

Sample Task**Questions****Questions with Answer Keys****MathonGo****Q1**

If $f(x) = \begin{cases} \lambda\sqrt{2x+3}, & 0 \leq x \leq 3 \\ \mu x + 12, & 3 < x \leq 9 \end{cases}$ is differentiable at $x = 3$, then the value of $\lambda + \mu$ is equal to

Q2

Let $f(x) = \begin{cases} \frac{1}{x^2} & : |x| \geq 1 \\ ax^2 + \beta & : |x| < 1 \end{cases}$. If $f(x)$ is continuous and differentiable at any point, then

- (1) $\alpha = 2, \beta = -1$
- (2) $\alpha = -1, \beta = 2$
- (3) $\alpha = 1, \beta = 0$
- (4) $\alpha = -2, \beta = 3$

Q3

If $f(x) = \begin{cases} \frac{\sqrt{4+ax} - \sqrt{4-ax}}{x}, & -1 \leq x < 0 \\ \frac{3x+2}{x-8}, & 0 \leq x \leq 1 \end{cases}$ is continuous in $[-1, 1]$, then the value of a is

- (1) 1
- (2) -1
- (3) $\frac{1}{2}$
- (4) $-\frac{1}{2}$

Q4

Sample Task**Questions****Questions with Answer Keys****MathonGo**

The value of $f(0)$ so that the function $f(x) = \frac{1 - \cos(1 - \cos x)}{x^4}$ is continuous everywhere is k , then value of $10k$ is

Q5

Let $f(x) = \begin{cases} \left(\frac{1 - \cos x}{(2\pi - x)^2}\right) \left(\frac{\sin^2 x}{\log(1 + 4\pi^2 - 4\pi x + x^2)}\right) & : x \neq 2\pi \\ \lambda & : x = 2\pi \end{cases}$ is continuous at $x = 2\pi$, then the value of λ is equal to

Q6

If $f(x) = \begin{cases} \frac{e^{[x] + |x|} - 1}{[x] + |x|} & : x \neq 0 \\ -1 & : x = 0 \end{cases}$ (where $[.]$ denotes the greatest integer function), then

(1) $f(x)$ is continuous at $x = 0$ (2) $\lim_{x \rightarrow 0^+} f(x) = -1$ (3) $\lim_{x \rightarrow 0^-} f(x) = 1$ (4) $\lim_{x \rightarrow 0^+} f(x) = 1$ **Q7**

Let $f(x) = \begin{cases} (1 + |\sin x|) \frac{l}{|\sin x|}, & \text{if } -\frac{\pi}{6} < x \leq 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & \text{if } 0 < x \leq \frac{\pi}{6} \\ m, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$. Then, the values of l and m are

(1) $l = -\frac{2}{3}, m = e^{\frac{2}{3}}$

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(2) $l = \frac{2}{3}, m = e^{-\frac{2}{3}}$

(3) $l = \frac{2}{3}, m = e^{\frac{2}{3}}$

(4) None of these

Q8

The function $f(x) = \{x\} \sin(\pi[x])$, where $[.]$ denotes the greatest integer function and $\{.\}$ is the fractional part function, is discontinuous at

(1) all x

(2) all integer points

(3) no x

(4) x which is not an integer

Q9

The function $f(x) = \lim_{n \rightarrow \infty} \cos^{2n}(\pi x) + [x]$ is (where, $[.]$ denotes the greatest integer function and $n \in N$)

(1) continuous at $x = 1$ but discontinuous at $x = \frac{3}{2}$

(2) continuous at $x = 1$ and $x = \frac{3}{2}$

(3) discontinuous at $x = 1$ and $x = \frac{3}{2}$

(4) discontinuous at $x = 1$ but continuous at $x = \frac{3}{2}$

Q10

Let $f(x) = -x^2 + x + p$, where p is a real number. If $g(x) = [f(x)]$ and $g(x)$ is discontinuous at $x = \frac{1}{2}$, then p cannot be (where $[.]$ represents the greatest integer function)

(1) $\frac{1}{2}$

Sample Task**Questions****Questions with Answer Keys****MathonGo**(2) $\frac{3}{4}$ (3) $\frac{7}{4}$ (4) $-\frac{1}{4}$ **Q11**

Let $f(x) = [x]\{x^2\} + [x][x^2] + \{x\}[x^2] + \{x\}\{x^2\}$, $\forall x \in [0,10]$ (where $[\cdot]$ and $\{\cdot\}$ are the greatest integer and fractional part functions respectively). The number of points of discontinuity of $f(x)$ is

Q12

Consider function $f(x) = \begin{cases} \frac{\tan\{2x-3\}}{x-2}, & x \in (2, \infty) \\ [x^2] + sgn(x), & x \in (-\infty, 2] \end{cases}$, then at $x = 2$

[Note: $\{k\}$ & $[k]$ denote fractional part & greatest integer function less than or equal to k respectively and sgn denotes signum part of function.]

