rightarrow 10\frac{(1-2^6)}{2^5-1}=a$

Middle term of the whole sequence $=T_{11}=a+20=10\frac{(1-2^6)}{2^5-1}+20$

$=10\left[\frac{-1}{2^5-1}\right]=\frac{10}{1-2^5}$

Q9

$\left(\frac{1}{a_1}-\frac{1}{a_2}\right)+\left(\frac{1}{a_2}-\frac{1}{a_3}\right)+\ldots\left(\frac{1}{a_9}-\frac{1}{a_{10}}\right)=\frac{9\text{ d}}{64}$

$\Rightarrow \frac{a_{10}-a_1}{a_1a_{10}}=\frac{9\text{ d}}{64}$

$\Rightarrow a_1a_{10}=64$

Also $(a_1+a_{10})\left(\frac{1}{a_1a_{10}}+\ldots\frac{1}{a_{10}a_1}\right)=\frac{a_1+a_{10}}{10}\left(\frac{1}{a_1}+\ldots\frac{1}{a_{10}}\right)$

$\Rightarrow 2\left(\frac{1}{a_1}+\ldots\frac{1}{a_{10}}\right)=\frac{a_1+a_{10}}{10}\left(\frac{1}{a_1}+\ldots\frac{1}{a_{10}}\right)$

$\Rightarrow a_1+a_{10}=20$

$a_1a_{10}=64$ ...(1)

$a_1+a_{10}=20$ ...(2)

From (1) & (2)

$a_1=4$ & $a_{10}=16$

$4\left(\frac{a_1}{a_{10}}+\frac{a_{10}}{a_1}\right)=4\left(\frac{4}{16}+\frac{16}{4}\right)=17$

Sum of digits of $17=1+7=8$

Q10

$a+b+c=25; 2a=2+b; c^2=18b$

$\Rightarrow 2a+2b+2c=50$

$\Rightarrow 2+b+2b+2c=50$

$\Rightarrow 3b+2c=48 \Rightarrow \frac{c^2}{6}+2c=48$

$\Rightarrow c^2+12c-48\times 6=0 \Rightarrow c^2+12c-24\times 12=0$

$\Rightarrow (c+24)(c-12)=0$

$\Rightarrow c=12,-24 \Rightarrow c=12$ (between 2 and 18)

$\Rightarrow b=\frac{c^2}{18}=\frac{144}{18}=8$

$\Rightarrow a=\frac{b+2}{2}=5$

$\Rightarrow a=5,\ b=8$ and $c=12$

$\Rightarrow c-a=7$

Q11

Given, harmonic mean $H=4$

We know that, $G^2=AH$

Since, $2A+G^2=27$

$\Rightarrow 2A+AH=27$

$\Rightarrow 2A+4A=27$

$\Rightarrow A=\frac{27}{6}=\frac{9}{2}=\frac{a+b}{2} \Rightarrow a+b=9$

$G^2=AH=\frac{9}{2}\times 4=18$

$\Rightarrow ab=18$

$|a-b|=\sqrt{(a+b)^2-4ab}=\sqrt{81-4\times 18}=3$

$\Rightarrow \alpha=9,\ \beta=3$

$\Rightarrow \frac{\alpha}{\beta}=3$

Q12

Let, $A_1, A_2, A_3, \ldots\ldots, A_{11}$ be 11 A.M.'s between 28 and 10

$\Rightarrow 28, A_1, A_2, \ldots\ldots, A_{11}, 10$ are in A.P

Let, d be the common difference of A.P.

Also, the number of terms $= 13$

$\Rightarrow 10 = \mathrm{T}_{13} = \mathrm{T}_1 + 12\mathrm{d} = 28 + 12\mathrm{d}$

$\Rightarrow d = \frac{10-28}{12} = \frac{-18}{12} = \frac{-3}{2}$

$\Rightarrow$ Number of integral A.M.'s are 5

Q13

In the n^{th} set of observation, the total observations are 'n'

and the first observation of n^{th} set is

$1 + 2 + 3 + \ldots + (n-1) + 1 = \frac{(n-1)n}{2} + 1 = a$

Sum of all the n observations in the n^{th} set

$= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}\left[2\left\{\frac{(n-1)n}{2} + 1\right\} + (n-1)\right]$

