Sample Task **Questions**

Questions with Answer Keys **MathonGo**

Q1

The value of x for which $\sin\left(\cot^{-1}(1+x)\right)=\cos\left(\tan^{-1}x\right)$ is

(1) $\frac{1}{2}$

(2) 1

(3) 0

(4) $-\frac{1}{2}$

Q2

The value of $\cos\left(\frac{1}{2}\cos^{-1}\left(\cos\left(\sin^{-1}\frac{\sqrt{63}}{8}\right)\right)\right)$ is

(1) $\frac{3}{16}$

(2) $\frac{3}{8}$

(3) $\frac{3}{4}$

(4) $\frac{3}{2}$

Q3

The value of $\sin^{-1}\sin 17+\cos^{-1}\cos 10$ is equal to

(1) 27

(2) -27

(3) $17-5\pi$

(4) $9\pi-27$

Q4

The value of $\sin\left\{\cot^{-1}\left[\cos\left(\cot^{-1}\left(\frac{1}{x}\right)\right)\right]\right\}$ is equal to $(x > 0)$

(1) $\sqrt{\frac{1+x^2}{2+x^2}}$

(2) $\sqrt{\frac{1-x^2}{2+x^2}}$

(3) $\sqrt{\frac{1+x^2}{2-x^2}}$

(4) $\sqrt{\frac{2+x^2}{1+x^2}}$

Q5

If the value of the expression $\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right)$ is in the form of $a+\sqrt{b}$ where $a,\ b \in Z,$ then the value of $\frac{a+b}{b}$ is

Q6

If $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3} = \sin^{-1}x$, then the value of x is

(1) 0

(2) $\frac{\left(\sqrt{5}-4\sqrt{2}\right)}{9}$

(3) $\frac{\left(\sqrt{5}+4\sqrt{2}\right)}{9}$

(4) $\frac{\pi}{2}$

Q7

The complete solution set of the inequality $\cos^{-1}\left(\cos 4\right) > 3x^2 - 4x$ is

(1) $\left(0, \frac{2+\sqrt{6\pi - 8}}{3}\right)$

(2) $\left(\frac{2-\sqrt{6\pi - 8}}{3}, 0\right)$

(3) $(-2, 2)$

(4) $\left(\frac{2-\sqrt{6\pi - 8}}{3}, \frac{2+\sqrt{6\pi - 8}}{3}\right)$

Q8

If $x = \sin\left(2\tan^{-1}3\right)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$, then

(1) $2x = 1 - y$

(2) $x^2 = 1 - 2y$

(3) $x^2 = 1 + y$

(4) $y^2 = 2x - 1$

Q9

$\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(1) 1

(2) 2

(3) $\frac{1}{4}$

(4) $\frac{5}{4}$

Q10

The value of $2\sin^{-1}\frac{4}{5}+2\sin^{-1}\frac{5}{13}+2\sin^{-1}\frac{16}{65}$ is equal to

(1) $\frac{3\pi}{2}$

(2) $\frac{\pi}{2}$

(3) π

(4) 2π

Q11

The number of solution of the equation $2\tan^{-1}x+\cot^{-1}x=\frac{7\pi}{6}$ is

(1) 0

(2) 1

(3) 2

(4) 3

Q12

The value of $\tan^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]\left(\forall x\in\left[0,\frac{\pi}{2}\right]\right)$ is equal to

(1) $\frac{x}{2}-\frac{\pi}{2}$

(2) $\frac{x}{2}+\frac{\pi}{2}$

(3) $\frac{x}{2}-\pi$

(4) $\frac{\pi}{2}-\frac{x}{2}$

Q13

If the equation $\sin^{-1}\left(4x^2 - 12x + 10\right) + \cos^{-1}\left(12x - 4x^2 - 10\right) + \lambda x = 0$ has a real solution, then λ is equal to