(1) $f(x)$ is continuous(2) $f(x)$ is discontinuous(3) $f(x)$ is differentiable, but $f'(x)$ is discontinuous

(4) None of these

Q13

Given $f(x) = \begin{cases} \sqrt{10 - x^2} & \text{if } -3 < x < 3 \\ 2 - e^{x-3} & \text{if } x \geq 3 \end{cases}$

The graph of $f(x)$ is -(1) continuous and differentiable at $x = 3$

Sample Task**Questions****Questions with Answer Keys****MathonGo**(2) continuous but not differentiable at $x = 3$ (3) differentiable but not continuous at $x = 3$ (4) neither differentiable nor continuous at $x = 3$ **Q14**

If $f(x) = \frac{1}{1-x}$, then the points of discontinuity of the function $f^{30}(x)$ where $f^n(x) = f \circ f \circ \dots \circ f(x)$ (n times) are

(1) $x = 2, 1$ (2) $x = 0, 1$ (3) $x = 1, 2$

(4) no points of discontinuity

Q15

Let f be a composite function of x defined by

$$f(u) = \frac{1}{u^2+u-2}, \quad u(x) = \frac{1}{x-1}$$

Then the number of points x where f is discontinuous is :

(1) 4

(2) 3

(3) 2

(4) 1

Q16

In $(0, 2\pi)$, the total number of points where $f(x) = \max \{\sin x, \cos x, 1 - \cos x\}$ is not differentiable, are equal to

(1) 3

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(2) 4

(3) 5

(4) 6

Q17

Consider the function $f(x) = \max \{|\sin x|, |\cos x|\}$, $\forall x \in [0, 3\pi]$. If λ is the number of points at which $f(x)$ is non-differentiable, then the value of $\frac{\lambda}{5}$ is

Q18

Let $f(x) = \max \{x^2 - 2|x|, |x|\}$ and $g(x) = \min \{x^2 - 2|x|, |x|\}$ then if $f(x)$ is not differentiable at 'p' number of points and $g(x)$ is non differentiable at 'q' number of points, then find $|p - q|$.

Q19

If $f(x) = \min \{\sqrt{9 - x^2}, \sqrt{1 + x^2}\}$, $\forall x \in [-3, 3]$, then the number of point(s) where $f(x)$ is non-differentiable is/are

(1) 4

(2) 3

(3) 2

(4) 0

Q20

Sample Task**Questions****Questions with Answer Keys****MathonGo**

Consider a function $g(x) = f(x - 2)$, $\forall x \in R$, where $f(x) = \begin{cases} \frac{1}{|x|} & : |x| \geq 1 \\ ax^2 + b & : |x| < 1 \end{cases}$. If $g(x)$ is continuous as well as differentiable for all x , then

- (1) $a = \frac{-1}{2}, b = \frac{3}{2}$
- (2) $a = \frac{1}{2}, b = \frac{3}{2}$
- (3) $a = \frac{-1}{2}, b = \frac{-3}{2}$
- (4) None of these

Q21

Let $f(x) = [n + p \sin x]$, $x \in (0, \pi)$, $n \in I$ and p is a prime number. The number of points where $f(x)$ is not differentiable is

(Here $[x]$ represents the greatest integer less than or equal to x)

- (1) $p - 1$
- (2) $p + 1$
- (3) $2p + 1$
- (4) $2p - 1$

Q22

The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ is not differentiable in the interval $(0, 2)$ is/are

- (1) 1
- (2) 2
- (3) 3

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(4) 4

Q23

Let $f(x) = 10 - |x - 5|$, $x \in R$, then the set of all values of x at which $f(f(x))$ is not differentiable is

- (1) $\{0,5, 10\}$
- (2) $\{5,10\}$
- (3) $\{0,5, 10,15\}$
- (4) $\{5,10,15\}$

Q24

Consider $f(x) = \begin{cases} -2, & -2 \leq x < 0 \\ x^2 - 2, & 0 \leq x \leq 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2)$, $g(x)$ is