$= \frac{n}{2}\left[n^2 - n + 2 + n - 1\right] = n\left(\frac{n^2+1}{2}\right)$

Mean $= \frac{n\frac{\left(n^2+1\right)}{2}}{n} = \frac{n^2+1}{2}$

Hence, for $n = 13$

Mean $= \frac{13^2+1}{2} = \frac{170}{2} = 85$

Q14

$S_1 \equiv 3,\ 7,\ 11,\ 15, \ldots\ldots \Rightarrow\ c.d = 4 \Rightarrow d_1 = 4$

$S_2 \equiv 1,\ 6,\ 11,\ 16, \ldots\ldots \Rightarrow\ c.d = 5 \Rightarrow d_2 = 5$

$\Rightarrow$Every 5^{th} term of S_1 and 4^{th} term of S_2 will be same

$\Rightarrow$ Terms common to both AP will have $a = 11$ and $d = 20$

Hence, $S_{20} = \frac{20}{2}[(2 \times 11) + (20 - 1)\,20]$

$= 10 \times 402$

$= 4020$

Q15

$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots + \infty$ ... (1)

$\frac{S}{5} = 0 + \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \ldots + \infty$ ... (2)

On subtracting both the eq (1) and (2), we get,

$\frac{4}{5}S = 1 + \frac{3}{5} + \frac{3}{5^2} + \ldots + \infty$

$S = \frac{35}{16}$

Q16

$S = \frac{7}{17} + \frac{77}{17^2} + \frac{777}{17^3} + \ldots\ldots.$

$\frac{S}{17} = \frac{7}{17^2} + \frac{77}{17^3} + \frac{777}{17^4} + \ldots\ldots..$

Subtracting both the above results, we get,

$\frac{16}{17}S = \frac{7}{17} + \frac{70}{17^2} + \frac{700}{17^3} + \ldots\ldots.$
$\left(\text{Infinite G. P. with r } = \frac{10}{17}\right)$
$\frac{16}{17}S = \frac{\frac{7}{17}}{1-\frac{10}{17}} = \frac{\frac{7}{17}}{\frac{7}{17}} = 1$
$S = \frac{17}{16}$

Q17

$x = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots.\infty$

$= \left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty\right) - \left(\frac{1}{2^4} + \frac{1}{4^4} + \ldots\infty\right)$

$= \frac{\pi^4}{90} - \frac{1}{16}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots..\infty\right)$

$= \frac{\pi^4}{90} - \frac{1}{16}.\frac{\pi^4}{90} = \frac{\pi^4}{96}.$

Q18

$S = \sum_{r=1}^{25} r(26-r) = 26\sum_{r=1}^{25} r - \sum_{r=1}^{25} r^2$
$= 26 \times \frac{25\times26}{2} - \frac{25\times26\times51}{6}$
$= \frac{25\times26}{2}\left(26 - \frac{51}{3}\right) = \frac{25\times \cancel{26}13}{\cancel{2}} \times 9$
$\Rightarrow \frac{S}{900} = \frac{13}{4} = 3.25$

Q19

$0.2 + 0.22 + 0.222 + \ldots n$ terms

$= 2(0.1 + 0.11 + 0.111 + \ldots n \text{ terms})$

$$= 2\left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n \text{ terms}\right)$$

$$= \frac{2}{9}\left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots n \text{ terms}\right)$$

$$= \frac{2}{9}\left(1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \ldots n \text{ terms}\right)$$

$$= \frac{2}{9}\left[n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots n\right)\right]$$

$$= \frac{2}{9}\left[n - \frac{1}{10}\frac{\left\{1 - \left(\frac{1}{10}\right)^n\right\}}{\left(1 - \frac{1}{10}\right)}\right]$$

$$= \frac{2}{9}\left[n - \frac{1}{10} \times \frac{10}{9} \cdot \left(\frac{10^n - 1}{10^n}\right)\right]$$

$$= \frac{2}{9}\left[n - \frac{1}{9}\left(1 - 10^{-n}\right)\right]$$

Q20

$$\mathrm{S}_{20} = \frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots. + \text{up to } 20 \text{ terms}$$

$$= \sum_{\mathrm{r}=1}^{20} \frac{(2\mathrm{r}+1)}{1^2+2^2+3^2+\ldots. \mathrm{r}^2}$$

$$= \sum_{\mathrm{r}=1}^{20} \frac{2\mathrm{r}+1}{\frac{\mathrm{r}}{6}(\mathrm{r}+1)(2\mathrm{r}+1)}$$

$$= 6\sum_{\mathrm{r}=1}^{20} \frac{1}{\mathrm{r}(\mathrm{r}+1)}$$

$$= 6\sum_{\mathrm{r}=1}^{20}\left(\frac{1}{\mathrm{r}} - \frac{1}{\mathrm{r}+1}\right)$$