(1) $\frac{\pi}{4}$

(2) $-\pi$

(3) $\frac{\pi}{2}$

(4) $\frac{-\pi}{2}$

Q14

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then

(1) $x^2 + y^2 + z^2 + xyz = 0$

(2) $x^2 + y^2 + z^2 + 2xyz = 0$

(3) $x^2 + y^2 + z^2 + xyz = 1$

(4) $x^2 + y^2 + z^2 + 2xyz = 1$

Q15

If $\left(\cot^{-1}x\right)^2 - 7\left(\cot^{-1}x\right) + 10 > 0$, then the range of x will be

(1) $(-\infty, \cot 2)$

(2) $(-\infty, \cot 5)$

(3) $(\cot 2, \cot 5)$

(4) $(\cot 2, \infty)$

Q16

The value of a for which $ax^2 + \sin^{-1}\left(x^2 - 2x + 2\right) + \cos^{-1}\left(x^2 - 2x + 2\right) = 0$ has a real solution, is

(1) $-\frac{2}{\pi}$

(2) $\frac{2}{\pi}$

(3) - $\frac{\pi}{2}$

(4) $\frac{\pi}{2}$

Q17

Number of real roots of the equation $\sin^{-1}\sin x = \cos^{-1}\cos 4$ in $[0, 2\pi]$ is

(1) 0

(2) 1

(3) 2

(4) more than 2

Q18

The greatest and the least value of $\left(\sin^{-1}x\right)^3 + \left(\cos^{-1}x\right)^3$ are

(1) $-\frac{\pi}{2}, \frac{\pi}{2}$

(2) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$

(3) $\frac{7\pi^3}{8}, \frac{\pi^3}{32}$

(4) None of these

Q19

The real solutions of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$ are

(1) -1, 0

(2) 0, 1

(3) -1, 1

(4) $-1,\ 2$

Q20

The number of integers for which the equation $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x = n$ has real solution(s) is

(1) 0

(2) 1

(3) 2

(4) 3

Q21

Let $a = \left(\sin^{-1}x\right)^{\sin^{-1}x}, b = \left(\sin^{-1}x\right)^{\cos^{-1}x},\ c = \left(\cos^{-1}x\right)^{\sin^{-1}x}, d = \left(\cos^{-1}x\right)^{\cos^{-1}x}$ and if $x \in (0, 1)$, then

(1) $a > b > d > c$

(2) $d > c > a > b$

(3) $b > a > d > c$

(4) $a < b < d < c$

Q22

The value of $tan^{-1}\left(\frac{9}{19}\right) + tan^{-1}\left(\frac{9}{49}\right) + tan^{-1}\left(\frac{9}{97}\right) + tan^{-1}\left(\frac{9}{163}\right) + \ \ldots \infty$ equals

(1) $\tan^{-1}(3)$

(2) $\tan^{-1}\left(\frac{1}{3}\right)$

(3) $\tan^{-1}\left(\frac{2}{3}\right)$

(4) $\tan^{-1}\left(\frac{3}{2}\right)$

Q23

If $y=\tan^{-1}\frac{1}{1+x+x^2}+\tan^{-1}\frac{1}{x^2+3x+3}+\tan^{-1}\frac{1}{x^2+5x+7}+\ldots+$ upto $2n$ terms ($\forall x \geq 0$), then $y(0)$ is

(1) $\tan^{-1}(n)$

(2) $\tan^{-1}(2n)$

(3) $2\tan^{-1}(n)$

(4) 0

Q24

The value of the expression $\cot^{-1}\frac{1}{2}+\cot^{-1}\frac{9}{2}+\cot^{-1}\frac{25}{2}+\cot^{-1}\frac{49}{2}+\ldots\ldots$ upto n terms is

(1) $\tan^{-1}2n$

(2) $\tan^{-1}(2n-1)$

(3) $\tan^{-1}n$

(4) $\tan^{-1}2n-\tan^{-1}1$

Q25

The value(s) of x satisfying the equation $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ is/are

(1) 0

(2) $\frac{1}{2}$

(3) $0, \frac{1}{2}$

(4) $-\frac{1}{2}$

Q26

Solution set of $\left[\sin^{-1}x\right] > \left[\cos^{-1}x\right]$, where [.] denotes the greatest integer function, is

(1) $\left[\frac{1}{\sqrt{2}}, 1\right]$

(2) $(\cos 1, \sin 1)$

(3) $[\sin 1, 1]$

(4) None of these

Q27

If $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots$ upto 100 terms, then α is:

(1) 1.01

(2) 1.00

(3) 1.02

(4) 1.03

Q28

For $k \in \mathbb{R}$, let the solutions of the equation $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right) = k, 0 < |x| < \frac{1}{\sqrt{2}}$ be α and β, where the inverse trigonometric functions take only principal values. If the solutions of the equation $x^2 - bx - 5 = 0$ are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to ______.