- (1) not differentiable at one point
- (2) differentiable at all points
- (3) not continuous
- (4) not differentiable at two points

Q25

Consider the function $f(x) = (x - 2) \left| x^2 - 3x + 2 \right|$, then the incorrect statement is

- (1) $f(x)$ is continuous at $x = 1$
- (2) $f(x)$ is continuous at $x = 2$
- (3) $f(x)$ is differentiable at $x = 1$
- (4) $f(x)$ is differentiable at $x = 2$

Q26

Sample Task**Questions****Questions with Answer Keys****MathonGo**

The number of points where the function, $f(x) = \cos |2018\pi - x| + \sin |2020\pi - x| + (x - \pi)|x^2 - 3\pi x + 2\pi^2|$ is non-differentiable is/are

- (1) 0
- (2) 1
- (3) 2
- (4) 3

Q27

Let $g(x) = \begin{cases} \frac{ax^2 + bx + c}{4 + (\cot x)^n}, & x \in \left(0, \frac{\pi}{4}\right) \\ 1, & \text{at } x = \frac{\pi}{4}, \text{ where } a, b, c \text{ are real constants and} \\ \frac{\sin x + \cos x + (\tan x)^n}{1 + c(\tan x)^n}, & x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \end{cases}$

$f(x) = \lim_{n \rightarrow \infty} g(x)$ If $\lim_{x \rightarrow \frac{\pi}{4}} f(x)$ exists, then c may be equal to

- (1) 2
- (2) $\frac{1}{2}$
- (3) 3
- (4) -1

Q28

Sample Task**Questions****Questions with Answer Keys****MathonGo**

Let $f(x) = \cos x$, $g(x) = \begin{cases} \min\{f(t) : 0 \leq t \leq x\}, & x \in [0, \pi] \\ \sin x - 1, & x > \pi \end{cases}$ then

- (1) $g(x)$ is discontinuous at $x = \pi$
- (2) $g(x)$ is continuous for $x \in (0, \infty)$
- (3) $g(x)$ is differentiable at $x = \pi$
- (4) $g(x)$ is differentiable for $x \geq 0$

Q29

Let $f(x)$ is a differentiable function such that $f(x+y) = f(x) + f(y) + 2xy \forall x, y \in R$ and $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 210$, then $f(2)$

is equal to

- (1) 20
- (2) 105
- (3) 424
- (4) none of these

Q30

Let $f: R \rightarrow R$ be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x) + f(y)}{3}$, $f(0) = 0$ and $f'(0) = 5$, then

- (1) $f(x)$ is a quadratic function
- (2) $f(x)$ is continuous but not differentiable
- (3) $f(x)$ is differentiable in R
- (4) $f(x)$ is bounded in R

Sample Task**Questions****Questions with Answer Keys****MathonGo****Answer Key****Q1 (8)****Q2 (2)****Q3 (4)****Q4 (1.25)****Q5 (0.5)****Q6 (4)****Q7 (3)****Q8 (3)****Q9 (4)****Q10 (1)****Q11 (0)****Q12 (2)****Q13 (2)****Q14 (2)****Q15 (2)****Q16 (1)****Q17 (43.20)****Q18 (2)****Q19 (1)****Q20 (1)****Q21 (4)****Q22 (3)****Q23 (1)****Q24 (1)****Q25 (3)****Q26 (3)****Q27 (4)****Q28 (2)****Q29 (3)****Q30 (3)**

Sample Task**Solutions****Hints and Solutions****MathonGo****Q1**

$\therefore f(x)$ is continuous at $x = 3$

$$\therefore LHL = RHL = f(3)$$

$$\lambda\sqrt{2(3)+3} = \mu(3) + 12 \Rightarrow \lambda = \mu + 4 \dots\dots(1)$$

Also $f(x)$ is differentiable at $x = 3$

$$\therefore \text{at } x = 3 \text{ LHD} = \text{RHD}$$

$$\frac{\lambda}{2\sqrt{2(3)+3}} \cdot 2 = \mu \Rightarrow \lambda = 3\mu \dots\dots(2)$$

By (1) and (2)

$$\mu = 2, \lambda = 6$$

Q2

The given function is clearly continuous at all points except possibly at $x = \pm 1$.

