

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q1**

If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

(1) $-\frac{y}{x}$

(2) $\frac{y}{x}$

(3) $-\frac{x}{y}$

(4) $\frac{x}{y}$

Q2

If $\log_{10}\left(\frac{x^3-y^3}{x^3+y^3}\right) = 2$, then $\frac{dy}{dx}$ is

(1) $\frac{x}{y}$

(2) $-\frac{y}{x}$

(3) $-\frac{x}{y}$

(4) $\frac{y}{x}$

Q3

If $y = a\cos(\log x) + b\sin(\log x)$, where a & b are parameters, then $x^2y'' + xy'$ is equal to

(1) y

(2) $-y$

(3) $2y$

(4) $-2y$

Q4**MathonGo**<https://www.mathongo.com>

Sample Task**Questions****Questions with Answer Keys****MathonGo**

If $\phi(x) = \log_8 \log_3 x$, then $\phi'(e)$ is equal to

(1) $e \log 8$

(2) $-e \log 8$

(3) $\frac{1}{e \log 8}$

(4) None of these

Q5

If $y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{1}{x \log_e 10} - \frac{\log_e 10}{x (\log_e x)^2}$

(2) $\frac{1}{x \log_e 10} - \frac{1}{x \log_{10} e}$

(3) $\frac{1}{x \log_e 10} - \frac{\log_e 10}{x (\log_e x)}$

(4) None of these

Q6

If $f: R \rightarrow R$ is a function defined as $f(x^3) = x^5$, $\forall x \in R - \{0\}$ and $f(x)$ is differentiable $\forall x \in R$ then the value of

$\frac{1}{4} f'(27)$ is equal to (here f' represents the derivative of f)

Q7

If $y = 2 + \sqrt{\sin x} + 2 + \sqrt{\sin x} + 2 + \sqrt{\sin x} + \dots \infty$, then the value of $\frac{dy}{dx}$ at $x = 0$ is

(1) 0

(2) 2

(3) $\frac{1}{2}$

Sample Task**Questions****Questions with Answer Keys****MathonGo**(4) $\frac{1}{3}$ **Q8**

If $x = 3\cos t$ and $y = 5\sin t$, where t is a parameter, then $9 \frac{d^2y}{dx^2}$ at $t = -\frac{\pi}{6}$ is equal to

Q9

If $y = x^2 + \frac{1}{x^2 + \frac{1}{x^2 + \dots + \infty}}$, then the value of $\frac{dy}{dx}$ is:

(1) $\frac{2xy}{2y-x^2}$

(2) $\frac{xy}{y+x^2}$

(3) $\frac{xy}{y-x^2}$

(4) $\frac{2xy}{2y+x^2}$

Q10

If $f(x) = \left(\frac{2+x}{1+x}\right)^{1+x}$, then $f'(0)$ is equal to

(1) $2\log 2$

(2) $\log 2$

(3) $3\log 2 - 1$

(4) $2\log 2 - 1$

Q11

If $x^3 + y^3 = t + \frac{4}{t}$ and $x^6 + y^6 = t^2 + \frac{16}{t^2}$ then find $\left| x^4 y^2 \frac{dy}{dx} \right|$.

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q12**

Let $f(x)$ be a polynomial of degree 3 such that $f(3) = 21$, $f'(3) = 30$, $f''(3) = 22$ and $f'''(3) = 6$. Find the value of $f'(2)$.

Q13

Find the value of $(fgh)'(0)$, if f , g and h are differentiable functions with $f(0) = 1$, $g(0) = 2$, $h(0) = 3$ and the derivatives of their pair wise products at $x = 0$ are $(fg)'(0) = 6$, $(gh)'(0) = 4$ and $(hf)'(0) = 5$.

Q14

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfy $f(x+y) = 2^x f(y) + 4^y f(x)$, $\forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14 \cdot \frac{f'(4)}{f'(2)}$ is equal to _____.

Q15

Find the value of $f^2(4) + g^2(4)$, if $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$.