Q29

The number of solutions of the equation $\sin^{-1}x = (\sin x)^{-1}$ is/are

(1) one

(2) two

(3) three

(4) zero

Q30

The number of real solutions (x, y) where $|y| = \sin x, y = \cos^{-1}(\cos x), -2\pi \le x \le 2\pi$, is

(1) 2

(2) 1

(3) 3

(4) 4

Answer Key

Q1 (4)	**Q2** (3)	**Q3** (4)	**Q4** (1)
Q5 (0.6)	**Q6** (3)	**Q7** (4)	**Q8** (4)
Q9 (2)	**Q10** (3)	**Q11** (1)	**Q12** (1)
Q13 (2)	**Q14** (4)	**Q15** (4)	**Q16** (3)
Q17 (1)	**Q18** (3)	**Q19** (1)	**Q20** (3)
Q21 (2)	**Q22** (4)	**Q23** (2)	**Q24** (1)
Q25 (1)	**Q26** (3)	**Q27** (1)	**Q28** (12)
Q29 (2)	**Q30** (1)		

Sample Task **Solutions**

Hints and Solutions **MathonGo**

Q1

Let, $\cot^{-1}(x+1) = \theta$ and $\tan^{-1} x = \phi$

Now, given equation becomes $\sin\theta = \cos\phi$

$\Rightarrow \frac{1}{\sqrt{1+(x+1)^2}} = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow (x+1)^2 + 1 = x^2 + 1 \Rightarrow x = -\frac{1}{2}$

Q2

The given trigonometric ratio

$= \cos\left(\frac{1}{2}\cos^{-1}\left(\cos\left(\cos^{-1}\frac{1}{8}\right)\right)\right)$

$= c \ \cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$

$= \sqrt{\frac{1+\cos\left(\cos^{-1}\frac{1}{8}\right)}{2}} = \frac{3}{4}$

Note: One may also proceed by writing the

ratio as $\cos\left(\frac{1}{2}\sin^{-1}\frac{\sqrt{63}}{8}\right)$.

Q3

$\sin^{-1}\sin 17 = \sin^{-1}\sin(17 - 5\pi + 5\pi)$

$= 5\pi - 17$

$\cos^{-1}(\cos 10) = \cos^{-1}\cos(10 - 3\pi + 3\pi)$

$= \cos^{-1}\cos\{3\pi + (10 - 3\pi)\}$

$= \cos^{-1}\{-\cos(10-3\pi)\}$

$= \pi - \cos^{-1}\cos(10-3\pi)$

$= \pi - (10-3\pi) = 4\pi - 10$

Hence, $\sin^{-1}\sin 17 + \cos^{-1}(\cos 10) = 9\pi - 27$

Q4

Let $\cot^{-1}\left(\frac{1}{x}\right) = \alpha \Rightarrow \tan\alpha = x$

So, $\cos\alpha = \frac{1}{\sqrt{1+x^2}}$

$\therefore \sin\left\{\cot^{-1}\left[\cos\left(\tan^{-1}x\right)\right]\right\}$

$= \sin\left(\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

Let $\cot^{-1}\frac{1}{\sqrt{1+x^2}} = \beta \Rightarrow \cot\beta = \frac{1}{\sqrt{1+x^2}}$

$\therefore \sin\beta = \frac{\sqrt{(1+x^2)}}{\sqrt{(2+x^2)}}$

$\therefore \sin\beta = \sin\left\{\cot^{-1}\left[\cos\left(\cot^{-1}\left(\frac{1}{x}\right)\right)\right]\right\} = \sqrt{\frac{1+x^2}{2+x^2}}$