For $f(x)$ to be continuous at $x = 1$, we must have

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x \rightarrow 1^-} \alpha x^2 + \beta = \lim_{x \rightarrow 1^+} \frac{1}{x^2}$$

$$\Rightarrow \alpha + \beta = 1 \dots (1)$$

Now, for $f(x)$ to be differentiable at $x = 1$, we must have

$$\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1}$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$\Rightarrow \lim_{x \rightarrow 1^-} \frac{\alpha x^2 + \beta - 1}{x-1} = \lim_{x \rightarrow 1^+} \frac{\frac{1}{x^2}-1}{x-1} \quad (\because \alpha + \beta = 1 \therefore \beta - 1 = -\alpha)$$

$$\Rightarrow \lim_{x \rightarrow 1^-} \frac{\alpha x^2 - \alpha}{x-1} = \lim_{x \rightarrow 1^+} \frac{\frac{1}{x^2}-1}{x-1}$$

$$\Rightarrow \lim_{x \rightarrow 1^-} \alpha(x+1) = \lim_{x \rightarrow 1^+} \frac{-2}{x^3} \Rightarrow 2\alpha = -2$$

$$\Rightarrow \alpha = -1$$

Putting $\alpha = -1$ in (1), we get $\beta = 2$

Q3

Given that $f(x)$ is continuous in the interval $[-1, 1]$, so, it will be continuous at $x = 0$.

$$\text{Now } f(0^+) = \frac{-1}{4}$$

$$f(0^-) = \lim_{x \rightarrow 0^-} \frac{2ax}{x(\sqrt{4+ax} + \sqrt{4-ax})} \quad (\text{rationalizing})$$

$$= \lim_{x \rightarrow 0^-} \frac{2a}{4} = \frac{a}{2}$$

$$\text{Now } f(0^+) = f(0^-) \quad (\because f(x) \text{ is continuous at point 0})$$

$$\Rightarrow \frac{a}{2} = \frac{-1}{4} \Rightarrow a = \frac{-1}{2}$$

Q4

If $f(x)$ is continuous at $x = 0$

$$f(x) = \lim_{x \rightarrow 0} f(x)$$

$$= \lim_{x \rightarrow 0} \frac{1-\cos(1-\cos x)}{x^4} \times \frac{1+\cos(1-\cos x)}{1+\cos(1-\cos x)}$$

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$$\lim_{x \rightarrow 0} \frac{\sin^2(1-\cos x)}{x^4 \cdot (1+\cos(1-\cos x))} \cdot \frac{(1-\cos x)^2}{(1-\cos x)^2}$$

$$= \lim_{x \rightarrow 0} \left[\frac{\sin(1-\cos x)}{(1-\cos x)} \right]^2 \times \lim_{x \rightarrow 0} \left(\frac{1-\cos x}{x^2} \right)^2 \times \lim_{x \rightarrow 0} \frac{1}{1+\cos(1-\cos x)}$$

$$= (1)^2 \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$\therefore k = \frac{1}{8} \Rightarrow 10k = \frac{5}{4} = 1.25$$

Q5

For the function $f(x)$ to be continuous at $x = 2\pi$

$$\lim_{x \rightarrow 2\pi} f(x) = f(2\pi)$$

$$\text{Now, } \lim_{x \rightarrow 2\pi} \frac{1-\cos x}{(2\pi-x)^2} \cdot \frac{\sin^2 x}{\log \{1 + (2\pi-x)^2\}} = \lambda$$

Putting $x = 2\pi + t$, we get,

$$\lim_{t \rightarrow 0} \frac{1-\cos t}{t^2} \cdot \frac{\sin^2 t}{\log(1+t^2)} = \lambda$$

$$\lim_{t \rightarrow 0} \frac{1}{2} \cdot \frac{\sin^2 t}{t^2} \cdot \frac{t^2}{\log(1+t^2)} = \lambda$$

$$\frac{1}{2} \times 1 \times 1 = \lambda$$

$$\Rightarrow \lambda = \frac{1}{2}$$

Q6

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$f(x) = \begin{cases} \frac{e^{[x]} + |x| - 1}{[x] + |x|} & x \neq 0 \\ -1 & x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{e^{[x]} + |x| - 1}{[x] + |x|} = \frac{e^{-1} - 1}{-1} = \frac{e-1}{e}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{e^{[x]} + |x| - 1}{[x] + |x|} = \lim_{x \rightarrow 0^+} \frac{e^x - 1}{x} = 1$$

$\therefore \text{LHL} \neq \text{RHL}$ at $x = 0 \Rightarrow f(x)$ is discontinuous at $x = 0$