Q16

If for $x \in (0, 1/4)$, derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$ then $g(x)$ is equal to.

$$(1) \frac{3}{1+9x^3}$$

$$(2) \frac{9}{1+9x^3}$$

$$(3) \frac{3x\sqrt{x}}{1-9x^3}$$

$$(4) \frac{3x}{1-9x^3}$$

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q17**

If $y = e^{nx}$, then $\left(\frac{d^2y}{dx^2}\right) \frac{d^2x}{dy^2}$ is equal to

(1) ne^{nx}

(2) ne^{-nx}

(3) 1

(4) $-ne^{-nx}$

Q18

Let $\phi(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^5}$, then $\frac{d}{dx}\phi(x)$ is equal to

(1) $\frac{1}{1+[\phi(x)]^5}$

(2) $\frac{1}{1+[f(x)]^5}$

(3) $1 + [\phi(x)]^5$

(4) $1 + f(x)$

Q19

If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

(1) $\frac{x}{y}$

(2) $\frac{y}{x}$

(3) $\frac{x+y}{xy}$

(4) xy

Q20

Sample Task**Questions****Questions with Answer Keys****MathonGo**

If $f(x) = x^3 + 3x + 1$ and $g(x)$ is the inverse function of $f(x)$, then the value of $g'(5)$ is equal to

- (1) 3
- (2) $\frac{1}{3}$
- (3) $\frac{1}{6}$
- (4) 6

Q21

Let $f: (-1, 1) \rightarrow \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then

- $$g'(0) =$$
- (1) 0
 - (2) -2
 - (3) 4
 - (4) -4

Q22

$f(x)$ and $g(x)$ are two differential function on $[0, 2]$ such that $f''(x) - g''(x) = 0$, $f'(1) = 2g'(1) = 4$, $f(2) = 3g(2) = 9$

then $f(x) - g(x)$ at $x = \frac{3}{2}$ is

- (1) 0
- (2) 2
- (3) 10
- (4) 5

Q23

Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

Sample Task**Questions****Questions with Answer Keys****MathonGo**(1) $\log 2$ (2) $-\log 2$ (3) -1 (4) 1 **Q24**

If $2y = \left(\cot^{-1} \left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x} \right) \right)^2$ for $x \in \left(0, \frac{\pi}{2} \right)$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{\pi}{6} - x$ (2) $2x - \frac{\pi}{3}$ (3) $x - \frac{\pi}{6}$

(4) None of these

Q25

If $y = y(x)$ is an implicit function of x such that $\log_e(x+y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to

Q26

Let, $f: R \rightarrow R$ be a function such that $f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$, $\forall x \in R$. Then $f(2)$ equals

(1) 30

(2) 8

(3) -4

(4) -2

Q27

Sample Task**Questions****Questions with Answer Keys****MathonGo**

Let f be a differentiable function such that $8f(x) + 6f\left(\frac{1}{x}\right) - x = 5$ ($x \neq 0$) and $y = x^2f(x)$, then $\frac{dy}{dx}$ at $x = -1$ is

$$(1) \frac{15}{14}$$

$$(2) -\frac{15}{14}$$

$$(3) -\frac{1}{14}$$

$$(4) \frac{1}{14}$$

Q28

For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $\left(1 + \log_e 2x\right)^2 \frac{dy}{dx}$ is equal to

$$(1) \log_e 2x$$

$$(2) \frac{x \log_e 2x - \log_e 2}{x}$$

$$(3) x \log_e 2x$$

$$(4) \frac{x \log_e 2x + \log_e 2}{x}$$

Q29

Let $y = f(x)$ is an invertible function satisfying $f(1) = 5$, $f'(1) = 2$, $f''(1) = 4$, then the absolute value of

$$2. (f^{-1})''(5)$$

Q30

If $y(x) = (x^x)^x$, $x > 0$ then $\frac{d^2y}{dx^2} + 20$ at $x = 1$ is equal to

Sample Task**Questions****Questions with Answer Keys****MathonGo****Answer Key****Q1 (3)****Q2 (4)****Q3 (2)****Q4 (3)****Q5 (1)****Q6 (3.75)****Q7 (4)****Q8 (40)****Q9 (1)****Q10 (4)****Q11 (4)****Q12 (11)****Q13 (16)****Q14 (248)****Q15 (32)****Q16 (2)****Q17 (4)****Q18 (3)****Q19 (2)****Q20 (3)****Q21 (4)****Q22 (4)****Q23 (3)****Q24 (4)****Q25 (40)****Q26 (4)****Q27 (3)****Q28 (2)****Q29 (1)****Q30 (16)**