Q5

We know that,

$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x}$

$\therefore \tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right) = \frac{1-\cos\left(\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)}{\sin\left(\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)}$

$= \frac{1-2/\sqrt{5}}{\sin\left(\sin^{-1}\left(1/\sqrt{5}\right)\right)}$

$= \frac{1-2/\sqrt{5}}{1/\sqrt{5}} = \sqrt{5} - 2$

Now, $a+\sqrt{b}=-2+\sqrt{5}$

$\Rightarrow a=-2$ and $b=5$

Hence, $\frac{a+b}{b}=\frac{3}{5}=0.6$

Q6

Given that, $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{1}x$

Taking sine on both sides

$\Rightarrow\left(\frac{1}{3}\sqrt{1-\frac{4}{9}}+\frac{2}{3}\sqrt{1-\frac{1}{9}}\right)=x$

$\Rightarrow\left(\frac{1}{3}\cdot\frac{\sqrt{5}}{3}+\frac{2}{3}\cdot\frac{\sqrt{8}}{3}\right)=x$

$\Rightarrow\left(\frac{\sqrt{5}+4\sqrt{2}}{9}\right)=x$

$\therefore\quad x=\left(\frac{\sqrt{5}+4\sqrt{2}}{9}\right)$

Q7

As, $\cos^{-1}(\cos 4)=\cos^{-1}\{\cos(2\pi-4)\}=2\pi-4$

$\Rightarrow\quad 2\pi-4>3x^2-4x$

$\Rightarrow\quad 3x^2-4x-(2\pi-4)<0$

$\Rightarrow \quad \frac{2-\sqrt{6\pi-8}}{3} < x < \frac{2+\sqrt{6\pi-8}}{3}$

Q8

Let, $x = \sin 2\theta$ (where $\tan\theta = 3$)

$\Rightarrow x = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{6}{1+9} = \frac{3}{5}$

If $\alpha = \tan^{-1}\frac{4}{3}$

$\Rightarrow y = \sin\left(\frac{\alpha}{2}\right) = \frac{1}{\sqrt{2}}\sqrt{1-\cos\alpha}$

$= \frac{1}{\sqrt{2}}\sqrt{1-\frac{3}{5}} = \frac{1}{\sqrt{5}}$

$\therefore y^2 = \frac{1}{5} \Rightarrow y^2 = 2x - 1$

Q9

Given,

$\tan\left(2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{\sqrt{5}}{2} + 2\tan^{-1}\frac{1}{8}\right)$

$= \tan\left(2\left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left(2\left(\tan^{-1}\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right]$

$= \tan\left[\tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right]$

$= \tan\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{2}\right)\right]$

$= \tan\left[\tan^{-1}\frac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}\right]$

$= \tan\tan^{-1}\frac{\frac{5}{4}}{\frac{5}{8}}$

$= \tan\tan^{-1} 2$

$= 2$

Q10

$2\left[\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}\right]$

$= 2\sin^{-1}\left(\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}+\frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right)+2\sin^{-1}\frac{16}{65}$

$= 2\sin^{-1}\left(\frac{48}{65}+\frac{15}{65}\right)+2\sin^{-1}\left(\frac{16}{65}\right)$

$= 2\left[\sin^{-1}\left(\frac{63}{65}\right)+\sin^{-1}\left(\frac{16}{65}\right)\right]$

$= 2\left[\cos^{-1}\left(\frac{16}{65}\right)+\sin^{-1}\left(\frac{16}{65}\right)\right]\left(\because \sin^{-1}x=\cos^{-1}\sqrt{1-x^2}\right)$

$= \pi$

Q11

$2\tan^{-1}x+\cot^{-1}x=\frac{7\pi}{6}$

$\Rightarrow \tan^{-1}x+\frac{\pi}{2}=\frac{7\pi}{6} \Rightarrow \tan^{-1}x=\frac{2\pi}{3}$

$\Rightarrow \tan^{-1}x \in \left(\frac{-\pi}{2},\ \frac{\pi}{2}\right)(x\in\phi)$