Q7

We have,

$$\begin{aligned} \lim_{h \rightarrow 0^+} f(0-h) &= \lim_{h \rightarrow 0^+} [1 + |\sin(-h)|]^{\frac{l}{|\sin(-h)|}} \\ &= \lim_{h \rightarrow 0^+} (1 + \sin h)^{\frac{l}{\sin h}} \\ &= \lim_{h \rightarrow 0^+} \left[(1 + \sin h)^{\frac{1}{\sin h}} \right]^l = e^l \end{aligned}$$

$$\begin{aligned} \lim_{h \rightarrow 0^+} f(0+h) &= \lim_{h \rightarrow 0^+} e^{\frac{\tan 2h}{\tan 3h}} \\ &= e^{\lim_{h \rightarrow 0^+} \left(\frac{\tan 2h}{2h} \times \frac{2}{3} \times \frac{3h}{\tan 3h} \right)} = e^{1 \times \frac{2}{3} \times 1} = e^{\frac{2}{3}} \end{aligned}$$

Also, $f(0) = m$ For $f(x)$ to be continuous at $x = 0$, we must have

$$\lim_{h \rightarrow 0^+} f(0-h) = \lim_{h \rightarrow 0^+} f(0+h) = f(0)$$

$$\Rightarrow e^l = e^{\frac{2}{3}} = m$$

$$\Rightarrow l = \frac{2}{3} \text{ and } m = e^{\frac{2}{3}}$$

Q8

$$f(x) = \{x\} \sin(\pi[x])$$

$$= \{x\} \sin(\text{integral multiple of } \pi)$$

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$\equiv 0$

Hence, $f(x)$ is continuous for all x .

Q9

$$f(x) = \lim_{n \rightarrow \infty} \{\cos^2(\pi x)\}^n + [x]$$

$$= \begin{cases} [x] & : \cos^2(\pi x) \in [0, 1) \\ 1 + [x] & : \cos^2(\pi x) = 1 \end{cases}$$

$$= \begin{cases} [x] & : x \notin I \\ 1 + [x] & : x \in I \end{cases}$$

For $x = 1$,

$$f(1^-) = [1^-] = 0, f(1^+) = [1^+] = 1, f(1) = 1 + [1] = 2$$

$\therefore f(1^+) \neq f(1^-) \Rightarrow f(x)$ is discontinuous at $x = 1$

For $x = \frac{3}{2}$,

$$f\left(\frac{3}{2}^+\right) = \left[\frac{3}{2}^+\right] = 1, f\left(\frac{3}{2}^-\right) = \left[\frac{3}{2}^-\right] = 1, f\left(\frac{3}{2}\right) = \left[\frac{3}{2}\right] = 1$$

$$\therefore f\left(\frac{3}{2}^-\right) = f\left(\frac{3}{2}^+\right) = f\left(\frac{3}{2}\right)$$

$\Rightarrow f(x)$ is continuous at $x = \frac{3}{2}$

Q10

$$f(x) = -\left(x - \frac{1}{2}\right)^2 + \left(p + \frac{1}{4}\right)$$

If $g(x)$ is discontinuous at $x = \frac{1}{2}$, then $f\left(\frac{1}{2}\right)$ should be an integer.

Hence, p cannot be $\frac{1}{2}$

Q11

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$f(x) = [x]([x^2] + [x^2]) + \{x\}([x^2] + \{x^2\})$$

$$= ([x] + \{x\})([x^2] + \{x^2\})$$

$$= x \cdot x^2 = x^3$$

Hence, $f(x)$ is continuous $\forall x \in [0, 10]$

Q12

$$\text{L.H.L.} = \lim_{h \rightarrow 0} [(2-h)^2] + sgn(2-h)$$

$$= 3 + 1 = 4$$

$$f(2) = 4 + 1 = 5$$

$\therefore f(x)$ is discontinuous.

Q13

$$f'(3^+) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(2-e^h)-1}{h} = -\lim_{h \rightarrow 0} \left(\frac{e^h-1}{h}\right) = -1$$

$$f'(3^-) = \lim_{h \rightarrow 0} \frac{f(3-h) - f(3)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{10-(3-h)^2}-1}{-h} = -\lim_{h \rightarrow 0} \frac{\sqrt{1+(6h-h^2)}-1}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{6h-h^2}{-h(\sqrt{1+6h-h^2}+1)} = \frac{-6}{2} = -3$$

Hence, $f'(3^+) \neq f(3^-)$

Q14

Clearly, $x = 1$ is a point of discontinuity of the function $f(x) = \frac{1}{1-x}$.

if $x \neq 1$, then

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$(f \circ f)(x) = f[f(x)] = f\left(\frac{1}{1-x}\right) = \frac{x-1}{x}$, which is discontinuous at $x = 0$.