$= 6\left\{\left[\frac{1}{1}-\frac{1}{2}\right]+\left[\frac{1}{2}-\frac{1}{3}\right]+\left[\frac{1}{3}-\frac{1}{4}\right]+\ldots.+\left[\frac{1}{20}-\frac{1}{21}\right]\right\}$

$= 6\left(1-\frac{1}{21}\right)$

$= \frac{6\times20}{21}$

$= \frac{120}{21}$

$= \frac{\mathrm{k}}{21}$

$\mathrm{k} = 120$

Q21

Given, $S=\sum_{n=1}^{9999}\frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}$

$=\sum_{n=1}^{9999}\frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}\times\frac{(\sqrt[4]{n+1}-\sqrt[4]{n})}{(\sqrt[4]{n+1}-\sqrt[4]{n})}$

$=\sum_{n=1}^{9999}\frac{(\sqrt[4]{n+1}-\sqrt[4]{n})}{(\sqrt{n}+\sqrt{n+1})(\sqrt{n+1}-\sqrt{n})}$

$=\sum_{n=1}^{9999}(\sqrt[4]{n+1}-\sqrt[4]{n})$

$=\left(2^{\frac{1}{4}}-1\right)+\left(3^{\frac{1}{4}}-2^{\frac{1}{4}}\right)+\left(4^{\frac{1}{4}}-3^{\frac{1}{4}}\right)+\ldots+\left(10000^{\frac{1}{4}}-9999^{\frac{1}{4}}\right)$

$=10000^{\frac{1}{4}}-1=10-1=9$

Q22

$T_r=\frac{1}{2}\left\{\frac{(r^2+r+1)-(r^2-r+1)}{(r^2+r+1)(r^2-r+1)}\right\}$

$=\frac{1}{2}\left\{\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right\}$

$=\frac{1}{2}\left\{\frac{1}{r^2-r+1}-\frac{1}{(r+1)^2-(r+1)+1}\right\}$

$=\frac{1}{2}\{f(r)-f(r+1)\}$

$S=\Sigma T_r=\sum_{r=1}^{80}\frac{1}{2}(f(\alpha)-f(\alpha+1))$
$=\frac{1}{2}\left\{f(1)-f\cancel{(2)}+f\cancel{(2)}-f\cancel{(3)}\ldots\ldots+\cancel{f(80)}-f(81)\right\}$
$=\frac{1}{2}(f(1)-f(81))=\frac{1}{2}\left(1-\frac{1}{81^2-81+1}\right)$
$=\frac{1}{2}\left(1-\frac{1}{6481}\right)\Rightarrow 6481S=\frac{6480}{2}=3240$

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Q23

r^{th} term, $\mathrm{T}_r=\frac{(1+2+3+\ldots\ldots+r)^2}{(1+3+5+\ldots+(2r-1))}$
$=\frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}=\frac{(r+1)^2}{4}=\frac{r^2+2r+1}{4}$
$S_{10}=\mathrm{K}=\frac{1}{4}\sum_{r=1}^{10}(r^2+2r+1)$
$\Rightarrow 4\mathrm{K}=\sum_{r=1}^{10}r^2+2\sum_{r=1}^{10}r+\sum_{r=1}^{10}1$
$=\frac{10\times11\times21}{6}+2\frac{10\times11}{2}+10$
$=385+110+10=505$
$\frac{4\mathrm{K}}{101}=5$

Q24

$\sum_{n=1}^{9}\frac{1}{2}\frac{n+2-n}{n(n+1)(n+2)}$

$=\frac{1}{2}\left(\left(\frac{1}{1.2}-\frac{1}{2.3}\right)+\left(\frac{1}{2.3}-\frac{1}{3.4}\right)+\ldots+\left(\frac{1}{9\times10}-\frac{1}{10\times11}\right)\right)$

$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{110}\right)=\frac{1}{2}\left(\frac{55-1}{110}\right)=\frac{27}{110}$

$\Rightarrow\quad q-p=110-27=83$

Q25

Given,

$S_n = (1^2 - 1 + 1)(1!) + (2^2 - 2 + 1)(2!) + \cdots + (n^2 - n + 1)(n!)$

Its general term t_r is given as,

$(r^2 - r + 1)r!$

$t_r = \left[(r^2 - 1) - (r - 2)\right](r!)$

$t_r = (r - 1)(r + 1)! - (r - 2)(r)!$

$S_n = \sum_{r=1}^{n} t_r$

$S_n = (0 - (-1)) + (3! - 0) + (2(4)! - 3!) + \ldots + (n - 1)(n + 1)! - (n - 2)n!$

$S_{50} = 1 + 49(51)!$.