Q12

$\tan^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$

$$= \tan^{-1}\left[\frac{(\sqrt{1-\sin x}+\sqrt{1+\sin x})}{(\sqrt{1-\sin x}-\sqrt{1+\sin x})}\,\frac{(\sqrt{1-\sin x}+\sqrt{1+\sin x})}{(\sqrt{1-\sin x}+\sqrt{1+\sin x})}\right]$$

$$= \tan^{-1}\left[\frac{(1-\sin x)+(1+\sin x)+2\sqrt{1-\sin^2 x}}{(1-\sin x)-(1+\sin x)}\right] = \tan^{-1}\left[\frac{2(1+\cos x)}{-2\sin x}\right]$$

$$= \tan^{-1}\left[\frac{-2\cos^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right] = \tan^{-1}\left(-\cot\frac{x}{2}\right) = \tan^{-1}\left[\cot\left(\pi - \frac{x}{2}\right)\right]$$

$$= \frac{\pi}{2} - \cot^{-1}\left[\cot\left(\pi - \frac{x}{2}\right)\right] = \frac{\pi}{2} - \left(\pi - \frac{x}{2}\right) = \frac{x}{2} - \frac{\pi}{2}$$

Q13

$$\sin^{-1}\left(1+(2x-3)^2\right) + \cos^{-1}\left(-1-(2x-3)^2\right) + \lambda x = 0$$

$x = \frac{3}{2}$

$\Rightarrow \frac{\pi}{2} + \pi + \frac{3\lambda}{2} = 0 \Rightarrow \lambda = -\pi.$

Q14

Given that $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$

$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = \cos^{-1}(-1)$

$\Rightarrow \cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}(-1) - \cos^{-1}(z)$

$\Rightarrow \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \cos^{-1}\{(-1)(z)\}$

$\Rightarrow xy - \sqrt{(1-x^2)(1-y^2)} = -2$

$\Rightarrow (xy+z) = \sqrt{(1-x^2)(1-y^2)}$

Squaring both sides we get $x^2+y^2+z^2+2xyz = 1$ Trick: Put $x = y = z = \frac{1}{2}$, so that

$\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2} = \pi$

Obviously (d) holds for these values of x, y, z.

Q15

$\left(\cot^{-1} x-5\right)\left(\cot^{-1} x-2\right)>0$

$\Rightarrow \cot^{-1} x \in(-\infty,\ 2) \cup(5,\ \infty)$

$\Rightarrow \cot^{-1} x \in(0,\ 2)$ (Taking intersection with range of $\cot^{-1} x$)

$\Rightarrow n \in(\cot 2,\ \infty)$

Q16

Here, $x^2-2x+2=(x-1)^2+1 \geq 1$

But, $-1 \leq\left(x^2-2x+2\right) \leq 1$

Which is possible only when

$x^2-2x+2=1$

$\Rightarrow x=1$

Then, $a(1)^2+\sin^{-1}(1)+\cos^{-1}(1)=0$

$\Rightarrow a+\frac{\pi}{2}+0=0$

$\Rightarrow a=-\frac{\pi}{2}$

Q17

$\cos^{-1}\cos 4 = 2\pi - 4 > \frac{\pi}{2}$ and $\sin^{-1}\sin x \leq \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\sin x = \cos^{-1}\cos 4$has no real root.

Q18

We have $\left(\sin^{-1} x\right)^3 + \left(\cos^{-1} x\right)^3$

$= \left(\sin^{-1} x + \cos^{-1} x\right)^3 - 3\sin^{-1} x\cos^{-1} x\left(\sin^{-1} x + \cos^{-1} x\right)$

$= \frac{\pi^3}{8} - 3\left(\sin^{-1} x\cos^{-1} x\right)\frac{\pi}{2}$

$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)$

$= \frac{\pi^3}{8} - \frac{3\pi^2}{4}\sin^{-1} x + \frac{3\pi}{2}\left(\sin^{-1} x\right)^2$

$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1} x\right)^2 - \frac{\pi}{2}\sin^{-1} x\right]$

$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32}$

$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1} x - \frac{\pi}{4}\right)^2$

$\therefore$ The least value is $\frac{\pi^3}{32}$

and since $\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \left(\frac{3\pi}{4}\right)^2$

$\therefore$ The greatest value is $\frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$.