If $x \neq 0$ and $x \neq 1$, then

$$(f \circ f \circ f)(x) = f[(f \circ f)(x)] = f\left(\frac{x-1}{x}\right) = x$$

Which is continuous everywhere.

Hence, $f^{30}(x) = x$, which is continuous everywhere.

So, the only points of discontinuity are $x = 0$ and $x = 1$

Q15

$\mu(x) = \frac{1}{x-1}$, which is discontinuous at $x = 1$

$$f(u) = \frac{1}{u^2+u-2} = \frac{1}{(u+2)(u-1)}$$

which is discontinuous at $u = -2, 1$

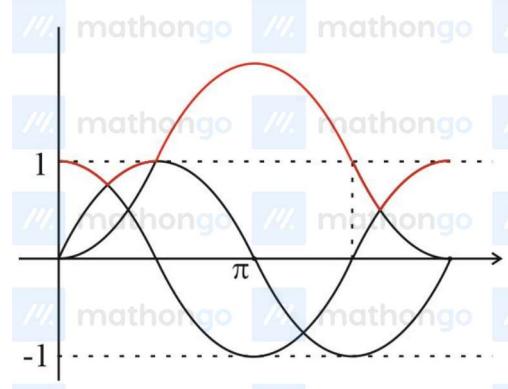
when $u = -2$, then $\frac{1}{x-1} = -2 \Rightarrow x = \frac{1}{2}$

when $u = 1$, then $\frac{1}{x-1} = 1 \Rightarrow x = 2$

Hence given composite function is discontinuous at three points, $x = 1, \frac{1}{2}$ and 2 .

Q16

The graph of $f(x) = \max.\{\sin x, \cos x, 1-\cos x\}$ is

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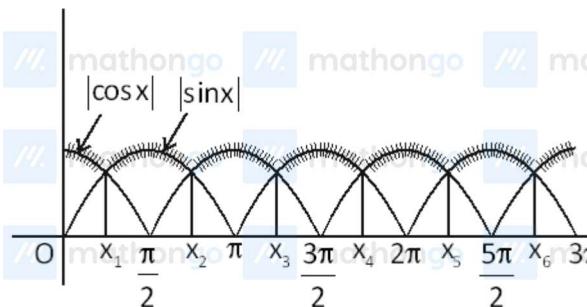
$\Rightarrow f(x)$ is not differentiable at $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{3}$.

$\Rightarrow f(x)$ is not differentiable at 3 points

Q17

Using graph of $|\sin x|$ & $|\cos x|$, we get,

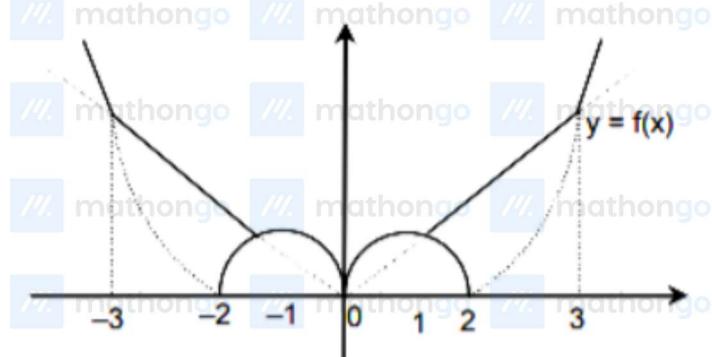
$f(x)$ is non differentiable at



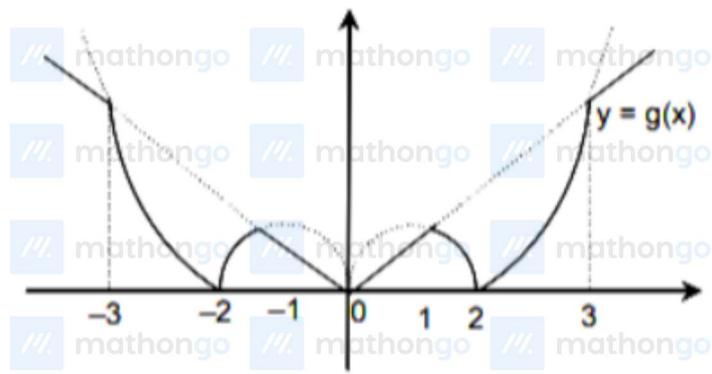
$$x_1 = \frac{\pi}{4}, x_2 = \frac{3\pi}{4}, x_3 = \frac{5\pi}{4}, x_4 = \frac{7\pi}{4}, x_5 = \frac{9\pi}{4}, x_6 = \frac{11\pi}{4}$$