Sample Task**Solutions****Hints and Solutions****MathonGo****Q1**

$$\therefore x = \frac{1-t^2}{1+t^2} \text{ and } y = \frac{2t}{1+t^2}$$

Put $t = \tan \theta$ in both the equations,

$$\text{we get } x = \frac{1-\tan^2 \theta}{1+\tan^2 \theta} = \cos 2\theta$$

$$\text{and } y = \frac{2\tan \theta}{1+\tan^2 \theta} = \sin 2\theta \dots (\text{ii})$$

On differentiating both the eqs.(i) and (ii), we get $\frac{dx}{d\theta} = -2 \sin 2\theta$ and $\frac{dy}{d\theta} = 2 \cos 2\theta$

$$\text{Therefore, } \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$$

Q2

$$\text{Given, } \log_{10} \left(\frac{x^3-y^3}{x^3+y^3} \right) = 2$$

$$\Rightarrow \left(\frac{x^3-y^3}{x^3+y^3} \right) = (10)^2 = 100$$

Applying componendo and dividendo, we get, $\frac{x^3}{y^3} = -\frac{101}{99} \Rightarrow \frac{x}{y} = \left(-\frac{101}{99} \right)^{\frac{1}{3}} \dots (\text{i})$

$$\text{Hence, } y = \left(-\frac{99}{101} \right)^{\frac{1}{3}} x$$

Differentiating, we get,

$$\frac{dy}{dx} = \left(-\frac{99}{101} \right)^{\frac{1}{3}} = \frac{y}{x} \text{ (from (i))}$$

Q3

$$y = a \cos(\log x) + b \sin(\log x) \text{ On differentiating w.r.t. } x, \text{ we get } y' = \frac{-a \sin(\log x)}{x} + \frac{b \cos(\log x)}{x}$$

$$\Rightarrow xy' = -a \sin(\log x) + b \cos(\log x)$$

Again, on differentiating w.r.t. x , we get

$$xy'' + y' = -a \cos(\log x) \frac{1}{x} - b \sin(\log x) \frac{1}{x}$$

$$\Rightarrow x^2y'' + y'x = -[a \cos(\log x) + b \sin(\log x)]$$

$$\Rightarrow x^2y'' + xy' = -y$$

Q4

Sample Task**Solutions****Hints and Solutions****MathonGo**

We have,

$$\phi(x) = \log_8 \log_3 x = \log_8 \left(\frac{\log x}{\log 3} \right)$$

$$= \log_8(\log x) - \log_8(\log 3)$$

$$= \frac{\log(\log x)}{\log 8} - \log_8(\log 3)$$

$$\phi'(x) = \frac{1}{\log 8} \cdot \frac{1}{\log x} \cdot \frac{1}{x} - 0$$

$$\therefore \phi'(e) = \frac{1}{\log 8} \cdot \frac{1}{\log e} \cdot \frac{1}{e} = \frac{1}{e \log 8}$$

Q5

$$y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$$

$$= \log_{10} x + \frac{\log_e 10}{\log_e x} + 1 + 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x} \log_{10} e - \frac{\log_e 10}{x (\log_e x)^2}$$

Q6

$$f(x^3) = x^5$$

On differentiating with respect to x

$$f'(x^3) \cdot 3x^2 = 5 \cdot x^4$$

$$f'(x^3) = \frac{5}{3}x^2$$

Putting $x = 3$, we get,

$$f'(27) = \frac{5}{3}(9) = 15$$

Q7

Given equation can be rewritten as

$$y = 2 + \sqrt{\sin x + y}$$

$$\Rightarrow (y - 2)^2 = \sin x + y$$

$$\Rightarrow y^2 - 4y + 4 = \sin x + y \dots (i)$$

Putting $x = 0$ in the equation (i), we get,

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$y^2 - 4y + 4 = 0 \Rightarrow y^2 - 5y + 4 = 0$$

$$\Rightarrow (y-1)(y-4) = 0$$

$$y = 1 \text{ or } y = 4$$

$$\therefore y > 2 \Rightarrow y = 4$$

Now differentiating equation (i) with respect to x , we get, $\frac{dy}{dx} = \frac{\cos x}{2y-5}$