Q19

$$\tan^{-1}\sqrt{\mathrm{x}(\mathrm{x}+1)} + \sin^{-1}\sqrt{\mathrm{x}^2+\mathrm{x}+1} = \frac{\pi}{2}$$

$$\Rightarrow \quad \tan^{-1}\sqrt{\mathrm{x}^2+\mathrm{x}} + \sin^{-1}\sqrt{\mathrm{x}^2+\mathrm{x}+1} = \frac{\pi}{2}$$

This equation holds true, if

$$\mathrm{x}^2+\mathrm{x} \geq 0 \text{ and } 0 \leq \mathrm{x}^2+\mathrm{x}+1 \leq 1$$

Now, $\quad x^2+x \geq 0$ and $0 \leq x^2+x+1 \leq 1$

$\Rightarrow \quad x^2+x \geq 0$ and $x^2+x+1 \leq 1 \quad [\because x^2+x+1>0$ for all $x]$

$\Rightarrow \quad x^2+x \geq 0$ and $x^2+x \leq 0$

$\Rightarrow \quad x^2+x=0 \quad \Rightarrow \quad x=0,-1.$

Clearly, these two values satisfy the given equation. Hence, $x=-1,\ 0$ are the solutions of the given equation.

Q20

Given equation is $\frac{\pi}{2}+\tan^{-1} x=n,\ \forall x \in[-1,1]$

Now LHS $\in\left[\frac{\pi}{4}, \frac{3 \pi}{4}\right]$

Integers in this interval are 1 & 2

Hence, there are 2 integers for which the equation has real solutions

Q21

It is given that $a=\left(\sin^{-1} x\right)^{\sin^{-1} x}$,

$b=\left(\sin^{-1} x\right)^{\cos^{-1} x}, c=\left(\cos^{-1} x\right)^{\sin^{-1} x}$

$d=\left(\cos^{-1} x\right)^{\cos^{-1} x}$

Also, here $x \in(0,1)$

$\Rightarrow \cos^{-1} x<\sin^{-1} x$

Also, $\cos^{-1} x>1$

and $\sin^{-1} x < 1$

$\therefore \left(\cos^{-1} x\right)^{\cos^{-1} x}$ is greatest and $\left(\sin^{-1} x\right)^{\cos^{-1} x}$ is least.

$\Rightarrow \ \left(\sin^{-1} x\right)^{\sin^{-1} x} < \left(\cos^{-1} x\right)^{\sin^{-1} x}$

$\Rightarrow d > c > a > b$

Q22

$S = \lim\limits_{m\to\infty} \sum_{n=1}^{m} tan^{-1}\left(\frac{9}{9n^2+3n+7}\right) = \lim\limits_{m\to\infty} \sum_{n=1}^{m} tan^{-1}\left(\frac{1}{1+n^2+\frac{n}{3}-\frac{2}{9}}\right)$

$= \lim\limits_{m\to\infty} \sum_{n=1}^{m} tan^{-1}\left(\frac{\left(n+\frac{2}{3}\right)-\left(n-\frac{1}{3}\right)}{1+\left(n+\frac{2}{3}\right)\left(n-\frac{1}{3}\right)}\right) = \lim\limits_{m\to\infty} \sum_{n=1}^{m}\left[tan^{-1}\left(n+\frac{2}{3}\right) - tan^{-1}\left(n-\frac{1}{3}\right)\right]$

Q23

$y = \tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} + \ldots + 2n$ terms

$= \tan^{-1}\frac{(x+1)-x}{1+x(1+x)} + \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)} + \ldots\ldots(2\text{n terms})$

$= \tan^{-1}(x+1) - \tan^{-1}x + \tan^{-1}(x+2) - \tan^{-1}(x+1) + \ldots + \tan^{-1}(x+2n) - \tan^{-1}(x+(2n-1))$