Hence, $\lambda = 6$

Q18

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$f(x)$ is not differentiable function at $x = -3, -1, 0, 1, 3$



$g(x)$ is not differentiable at $x = -3, -2, -1, 0, 1, 2, 3$

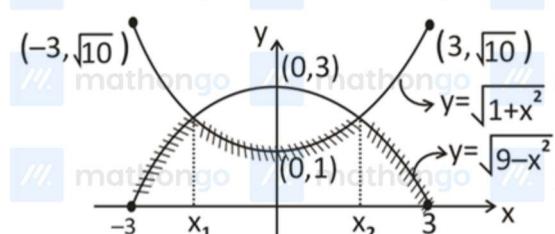
Q19

Using graph of

$$y = \sqrt{9 - x^2} \Rightarrow x^2 + y^2 = 9,$$

$$y = \sqrt{1 + x^2} \Rightarrow y^2 - x^2 = 1$$

In the figure, we can see the graph of $f(x)$



Clearly, $f(x)$ is non-differentiable at

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$$x = -3, x_1, x_2, 3 \text{ (4 points)}$$

because there are sharp points at x_1, x_2 and at $x = 3, x = -3$ there are vertical tangents.

Q20

For $g(x)$ to be continuous and differentiable $\forall x \in R$, $f(x)$ must be continuous and differentiable $\forall x \in R$

Since, $f(x)$ is continuous for all x , therefore, it is continuous at $x = 1$ also.

$$\therefore f(1) = \lim_{h \rightarrow 0^+} f(1-h) = 1 = \lim_{h \rightarrow 0^+} [a(1-h)^2 + b]$$

$$\Rightarrow a + b = 1 \dots (1)$$

Also, $f(x)$ is differentiable at $x = 1 \Rightarrow f'(1^-) = f'(1^+)$

$$\Rightarrow \lim_{h \rightarrow 0^+} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0^+} \frac{a(1-h)^2 + b - 1}{-h} = \lim_{h \rightarrow 0^+} \frac{\frac{1}{|1+h|} - 1}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0^+} \frac{(a+b-1) + (h^2 - 2h)a}{-h} = \lim_{h \rightarrow 0^+} \frac{1-1-h}{h(1+h)}$$

$$\Rightarrow 2a = -1 \text{ (Using } a + b = 1\text{)}$$

$$\therefore a = \frac{-1}{2}$$

$$\text{Hence, } a + b = 1 \Rightarrow b = 1 - a = \frac{3}{2}.$$

Q21

$$f(x) = n + [p \sin x]$$

As we know the greatest integer function is discontinuous at integer.

So, $f(x)$ is discontinuous when $p \sin x$ is integer.

In $(0, \frac{\pi}{2})$, $f(x)$ is discontinuous when $p \sin x = 1, 2, \dots, p-1$

and in $(\frac{\pi}{2}, \pi)$, $f(x)$ is discontinuous when $p \sin x = 1, 2, \dots, p-1$

and at $\frac{\pi}{2}$, $p \sin x = p$

Sample Task**Solutions****Hints and Solutions****MathonGo**

So total number of points of discontinuity is $p - 1 + p - 1 + 1 = 2p - 1$

As we know function is not differentiable at a point when function is not continuous at that point.

So total number of point of where $f(x)$ is not differentiable is $2p - 1$

Q22

The curve will have sharp corner at $x = 0.5$ and at $x = 1$ and a point of discontinuity at $x = \frac{\pi}{2}$.

Hence, total number of points where the function is not differentiable is 3.