Putting $x = 0, y = 4$

$$\frac{dy}{dx} = \frac{\cos(0)}{2(4)-5} = \frac{1}{3}$$

Q8

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{5 \cos t}{-3 \sin t} = \frac{-5}{3} \cot(t)$$

$$\frac{d^2y}{dx^2} = \frac{-5}{3} \cdot (-\operatorname{cosec}^2 t) \cdot \frac{dt}{dx} = \frac{5}{3} \left(\frac{1}{\sin^2 t} \right) \left(\frac{1}{-3 \sin t} \right)$$

$$= \frac{5}{9} (8)$$

$$\text{Hence, } 9 \frac{d^2y}{dx^2} = 40$$

Q9

$$y = x^2 + \frac{1}{y}$$

$$y^2 = x^2 y + 1$$

$$2y \frac{dy}{dx} = y \cdot 2x + x^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2xy}{2y-x^2}$$

Q10

By taking logarithm on both sides,

$$\log(x) = (1+x) [\log(2+x) - \log(1+x)]$$

On differentiating, we get,

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$\frac{1}{f(x)} \cdot f'(x) = \frac{(1+x)}{(2+x)} + \log(2+x) - \frac{(1+x)}{(1+x)} - \log(1+x)$$

Now, put $x = 0$

$$f'(0) = f(0) \left\{ \frac{1+\log 2}{2} - 1 \right\} = f(0) \left[\log 2 - \frac{1}{2} \right]$$

$$= 2 \log 2 - 1 (\because f(0) = 2)$$

Q11

$$x^3 + y^3 = t + \frac{4}{t}$$

$$x^6 + y^6 + 2x^3y^3 = t^2 + \frac{16}{t^2} + 8$$

... [By squaring both the sides]

$$\Rightarrow \left(t^2 + \frac{16}{t^2} \right) + 2x^3y^3 = t^2 + \frac{16}{t^2} + 8$$

$$\Rightarrow x^3y^3 = 4 \quad \dots (i)$$

Differentiating with respect to x we get

$$x^3 \left(3y^2 \frac{dy}{dx} \right) + y^3 \cdot (3x^2) = 0$$

$$\Rightarrow 3x^3y^2 \frac{dy}{dx} = -3x^2y^3$$

$$\Rightarrow x^3y^2 \frac{dy}{dx} = -x^2y^3$$

$$\Rightarrow x^4y^2 \frac{dy}{dx} = -x^3y^3$$

$$\Rightarrow x^4y^2 \frac{dy}{dx} = -4 \quad \dots [\text{From (i)}]$$

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Sample Task**Solutions****Hints and Solutions****MathonGo****Q12**

Let $f(x) = a(x - 3)^3 + b(x - 3)^2 + c(x - 3) + d$

$$f(3) = 21 \Rightarrow d = 21$$

$$f'(3) = 30 \Rightarrow c = 30$$

$$f''(3) = 22 \Rightarrow b = 11$$

$$f'''(3) = 6 \Rightarrow a = 1$$

$$f(x) = (x - 3)^3 + 11(x - 3)^2 + 30(x - 3) + 21$$

$$\Rightarrow f'(x) = 3(x - 3)^2 + 22(x - 3) + 30$$

$$\Rightarrow f'(2) = 11$$

Q13

$$y = fgh$$

$$\frac{dy}{dx} = f'gh + fg'h + fgh'$$

$$= \frac{1}{2}(2f'gh + 2fg'h + 2fgh')$$

$$= \frac{1}{2}(h(f'g + g'f) + g(f'h + fh') + f(g'h + gh))$$

$$= \frac{1}{2}[h \cdot (fg)' + g \cdot (fh)' + f \cdot (gh)']$$

$$(fgh)'(0) = \frac{1}{2}[h(0)(fg)'(0) + g(0)(fh)'(0) + f(0)(gh)'(0)]$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$= \frac{1}{2}(3 \times 6 + 2 \times 5 + 1 \times 4)$$