Q26

Let $\sqrt{a_1} = b_1;$

$\sqrt{a_2 - 1} = b_2;$

$\sqrt{a_3 - 2} = b_3;$

$\sqrt{a_n - (n - 1)} = b_n$

$\therefore$ LHS $= b_1 + b_2 + \ldots + b_n = \frac{1}{2}\left[b_1^2 + (b_2^2 + 1) + (b_3^2 + 2) + \ldots + (b_n^2 + (n - 1))\right] - \frac{n(n-3)}{4}$

$\therefore\ \Sigma b_i = \frac{1}{2}\left[(b_1^2 + b_2^2 + b_3^2 + \ldots + b_n^2) + (1 + 2 + 3 + \ldots + (n - 1))\right] - \frac{n(n-3)}{4}$

$\Rightarrow 2\Sigma b_i = \Sigma b_i^2 + \frac{n(n-1)}{2} - \frac{n(n-3)}{2}$

$\Rightarrow 2\Sigma b_i = \Sigma b_i^2 + n$

$\therefore\ \Sigma b_i^2 - 2\Sigma b_i + \Sigma 1 = 0$

$\therefore \sum_{i=1}^{n} (b_i - 1)^2 = 0$

$b_1 - 1 = 0 \Rightarrow b_1^2 = a_1 = 1$

$b_2 - 1 = 0 \Rightarrow b_2^2 = a_2 - 1 = 1 \Rightarrow a_2 = 2$

$b_3 - 1 = 0 \Rightarrow b_3^2 = a3 - 2 = 1 \Rightarrow a_3 = 3$ and so on.

Hence, $a_n = n$.

$\therefore \sum_{i=1}^{100} a_i = 1 + 2 + 3 + \ldots + 100 = 5050$

Q27

$a_k = 2a_{k-1} - a_{k-2}$

$\Rightarrow a_1, a_2, \ldots, a_{11}$ are in AP with let common difference be d

$\therefore\ \frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11\mathrm{d}^2 + 110ad}{11} = 90$

$\Rightarrow 225 + 35\mathrm{d}^2 + 150\mathrm{d} = 90$

$35\mathrm{d}^2 + 150\mathrm{d} + 135 = 0$

$\Rightarrow \mathrm{d} = -3, -\frac{9}{7}$

Given, $a_2 < \frac{27}{2} \therefore \text{d} = -3 \text{ and d} \neq -\frac{9}{7}$

Hence, $a_1 + a_2 + \ldots\ldots a_{11} = \frac{11}{2}[2 \times 15 + 10(-3)] = 0$

Q28

$\frac{1.2}{1!} + \frac{2.3}{2!} + \frac{3.4}{3!} + \frac{4.5}{4!} + \ldots\infty,$

Here, $T_n = \frac{n(n+1)}{n!} = \frac{n+1}{(n-1)!} = \frac{(n-1)+2}{(n-1)!}$

$= \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$

$\Rightarrow \quad S = \Sigma T_n = e + 2e = 3e$

Q29

$\because \; A.M. \geq G.M.$

$\Rightarrow \frac{a^{-6}+a^{-4}+a^{-4}+a^{-3}+a^{-3}+1+a^{10}+a^{10}}{8} \geq 1$

$\therefore$ Minimum value $= 8$

Q30

Given, $a + b + c = 3$

$\Rightarrow 2 \cdot \frac{a}{2} + 3 \cdot \frac{b}{3} + 2 \cdot \frac{c}{2} = 3$

$\frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}}{7} \geq \left(\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2\right)^{\frac{1}{7}}$

$\frac{a+b+c}{7} \geq \left(\frac{a^2b^3c^2}{2^4 3^3}\right)^{\frac{1}{7}}$

$\left(\frac{3}{7}\right)^7 \geq \frac{a^2b^3c^2}{2^4 3^3}$

Hints and Solutions **MathonGo**

$$\Rightarrow a^2b^3c^2 \leq \frac{3^7}{7^7} \times 2^4 \times 3^3 = \frac{2^4 \times 3^{10}}{7^7}$$

Hence, maximum value $= \frac{2^4 \times 3^{10}}{7^7}$