$= \tan^{-1}(x+2n) - \tan^{-1}x$

$y(0) = \tan^{-1}(2n)$

Q24

Given expression $= \tan^{-1} 2 + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{2}{25} + \tan^{-1}\frac{2}{49} + \ldots\ldots$

General term $= \frac{2}{(2n-1)^2} = \frac{2}{4n^2-4n+1} = \frac{2}{1+4n(n-1)} = \frac{2n-(2n-2)}{1+2n(2n-2)}$

$T_n = \tan^{-1} 2n - \tan^{-1}(2n-2)$

$\therefore$ Sum of the series

$= \tan^{-1} 2 - \tan^{-1} 0 + \tan^{-1} 4 - \tan^{-1} 2 + \tan^{-1} 6 - \tan^{-1} 4 + \ldots \tan^{-1} 2\mathrm{n} - \tan^{-1}(2\mathrm{n} - 2)$

$= \tan^{-1} 2n - \tan^{-1} 0 = \tan^{-1} 2n$

Q25

We have,

$\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1} x$

$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} + 2\sin^{-1} x\right)$

$\Rightarrow 1 - x = \cos\left(2\sin^{-1} x\right)$

$\Rightarrow 1 - x = \cos\left\{\cos^{-1}\left(1 - 2x^2\right)\right\}\left[\because 2\sin^{-1} x = \cos^{-1}\left(1 - 2x^2\right)\right]$

$\Rightarrow 1 - x = \left(1 - 2x^2\right)$

$\Rightarrow x = 2x^2 \Rightarrow x(2x - 1) = 0 \Rightarrow x = 0, \frac{1}{2}$

For, $x = \frac{1}{2}$, we have

LHS $= \sin^{-1}(1-x) - 2\sin^{-1} x$

$= \sin^{-1} \frac{1}{2} - 2\sin^{-1} \frac{1}{2} = -\sin^{-1} \frac{1}{2} = \frac{-\pi}{6} \neq \mathrm{R.H.S.}$

So, $x = \frac{1}{2}$ is not a root of the given equation.

Clearly, $x = 0$ satisfies the equation

Here, $x = 0$ is the root of the given equation.

Q26

$\because \left[\sin^{-1} x\right] > \left[\cos^{-1} x\right]$

$\Rightarrow \quad x > 0$

Here, $\left[\cos^{-1}\ x\right]=\begin{cases} 0, & x\in(\cos 1,\ 1] \\ 1, & x\in(0\ ,\ \cos 1] \end{cases}$

and $\left[\sin^{-1}\ x\right]=\begin{cases} 0\ , & x\in(0,\ \sin 1\ ) \\ 1, & x\in[\sin 1,\ 1] \end{cases}$

$\therefore\ \ x\in[\sin 1,\ 1]$

Q27

Given $\cot^{-1}(\alpha)=\cot^{-1}(2)+\cot^{-1}(8)+\cot^{-1}(18)+\ldots.$

$=\sum_{n=1}^{100}\tan^{-1}\left(\frac{2}{4n^2}\right)$

$=\sum_{n=1}^{100}\tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$=\sum_{n=1}^{100}\tan^{-1}(2n+1)-\tan^{-1}(2n-1)$

$=\tan^{-1}201-\tan^{-1}1$

$=\tan^{-1}\left(\frac{200}{202}\right)$

$\therefore\cot^{-1}\left(\alpha\right)=\cot^{-1}\left(\frac{202}{200}\right)$

$\alpha=1.01$

Q28

Given,

$\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right)=k$

Now simplifying $\cos\left(\sin^{-1}x\right)=\cos\left(\cos^{-1}\sqrt{1-x^2}\right)=\sqrt{1-x^2}$

So, $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right)=k$

becomes $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\sqrt{1-x^2}\right)\right)\right)=k$

And now solving $\cot\left(\tan^{-1}\sqrt{1-x^2}\right)=\cot\cot^{-1}\left(\sqrt{\frac{1}{\sqrt{1-x^2}}}\right)=\frac{1}{\sqrt{1-x^2}}$

So, $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\sqrt{1-x^2}\right)\right)\right)=k$ becomes

$\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)=k$

Now solving $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)=\frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}$