Q23

$$\begin{aligned} f(f(x)) &= 10 - |f(x) - 5| = 10 - |10 - |x - 5|| - 5 \\ &= 10 - |5 - |x - 5|| \Rightarrow 10 - ||x - 5|| - 5 \end{aligned}$$

Points where $f(f(x))$ is non-differentiable are

$$x - 5 = 0 \text{ & } |x - 5| = 5$$

$$x = 5 \text{ & } x = 0, 10$$

Q24

$$\text{Given, } f(x) = \begin{cases} -2, & -2 \leq x < 0 \\ x^2 - 2, & 0 \leq x \leq 2 \end{cases}$$

$$\therefore |f(x)| = \begin{cases} 2, & -2 \leq x < 0 \\ 2 - x^2, & 0 \leq x < \sqrt{2} \\ x^2 - 2, & \sqrt{2} \leq x \leq 2 \end{cases}$$

$$\text{And } f(|x|) = x^2 - 2, x \in [-2, 2]$$

$$\therefore g(x) = \begin{cases} x^2, & -2 \leq x < 0 \\ 0, & 0 \leq x < \sqrt{2} \\ 2(x^2 - 2), & \sqrt{2} \leq x \leq 2 \end{cases}$$

$$g'(x) = \begin{cases} 2x, & -2 < x < 0 \\ 0, & 0 < x < \sqrt{2} \\ 4x, & \sqrt{2} < x < 2 \end{cases}$$

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$$g'(0^-) = 0 = g'(0^+)$$

$$g'(\sqrt{2}^-) = 0, g'(\sqrt{2}^+) = 4\sqrt{2}$$

So, $g(x)$ is not differentiable at $x = \sqrt{2}$

Q25

$\because x - 2$ & $|x^2 - 3x + 2|$ both are continuous $\forall x \in R$

$\therefore f(x)$ is continuous $\forall x \in R$

Now, $|x^2 - 3x + 2| = |(x-1)(x-2)|$

$$\therefore f(x) = \begin{cases} (x-2)(x-1)(x-2) & : x \in (-\infty, 1) \cup (2, \infty) \\ -(x-2)(x-1)(x-2) & : x \in [1, 2] \end{cases}$$

$$f(x) = \begin{cases} (x-1)(x-2)^2 & : x \in (-\infty, 1) \cup (2, \infty) \\ -(x-1)(x-2)^2 & : x \in [1, 2] \end{cases}$$

$$f'(x) = \begin{cases} (x-1)2(x-2) + (x-2)^2 & : x \in (-\infty, 1) \cup (2, \infty) \\ -(x-1)2(x-2) - (x-2)^2 & : x \in (1, 2) \end{cases}$$

Clearly, $f'(2^-) = f'(2^+) = 0 \Rightarrow f(x)$ is differentiable at $x = 2$

and $f'(1^-) = 1, f'(1^+) = -1 \Rightarrow f(x)$ is non-differentiable at $x = 1$

Q26

$x = 2020\pi, 2\pi$ are points of non-differentiability

Q27

$$g(x) = \begin{cases} \frac{ax^2+bx}{(\cot x)^n} + c & \text{if } x < \frac{\pi}{4} \\ \frac{4}{1+\frac{4}{(\cot x)^n}} = c & \text{if } x > \frac{\pi}{4} \\ \frac{\sin x + \cos x + 1}{(\tan x)^n} = \frac{1}{c} & \text{if } x > \frac{\pi}{4} \\ \frac{1}{(\tan x)^n} + c & \end{cases}$$

limit to exist $c = \frac{1}{c} \Rightarrow c^2 = 1, c = \pm 1$

Sample Task**Solutions****Hints and Solutions****MathonGo****Q28**

$$f(x) = \cos x$$

$$f'(x) = -\sin x < 0$$

So, function is decreasing in $x \in [0, \pi]$

$$g(x) = \begin{cases} f(x), & x \in [0, \pi] \\ \sin x - 1, & x > \pi \end{cases}$$

$$\therefore \text{L. H. L.} = \text{R. H. L.} = f(\pi) = -1.$$

Q29

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(x) + f(h) + 2xh - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(h)}{h} + 2x \right) = 210 + 2x$$

$$f(x) = 210x + x^2 + C$$

$$f(0) = 0 \Rightarrow C = 0$$

$$f(x) = 210x + x^2$$

$$\therefore f(2) = 424$$

Q30

$$\text{We have, } f\left(\frac{x+y}{3}\right) = \frac{f(x) + f(y)}{3}, f(0) = 0 \text{ and } f'(0) = 5$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f\left(\frac{3x+3h}{3}\right) - f\left(\frac{3x}{3}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{f(3x) + f(3h)}{3} - \frac{f(3x) + f(0)}{3}}{h} \end{aligned}$$

Sample Task**Solutions****Hints and Solutions****MathonGo**

$$= \lim_{h \rightarrow 0} \frac{f(3h) - f(0)}{3h} = f'(0) = 5$$

$$\therefore f(x) = 5x + c$$

$$\therefore f(0) = 0 \Rightarrow c = 0$$

Hence, $f(x) = 5x$