$$= \frac{1}{2}(18 + 10 + 4) = \frac{32}{2} = 16$$

Q14

Given, $f(x + y) = 2^x f(y) + 4^y (f(x))$

Put $y = 2$ we get,

$$f(x + 2) = 2^x \times 3 + 16f(x)$$

$$f'(x + 2) = 16f'(x) + 3 \times 2^x \ln 2$$

Now put $x = 2$ we get,

$$f'(4) = 16f'(2) + 12 \ln 2 \quad \dots(i)$$

Similarly, $f(y + 2) = 4f(y) + 3 \times 4^y$

$$f'(4) = 4f'(y) + 3 \times 4^y \ln 4$$

$$f'(4) = 4f'(2) + 96 \ln 2 \quad \dots(ii)$$

solving eq. (i) and (ii), we get

$$f'(2) = 7 \ln 2$$

from equation (i), we get

$$f'(4) = 124 \ln 2$$

$$\text{Now } \Rightarrow 14 \times \frac{f'(4)}{f'(2)}$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$14 \times \frac{124 \ln 2}{7 \ln 2} = 248$$

Q15

$$f^2(4) + g^2(4) = ?$$

$$\frac{d}{dx}[f^2(x) + g^2(x)] = 2f(x)f'(x) + 2g(x)g'(x)$$

$$= 2 f(x)g(x) + 2g(x)(-f(x))$$

$$= 0$$

$$\therefore f^2(x) + g^2(x) = \text{constant}$$

$$\therefore f^2(4) + g^2(4) = f^2(2) + g^2(2)$$

$$= 4^2 + (f'(2))^2$$

$$= 4^2 + 4^2 = 32$$

Q16

$$y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x}\right)$$

$$= \tan^{-1}\left(\frac{2\left(3x^{\frac{3}{2}}\right)}{1-\left(3x^{\frac{3}{2}}\right)^2}\right) = 2\tan^{-1}\left(3x^{\frac{3}{2}}\right)$$

$$\frac{dy}{dx} = \frac{2 \cdot \left(3x^{\frac{1}{2}}\right)\left(\frac{3}{2}\right)}{1+9x^2} = \frac{9}{1+9x^2}\sqrt{x}$$

$$\therefore g(x) = \frac{9}{1+9x^3}$$

Q17

$$y = e^{nx} \Rightarrow \ln y = nx$$

$$\text{Now, } \frac{d^2y}{dx^2} = n^2 e^{nx} = n^2 y$$

$$\text{and } \frac{d^2x}{dy^2} = \frac{1}{ny^2}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right) \left(\frac{d^2x}{dy^2}\right) = \frac{n}{y} = -ne^{-nx}$$

Sample Task**Solutions****Hints and Solutions****MathonGo****Q18**

We have, $\phi(x) = f^{-1}(x)$

$$\Rightarrow x = f[\phi(x)]$$

On differentiating both sides w.r.t. x , we get $1 = f'[\phi(x)] \cdot \phi'(x) \Rightarrow \phi'(x) = \frac{1}{f'[\phi(x)]} \dots (i)$

Since, $f'(x) = \frac{1}{1+x^5}$ (given)

$$f'(\phi(x)) = \frac{1}{1+[\phi(x)]^5}$$

From Equation (i),

$$\phi'(x) = \frac{1}{f'[\phi(x)]} = 1 + [\phi(x)]^5$$

Q19

$$m \log x + n \log y = (m+n) \log(x+y)$$

$$\Rightarrow \frac{m}{x} + \frac{n}{y} \frac{dy}{dx} = \frac{m+n}{x+y} \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \left(\frac{n}{y} - \frac{m+n}{x+y}\right) \frac{dy}{dx} = \frac{m+n}{x+y} - \frac{m}{y}$$

$$\Rightarrow \frac{nx-my}{y(x+y)} \frac{dy}{dx} = \frac{nx-my}{(x+y)x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Q20