So, $\frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}=k$

$\Rightarrow 1-2x^2=k^2\left(1-x^2\right)$

$\Rightarrow\left(k^2-2\right)x^2=k^2-1$

$\Rightarrow x^2=\frac{k^2-1}{k^2-2}$

So, roots are $\alpha=\sqrt{\frac{k^2-1}{k^2-2}}\Rightarrow\alpha^2=\frac{k^2-1}{k^2-2}$

And $\beta=\sqrt{\frac{k^2-1}{k^2-2}}\Rightarrow\beta^2=\frac{k^2-1}{k^2-2}$

Now finding $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=2\left(\frac{k^2-2}{k^2-1}\right)$ and $\frac{\alpha}{\beta}=-1$

So, sum of roots of $x^2-bx-5=0$ will be $=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{\alpha}{\beta}=b$

$\Rightarrow\frac{2\left(k^2-2\right)}{k^2-1}-1=b\qquad\cdots(1)$

Product of roots of $x^2-bx-5=0$ will be $=\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)\frac{\alpha}{\beta}=-5$

$\Rightarrow\frac{2\left(k^2-2\right)}{k^2-1}(-1)=-5$

$\Rightarrow 2k^2 - 4 = 5k^2 - 5$

$\Rightarrow 3k^2 = 1 \Rightarrow k^2 = \frac{1}{3} \quad \cdots \text{Put in } (1)$

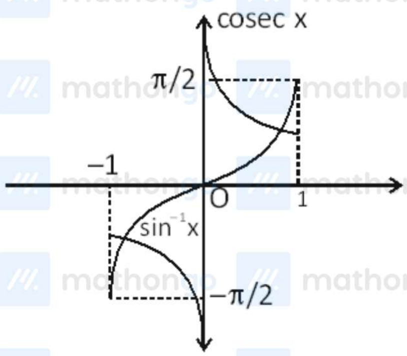
$\Rightarrow b = \frac{2\left(k^2-2\right)}{k^2-1} - 1 = 5 - 1 = 4$

$\frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$

Q29

$\because (\sin x)^{-1} = \frac{1}{\sin x} = \operatorname{cosec} x$

Now, from graphs of $\sin^{-1} x$, $\operatorname{cosec} x$



Clearly, both graph intersects at two points

$\therefore$ two solutions

Q30

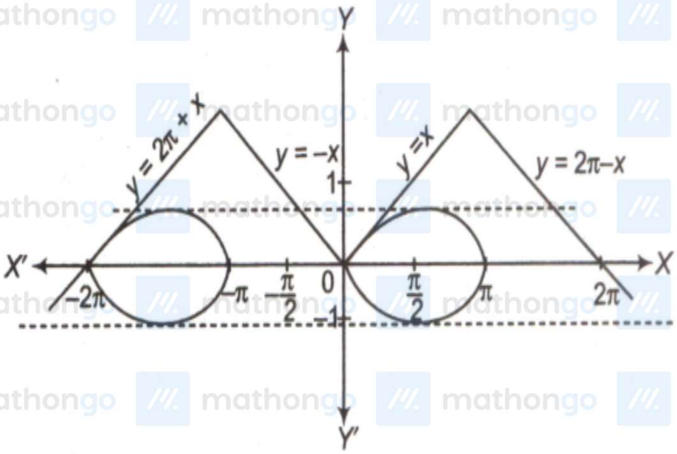
In $[0, \pi]$, $|y| = \sin x$, $y = \cos^{-1}(\cos x) = x$

In $[\pi, 2\pi]$, $|y| = \sin x$, $y = \cos^{-1}\{\cos(2\pi - x)\} = 2\pi - x$

In $[-\pi, 0]$, $|y| = \sin x$, $y = \cos^{-1}\{\cos(-x)\} = -x$

In $[-2\pi, -\pi]$, $|y| = \sin x$, $y = \cos^{-1}\{\cos(2\pi + x)\} = 2\pi + x$

Plotting the graphs, we have



There are 2 solutions, i.e., $(0, 0)$ and $(-2\pi, 0)$.