$\therefore f(x), g(x)$ are inverse function of each other

$$\therefore g'(f(x)) = \frac{1}{f'(x)}$$

$$\text{Now put } x = 1; g'(f(1)) = \frac{1}{f'(1)}$$

$$\text{Hence, } g'(5) = \frac{1}{(3x^2+3)_{x=1}} = \frac{1}{6}$$

Q21

$$g'(x) = 2[f(2f(x)+2)]f'(2f(x)+2)(2f'(x))$$

$$\Rightarrow g'(0) = 2[f(2f(0)+2)]f'(2f(0)+2)(2f'(0))$$

$$= 4(f(0))f'(0)^2 = 4(-1)(1)^2 = -4$$

Sample Task**Solutions****Hints and Solutions****MathonGo****Q22**

$$f''(x) - g''(x) = 0 \Rightarrow f'(x) - g'(x) = \text{constant.}$$

Putting $x = 1$, we get $f'(1) - g'(1) = C \Rightarrow C = 2g'(1) - g'(1)$

$$= g'(1) = 2$$

$$\text{So } f'(x) - g'(x) = 2 \Rightarrow f(x) - g(x) = 2x + C'.$$

Putting $x = 2$ we have $f(2) - g(2) = 4 + C \Rightarrow 4 +$

$$C' = 3g(2) - g(2) = 2g(2) = 6 \Rightarrow C' = 6 - 4 = 2$$

Hence $f(x) - g(x) = 2x + 2$. At $x = \frac{3}{2}$

$$f\left(\frac{3}{2}\right) - g\left(\frac{3}{2}\right) = 2 \cdot \frac{3}{2} + 2 = 5$$

Q23

Putting $x = 1$ in $x^{2x} - 2x^x \cot y - 1 = 0$

$$\text{we get } 1 - 2 \cot y - 1 = 0$$

$$\Rightarrow \cot y = 0 \Rightarrow y = \pi/2$$

Using the formula,

$$\frac{d}{dx} [f(x)^{g(x)}] = g(x)f(x)^{g(x)-1}f'(x) + (g'(x)\log$$

$f(x))f(x)^{g(x)}$, we have

$$2x \cdot x^{2x-1}(1) + (2\log x)x^{2x} - 2[x \cdot x^{x-1}(1) + (\log x)$$

$$x^x] \cot y + 2x^x (\cosec^2 y) y'(x) = 0$$

Putting $x = 1, y = \pi/2$, we get

$$2 + 0 - 2(1)(0) + 2 \cosec^2(\pi/2)y'(1) = 0$$

$$\Rightarrow y'(1) = -1$$

Q24

$$2y = \left(\cot^{-1} \frac{\sqrt{3} \cos x + \sin x}{\cos x - \sqrt{3} \sin x} \right)^2$$

$$= \left(\cot^{-1} \left(\frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \tan x} \right) \right)^2$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$= \left(\cot^{-1} \tan\left(\frac{\pi}{3} + x\right) \right)^2$$

$$= \left(\cot^{-1} \cot\left(\frac{\pi}{2} - \left(\frac{\pi}{3} + x\right)\right) \right)^2$$

$$\Rightarrow 2y = \begin{cases} \left(\frac{\pi}{6} - x\right)^2, & x \in \left(0, \frac{\pi}{6}\right) \\ \left(\pi + \frac{\pi}{6} - x\right)^2, & x \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right) \end{cases}$$

$$\Rightarrow \frac{2dy}{dx} = 2\left(\frac{\pi}{6} - x\right).(-1) \Rightarrow \frac{dy}{dx} = x - \frac{\pi}{6}, x \in \left(0, \frac{\pi}{6}\right)$$

$$\text{And } \frac{dy}{dx} = x - \frac{7\pi}{6}, x \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$$

Left Hand and Right Hand Derivatives are not same so function is non derivable at $x = \frac{\pi}{6}$.

Hence, $\frac{dy}{dx}$ does not exist for all values in the given interval.

Q25

Given:

$$\log_e(x+y) = 4xy$$

When $x = 0$, then $y = 1$

$$\log_e(x+y) = 4xy$$

$$\Rightarrow x+y = e^{4xy}$$

Now differentiate w.r.t. x

$$1+y' = e^{4xy}(4y+4xy') \dots (i)$$

$$\text{At } (0, 1) \Rightarrow y'(0) + 1 = 4 \Rightarrow y'(0) = 3$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

Now, again differentiate equation (i), we get

$$y'' = e^{4xy} (4y + 4xy')^2 + e^{4xy} (4y' + 4y' + 4xy'')$$

At $(0, 1)$

$$y''(0) = 1(4 \times 1 + 0)^2 + 1(4 \times 3 + 4 \times 3 + 0)$$

$$\Rightarrow y''(0) = 16 + 24 = 40$$

$$\Rightarrow y''(0) = 40$$

Q26

$$\text{Let, } f(x) = x^3 + ax^2 + bx + c$$

$$\Rightarrow f'(x) = 3x^2 + 2ax + b$$

$$\Rightarrow f''(x) = 6x + 2a$$

$$\Rightarrow f'''(x) = 6$$

According to the question,

$$a = f'(1) = 3 + 2a + b \Rightarrow a + b = -3 \dots (i)$$

$$b = f''(2) = 12 + 2a \Rightarrow 2a - b = -12 \dots (ii)$$

$$c = f'''(3) \Rightarrow c = 6$$

Solving equations (i) & (ii), we get, $a = -5$ & $b = 2$

$$\Rightarrow f(x) = x^3 - 5x^2 + 2x + 6$$

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$$\therefore f(2) = 8 - 20 + 4 + 6 = -2.$$

Q27

$$8f(x) + 6f\left(\frac{1}{x}\right) - x = 5 \text{ (i)}$$

Replace x by $\frac{1}{x}$, we have $8f\left(\frac{1}{x}\right) + 6f(x) - \frac{1}{x} = 5$ (ii)

Solving (i) and (ii) for $f(x)$ and $f(1/x)$, we have

$$14f(x) = 5 - \frac{3}{x} + 4x$$

$$\Rightarrow 14f'(x) = \frac{3}{x^2} + 4$$

$$\text{So } f(-1) = \frac{1}{14}[5 + 3 - 4] = \frac{2}{7}$$

$$\text{and } f'(-1) = \frac{1}{14}[3 + 4] = \frac{1}{2}$$

Differentiating $y = x^2 f(x)$, we have $\frac{dy}{dx} = 2xf(x) + x^2 f'(x)$

$$\Rightarrow \frac{dy}{dx} \Big|_{x=-1} = -2f(-1) + f'(-1) = -\frac{4}{7} + \frac{1}{2} = -\frac{1}{14}$$

Q28

$$\text{Given, } (2x)^{2y} = 4 \cdot e^{2x-2y}$$

Taking natural logarithm on both sides, we get

$$2y \log_e 2x = \log_e 4 + (2x - 2y)$$

$$\Rightarrow 2y = \left(\frac{\log_e 4 + 2x}{1 + \log_e 2x} \right)$$

Differentiating both sides with respect to x , we get

$$\frac{2dy}{dx} = \frac{(1 + \log_e 2x) \cdot 2 - (\log_e 4 + 2x) \frac{1}{x}}{(1 + \log_e 2x)^2} \quad (\text{Using quotient rule})$$

$$\Rightarrow (1 + \log_e 2x)^2 \frac{dy}{dx} = \left(\frac{x \log_e 2x - \log_e 2}{x} \right).$$

Sample Task**Solutions****Hints and Solutions****MathonGo****Q29**

$$\text{let } b^{-1}(x) = g(x)$$

$$g(b(x)) = x$$

$$g'(f(x))b'(x) = 1$$

$$g'(f(x)) = \frac{1}{b'(x)}$$

$$g''(f(x))f'(x) = \frac{-b''(x)}{(f'(x))^2}$$

$$\text{If } f(x) = 5, x = 1$$

$$\therefore g''(5) = -\frac{f''(1)}{(f'(1))^3} = \frac{-4}{8} = \frac{-1}{2}$$

$$|2g''(5)| = 1$$

Q30

Given,

$$y(x) = (x^x)^x$$

Taking \log_e both side

$$\ln y(x) = x^2 \cdot \ln x$$

Now differentiating both side w.r.t x we get,

$$\frac{1}{y(x)} \cdot y'(x) = \frac{x^2}{x} + 2x \cdot \ln x$$

$$y'(x) = y(x)[x + 2x \ln x] \dots\dots (i)$$

$$\text{Given } y(1) = 1, \text{ so } y'(1) = 1$$

Now rewriting equation (i) again we get,

$$\frac{dx}{dy} = \frac{1}{x^{x^2+1} (1+2 \ln x)}$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$\text{Now } \frac{d^2x}{dy^2} = \frac{d}{dx} \left(\left(x^{x^2+1} (1 + 2 \ln x) \right)^{-1} \right) \frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{-x^{x^2} (1+2 \ln x) (x^2+3+2x^2 \ln x)}{(x^{x^2} (1+2 \ln x))^3} \times 1$$

$$\Rightarrow \left(\frac{d^2x}{dy^2} \right)_{x=1} = -4$$

$$\text{So, } \left(\frac{d^2x}{dy^2} \right)_{x=1} + 20 = -4 + 20 = 16$$

